Hadron gas and repulsive interactions

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FIAS Frankfurt Institute for Advanced Studies

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

MIAPP Munich Institute for Astro- and Particle Physics

Outline

1. **Hadron resonance gas (HRG) model**
   - Hadron yields in heavy-ion collisions
   - Lattice QCD equation of state

2. **HRG model with repulsive interactions**
   - Susceptibilities of conserved charges
   - Fourier coefficients at imaginary chemical potential

3. **Hagedorn resonance gas with repulsive interactions and the QCD transition**

Hadron resonance gas model

Ideal HRG: Equation of state of hadronic matter as a multi-component non-interacting gas of known hadrons and resonances

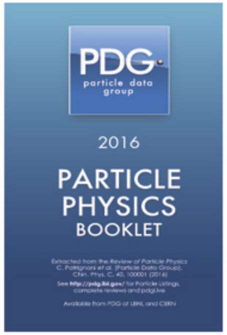
$$\ln Z \approx \sum_{i \in M,B} \ln Z_i^{id} = \sum_{i \in M,B} V \frac{d_i m_i^2 T}{2\pi^2} K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}}$$

Dashen, Ma, Bernstein (1969): Inclusion of narrow resonances as free, non-interacting particles models attractive interactions which they mediate [S-matrix formulation of statistical mechanics, PRC 187, 345 (1969)]

Particle list usually consists of established hadrons and resonances from PDG: ~400 species

p	$1/2^+$	****	$\Delta(1232)$	$3/2^+$	****	Σ^+	$1/2^+$	****	Ξ^0	$1/2^+$	****
n	$1/2^+$	****	$\Delta(1600)$	$3/2^+$	****	Σ^0	$1/2^+$	****	Ξ^-	$1/2^+$	****
$N(1440)$	$1/2^+$	****	$\Delta(1620)$	$1/2^-$	****	Σ^-	$1/2^+$	****	$\Xi(1530)$	$3/2^+$	****
$N(1520)$	$3/2^-$	****	$\Delta(1700)$	$3/2^-$	****	$\Sigma(1385)$	$3/2^+$	****	$\Xi(1620)$		*
$N(1535)$	$1/2^-$	****	$\Delta(1750)$	$1/2^+$	*	$\Sigma(1480)$		*	$\Xi(1690)$		***
$N(1650)$	$1/2^-$	****	$\Delta(1900)$	$1/2^-$	***	$\Sigma(1560)$		**	$\Xi(1820)$	$3/2^-$	***
$N(1675)$	$5/2^-$	****	$\Delta(1905)$	$5/2^+$	****	$\Sigma(1580)$	$3/2^-$	*	$\Xi(1950)$		***
$N(1680)$	$5/2^+$	****	$\Delta(1910)$	$1/2^+$	****	$\Sigma(1620)$	$1/2^-$	*	$\Xi(2030)$	$\geq \frac{5}{2}?$	***
$N(1700)$	$3/2^-$	***	$\Delta(1920)$	$3/2^+$	***	$\Sigma(1660)$	$1/2^+$	***	$\Xi(2120)$		*
$N(1710)$	$1/2^+$	****	$\Delta(1930)$	$5/2^-$	***	$\Sigma(1670)$	$3/2^-$	****	$\Xi(2250)$		**
$N(1720)$	$3/2^+$	****	$\Delta(1940)$	$3/2^-$	**	$\Sigma(1690)$		**	$\Xi(2370)$		**
$N(1860)$	$5/2^+$	**	$\Delta(1950)$	$7/2^+$	****	$\Sigma(1730)$	$3/2^+$	*	$\Xi(2500)$		*
$N(1875)$	$3/2^-$	***	$\Delta(2000)$	$5/2^+$	**	$\Sigma(1750)$	$1/2^-$	***			
$N(1880)$	$1/2^+$	***	$\Delta(2150)$	$1/2^-$	*	$\Sigma(1770)$	$1/2^+$	*	Ω^-	$3/2^+$	****
$N(1895)$	$1/2^-$	****	$\Delta(2200)$	$7/2^-$	***	$\Sigma(1775)$	$5/2^-$	****	$\Omega(2250)^-$		***
$N(1900)$	$3/2^+$	****	$\Delta(2300)$	$9/2^+$	**	$\Sigma(1840)$	$3/2^+$	*	$\Omega(2380)^-$		**
$N(1990)$	$7/2^+$	**	$\Delta(2350)$	$5/2^-$	*	$\Sigma(1880)$	$1/2^+$	**	$\Omega(2470)^-$		**

• $\pi^\pm$	$1^-(0^-)$	• $\phi(1680)$	$0^-(1^{--})$	• $K^\pm$	
• π^0	$1^-(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$	• K^0	
• η	$0^+(0^{-+})$	• $\rho(1700)$	$1^+(1^{--})$	• K^0_S	
• $f_0(500)$	$0^+(0^{++})$	$a_2(1700)$	$1^-(2^{++})$	• K^0_L	
• $\rho(770)$	$1^+(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$	• $K_0^*(700)$	
• $\omega(782)$	$0^-(1^{--})$	$\eta(1760)$	$0^+(0^{-+})$	• $K^*(892)$	
• $\eta'(958)$	$0^+(0^{-+})$	• $\pi(1800)$	$1^-(0^{-+})$	• $K_1(1270)$	
• $f_0(980)$	$0^+(0^{++})$	$f_2(1810)$	$0^+(2^{++})$	• $K_1(1400)$	$1/2(1^+)$
• $a_0(980)$	$1^-(0^{++})$	$X(1835)$	$?^?(0^{-+})$	• $K^*(1410)$	$1/2(1^-)$
• $\phi(1020)$	$0^-(1^{--})$	$X(1840)$	$?^?(?^{??})$	• $K_0^*(1430)$	$1/2(0^+)$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$	• $K_2^*(1430)$	$1/2(2^+)$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$	$K(1460)$	$1/2(0^-)$
• $a_1(1260)$	$1^-(1^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$	$K_2(1580)$	$1/2(2^-)$
• $f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$	$K(1630)$	$1/2(?^?)$

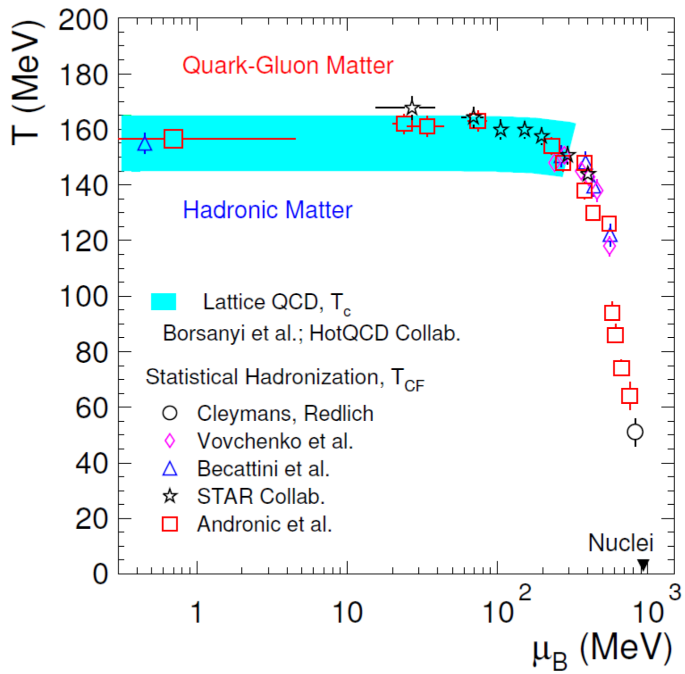


HRG model and heavy-ion collisions

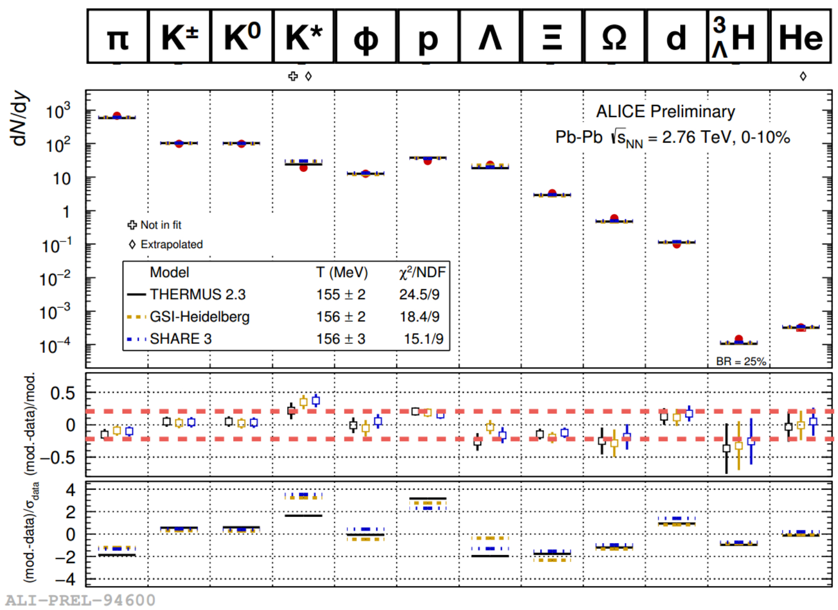
A simple and successful model to describe hadron abundances

$$N_i^{\rm hrg} = V \frac{d_i m_i^2 T}{2\pi^2} K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}}, \;\; N_i^{\rm tot} = N_i^{\rm hrg} + \sum_j BR(j \to i) N_j^{\rm hrg}$$

Fits extract $T, \mu_B, \ldots$ by minimizing $\chi^2 = \sum_i \frac{(N_i^{\rm mod} - N_i^{\rm exp})^2}{(\sigma_i^{\rm exp})^2}$



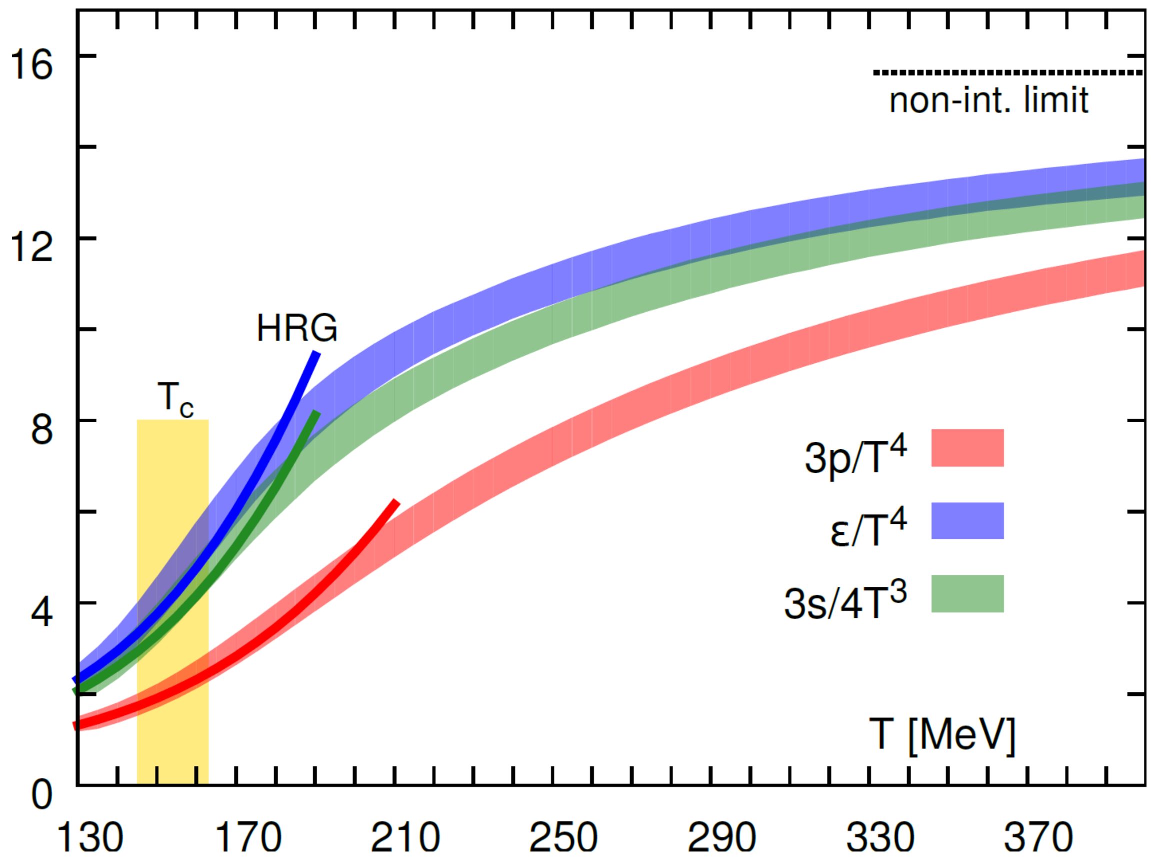
[A. Andronic et al., 1710.09425]



[ALICE collaboration (SQM2015)]

Describes yields on a 10% level and maps HIC on the QCD phase diagram

HRG model and lattice QCD equation of state

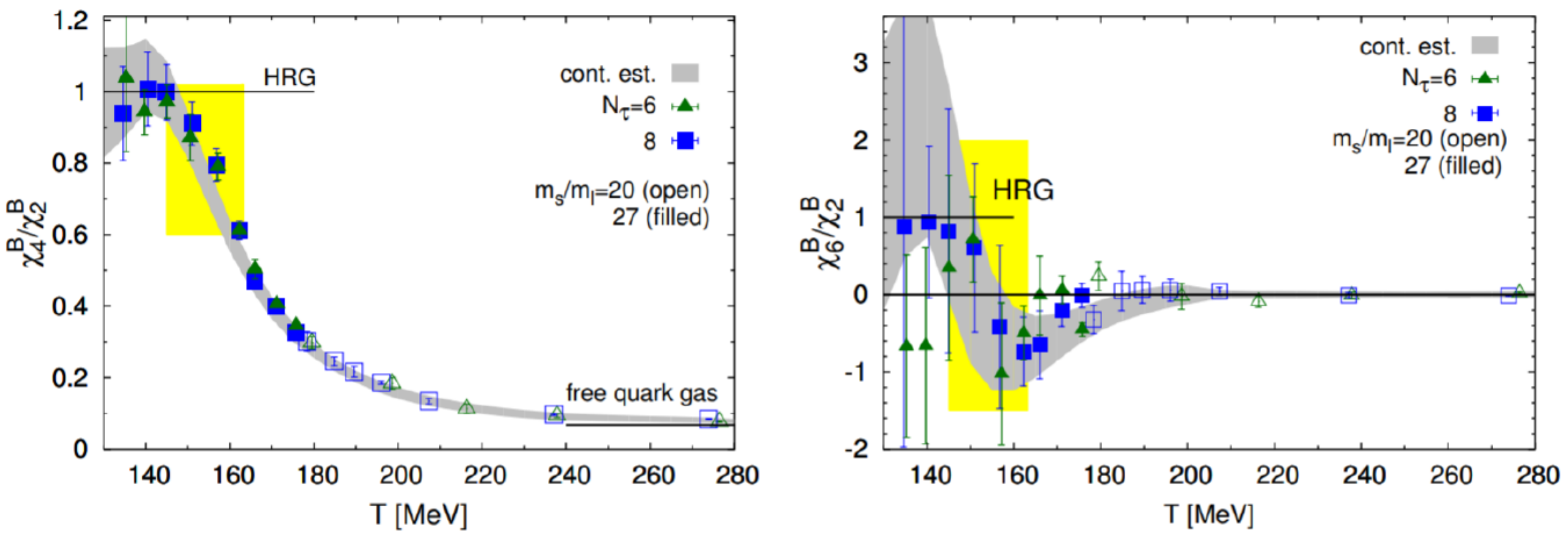


[HotQCD collaboration, 1407.6387; similar results from Wuppertal-Budapest collab., 1309.5258]

HRG describes quite well LQCD thermodynamic functions below and in the vicinity of the pseudocritical temperature

HRG and fluctuations of conserved charges

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \, \partial(\mu_S/T)^m \, \partial(\mu_Q/T)^n}$$



[HotQCD collaboration, 1701.04325]

Standard HRG model rapidly breaks down in the vicinity of T_{pc} for the description of higher-order susceptibilities

Extensions of the HRG model

- Quantum statistics

  Important at high μ_B, irrelevant for most observables at $\mu_B = 0$

- Treatment of finite resonance widths

  Zero-width, (energy-dependent) Breit-Wigner, phase shifts

  Relevant for precision HRG model applications, e.g.

  low-pT pion spectra from resonance decays [Huovinen et al., 1608.06817]

  a possible resolution of the proton yield anomaly at LHC [**V.V.** et al., 1807.02079]

- **Repulsive interactions**

  From uncorrelated to correlated gas of hadrons

- **Inclusion of the exponential Hagedorn mass spectrum**

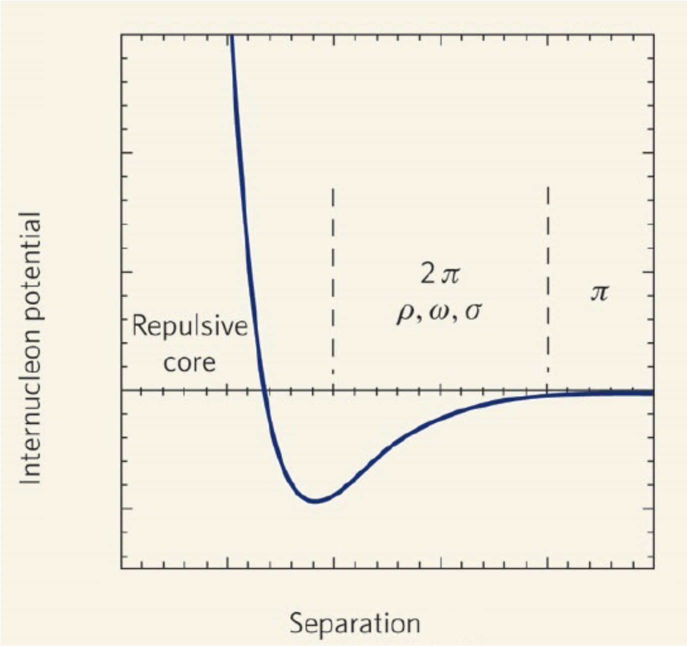
  Modeling QCD transition with a single partition function

Nucleon-nucleon interaction

Many hadronic interactions described by resonance formation... however

Nucleon-nucleon potential:

- Repulsive core at small distances
- Attraction at intermediate distances
- No resonance structure
- Suggestive similarity to VDW interactions
- Could nuclear matter be described by VDW equation?



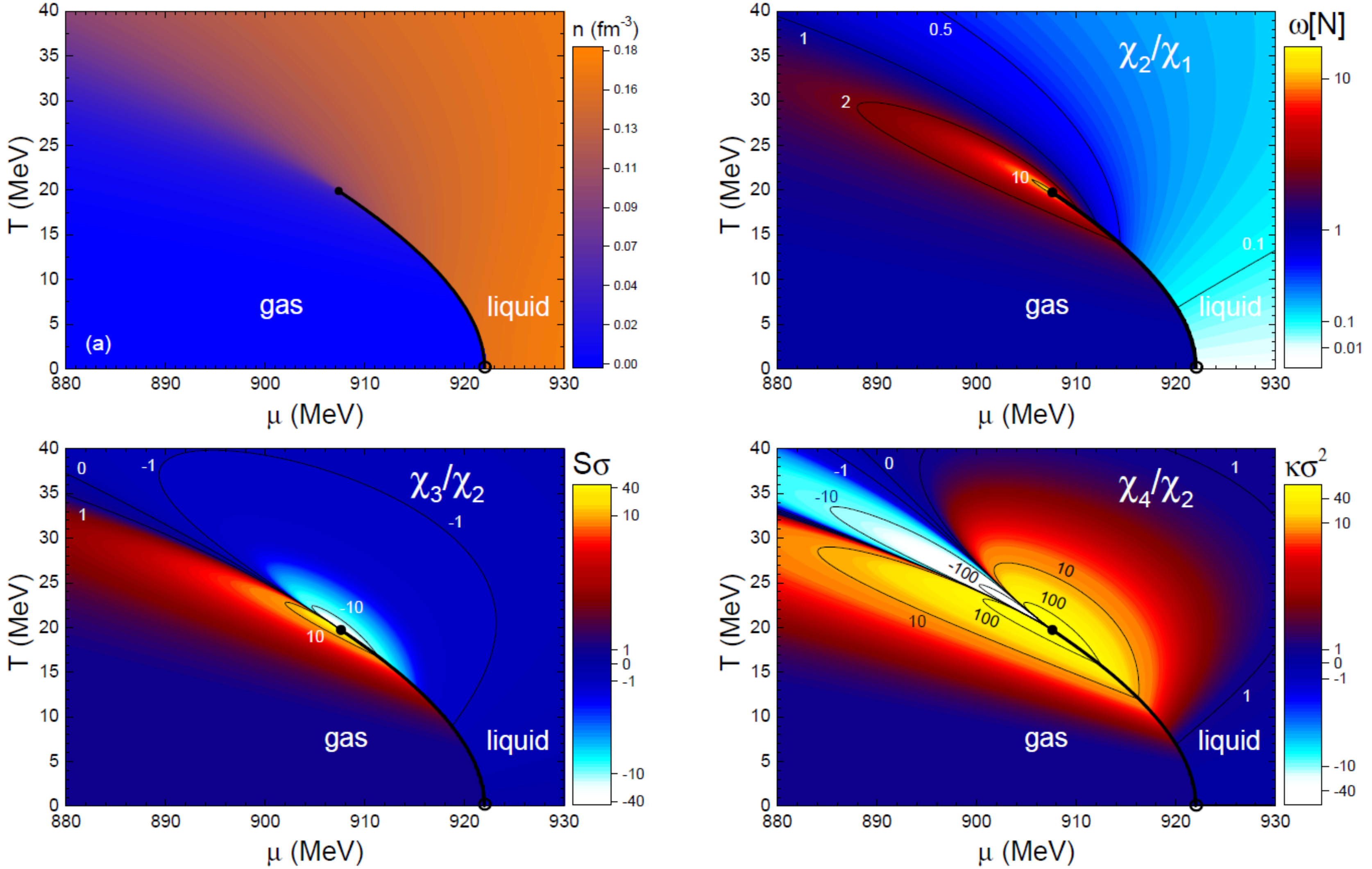
Nuclear matter with quantum van der Waals equation

$$p(T,n) \;=\; p_q^{\rm id}\left(T, \frac{n}{1-bn}\right) - a\,n^2$$

$$E/A = -16~\mathrm{MeV},~~ n_0 = 0.16~\mathrm{fm}^{-3} \quad \Rightarrow \quad a_{NN} = 329~\mathrm{MeV\,fm}^3,~~ b_{NN} = 3.42~\mathrm{fm}^3$$

V.V., Anchishkin, Gorenstein, PRC '15; Redlich, Zalewski, APPB '16.

QvdW nucleon fluid: μ_B-T phase diagram

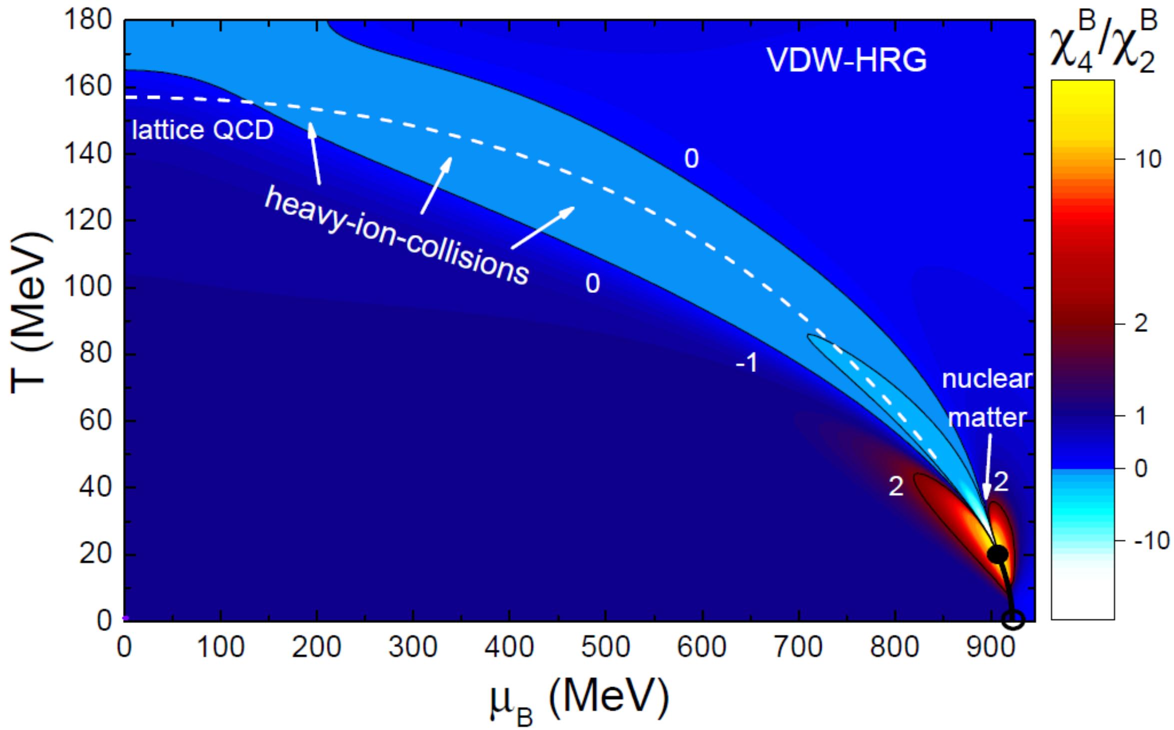


V.V., D. Anchishkin, M. Gorenstein, R. Poberezhnyuk, PRC 91, 064314 (2015)

QvdW-HRG model

HRG with additional QvdW interaction terms for all pairs of (anti)baryons

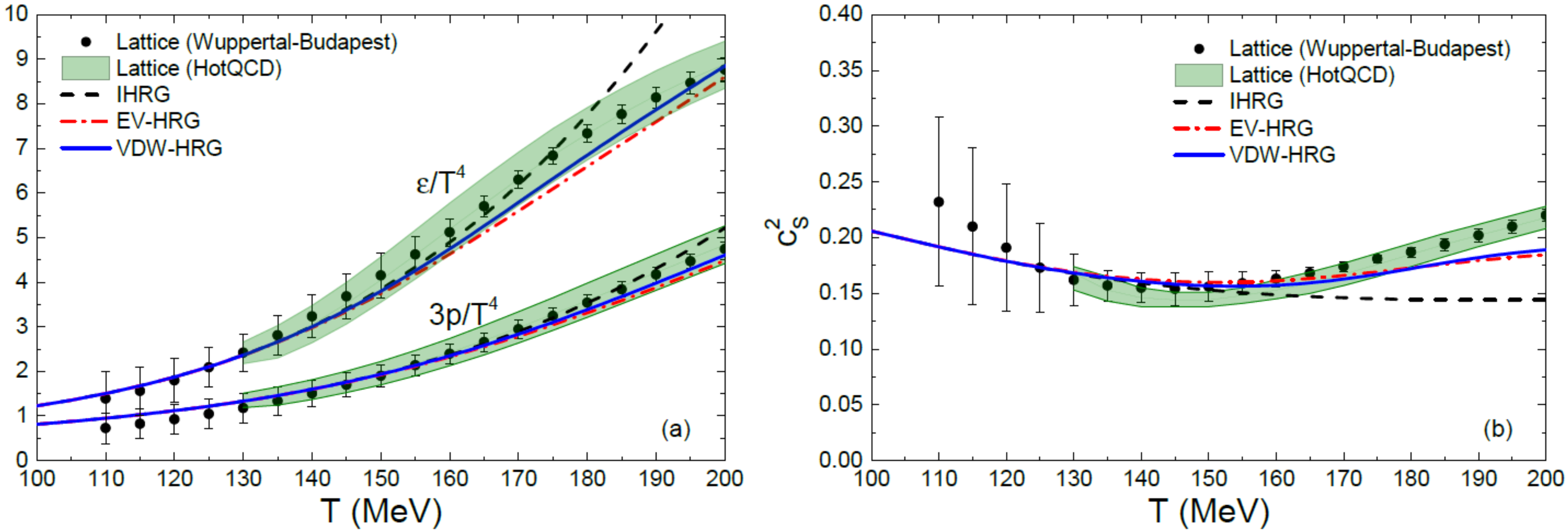
$$p = p_M^{\rm id} + p_B^{\rm vdW} + p_{\bar{B}}^{\rm vdW} \qquad\qquad p_B^{\rm vdW} \simeq \frac{T n_B}{1 - b\, n_B} - a\, n_B^2$$



[**V.V.**, Gorenstein, Stoecker, PRL 118, 182301 (2018)]

Effects of interactions extend to $\mu_B = 0$

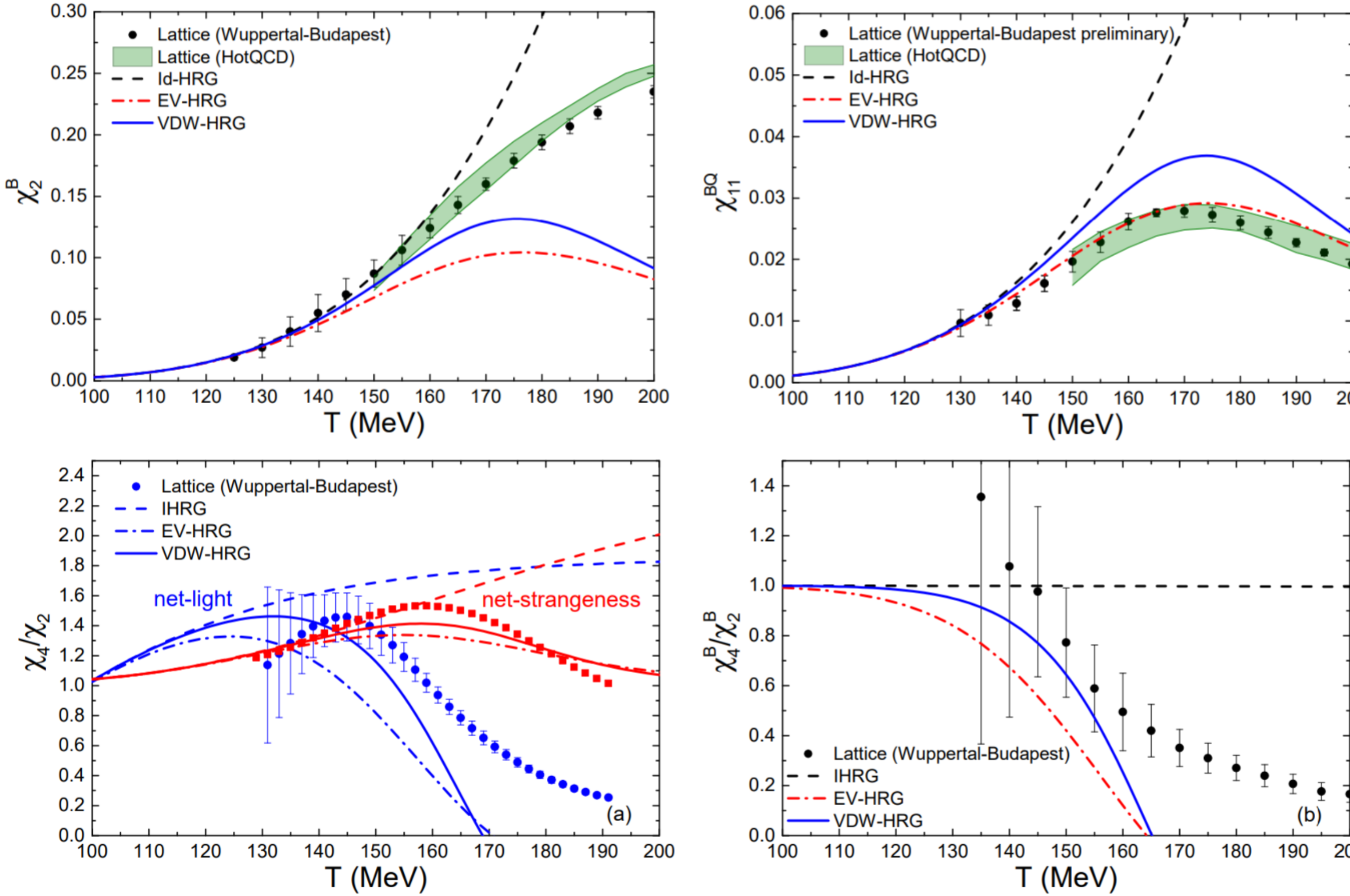
QvdW-HRG model: thermodynamics at $\mu_B = 0$



- **VDW-HRG:** b = 3.42 fm^3, a = 329 MeV fm^3
- **EV-HRG:** b = 3.42 fm^3, a = 0
- **IHRG:** b = 0, a = 0

Repulsive interactions (eigenvolumes) drive the effects at $\mu_B = 0$

QvdW-HRG model: susceptibilities at $\mu_B = 0$



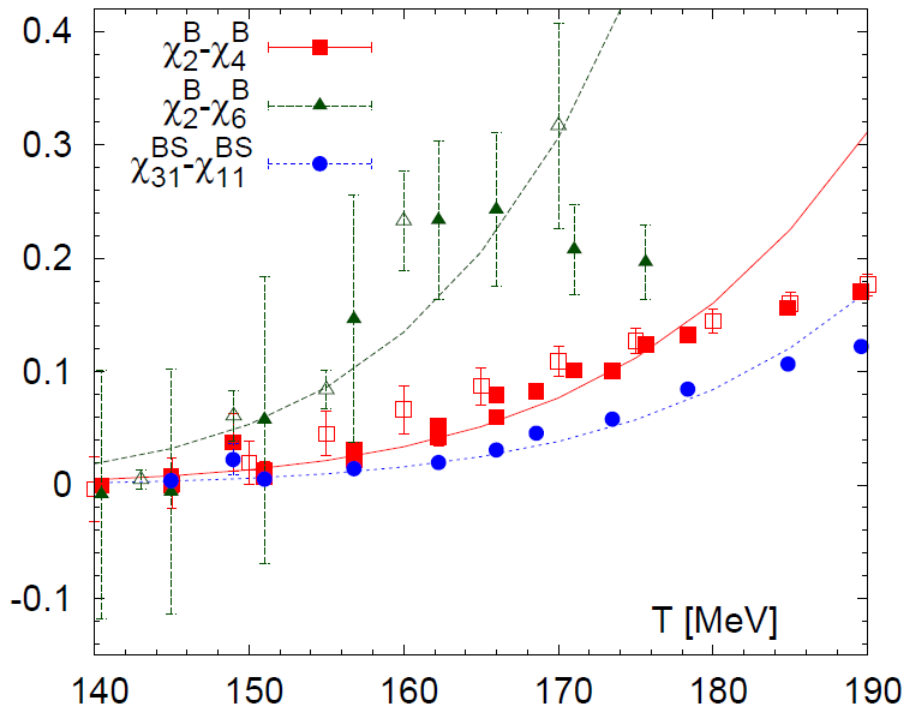
Repulsive interactions in the mean-field model

HRG with repulsive mean-field interactions between baryons

Huovinen, Petreczky, 1708.00879

$$p_B + \bar{p}_B = T\,(n_B + \bar{n}_B) + \frac{K}{2}(n_B^2 + \bar{n}_B^2),$$

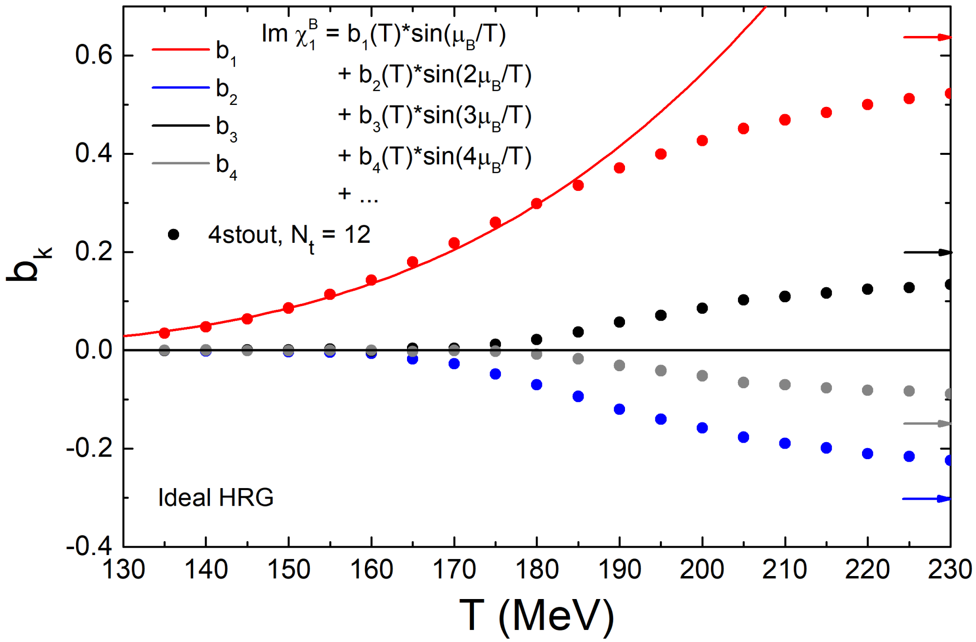
where $K = 450$ MeV fm^3 based on empirical NN phase shifts.



"Signals of deconfinement" interpreted in terms of repulsive baryonic interactions

Fourier coefficients at imaginary μ_B

Net baryon density: $$\frac{\rho_B(T,\mathrm{i}\tilde{\mu}_B)}{T^3} = \mathrm{i}\sum_{k=1}^{\infty} b_k(T)\,\sin\left(\frac{k\,\tilde{\mu}_B}{T}\right)$$



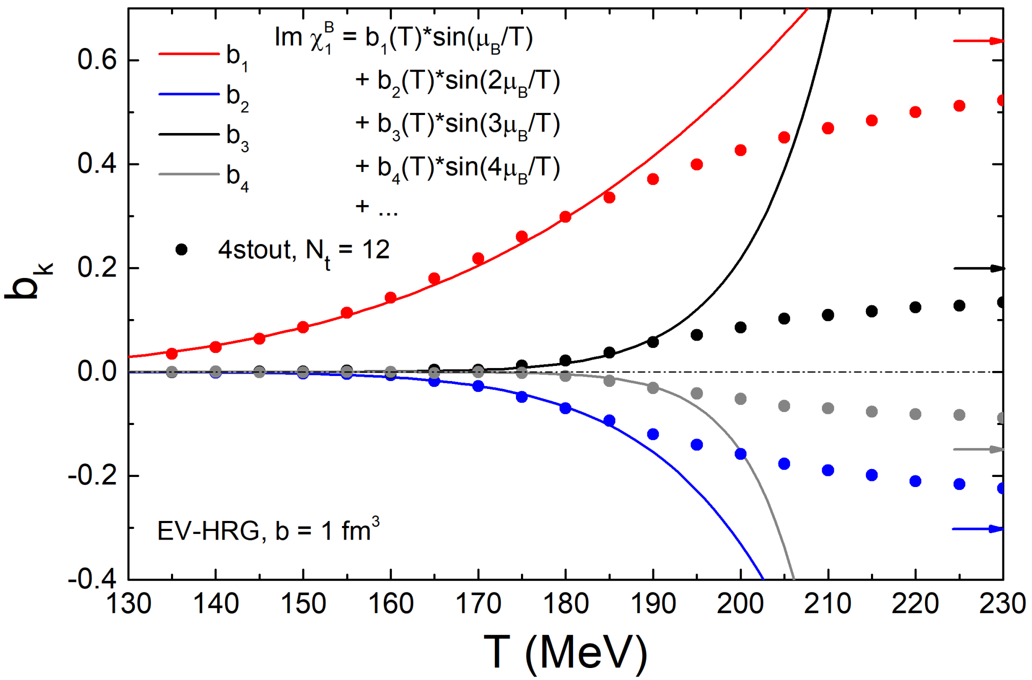
S. Borsanyi et al. [Wuppertal-Budapest collaboration], QM2017

- Consistent with HRG at low temperatures
- Consistent with approach to the Stefan-Boltzmann limit
- Ideal HRG: $$\frac{\rho_B}{T^3} = b_1(T)\,\sinh\left(\frac{\mu_B}{T}\right)$$

HRG with repulsive baryonic interactions

Repulsive interactions with excluded volume (EV-HRG): $V \to V - bN$

[Hagedorn, Rafelski, ’80; Dixit, Karsch, Satz, ’81; Cleymans et al., ‘86; Rischke et al., Z. Phys. C ‘91]



HRG with baryonic EV:

$$p_B(T,\mu_B) = p_B^{\rm id}(T,\mu_B - b p_B)$$

$$b_k^{ev}(T) = (-1)^{k-1}\,\frac{2\,k^k}{k!}\,(b\,T^3)^{k-1}\,\left[\frac{\phi_B(T)}{T^3}\right]^k$$

V.V., A. Pasztor, Z. Fodor, S.D. Katz, H. Stoecker, 1708.02852

- Non-zero $b_k(T)$ for $k \geq 2$ signal deviation from ideal HRG
- EV interactions between baryons ($b \approx 1$ fm^3) reproduce lattice trend

Hagedorn resonance gas model with repulsive interactions

exactly solvable model for (phase) transition between hadrons and QGP

[Gorenstein, Petrov, Zinovjev, PLB '81; Gorenstein, W. Greiner, Yang, JPG '98; Ferroni, Koch, PRC '09]

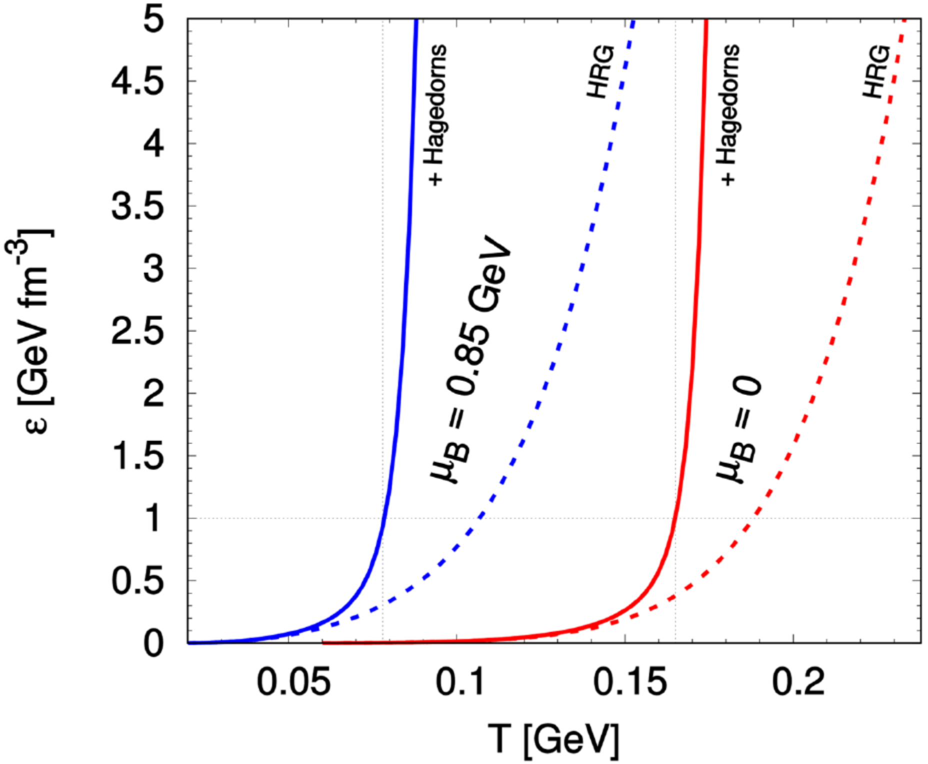
Here the model is explored in the context of lattice QCD

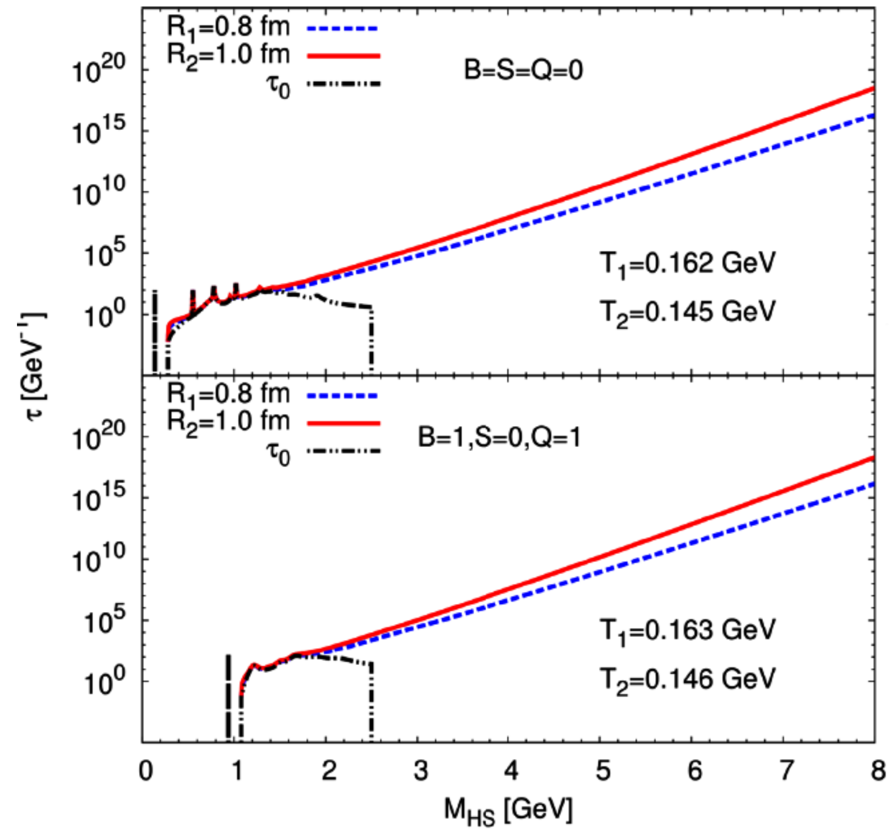
V. Vovchenko, M.I. Gorenstein, C. Greiner, H. Stoecker, arxiv:1811.XXXXX

Hagedorn resonance gas

HRG + exponential Hagedorn mass spectrum, e.g. as obtained from the bootstrap equation [Hagedorn '65; Frautschi, '71]

$$\rho(m) = A\, m^{-\alpha} \exp(m/T_H)$$





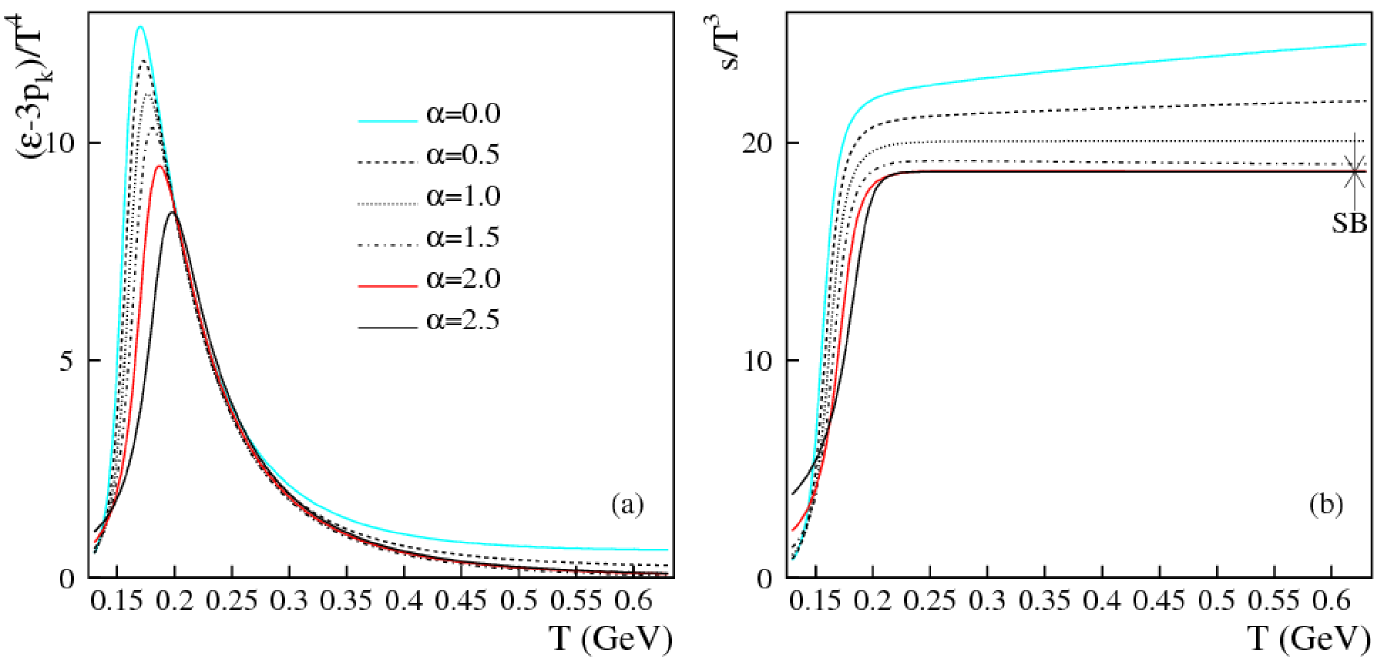
[Beitel, Gallmeister, Greiner, 1402.1458]

If Hagedorns are point-like, *T_H is the limiting temperature*

From limiting temperature to crossover

- A gas of extended objects → excluded volume
- Exponential spectrum of *compressible* QGP bags
- Both phases described by single partition function

[Gorenstein, Petrov, Zinovjev, PLB '81; Gorenstein, W. Greiner, Yang, JPG '98; I. Zakout et al., NPA '07]



[Ferroni, Koch, PRC 79, 034905 (2009)]

Crossover transition in bag-like model qualitatively compatible with LQCD

Model formulation

Thermodynamic system of known hadrons and quark-gluon bags

Mass-volume density: $\rho(m, v; \lambda_B, \lambda_Q, \lambda_S) = \rho_H + \rho_Q$

$$\rho_H(m, v; \lambda_B, \lambda_Q, \lambda_S) = \sum_{i \in \mathrm{HRG}} \lambda_B^{b_i} \lambda_Q^{q_i} \lambda_S^{s_i} d_i \, \delta(m - m_i) \, \delta(v - v_i)$$

PDG hadrons

$$\rho_Q(m, v; \lambda_B, \lambda_Q, \lambda_S) = C \, v^\gamma \, (m - Bv)^\delta \, \exp\left\{ \frac{4}{3} [\sigma_Q v]^{1/4} \, (m - Bv)^{3/4} \right\} \theta(v - V_0) \, \theta(m - Bv)$$

Quark-gluon bags [J. Kapusta, PRC '81; Gorenstein+, ZPC '84]

Non-overlapping particles → **isobaric (pressure) ensemble**

[Gorenstein, Petrov, Zinovjev, PLB '81]

$$\hat{Z}(T, s, \lambda_B, \lambda_Q, \lambda_S) = \int_0^\infty Z(T, V, \lambda_B, \lambda_Q, \lambda_S) \, e^{-sV} \, dV = [s - f(T, s, \lambda_B, \lambda_Q, \lambda_S)]^{-1}$$

$$f(T, s, \lambda_B, \lambda_Q, \lambda_S) = \int dv \int dm \, \rho(m, v; \lambda_B, \lambda_Q, \lambda_S) \, e^{-v s} \, \phi(T, m)$$

The system pressure is $p = Ts^*$ with s^* being the *rightmost* singularity of $\hat{Z}$

Mechanism for transition to QGP

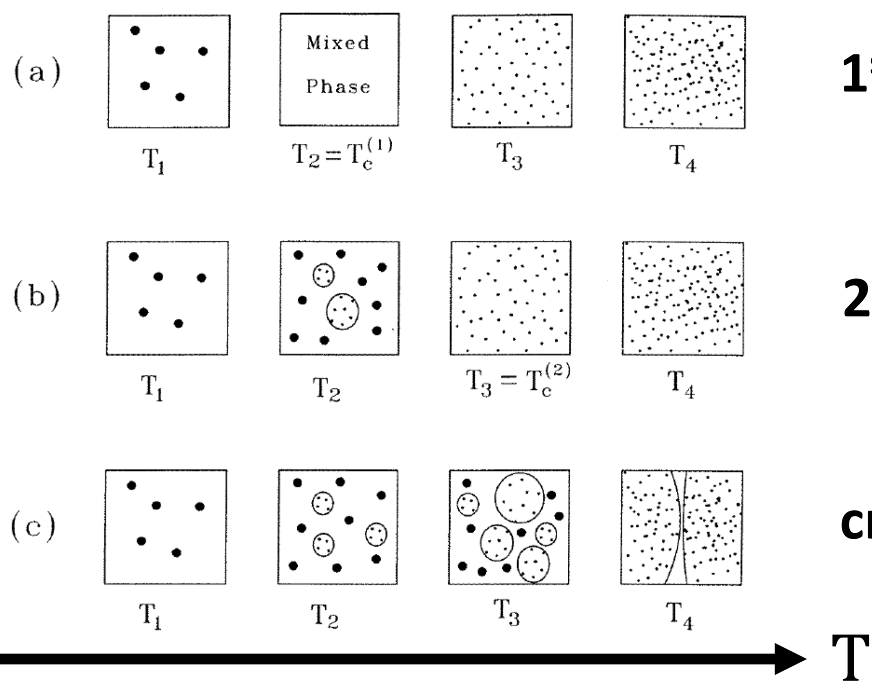
The isobaric partition function, $\hat{Z}(T,s,\lambda) = [s - f(T,s,\lambda)]^{-1}$, has

- pole singularity $s_H = f(T, s_H, \lambda)$ **"hadronic" phase**
- singularity s_B in the function $f(T,s,\lambda)$ due to the exponential spectrum

$$p_B = T\, s_B = \frac{\sigma Q}{3} T^4 - B$$

MIT bag model EoS for QGP

[Chodos+, PRD '74; Baacke, APPB '77]



1st order PT

"collision" of singularities

$$s_H(T_C) = s_B(T_C)$$

2nd order PT

crossover $s_H(T) > s_B(T)$ at all T

Crossover transition

Type of transition is determined by exponents γ and δ of bag spectrum

Crossover seen in lattice, realized in model for $\gamma + \delta \geq -3$ and $\delta \geq -7/4$

[Begun, Gorenstein, W. Greiner, JPG '09]

Transcendental equation for pressure:

$$p(T,\lambda_B,\lambda_Q,\lambda_S) = T \sum_{i\in \mathrm{HRG}} d_i \, \phi(T,m) \, \lambda_B^{b_i} \, \lambda_Q^{q_i} \, \lambda_S^{s_i} \, \exp\left(-\frac{m_i p}{4BT}\right)$$

$$+ \frac{C}{\pi} \, T^{5+4\delta} \, [\sigma_Q]^{\delta+1/2} \, [B + \sigma_Q T^4]^{3/2} \, \left(\frac{T}{p - p_B}\right)^{\gamma+\delta+3} \Gamma\left[\gamma + \delta + 3, \frac{(p-p_B)V_0}{T}\right]$$

Solved numerically

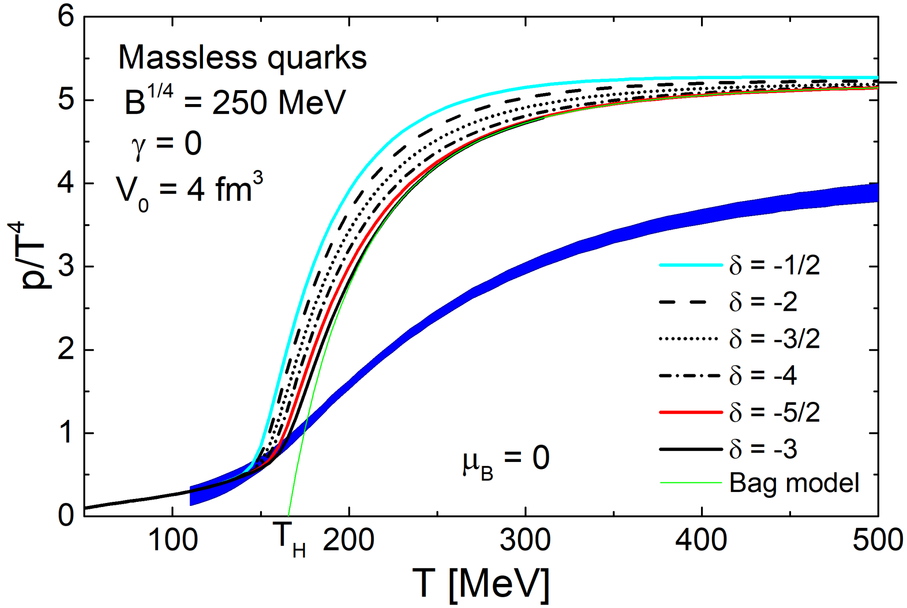
Calculation setup:

$$\gamma = 0, \quad -3 \leq \delta \leq -\frac{1}{2}, \quad B^{1/4} = 250 \text{ MeV}, \quad C = 0.03 \text{ GeV}^{-\delta+2}, \quad V_0 = 4 \text{ fm}^3$$

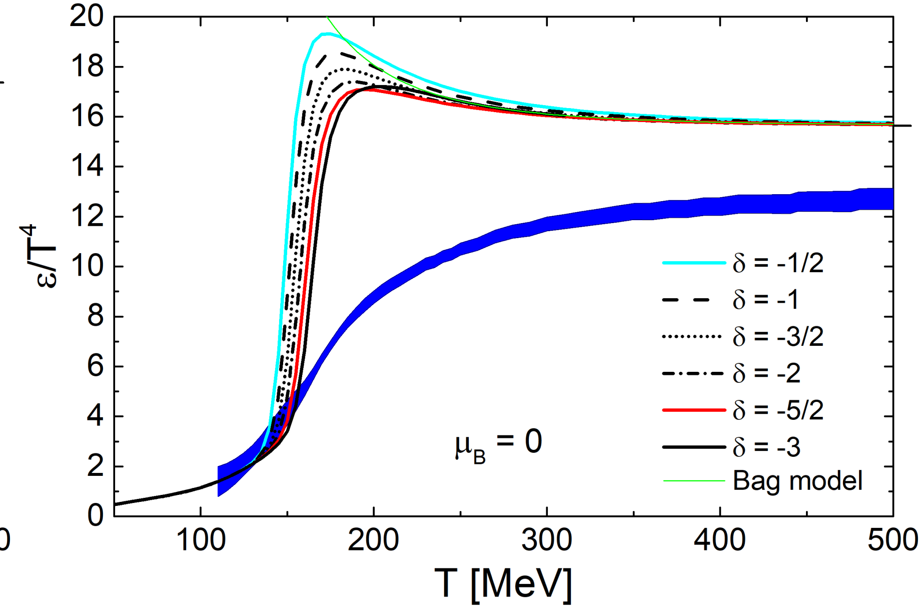
$$T_H = \left(\frac{3B}{\sigma_Q}\right)^{1/4} \simeq 165 \text{ MeV}$$

Thermodynamic functions

Pressure p/T^4



Energy density ε/T^4

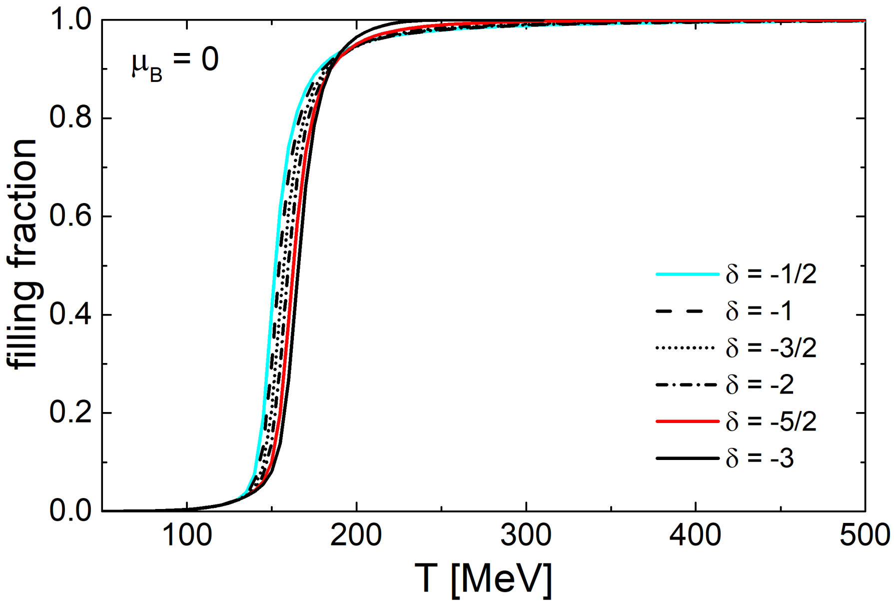


Lattice data from 1309.5258 (Wuppertal-Budapest)

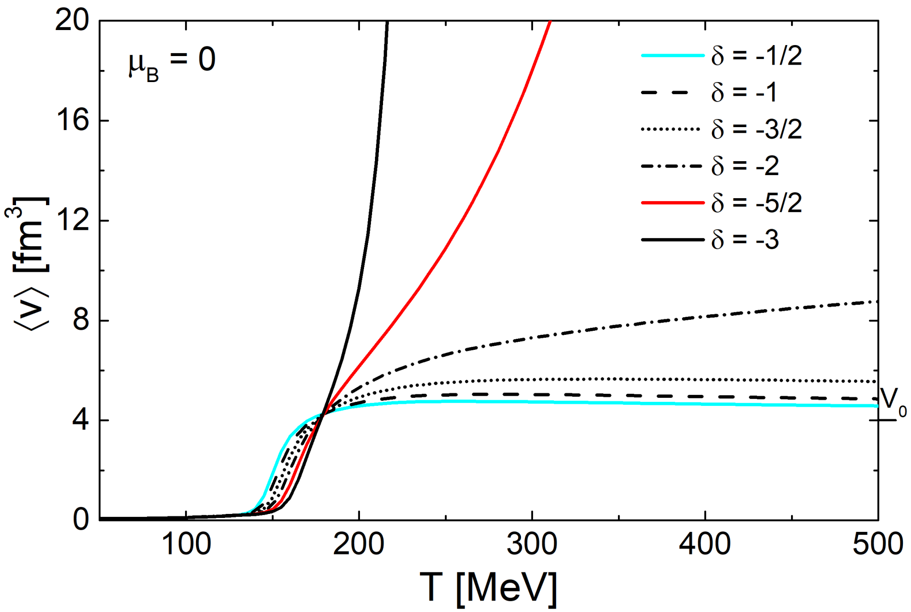
- Crossover transition towards bag model EoS
- Dependence on δ is mild
- Approach to the Stefan-Boltzmann limit is too fast
- Peak in energy density, not seen on the lattice

Nature of the transition

Filling fraction = $\frac{<V_{had}>}{V}$



Mean hadron volume $< v >$

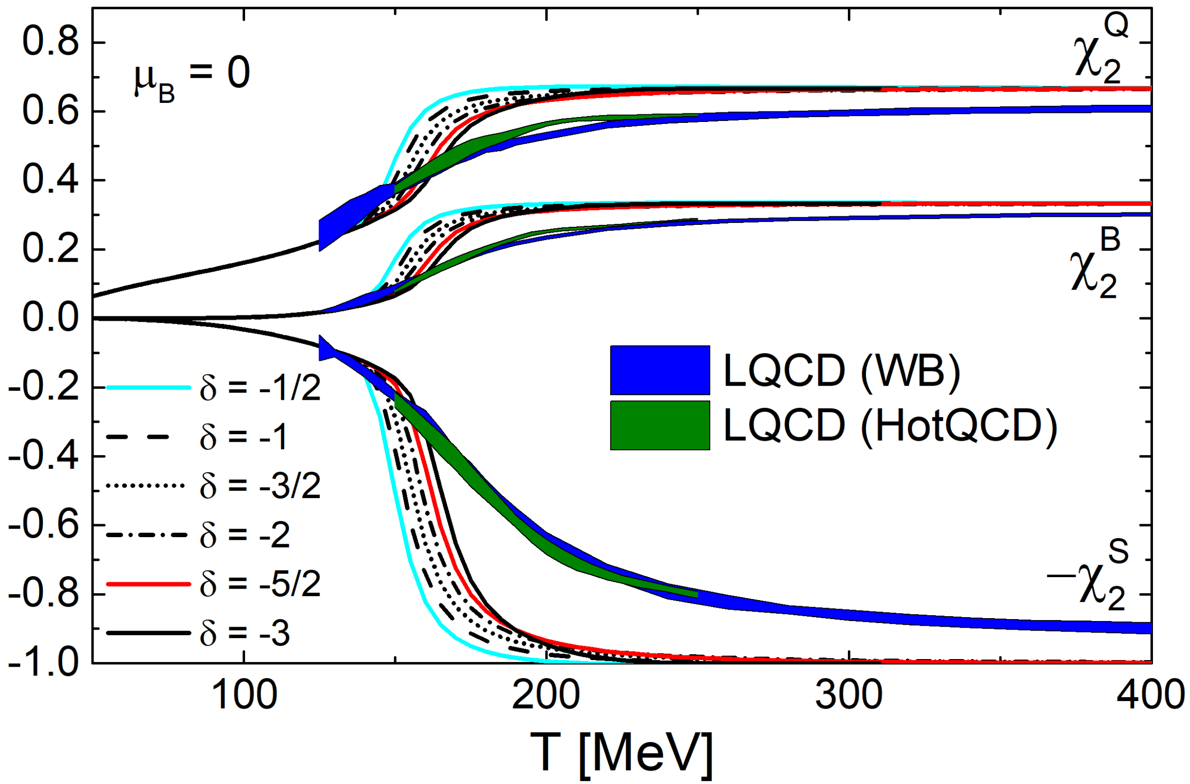


- Bags occupy almost whole space at large temperatures
- Strongest changes take place in the vicinity of T_H
- At $\delta < -7/4$ and $T \to \infty$ whole space — large bags with QGP

Conserved charges susceptibilities

$$\chi^{BSQ}_{lmn} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \, \partial(\mu_S/T)^m \, \partial(\mu_Q/T)^n}$$

Available from lattice QCD, not considered in this type of model before



Qualitatively compatible with lattice QCD

Bag model with massive quarks

Main source of quantitative disagreement comes from inaccuracy of the standard MIT bag model with massless quarks for describing QGP

Quasiparticle models suggest sizable thermal masses of quarks and gluons in high-temperature QGP [Peshier et al., PLB '94; PRC '00; PRC '02]

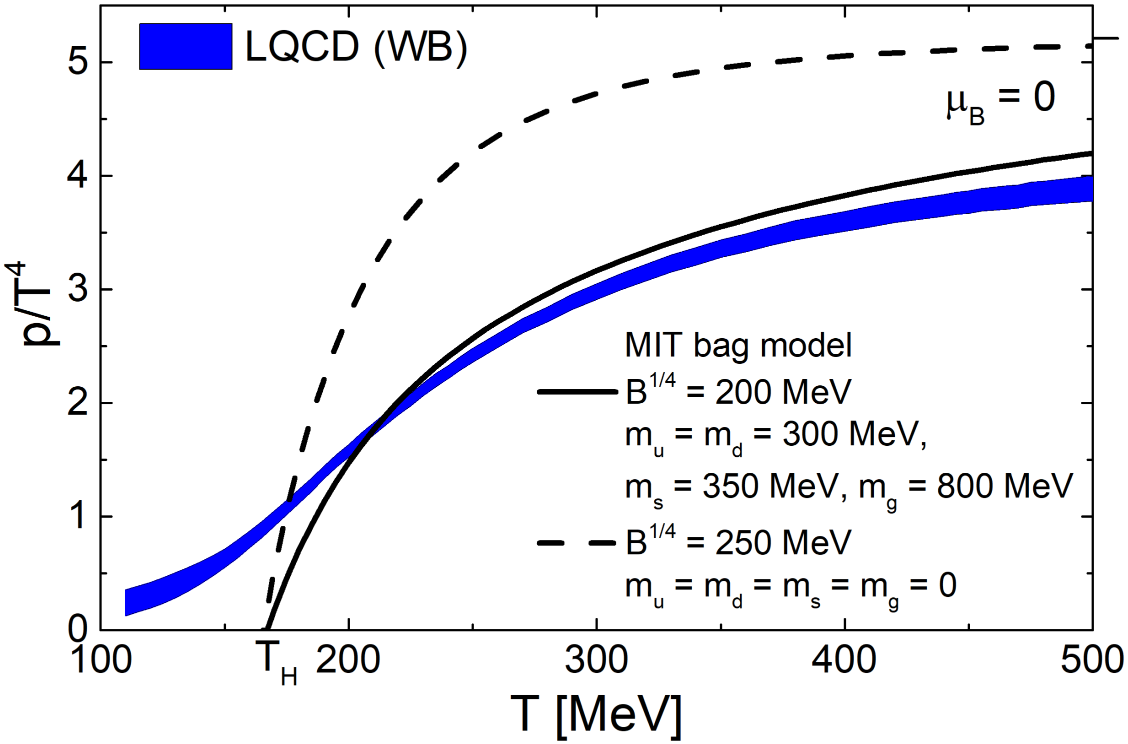
Heavy-bag model: bag model EoS with non-interacting *massive* quarks and gluons and the bag constant [Ivanov et al., PRC 72, 025804 (2005)]

Massive quarks and gluons instead of massless ones:

$$\sigma_Q(T,\lambda_B,\lambda_Q,\lambda_S) = \frac{8}{\pi^2\,T^4}\int_0^\infty dk \frac{k^4}{\sqrt{k^2+m_g^2}}\left[\exp\left(\frac{\sqrt{k^2+m_g^2}}{T}\right)-1\right]^{-1}$$
$$+\sum_{f=u,d,s}\frac{3}{\pi^2\,T^4}\int_0^\infty dk \frac{k^4}{\sqrt{k^2+m_f^2}}\left[\lambda_f^{-1}\exp\left(\frac{\sqrt{k^2+m_f^2}}{T}\right)+1\right]^{-1}$$
$$+\sum_{f=u,d,s}\frac{3}{\pi^2\,T^4}\int_0^\infty dk \frac{k^4}{\sqrt{k^2+m_f^2}}\left[\lambda_f\exp\left(\frac{\sqrt{k^2+m_f^2}}{T}\right)+1\right]^{-1}$$

Bag model with massive quarks

Introduction of constituent masses leads to much better description of QGP



Parameters for the crossover model:

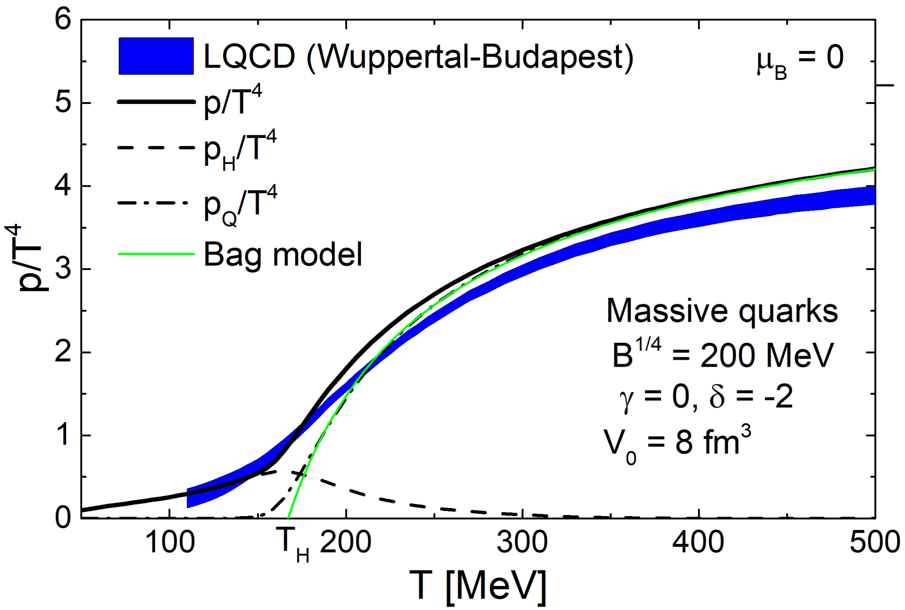
$$m_u = m_d = 300 \text{ MeV}, \quad m_s = 350 \text{ MeV}, \quad m_g = 800 \text{ MeV}, \quad B^{1/4} = 200 \text{ MeV}$$

$$\gamma = 0, \quad \delta = -2, \quad C = 0.03, \quad V_0 = 8 \text{ fm}^3$$

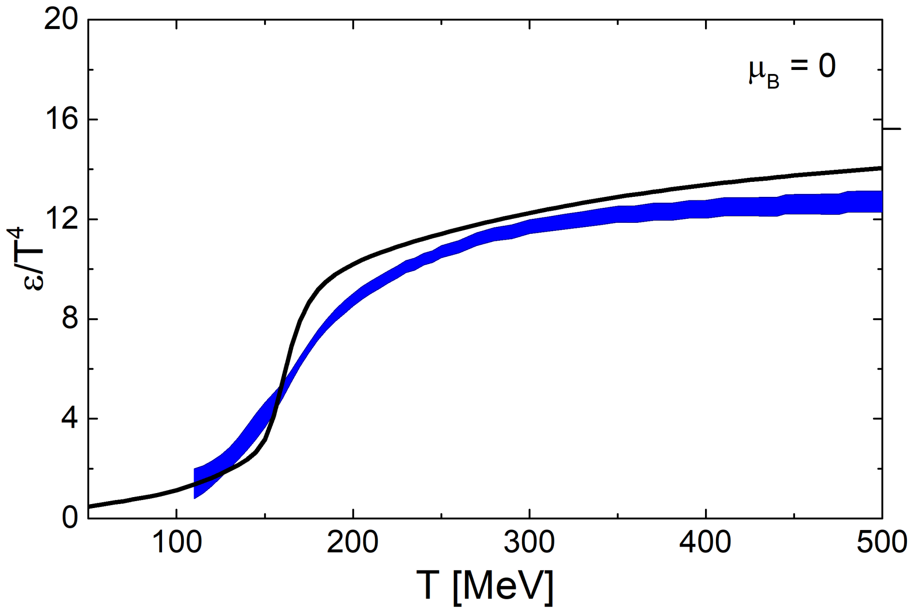
$$T_H \simeq 167 \text{ MeV}$$

Hagedorn model: Thermodynamic functions

Pressure p/T^4



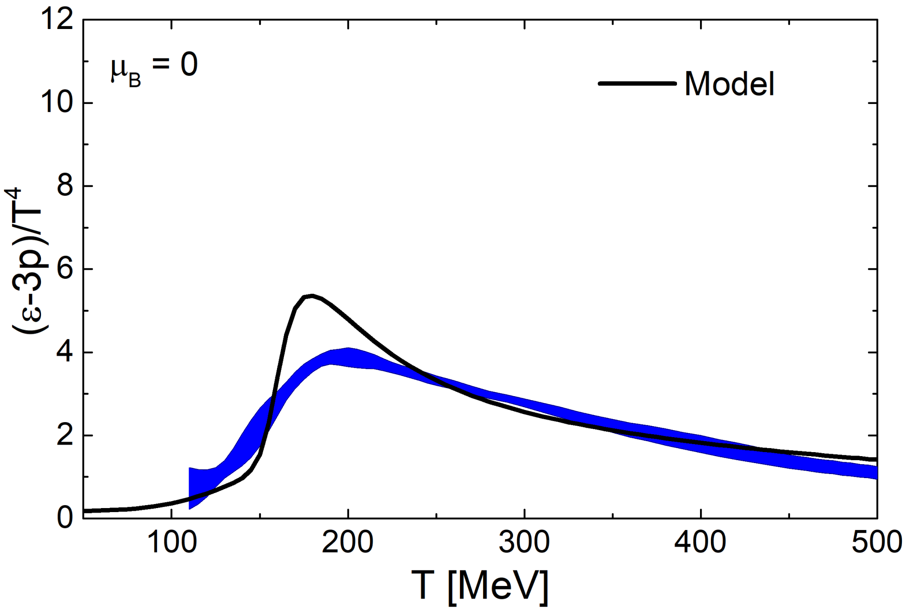
Energy density ε/T^4



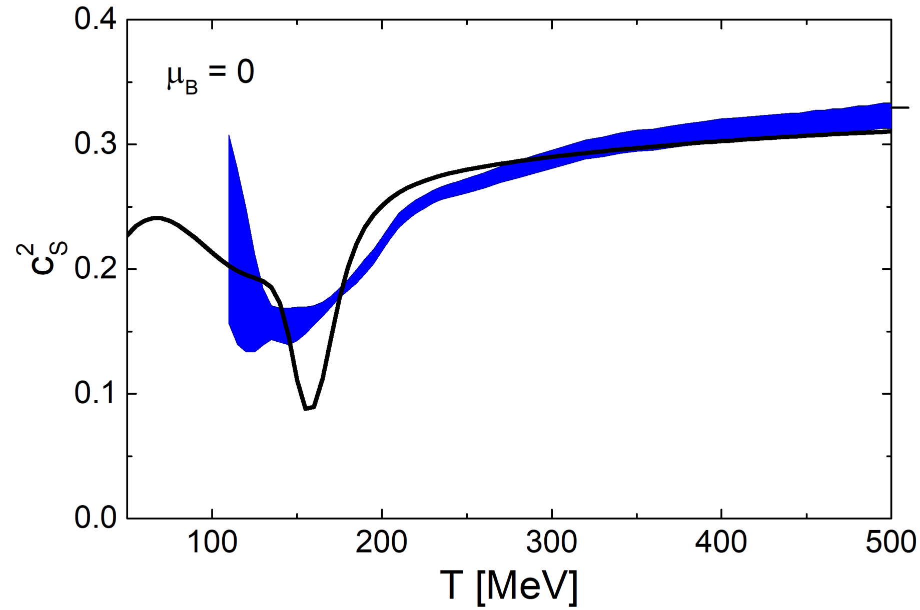
- Semi-quantitative description of lattice data
- Peak in energy density gone!

Hagedorn model: Thermodynamic functions

Trace anomaly $(\varepsilon - 3p)/T^4$

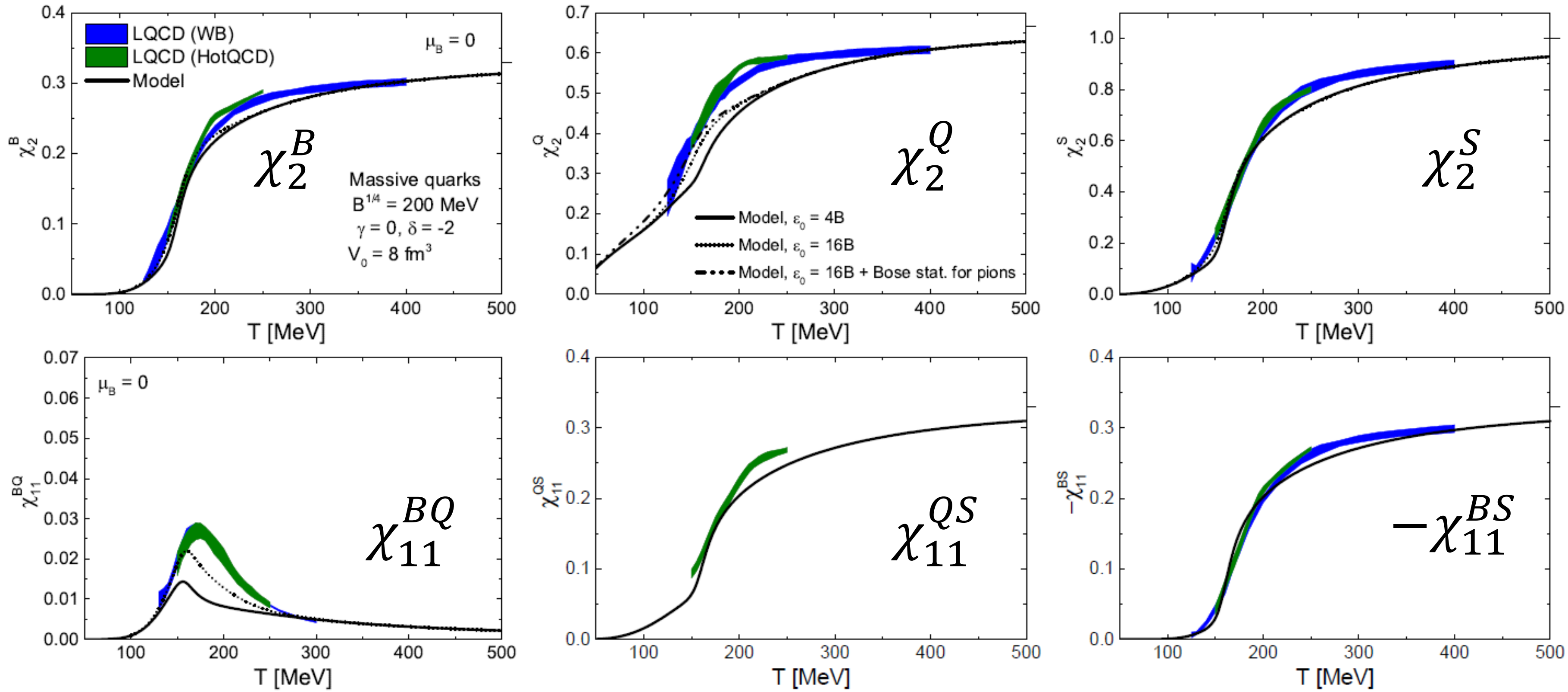


Speed of sound $c_s^2 = dp/d\varepsilon$



Hagedorn model: Susceptibilities

$$\chi^{BSQ}_{lmn} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l\, \partial(\mu_S/T)^m\, \partial(\mu_Q/T)^n}$$



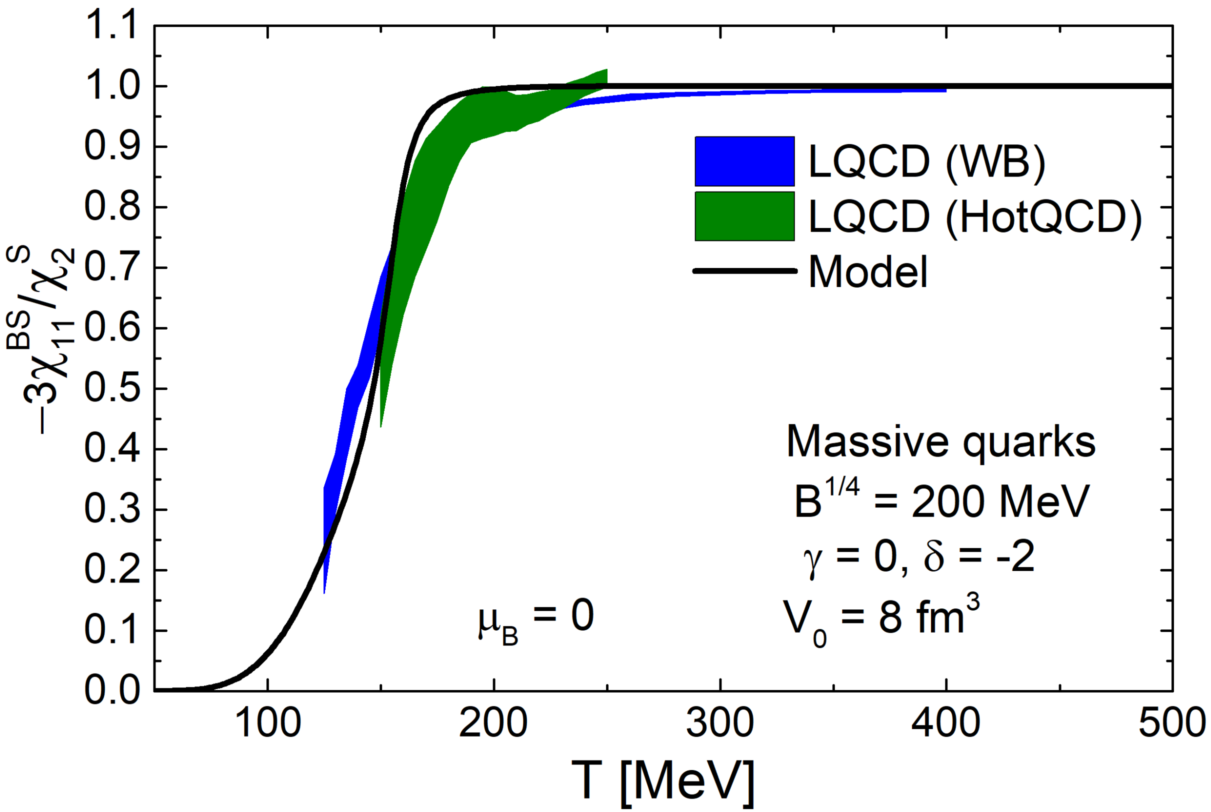
Lattice data from 1112.4416 (Wuppertal-Budapest), 1203.0784 (HotQCD)

Hagedorn model: Baryon-strangeness ratio

$$C_{BS} = -\frac{3\chi_{11}^{BS}}{\chi_2^S}$$

Useful diagnostic of QCD matter

[V. Koch, Majumder, Randrup, PRL 95, 182301 (2005)]

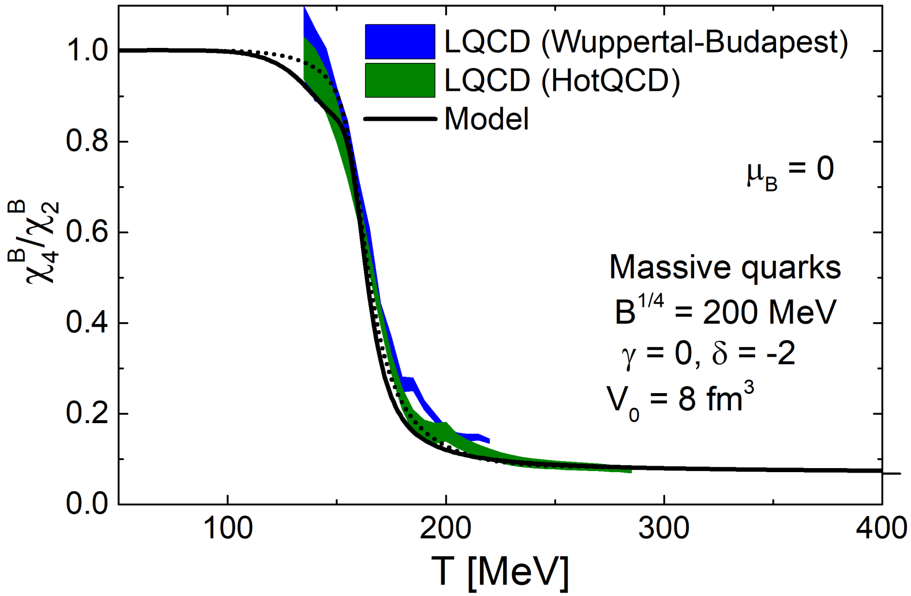


Well consistent with lattice QCD

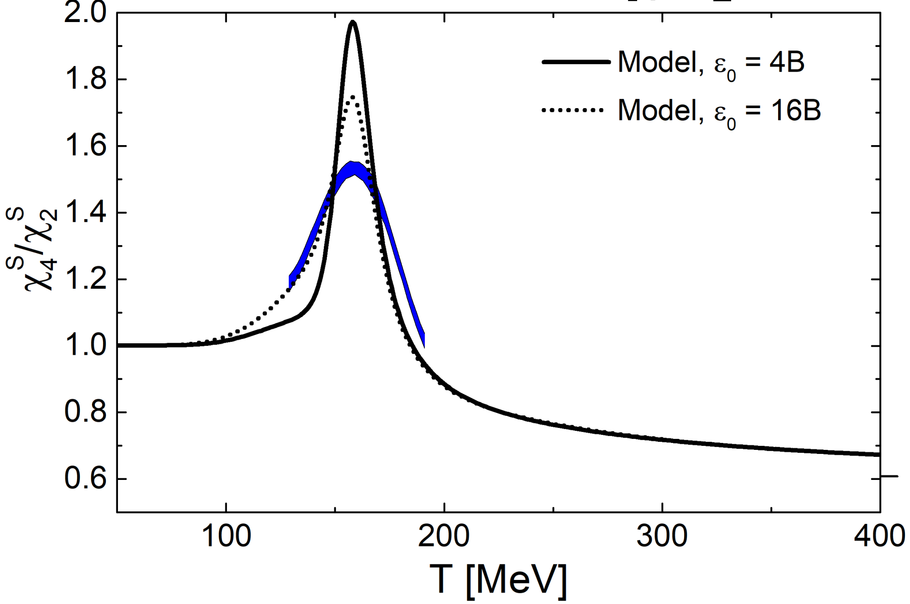
Hagedorn model: Higher-order susceptibilities

Higher-order susceptibilities are particularly sensitive probes of the parton-hadron transition and possible remnants of criticality at $\mu_B = 0$

net baryon χ_4^B/χ_2^B



net strangeness χ_4^S/χ_2^S

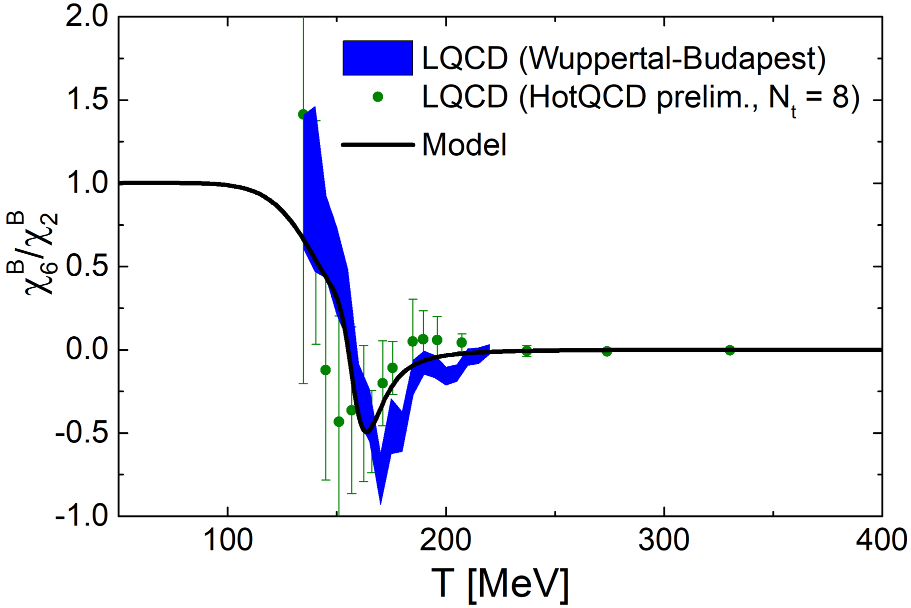


Lattice data from 1305.6297 & 1805.04445 (Wuppertal-Budapest), 1708.04897 (HotQCD)

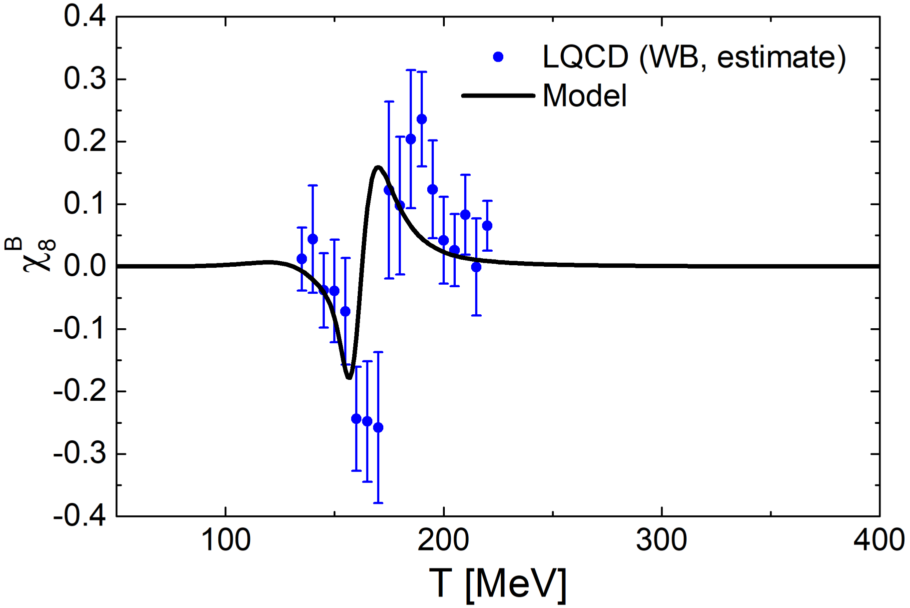
- Drop of χ_4^B/χ_2^B caused by repulsive interactions which ensure crossover transition to QGP
- Peak in χ_4^S/χ_2^S is an interplay of the presence of multi-strange hyperons and repulsive interactions

Hagedorn model: Higher-order susceptibilities

net baryon χ_6^B/χ_2^B



net baryon χ_8^B



Lattice data from 1805.04445 (Wuppertal-Budapest), 1708.04897 (HotQCD)

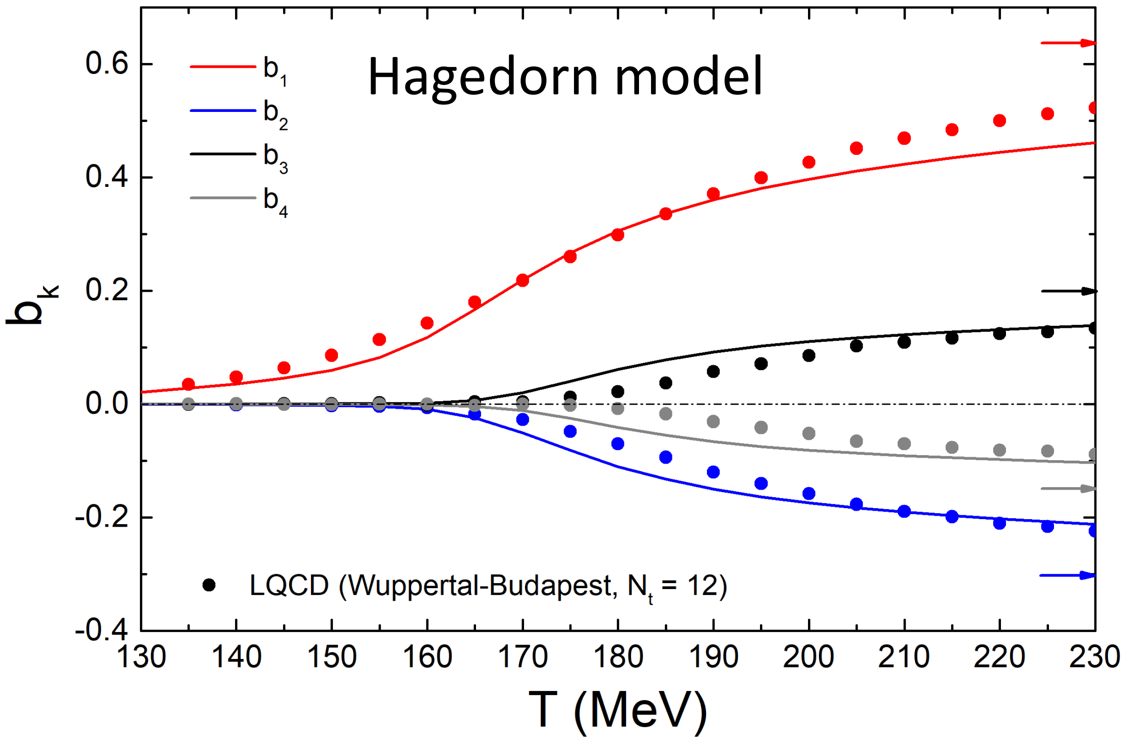
- Strong non-monotonic dependence of higher-order baryon number susceptibilities χ_6^B/χ_2^B and χ_8^B well reproduced by the crossover model
- No critical point signal in lattice data?

Hagedorn model: Fourier coefficients

Additional model test provided by imaginary μ_B lattice data, where Fourier coefficients of the net baryon density were computed

[Vovchenko, Pasztor, Fodor, Katz, Stoecker, 1708.02852]

$$\left.\frac{\rho_B(T,\mu_B)}{T^3}\right|_{\mu_B=i\theta_B T} = i\sum_{k=1}^{\infty} b_k(T)\sin(k\theta_B)$$



Summary

- Higher-order susceptibilities at $\mu_B = 0$ are sensitive to hadronic interactions
- Deviations of LQCD susceptibilities from uncorrelated hadron gas picture can be understood in terms of repulsive baryonic interactions (excluded volume, mean field, etc.)
- Inclusion of repulsive interactions in the Hagedorn bag-like model allow to obtain crossover transition compatible with lattice QCD
- No unambiguous sign of criticality in higher-order LQCD susceptibilities

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Thanks for your attention!