Thermal model fits: an overview

Volodymyr Vovchenko

Goethe University Frankfurt & Frankfurt Institute for Advanced Studies

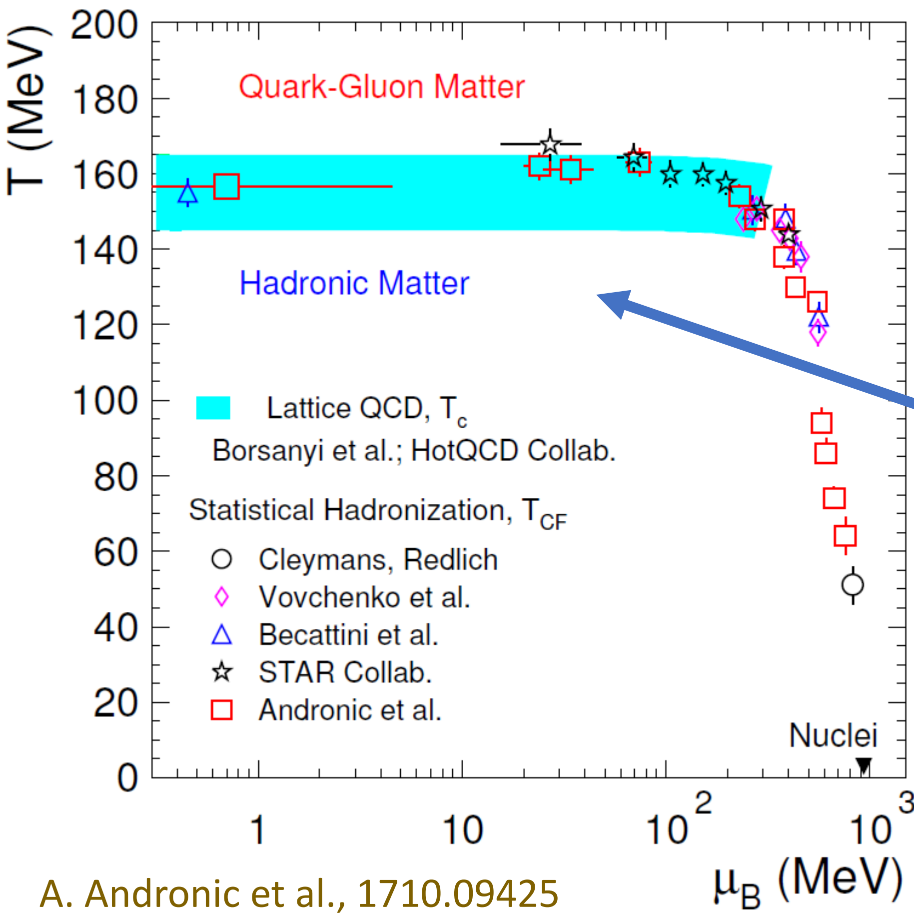
Light up 2018 — An ALICE and theory workshop, CERN

June 14, 2018

The conventional picture

Thermal fits map heavy-ion collisions to the QCD phase diagram

$$N_i^{\text{mod}} = N_i^{\text{hrg}} + \sum_j BR(j \to i) N_j^{\text{hrg}}, \qquad N_i^{\text{hrg}} = V \frac{d_i m_i^2 T}{2\pi^2} K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}}$$



A. Andronic et al., 1710.09425

Fits minimize

$$\chi^2 = \sum_i \frac{(N_i^{\text{mod}} - N_i^{\text{exp}})^2}{(\sigma_i^{\text{exp}})^2}$$

Conventional picture based on chemical equilibrium (ideal) HRG model fits

Many aspects of the thermal fits

Alternative/extended scenarios:

- chemical non-equilibrium (γ_q, γ_s)
- hadronic phase influence
- flavor hierarchy at freeze-out
- light nuclei

Systematic uncertainties in the HRG model:

- hadron spectrum and decay channels
- treatment of finite resonance widths
- excluded volume/van der Waals interactions

Description of small systems:

- exact conservation of conserved charges (canonical ensemble)

Commonly used tools for thermal fits

1) **SHARE 3** [G. Torrieri, J. Rafelski, M. Petran, et al.]
   Fortran/C++. Chemical (non-)equilibrium, fluctuations, charm, nuclei
   open source: http://www.physics.arizona.edu/~gtshare/SHARE/share.html

2) **THERMUS 4** [S. Wheaton, J. Cleymans, B. Hippolyte, et al.]
   C++/ROOT. Canonical ensemble, EV corrections, charm, nuclei
   open source: https://github.com/thermus-project/THERMUS

3) **GSI-Heidelberg** code [A. Andronic et al.] **not open source**

4) **Florence** code [F. Becattini et al.] **not open source**

Commonly used tools for thermal fits

1) **SHARE 3** [G. Torrieri, J. Rafelski, M. Petran, et al.]
Fortran/C++. Chemical (non-)equilibrium, fluctuations, charm, nuclei
open source: http://www.physics.arizona.edu/~gtshare/SHARE/share.html

2) **THERMUS 4** [S. Wheaton, J. Cleymans, B. Hippolyte, et al.]
C++/ROOT. Canonical ensemble, EV corrections, charm, nuclei
open source: https://github.com/thermus-project/THERMUS

3) **GSI-Heidelberg** code [A. Andronic et al.] **not open source**

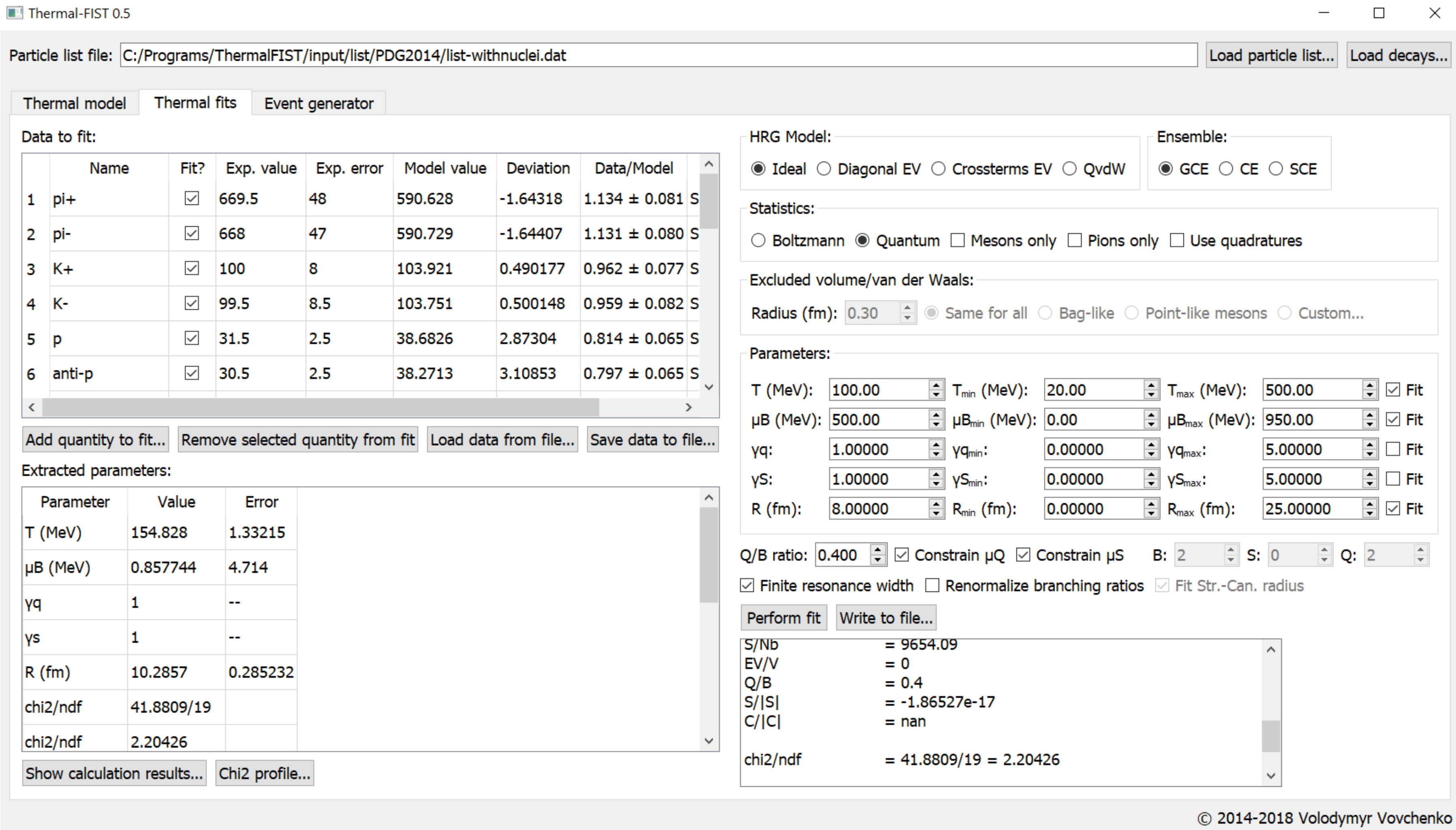
4) **Florence** code [F. Becattini et al.] **not open source**

New development:

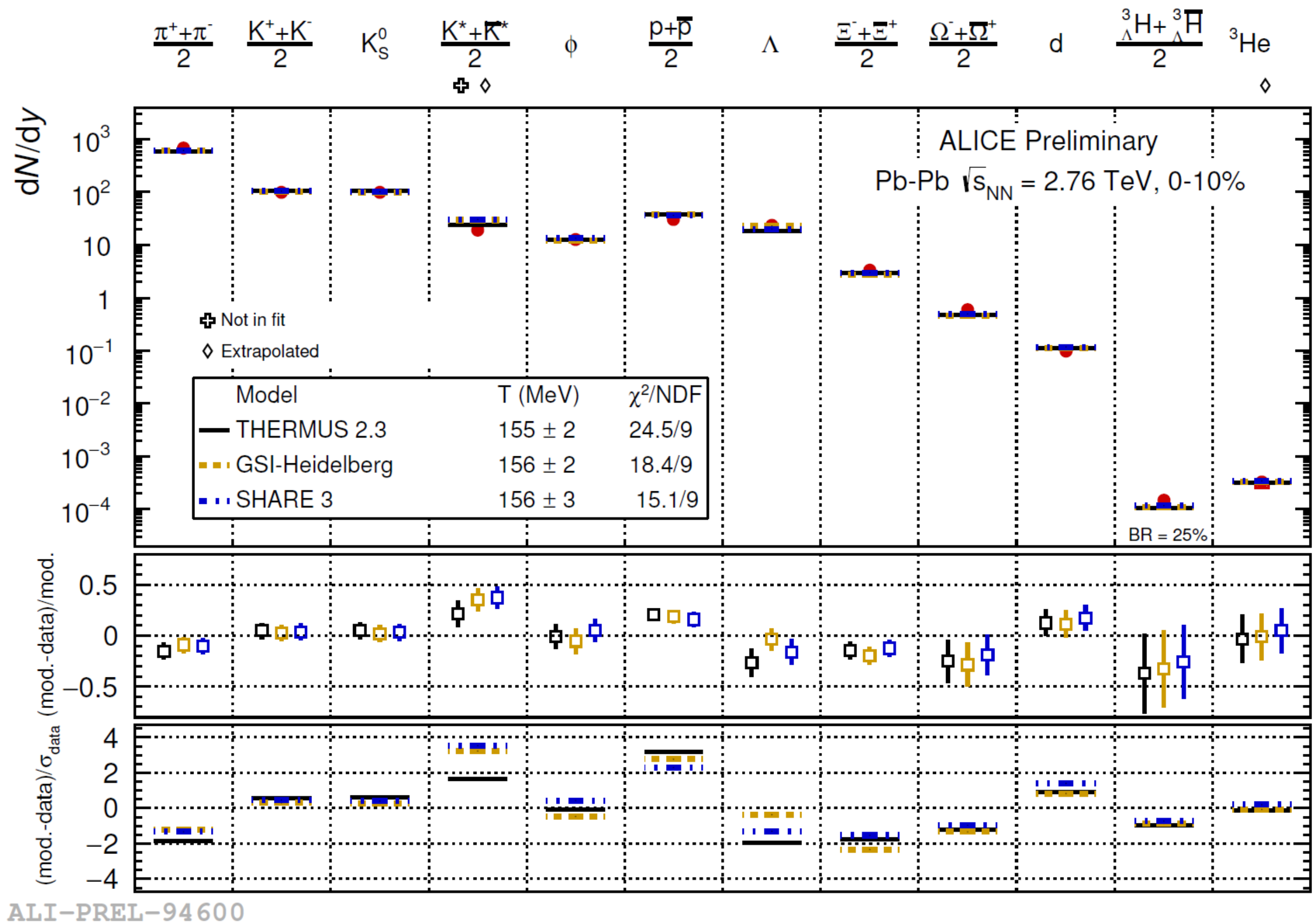
Thermal-FIST v0.5 (or simply "The FIST") [**V.V.**, H. Stoecker]
C++. Chemical (non-)equilibrium, EV/vdW corrections, Monte Carlo, (higher-order) fluctuations, canonical ensemble, combinations of effects
open source: https://github.com/vlvovch/Thermal-FIST

Thermal-FIST

Graphical user interface for *general-purpose* thermal fits and more



Standard picture for Pb+Pb @ 2.76 TeV



ALICE collaboration (SQM 2015)

Similar results with *Thermal-FIST* and *Florence codes* [Becattini et al., 1605.09694]

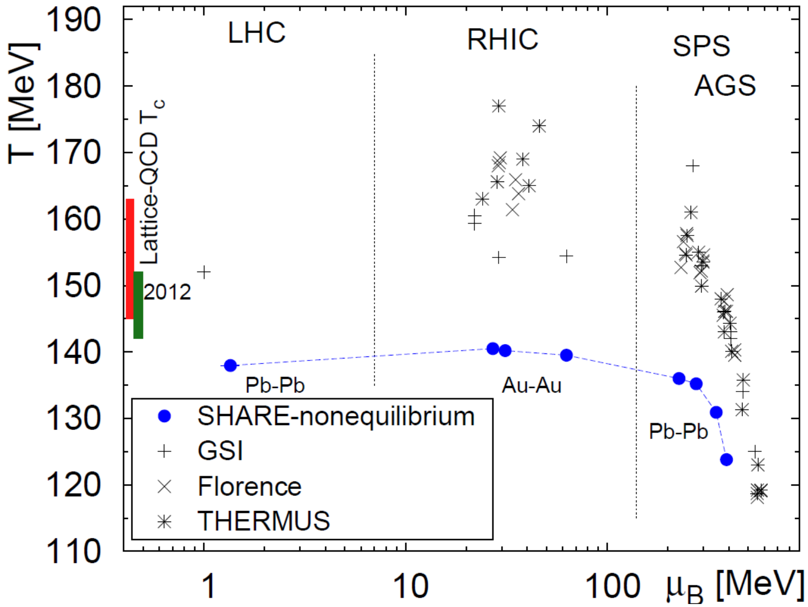
Consistent picture between codes for chem. equilibrium ideal HRG

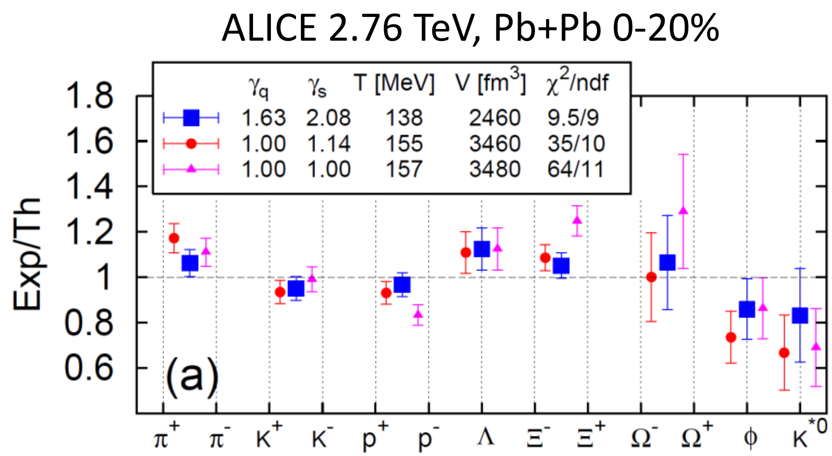
Alternative/extended scenarios

Chemical non-equilibrium model

In chemical non-equilibrium scenario $N_i^{\mathrm{hrg}} \propto (\gamma_q)^{|q_i|}(\gamma_s)^{|s_i|}$

E.g. hadronization of chem. non-eq. supercooled QGP [Letessier, Rafelski, '99]





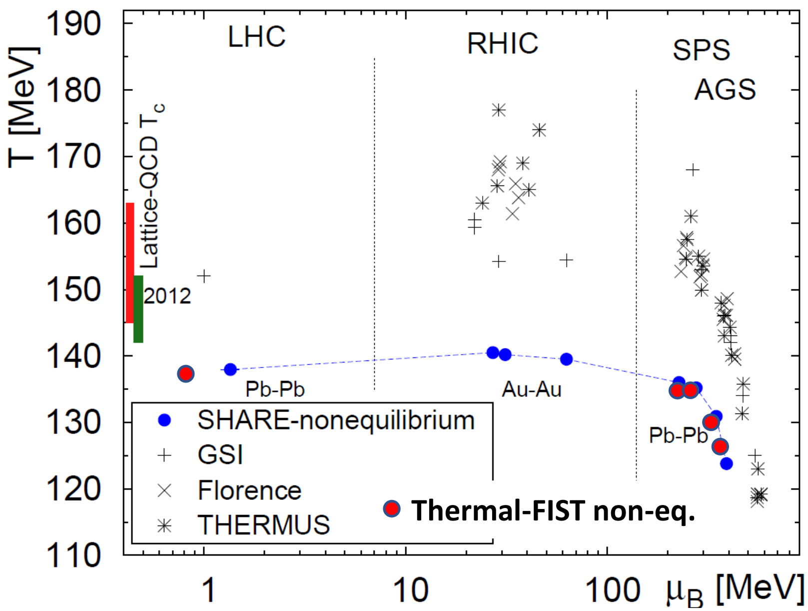
[M. Petran et al., 1303.2098]

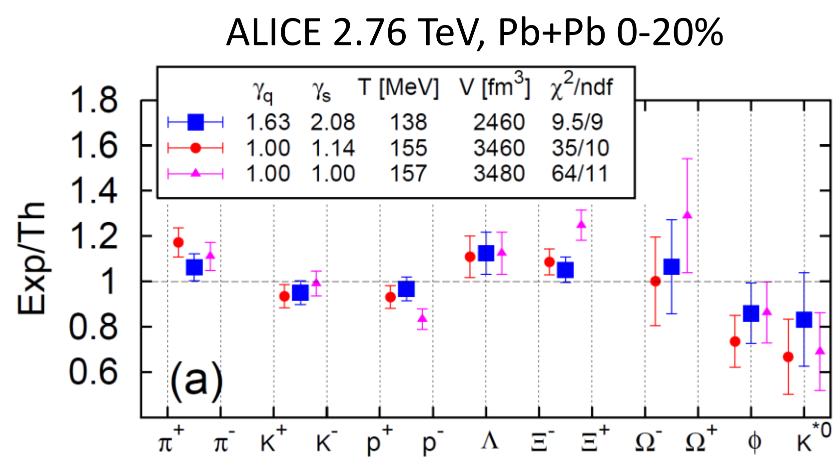
- smaller reduced χ^2 compared to chem. equilibrium scenario
- describes p_T-spectra of many hadrons [V. Begun et al., 1312.1487, 1405.7252]
- $\gamma_q = 1.63$ => $\mu_\pi \approx 135\ MeV \approx m_\pi$ => pion BEC? [V. Begun et al., 1503.04040]
- However, $\gamma_q \approx \gamma_s \approx 1$ when light nuclei included in fit [M. Floris, 1408.6403]

Chemical non-equilibrium model

In chemical non-equilibrium scenario $N_i^{\mathrm{hrg}} \propto (\gamma_q)^{|q_i|}(\gamma_s)^{|s_i|}$

E.g. hadronization of chem. non-eq. supercooled QGP [Letessier, Rafelski, '99]





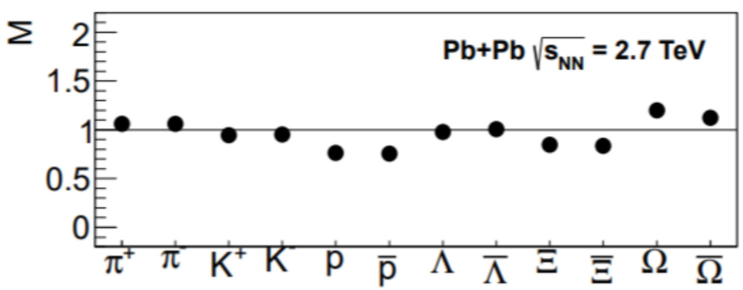
[M. Petran et al., 1303.2098]

- smaller reduced χ^2 compared to chem. equilibrium scenario
- describes p_{T}-spectra of many hadrons [V. Begun et al., 1312.1487, 1405.7252]
- $\gamma_q = 1.63$ => $\mu_\pi \approx 135\ MeV \approx m_\pi$ => pion BEC? [V. Begun et al., 1503.04040]
- However, $\gamma_q \approx \gamma_s \approx 1$ when light nuclei included in fit [M. Floris, 1408.6403]

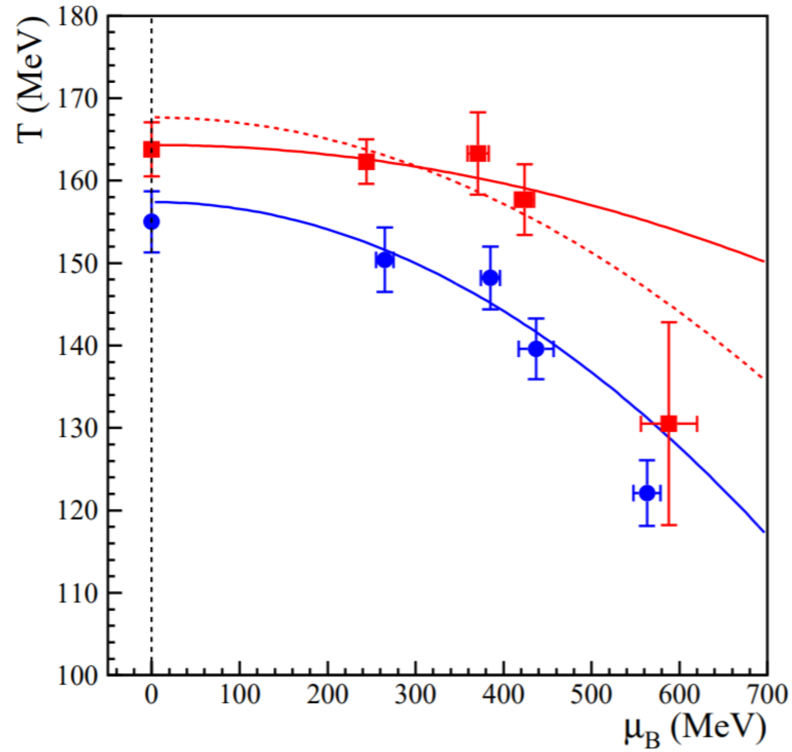
Influence of the hadronic phase

Modification of hadron yields in non-equilibrium hadronic phase

$B\bar{B}$ annihilation reduces (anti)proton yields [Steinheimer et al., 1203.5302]



	T(MeV)	μ_B(MeV)	γ_S	χ^2/NDF
Pb-Pb 20% central $\sqrt{s_{\rm NN}} = 2.7$ TeV				
Std. fit	156 ± 5	1 ± 12	1.09 ± 0.07	26.5/9
Mod. fit	166 ± 3	2 ± 6	0.98 ± 0.04	11.5/9
Pb-Pb 5% central $\sqrt{s_{\rm NN}} = 17.3$ GeV				
Std. fit	151 ± 4	266 ± 9	0.91 ± 0.05	26.9/11
Mod. fit	163 ± 4	250 ± 9	0.83 ± 0.04	20.4/11
Pb-Pb 5% central $\sqrt{s_{\rm NN}} = 8.7$ GeV				
Std. fit	148 ± 4	385 ± 11	0.78 ± 0.06	17.9/9
Mod. fit	161 ± 6	376 ± 15	0.72 ± 0.06	25.9/9
Pb-Pb 5% central $\sqrt{s_{\rm NN}} = 7.6$ GeV				
Std. fit	140 ± 1	437 ± 5	0.91 ± 0.01	22.4/7
Mod. fit	156 ± 5	426 ± 4	0.81 ± 0.00	14.7/7



[Becattini et al., 1212.2431, 1605.09694]

- somewhat better χ^2 and increase in T_{ch} by 10-15 MeV
- no backreaction, e.g. $5M \to B\bar{B}$, in UrQMD. What is its role?

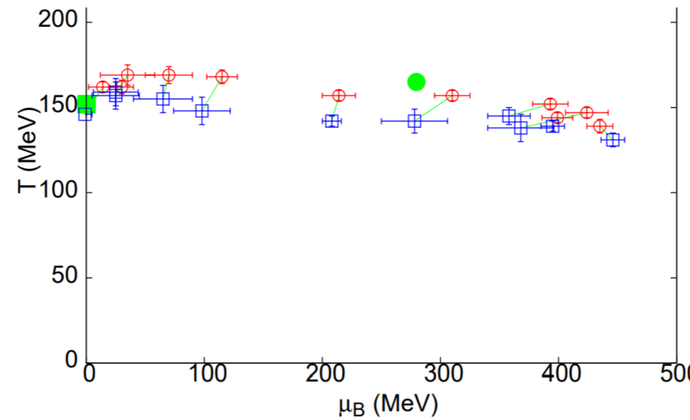
Flavor hierarchy at freeze-out

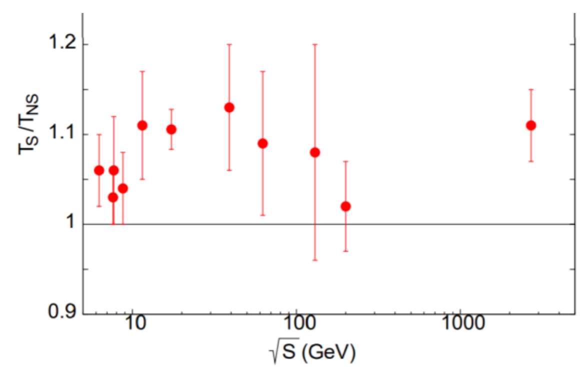
QCD transition is a broad crossover

=> different "T_c" for different observables

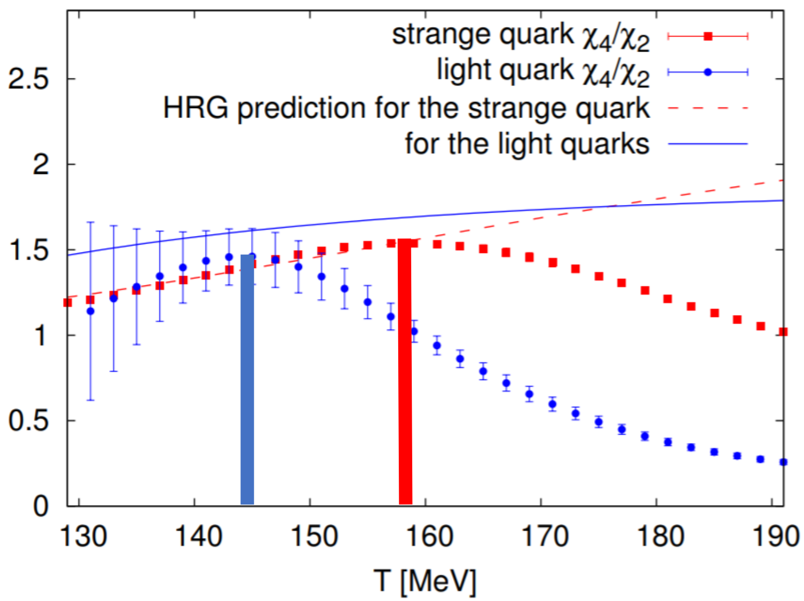
2CFO scheme

[S. Chatterjee et al., 1306.2006]





strange vs light number susceptibility



[R. Bellwied et al., 1305.6297]

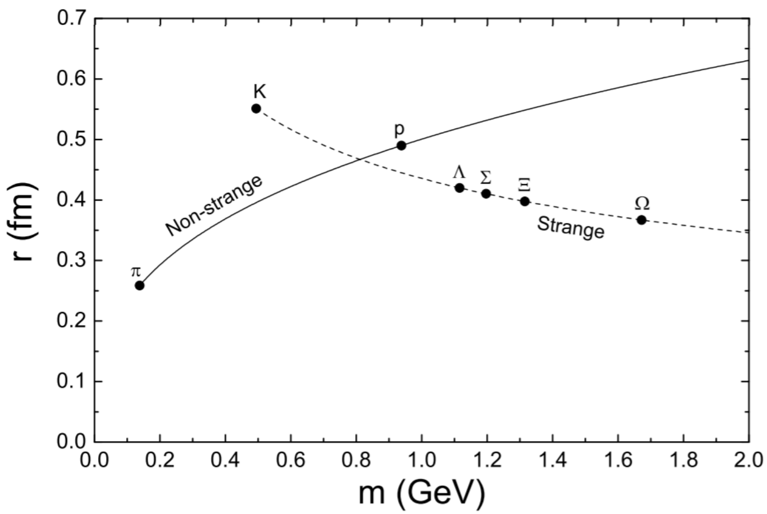
- higher T_f for strange particles than for non-strange
- effect may disappear if more strange baryons included

[Bazavov et al., 1404.6511, S. Chatterjee, 1708.08152]

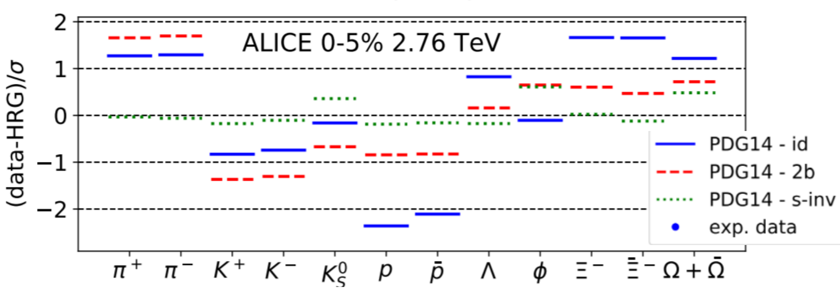
Flavor hierarchy in hadron sizes

Alternative: Flavor hierarchy in hadron sizes [P. Alba et al., 1606.06542]

$v_i \propto m_i$ for non-strange, $v_i \propto m_i^{-1}$ for strange, excluded-volume HRG



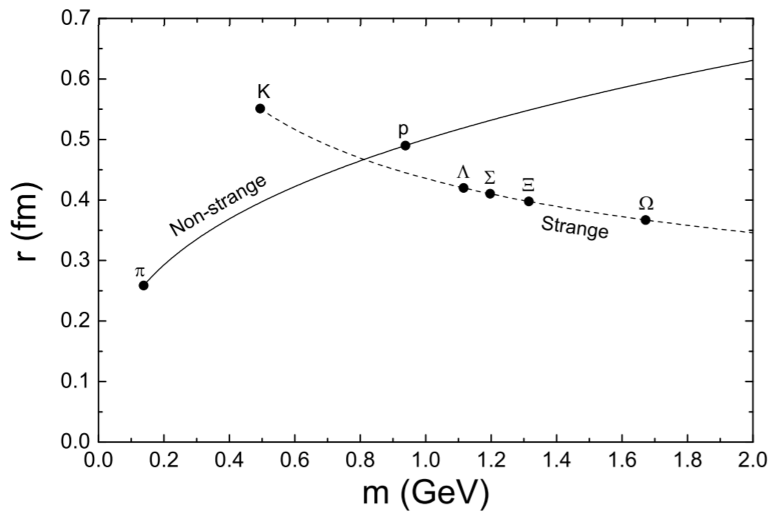
ALICE 0-5%:
χ^2/N_{dof} = **0.88/7**

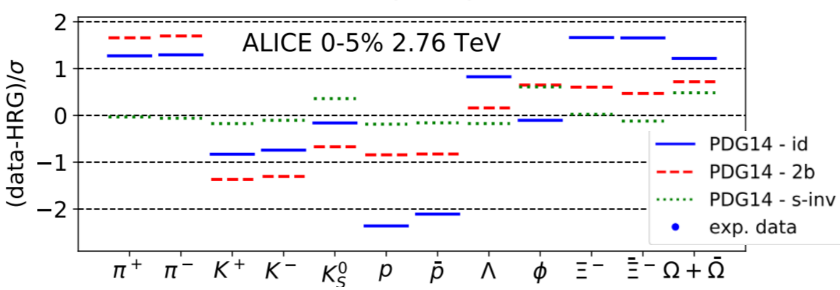


Flavor hierarchy in hadron sizes

Alternative: Flavor hierarchy in hadron sizes [P. Alba et al., 1606.06542]

$v_i \propto m_i$ for non-strange, $v_i \propto m_i^{-1}$ for strange, excluded-volume HRG





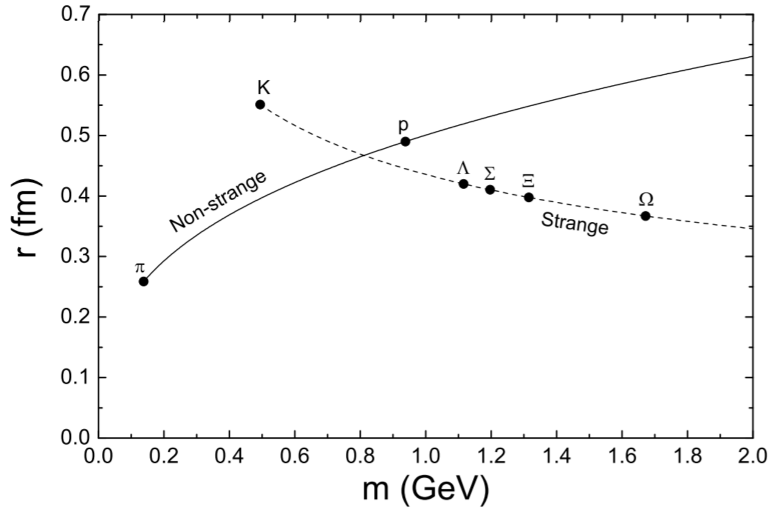
ALICE 0-5%:
χ^2/N_{dof} = **0.88/7**

	χ^2/N_{dof}	T (MeV)
ALICE 5-10%	$1.022/7 \simeq 0.14$	154.3±2.3
ALICE 10-20%	$2.7/9 \simeq 0.30$	156.7±1.6
ALICE 20-30%	$6.08/8 \simeq 0.76$	158.4±1.8
ALICE 30-40%	$6.9/8 \simeq 0.86$	158.7±1.9
ALICE 40-50%	$3.07/8 \simeq 0.38$	158.0±1.8
ALICE 50-60%	$4.42/8 \simeq 0.55$	155.3±2.0
ALICE 60-70%	$8.09/8 \simeq 1.01$	153.2±2.9
ALICE 70-80%	$5.01/8 \simeq 0.62$	161.2±4.5

Flavor hierarchy in hadron sizes

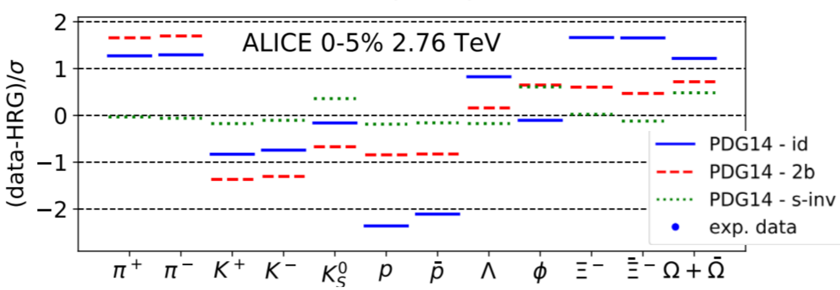
Alternative: Flavor hierarchy in hadron sizes [P. Alba et al., 1606.06542]

$v_i \propto m_i$ for non-strange, $v_i \propto m_i^{-1}$ for strange, excluded-volume HRG



ALICE 0-5%:
χ^2/N_{dof} = **0.88/7**

	$\chi^2/N_{\rm dof}$	T (MeV)
ALICE 5-10%	$1.022/7 \simeq 0.14$	154.3 ± 2.3
ALICE 10-20%	$2.7/9 \simeq 0.30$	156.7 ± 1.6
ALICE 20-30%	$6.08/8 \simeq 0.76$	158.4 ± 1.8
ALICE 30-40%	$6.9/8 \simeq 0.86$	158.7 ± 1.9
ALICE 40-50%	$3.07/8 \simeq 0.38$	158.0 ± 1.8
ALICE 50-60%	$4.42/8 \simeq 0.55$	155.3 ± 2.0
ALICE 60-70%	$8.09/8 \simeq 1.01$	153.2 ± 2.9
ALICE 70-80%	$5.01/8 \simeq 0.62$	161.2 ± 4.5



	$\chi^2/N_{\rm dof}$
STAR 200	$16.8/8 \simeq 2.1$
NA49 158	$70.1/10 \simeq 7.01$
NA49 80	$71.7/7 \simeq 10.2$
NA49 40	$44.2/8 \simeq 5.5$
NA49 30	$29.6/7 \simeq 4.2$

→

$\chi^2/N_{\rm dof}$
$6.5/8 \simeq 0.82$
$57.4/10 \simeq 5.7$
$28.3/7 \simeq 4.05$
$15.1/8 \simeq 1.8$
$7.08/7 \simeq 1.01$

- Significant improvement in fit quality across $\sqrt{s}$ and centralities
- Reflects systematics in data, exact physical reasons to be clarified

Hierarchy in baryon number?

Considering the ALICE 2.76 TeV Pb+Pb 0-10% data in ideal HRG model...

1) Fit of mesons + baryons + nuclei: $T_{ch} = 155 \pm 2$ MeV, χ^2/N_{dof} = 41.9/20

2) Fit of mesons + baryons: $T_{ch} = 155 \pm 2$ MeV, χ^2/N_{dof} = 36.7/12

Hierarchy in baryon number?

Considering the ALICE 2.76 TeV Pb+Pb 0-10% data in ideal HRG model…

1) Fit of mesons + baryons + nuclei: $T_{ch} = 155 \pm 2$ MeV, χ^2/N_{dof} = 41.9/20

2) Fit of mesons + baryons: $T_{ch} = 155 \pm 2$ MeV, χ^2/N_{dof} = 36.7/12

3) Fit of mesons $(\pi^{\pm}, K^{\pm}, K_0^S, \phi)$: $T_{ch} = \mathbf{141} \pm 9$ MeV, χ^2/N_{dof} = **3.7/4**

Hierarchy in baryon number?

Considering the ALICE 2.76 TeV Pb+Pb 0-10% data in ideal HRG model…

1) Fit of mesons + baryons + nuclei: $T_{ch} = 155 \pm 2$ MeV, χ^2/N_{dof} = 41.9/20

2) Fit of mesons + baryons: $T_{ch} = 155 \pm 2$ MeV, χ^2/N_{dof} = 36.7/12

3) Fit of mesons $(\pi^{\pm}, K^{\pm}, K_0^S, \phi)$: $T_{ch} = \mathbf{141} \pm 9$ MeV, χ^2/N_{dof} = **3.7/4**

4) Fit of baryons $(p, \Lambda, \Xi, \Omega)$: $T_{ch} = \mathbf{192} \pm 14$ MeV, χ^2/N_{dof} = **15.3/6**

Hierarchy in baryon number?

Considering the ALICE 2.76 TeV Pb+Pb 0-10% data in ideal HRG model…

1) Fit of mesons + baryons + nuclei: $T_{ch} = 155 \pm 2$ MeV, χ^2/N_{dof} = 41.9/20

2) Fit of mesons + baryons: $T_{ch} = 155 \pm 2$ MeV, χ^2/N_{dof} = 36.7/12

3) Fit of mesons $(\pi^{\pm}, K^{\pm}, K_0^S, \phi)$: $T_{ch} = \mathbf{141} \pm 9$ MeV, χ^2/N_{dof} = **3.7/4**

4) Fit of baryons $(p, \Lambda, \Xi, \Omega)$: $T_{ch} = \mathbf{192} \pm 14$ MeV, χ^2/N_{dof} = **15.3/6**

5) Fit of nuclei (d, ^{3}He, $^3_\Lambda\mathrm{H}$, ^{4}He): $T_{ch} = \mathbf{161} \pm 4$ MeV, χ^2/N_{dof} = **2.4/6**

Similar results at other centralities

Rather different fit temperatures in different baryon number sectors…

More tension in the baryonic sector

Systematic uncertainties in the HRG model

Input hadron list and decay channels

- High-mass resonances and their decay channels poorly known
- Evidence for missing strange baryons for lattice QCD
  [A. Bazavov et al., 1404.6511; P. Alba et al., 1702.0113; S. Chatterjee, 1708.08152]

Modeling finite resonance widths

- Zero-width approx., energy (in)dependent Breit-Wigner, phase shifts

Excluded volume/van der Waals interaction effects

- Thermal fits affected when EV parameters differ between hadrons
  [**V.V.**, H. Stoecker, 1512.08046, 1606.06218]

In-medium hadron masses

- In-medium masses due to interactions/chiral symmetry restoration
  [D. Zschiesche et al., nucl-th/0209022; G. Aarts et al., 1703.09246]
- Needs reconciliation with vacuum masses actually measured

Modeling finite resonance widths

$$n_i(T,\mu;m_i) \to \int_{m_i^{\min}}^{m_i^{\max}} dm\, \rho_i(m)\, n_i(T,\mu;m)$$

1) Zero-width approximation $\rho_i(m) = \delta(m - m_i)$

Simplest possibility, used commonly in LQCD comparisons

2) Breit-Wigner (BW) in $\pm 2\Gamma_i$ interval $\rho_i(m) = A_i \dfrac{2\, m\, m_i\, \Gamma_i}{(m^2 - m_i^2)^2 + m_i^2\, \Gamma_i^2}$

Popular choice in thermal fits (THERMUS, Florence code, Thermal-FIST)

Could be overestimating density near threshold

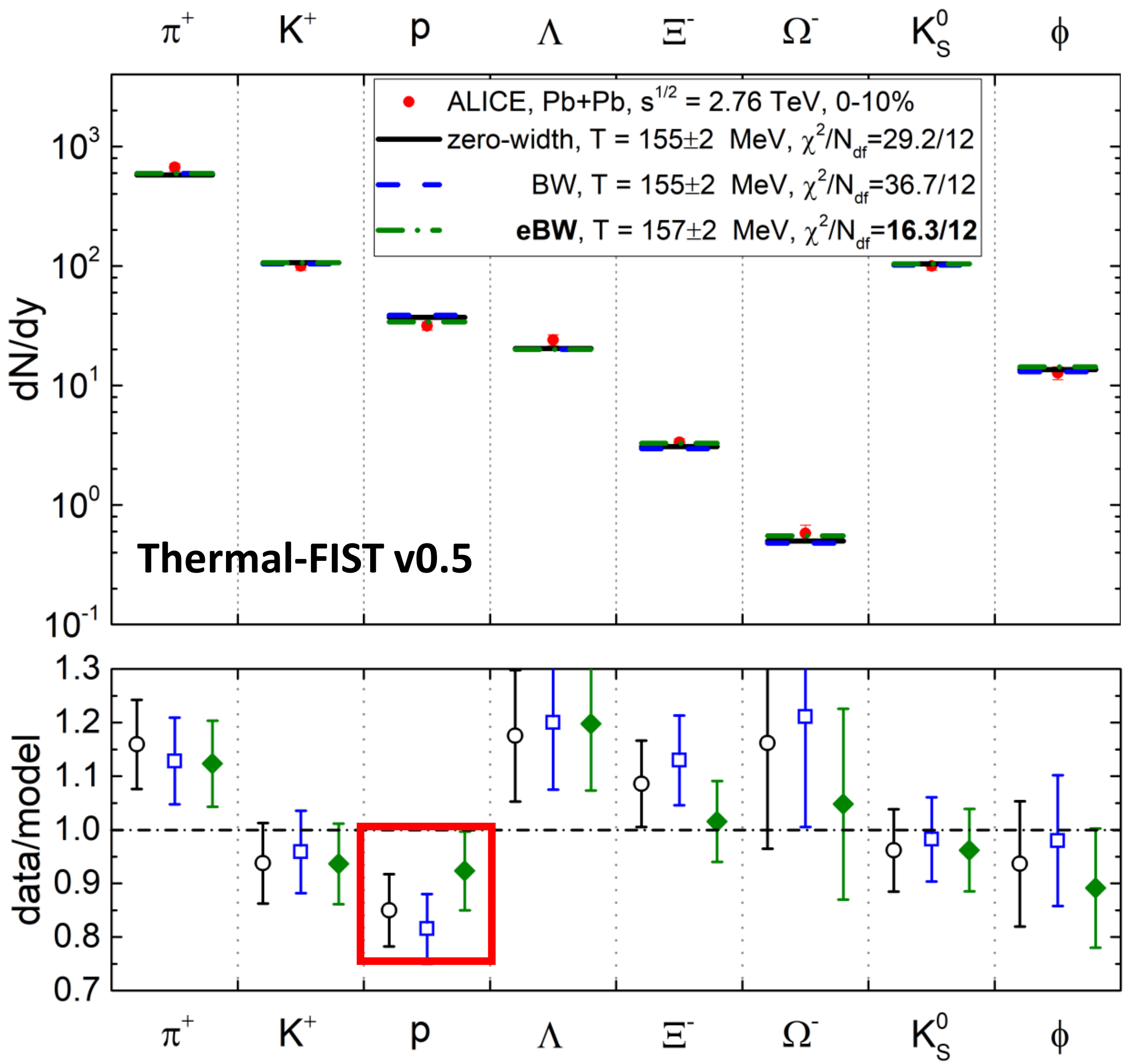
3) Energy-dependent Breit-Wigner (eBW) $\Gamma_i(m) = \sum_j \Gamma_{i\to j}(m)$

$$\Gamma_{i\to j}(m) = b_{i\to j}\, \Gamma_i \left[1 - \left(\frac{m_{i\to j}^{\rm thr}}{m}\right)^2\right]^{l_{ij}+1/2}$$

suppression at threshold (as in SHARE)

+ *m*-dependent decay feeddown $N_i^{\rm tot} = N_i^{\rm hrg} + \sum_{j\in {\rm pdg}} \int dm\, {\rm BR}(j\to i;m)\, \rho_j(m)\, \dfrac{dN_j^{\rm hrg}}{dm}$

Modeling widths: effect on thermal fits



Significant improvement in the eBW scheme due to a reduced proton feeddown from Δ and N*

Modeling of wide resonances important!!

[**V.V.** et al., in preparation]

Excluded volume corrections

Notion that hadrons have finite eigenvolume suggested a while ago

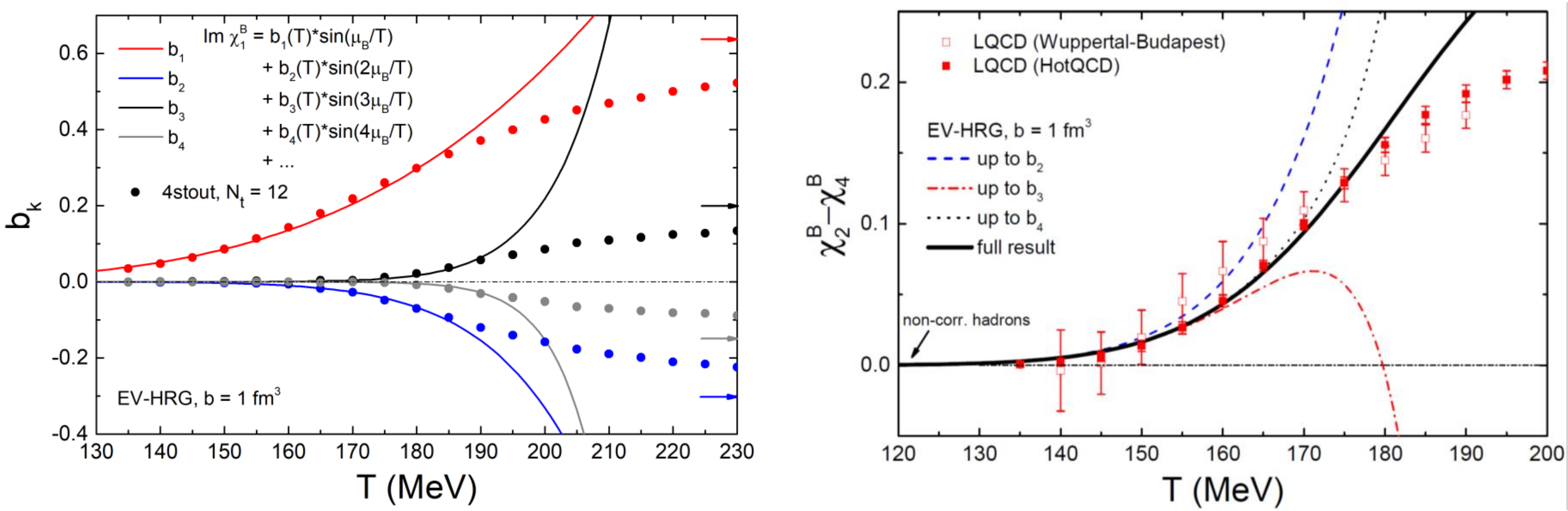
[R. Hagedorn, J. Rafelski, PLB ‘80]

Excluded volume model: $V \to V - bN \quad \Rightarrow$ repulsive interactions

[D. Rischke et al., Z. Phys. C ‘91]

Whether EV corrections are needed at all has been debated…

Recent lattice data favor EV-like effects in baryonic interactions



V.V., A. Pasztor, Z. Fodor, S.D. Katz, H. Stoecker, 1708.02852

but not much info regarding (non-)existence of EV effects for mesons

“One size fits them all” scenario

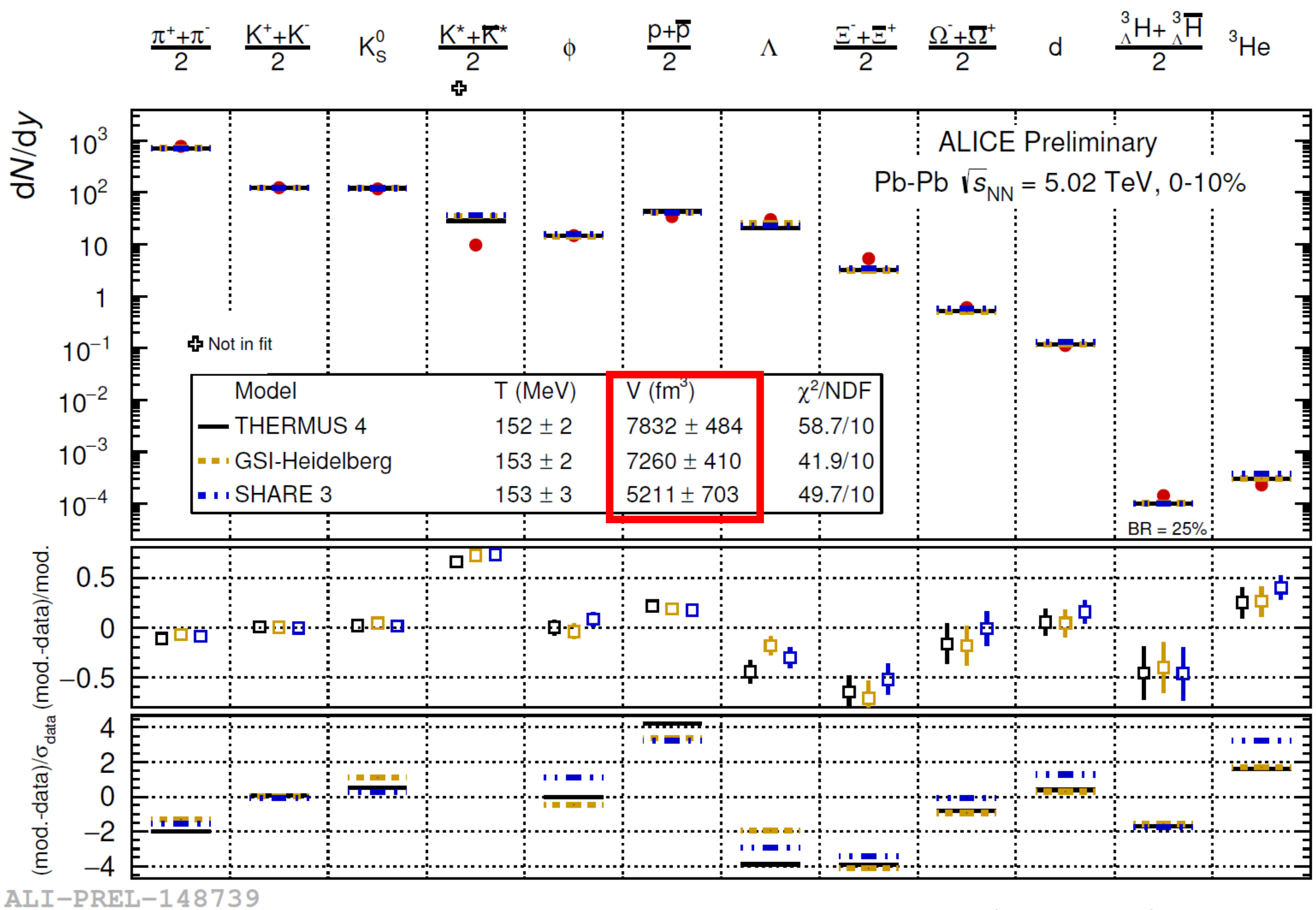
EV model: $N_i \propto \exp\left(-v_i \frac{p}{T}\right)$ $\leftarrow$ larger hadrons suppressed

EV effects cancel out in hadron yield ratios if $v_i \equiv v$, volume renormalized

"One size fits them all" scenario

EV model: $N_i \propto \exp\left(-v_i \frac{p}{T}\right)$ ← larger hadrons suppressed

EV effects cancel out in hadron yield ratios if $v_i \equiv v$, volume renormalized



Model	T (MeV)	V (fm^3)	χ^2/NDF
THERMUS 4	152 ± 2	7832 ± 484	58.7/10
GSI-Heidelberg	153 ± 2	7260 ± 410	41.9/10
SHARE 3	153 ± 3	5211 ± 703	49.7/10

GSI-HD, THERMUS:

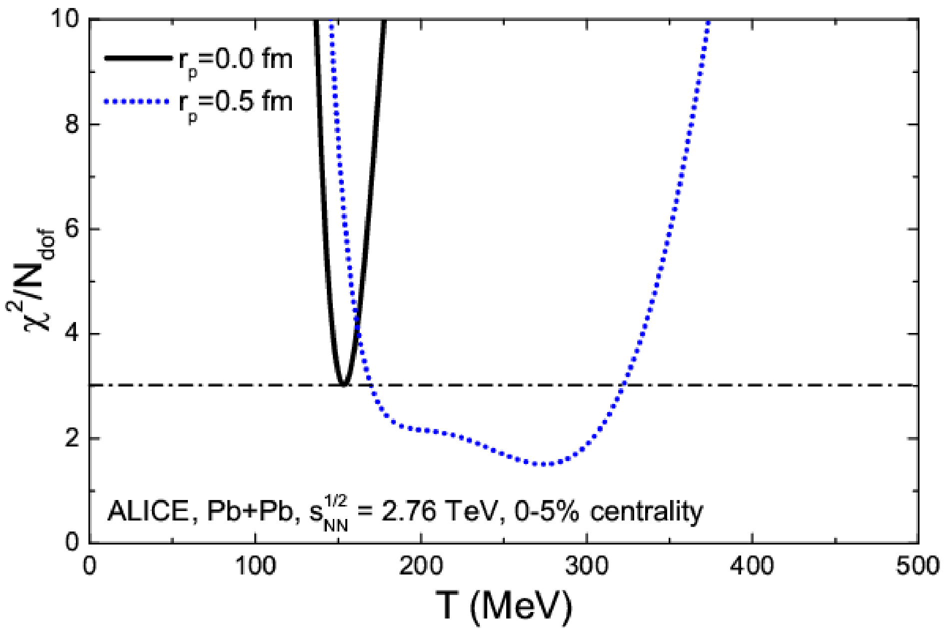
$r = 0.3$ fm for *all* mesons, baryons, and *light nuclei*

SHARE:

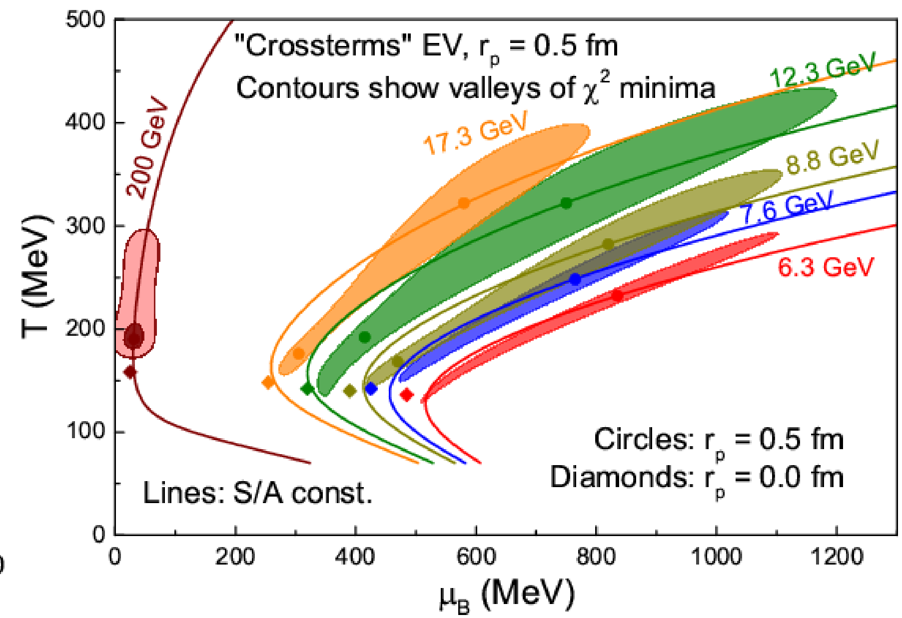
no EV effects

Another extreme: bag model scaling

Bag model: $v_i \propto m_i$ [Chodos et al., PRD ’74; Kapusta et al., NPA ’83, PRC ’15]



[**V.V.**, H. Stoecker, 1512.08046]



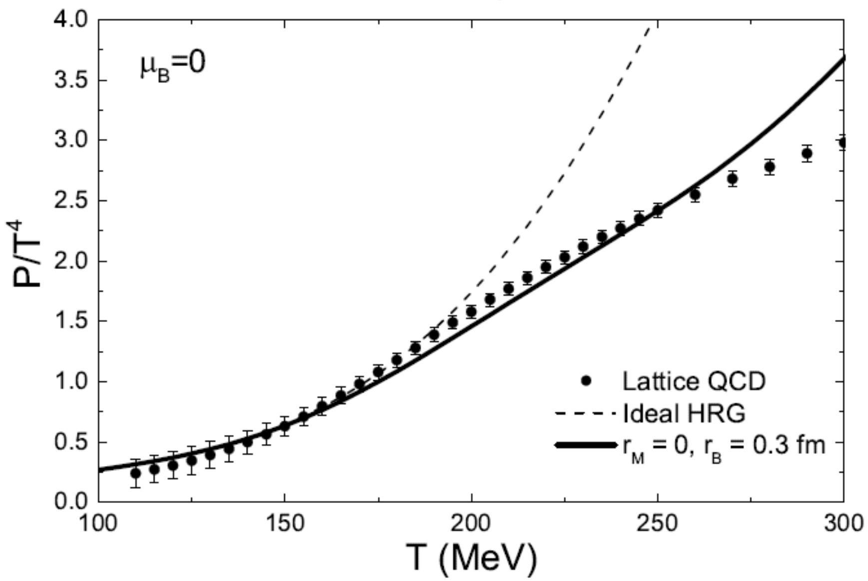
[**V.V.**, H. Stoecker, 1606.06218]

Extraction of T and μ can be quite sensitive w.r.t EV corrections,
but entropy per baryon, S/A, is a robust observable

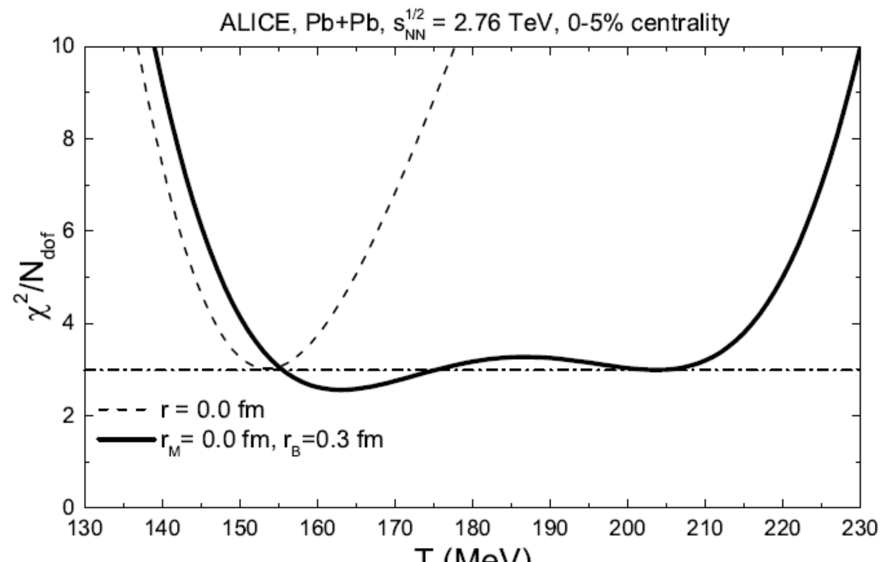
More moderate: two-component model

Two-component model: $r_M = 0$ fm, $r_B = 0.3$ fm [Andronic et al., 1201.0693]

Pressure p/T^4



ALICE data fit (just hadrons)



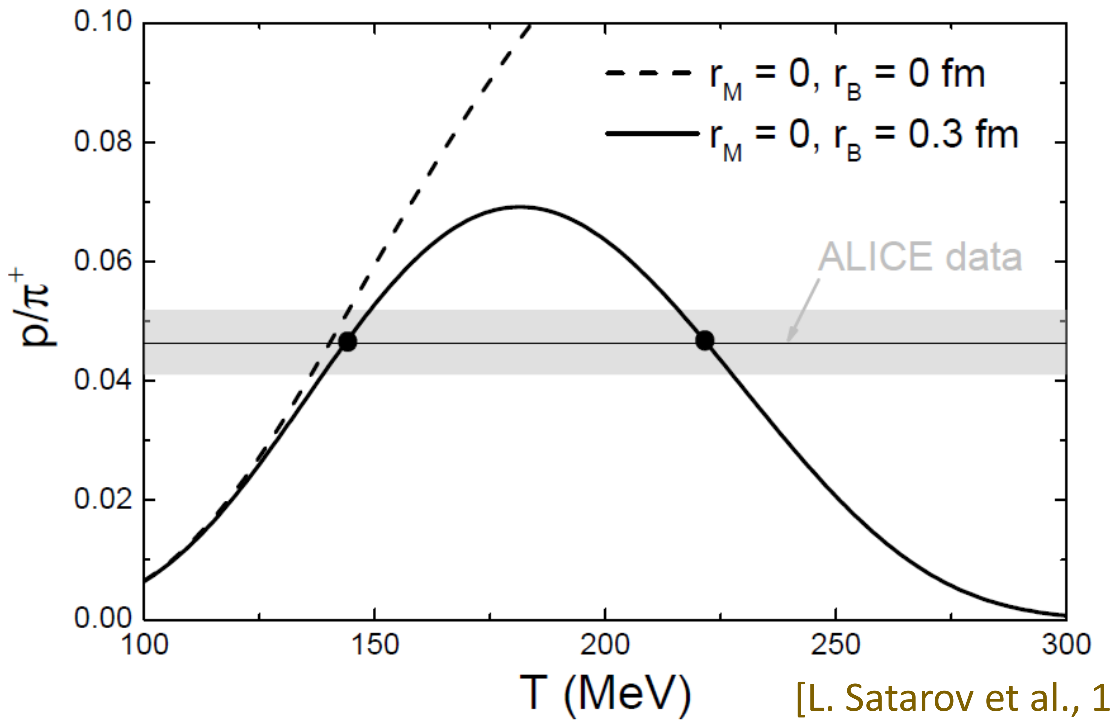
[**V.V.**, H. Stoecker, 1512.08046]

Origin of the two minima

Where does the 2nd minimum come from?

Consider p/π ratio in the EV model

$$\frac{n_p^{ev}}{n_\pi^{ev}} = \frac{n_p^{id}}{n_\pi^{id}} \, e^{(v_\pi - v_p)P/T}$$



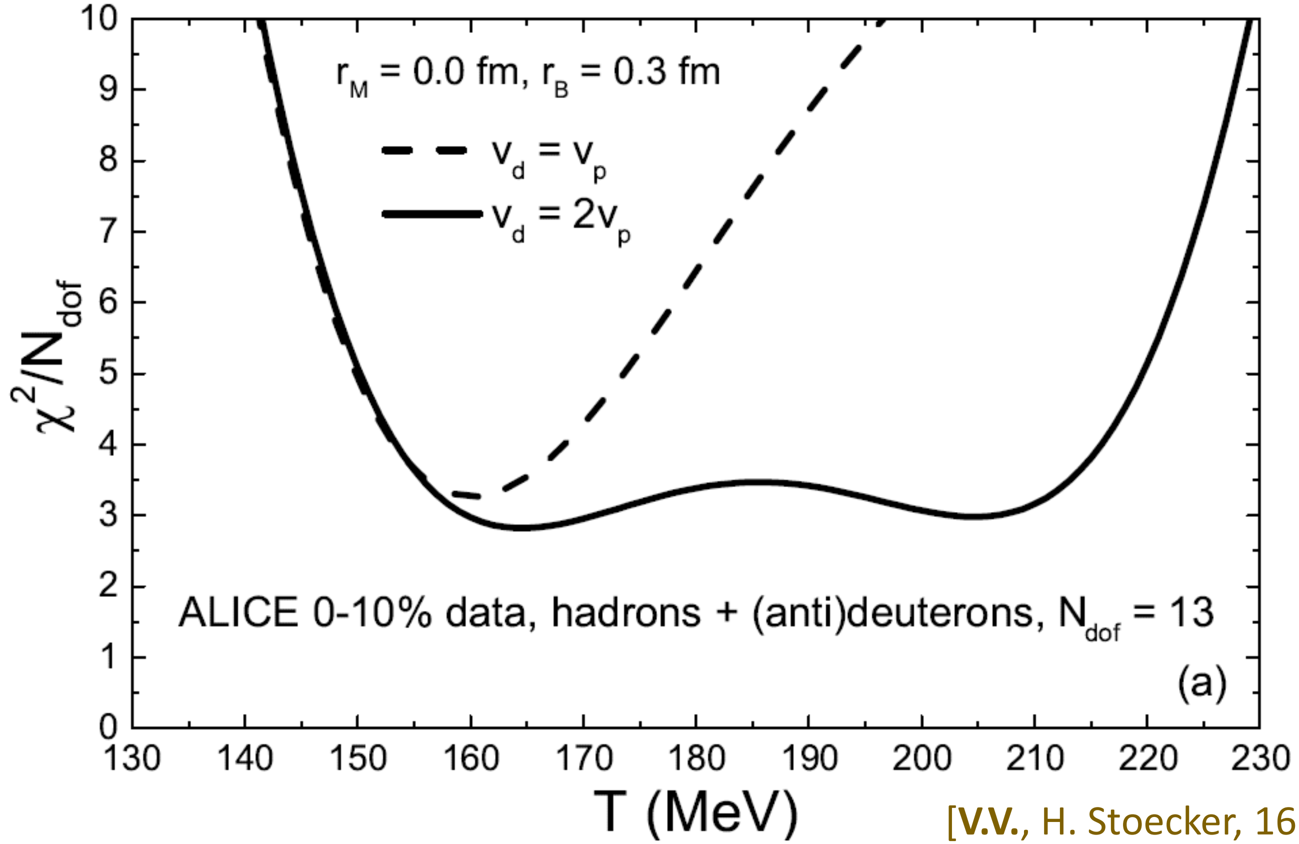
[L. Satarov et al., 1610.08753]

Non-monotonic behavior when $v_\pi < v_p$ which yields two solutions

Light nuclei and EV corrections

Could light nuclei stabilize the fit? Let us add deuteron into the fit

Two options: $v_d = v_p$ and $v_d = 2v_p$



[**V.V.**, H. Stoecker, 1610.02346]

The 2nd minimum strikes again

Small systems

thermal model applied also for small systems, even for elementary reactions like $e^+e^-, pp, p\bar{p}$

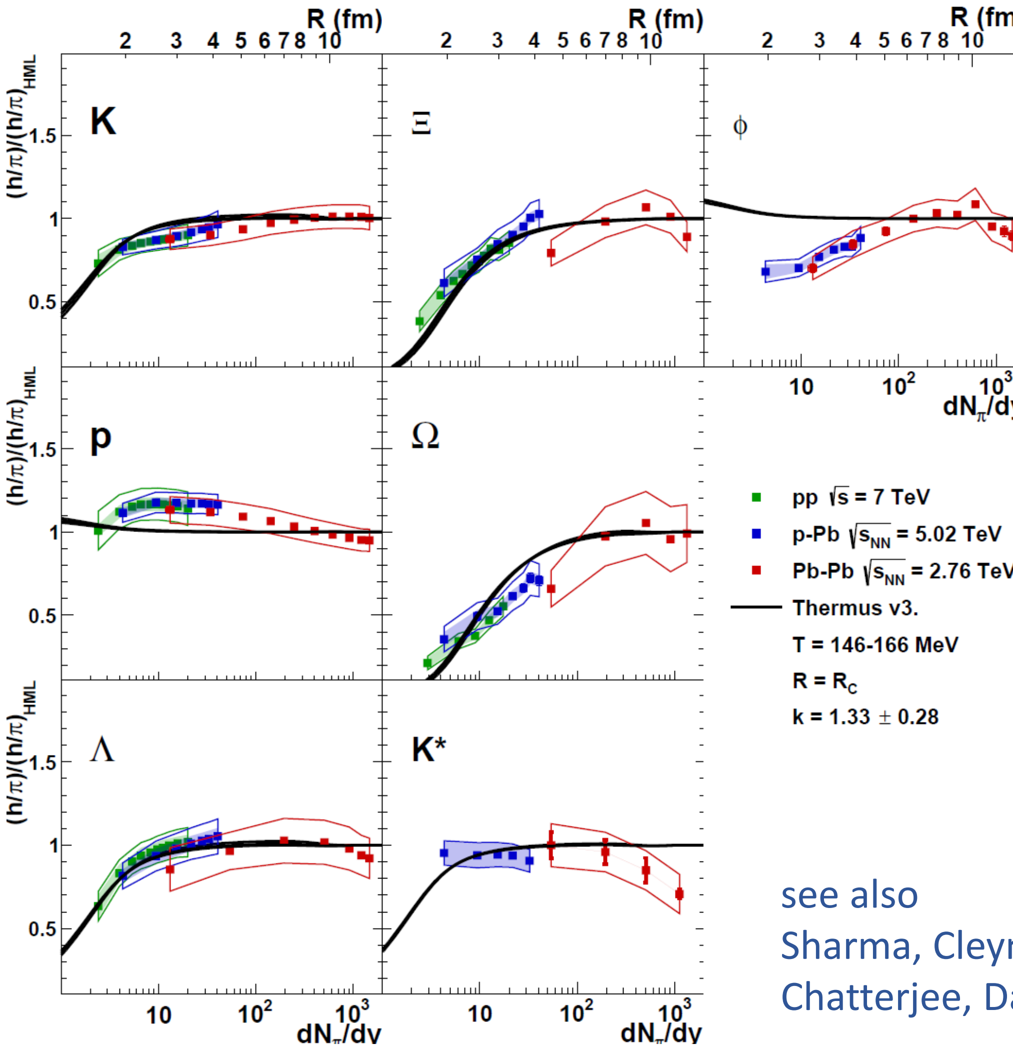
[Becattini et al., ZPC '95, ZPC '97]

canonical treatment of (some) conserved charges needed when the reaction volume is small, suppresses yields

[Rafelski, Danos, et al., PLB '80]

Small systems at LHC

Multiplicity dependence within strangeness-canonical ensemble



[Vislavicius, Kalweit, 1610.03001]

- general trend for most hadrons captured by SCE
- notable exception: ϕ
- problems with ϕ in small systems were pointed out before

[Becattini et al., hep-ph/0511092]

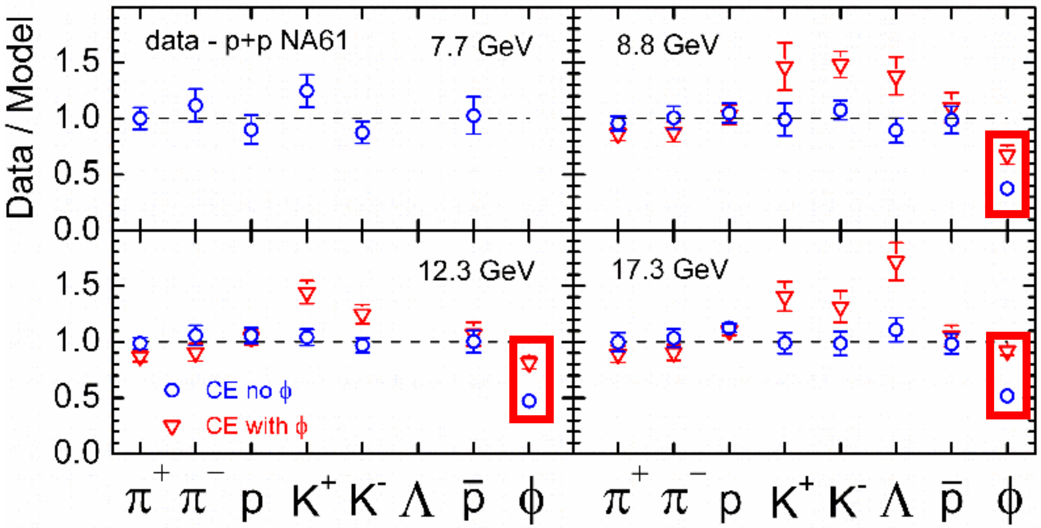
see also
Sharma, Cleymans, Hippolyte, 1803.05409
Chatterjee, Dash, Mohanty, 1608.00643

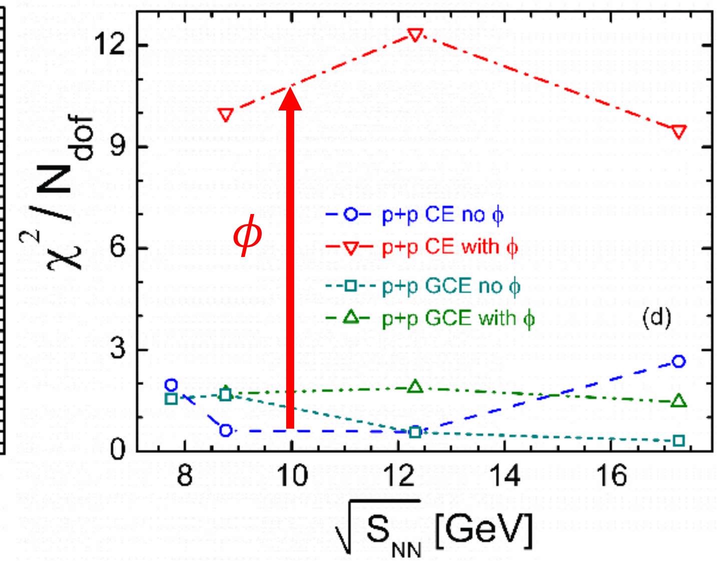
Small systems at SPS

NA61/SHINE: yields in inelastic p+p collisions at $\sqrt{s_{NN}} = 6.6 - 17.3$ GeV

[NA61/SHINE collaboration, 1310.2417, 1705.02467, 1711.09633]

collaboration reports 4π yields => natural to apply canonical ensemble





[Begun, **V.V.**, Gorenstein, Stoecker, 1805.01901]

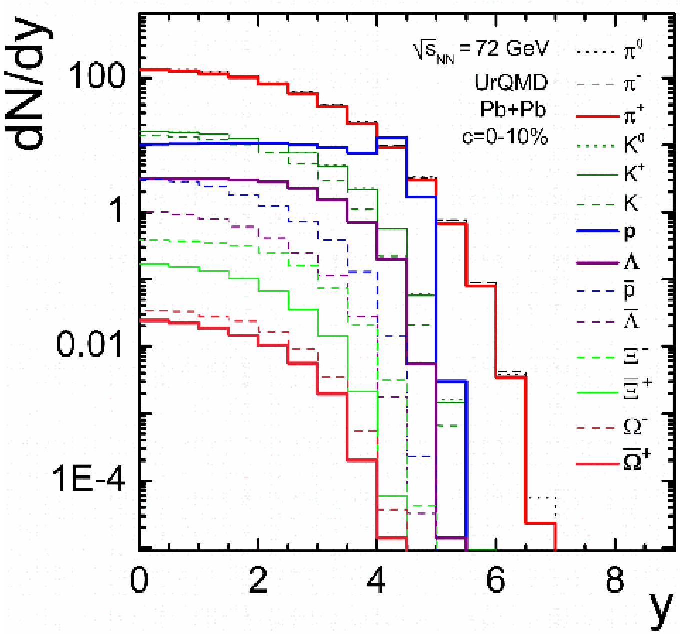
- CE fails when ϕ included
- GCE much better than CE with ϕ, for 4π yields!!
- Non-statistical fluctuations? Centrality selection may help…

Rapidity scan

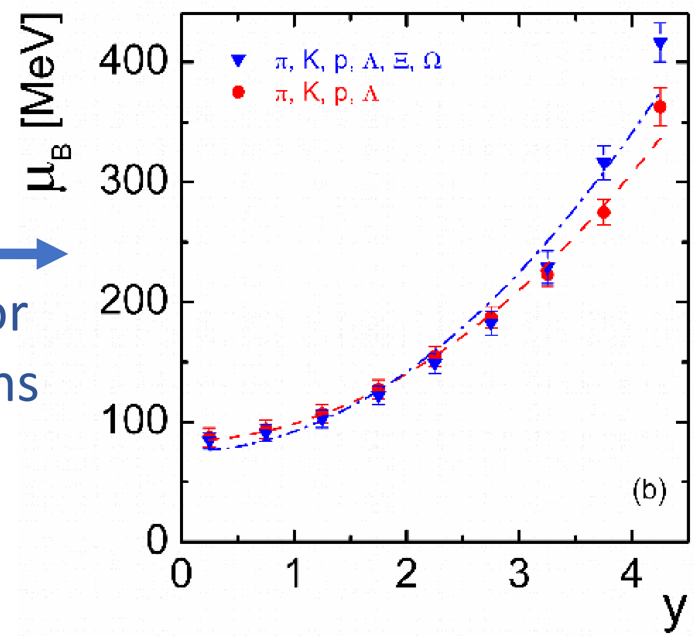
Fireballs at midrapidity: $\mu_B(y_s) \approx \mu_B(0) + b\, y_s^2$

RHIC @ $\sqrt{s_{NN}} = 200$ GeV: $\mu_B(y_s) \approx 25 + 11 y_s^2$ [MeV]   [Becattini et al., 0709.2599]

Example: AFTER@LHC project: Pb+Pb collisions @ $\sqrt{s_{NN}} = 72$ GeV



Thermal fits for different dy bins



[Begun, Kikola, **V.V.**, Wielanek, 1806.01303]

Rapidity scan: complementary approach to scan QCD phase diagram

see also Li, Kapusta, 1604.08525; Brewer, Mukherjee, Rajagopal, Yin, 1804.10215

Summary

- Thermal model is a simple model for particle production, but has surprisingly many important details
- Different thermal model codes yield overall consistent results, when the same physical input used.
- New **Thermal-FIST** package provides most of the features used in thermal model analysis in a convenient way.
- Understanding effects of wide resonances and excluded volume interactions is important for precision studies
- Rapidity scan of hadron chemistry provides complementary approach to scan QCD phase diagram

Summary

- Thermal model is a simple model for particle production, but has surprisingly many important details
- Different thermal model codes yield overall consistent results, when the same physical input used.
- New **Thermal-FIST** package provides most of the features used in thermal model analysis in a convenient way.
- Understanding effects of wide resonances and excluded volume interactions is important for precision studies
- Rapidity scan of hadron chemistry provides complementary approach to scan QCD phase diagram

Thanks for your attention!

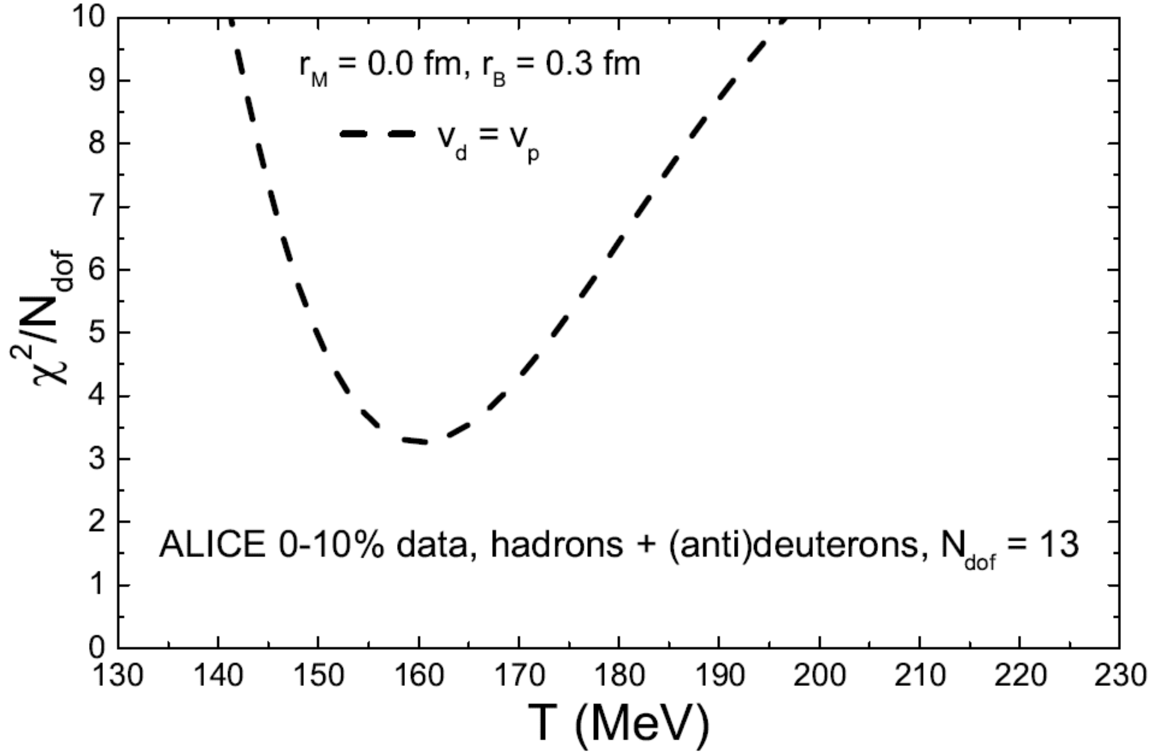
Backup slides

Light nuclei and EV corrections

Could light nuclei stabilize the fit?

Let us now add deuteron into the fit

First assume for simplicity $v_d = v_p$, i.e. $r_p = r_d = 0.3$ fm

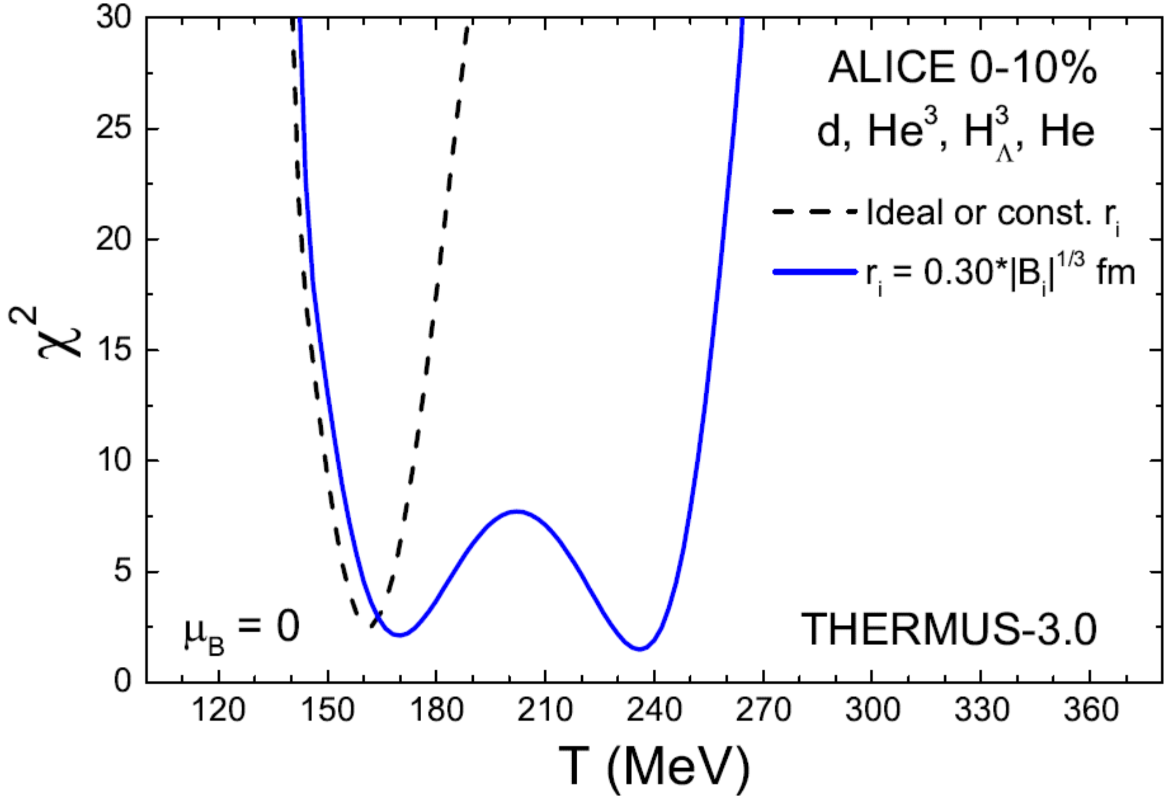


Thermal fits are stabilized?!

Fitting light nuclei only

One could forget about the hadrons and fit just the light nuclei

Advantage: No dependence on high-mass resonance spectrum and feeddown



Ideal HRG (or $v_i = \text{const.}$): $T_f = 160 \pm 5$ MeV

EV-HRG with $v_i = v|A_i|$: $T_f = 160 - 250$ MeV

Disadvantage: Fits are even more sensitive to EV corrections