Thermal model fits: an overview

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The conventional picture

Thermal fits map heavy-ion collisions to the QCD phase diagram

\[ N_i^{\text{mod}} = N_i^{\text{hrg}} + \sum_j BR(j \rightarrow i) N_j^{\text{hrg}}, \quad N_i^{\text{hrg}} = V \frac{d_i m_i^2 T}{2 \pi^2} K_2 \left( \frac{m_i}{T} \right) e^{\frac{\mu_i}{T}} \]

Fits minimize

\[ \chi^2 = \sum_i \frac{(N_i^{\text{mod}} - N_i^{\text{exp}})^2}{(\sigma_i^{\text{exp}})^2} \]

Conventional picture based on chemical equilibrium (ideal) HRG model fits

A. Andronic et al., 1710.09425
Many aspects of the thermal fits

Alternative/extended scenarios:
• chemical non-equilibrium ($\gamma_q, \gamma_s$)
• hadronic phase influence
• flavor hierarchy at freeze-out
• light nuclei

Systematic uncertainties in the HRG model:
• hadron spectrum and decay channels
• treatment of finite resonance widths
• excluded volume/van der Waals interactions

Description of small systems:
• exact conservation of conserved charges (canonical ensemble)
Commonly used tools for thermal fits

1) **SHARE 3** [G. Torrieri, J. Rafelski, M. Petran, et al.]
   
   *Fortran/C++.* Chemical (non-)equilibrium, fluctuations, charm, nuclei
   
   **open source:** http://www.physics.arizona.edu/~gtshare/SHARE/share.html

2) **THERMUS 4** [S. Wheaton, J. Cleymans, B. Hippolyte, et al.]
   
   *C++/ROOT.* Canonical ensemble, EV corrections, charm, nuclei
   
   **open source:** https://github.com/thermus-project/THERMUS

3) **GSI-Heidelberg code** [A. Andronic et al.] **not open source**

4) **Florence code** [F. Becattini et al.] **not open source**
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**New development:**

**Thermal-FIST v0.5** (or simply “The FIST”)  [V.V., H. Stoecker]
*C++. Chemical (non-)equilibrium, EV/vdW corrections, Monte Carlo, (higher-order) fluctuations, canonical ensemble, combinations of effects*
**open source:** [https://github.com/vlvovch/Thermal-FIST](https://github.com/vlvovch/Thermal-FIST)
Thermal-FIST

Graphical user interface for *general-purpose* thermal fits and more
Standard picture for Pb+Pb @ 2.76 TeV

Similar results with *Thermal-FIST* and *Florence codes* [Becattini et al., 1605.09694]

Consistent picture between codes for chem. equilibrium ideal HRG
Alternative/extended scenarios
Chemical non-equilibrium model

In chemical non-equilibrium scenario $N_i^{hrg} \propto (\gamma_q)|q_i|(\gamma_s)|s_i|$

E.g. hadronization of chem. non-eq. supercooled QGP [Letessier, Rafelski, ‘99]

**Figure 1:**
- Lattice QCD $T_c$
- RHIC
- SPS
- AGS
- SHARE-nonequilibrium
- GSI
- Florence
- THERMUS

**Table 1:**

<table>
<thead>
<tr>
<th>$\gamma_q$</th>
<th>$\gamma_s$</th>
<th>$T$ [MeV]</th>
<th>$V$ [fm$^3$]</th>
<th>$\chi^2/ndf$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.63</td>
<td>2.08</td>
<td>138</td>
<td>2460</td>
<td>9.5/9</td>
</tr>
<tr>
<td>1.00</td>
<td>1.14</td>
<td>155</td>
<td>3460</td>
<td>35/10</td>
</tr>
<tr>
<td>1.00</td>
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<td>157</td>
<td>3480</td>
<td>64/11</td>
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</tbody>
</table>

- smaller reduced $\chi^2$ compared to chem. equilibrium scenario
- describes $p_T$-spectra of many hadrons [V. Begun et al., 1312.1487, 1405.7252]
- $\gamma_q = 1.63 \Rightarrow \mu_\pi \approx 135$ MeV $\approx m_\pi \Rightarrow$ pion BEC? [V. Begun et al., 1503.04040]
- However, $\gamma_q \approx \gamma_s \approx 1$ when light nuclei included in fit [M. Floris, 1408.6403]
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Influence of the hadronic phase

Modification of hadron yields in non-equilibrium hadronic phase

$B\bar{B}$ annihilation reduces (anti)proton yields  

[Steinheimer et al., 1203.5302]

• somewhat better $\chi^2$ and increase in $T_{ch}$ by 10-15 MeV
• no backreaction, e.g. $5M \rightarrow B\bar{B}$, in UrQMD. What is its role?

[Becattini et al., 1212.2431, 1605.09694]
Flavor hierarchy at freeze-out

QCD transition is a broad crossover

=> different \( T_c \) for different observables

strange vs light number susceptibility

\[ \frac{\chi_4}{\chi_2} \]

[R. Bellwied et al., 1305.6297]

- higher \( T_f \) for strange particles than for non-strange
- effect may disappear if more strange baryons included

[Bazavov et al., 1404.6511, S. Chatterjee, 1708.08152]
Flavor hierarchy in hadron sizes

Alternative: Flavor hierarchy in hadron sizes

\[ \nu_i \propto m_i \text{ for non-strange, } \nu_i \propto m_i^{-1} \text{ for strange, excluded-volume HRG} \]

ALICE 0-5%:
\[ \chi^2 / N_{\text{dof}} = 0.88 / 7 \]
**Flavor hierarchy in hadron sizes**

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<tr>
<th>ALICE 5-10%</th>
<th>1.022/7 ( \simeq 0.14 )</th>
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<tr>
<td>ALICE 10-20%</td>
<td>2.7/9 ( \simeq 0.30 )</td>
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<tr>
<td>ALICE 20-30%</td>
<td>6.08/8 ( \simeq 0.76 )</td>
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<td>ALICE 30-40%</td>
<td>6.9/8 ( \simeq 0.86 )</td>
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<tr>
<td>ALICE 40-50%</td>
<td>3.07/8 ( \simeq 0.38 )</td>
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<td>ALICE 50-60%</td>
<td>4.42/8 ( \simeq 0.55 )</td>
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<td>ALICE 60-70%</td>
<td>8.09/8 ( \simeq 1.01 )</td>
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\[ T \text{ (MeV)} \]

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ALICE 0-5%:
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- Significant improvement in fit quality across \( \sqrt{s} \) and centralities
- Reflects systematics in data, exact physical reasons to be clarified
Hierarchy in baryon number?

Considering the ALICE 2.76 TeV Pb+Pb 0-10% data in ideal HRG model...

1) Fit of mesons + baryons + nuclei: $T_{ch} = 155 \pm 2$ MeV, $\chi^2/N_{dof} = 41.9/20$

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3) Fit of mesons ($\pi^\pm, K^\pm, K_0^S, \phi$): $T_{ch} = 141 \pm 9$ MeV, $\chi^2/N_{dof} = 3.7/4$
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5) Fit of nuclei (\( d, ^3\text{He}, ^3\Lambda\text{H}, ^4\text{He} \)): \( T_{ch} = 161 \pm 4 \text{ MeV}, \chi^2/N_{dof} = 2.4/6 \)

Rather different fit temperatures in different baryon number sectors...

More tension in the baryonic sector
Systematic uncertainties in the HRG model

Input hadron list and decay channels
• High-mass resonances and their decay channels poorly known
• Evidence for missing strange baryons for lattice QCD
  [A. Bazavov et al., 1404.6511; P. Alba et al., 1702.0113; S. Chatterjee, 1708.08152]

Modeling finite resonance widths
• Zero-width approx., energy (in)dependent Breit-Wigner, phase shifts

Excluded volume/van der Waals interaction effects
• Thermal fits affected when EV parameters differ between hadrons
  [V.V., H. Stoecker, 1512.08046, 1606.06218]

In-medium hadron masses
• In-medium masses due to interactions/chiral symmetry restoration
  [D. Zschiesche et al., nucl-th/0209022; G. Aarts et al., 1703.09246]
• Needs reconciliation with vacuum masses actually measured
Modeling finite resonance widths

\[ n_i(T, \mu; m_i) \to \int_{m_i^{\text{min}}}^{m_i^{\text{max}}} dm \rho_i(m) n_i(T, \mu; m) \]

1) Zero-width approximation \( \rho_i(m) = \delta(m - m_i) \)
   Simplest possibility, used commonly in LQCD comparisons

2) Breit-Wigner (BW) in \( \pm 2\Gamma_i \) interval \( \rho_i(m) = A_i \frac{2m m_i \Gamma_i}{(m^2 - m_i^2)^2 + m_i^2 \Gamma_i^2} \)
   Popular choice in thermal fits (THERMUS, Florence code, Thermal-FIST)
   Could be overestimating density near threshold

3) Energy-dependent Breit-Wigner (eBW) \( \Gamma_i(m) = \sum_j \Gamma_{i \to j}(m) \)
   \[ \Gamma_{i \to j}(m) = b_{i \to j} \Gamma_i \left[ 1 - \left( \frac{m_{\text{thr}}^{i \to j}}{m} \right)^2 \right]^{l_j + 1/2} \]
   suppression at threshold (as in SHARE)

+ \( m \)-dependent decay feeddown
   \[ N^{\text{tot}}_i = N^{\text{hrg}}_i + \sum_{j \in \text{pdg}} \int dm \text{BR}(j \to i; m) \rho_j(m) \frac{dN_{i \to j}^{\text{hrg}}}{dm} \]
Modeling widths: effect on thermal fits

Significant improvement in the eBW scheme due to a reduced proton feeddown from $\Delta$ and $N^*$

Modeling of wide resonances important!!

[VV. et al., in preparation]
Excluded volume corrections

Notion that hadrons have finite eigenvolume suggested a while ago

Excluded volume model: \( V \rightarrow V - bN \) \( \Rightarrow \) repulsive interactions

Whether EV corrections are needed at all has been debated...

Recent lattice data favor EV-like effects in baryonic interactions

V.V., A. Pasztor, Z. Fodor, S.D. Katz, H. Stoecker, 1708.02852

but not much info regarding (non-)existence of EV effects for mesons
“One size fits them all” scenario

EV model: \( N_i \propto \exp\left(-v_i \frac{p}{T}\right) \) ← larger hadrons suppressed

EV effects cancel out in hadron yield ratios if \( v_i \equiv v \), volume renormalized
"One size fits them all" scenario

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EV effects cancel out in hadron yield ratios if \( v_i \equiv v \), volume renormalized

GSI-HD, THERMUS:
\( r = 0.3 \text{ fm for all mesons, baryons, and light nuclei} \)

SHARE:
no EV effects
Another extreme: bag model scaling

Bag model: $v_i \propto m_i$  

[Chodos et al., PRD ’74; Kapusta et al., NPA ’83, PRC ’15]

Extraction of $T$ and $\mu$ can be quite sensitive w.r.t EV corrections, but entropy per baryon, $S/A$, is a robust observable
More moderate: two-component model

Two-component model: $r_M = 0$ fm, $r_B = 0.3$ fm  

[Andronic et al., 1201.0693]

Pressure $p/T^4$

$\mu_B=0$

[ALICE, Pb+Pb, $s_{NN}^{1/2}$ = 2.76 TeV, 0-5% centrality

$\chi^2/N_{\text{df}}$

[V.V., H. Stoecker, 1512.08046]
Origin of the two minima

Where does the 2nd minimum come from?
Consider \( \frac{p}{\pi} \) ratio in the EV model

\[
\frac{n_p^{ev}}{n_{\pi}^{ev}} = \frac{n_p^{id}}{n_{\pi}^{id}} e^{(\nu_{\pi} - \nu_p)P/T}
\]

- \( r_M = 0, r_B = 0 \) fm
- \( r_M = 0, r_B = 0.3 \) fm

Non-monotonic behavior when \( \nu_{\pi} < \nu_p \) which yields two solutions

[L. Satarov et al., 1610.08753]
Light nuclei and EV corrections

Could light nuclei stabilize the fit? Let us add deuteron into the fit

Two options: $v_d = v_p$ and $v_d = 2v_p$

The 2nd minimum strikes again

ALICE 0-10% data, hadrons + (anti)deuterons, $N_{dof} = 13$

[V.V., H. Stoecker, 1610.02346]
Small systems

thermal model applied also for small systems, even for elementary reactions like $e^+ e^-, pp, p\bar{p}$

[Becattini et al., ZPC ‘95, ZPC ‘97]

canonical treatment of (some) conserved charges needed when the reaction volume is small, suppresses yields

[Rafelski, Danos, et al., PLB ‘80]
Small systems at LHC

Multiplicity dependence within strangeness-canonical ensemble

- general trend for most hadrons captured by SCE
- notable exception: $\phi$
- problems with $\phi$ in small systems were pointed out before

[Becattini et al., hep-ph/0511092]

see also
Sharma, Cleymans, Hippolyte, 1803.05409
Chatterjee, Dash, Mohanty, 1608.00643

[Vislavicius, Kalweit, 1610.03001]
Small systems at SPS

NA61/SHINE: yields in inelastic p+p collisions at $\sqrt{S_{NN}} = 6.6 - 17.3$ GeV

[NA61/SHINE collaboration, 1310.2417, 1705.02467, 1711.09633]

collaboration reports $4\pi$ yields $\Rightarrow$ natural to apply canonical ensemble

![Graph showing data points and model predictions for inelastic p+p collisions at different energies.]

[Begun, V.V., Gorenstein, Stoecker, 1805.01901]

- CE fails when $\phi$ included
- GCE much better than CE with $\phi$, for $4\pi$ yields!!
- Non-statistical fluctuations? Centrality selection may help…
Rapidity scan

Fireballs at midrapidity: \( \mu_B(y_s) \approx \mu_B(0) + b \, y_s^2 \)

RHIC @ \( \sqrt{s_{NN}} = 200 \text{ GeV} \): \( \mu_B(y_s) \approx 25 + 11y_s^2 \) [MeV]  [Becattini et al., 0709.2599]

Example: AFTER@LHC project: Pb+Pb collisions @ \( \sqrt{s_{NN}} = 72 \text{ GeV} \)

Thermal fits for different dy bins

[ Begun, Kikola, V.V., Wielanek, 1806.01303 ]

Rapidity scan: complementary approach to scan QCD phase diagram

see also Li, Kapusta, 1604.08525; Brewer, Mukherjee, Rajagopal, Yin, 1804.10215
Summary

• Thermal model is a simple model for particle production, but has surprisingly many important details

• Different thermal model codes yield overall consistent results, when the same physical input used.

• New **Thermal-FIST** package provides most of the features used in thermal model analysis in a convenient way.

• Understanding effects of wide resonances and excluded volume interactions is important for precision studies

• Rapidity scan of hadron chemistry provides complementary approach to scan QCD phase diagram
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Thanks for your attention!
Backup slides
Light nuclei and EV corrections

Could light nuclei stabilize the fit?

Let us now add deuteron into the fit

First assume for simplicity $v_d = v_p$, i.e. $r_p = r_d = 0.3$ fm

$\chi^2/N_{\text{dof}}$

ALICE 0-10\% data, hadrons + (anti)deuterons, $N_{\text{dof}} = 13$

$r_M = 0.0$ fm, $r_B = 0.3$ fm

$\chi_d = \chi_p$

T (MeV)

Thermal fits are stabilized?!
Fitting light nuclei only

One could forget about the hadrons and fit just the light nuclei.

**Advantage:** No dependence on high-mass resonance spectrum and feeddown.

---

Ideal HRG (or $v_i = \text{const.}$): $T_f = 160 \pm 5$ MeV

EV-HRG with $v_i = v |A_i|$: $T_f = 160 - 250$ MeV

**Disadvantage:** Fits are even more sensitive to EV corrections.