Time dependence and fluctuations of partition into spectators and participants in heavy-ion collisions

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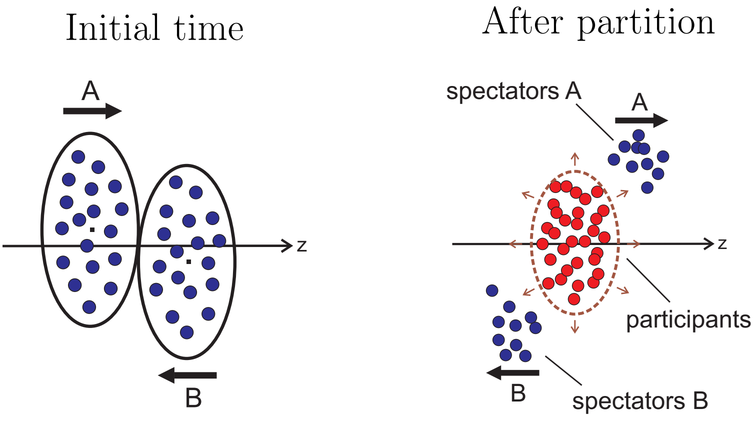
VI Young Scientists Conference
"Problems of Theoretical Physics"

Kyiv, Ukraine
25 November 2014

Outline

- Introduction
- Model for time-dependent description of spectator-participant partition process
- Longitudinal fluctuations of center of mass of participants
- Summary

Partition into spectators and participants



- Partition happens at the earliest stage of heavy-ion collision
- Some models (e.g. hydro) describe participants only after partition stage
- A lot of questions regarding partition stage: time scales, centrality dependence, event-by-event asymmetry due to fluctuations

Kinetic description

How to study time dependence?

Use kinetic description and partition distribution function into nucleons from projectile (A) and target (B):

$$f(t, \mathbf{r}, \mathbf{p}) = f_A(t, \mathbf{r}, \mathbf{p}) + f_B(t, \mathbf{r}, \mathbf{p}).$$

Considering just nucleon-nucleon reactions in initial stage describe $f_{A(B)}$ with relativistic Boltzmann equation:

$$p^\mu \partial_\mu f_{A(B)}(t, \mathbf{r}, \mathbf{p}) = C[f](t, \mathbf{r}, \mathbf{p}).$$

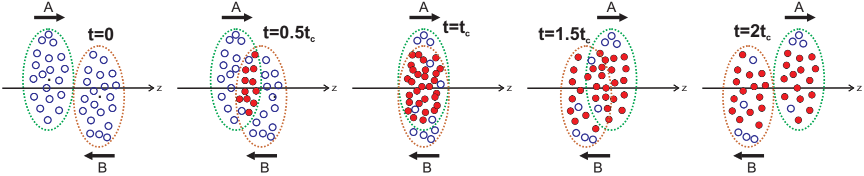
Initial conditions

- Lorentz-boosted Woods-Saxon
- Momentum distribution with neglected Fermi motion

$$f_{A(B)}(t_0, \mathbf{r}, \mathbf{p}) = \rho^{WS}_{A(B)}(\mathbf{r})\, \delta^2(p_\perp)\, \delta(p_z - p_{A(B)}).$$

Ballistic mode

Simplest approximation: no transverse motion and no stopping



Described by collisionless Boltzmann equation:

$$p^\mu \partial_\mu f^{(0)}_{A(B)}(t, \mathbf{r}, \mathbf{p}) = 0.$$

Solution

$$f^{(0)}_{A(B)}(t, \mathbf{r}, \mathbf{p}) = \rho^{(0)}_{A(B)}(t, \mathbf{r})\, \delta^2(p_\perp)\, \delta(p_z - p_{A(B)}),$$

$$\rho^{(0)}_{A(B)}(t, \mathbf{r}) = \gamma_0\, \rho_{WS}(x \mp b/2, y, \gamma_0[z - v_{A(B)}(t - t_c)]),$$

$$\gamma_0 = (1 - v_0^2)^{-1/2},\ t_c = R/(\gamma_0 v_0),\ b \text{ -- impact parameter.}$$

Partition into spectators and participants

But how to see the spectator-participant separation process?

Time-dependent partition of distribution function into two subsystems:
"Spectators" – nucleons which did not interact before t
Participants – nucleons which have interacted before t

"Spectators" distribution function

- assume only head-on collisions with incoming nucleons
- local molecular chaos $\rightarrow$ Boltzmann collision integral
- only "loss" collision term
- momentum distribution does not change

$$f^S_{A(B)}(t, \mathbf{r}, \mathbf{p}) = \rho^S_{A(B)}(t, \mathbf{r})\, \delta^2(p_\perp)\, \delta(p_z - p_{A(B)}),$$

$$p_0^\mu \partial_\mu \rho^S_A(t, \boldsymbol{r}) = -2\sigma_{NN} p_0 \rho^S_A(t, \boldsymbol{r})\, \rho^{(0)}_B(t, \boldsymbol{r}),$$

$$\rho^S_{A(B)}(t_0, \boldsymbol{r}) = \rho^{(0)}_{A(B)}(t_0, \boldsymbol{r})$$

At $t \rightarrow \infty$ the $f^S_{A(B)}(t, \mathbf{r}, \mathbf{p})$ describes spectators of collision

Partition into spectators and participants

Solution for "spectators"

$$f^S_{A(B)}(t,\mathbf{r},\mathbf{p}) = \rho^S_{A(B)}(t,\mathbf{r})\,\delta^2(p_\perp)\,\delta(p_z - p_{A(B)}),$$

$$\rho^S_A(t,\boldsymbol{r}) = \rho^{(0)}_A(t,\boldsymbol{r})\,\exp\left\{-2\sigma_{NN}v_0\int_{t_0}^{t} dt'\rho^{(0)}_B[t',\boldsymbol{r}-\boldsymbol{v}_A(t-t')]\right\},$$

At $t \to \infty$ the $f^S_{A(B)}(t,\mathbf{r},\mathbf{p})$ describes spectators of collision

Transverse distribution of spectators

$$T^{\rm spec}_{A(B)}(x,y) = \lim_{t\to\infty}\int d\boldsymbol{p}\int dz\, f^S_{A(B)}(t,\boldsymbol{r},\boldsymbol{p})$$

Calculating results in

$$T^{\rm spec}_{A(B)}(x,y) = T_{A(B)}(x-b/2,y)\,\exp\left\{-\sigma_{NN}T_{B(A)}(x+b/2,y)\right\}.$$

Consistent with Glauber-Sitenko approach (optical limit)
Some similarities to other initial state models (CGC, string rope model etc.)

Temporal scaling

Transverse distribution of "spectators" can be calculated at any time

Transverse distribution of "spectators"

$$
\begin{aligned}
T^{s}_{A(B)}(\tilde{t};x,y) &= \int d\tilde{z}\rho_{_{WS}}[x \mp b/2, y, \tilde{z} \mp R_0(\tilde{t}-1)] \times \\
&\exp\left\{-2\sigma_{_{NN}} R_0 \int_{-\infty}^{\tilde{t}} d\tilde{t}'\rho_{_{WS}}[x \pm b/2, y, \tilde{z} \mp R_0(\tilde{t}+1) \pm 2R_0\tilde{t}']\right\},
\end{aligned}
$$

where $\tilde{t} = t/t_c$ and

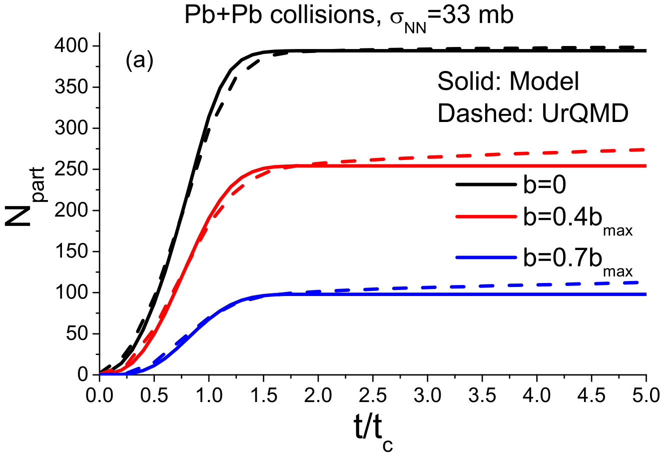
$$
t_c = \frac{R}{\gamma_0 v_0}.
$$

t_c is a natural energy-dependent temporal scale of initial stage.

Time dependence of number of participants

Number of participants (net baryon charge)

$$N_{\text{part}}(t) = 2A - \int dxdy \left[T_A^s(t;x,y) + T_B^s(t;x,y) \right].$$



Angular momentum

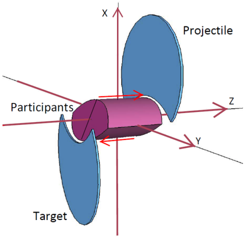
Angular momentum of participants

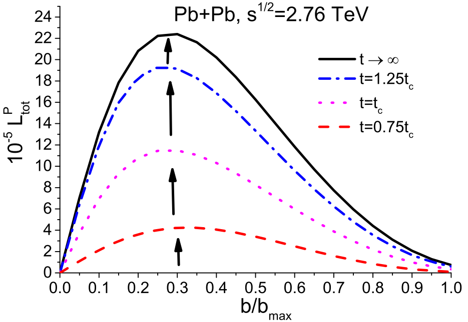
From angular momentum conservation

$$L_{\rm tot} = p^z_{\rm in} \int dxdy\, x \left[T_A(x-b/2,y) - T_B(x+b/2,y)\right],$$

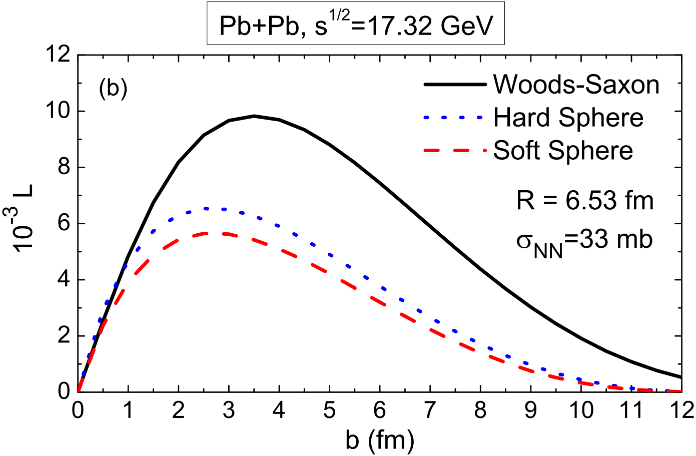
$$L^S_{\rm tot}(t) = p^z_{\rm in} \int dxdy\, x \left[T^S_A(t;x,y) - T^S_B(t;x,y)\right],$$

$$L^P_{\rm tot}(t) = L_{\rm tot} - L^S_{\rm tot}(t).$$



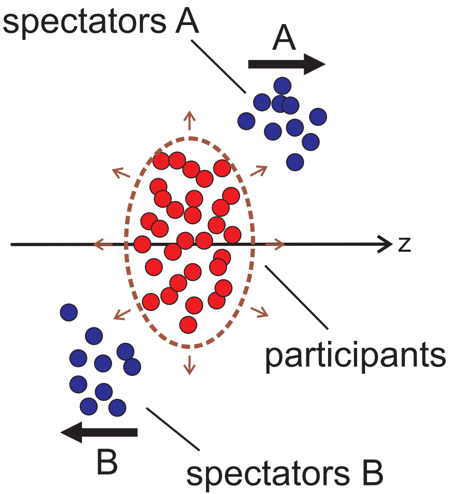


Non-zero angular momentum indicates presence of rotation and shear



Significant quantitative difference but qualitatively same.
Initial hydro state in non-central collisions must have shear

Participant center-of-mass system

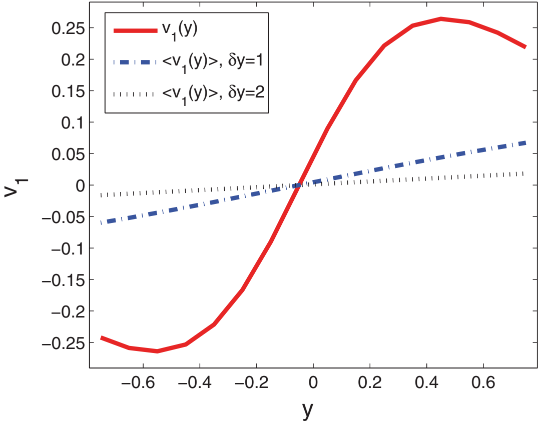


Participant sub-system moves with rapidity $y_{c.m.}$
In general $\langle y_{c.m} \rangle = 0$, but $\langle \delta y_{c.m.}^2 \rangle \neq 0$!
Consequently, participant c.m.s. $\neq$ collider c.m.s. event-by-event

Participant center-of-mass system

$y_{c.m}$ fluctuations may have a substantial influence on fluid dynamical predictions!

$$\text{Assuming } f(y_{c.m}) = \frac{1}{\sqrt{2\pi}\,\delta y}\, e^{\frac{-y_{c.m}^2}{2(\delta y)^2}}$$



Csernai, Magas, Stöcker, Strottman, Phys. Rev. C84, 024914 (2011)

Participant rapidity from spectator numbers

Three subsystems:
target spectators (A) + projectile spectators (B) + participants (P)

Four-momentum conservation (collider c.m.s.)

$$\begin{aligned} E_{\mathrm{tot}} &= E_A + E_B + E_P, \\ P^z_{\mathrm{tot}} &= P^z_A + P^z_B + P^z_P = 0\,. \end{aligned}$$

Participant rapidity from spectator numbers N_A and N_B

$$y_{c.m.} = y_P = \mathrm{arctanh}\left(\frac{P^z_P}{E_P}\right) = \mathrm{arctanh}\left(\frac{N_B - N_A}{2A - N_A - N_B} v_{in}\right)$$

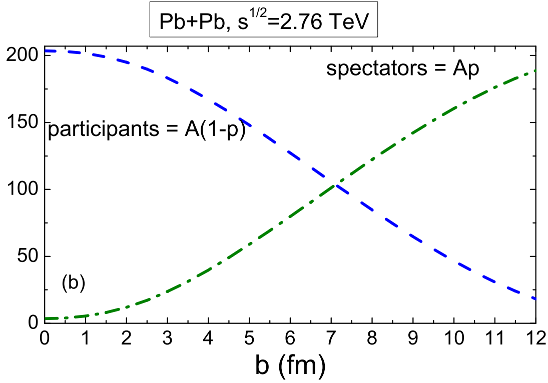
Allows for experimental measurement of $y_{c.m.}$ fluctuations using ZDC.
Csernai, Eyyubova, Magas, Phys. Rev. C86, 024912 (2012)

Spectator probability

Probability of nucleon to be a spectator

$$p = \frac{1}{A} \int dx\, dy\, T_A(x - b/2, y) \exp\left\{-\sigma_{NN} T_{B(A)}(x + b/2, y)\right\}.$$

$T_{A(B)}$ – thickness function,
σ_{NN} – nucleon-nucleon cross section.



Spectator number probability

Probability of $N_{A(B)}$ spectators

For uncorrelated initial nucleons we have binomial distribution

$$
\begin{aligned}
p(N_A) &= \binom{A}{N_A} p^{N_A} (1-p)^{A-N_A}\,, \\
p(N_B) &= \binom{A}{N_B} p^{N_B} (1-p)^{A-N_B}\,.
\end{aligned}
$$

Allow N_A and N_B to take continuous values,
Gaussian approximation for large Ap and $A(1-p)$

$$
p(N_{A(B)}) \Rightarrow \rho(N_{A(B)}) = \frac{\exp\left(-\frac{(N_{A(B)}-Ap)^2}{2Ap(1-p)}\right)}{\sqrt{2\pi Ap(1-p)}}\,.
$$

Rapidity distribution

Approximation: $\rho(N_A, N_B) \approx \rho(N_A)\,\rho(N_B)$.

$$f_P(y) = \int dN_A \int dN_B\, \rho(N_A)\,\rho(N_B)\delta\left[\, y - y_P(N_A, N_B),\, \right]$$

$$y_P(N_A, N_B) = \operatorname{arctanh}\left(\frac{N_B - N_A}{2A - N_A - N_B} v_{in}\right).$$

After integration

Rapidity distribution (general expression)

$$f_P(y) \;=\; \sqrt{\frac{A(1-p)}{\pi p}}\, \frac{v_{\rm in}^2\, \exp\left[-\frac{A(1-p)}{p}\, \frac{\tanh^2 y}{v_{\rm in}^2 + \tanh^2 y}\right]}{\cosh^2 y \left[v_{\rm in}^2 + \tanh^2 y\right]^{\frac{3}{2}}}.$$

Rapidity distribution

Ultrarelativistic limit: $v_{\text{in}} \to 1$

Rapidity distribution in ultra-relativistic limit

$$f_P^{\mathrm{UR}}(y) = \sqrt{\frac{A(1-p)}{\pi p}} \frac{\exp\left[-\frac{A(1-p)}{p} \frac{\tanh^2 y}{1+\tanh^2 y}\right]}{\cosh^2 y \left[1 + \tanh^2 y\right]^{\frac{3}{2}}} .$$

Weak energy dependence, only through σ_{NN}

Rapidity distribution

Ultrarelativistic limit: $v_{\rm in} \to 1$

Rapidity distribution in ultra-relativistic limit

$$f_P^{\rm UR}(y) = \sqrt{\frac{A(1-p)}{\pi p}}\,\frac{\exp\left[-\frac{A(1-p)}{p}\,\frac{\tanh^2 y}{1+\tanh^2 y}\right]}{\cosh^2 y\left[1+\tanh^2 y\right]^{\frac{3}{2}}}\,.$$

Weak energy dependence, only through σ_{NN}

Near mid-rapidity y is small $\Rightarrow$ $\tanh y \approx y$, $\cosh y \approx 1$

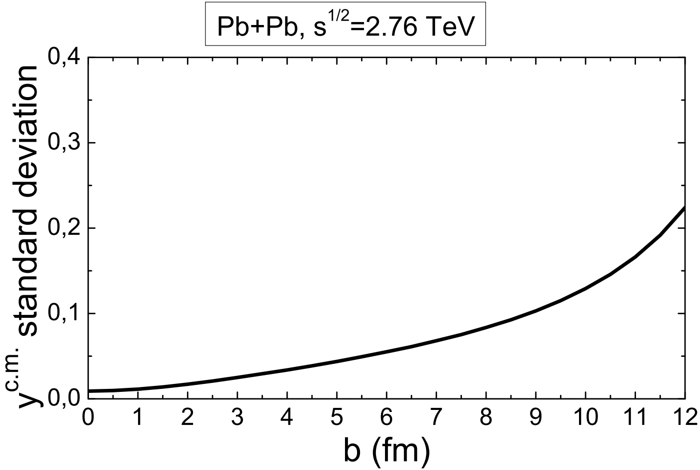
Rapidity distribution at mid-rapidity

$$f_P(y) = \sqrt{\frac{A(1-p)}{\pi p\, v_{\rm in}^2}}\,\exp\left[-\frac{A(1-p)}{p\, v_{\rm in}^2}\,y^2\right]\,.$$

Gaussian distribution with $\delta y^2 = \dfrac{p\, v_{\rm in}^2}{2A(1-p)}$

Distribution at mid-rapidity

$$\delta y^2 = \frac{p\, v_{\rm in}^2}{2A(1-p)}$$

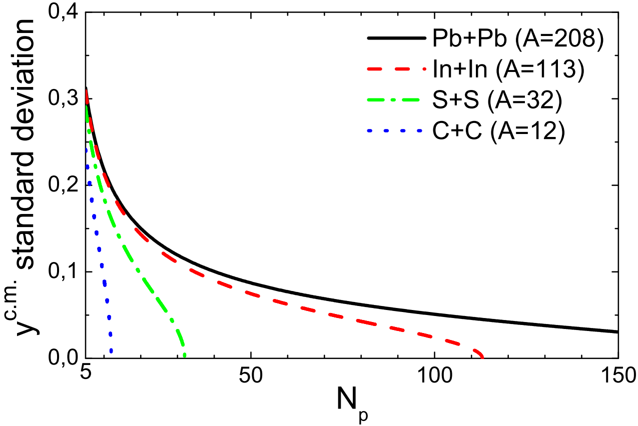


Distribution in form of Gaussian works well for most conditions!

Dependence on mass number A

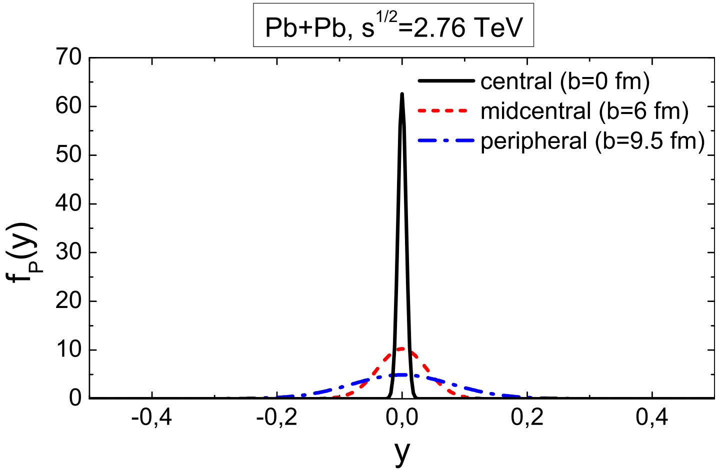
In terms of av. number of participants $N_p = A(1-p)$ from nucleus

$$\delta y^2 = \frac{v_{\text{in}}^2}{2}\left(\frac{1}{N_p} - \frac{1}{A}\right)$$



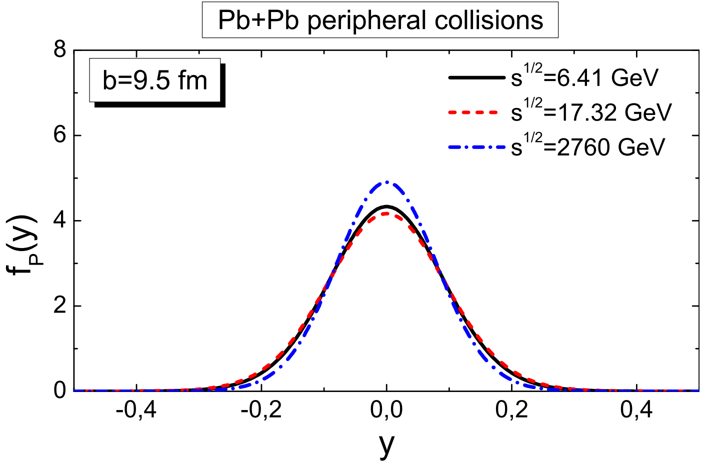
Fluctuations are smaller in “equivalent” collisions with smaller mass number

Rapidity distribution: centrality dependence



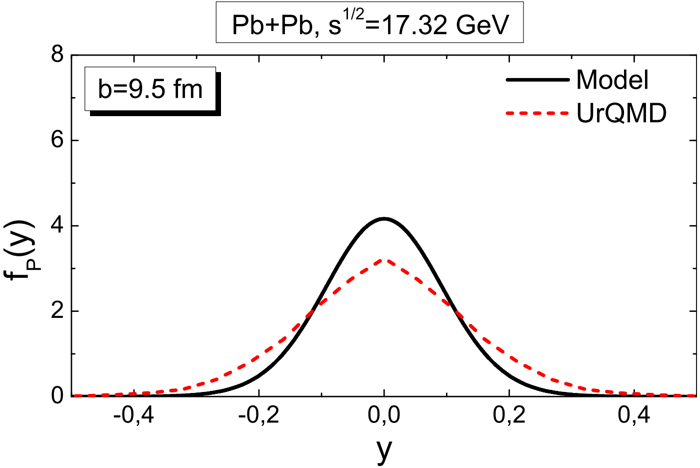
Strong dependence on centrality

Rapidity distribution: energy dependence



Weak dependence on collision energy

Comparison with microscopic model



Conclusions

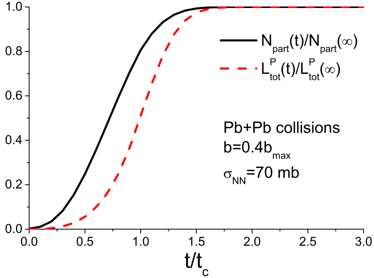
1. The initial spectator-participant partition stage is characterized by the energy-dependent temporal scale t_c and partition process into spectators and participants becomes complete at $t \simeq 1.5 t_c$.
2. Participants have a significant total angular momentum in non-central collisions. This indicates presence of rotation and shear in initial hydro state.
3. Fluctuations of the rapidity of the formed participant system are described by Gaussian and increase significantly with impact parameter.
4. $y^{c.m.}$-fluctuations need to be taken into account for a proper analysis of rapidity-dependent observables such as collective flow.

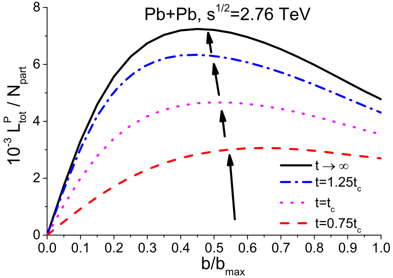
More details in:
Phys. Rev. C 88, 014901 (2013) and Phys. Rev. C 90, 044907 (2014)

Thanks for your attention!

Backup slides

Angular momentum per baryon charge



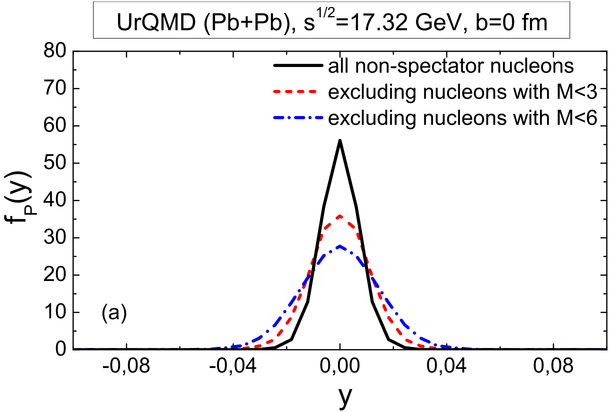


Angular momentum increases at later times than number of participants

Angular momentum per participant stays significant in peripheral collisions.

Fluctuations for different definitions of "participants"

The separation between spectators and participants is not trivial.



If we exclude nucleons with $M_{\text{coll}} < 6$ from participants the distribution width doubles! The effect is smaller in peripheral collisions.