QCD phase structure at finite baryon density from fluctuations

Volodymyr Vovchenko (Berkeley Lab)

The 1st Workshop on Physics at High Baryon Density, UCLA

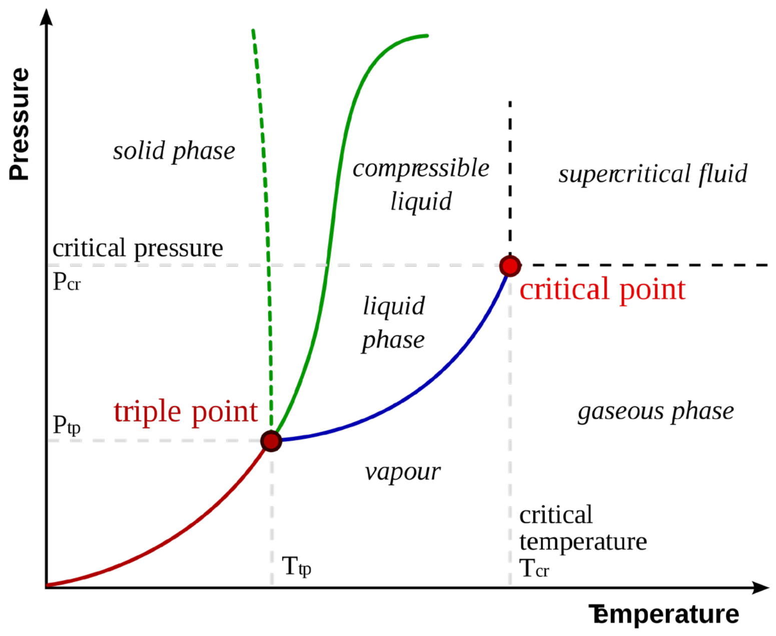
March 19, 2022

Unterstützt von / Supported by

Alexander von Humboldt Stiftung/Foundation

QCD phase diagram

Ordinary fluid (e.g. water)



QCD

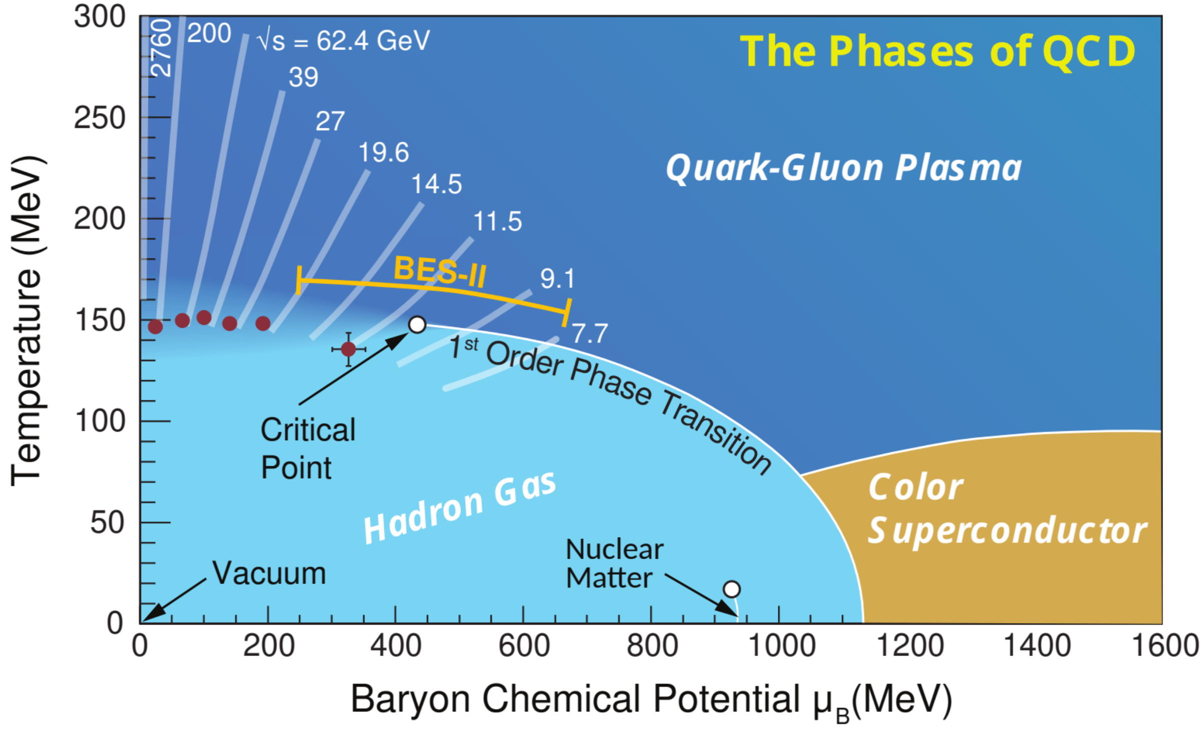


Figure from Bzdak et al., Phys. Rept. '20

- Dilute hadron gas at low T/n_B due to confinement, quark-gluon plasma high T/n_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured

What is the nature of the quark-hadron transition?

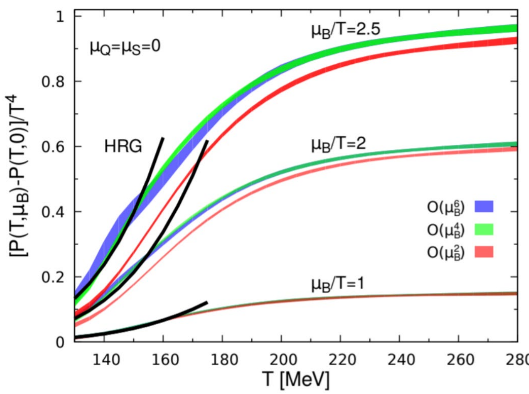
Lattice-based constraints on the QCD critical point

Indirect lattice QCD methods offer glimpse into small μ_B/T

- Taylor expansion around $\mu_B/T\!=\!0$

$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \frac{\chi_2^B(T,0)}{2!}(\mu_B/T)^2 + \frac{\chi_4^B(T,0)}{4!}(\mu_B/T)^4 + \dots$$

No hints for the critical point at T $>$ 135 MeV
Critical point $\mu_B/T < 3$ disfavored

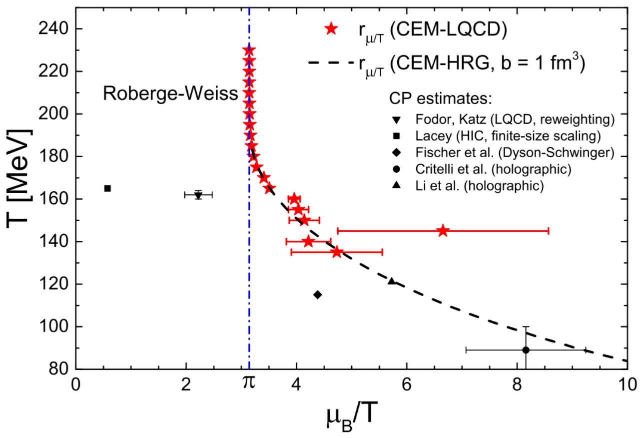


[HotQCD Coll., PRD 95, 054504 (2017)
Budapest-Wuppertal Coll., PRL 126, 232001 (2021)]

- Relativistic virial expansion in fugacities via analytic continuation from imaginary μ_B/T

$$\frac{p(T,\mu_B)}{T^4} = \sum_{k=0}^{\infty} p_k(T)\cosh\left(\frac{k\,\mu_B}{T}\right)$$

Expansion sees singularity in the complex plane, Im $[\mu_B/T] = \pi$
Critical point at $\mu_B/T < \pi$ disfavored



[V.V., Steinheimer, Philipsen, Stoecker, PRD 97, 114030 (2018)]

Critical point, if it exists, likely located at high baryon density

QCD phase diagram with heavy-ion collisions

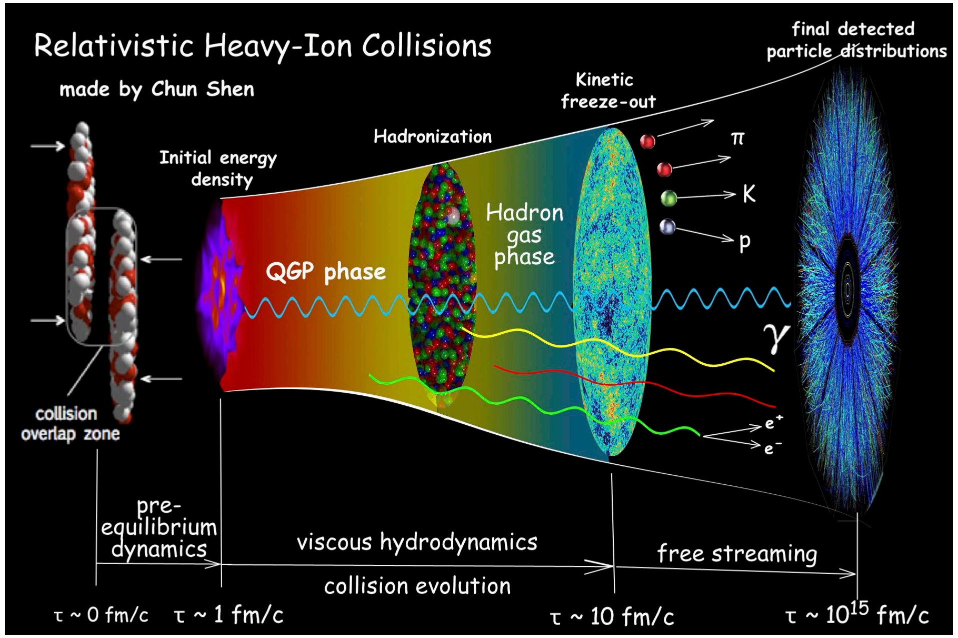
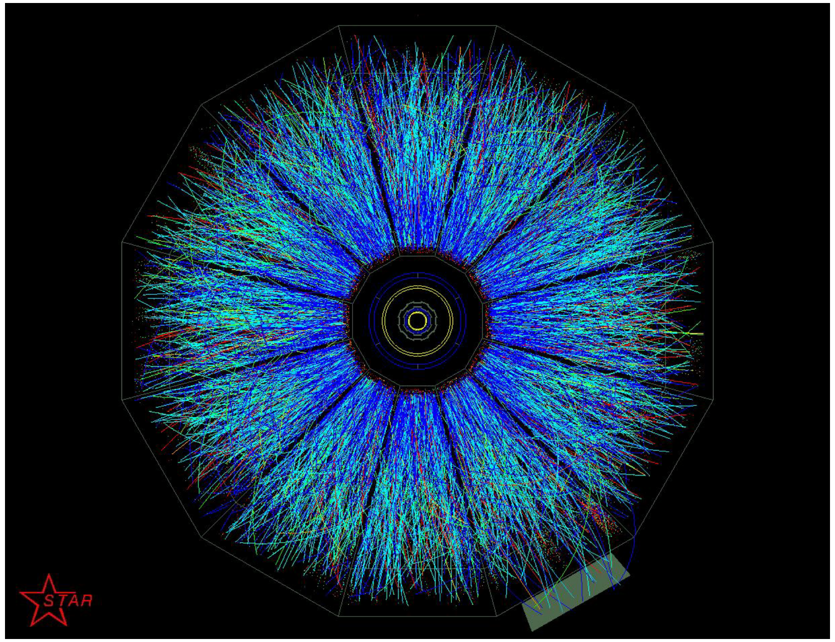


Figure from Bzdak et al., Phys. Rept. '20



STAR event display

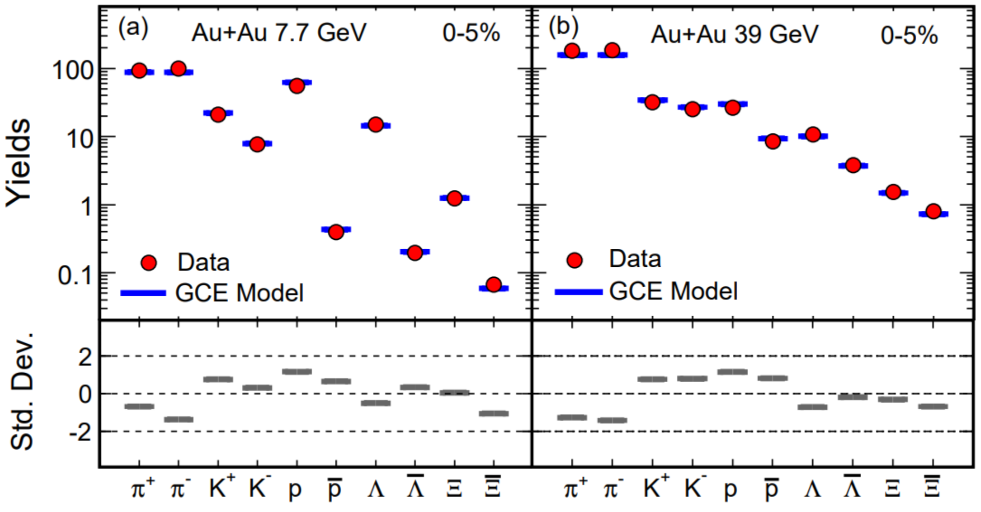
Thousands of particles created in relativistic heavy-ion collisions

↓

Apply concepts of statistical mechanics

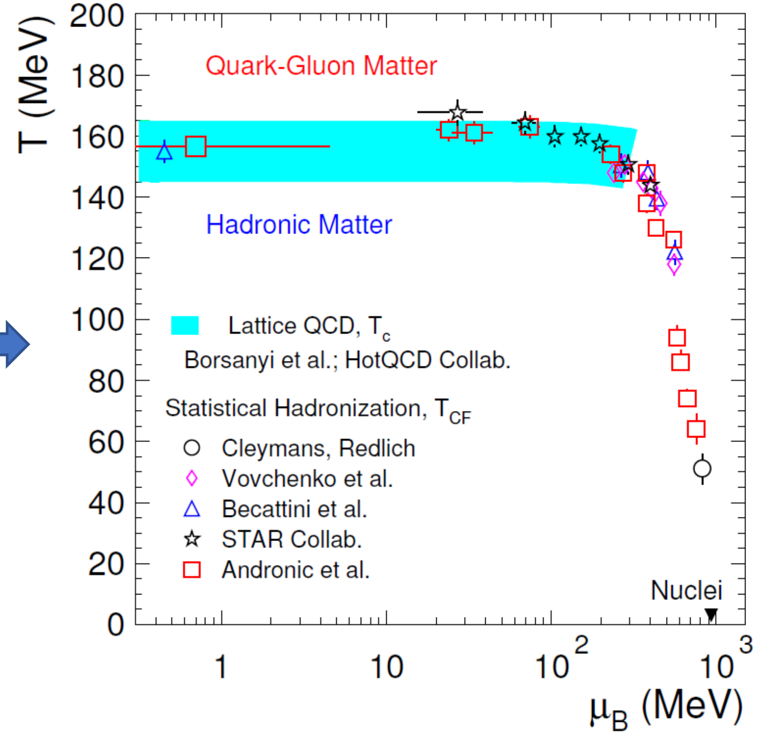
Mapping heavy-ion collisions onto the QCD phase diagram

Fit hadron yields with the thermal model: $N_i = V \frac{d_i m_i^2 T}{2\pi^2} K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}} + \text{feedown}, \qquad \mu_i = b_i \mu_B + q_i \mu_Q + s_i \mu_S$



STAR Collaboration, Phys. Rev. C 96, 044904 (2017)

$\boxed{\begin{array}{c} T(\sqrt{s_{NN}}) \\ \mu_B(\sqrt{s_{NN}}) \end{array}}$



A. Andronic et al., Nature 561, 321 (2018)

$\sqrt{s_{NN}} \searrow \quad \Longrightarrow \quad \mu_B \nearrow$

For differential observables (spectra, flow, ...) use relativistic hydrodynamics

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

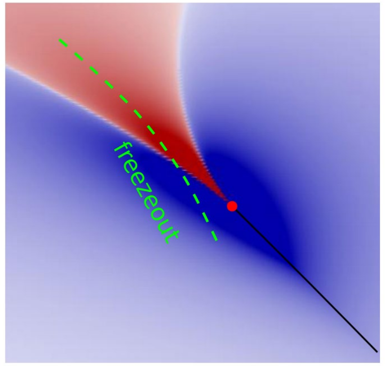
Grand partition function

$$\ln Z^{\mathrm{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\mathrm{ce}}(T,V,N)\right]$$

$$\boxed{\kappa_n \propto \frac{\partial^n(\ln Z^{\mathrm{gce}})}{\partial\mu^n}}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- **(QCD) critical point** – large correlation length, critical fluctuations of baryon number



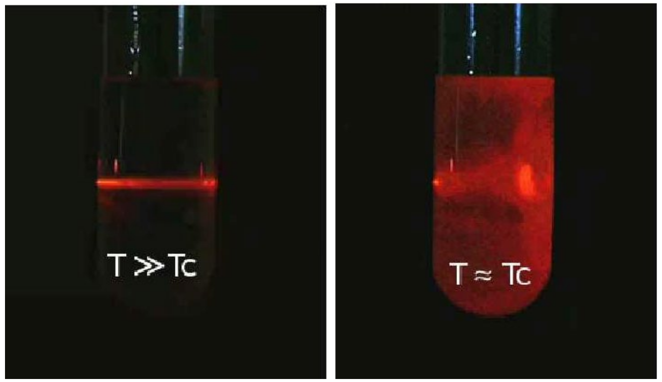
M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

$$\xi \to \infty$$

Looking for non-monotonic dependence of κ_4

Critical opalescence



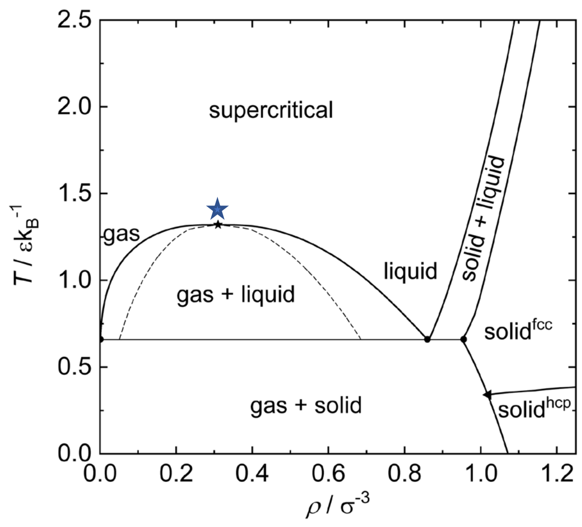
Example: Lennard-Jones fluid

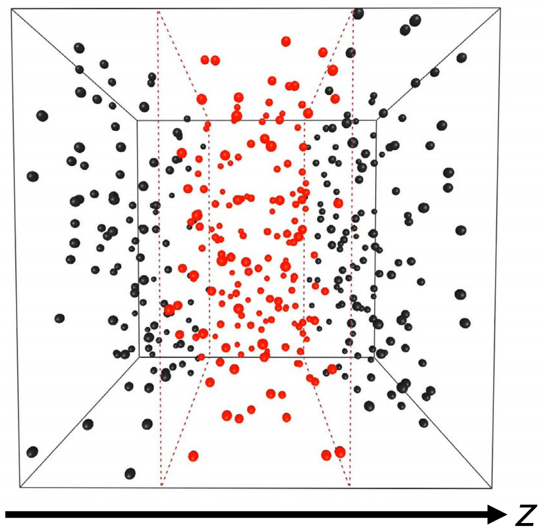
V. Kuznietsov, O. Savchuk, M.I. Gorenstein, V. Koch, VV, arXiv:2201.08486

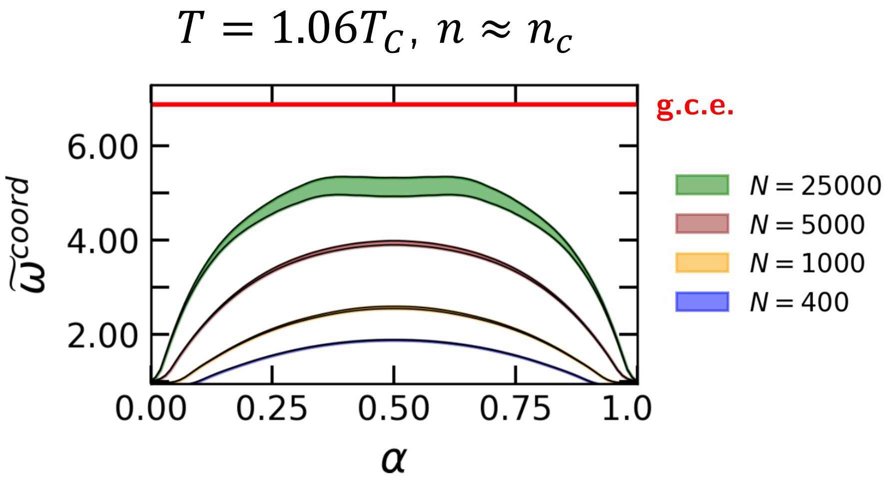
Classical molecular dynamics simulations of a **Lennard-Jones fluid** near the critical point of the liquid-gas transition

$$V_{\mathrm{LJ}}(r) = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$

$$\tilde{\omega}^{\mathrm{coord}} = \frac{1}{1-\alpha}\,\frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle}$$







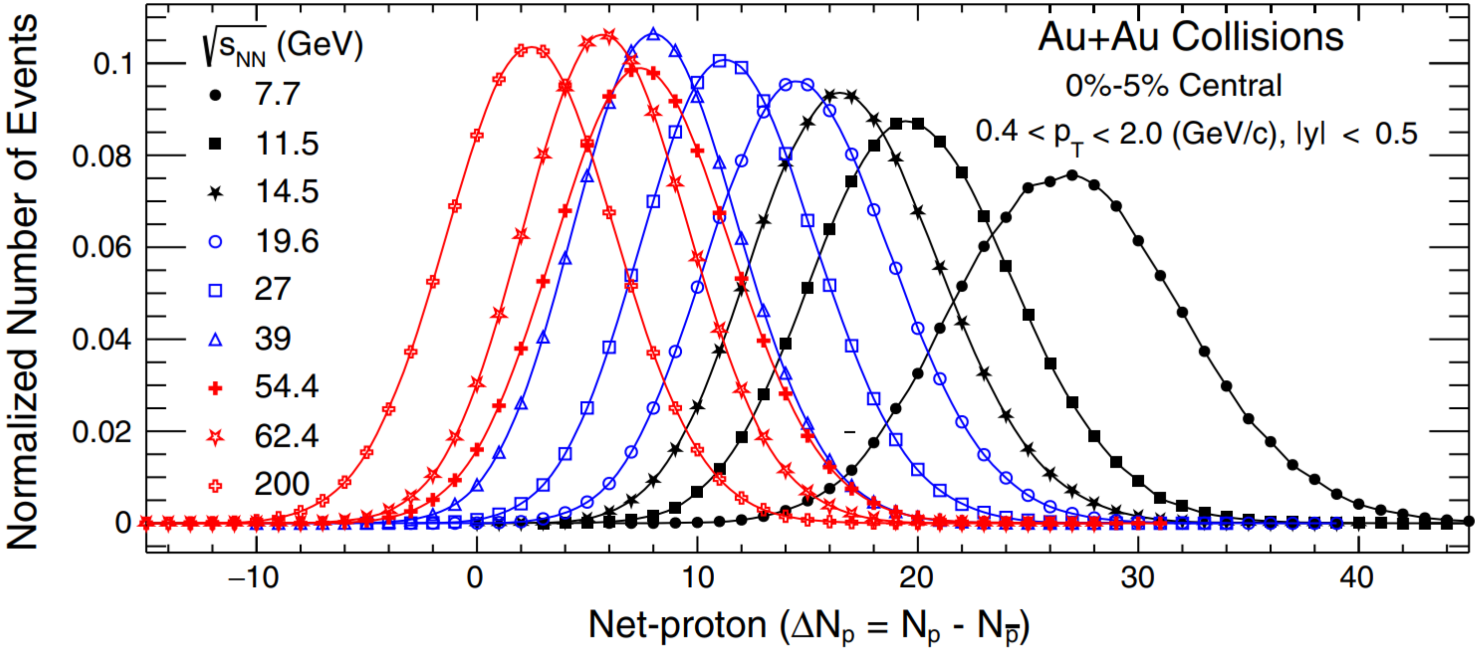
NB: signal disappears in the momentum space in absence of collective flow and expansion

Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons

$$P(\Delta N_p) \sim \frac{N_{\text{events}}(\Delta N_p)}{N^{total}_{events}}$$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)

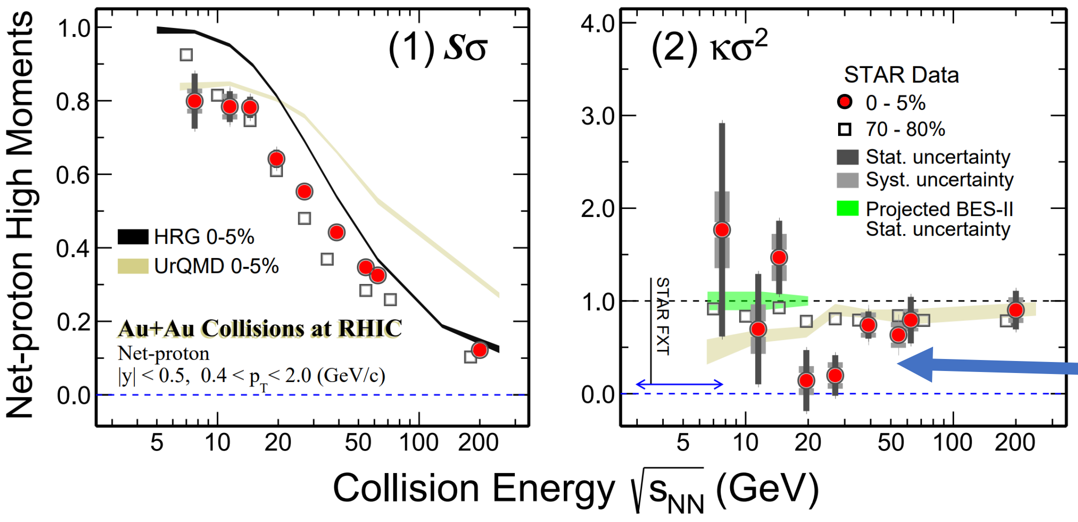


Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

$$\frac{\kappa_2}{\langle N \rangle}, \qquad \frac{\kappa_3}{\kappa_2}, \qquad \frac{\kappa_4}{\kappa_2}$$

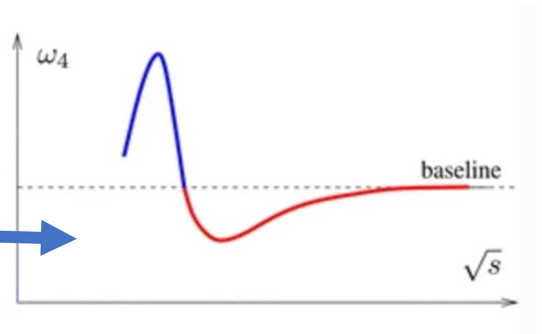
Experimental measurements

Beam energy scan in search for the critical point (STAR Coll.)



Phys. Rev. Lett. 126, 092301 (2021)

?



M. Stephanov, Phys. Rev. Lett. (2011)

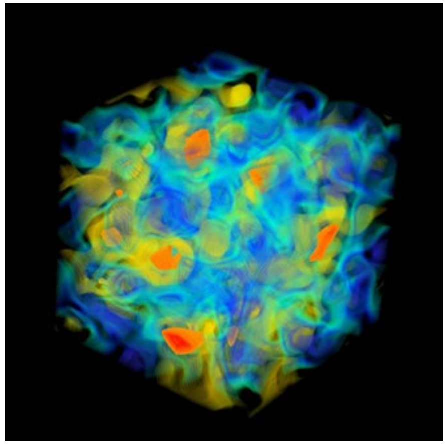
Reduced errors (better statistics) to come soon from beam energy scan II program

Can we learn more from the more accurate data available for κ_2 and κ_3?

Other measurements: LHC-ALICE, GSI-HADES & CERN-NA61/SHINE Collaborations

Theory vs experiment: Challenges for fluctuations

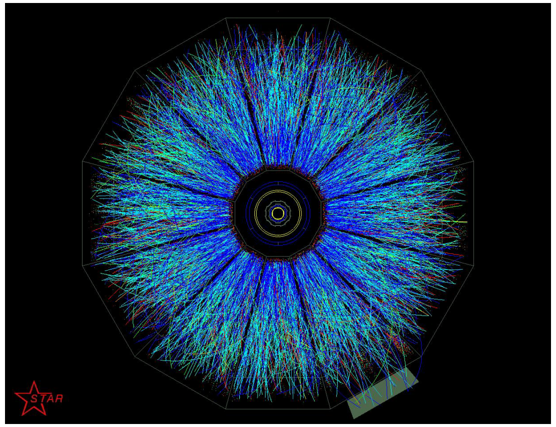
Theory



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

Need dynamical description

Dynamical approaches to the QCD critical point search

1. Dynamical model calculations of critical fluctuations
   - Fluctuating hydrodynamics
   - Equation of state with tunable critical point [P. Parotto et al, Phys. Rev. C 101, 034901 (2020)]
   - Predict CP signatures dependent on its location

   Under development within the Beam Energy Scan Theory (BEST) Collaboration

   BEST COLLABORATION [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]

2. **Deviations from precision calculations of the non-critical baseline**
   - Include essential non-critical contributions to (net-)proton number cumulants
   - Exact baryon conservation + hadronic interactions (hard core repulsion)
   - Based on realistic hydrodynamic simulations tuned to bulk data

   [VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

Excluded volume effect

Incorporate repulsive baryon (nucleon) hard core via excluded volume

VV, M.I. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

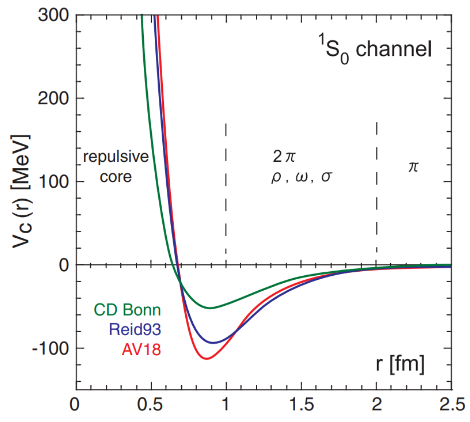
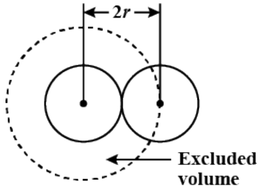


Figure from Ishii et al., PRL '07

Amounts to a van der Waals correction for baryons in the HRG model

$$V \to V - bN$$



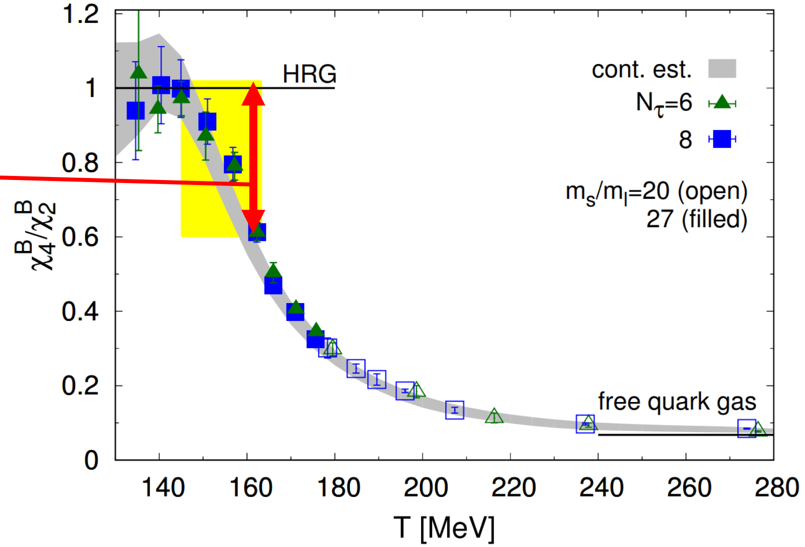
$$p^{\rm ev}_{B(\bar{B})} = p^{\rm id}_{B(\bar{B})}\, e^{-b p^{\rm ev}_{B(\bar{B})}/T}$$

- Net baryon kurtosis suppressed as in lattice QCD

$$\frac{\chi_4^B}{\chi_2^B} \simeq 1 - 12 b \phi_B(T) + O(b^2)$$

- Reproduces virial coefficients of baryon interaction from lattice QCD

Excluded volume from lattice QCD: $\boxed{\mathrm{b} \approx 1\ \mathrm{fm}^3}$



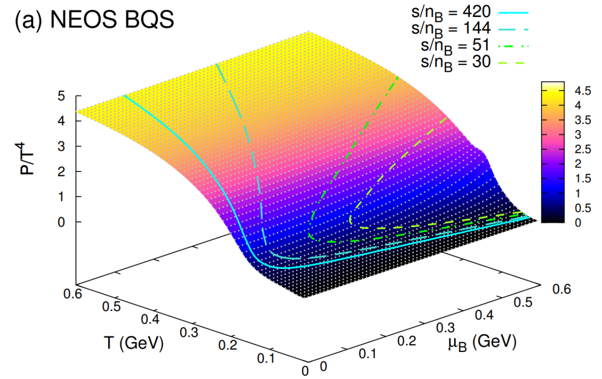
VV, A. Pasztor, S. Katz, Z. Fodor, H. Stoecker, Phys. Lett. B 755, 71 (2017)

RHIC-BES: Hydrodynamic description in non-critical scenario

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- Collision geometry based 3D initial state
  - Constrained to net proton distributions [Shen, Alzhrani, Phys. Rev. C '20]
- Viscous hydrodynamics evolution – MUSIC-3.0
  - Energy-momentum and baryon number conservation
  - Crossover equation of state based on lattice QCD

[Monnai, Schenke, Shen, Phys. Rev. C '19; also Noronha-Hostler, Parotto, Ratti, Stafford, Phys. Rev. C '19]



- Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3

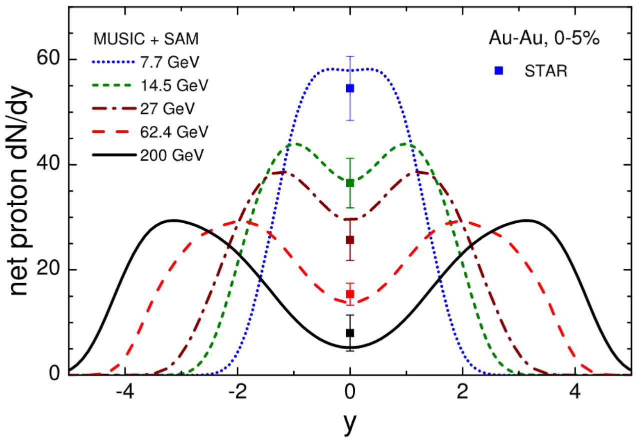
$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x)\, p^\mu \frac{d_j\, \lambda_j^{\rm ev}(x)}{(2\pi)^3} \exp\left[\frac{\mu_j(x) - u^\mu(x) p_\mu}{T(x)}\right].$$

- Particlization respects QCD-based baryon number distribution
  - Incorporated via baryon excluded volume b = 1 fm^3

[VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]

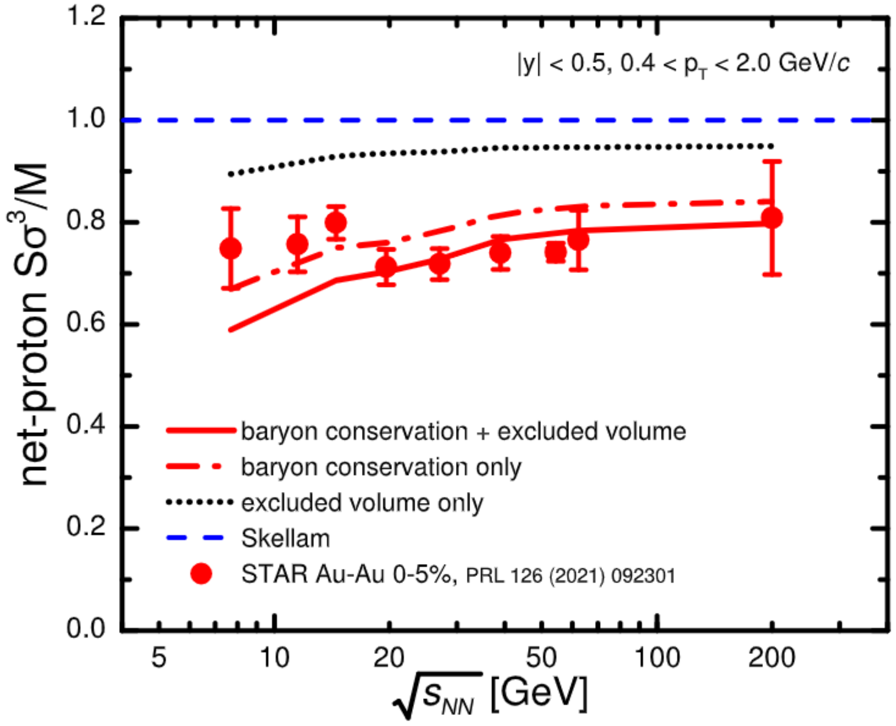
- Incorporates exact global baryon conservation via a method SAM-2.0

[VV, Phys. Rev. C 105, 014903 (2022)]

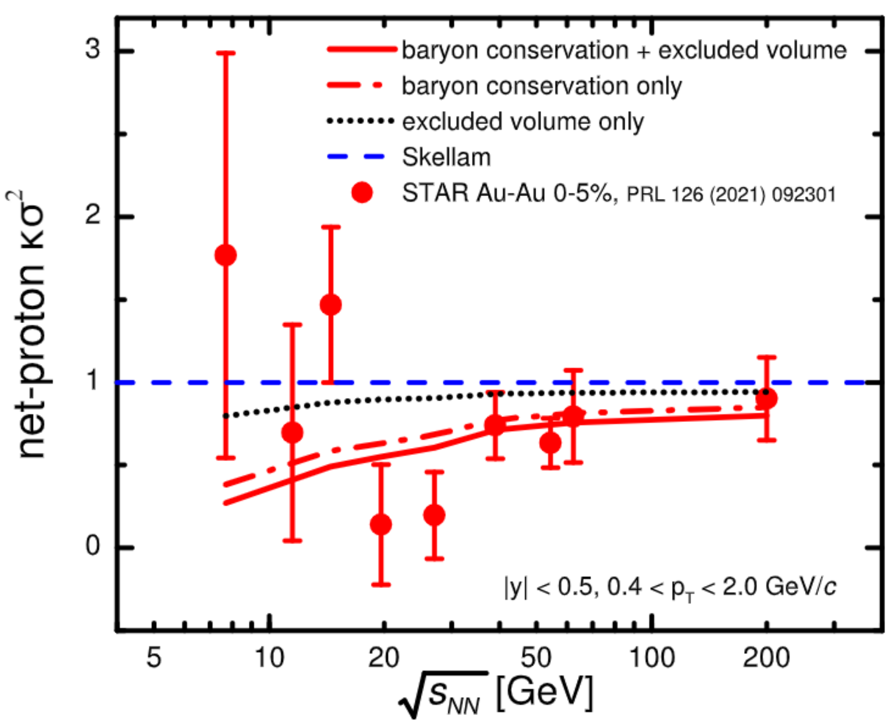


Net proton cumulant ratios

κ_3/κ_1



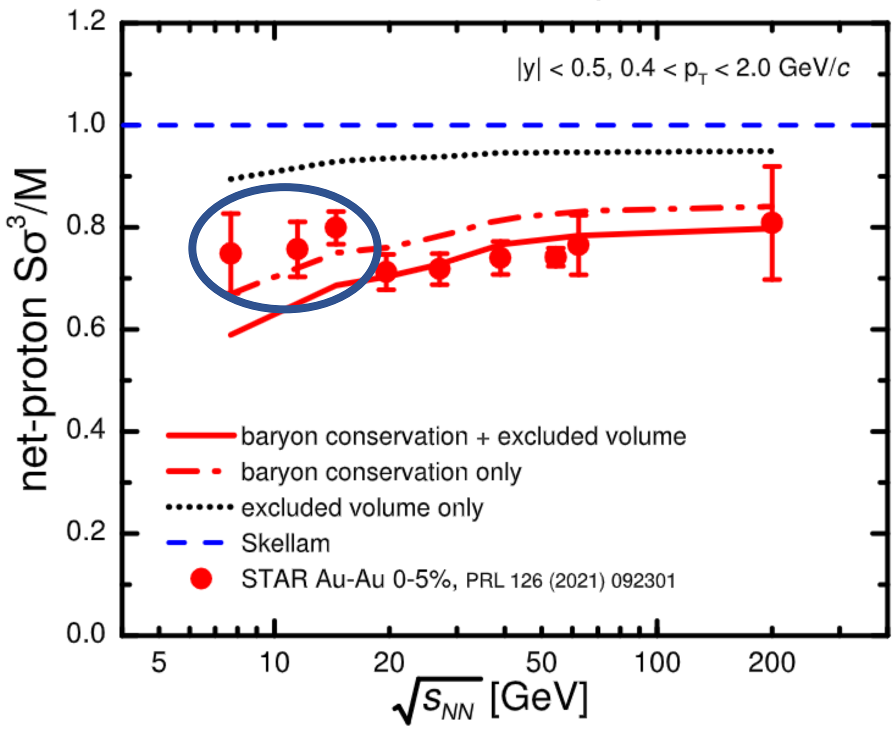
κ_4/κ_2



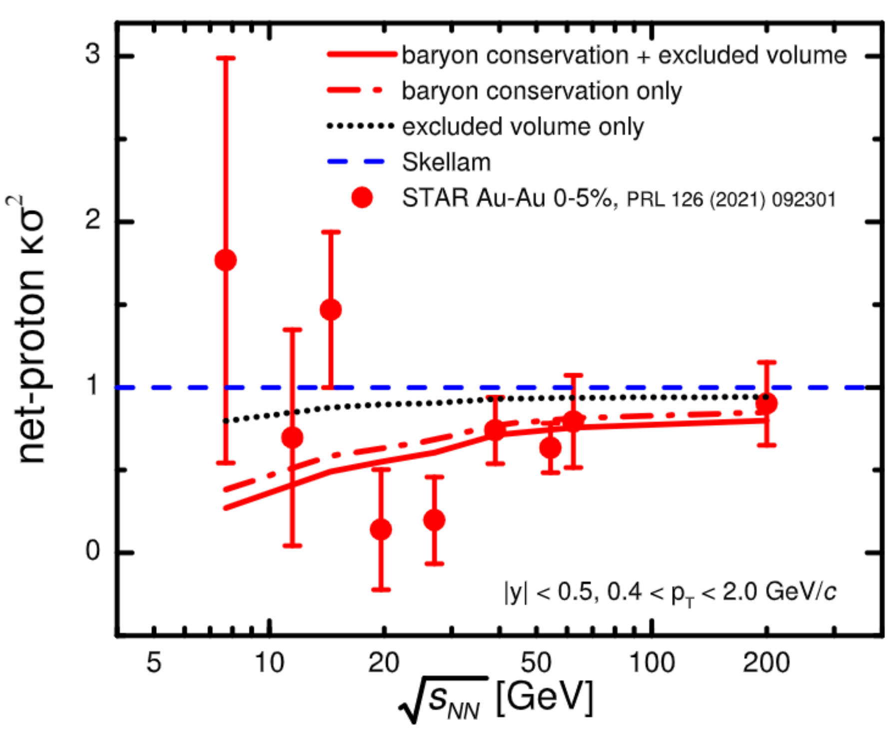
- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (baryon conservation and repulsion)
- Effect from baryon conservation is larger than from repulsion
- Excess of skewness in data at $\sqrt{s_{NN}} < 20$ GeV – *hint of attractive interactions?* ***Critical point?***

Net proton cumulant ratios

κ_3/κ_1



κ_4/κ_2



- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (baryon conservation and repulsion)
- Effect from baryon conservation is larger than from repulsion
- Excess of skewness in data at $\sqrt{s_{NN}} < 20$ GeV – *hint of attractive interactions?* ***Critical point?***

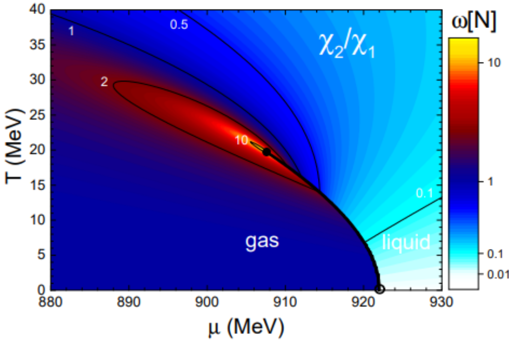
Correlation Functions

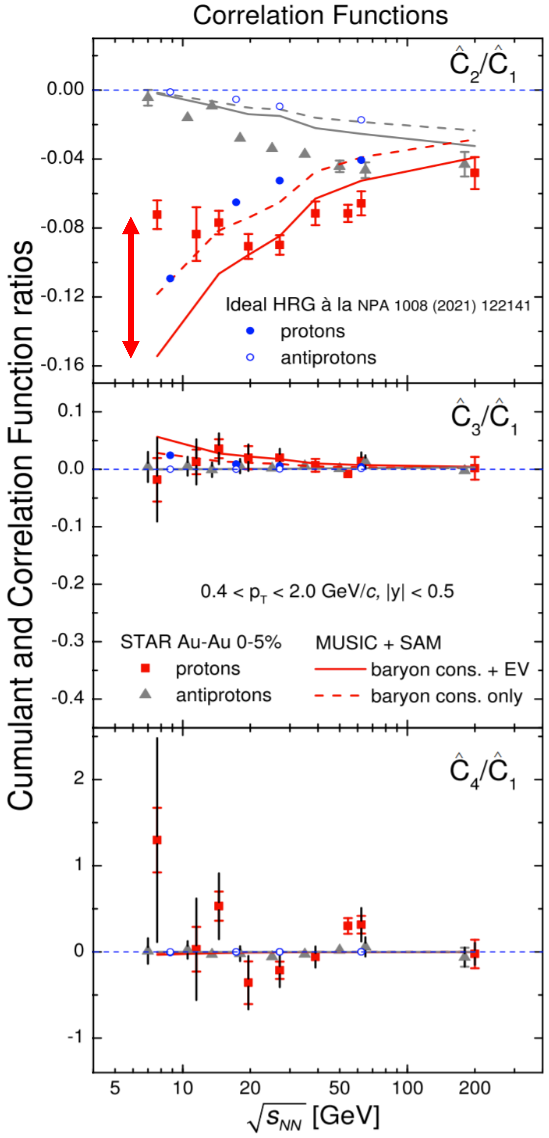
- Analyze genuine multi-particle correlations via **factorial cumulants $\hat{C}_n$** [Bzdak, Koch, Strodthoff, Phys. Rev. C '17]

$$\hat{C}_1 = \kappa_1, \qquad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$
$$\hat{C}_2 = -\kappa_1 + \kappa_2, \qquad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$

- Three- and four-particle correlations are small without a CP
  - Multi-particle correlations expected near the critical point [Ling, Stephanov, PRC '15]

- Signals from the data at $\sqrt{s_{NN}} \leq 20$ GeV
  - Excess of two-proton correlations
  - Possibility of significant 4-proton correlations
  - ***Critical point?***





Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV

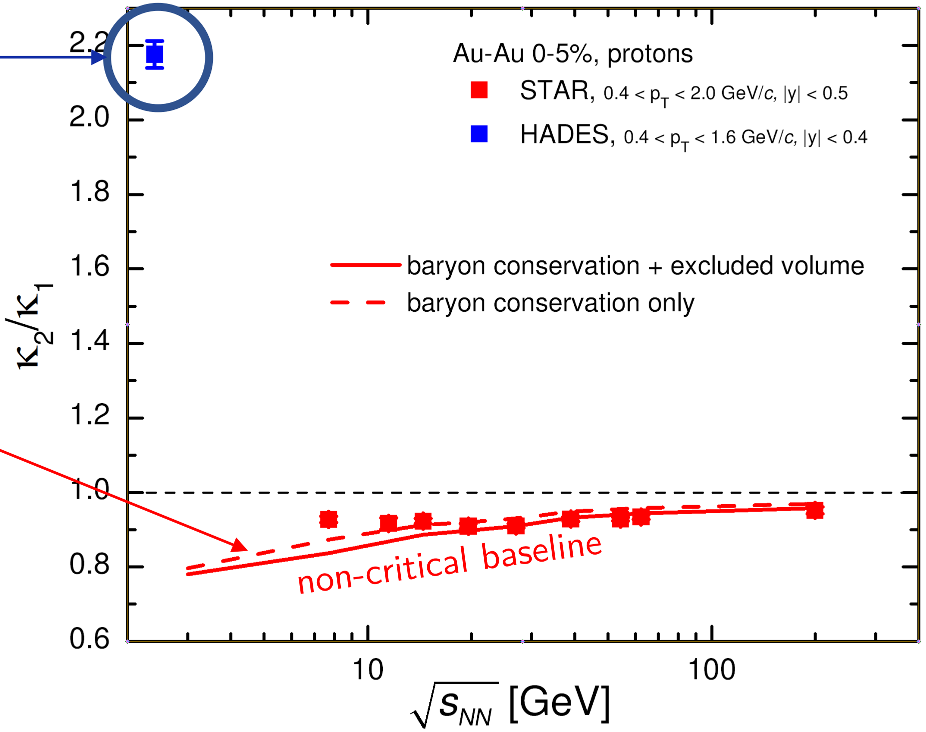
- Intriguing hint from HADES @ $\sqrt{s_{NN}} = 2.4$ GeV: huge proton variance!

  [HADES Collaboration, Phys. Rev. C 102, 024914 (2020)]

- No change of trend in the non-critical baseline

- Additional mechanisms:
  - Nuclear liquid-gas transition
  - Light nuclei formation

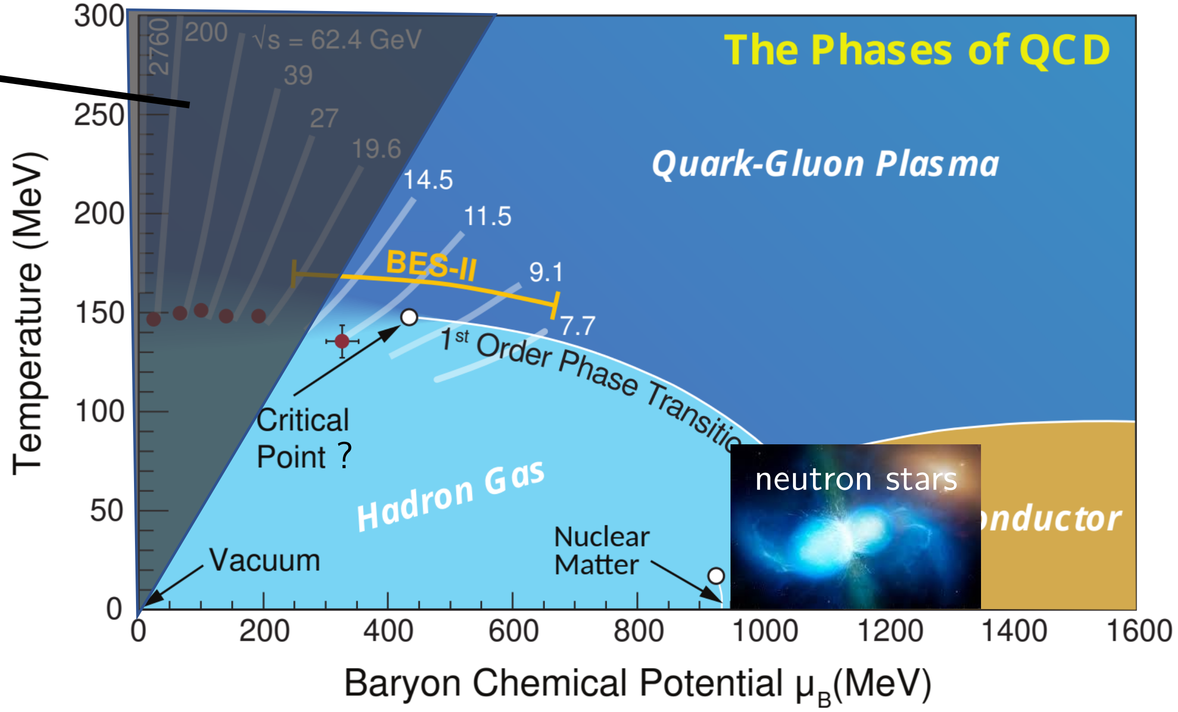
- Fill the gap with ongoing/future data from STAR fixed target, future experiments like **CBM-FAIR**



Naïvely, could indicate approach of the QCD critical point in baryon-rich regime

QCD phase structure: What we learned so far

Critical point disfavored

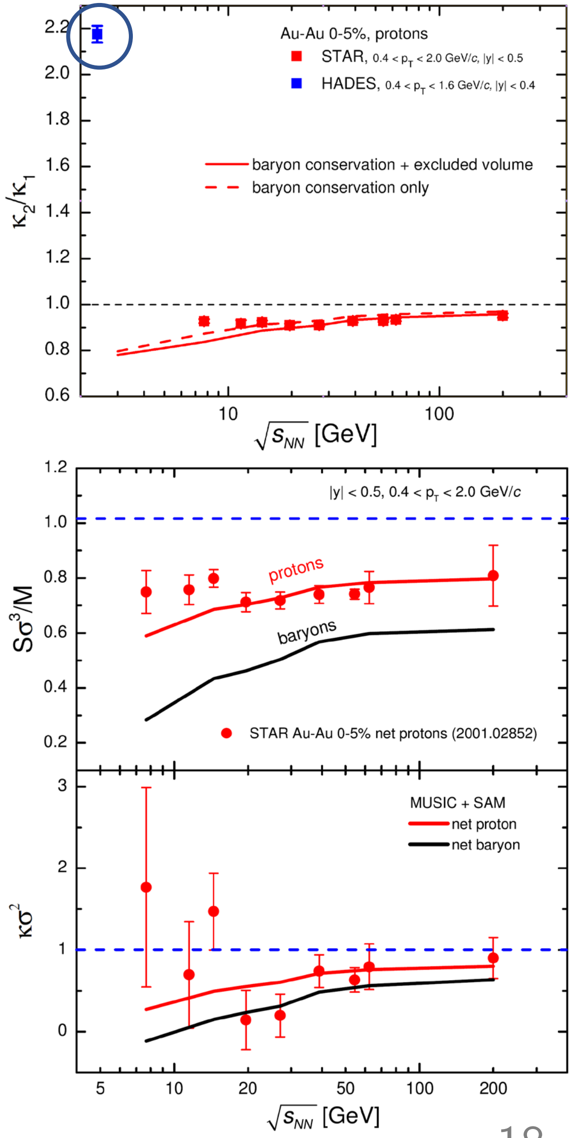


- Data at high energies ($\sqrt{s_{NN}} \geq 20$ GeV) consistent with "non-critical" physics
- Disfavors critical point at $\mu_B/T < 2$-3, consistent with what we know from lattice QCD
- Interesting physics at high densities probed by future experiments, neutron stars & their mergers

CBM, STAR-FXT are in the right spot!

Summary

- Fluctuations are a powerful tool to explore the QCD phase diagram
  - Heavy-ion data are described quantitatively at $\sqrt{s_{NN}} \geq 20$ GeV ($\mu_B/T < 3$) without critical point
  - Possible critical point signals at $\sqrt{s_{NN}} < 14.5$ GeV
- Outlook: Lower collision energies $\sqrt{s_{NN}} \leq 7.7$ GeV (e.g. with CBM)
  - Close the gap between HADES/STAR FXT and BES
  - Multi-particle correlations (higher-order factorial cumulants)
  - Baryons from protons
  - Connections to neutron star phenomena



Thanks for your attention!