Critical point of nuclear matter

and beam energy dependence of net proton number fluctuations

Volodymyr Vovchenko

ITP & FIAS, Goethe University Frankfurt

V.V., M.I. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

V.V., L. Jiang, M.I. Gorenstein, H. Stoecker, arXiv:1711.07260

NA61/SHINE Analysis/Software/Calibration Meeting

Monbachtal, Germany

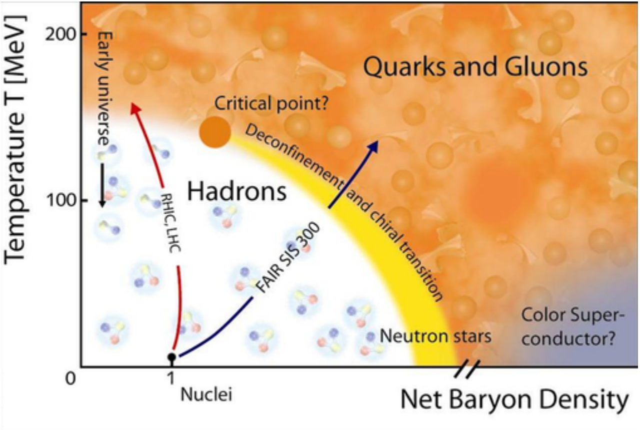
February 20, 2017

Outline

- Motivation: Exploring the QCD phase diagram with fluctuations
- Nuclear matter and the QCD phase diagram
  - Nucleon-nucleon interaction
  - Nuclear matter with quantum van der Waals equation
  - van der Waals interaction in Hadron Resonance Gas
- Beam energy dependence of net proton number fluctuations
- Summary

Exploring the QCD phase diagram with fluctuations

The QCD phase diagram has many unknowns at finite density



Does the critical point exists? Use *fluctuations* as its signal!

Theory: $\chi^{(n)} = \partial^n(p/T^4)/\partial(\mu/T)^n \sim \xi^k$, $\xi \to \infty$ at the CP

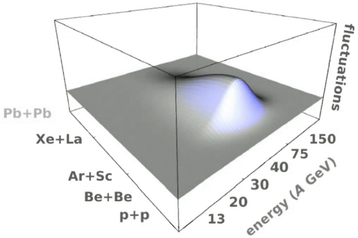
In heavy-ion collisions $\xi \lesssim 2-3$ fm [M. Stephanov, PRL '09]

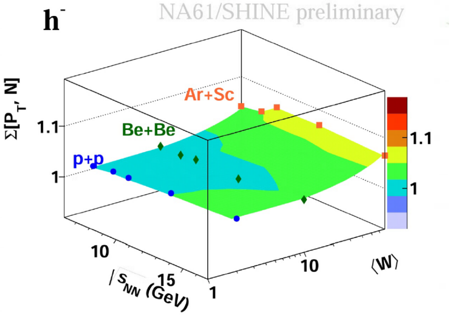
NA61/SHINE: No critical point seen so far

M. Gazdzicki [NA61/SHINE collaboration], EMMI Workshop, 12 Feb 2018

SEARCH FOR CRITICAL POINT: FLUCTUATIONS VS $\sqrt{s_{NN}}$ AND A

CP ⇒ "FLUCTUATION HILL"



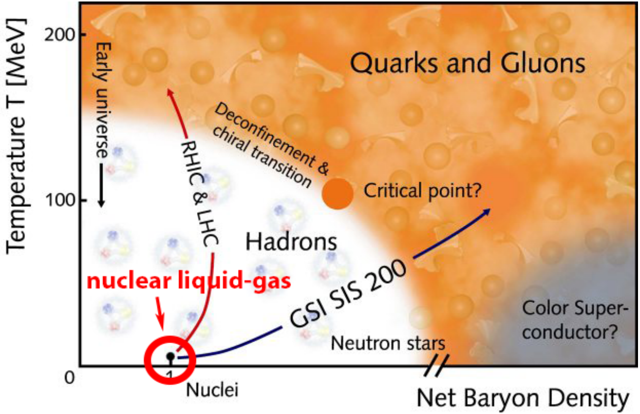


NO INDICATION FOR CRITICAL POINT SO FAR

No signs of CP at $\mu_B/T < 2$ from lattice QCD as well [HotQCD, 1701.04325]

Critical point of nuclear matter

The QCD phase diagram



is known to contain the critical point of nuclear matter at $T_c \sim 15$ MeV and $(\mu_B/T)_c \sim 40 \quad \Rightarrow \quad$ *way beyond current lattice methods*

It is the only QCD critical point we know is there...

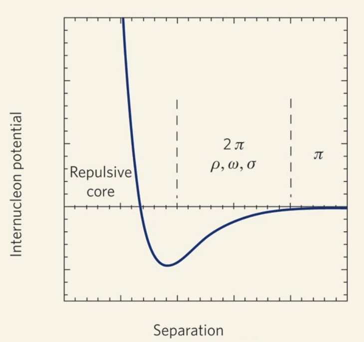
How does it influence the fluctuation observables in heavy-ion collisions?

Nucleon-nucleon interaction

Nuclear liquid-gas transition appears due to the vdW type structure of the nucleon-nucleon interaction

Nucleon-nucleon potential:

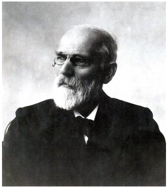
- Repulsive core at small distances
- Attraction at intermediate distances
- Suggestive similarity to vdW interactions



Could nuclear matter be described by the van der Waals equation?

van der Waals equation

$$P(T,V,N) = \frac{NT}{V-bN} - a\frac{N^2}{V^2}$$



Formulated in 1873.

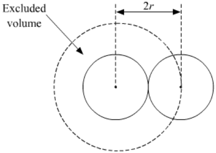
Simplest model which contains attractive and repulsive interactions

Contains 1st order phase transition and critical point

Nobel Prize in 1910.

1. Short-range repulsion: excluded volume (EV) procedure

   $V \to V - bN, \qquad b = 4\frac{4\pi r_c^3}{3}$

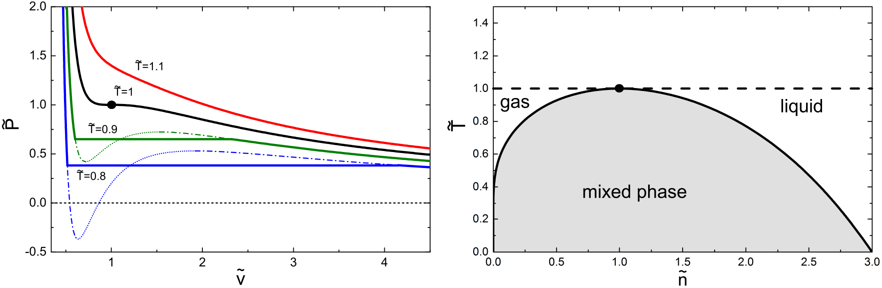


2. Intermediate range attraction in mean-field approx.

   $P \to P - a\,n^2, \qquad a = \pi\int_{2r_c}^{\infty}|U_{12}(r)|r^2dr$

van der Waals isotherms

- vdW isotherms show irregular behavior below certain temperature T_C
- Below T_C isotherms are corrected by Maxwell's rule of equal areas
- Results in appearance of mixed phase



Critical point

$$\frac{\partial p}{\partial v} = 0, \quad \frac{\partial^2 p}{\partial v^2} = 0, \quad v = V/N$$

$$p_C = \frac{a}{27b^2},\ n_C = \frac{1}{3b},\ T_C = \frac{8a}{27b}$$

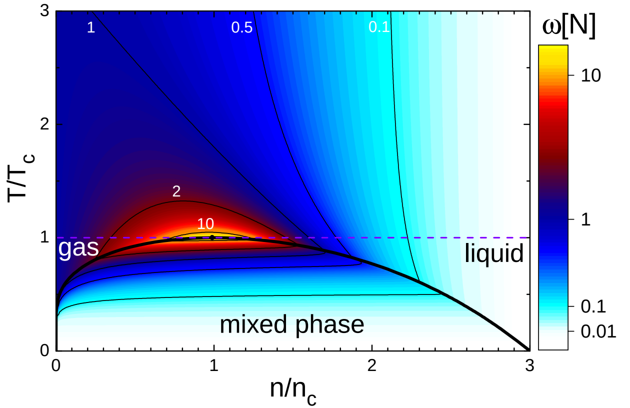
Reduced variables

$$\tilde{p} = \frac{p}{p_C},\ \tilde{n} = \frac{n}{n_C},\ \tilde{T} = \frac{T}{T_C}$$

Scaled variance for classical VDW equation

Particle number fluctuations in a classical vdW gas within the GCE

$$\omega[N] = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} = \frac{\chi_2}{\chi_1} = \left[\frac{1}{(1-bn)^2} - \frac{2an}{T}\right]^{-1}$$



- Repulsive interactions suppress N-fluctuations
- Attractive interactions enhance N-fluctuations

V.V., Anchishkin, Gorenstein, J. Phys. A 48, 305001 (2015)

Quantum statistical van der Waals fluid

Nuclear matter applications require explicit treatment of *Fermi statistics*

Free energy of classical vdW fluid:

$$F(T,V,N) = F^{\rm id}(T, V-bN, N) - a\,\frac{N^2}{V}$$

Ansatz: $F^{\rm id} \Rightarrow F^{\rm id}_q(T, V-bN, N)$ is free energy of ideal *quantum* gas

Quantum van der Waals equation:

$$p(T,n) = p_q^{\rm id}\left(T, \frac{n}{1-bn}\right) - an^2$$

$p_q^{\rm id}(T,n)$ corresponds to Fermi-Dirac or Bose-Einstein distribution

Model properties:

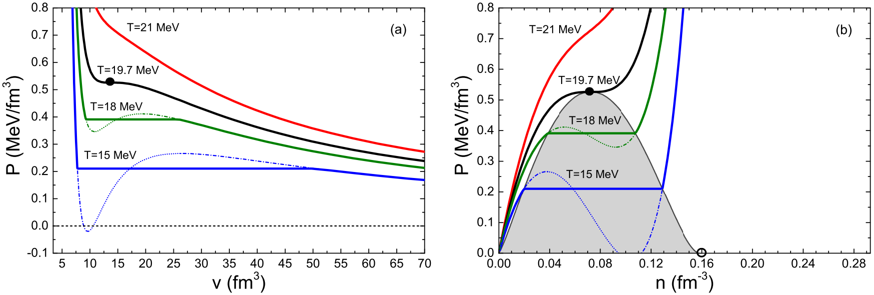
- Reduces to the classical vdW equation when quantum statistics are negligible
- Reduces to ideal quantum gas for $a=0$ and $b=0$
- Entropy density non-negative and $s \to 0$ with $T \to 0$

V.V., Anchishkin, Gorenstein, Phys. Rev. C 91, 064314 (2015)

QvdW gas of nucleons: pressure isotherms

a and b fixed to reproduce saturation density and binding energy:

$n_0 = 0.16$ fm^{-3}, $E/A = -16$ MeV $\Rightarrow$ $a \cong 329$ MeV fm^3 and $b \cong 3.42$ fm^3



Behavior qualitatively same as for Boltzmann case

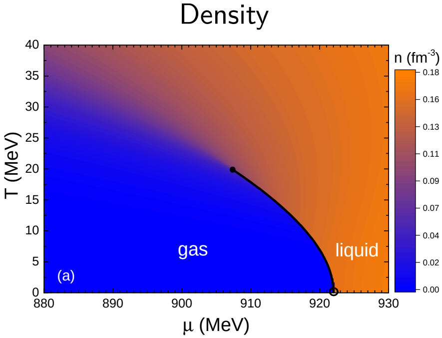
Mixed phase results from Maxwell construction

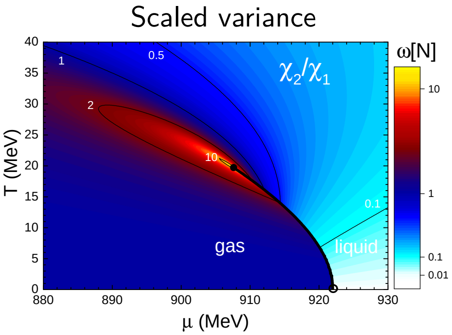
Critical point at $T_c \cong 19.7$ MeV and $n_c \cong 0.07$ fm^{-3}

Experimental estimate[1]: $T_c = 17.9 \pm 0.4$ MeV, $n_c = 0.06 \pm 0.01$ fm^{-3}

[1] J.B. Elliot, P.T. Lake, L.G. Moretto, L. Phair, Phys. Rev. C 87, 054622 (2013)

QvdW gas of nucleons: (T, μ) plane





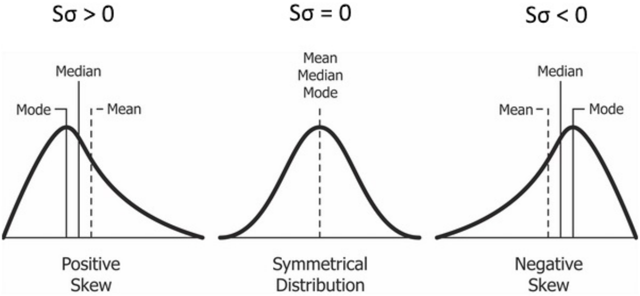
A familiar critical point signal in the scaled variance

V.V., D. Anchishkin, M. Gorenstein, R. Poberezhnyuk, PRC 91, 064314 (2015)

Non-Gaussian fluctuations: Skewness

(Normalized) skewness measures the degree of asymmetry of distribution

$$S\sigma = \frac{\kappa_3}{\kappa_2} = \frac{\langle(\Delta N)^3\rangle}{\sigma^2} .$$



Baselines:

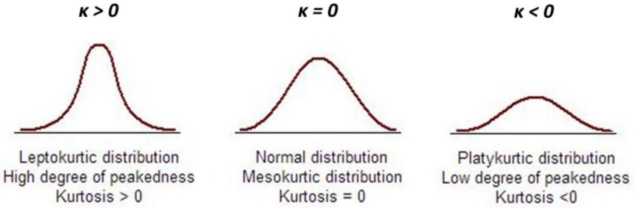
- Gaussian: $S\sigma = 0$
- Poisson: $S\sigma = 1 \quad \leftarrow \quad$ ideal gas in grand canonical ensemble

At CP: $S\sigma \sim \xi^{4.5}$

Non-Gaussian fluctuations: Kurtosis

(Normalized) kurtosis measures "peakedness" of distribution

$$\kappa\sigma^2 \;=\; \frac{\kappa_4}{\kappa_2} = \frac{\langle(\Delta N)^4\rangle \;-\; 3\,\langle(\Delta N)^2\rangle^2}{\sigma^2}\;.$$

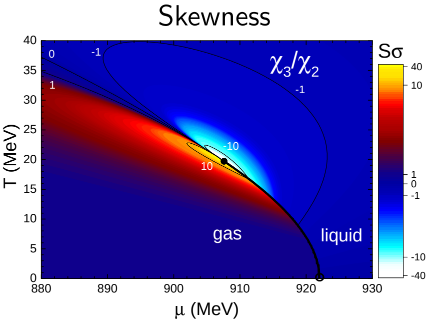


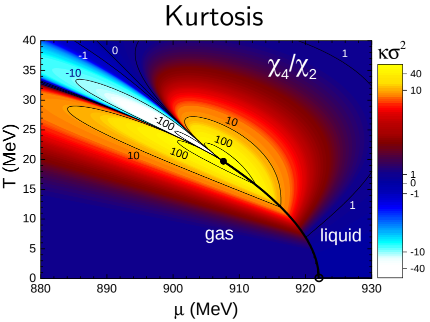
Baselines:

- Gaussian: $\kappa\sigma^2 = 0$
- Poisson: $\kappa\sigma^2 = 1$ $\quad\leftarrow\quad$ ideal gas in grand canonical ensemble

At CP: $\kappa\sigma^2 \sim \xi^7$

QvdW gas of nucleons: skewness and kurtosis



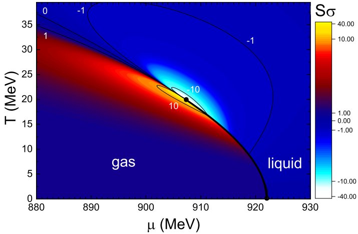


Signals from skewness and kurtosis appear to survive far beyond the nuclear matter region!

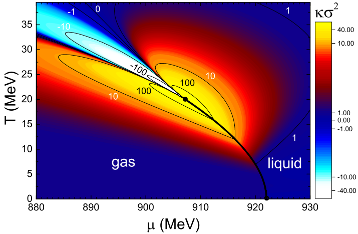
V.V., D. Anchishkin, M. Gorenstein, R. Poberezhnyuk, PRC 91, 064314 (2015)

QvdW gas of nucleons: skewness and kurtosis

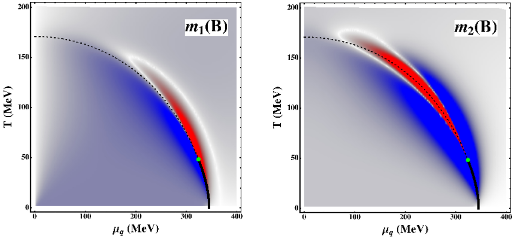
QvdW Skewness



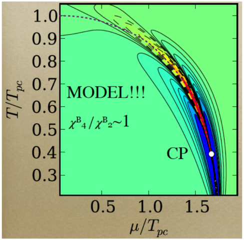
QvdW Kurtosis



NJL, J.W. Chen et al., PRD 93, 034037 (2016)



PQM, V. Skokov, QM2012



Fluctuation patterns in vdW very similar to effective QCD models

Fluctuation signals from nuclear matter critical point and from QCD critical point may very well look alike

QvdW-HRG model

Further applications require treatment of full hadron spectrum

QvdW-HRG model [**V.V.**, M.I. Gorenstein, H. Stoecker, PRL 118, 182301 (2017)]

- Hadron Resonance Gas (HRG) with attractive and repulsive vdW interactions between baryons
- vdW parameters $a = 329$ MeV fm^3 and $b = 3.42$ fm^3 tuned to nuclear ground state properties
- Critical point of nuclear matter at $T_c \simeq 19.7$ MeV, $\mu_c \simeq 908$ MeV

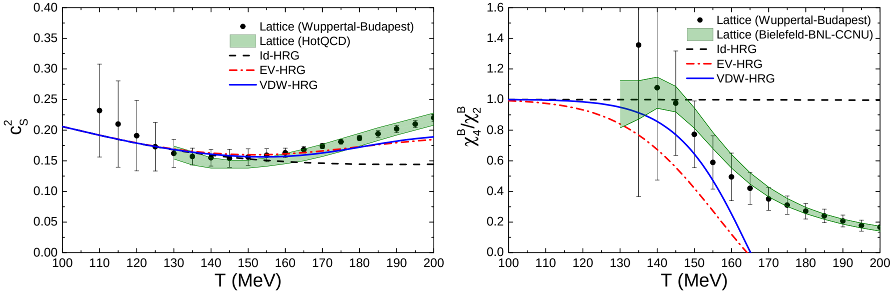
Three independent subsystems: mesons + baryons + antibaryons

$$p(T,\boldsymbol{\mu}) = P_M(T,\boldsymbol{\mu}) + P_B(T,\boldsymbol{\mu}) + P_{\bar{B}}(T,\boldsymbol{\mu}),$$

$$P_M(T,\boldsymbol{\mu}) = \sum_{j\in M} p_j^{\rm id}(T,\mu_j) \quad \text{and} \quad P_B(T,\boldsymbol{\mu}) = \sum_{j\in B} p_j^{\rm id}(T,\mu_j^{B*}) - a\, n_B^2$$

QvdW-HRG model at $\mu_B = 0$

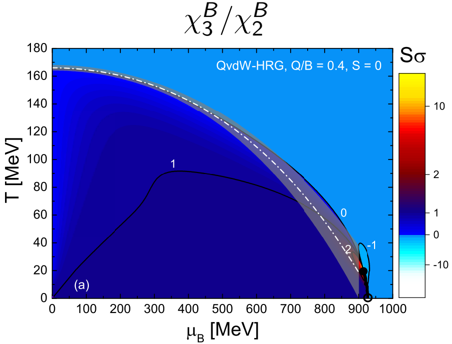
At $\mu = 0$ the model can be confronted with the lattice data

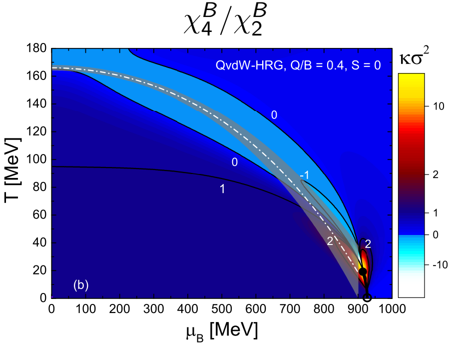


Strong effects even at $\mu_B = 0$!

What about intermediate collision energies?

QvdW-HRG model: fluctuations in T-μ_B plane





Chemical freeze-out curve from *Cleymans et al., Phys. Rev. C 73, 034905 (2006)*

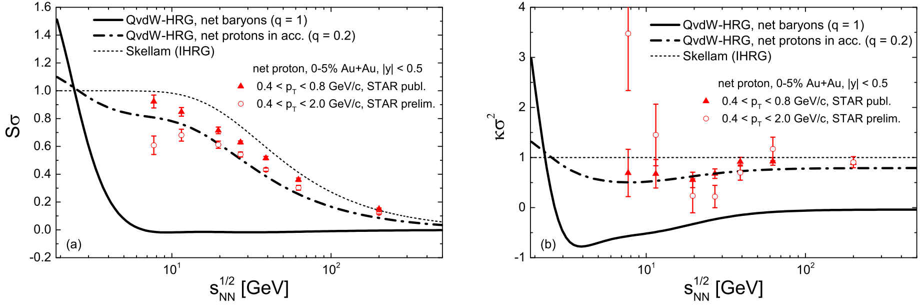
Critical point of nuclear matter shines brightly in fluctuation observables, across the whole region of phase diagram probed by heavy-ion collisions

QvdW-HRG model: collision energy dependence

Calculating fluctuations along the “freeze-out” curve

Acceptance effects (protons instead of baryons, momentum cut) modeled *schematically*, by applying the *binomial filter* [M. Kitazawa, M. Asakawa, PRC ’12; A. Bzdak, V. Koch, PRC ’12]

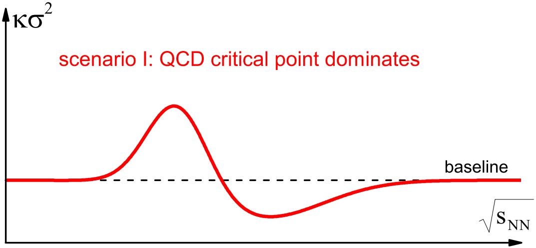
χ_3^B/χ_2^B χ_4^B/χ_2^B



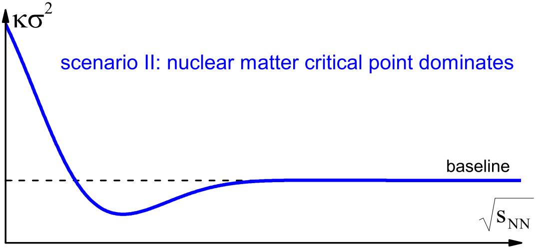
Effects of nuclear liquid-gas criticality:

- Non-monotonic collision energy dependence
- Net proton quite different from net baryon

Scenarios for collision energy dependence



M. Stephanov, JPG "11



V.V., *L. Jiang, M. Gorenstein, H. Stoecker, 1711.07260*

Can the scenarios be distinguished? Need data at lower energies...

Opportunities for HADES, CBM, NA61/SHINE, STAR!

Summary

- Nuclear liquid-gas criticality causes non-monotonic energy dependence of skewness and kurtosis
- Enhancement of fluctuations expected as collision energy is lowered, new data can verify this
- Net proton fluctuations $\neq$ Net baryon fluctuations, for more reasons than just baryon number conservation

Summary

- Nuclear liquid-gas criticality causes non-monotonic energy dependence of skewness and kurtosis
- Enhancement of fluctuations expected as collision energy is lowered, new data can verify this
- Net proton fluctuations $\neq$ Net baryon fluctuations, for more reasons than just baryon number conservation

Thanks for your attention!

Backup slides

Strongly intensive measures near CP

Strongly intensive (SI) measures [M.I. Gorenstein, M. Gazdzicki, PRC '11]

- Independent of volume fluctuations, mitigate impact parameter fluctuations
- Can be constructed from moments of two extensive quantities

$$\Delta[A,B] = C_{\Delta}^{-1}\left[\langle A\rangle\omega[B] - \langle B\rangle\omega[A]\right]$$
$$\Sigma[A,B] = C_{\Sigma}^{-1}\left[\langle A\rangle\omega[B] + \langle B\rangle\omega[A] - 2(\langle AB\rangle - \langle A\rangle\langle B\rangle)\right]$$

- For most models without PT and CP equal/close to unity
- Supposedly show critical behavior, but no model calculation
- Used in search for CP, e.g. NA61/SHINE program[1]

SI measures of excitation energy and particle number fluctuations in vdW

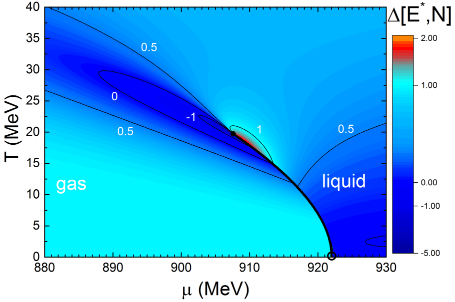
$$\Delta[E^*,N] \;=\; 1 \;-\; \frac{an(2\bar{\epsilon}_{\rm id} - 3an)}{\overline{\epsilon^2_{\rm id}} - \bar{\epsilon}^2_{\rm id}}\,\omega[N], \quad \Sigma[E^*,N] \;=\; 1 + \frac{a^2n^2}{\overline{\epsilon^2_{\rm id}} - \bar{\epsilon}^2_{\rm id}}\,\omega[N].$$

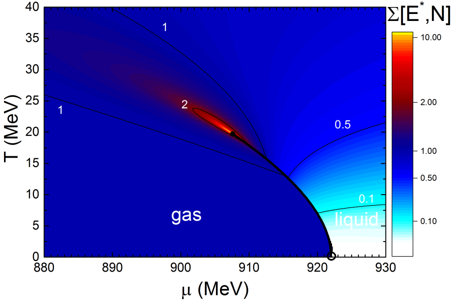
- Critical behavior is present due to criticality of $\omega[N]$ term[2]
- If a=0 then no signal at all! Deviations really stem from criticality.

[1] Gazdzicki, Seyboth, APP '15; E. Andronov, 1610.05569; A. Seryakov, 1704.00751

[2] V.V., Poberezhnyuk, Anchishkin, Gorenstein, J. Phys. A 49, 015003 (2016)

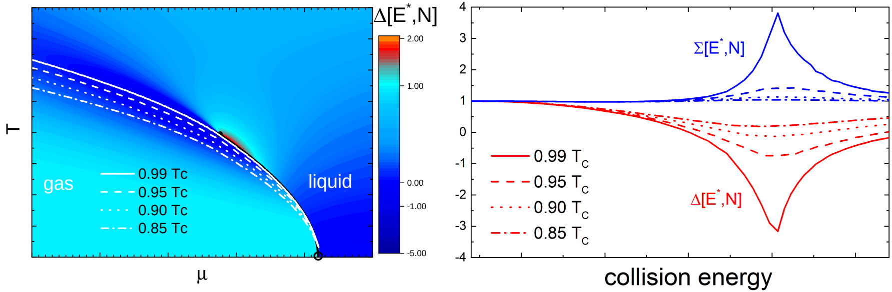
Strongly intensive measures in T-μ plane: Nuclear matter





- Both $\Delta[E^*, N]$ and $\Sigma[E^*, N]$ signal nuclear liquid-gas criticality
- $\Sigma[E^*, N] > 0$ always. However, $\Delta[E^*, N]$ can be both positive and negative

Strongly intensive measures in T-μ plane: Nuclear matter



- Both $\Delta[E^*, N]$ and $\Sigma[E^*, N]$ signal nuclear liquid-gas criticality
- $\Sigma[E^*, N] > 0$ always. However, $\Delta[E^*, N]$ can be both positive and negative
- Non-monotonous energy/system-size dependence of $\Delta[E^*, N]$ and $\Sigma[E^*, N]$ in a scenario with CP
- $\Delta[E^*, N]$ is more sensitive than $\Sigma[E^*, N]$ to proximity of CP