

QCD phase structure from fluctuations in heavy-ion collisions

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QCD phase structure

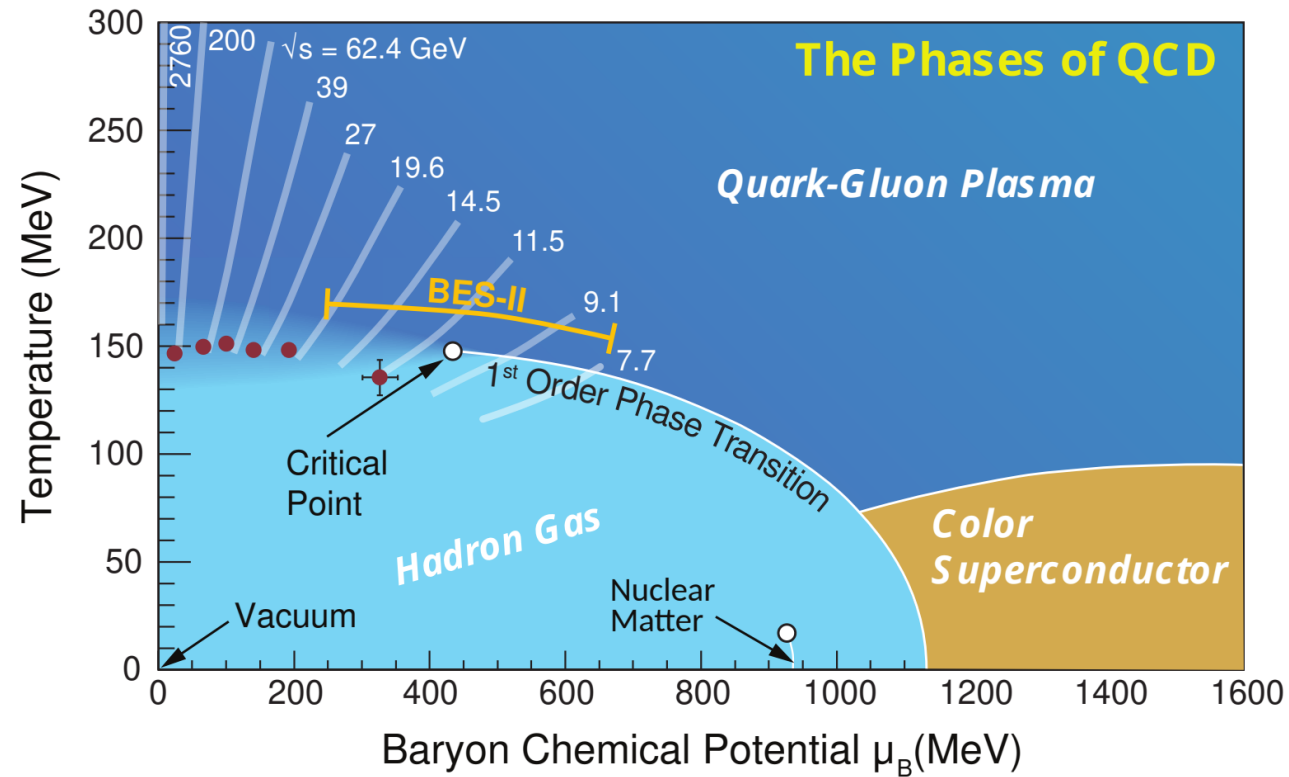


Figure from Bzdak et al., Phys. Rept. '20

- Dilute hadron gas at low T & ρ_B due to confinement, quark-gluon plasma high T & ρ_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured

Is there a critical point and how to find it with heavy-ion collisions?

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln \langle e^{tN} \rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

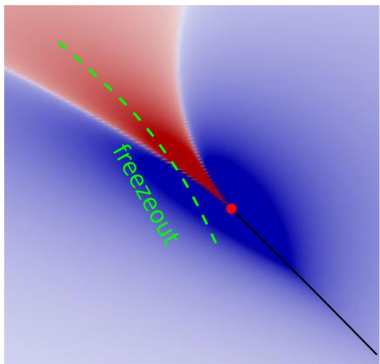
$$\kappa_n \propto \frac{\partial^n (\ln Z^{\text{gce}})}{\partial \mu^n}$$

Grand partition function

$$\ln Z^{\text{gce}}(T, V, \mu) = \ln \left[\sum_N e^{\mu N/T} Z^{\text{ce}}(T, V, N) \right]$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- **(QCD) critical point** – large correlation length, critical fluctuations of baryon number



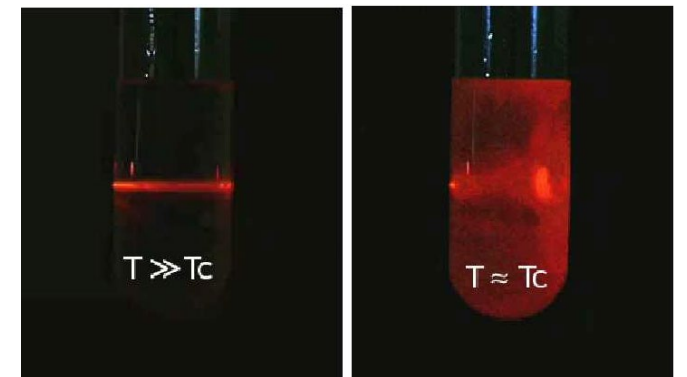
M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

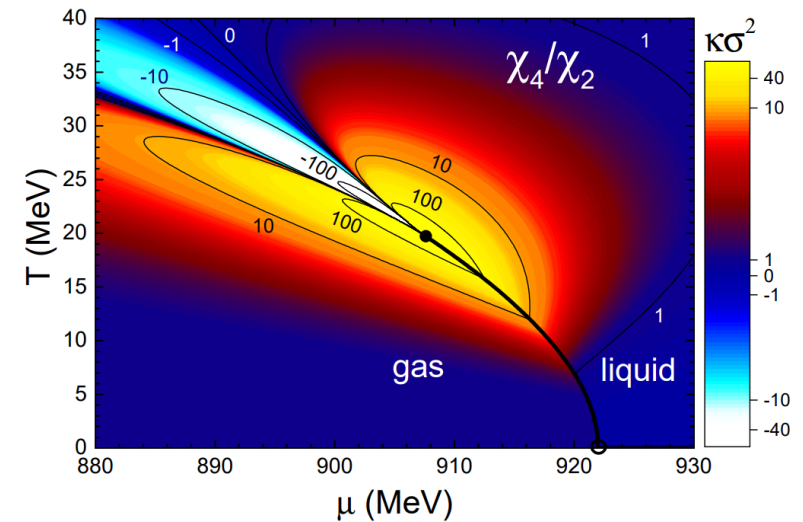
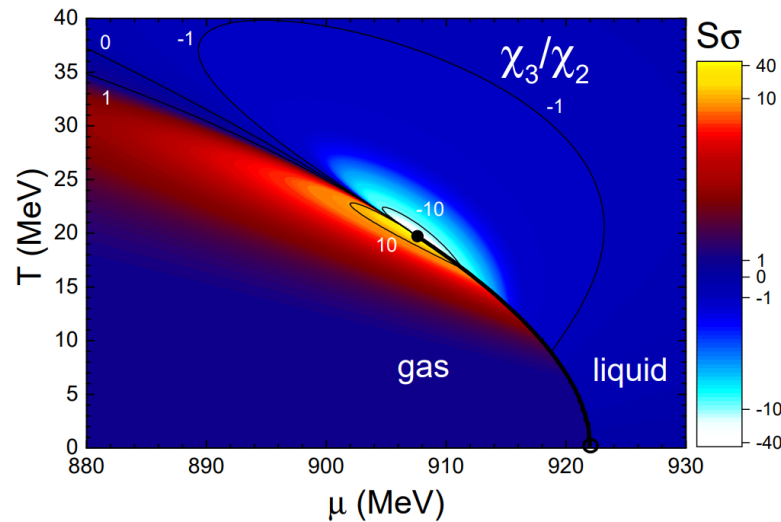
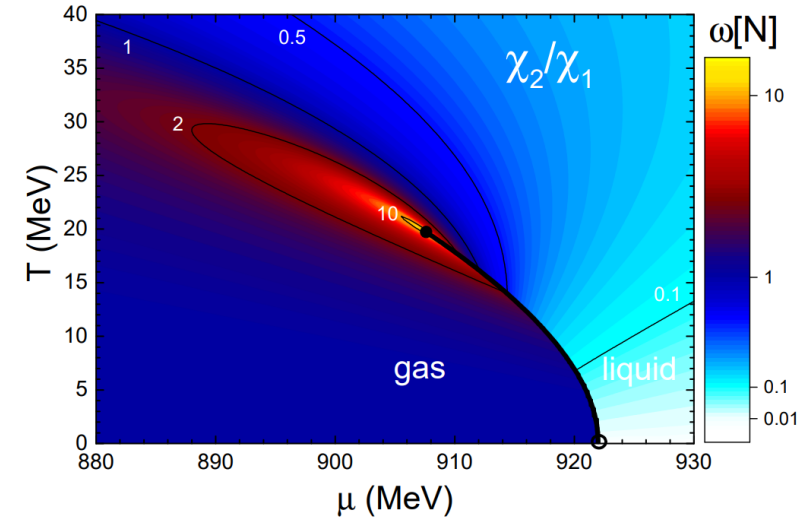
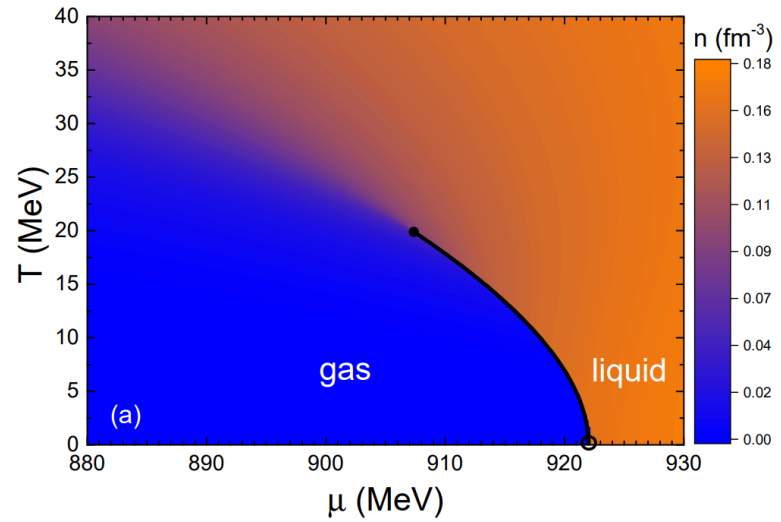
$$\xi \rightarrow \infty$$

Looking for enhanced fluctuations
and non-monotonicities

Critical opalescence



Example: Nuclear liquid-gas transition



VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Example: Lennard-Jones fluid

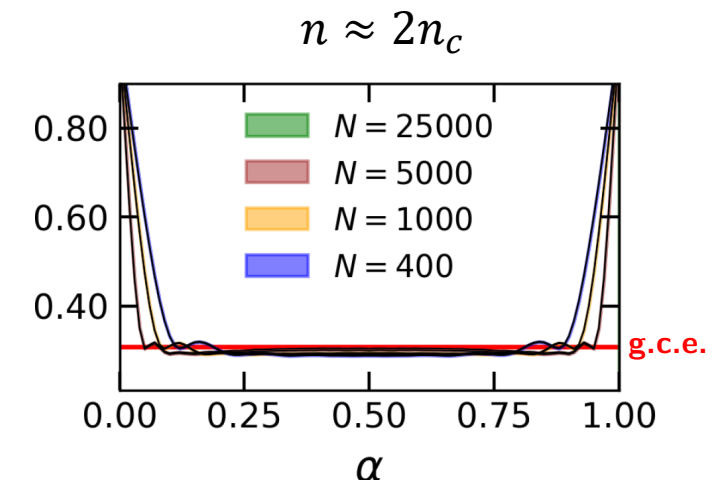
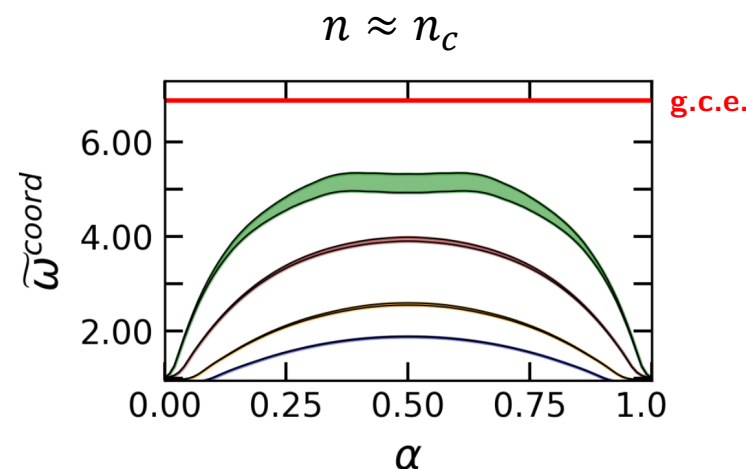
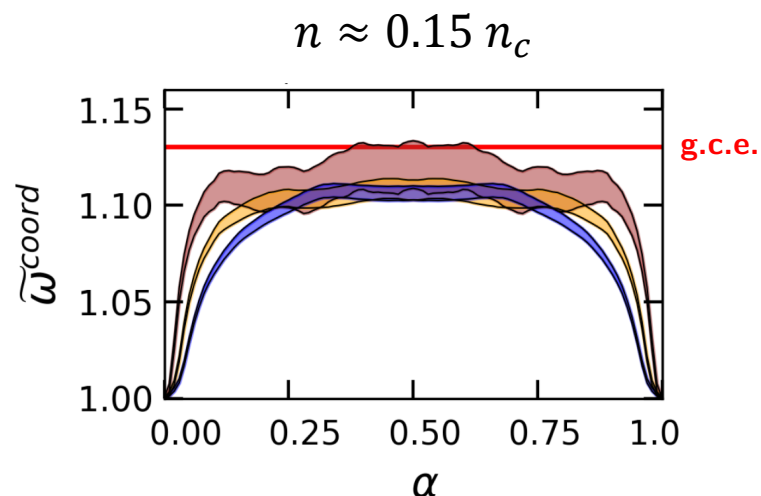
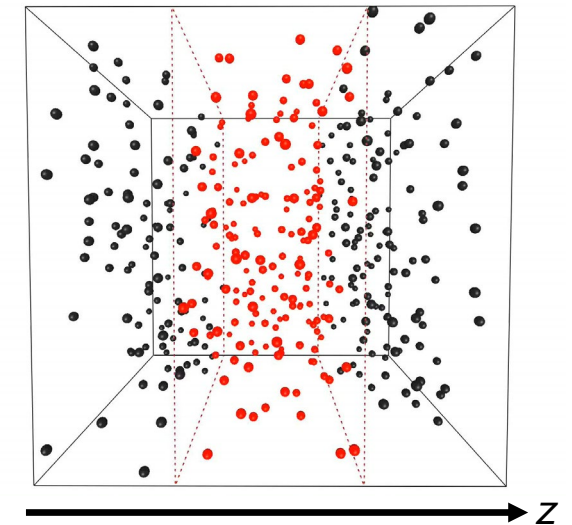
Kuznetsov, Savchuk, Gorenstein, Koch, VV, Phys. Rev. C 105, 044903 (2022)

Classical molecular dynamics simulations* of a **Lennard-Jones fluid** along the (super)critical isotherm of the liquid-gas transition

Microcanonical (const. EVN) ensemble with periodic boundary conditions

Variance of conserved particle number distribution inside coordinate space subvolume $|z| < z^{max}$ as time average

$$\tilde{\omega}^{\text{coord}} = \frac{1}{1 - \alpha} \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$

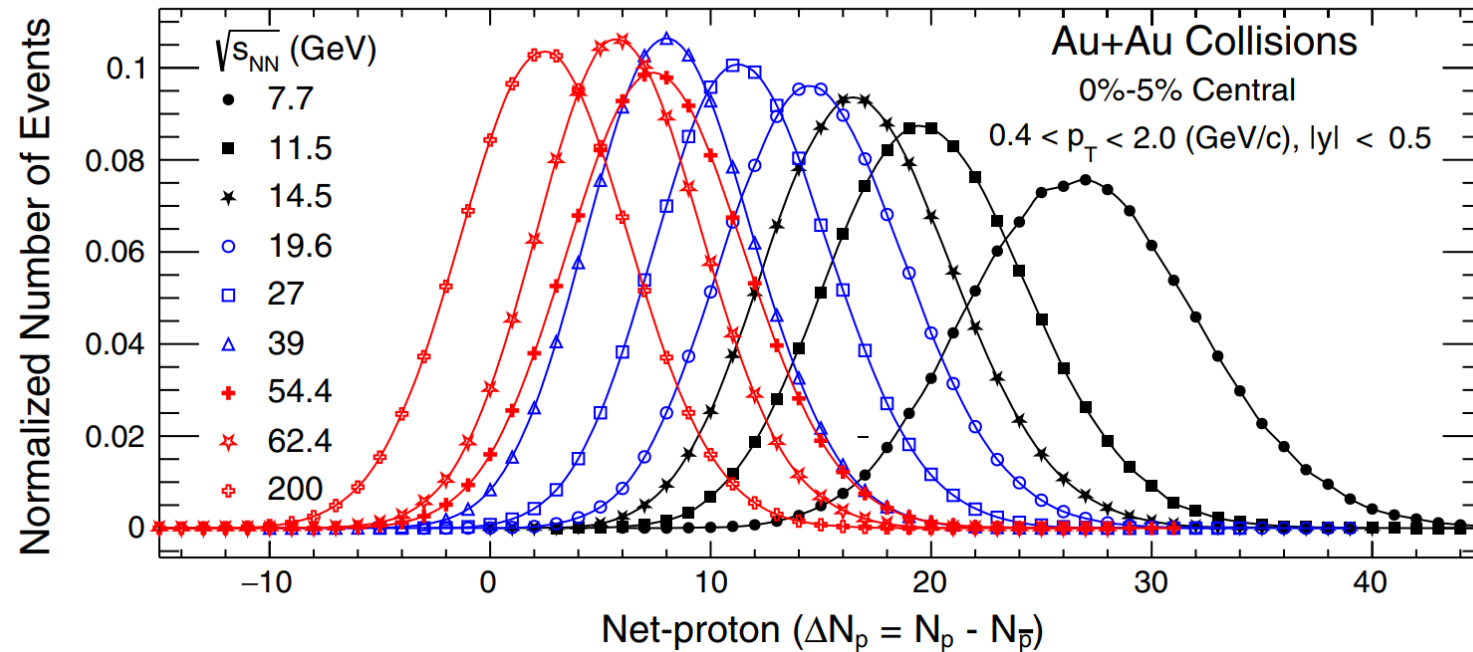


*Molecular dynamics code from <https://github.com/vlvovch/lennard-jones-cuda>

Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons $P(\Delta N_p) \sim \frac{N_{\text{events}}(\Delta N_p)}{N_{\text{events}}^{\text{total}}}$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)

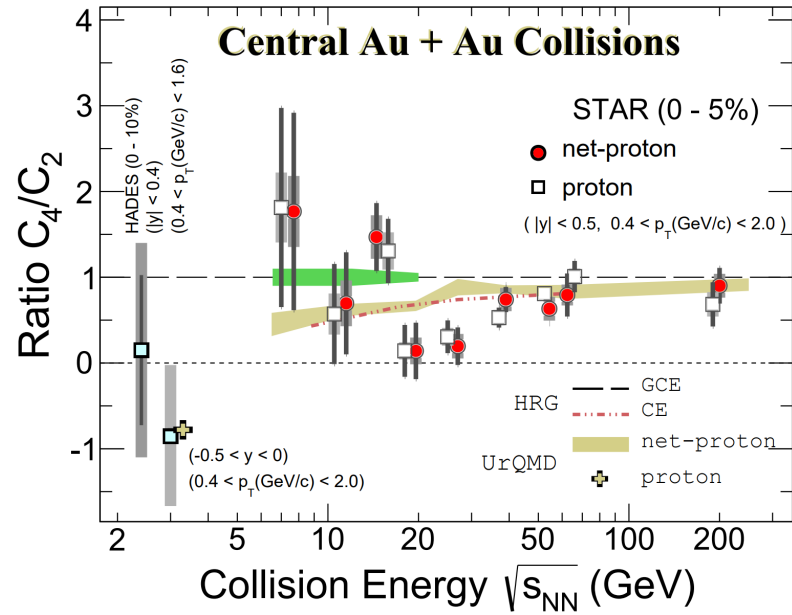


Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

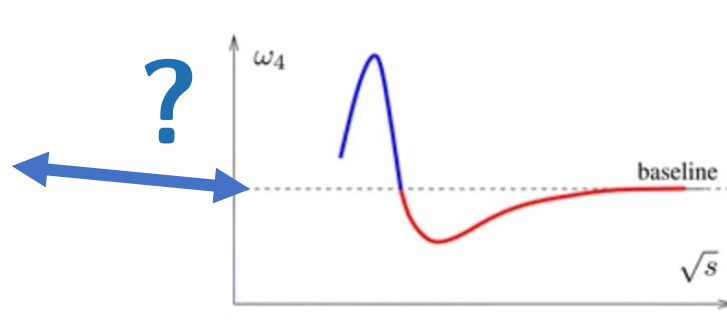
$$\frac{\kappa_2}{\langle N \rangle}, \quad \frac{\kappa_3}{\kappa_2}, \quad \frac{\kappa_4}{\kappa_2}$$

Experimental measurements

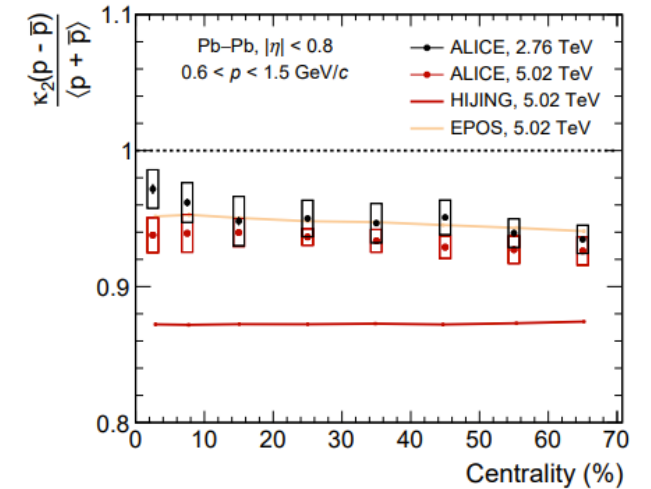
Beam energy scan in search for the critical point (STAR Coll.)



STAR Coll., Phys. Rev. Lett. 126, 092301 (2021); arXiv:2112.00240



M. Stephanov, Phys. Rev. Lett. (2011)



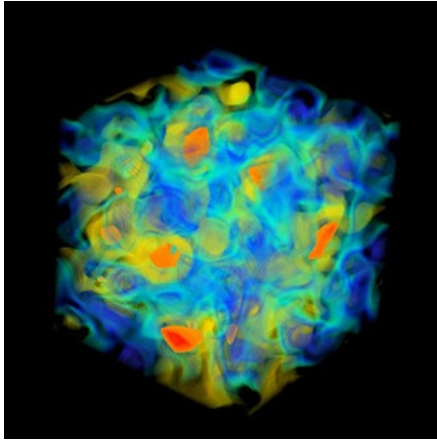
ALICE Coll., arXiv:2206.03343

Reduced errors (better statistics), more energies, to come soon from RHIC-BES-II program, STAR-FXT etc.

Can we learn more from the more accurate data available for κ_2 and κ_3 ?

Theory vs experiment: Challenges for fluctuations

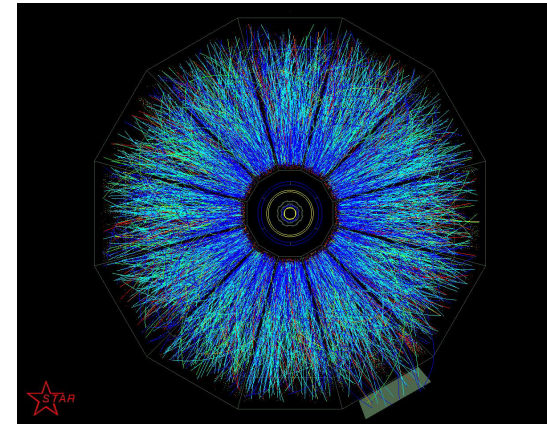
Theory



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

Need dynamical description

Dynamical approaches to the QCD critical point search

1. Dynamical model calculations of critical fluctuations

- Fluctuating hydrodynamics
- Equation of state with tunable critical point [P. Parotto et al, Phys. Rev. C 101, 034901 (2020)]

Under development within the Beam Energy Scan Theory (BEST) Collaboration



[X. An et al., Nucl. Phys. A 1017, 122343 (2022)]

2. Molecular dynamics with a critical point

V. Kuznetsov et al., Phys. Rev. C 105, 044903 (2022)

3. Deviations from precision calculations of non-critical fluctuations

- Include essential non-critical contributions to (net-)proton number cumulants
- Exact **baryon conservation** + **hadronic interactions** (hard core repulsion)
- Based on realistic hydrodynamic simulations tuned to bulk data

[VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

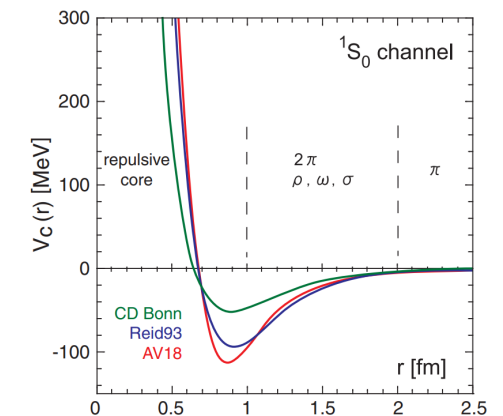


Figure from Ishii et al., PRL '07

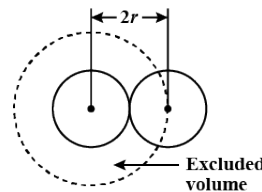
Excluded volume effect

Incorporate repulsive baryon (nucleon) hard core via **excluded volume**

VV, M.I. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

Amounts to a van der Waals correction for baryons in the HRG model

$$V \rightarrow V - bN$$



$$p_{B(\bar{B})}^{\text{ev}} = p_{B(\bar{B})}^{\text{id}} e^{-bp_{B(\bar{B})}^{\text{ev}}/T}$$

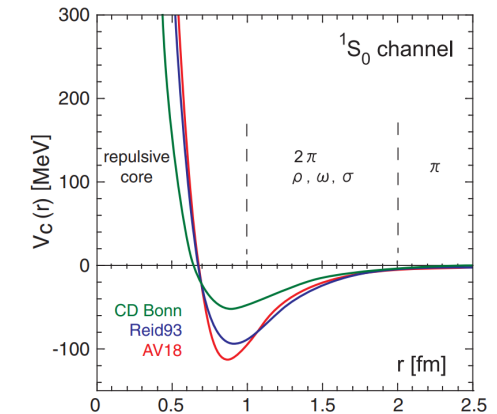


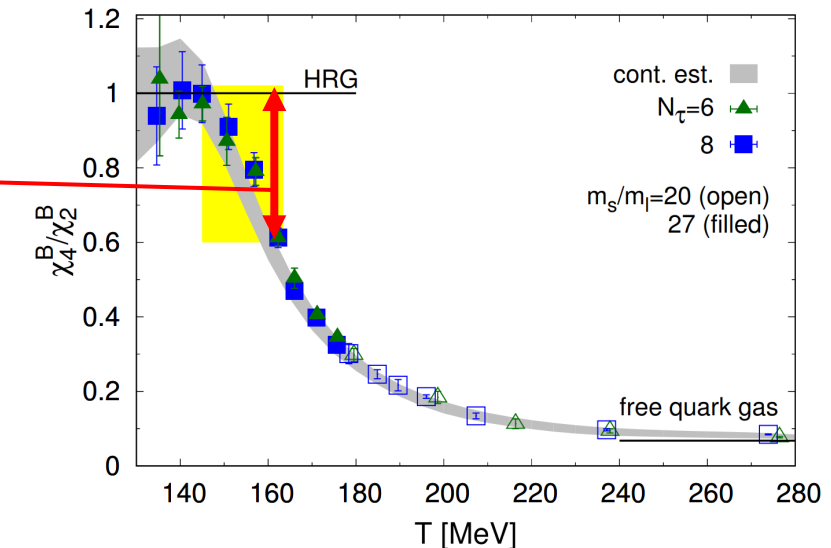
Figure from Ishii et al., PRL '07

- Net baryon kurtosis suppressed as in lattice QCD

$$\frac{\chi_4^B}{\chi_2^B} \simeq 1 - 12b\phi_B(T) + O(b^2)$$

- Reproduces virial coefficients of baryon interaction from lattice QCD

Excluded volume from lattice QCD: $b \approx 1 \text{ fm}^3$



Hydrodynamic description within non-critical physics

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- Collision geometry based 3D initial state
 - Constrained to net proton distributions [Shen, Alzhroni, Phys. Rev. C '20]

- Viscous hydrodynamics evolution – MUSIC-3.0

- Energy-momentum and baryon number conservation
- Crossover equation of state based on lattice QCD
[Monnai, Schenke, Shen, Phys. Rev. C '19]

- Cooper-Frye particlization at $\epsilon_{sw} = 0.26 \text{ GeV/fm}^3$

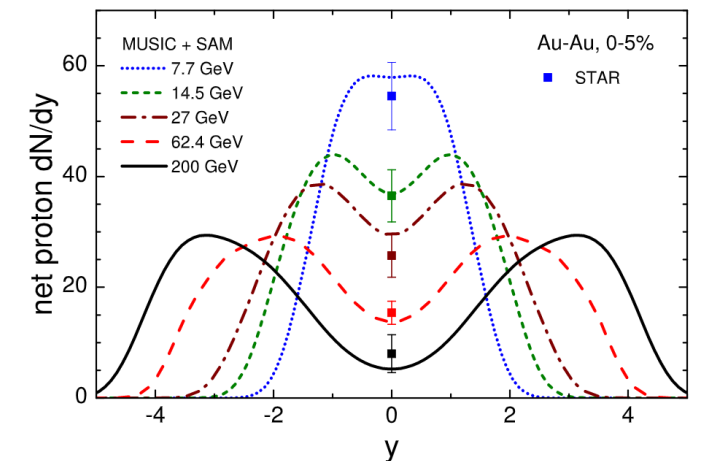
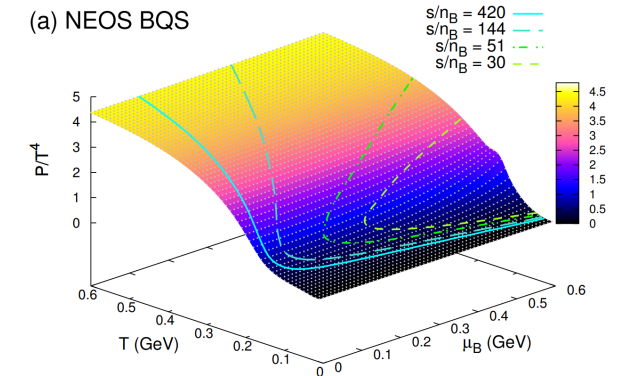
$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x) p^\mu \frac{d_j \lambda_j^{\text{ev}}(x)}{(2\pi)^3} \exp \left[\frac{\mu_j(x) - u^\mu(x) p_\mu}{T(x)} \right].$$

- Particlization respects QCD-based baryon number distribution

- Incorporated via baryon excluded volume $b = 1 \text{ fm}^3$
[VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]

- Incorporates exact global baryon conservation via a method SAM-2.0

[VV, Phys. Rev. C 105, 014903 (2022)]



Calculating cumulants from hydrodynamics

- Strategy:
 1. Calculate proton cumulants in the experimental acceptance in the grand-canonical limit
 2. Apply correction for the exact global baryon number conservation

First step:

- Sum contributions from each hypersurface element x_i at freeze-out
 - Cumulants of joint (anti)proton/(anti)baryon distribution

$$\kappa_{n,m}^{B^\pm, p^\pm, \text{gce}}(\Delta p_{\text{acc}}) = \sum_{i \in \sigma} \delta \kappa_{n,m}^{B^\pm, p^\pm, \text{gce}}(x_i; \Delta p_{\text{acc}})$$

$$p_{\text{acc}}(x_i; \Delta p_{\text{acc}}) = \frac{\int_{p \in \Delta p_{\text{acc}}} \frac{d^3 p}{\omega_p} \delta \sigma_\mu(x_i) p^\mu f[u^\mu(x_i) p_\mu; T(x_i), \mu_j(x_i)]}{\int \frac{d^3 p}{\omega_p} \delta \sigma_\mu(x_i) p^\mu f[u^\mu(x_i) p_\mu; T(x_i), \mu_j(x_i)]}.$$


- To compute each contribution
 - GCE susceptibilities $\chi^{B^\pm}(x_i)$ define the distribution of the emitted (anti)baryons
 - Each baryon ends up in acceptance Δp_{acc} with binomial probability via the Cooper-Frye formula
 - Each baryon is a proton with probability $q(x_i) = \langle N_p(x_i) \rangle / \langle N_B(x_i) \rangle$

Correcting for baryon number conservation with SAM-2.0

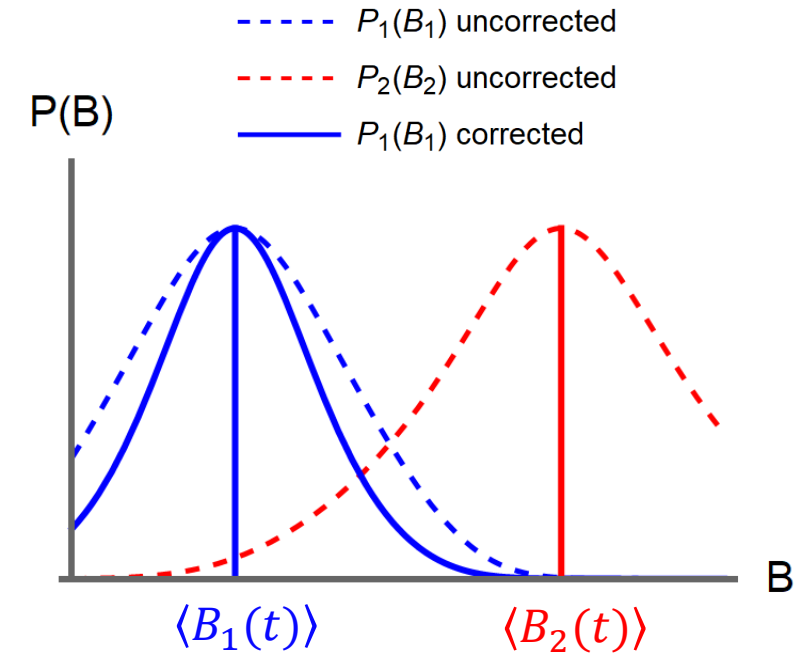
VV, arXiv:2107.00163 (to appear in PRC)

$$P_1^{\text{ce}}(B_1) \propto \sum_{B_1, B_2} P_1^{\text{gce}}(B_1) P_2^{\text{gce}}(B_2) \times \delta_{B, B_1+B_2}$$

SAM-1.0: uniform thermal system and **coordinate** space

SAM-2.0: apply the correction for *arbitrary* distributions inside and outside the acceptance that are peaked at the mean

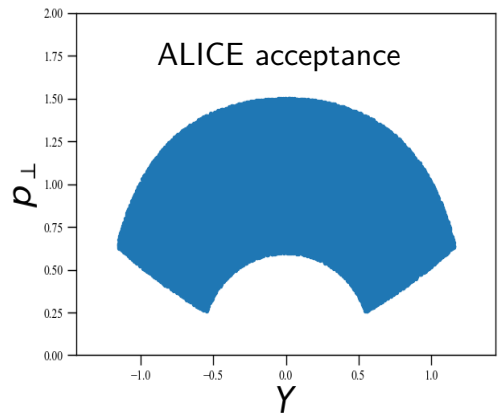
- Spatially inhomogeneous systems (e.g. RHIC)
- Momentum space
- Non-conserved quantities (e.g. proton number)
- Map “grand-canonical” cumulants inside and outside the acceptance to the “canonical” cumulants inside the acceptance



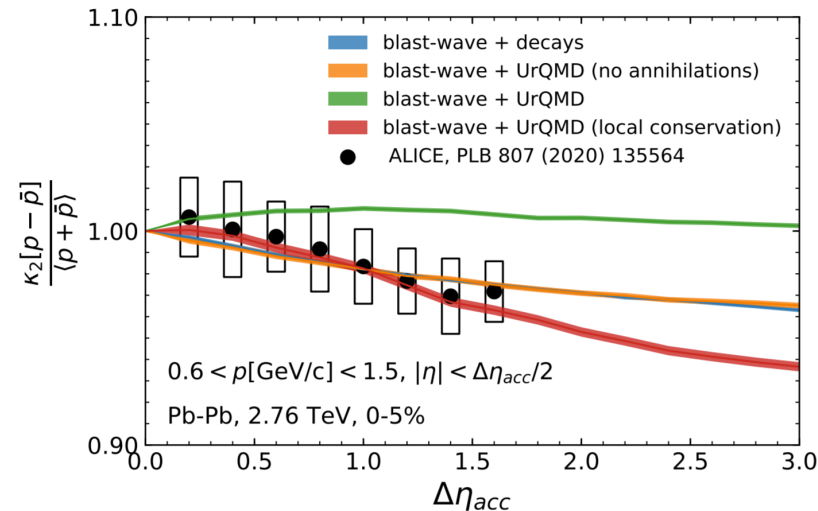
$$\kappa_{p,B}^{\text{in,ce}} = \text{SAM} \left[\kappa_{p,B}^{\text{in,gce}}, \kappa_{p,B}^{\text{out,gce}} \right]$$

Net-particle fluctuations at the LHC (blast-wave)

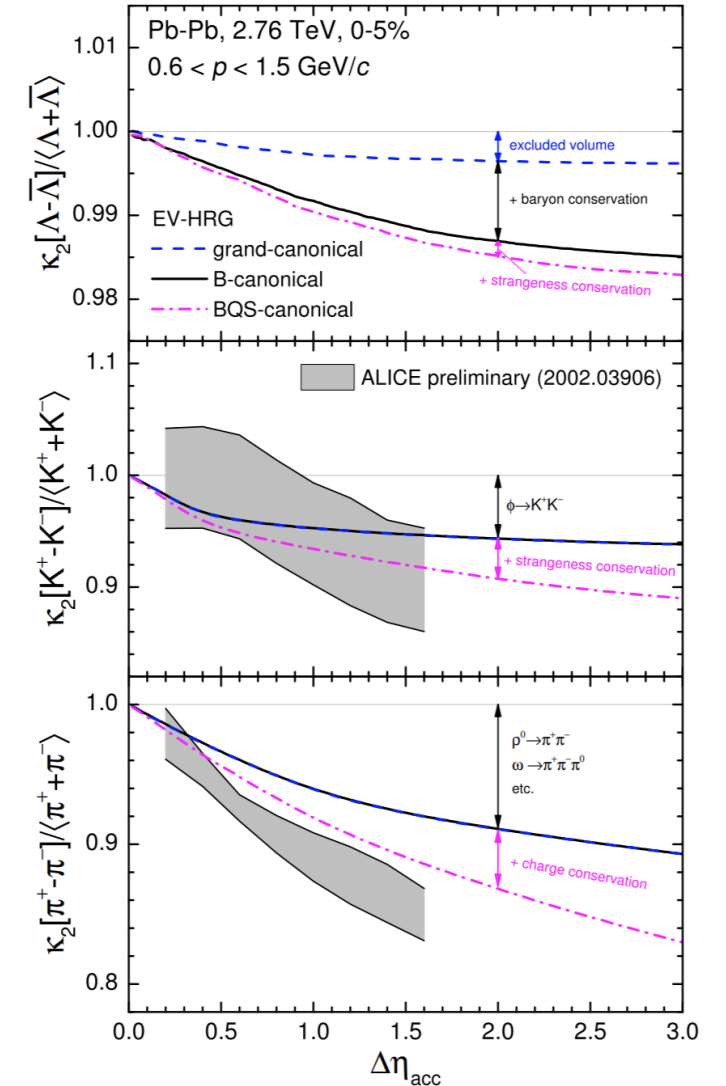
- Net protons described within errors and consistent with either
 - global** baryon conservation without $B\bar{B}$ annihilations
see e.g. ALICE Coll. arXiv:2206.03343
 - or **local** baryon conservation with $B\bar{B}$ annihilations
O. Savchuk et al., Phys. Lett. B 827, 136983 (2022)



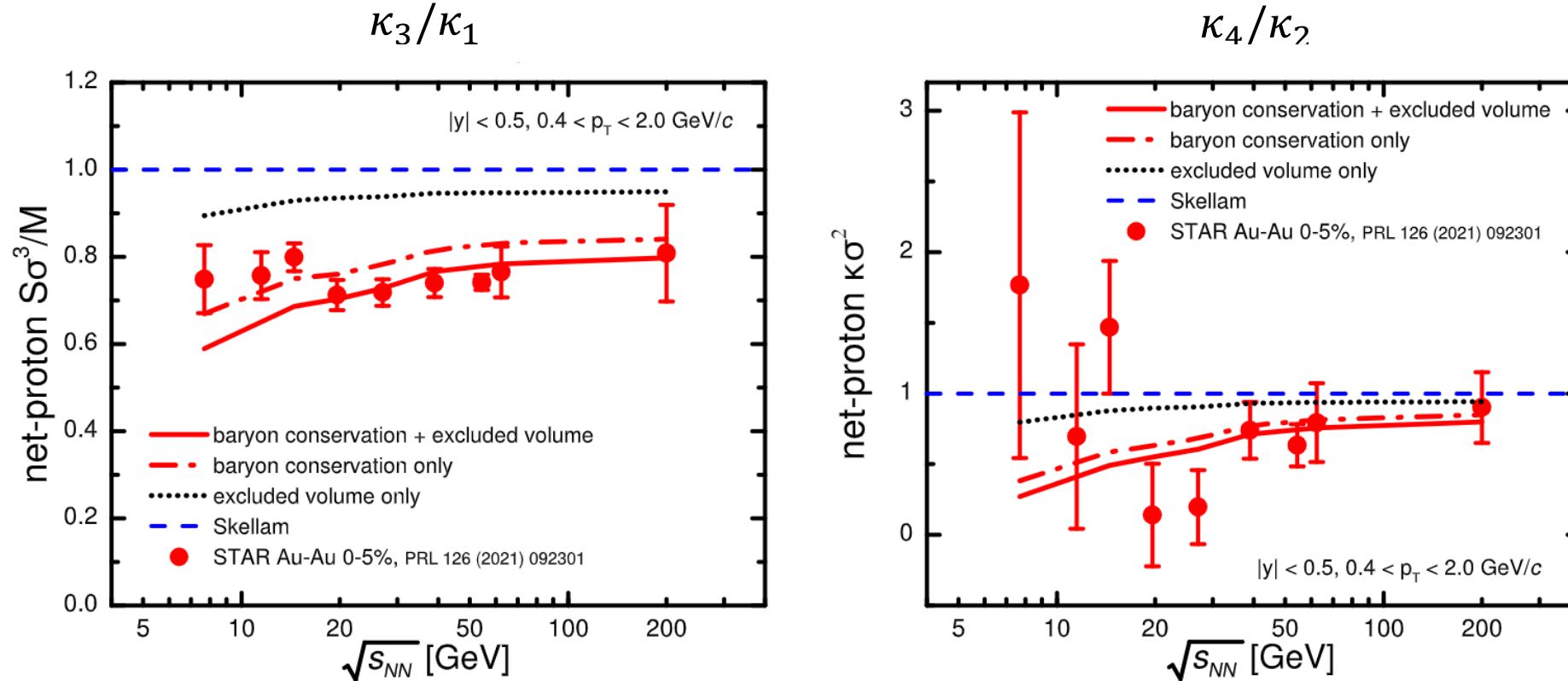
$0.6 < p < 1.5 \text{ GeV}/c$, $\Delta\eta_{acc} = 1.6$



- Large effect from resonance decays for pions and kaons + exact conservation of electric charge/strangeness

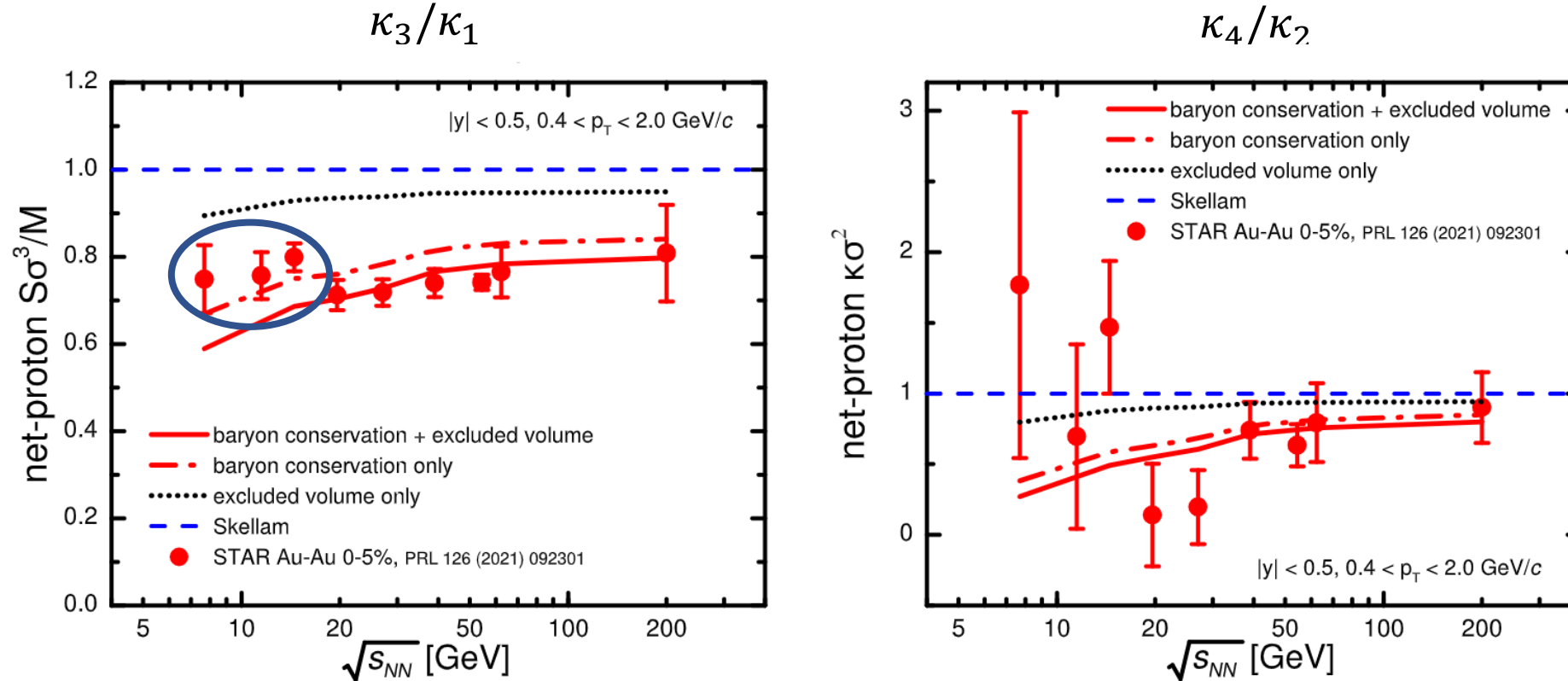


RHIC-BES: Net proton cumulant ratios (MUSIC)



- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (baryon conservation and repulsion)
- Effect from baryon conservation is larger than from repulsion
- Excess of skewness in data at $\sqrt{s_{NN}} < 20$ GeV – *hint of attractive interactions?*

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Correlation Functions

- Analyze genuine multi-particle correlations via **factorial cumulants** $\hat{C}_n$
[Bzdak, Koch, Strodthoff, Phys. Rev. C '17]

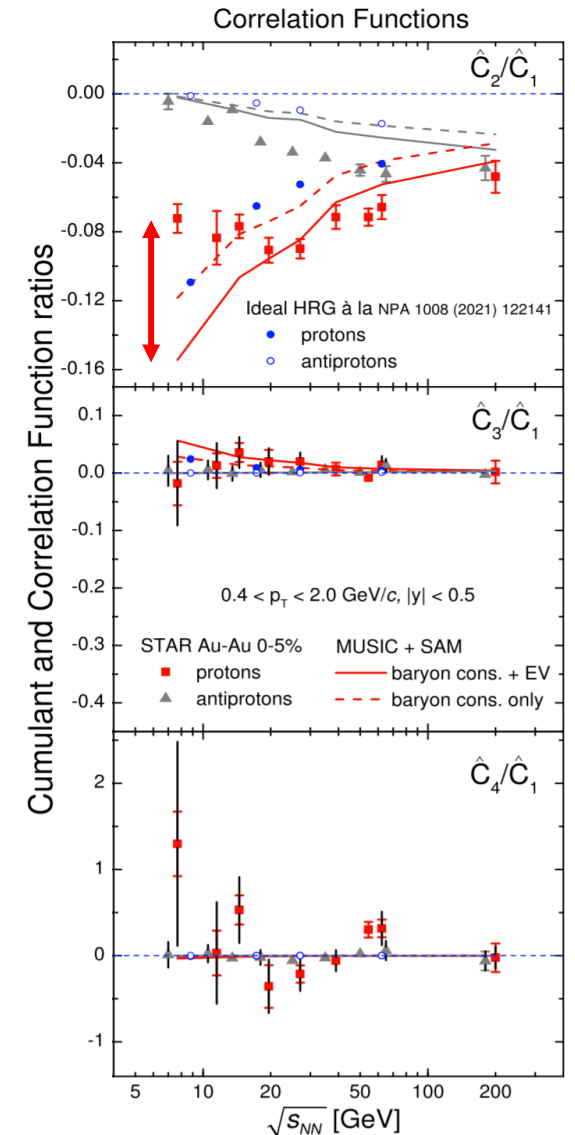
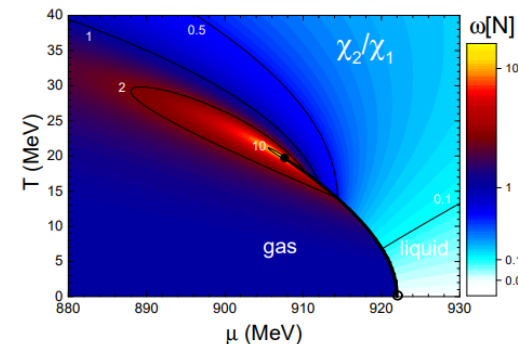
$$\begin{aligned}\hat{C}_1 &= \kappa_1, & \hat{C}_3 &= 2\kappa_1 - 3\kappa_2 + \kappa_3, \\ \hat{C}_2 &= -\kappa_1 + \kappa_2, & \hat{C}_4 &= -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.\end{aligned}$$

$$\hat{C}_n^{\text{cons}} \propto \alpha^n, \quad \hat{C}_n^{\text{EV}} \propto b^n$$

[Bzdak, Koch, Skokov, EPJC '17]

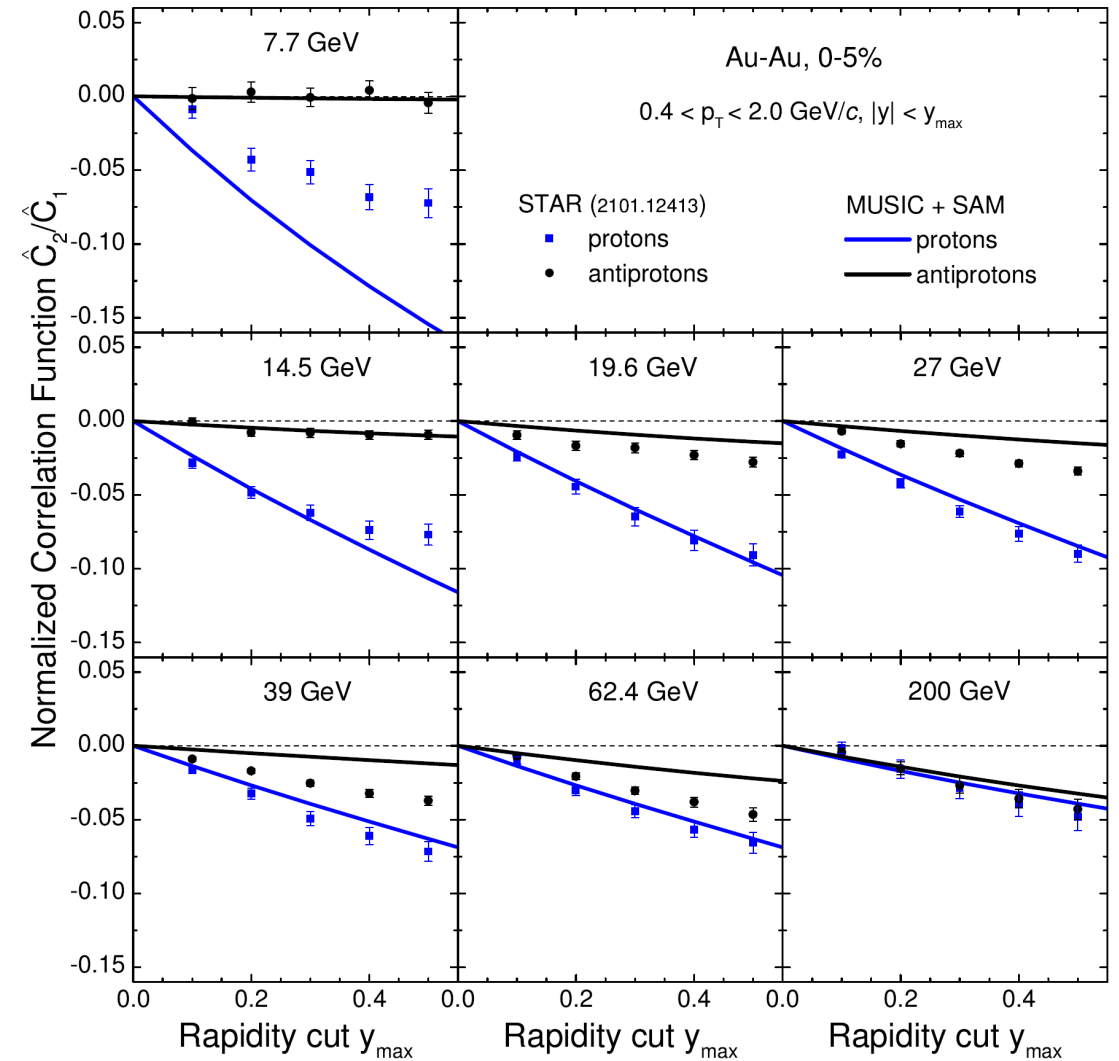
[VV et al, PLB '17]

- Three- and four-particle correlations are small without a CP
 - Multi-particle correlations expected near the critical point [Ling, Stephanov, PRC '15]
- Signals from the data at $\sqrt{s_{NN}} \leq 20$ GeV
 - Excess of two-proton correlations
 - Possibility of significant 4-proton correlations
 - Critical point?**



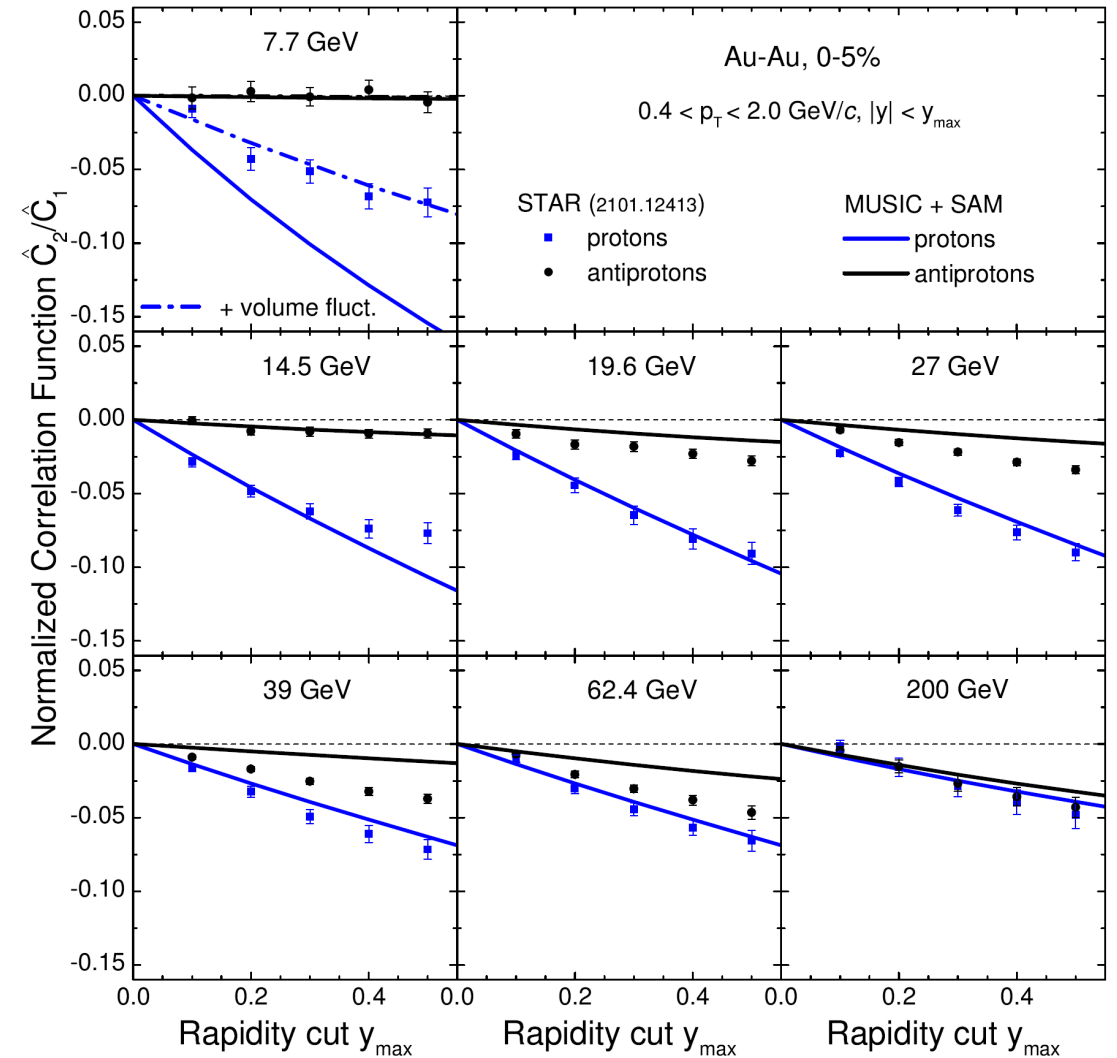
Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?



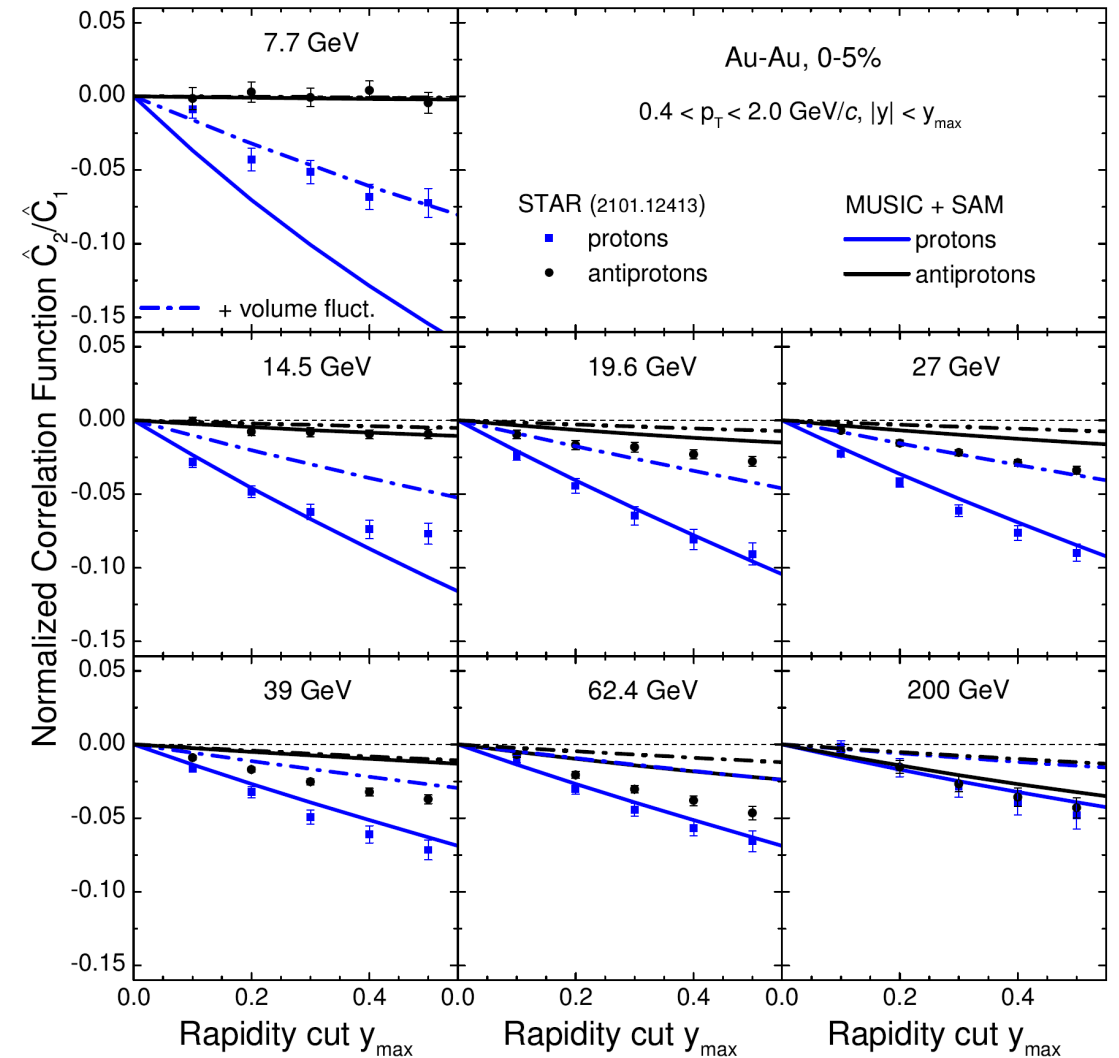
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- **Volume fluctuations?** [Skokov, Friman, Redlich, PRC '13]
 - $C_2/C_1 \neq C_1 * v_2$



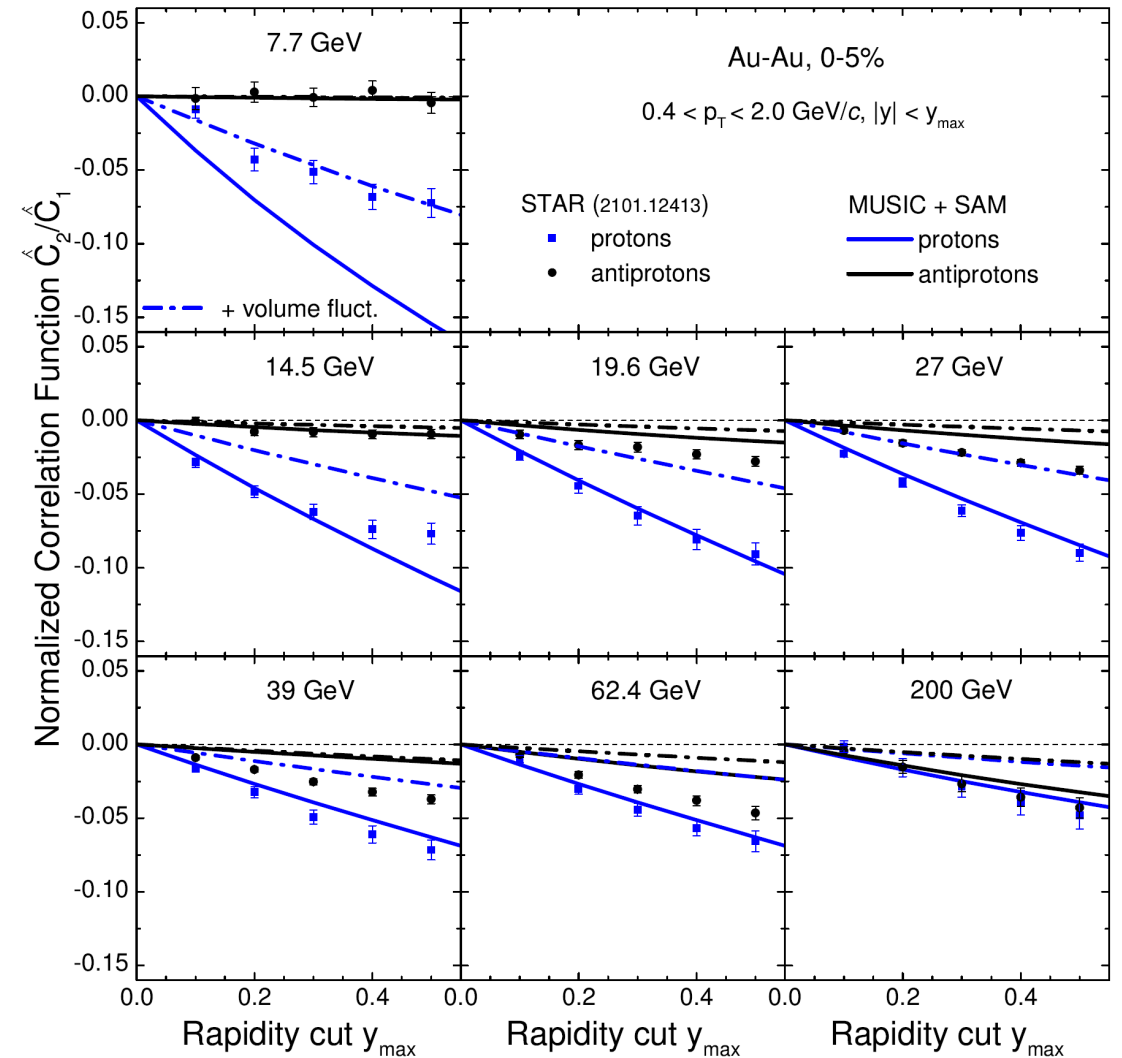
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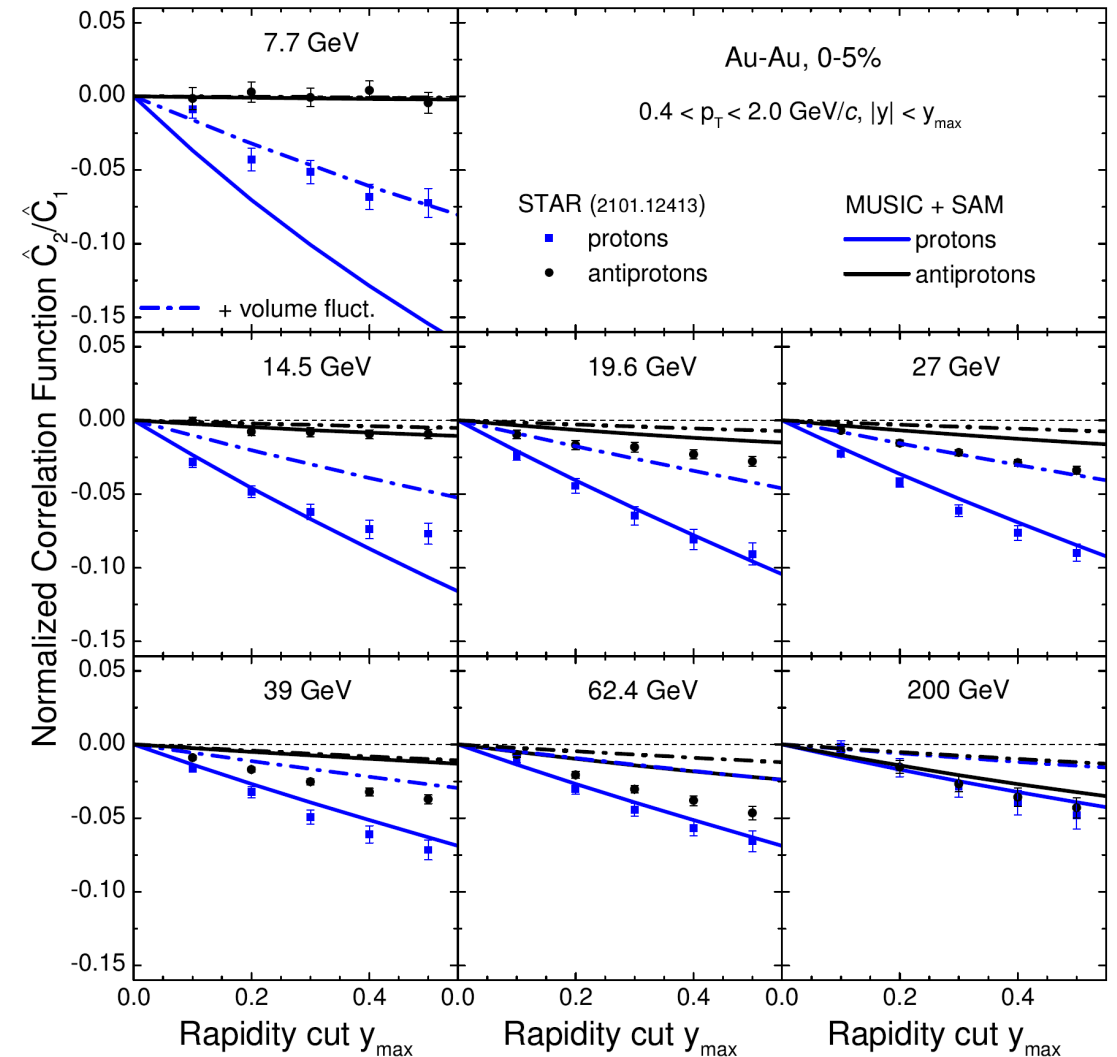
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- **Exact electric charge conservation?**
 - Worsens the agreement at $\sqrt{s_{NN}} \leq 14.5$, higher energies virtually unaffected



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- Exact electric charge conservation?
 - Worsens the agreement at $\sqrt{s_{NN}} \leq 14.5$, higher energies virtually unaffected
- **Attractive interactions?**
 - Could work if baryon repulsion turns into attraction in the high- μ_B regime
 - **Critical point?**

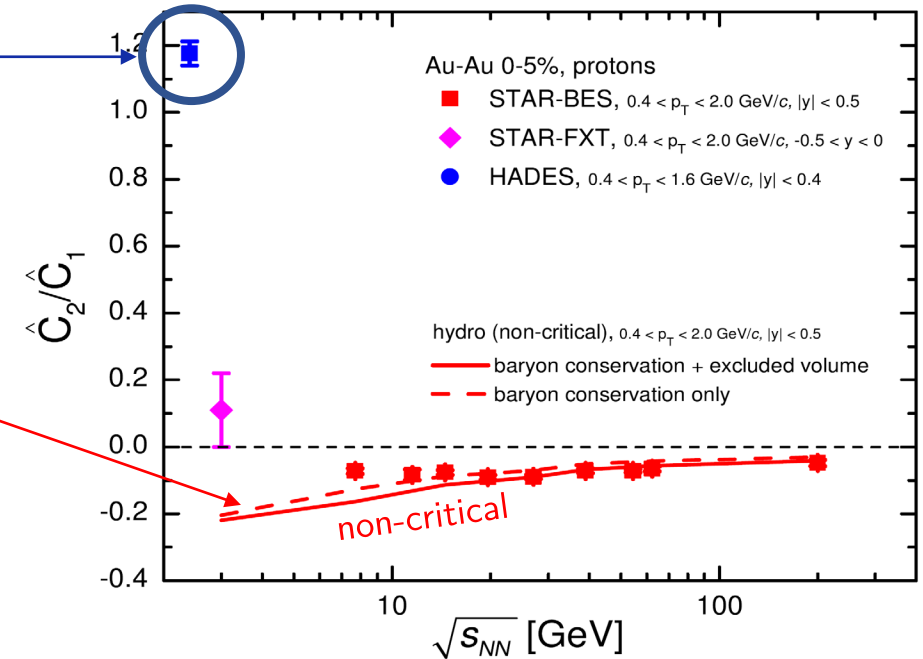


Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV

- Intriguing hint from HADES @ $\sqrt{s_{NN}} = 2.4$ GeV: huge excess of two-proton correlations!

[HADES Collaboration, Phys. Rev. C 102, 024914 (2020)]

- No change of trend in the non-critical reference
- Additional mechanisms:
 - Nuclear liquid-gas transition
 - Light nuclei formation
- Fill the gap with ongoing/future data from STAR-FXT (e.g. [arXiv:2112.00240](https://arxiv.org/abs/2112.00240)), future experiments like CBM-FAIR



Take a closer look at the HADES data

Thermodynamic analysis of HADES data

VV, Koch, arXiv:2204.00137

- **Single freeze-out scenario:** Emission from Siemens-Rasmussen hypersurface with Hubble-like flow

→ Pion and proton spectra o.k.

[S. Harabasz et al., PRC 102, 054903 (2020)]

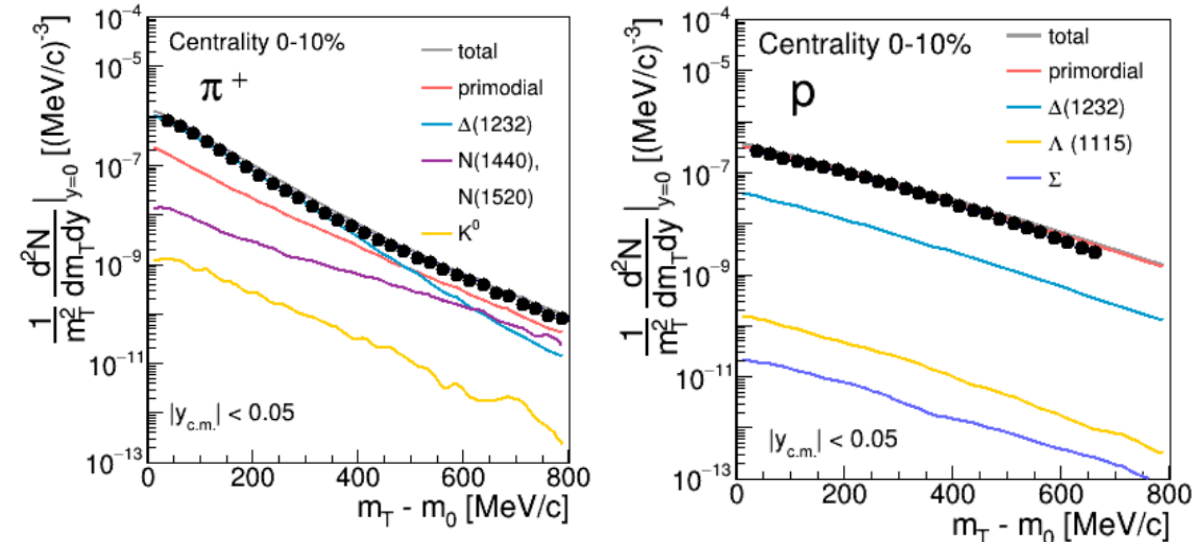
- Uniform $T \approx 70$ MeV, $\mu_B \approx 875$ MeV across the fireball

[A. Motornenko et al., PLB 822, 136703 (2021)]

- **Fluctuations:**

- Same as before but incorporate additional binomial filtering to account for protons bound in light nuclei
- Uniform fireball → Final proton cumulants are linear combinations of baryon susceptibilities χ_n^B at freeze-out

$$\kappa_n^p = \sum_{m=1}^n \alpha_{n,m} \chi_m^B$$



Extract χ_n^B directly from experimental data

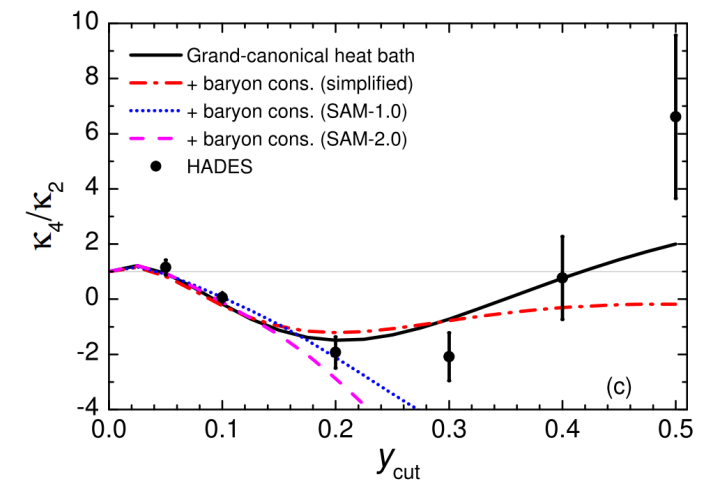
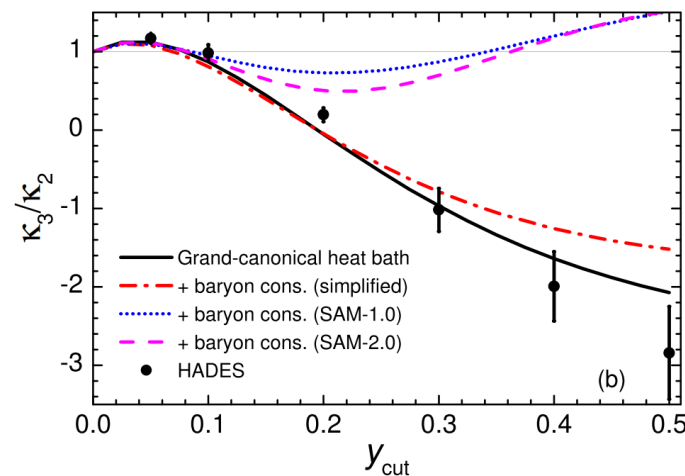
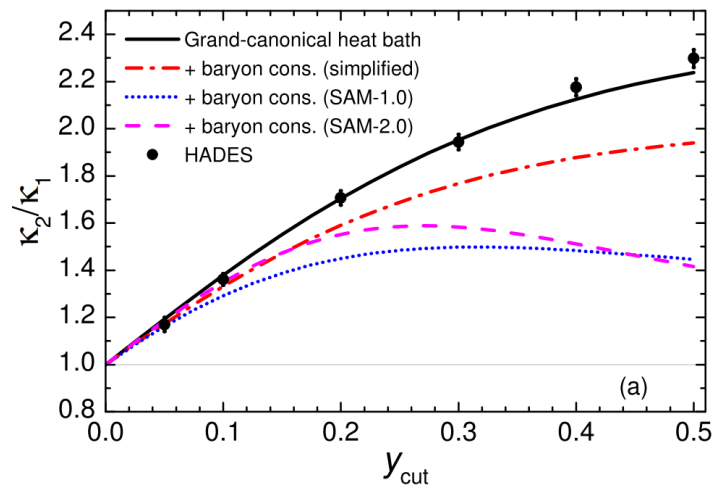
A closer look at the HADES data

VV, Koch, arXiv:2204.00137

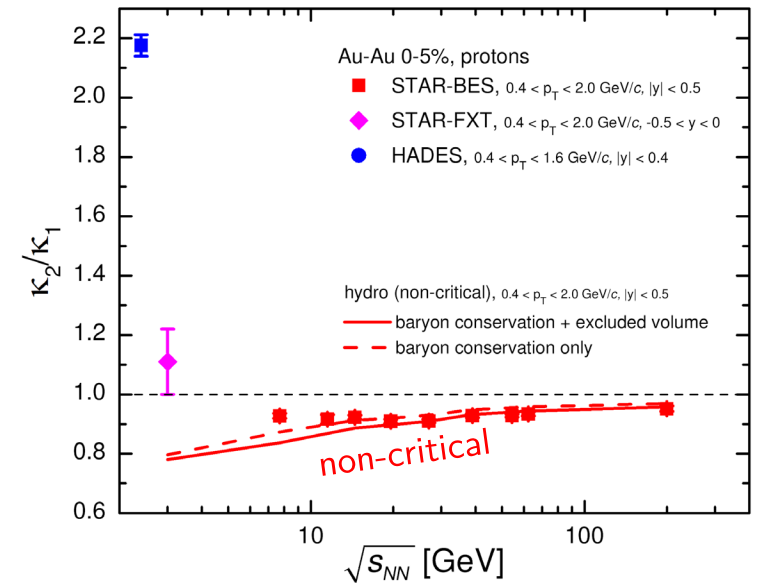
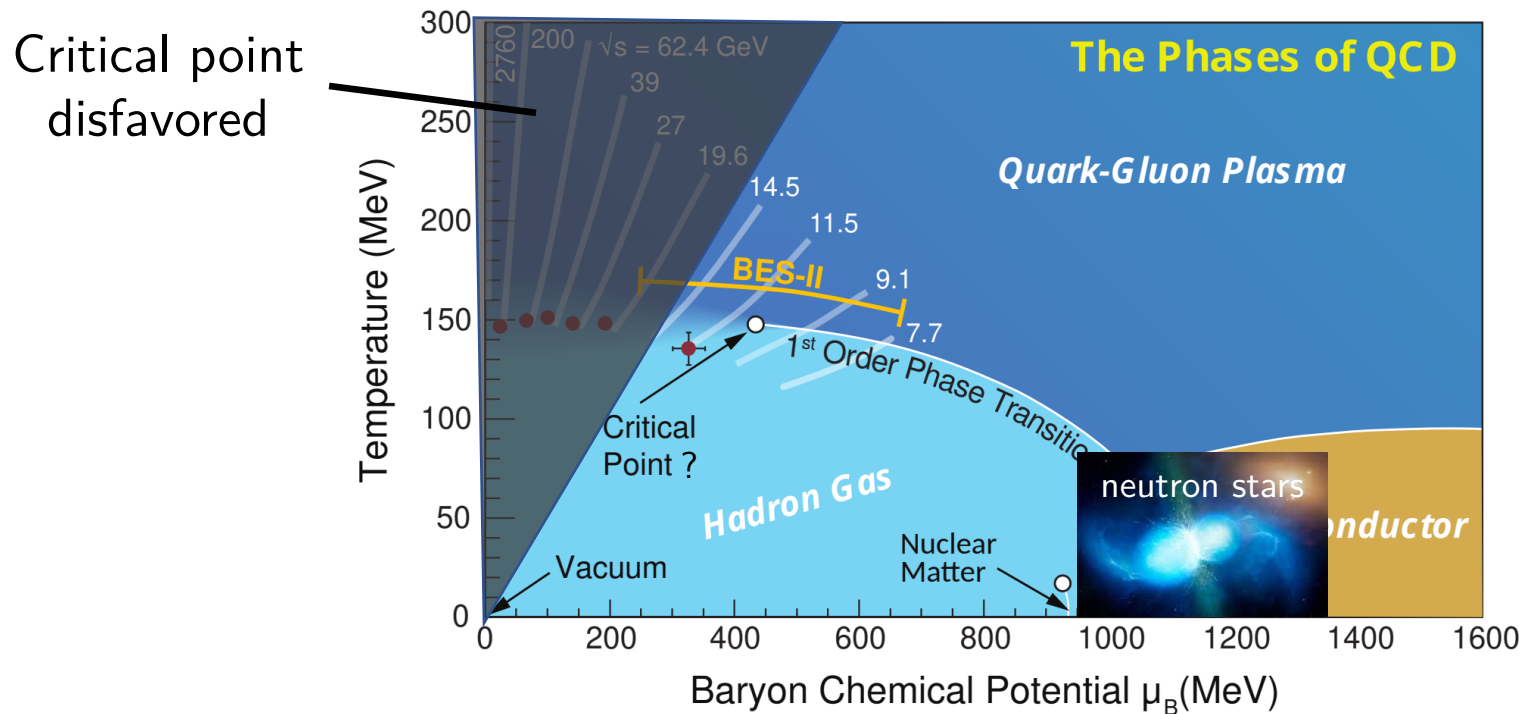
- Fit baryon susceptibilities to data within a fireball model (Siemens-Rasmussen*)
- In the grand-canonical limit (no baryon conservation, small y_{cut}) the data are described well with

$$\frac{\chi_2^B}{\chi_1^B} = 9.35 \pm 0.40, \quad \frac{\chi_3^B}{\chi_2^B} = -39.6 \pm 7.2, \quad \frac{\chi_4^B}{\chi_2^B} = 1130 \pm 488 \quad \text{i.e.} \quad \chi_4^B \gg -\chi_3^B \gg \chi_2^B \gg \chi_1^B$$

- Could be indicative of a *critical point* near the HADES freeze-out at $T \sim 70$ MeV, $\mu_B \sim 875$ MeV
- However, the results for $y_{cut} > 0.2$ are challenging to describe with baryon conservation included



Summary: What we learned so far from fluctuations



- Data at high energies ($\sqrt{s_{NN}} \geq 20$ GeV) consistent with “non-critical” physics
 - Disfavors QCD critical point at $\mu_B/T < 2-3$, consistent with what we know from lattice QCD
- Interesting indications for (multi)-proton correlations at $\sqrt{s_{NN}} \leq 7.7$ GeV

Thanks for your attention!