van der Waals Interactions in Hadron Resonance Gas:

From Nuclear Matter to Lattice QCD

Volodymyr Vovchenko

ITP & FIAS, Goethe University Frankfurt, Germany

- V.V, M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)
- V.V, Phys. Rev. C 96, 015206 (2017)

Thanks to: P. Alba, S. Borsányi, Z. Fodor, M. Gorenstein, S. Katz, A. Motornenko, A. Pásztor, L. Satarov, H. Stoecker

CPOD 2017, Stony Brook, NY, USA

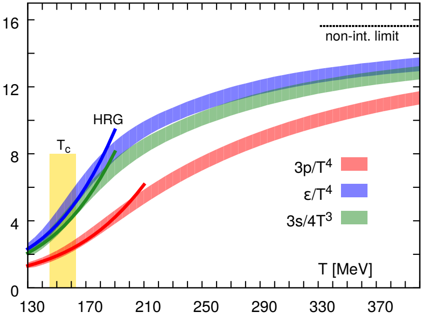
August 7, 2017

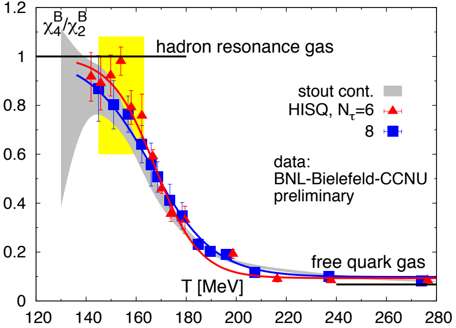
Outline

- Motivation
- (Quantum) van der Waals equation in QCD
  - Nuclear matter
  - Baryonic sector in HRG
  - Fluctuations of conserved charges and Lattice QCD
- Extensions
  - Strangeness dependent interactions
  - Beyond van der Waals
  - Beth-Uhlenbeck approach and quantum excluded volume
- Imaginary chemical potential
  - Hadronic description with repulsive BB interactions
  - Estimation of parameters from Lattice QCD
- Summary

QCD equation of state at $\mu = 0$

Lattice simulations provide equation of state at $\mu_B = 0$[1]





Common model for confined phase is ideal HRG: non-interacting gas of known hadrons and resonances

- Good description of thermodynamic functions up to 180 MeV
- Rapid breakdown in crossover region for description of susceptibilities[2]
- Often interpreted as clear signal of deconfinement...
- But what is the role of hadronic interactions beyond those in ideal HRG?

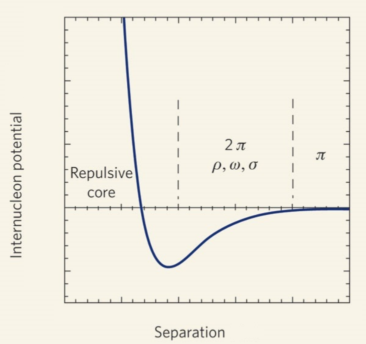
[1] Bazavov et al., PRD 90, 094503 (2014); Borsanyi et al., PLB 730, 99 (2014)
[2] Ding, Karsch, Mukherjee, IJMPE 24, 1530007 (2015)

Nucleon-nucleon interaction

Many hadronic interactions described by resonance formation... however

Nucleon-nucleon potential:

- Repulsive core at small distances
- Attraction at intermediate distances
- No resonance structure
- Suggestive similarity to vdW interactions
- Could nuclear matter be described by vdW equation?



Nuclear matter with quantum van der Waals (QvdW) equation

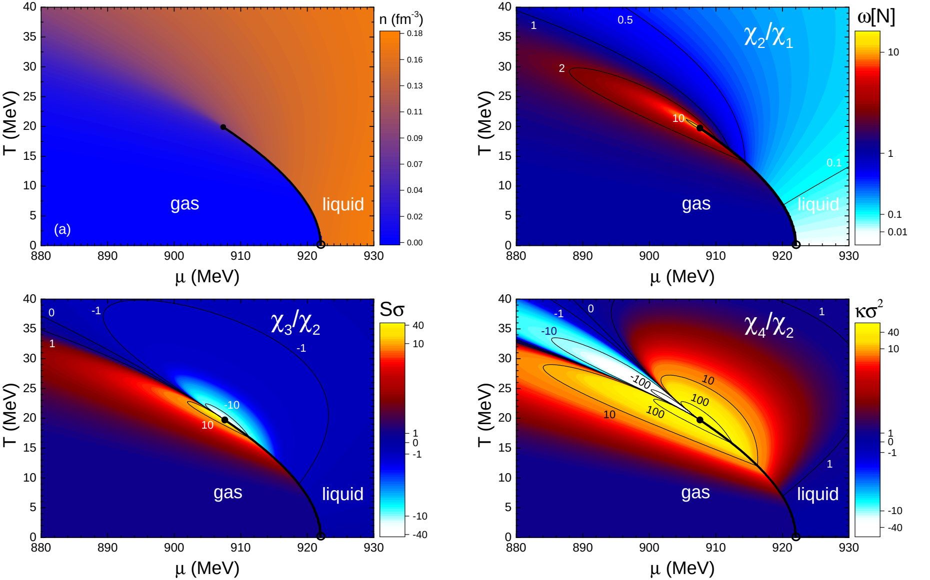
$$p(T, n) = p_q^{\rm id}\left(T, \frac{n}{1 - bn}\right) - a\, n^2$$

$$E/A = -16 \text{ MeV}, \;\; n_0 = 0.16 \text{ fm}^{-3} \quad \Rightarrow \quad a_{NN} = 329 \text{ MeV fm}^3, \;\; b_{NN} = 3.42 \text{ fm}^3$$

V.V., Anchishkin, Gorenstein, PRC '15; Redlich, Zalewski, APPB '16.

QvdW gas of nucleons: (T, μ) plane

(T, μ) plane: structure of critical fluctuations $\chi_i = \partial^i (p/T^4)/\partial(\mu/T)^i$



V.V., D. Anchishkin, M. Gorenstein, R. Poberezhnyuk, PRC 91, 064314 (2015)

van der Waals interactions in hadron resonance gas

Let us now include nuclear matter physics into HRG...

(Q)vdW-HRG model[1]

- Identical vdW interactions between all baryons
- Baryon-antibaryon, meson-meson, meson-baryon vdW terms neglected
- Baryon vdW parameters extracted from ground state of nuclear matter ($a = 329$ MeV fm^3, $b = 3.42$ fm^3)

Three independent subsystems: mesons + baryons + antibaryons

$$p(T, \boldsymbol{\mu}) = P_M(T, \boldsymbol{\mu}) + P_B(T, \boldsymbol{\mu}) + P_{\bar{B}}(T, \boldsymbol{\mu}),$$

$$P_M(T, \boldsymbol{\mu}) = \sum_{j \in M} p_j^{\rm id}(T, \mu_j) \quad \text{and} \quad P_B(T, \boldsymbol{\mu}) = \sum_{j \in B} p_j^{\rm id}(T, \mu_j^{B*}) - a\, n_B^2$$

In this simplest setup model is essentially "parameter-free"

Quite different from "standard scenario" for hadronic interactions in HRG: no attraction and constant radius for all mesons, baryons, light nuclei etc.[2]

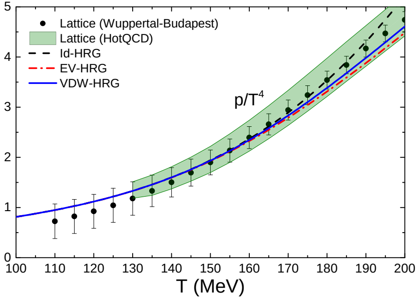
[1] V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

[2] Andronic et al., PLB '12; Redlich, Zalewski, PRC '16; Cleymans, Randrup, EPJ '16

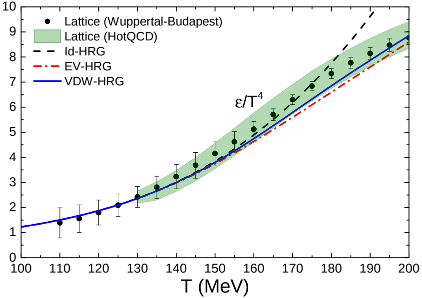
vdW-HRG at $\mu_B = 0$: thermodynamic functions

Comparison of vdW-HRG with lattice QCD at $\mu_B = 0$

Pressure p/T^4



Energy density ε/T^4

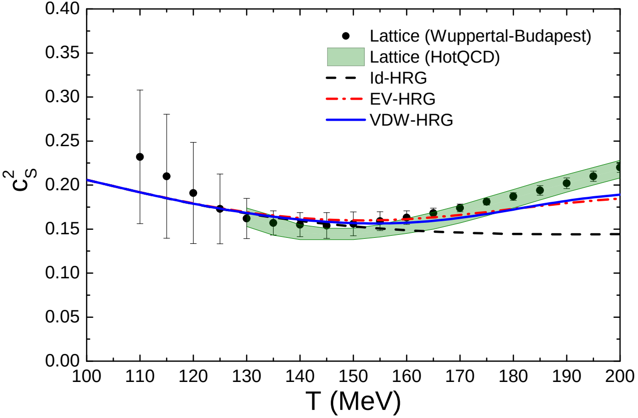


- vdW-HRG does not spoil existing agreement of Id-HRG with LQCD despite significant excluded-volume interactions between baryons
- Not surprising: matter meson-dominated at $\mu_B = 0$

V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

vdW-HRG at $\mu_B = 0$: speed of sound

Speed of sound $c_s^2 = \frac{dp}{d\varepsilon}$



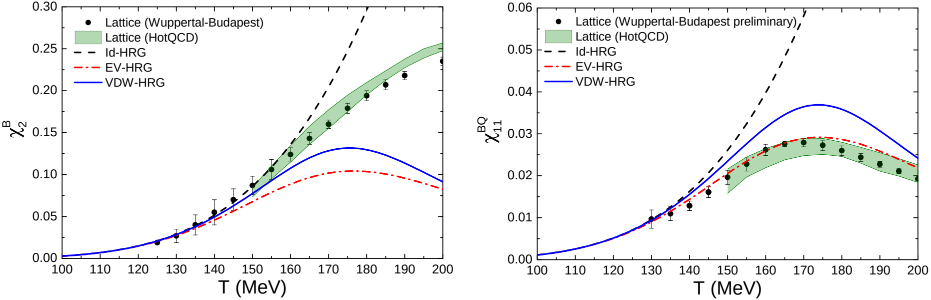
- Monotonic decrease in Id-HRG, at odds with lattice
- Minimum for EV-HRG/VDW-HRG at 150-160 MeV
- No acausal behavior, often an issue in models with eigenvolumes

V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

vdW-HRG at $\mu_B = 0$: baryon number fluctuations

Susceptibilities: $\chi^{BSQ}_{lmn} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \, \partial(\mu_S/T)^m \, \partial(\mu_Q/T)^n}$

Net-baryon χ^B_2 | Baryon-charge correlator χ^{BQ}_{11}



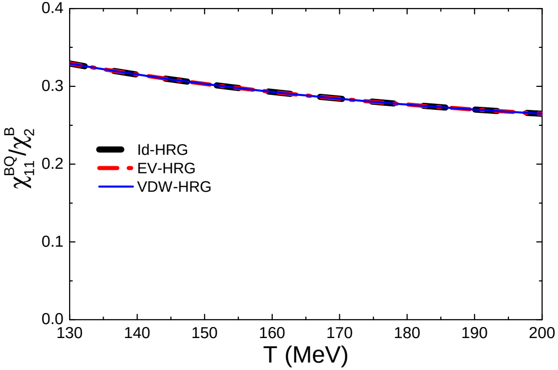
- Very different qualitative behavior between Id-HRG and vdW-HRG
- For χ^B_2 lattice data is between Id-HRG and vdW-HRG at high T
- For χ^{BQ}_{11} lattice data is below all models, closer to EV-HRG

V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

vdW-HRG at $\mu_B = 0$: χ_{11}^{BQ}/χ_2^B

Ratio χ_{11}^{BQ}/χ_2^B

$$\left(\frac{\chi_{11}^{BQ}}{\chi_2^B}\right)_{\rm vdW} = \frac{\sum_{i\in B} Q_i\, n_i^{\rm id}\, f(n_B)}{\sum_{i\in B} n_i^{\rm id}\, f(n_B)} = \frac{\sum_{i\in B} Q_i\, n_i^{\rm id}}{\sum_{i\in B} n_i^{\rm id}} = \left(\frac{\chi_{11}^{BQ}}{\chi_2^B}\right)_{\rm ideal}$$



- The χ_{11}^{BQ}/χ_2^B ratio is unaffected within vdW-HRG model

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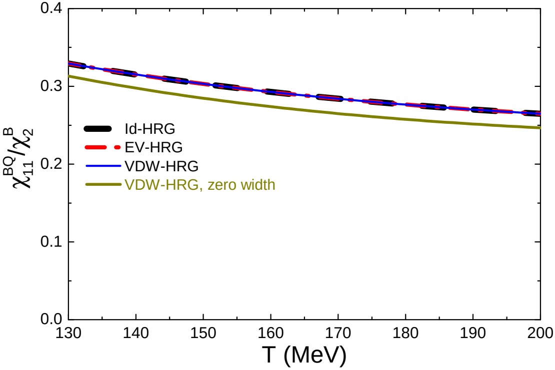

- The χ_{11}^{BQ}/χ_2^B ratio is unaffected within vdW-HRG model
- Affected by modeling of resonance widths

vdW-HRG at $\mu_B = 0$: χ_{11}^{BQ}/χ_2^B

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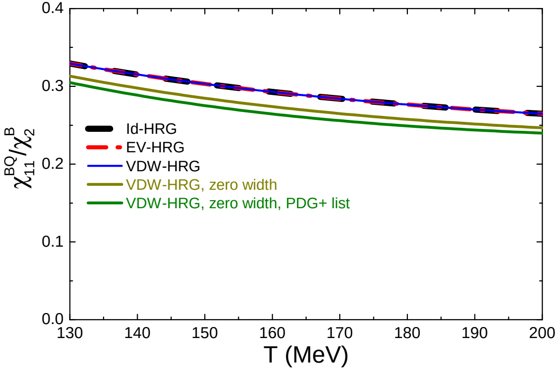
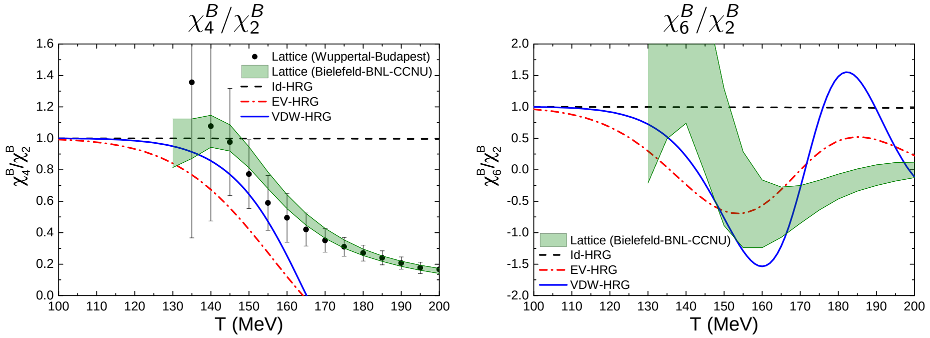


- The χ_{11}^{BQ}/χ_2^B ratio is unaffected within vdW-HRG model
- Affected by modeling of resonance widths
- Quark model states and/or flavor dependent interactions improve description[1]

[1] P. Alba, SQM 2017

vdW-HRG at $\mu_B = 0$: baryon number fluctuations

Higher-order fluctuations are expected to be even more sensitive



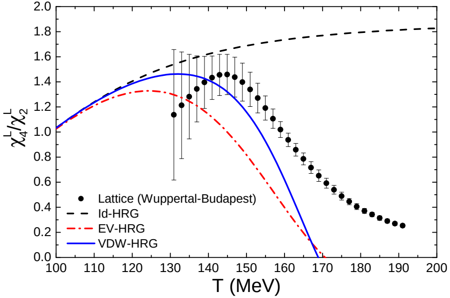
- χ_4^B deviates from χ_2^B at high enough T, they stay equal in Id-HRG
- Cannot be related only to onset of deconfinement
- vdW-HRG predicts strong non-monotonic behavior for χ_6^B/χ_2^B

V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

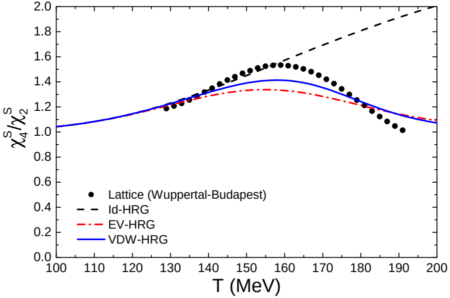
vdW-HRG at $\mu_B = 0$: net-light and net-strangeness

Fluctuations of net-light $L = (u+d)/2 = (3B+S)/2$ and net-strangeness

χ_4^L/χ_2^L



χ_4^S/χ_2^S

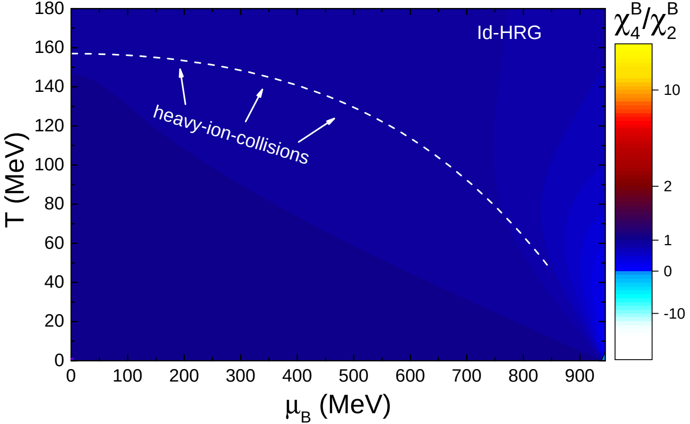


- Lattice shows peaked structures in crossover regions
- Not at all reproduced by ideal HRG, signal for deconfinement?[1]
- Peaks at different T for net-L and net-S ⇒ flavor hierarchy?[2]
- vdW-HRG also shows peaks and flavor hierarchy ⇒ cannot be traced back directly to deconfinement

[1] S. Ejiri, F. Karsch, K. Redlich, PLB 633, 275 (2006)
[2] Bellwied et al., PRL 111, 202302 (2013)

vdW-HRG at finite μ_B

Net-baryon fluctuations in T-μ_B plane: χ_4^B/χ_2^B



- Almost no effect in Id-HRG, only Fermi statistics...

V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)
see also A. Mukherjee et al., 1611.10144

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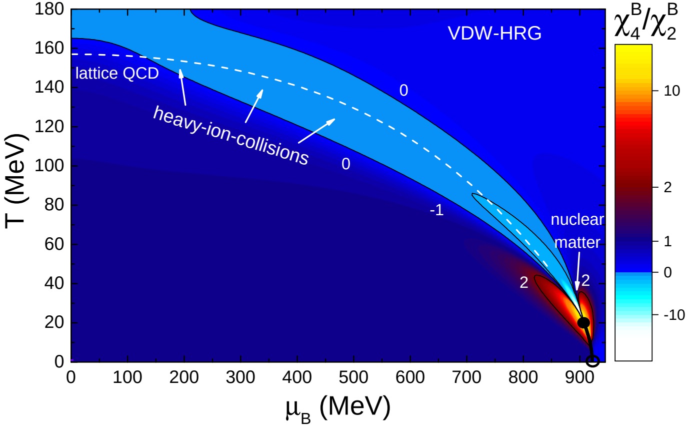


- Almost no effect in Id-HRG, only Fermi statistics...
- Rather rich structure for vdW-HRG, huge effect of vdW interactions!
- Fluctuations seen at RHIC are remnants of nuclear liquid-gas PT?

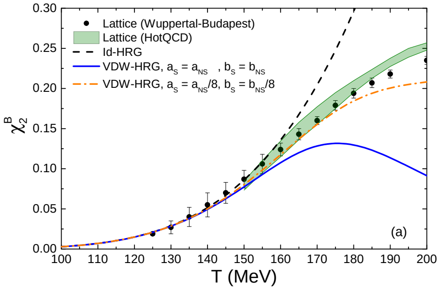
V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)
see also A. Mukherjee et al., 1611.10144

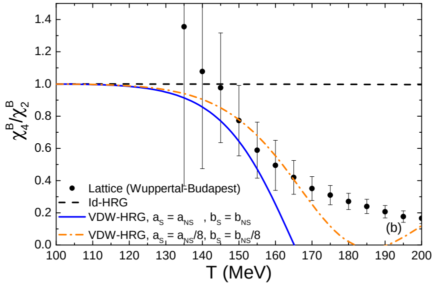
Extensions

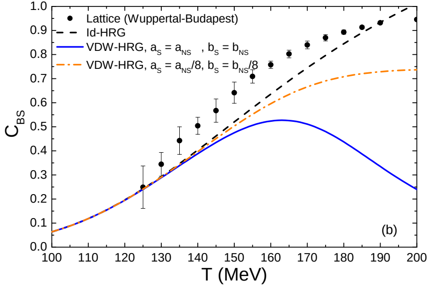
I. Strangeness dependent interactions

Effect of reducing vdW interactions involving strange baryons

- Twice smaller radius for strange baryons
- Illustrative calculation
- Most observables improved
- Needs more systematic study







V.V., A.Motornenko, P.Alba, M.I.Gorenstein, L.M.Satarov, H.Stoecker, 1707.09215

II. Beyond van der Waals

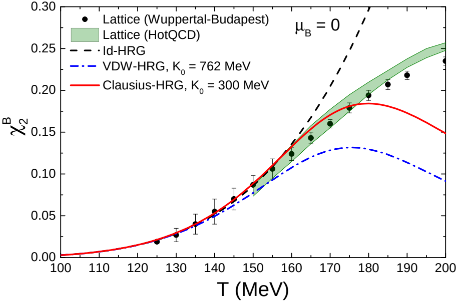
From van der Waals equation to Clausius equation:

$$p = \frac{nT}{1 - bn} - a\,n^2 \qquad \Rightarrow \qquad p = \frac{nT}{1 - bn} - \frac{a\,n^2}{1 + cn}$$

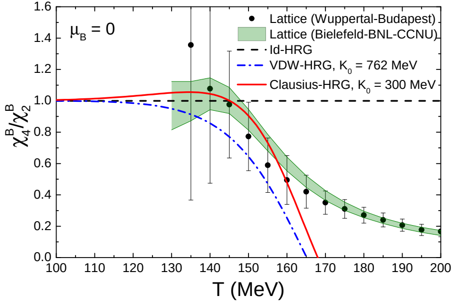
Nuclear incompressibility K_0: from 762 MeV in vdW to 300 MeV in Clausius

Clausius-HRG: baryon-baryon interactions in HRG with Clausius equation

net baryon χ_2^B



net baryon χ_4^B/χ_2^B



- Clausius-HRG yields improved K_0 and improved description of LQCD
- Behavior of LQCD observables correlates with nuclear matter properties

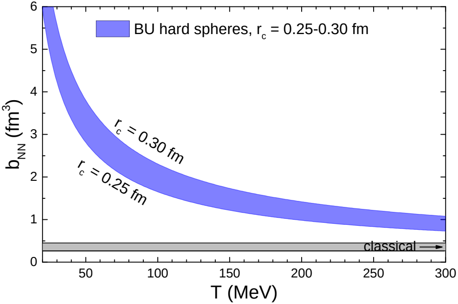
V.V., Phys. Rev. C 96, 015206 (2017)

III. Hard-core repulsion: classical vs Beth-Uhlenbeck

Nucleon $\lambda_{dB} = \sqrt{2\pi/(mT)} \simeq 1.3$ fm at $T = 150$ MeV, comparable to r_c

QM approach to NN hard-core repulsion: Beth-Uhlenbeck (BU) formula

$$\Delta p^{\rm int} \propto B_2(T) = \frac{T}{2\pi^3} \int_{2m_N}^{\infty} d\varepsilon\, \varepsilon^2\, K_2(\varepsilon/T) \sum_{J,T} (2J+1)\,(2T+1)\, \frac{\partial \delta_{J,T}(\varepsilon)}{\partial \varepsilon}$$



$$\delta_{J,T}(\varepsilon) = \arctan \left\{ \frac{j_L[2r_c q(\varepsilon)]}{y_L[2r_c q(\varepsilon)]} \right\}$$

Beth-Uhlenbeck eigenvolume:

$$b = b(T) = -B_2(T)/[n^{\rm id}(T)]^2$$

Classical eigenvolume:

$$p = p^{\rm id}(T, \mu - bp), \quad b = \frac{16\pi r_c^3}{3}$$

- EV of nucleon-nucleon interaction is strongly T-dependent due to QM effects
- Classical approach with $r_c \simeq 0.25$-0.3 fm[1,2] underestimates EV by factor 3-4

[1] NN-scattering data analysis: R. B. Wiringa et al., Phys. Rev. C **51**, 38 (1995)

[2] A. Andronic et al., Phys. Lett. B **718**, 80 (2012); K. Redlich, K. Zalewski, PRC '16

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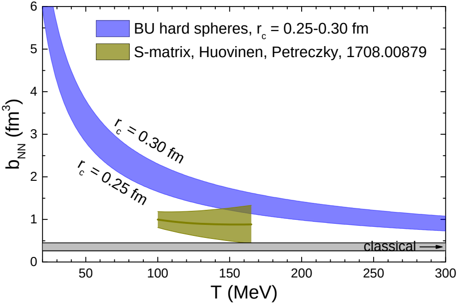

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III. Hard-core repulsion: classical vs Beth-Uhlenbeck

Now let us use this T-dependent eigenvolume $b(T)$ to model BB repulsion in HRG

$$\text{BU-HRG:} \quad P_B(T,\mu) = \sum_{i \in B} p_i^{\rm id}(T,\mu_B) - T\, b(T) \sum_{i,j} \phi_i(T)\, \phi_j(T)\, \lambda_B^2$$

$$\text{EV-HRG:} \quad P_B(T,\mu) = \sum_{i \in B} p_i^{\rm id}[T, \mu_B - b(T)\, P_B]$$

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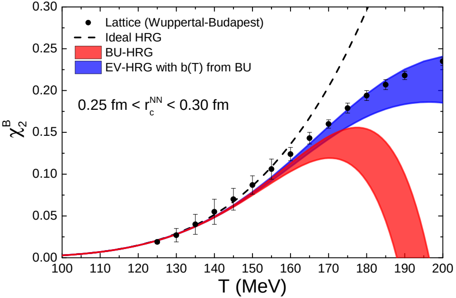
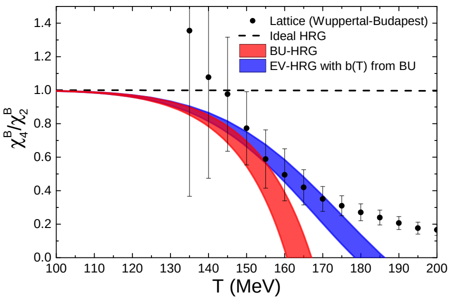
net baryon χ_2^B



net baryon χ_4^B/χ_2^B



- 2nd order expansion breaks down at high T, higher orders matter!
- EV-HRG with BU-motivated $b \sim 1-2$ fm^3 describes LQCD fairly well

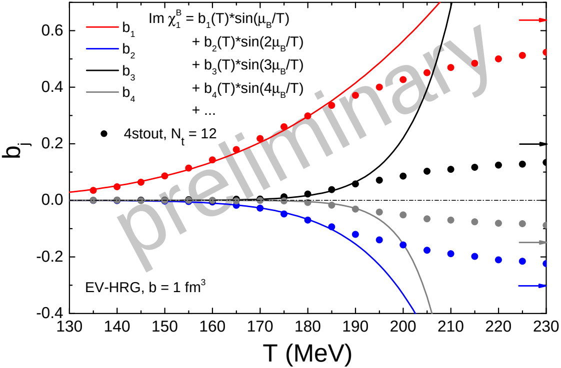
Imaginary μ_B

Repulsive baryonic interactions and imaginary μ_B

Lattice QCD is problematic at real μ but tractable at imaginary μ

E.g., net-baryon density is imaginary and has trigonometric series form

$$\mu_B \to i\tilde{\mu}_B \quad \Rightarrow \quad \frac{n_B(T, i\tilde{\mu}_B)}{T^3} = i \sum_{j=1}^{\infty} b_j(T) \sin(j\,\tilde{\mu}_B/T)$$



$$b_1^{\mathrm{ev}}(T) = 2\,\frac{\phi_B(T)}{T^3}$$

$$b_2^{\mathrm{ev}}(T) = -4\,[b\phi_B(T)]\,\frac{\phi_B(T)}{T^3}$$

$$b_3^{\mathrm{ev}}(T) = 9\,[b\,\phi_B(T)]^2\,\frac{\phi_B(T)}{T^3}$$

$$b_4^{\mathrm{ev}}(T) = -\frac{64}{3}\,[b\,\phi_B(T)]^3\,\frac{\phi_B(T)}{T^3}$$

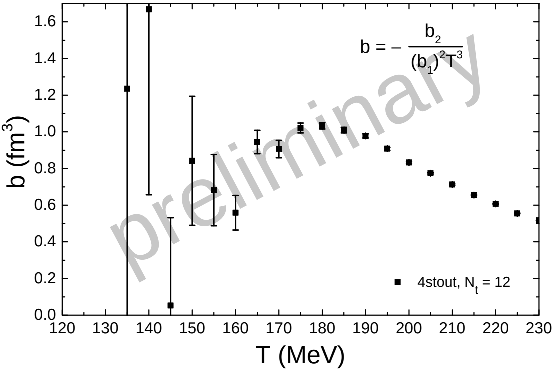
- Non-zero $b_j(T)$ for $j \geq 2$ signal deviations from ideal HRG
- Addition of EV interactions between baryons reproduces lattice trend

V.V., A. Pásztor, Z. Fodor, S. Katz, H. Stoecker, in preparation; S. Borsányi, QM2017

"Excluded volume" parameter from imaginary μ_B data

"Excluded volume" parameter of BB interactions can be estimated from lattice

$$b(T) = -\frac{b_2(T)}{[b_1(T)]^2\, T^3}$$



- $b(T)$ mostly consistent with 1 fm^3 at $T < 190$ MeV
- $b \sim 1/T^3$ at high T: limiting SB-type behavior

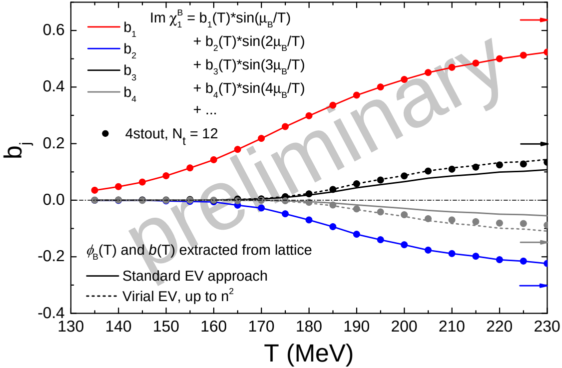
V.V., A. Pásztor, Z. Fodor, S. Katz, H. Stoecker, in preparation; S. Borsányi, QM2017

Constraining model parameters from imaginary μ_B data

Constrain parameters of model with repulsive baryon interactions from lattice

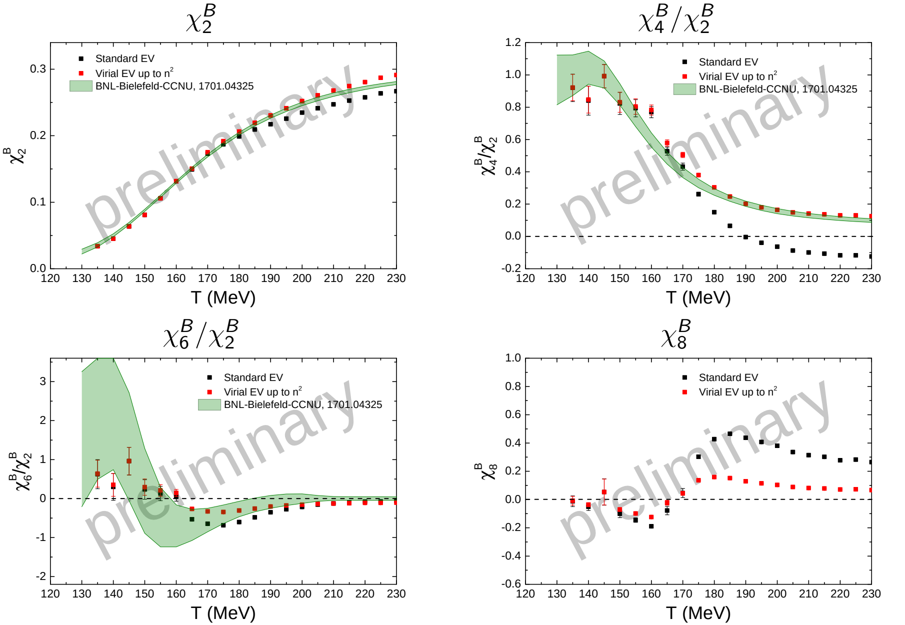
$$P_B^{\mathrm{EV}} = \frac{T\, n_B}{1 - b(T)\, n_B} = T\, n_B + T\, b(T)\, n_B^2 + \ldots$$

- Fix "baryon spectrum" and BB EV parameter from b_1 and b_2
- Predict b_3 and b_4



- EV approach appears to have predictive power, even at high T
- Variations on EV mechanism should be considered
- Lattice data for b_3 and b_4 lies between two EV-type models predictions

EV & imaginary μ: Predictions for net baryon susceptibilities



Summary

- van der Waals interactions between baryons in HRG change qualitative behavior of fluctuations of conserved charges in the crossover region
- Interpretation within standard ideal HRG should be done with extreme care, not clear how useful are ideal HRG vs LQCD comparisons in crossover region
- Nuclear liquid-gas transition manifests itself into non-trivial net-baryon fluctuations in regions of phase diagram probed by heavy-ion collisions
- LQCD data at imaginary μ_B suggests presence of repulsive baryonic interactions with 2nd virial coefficient $b \sim 1$ fm^3 in the crossover region
- LQCD observables at imaginary μ are promising in studying hadronic interactions

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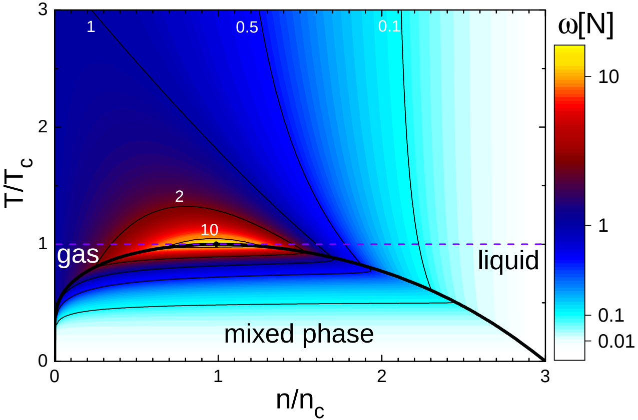
Thanks for your attention!

Backup slides

Scaled variance for classical vdW equation

Particle number fluctuations in classical vdW gas within GCE

$$\omega[N] = \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle} = \frac{\chi_2}{\chi_1} = \left[\frac{1}{(1-bn)^2} - \frac{2an}{T}\right]^{-1}$$



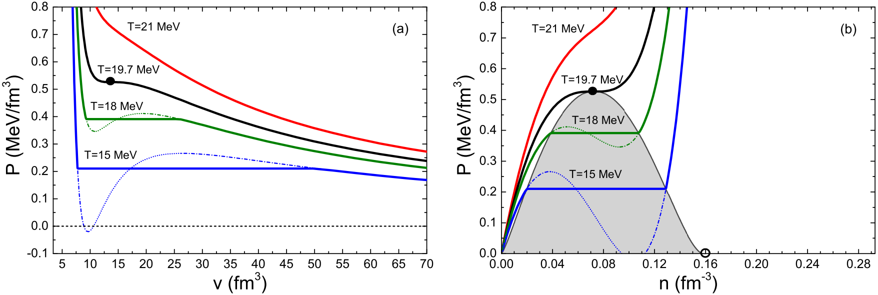
- Repulsive interactions suppress N-fluctuations
- Attractive interactions enhance N-fluctuations

V.V., Anchishkin, Gorenstein, J. Phys. A 48, 305001 (2015)

vdW gas of nucleons: pressure isotherms

a and b fixed to reproduce saturation density and binding energy:

$$n_0 = 0.16 \text{ fm}^{-3},\ E/A = -16 \text{ MeV} \Rightarrow a \cong 329 \text{ MeV fm}^3 \text{ and } b \cong 3.42 \text{ fm}^3$$



Behavior qualitatively same as for Boltzmann case

Mixed phase results from Maxwell construction

Critical point at $T_c \cong 19.7$ MeV and $n_c \cong 0.07$ fm^{-3}

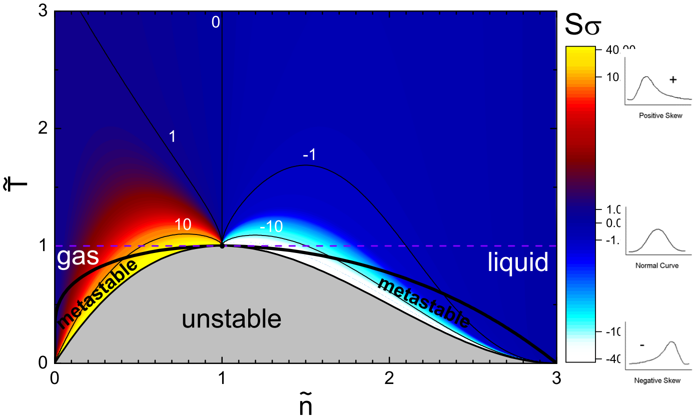
Experimental estimate[1]: $T_c = 17.9 \pm 0.4$ MeV, $n_c = 0.06 \pm 0.01$ fm^{-3}

[1] J.B. Elliot, P.T. Lake, L.G. Moretto, L. Phair, Phys. Rev. C 87, 054622 (2013)

Skewness

Higher-order (non-gaussian) fluctuations are even more sensitive

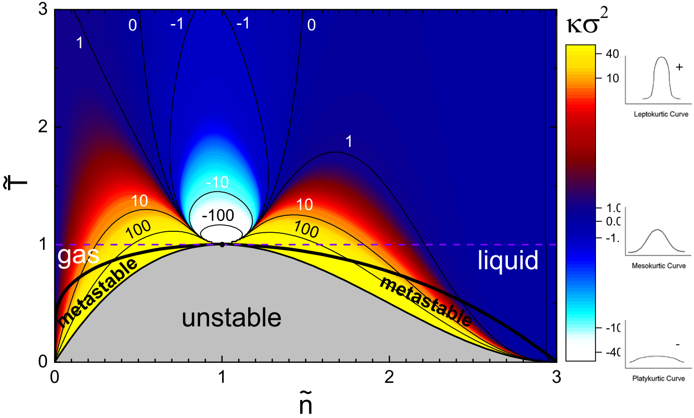
Skewness: $$S\sigma = \frac{\langle(\Delta N)^3\rangle}{\sigma^2} = \omega[N] + \frac{T}{\omega[N]}\left(\frac{\partial \omega[N]}{\partial \mu}\right)_T$$ asymmetry



Skewness is

- Positive (right-tailed) in gaseous phase
- Negative (left-tailed) in liquid phase

Kurtosis: $\kappa\sigma^2 = \frac{\langle(\Delta N)^4\rangle - 3\langle(\Delta N)^2\rangle^2}{\sigma^2}$ peakedness



Kurtosis is negative (flat) above critical point (crossover), positive (peaked) elsewhere and very sensitive to the proximity of the critical point

V. Vovchenko et al., J. Phys. A 015003, 49 (2016)

Quantum statistical van der Waals fluid

Free energy of classical vdW fluid:

$$F(T,V,N) = F^{\rm id}(T, V-bN, N) - a\,\frac{N^2}{V}$$

Ansatz: $F^{\rm id}(T, V-bN, N)$ is free energy of ideal *quantum* gas

$$\text{Pressure:} \qquad p = -\left(\frac{\partial F}{\partial V}\right)_{T,N} = p^{\rm id}(T,\mu^*) - a\,n^2$$

$$\text{Particle density:} \qquad n = \left(\frac{\partial p}{\partial \mu}\right)_{T} = \frac{n^{\rm id}(T,\mu^*)}{1 + b\,n^{\rm id}(T,\mu^*)}$$

$$\text{Shifted chemical potential:} \qquad \mu^* = \mu - b\,p - a\,b\,n^2 + 2\,a\,n$$

Model properties:

- Reduces to classical vdW equation when quantum statistics are negligible
- Reduces to ideal quantum gas for $a = 0$ and $b = 0$
- Entropy density non-negative and $s \to 0$ with $T \to 0$

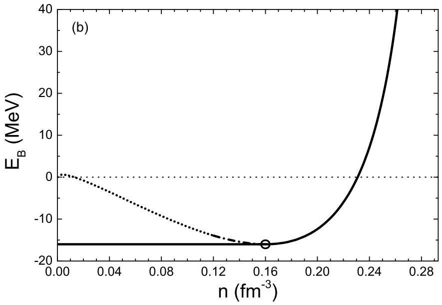
V.V., Anchishkin, Gorenstein, JPA '15 and PRC '15; Redlich, Zalewski, APPB '16.
a=0 $\Rightarrow$ excluded-volume model, D. Rischke et al., ZPC '91

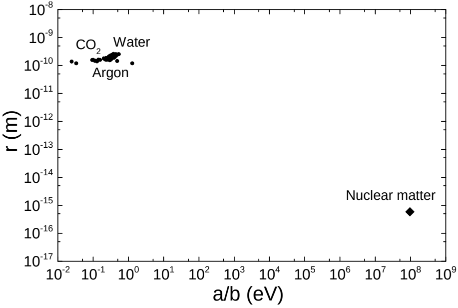
vdW gas of nucleons: zero temperature

How to fix a and b? For classical fluid usually tied to CP location.
Different approach: Reproduce saturation density and binding energy

From $E_B = E/A \cong -16$ MeV and $n = n_0 \cong 0.16$ fm^{-3} at $T = 0$ and $p = 0$

$a \cong 329$ MeV fm^3 and $b \cong 3.42$ fm^3



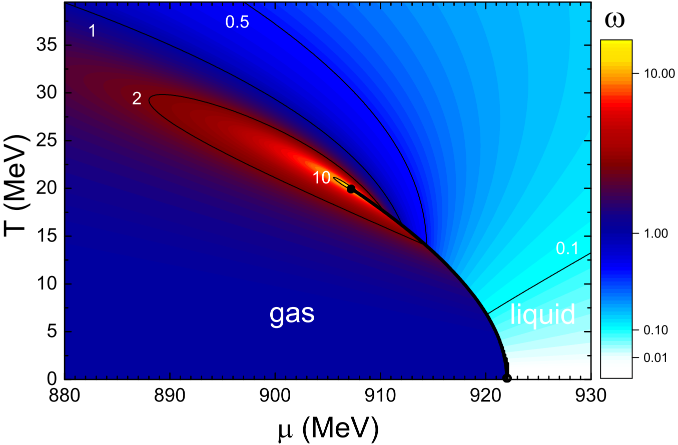


Mixed phase at $T = 0$ is specific:
A mix of vacuum ($n = 0$) and liquid at $n = n_0$

vdW eq. now at very different scale!

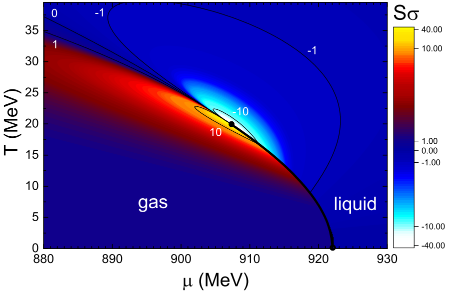
Scaled variance in quantum vdW:

$$\omega[N] = \omega_{\rm id}(T,\mu^*) \left[\frac{1}{(1-bn)^2} - \frac{2an}{T}\,\omega_{\rm id}(T,\mu^*)\right]^{-1}$$



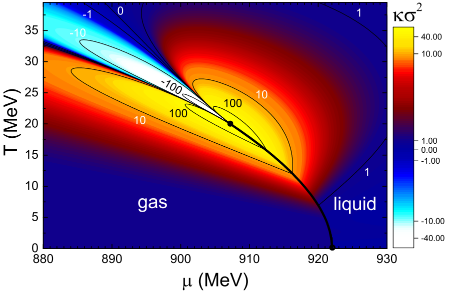
Skewness

$$S\sigma = \omega[N] + \frac{T}{\omega[N]} \left(\frac{\partial \omega[N]}{\partial \mu} \right)_T$$



Kurtosis

$$\kappa\sigma^2 = (S\sigma)^2 + T \left(\frac{\partial [S\sigma]}{\partial \mu} \right)_T$$

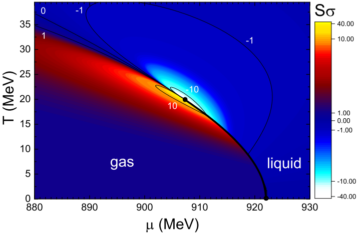


For skewness and kurtosis singularity is rather specific: sign depends on the path of approach

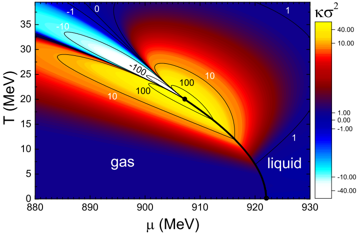
V. Vovchenko et al., Phys. Rev. C 92, 054901 (2015)

vdW gas of nucleons: skewness and kurtosis

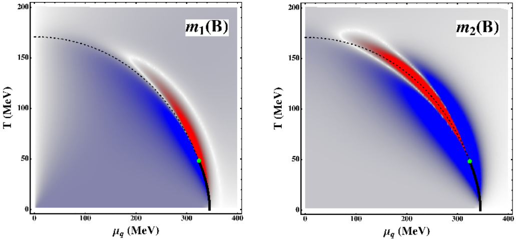
vdW Skewness



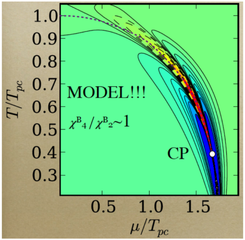
vdW Kurtosis



NJL, J.W. Chen et al., PRD 93, 034037 (2016)



PQM, V. Skokov, QM2012



Fluctuation patterns in vdW very similar to effective QCD models

Strongly intensive measures near CP

Strongly intensive (SI) measures: Gorenstein, Gazdzicki, PRC 84, 014904 (2011)

- Independent of volume fluctuations, mitigate impact parameter fluctuations
- Can be constructed from moments of two extensive quantities

$$\Delta[A,B] = C_\Delta^{-1}\left[\langle A\rangle\omega[B] - \langle B\rangle\omega[A]\right]$$
$$\Sigma[A,B] = C_\Sigma^{-1}\left[\langle A\rangle\omega[B] + \langle B\rangle\omega[A] - 2(\langle AB\rangle - \langle A\rangle\langle B\rangle)\right]$$

- For most models without PT and CP equal/close to unity
- Supposedly show critical behavior, but no model calculation
- Used in search for CP, e.g. NA61/SHINE program[1]

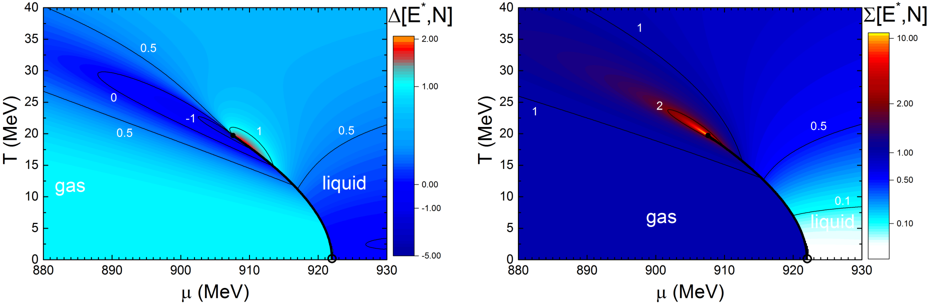
SI measures of excitation energy and particle number fluctuations in cl. vdW

$$\Delta[E^*,N] \;=\; 1 \;-\; \frac{an(2\bar{\epsilon}_{\rm id} - 3an)}{\overline{\epsilon^2_{\rm id}} - \bar{\epsilon}^2_{\rm id}}\,\omega[N], \quad \Sigma[E^*,N] \;=\; 1 + \frac{a^2n^2}{\overline{\epsilon^2_{\rm id}} - \bar{\epsilon}^2_{\rm id}}\,\omega[N].$$

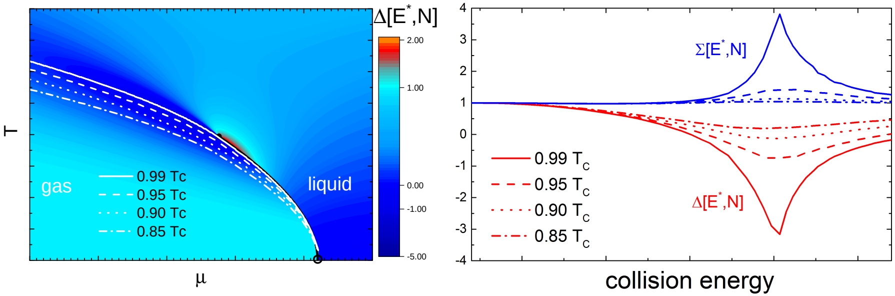
- Critical behavior is present due to criticality of $\omega[N]$ term[2]
- If a=0 then no signal at all! Deviations really stem from criticality.

[1] Gazdzicki, Seyboth, APP '15; E. Andronov, 1610.05569; A. Seryakov, 1704.00751
[2] V.V., Poberezhnyuk, Anchishkin, Gorenstein, J. Phys. A 49, 015003 (2016)

Strongly intensive measures in T-μ plane: Nuclear matter



- Both $\Delta[E^*, N]$ and $\Sigma[E^*, N]$ signal nuclear liquid-gas criticality
- $\Sigma[E^*, N] > 0$ always. However, $\Delta[E^*, N]$ can be both positive and negative

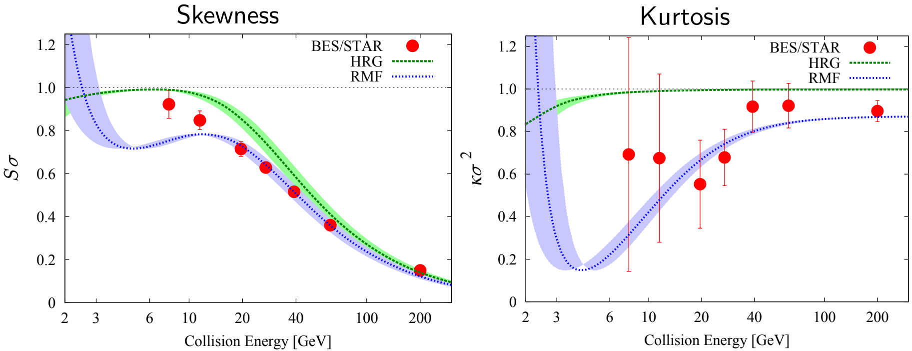


- Both $\Delta[E^*, N]$ and $\Sigma[E^*, N]$ signal nuclear liquid-gas criticality
- $\Sigma[E^*, N] > 0$ always. However, $\Delta[E^*, N]$ can be both positive and negative
- Non-monotonous energy/system-size dependence of $\Delta[E^*, N]$ and $\Sigma[E^*, N]$ in a scenario with CP
- $\Delta[E^*, N]$ is more sensitive than $\Sigma[E^*, N]$ to proximity of CP

Net-baryon fluctuations and nuclear matter

Are NN interactions relevant for heavy-ion collisions?

Net-nucleon fluctuations within RMF (σ-ω model) of nuclear matter along line of "chemical freeze-out"

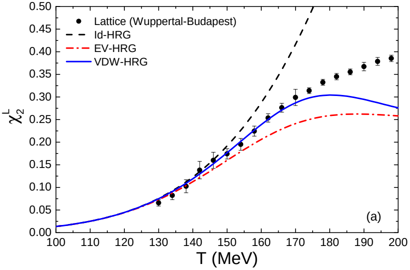


A notable effect in fluctuations even at $\mu_B \simeq 0$

Reconciliation of HRG with nuclear matter can be interesting

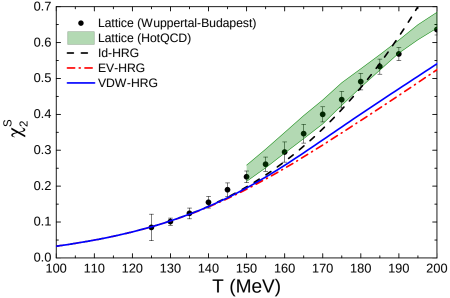
K. Fukushima, PRC 91, 044910 (2015)

vdW-HRG at $\mu = 0$: net-light and net-strangeness



- Net number of light quarks χ_2^L
- $L = (u + d)/2 = (3B + S)/2$
- Improved description in vdW-HRG

- Net-strangeness χ_2^S
- Underestimated by HRG models, similar for χ_{11}^{BS}
- Extra strange states?[1]
- Weaker vdW interactions for strange baryons?[2]



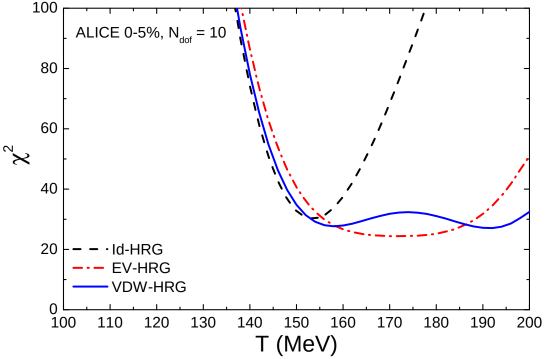
[1] Bazavov et al., PRL 113, 072001 (2014)

[2] Alba, Vovchenko, Gorenstein, Stoecker, arXiv:1606.06542

vdW-HRG: influence on hadron ratios

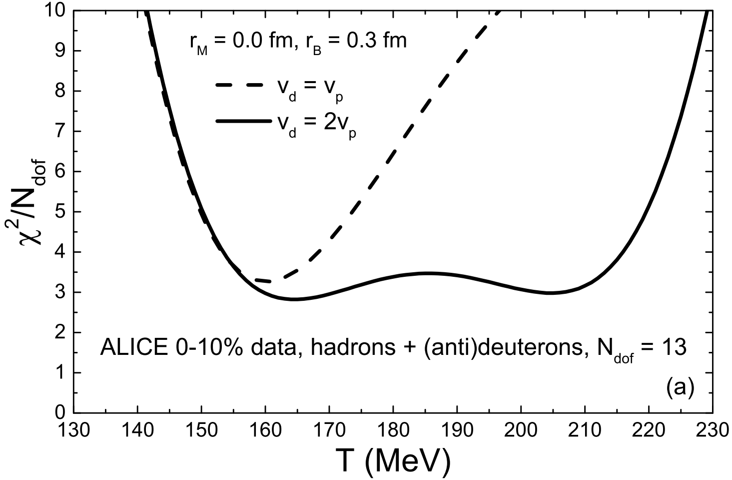
vdW interactions change relative hadron yields in HRG

Thermal model fit to ALICE (Pb+Pb @ 2.76 TeV) yields: from π to Ω



- Fit quality slightly better in EV-HRG/VDW-HRG vs Id-HRG but very different picture!
- All temperatures between 150 and 200 MeV yield similarly fair data description in vdW-HRG
- Results likely to be sensitive to further modifications, e.g for strangeness

Thermal fits and light nuclei



- Picture changes dramatically when changing deuteron eigenvolume to two times the proton one

V.V., H. Stoecker, 1610.02346