Exploring the QCD Phase Diagram with Fluctuation Observables

Volodymyr Vovchenko

Institut für Theoretische Physik, Goethe University Frankfurt, Germany
Frankfurt Institute for Advanced Studies, Germany

- Lattice QCD context: Going from zero to finite μ_B
- QCD phase structure from experimental measurements

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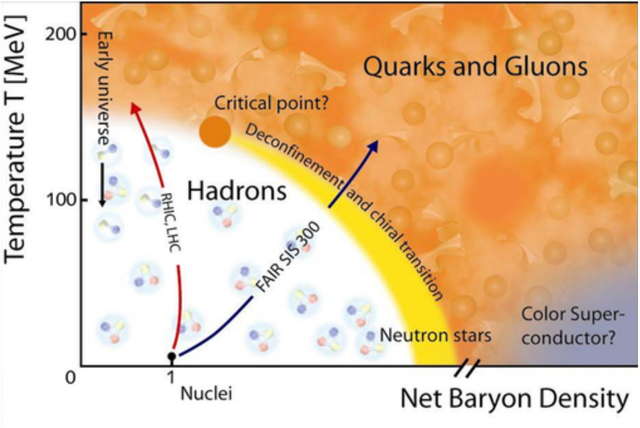
Exploring the QCD phase diagram

The QCD phase diagram has many unknowns at finite density



A ton of information is carried by the *fluctuation* observables

Fluctuation observables

Higher-order moments of fluctuations

Let N be a random variable and $P(N)$ its probability distribution.

$$k\text{-th moment:} \quad \langle N^k \rangle = \sum_N N^k P(N)$$

$$k\text{-th cumulant:} \quad \kappa_k : \quad \log\langle e^{tN} \rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

$$\text{Mean:} \quad M = \langle N \rangle$$

$$\text{Variance:} \quad \sigma^2 = \langle (\Delta N)^2 \rangle = \langle (N - \langle N \rangle)^2 \rangle$$

Cumulant ratios:

Scaled variance: $\dfrac{\sigma^2}{M} = \dfrac{\kappa_2}{\kappa_1} = \dfrac{\sigma^2}{\langle N \rangle}$ — width

(Scaled) **skewness:** $S\sigma = \dfrac{\kappa_3}{\kappa_2} = \dfrac{\langle (\Delta N)^3 \rangle}{\sigma^2}$ — asymmetry

(Scaled) **kurtosis:** $\kappa\sigma^2 = \dfrac{\kappa_4}{\kappa_2} = \dfrac{\langle (\Delta N)^4 \rangle - 3\langle (\Delta N)^2 \rangle^2}{\sigma^2}$ — peakedness

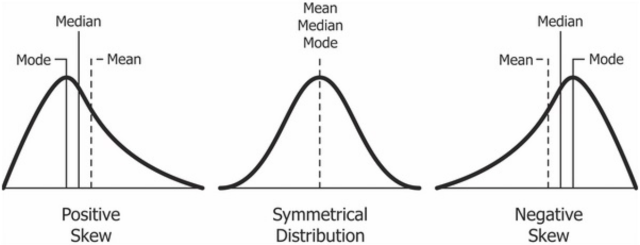
and so on...

Non-Gaussian fluctuations: Skewness

(Normalized) skewness measures the degree of asymmetry of distribution

$$S\sigma \;=\; \frac{\kappa_3}{\kappa_2} = \frac{\langle(\Delta N)^3\rangle}{\sigma^2} \;.$$

$S\sigma > 0$ $S\sigma = 0$ $S\sigma < 0$



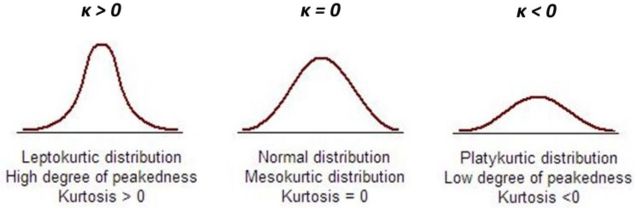
Baselines:

- Gaussian: $S\sigma = 0$
- Poisson: $S\sigma = 1$ $\leftarrow$ ideal gas in grand canonical ensemble

Non-Gaussian fluctuations: Kurtosis

(Normalized) kurtosis measures "peakedness" of distribution

$$\kappa\sigma^2 \;=\; \frac{\kappa_4}{\kappa_2} = \frac{\langle(\Delta N)^4\rangle \;-\; 3\,\langle(\Delta N)^2\rangle^2}{\sigma^2}\;.$$



Baselines:

- Gaussian: $\kappa\sigma^2 = 0$
- Poisson: $\kappa\sigma^2 = 1 \quad \leftarrow \quad$ ideal gas in grand canonical ensemble

Fluctuations in thermodynamic systems

In thermodynamics fluctuations are related to susceptibilities $\chi^{(n)}$

$$\langle N^k \rangle = \frac{\sum_N N^k e^{\frac{\mu N}{T}} Z_N}{\sum_N e^{\frac{\mu N}{T}} Z_N} \qquad \Rightarrow \qquad \kappa_n \sim \chi^{(n)} = \frac{\partial^n (p/T^4)}{\partial (\mu/T)^n}$$

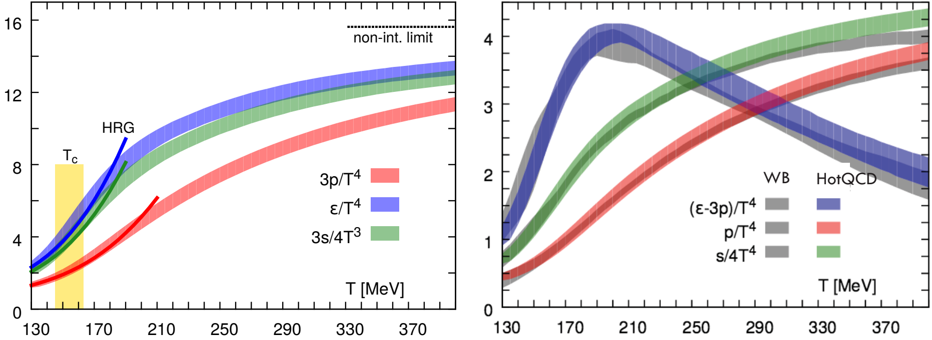
$$\frac{\sigma^2}{M} = \frac{\chi^{(2)}}{\chi^{(1)}}, \qquad S\sigma = \frac{\chi^{(3)}}{\chi^{(2)}}, \qquad \kappa\sigma^2 = \frac{\chi^{(4)}}{\chi^{(2)}},$$

Since they are determined by the derivatives of the thermodynamic potential, fluctuations can probe regions of the phase diagram which are otherwise inaccessible

Fluctuation observables and Lattice QCD

QCD equation of state at $\mu = 0$

Lattice QCD provides the equation of state at $\mu_B = 0$ from first principles



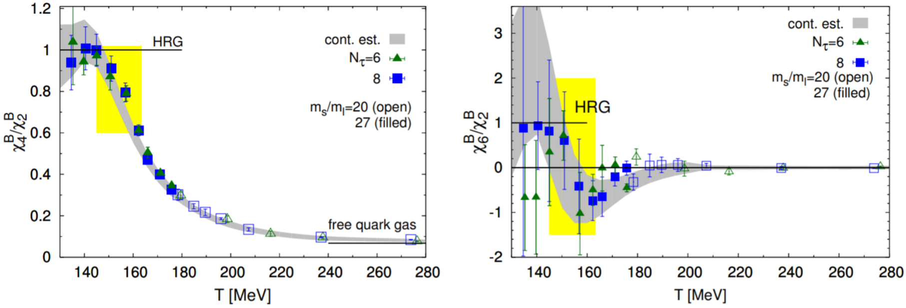
- QCD exhibits crossover-type transition at $T \sim 140 - 190$ MeV
- But not an actual phase transition, at least not at $\mu = 0$
- Basic thermodynamic quantities described well by ideal hadron resonance gas model below and in the crossover region

Bazavov et al. [HotQCD Collaboration], PRD 90, 094503 (2014)
Borsanyi et al. [Wuppertal-Budapest Collaboration], PLB 730, 99 (2014)

Lattice QCD susceptibilities at $\mu = 0$

Susceptibilities of the conserved charges at $\mu = 0$ are calculated as well:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \, \partial(\mu_S/T)^m \, \partial(\mu_Q/T)^n}$$



- Fluctuations probe finer details of the EoS
- Ideal HRG breaks down rapidly at $T \sim 150$ MeV
- This deviation commonly attributed to onset of deconfinement

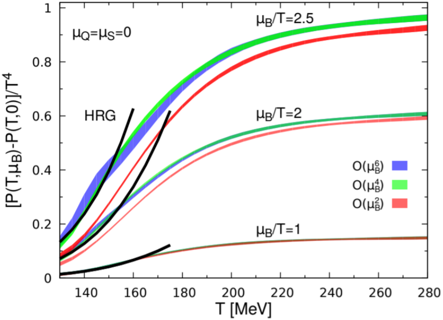
Bazavov et al. [BNL-Bielefeld-CCNU Collaboration], PRD 95, 054504 (2017)

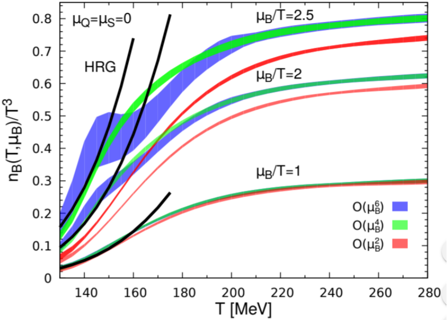
QCD equation of state at small but finite μ

LQCD simulations are restricted to $\mu = 0$ region due to the sign problem

Fluctuation observables at $\mu_B = 0$ provide EoS at finite μ_B via Taylor expansion:

$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \frac{\chi_2^B(T,0)}{2!}(\mu_B/T)^2 + \frac{\chi_4^B(T,0)}{4!}(\mu_B/T)^4 + \dots$$





- QCD EoS now available up to $\mu_B/T \simeq 2$ via Taylor expansion
- Deviations from ideal HRG set in earlier for a larger μ_B/T

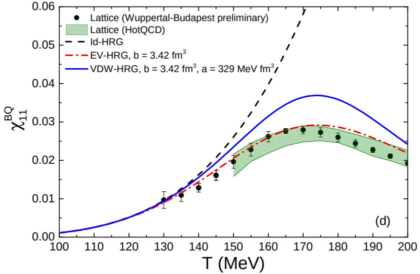
Bazavov et al. [BNL-Bielefeld-CCNU Collaboration], PRD 95, 054504 (2017)

Baryon number susceptibilities and baryonic interactions

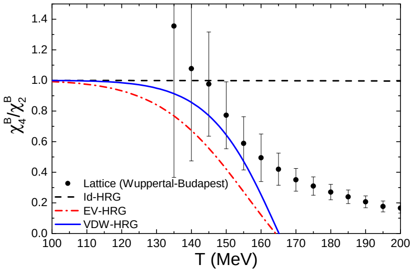
Fluctuation observables from lattice constrain phenomenological models

Example: HRG model with attractive and repulsive van der Waals interactions between baryons [**V.V.**, M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)]

χ_{11}^{BQ}



χ_4^B/χ_2^B



Initial deviation from Poisson statistics captured by repulsive interactions

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More recent developments:

V.V., A. Motornenko, M. Gorenstein, H. Stoecker, Phys. Rev. C 97, 035202 (2018)

P. Huovinen, P. Petreczky, Phys. Lett. B 777, 125 (2018)

S. Samanta, B. Mohanty, Phys. Rev. C 97, 015201 (2018)

Cluster Expansion Model (CEM)

Cluster Expansion Model combines repulsive baryonic interactions with deconfinement at high T [**V.V.**, Steinheimer, Philipsen, Stoecker, 1711.01261]

- Relativistic fugacity expansion for baryon density

$$\frac{\rho_B(T)}{T^3} = \chi_1^B(T) = \sum_{k=1}^{\infty} b_k(T) \sinh(k\, \mu_B / T)$$

- $b_1(T)$ and $b_2(T)$ are model input imaginary μ_B lattice data[1]

- Higher order coefficients given by the excluded volume type expression, which is matched to the Stefan-Boltzmann limit of massless quarks

$$b_k(T) = \alpha_k^{SB} \frac{[b_2(T)]^{k-1}}{[b_1(T)]^{k-2}}, \qquad \alpha_k^{SB} = \frac{[b_1^{SB}(T)]^{k-2}}{[b_2^{SB}(T)]^{k-1}} b_k^{SB}(T)$$

- **Physical picture:** Hadron gas with residual repulsion at moderate T, weakly interacting quarks and gluons at high T

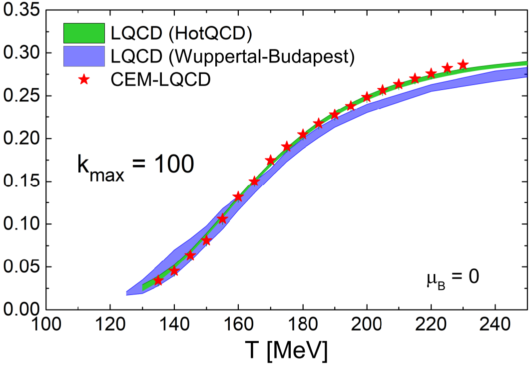
[1] **V.V.**, A. Pásztor, Z. Fodor, S. Katz, H. Stoecker, 1708.02852; S. Borsányi, QM2017

CEM: Baryon number fluctuations

Baryon number susceptibilities at $\mu_B = 0$:

$$\chi_{2n}^B(T) \equiv \left.\frac{\partial^{2n}(p/T^4)}{\partial(\mu_B/T)^{2n}}\right|_{\mu_B=0} = \sum_{k=1}^{\infty} k^{2n-1}\, b_k(T) \simeq \sum_{k=1}^{k_{\max}} k^{2n-1}\, b_k(T).$$

χ_2^B

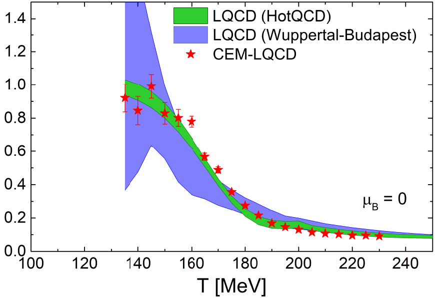


Implicit evidence for consistency between WB and HotQCD data

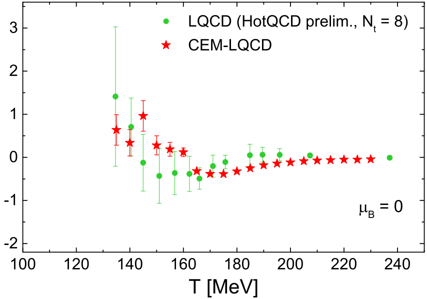
CEM: 4th and 6th order ratios

$$\chi^B_{2n}(T) \equiv \frac{\partial^{2n}(p/T^4)}{\partial(\mu_B/T)^{2n}}\bigg|_{\mu_B=0} = \sum_{k=1}^{\infty} k^{2n-1}\, b_k(T) \simeq \sum_{k=1}^{k_{\max}} k^{2n-1}\,.$$

χ^B_4/χ^B_2



χ^B_6/χ^B_2



Lattice QCD data on fluctuation observables validate CEM

LQCD data from 1507.04627 (Wuppertal-Budapest), 1701.04325 & 1708.04897 (HotQCD)

Radius of convergence

Taylor expansion of QCD pressure:

$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \frac{\chi_2^B(T)}{2!}(\mu_B/T)^2 + \frac{\chi_4^B(T)}{4!}(\mu_B/T)^4 + \dots$$

Radius of convergence $r_{\mu/T}$ of the expansion is the distance to the nearest singularity of p/T^4 in the *complex* μ_B/T plane at a given temperature T

If the nearest singularity is at a real μ_B/T value, this could point to the QCD critical point

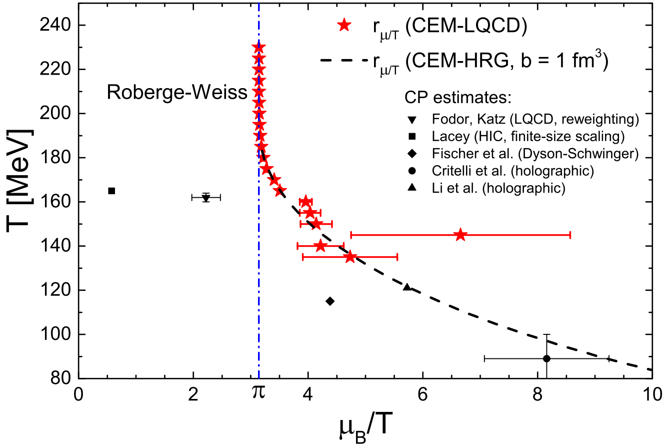
Lattice QCD strategy: Estimate $r_{\mu/T}$ from few leading terms

M. D'Elia et al., 1611.08285; S. Datta et al., 1612.06673; A. Bazavov et al., 1701.04325

Baryon number fluctuations at $\mu_B = 0$ determine $r_{\mu/T}$

$$r_{\mu/T} = \lim_{n\to\infty} \inf \left| \frac{(2n)!}{\chi_{2n}^B} \right|^{1/(2n)}$$

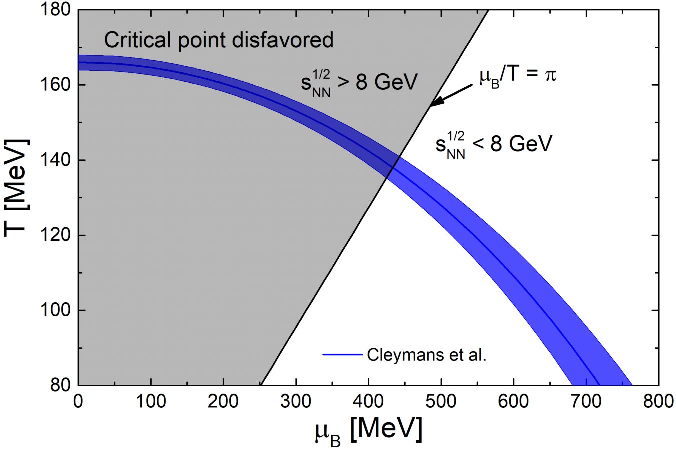
Radius of convergence in CEM



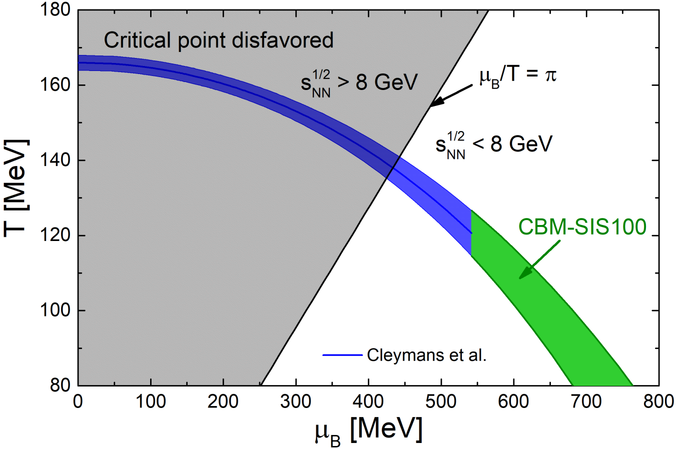
- LQCD susceptibilities at $\mu = 0$ are consistent with the Roberge-Weiss type transition in the complex μ_B/T plane [Roberge, Weiss, NPB '86]

- No evidence for a phase transition at $\mu_B/T \lesssim \pi$

Where to look for a critical point



Where to look for a critical point



CBM may be in the right spot!

Probing phase diagram with fluctuation measurements in heavy-ion collisions

Fluctuations and critical point

Lattice QCD data on susceptibilities

$$\chi^{(n)} = \frac{\partial^n (p/T^4)}{\partial(\mu/T)^n}$$

at $\mu = 0$ may not locate the critical point and/or a phase transition.

In the vicinity of a critical point, however, cumulants are proportional to increasing powers of the correlation length [M. Stephanov, PRL (2009)]

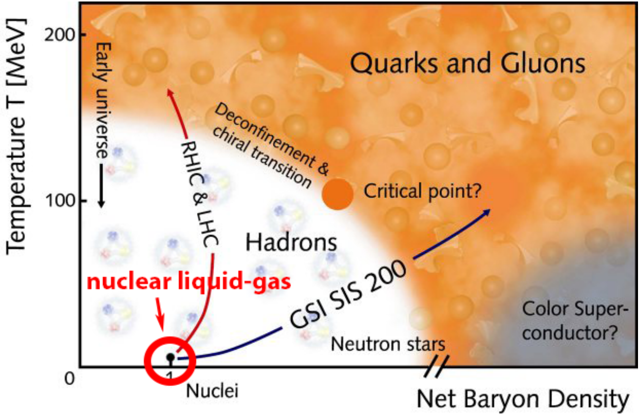
$$\chi^{(2)} \sim \xi^2$$
$$\chi^{(3)} \sim \xi^{4.5}$$
$$\chi^{(4)} \sim \xi^7$$

Infinite system: $\xi \to \infty$ at CP
Heavy-ion collisions: $\xi \lesssim 2 - 3$ fm [B. Berdnikov, K. Rajagopal, PRD (2000)]

Fluctuation observables at finite μ_B probe critical behavior and can be measured in heavy-ion collisions see also next talk by X. Luo

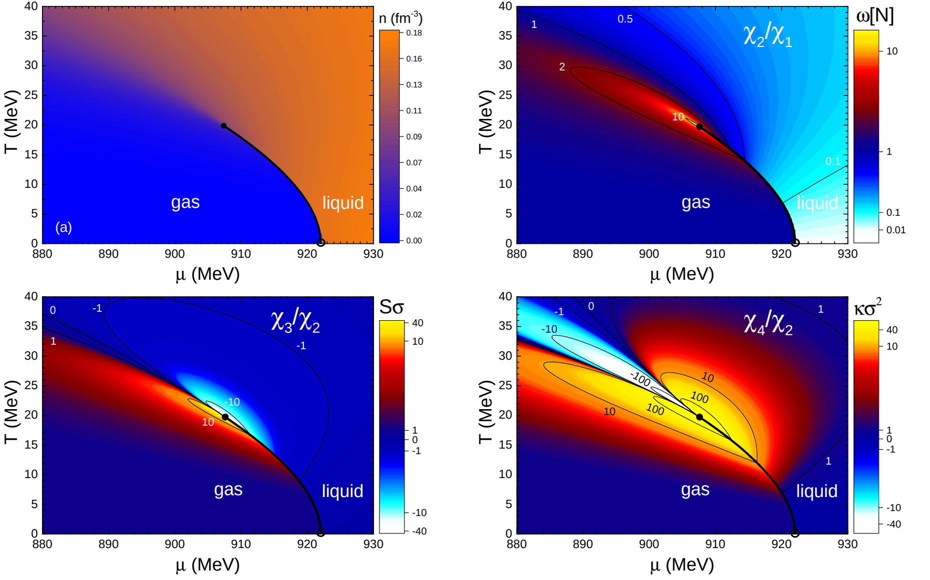
Example: Critical point of nuclear matter

The QCD phase diagram



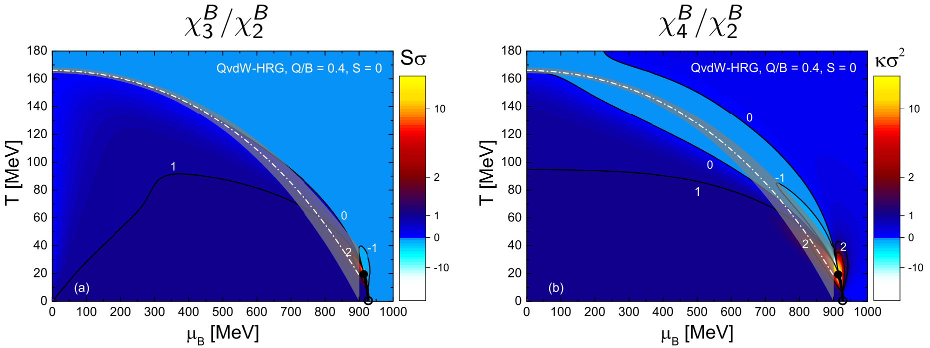
is known to contain the critical point of nuclear matter at $T_c \sim 15$ MeV and *real* $(\mu_B/T)_c \sim 40$

Quantum van der Waals model of nuclear matter



V.V., D. Anchishkin, M. Gorenstein, R. Poberezhnyuk, PRC 91, 064314 (2015)

QvdW-HRG model: fluctuations in T-μ_B plane



Chemical freeze-out curve from *Cleymans et al., Phys. Rev. C 73, 034905 (2006)*

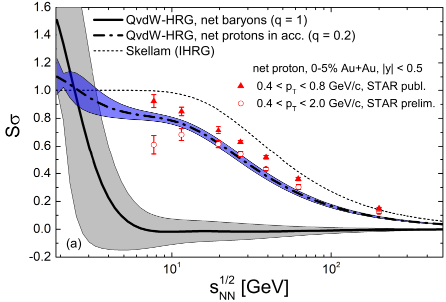
Critical point of nuclear matter shines brightly in fluctuation observables, across the whole region of phase diagram probed by heavy-ion collisions

QvdW-HRG model: collision energy dependence

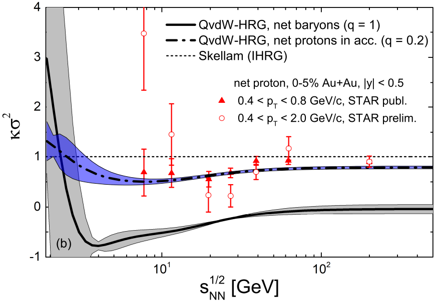
Calculating fluctuations along the “freeze-out” curve

Acceptance effects (protons instead of baryons, momentum cut) modeled *schematically*, by applying the *binomial filter* [M. Kitazawa, M. Asakawa, PRC ’12; A. Bzdak, V. Koch, PRC ’12]

χ_3^B/χ_2^B



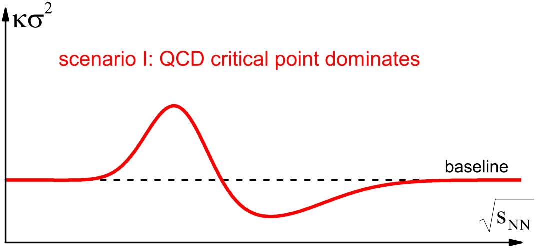
χ_4^B/χ_2^B



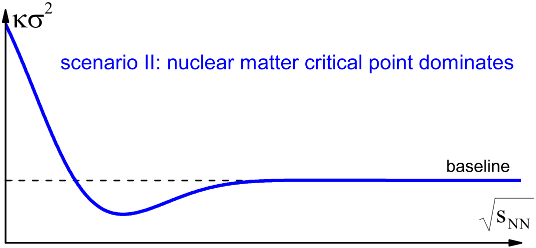
Effects of nuclear liquid-gas criticality:

- Non-monotonic collision energy dependence
- Net proton quite different from net baryon

Scenarios for collision energy dependence



M. Stephanov, JPG "11



V.V., *L. Jiang, M. Gorenstein, H. Stoecker, 1711.07260*

Can the scenarios be distinguished? Need data at lower energies...

Opportunities for HADES, CBM, NA61/SHINE, STAR!

Summary

- Fluctuation observables from lattice QCD provide the QCD equation of state at small μ_B/T and constrain phenomenological models
- Onset of deviations from Poisson statistics is explained by repulsive baryonic interactions
- Susceptibilities at $\mu = 0$ are consistent with a Roberge-Weiss like transition in the complex μ_B/T plane, no hints for critical point at $\mu_B/T \lesssim \pi$
- Signals from nuclear liquid-gas criticality are surprisingly strong
- Net proton fluctuations $\neq$ Net baryon fluctuations, for more reasons than just baryon number conservation

Summary

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Thanks for your attention!

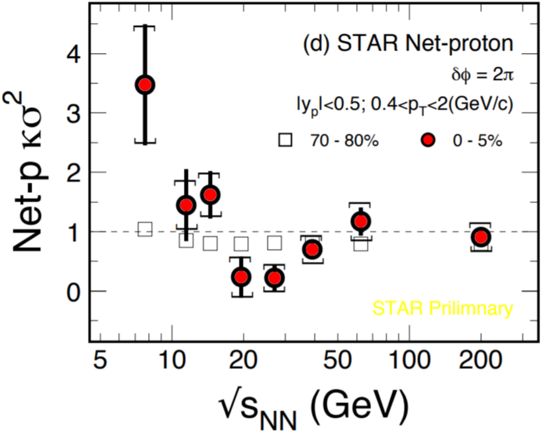
Backup slides

Search for CP in heavy-ion collision experiments

Experimental search for QCD CP using non-Gaussian fluctuations is underway

Measurements at STAR and NA61/SHINE experiments

X. Luo, CPOD2014, QM2015



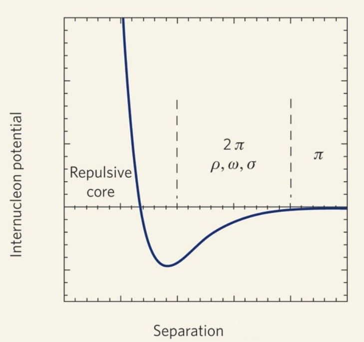
Interpretation challenging, many "background" things contribute

Nucleon-nucleon interaction

Nuclear liquid-gas transition appears due to the vdW type structure of the nucleon-nucleon interaction

Nucleon-nucleon potential:

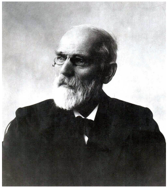
- Repulsive core at small distances
- Attraction at intermediate distances
- Suggestive similarity to vdW interactions



Could nuclear matter be described by the van der Waals equation?

van der Waals equation

$$P(T, V, N) = \frac{NT}{V - bN} - a\frac{N^2}{V^2}$$



Formulated in 1873.

Simplest model which contains attractive and repulsive interactions

Contains 1st order phase transition and critical point

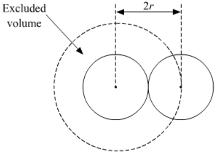
Nobel Prize in 1910.

1. Short-range repulsion: excluded volume (EV) procedure

   $$V \to V - bN, \qquad b = 4\frac{4\pi r_c^3}{3}$$

2. Intermediate range attraction in mean-field approx.

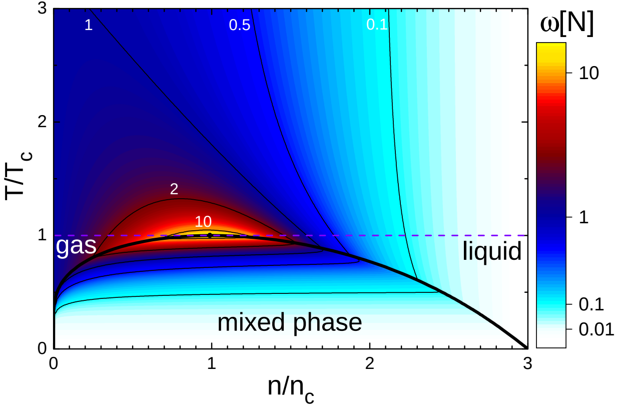
   $$P \to P - a\,n^2, \qquad a = \pi \int_{2r_c}^{\infty} |U_{12}(r)| r^2 dr$$



Scaled variance for classical VDW equation

Particle number fluctuations in a classical vdW gas within the GCE

$$\omega[N] = \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle} = \frac{\chi_2}{\chi_1} = \left[\frac{1}{(1-bn)^2} - \frac{2an}{T}\right]^{-1}$$



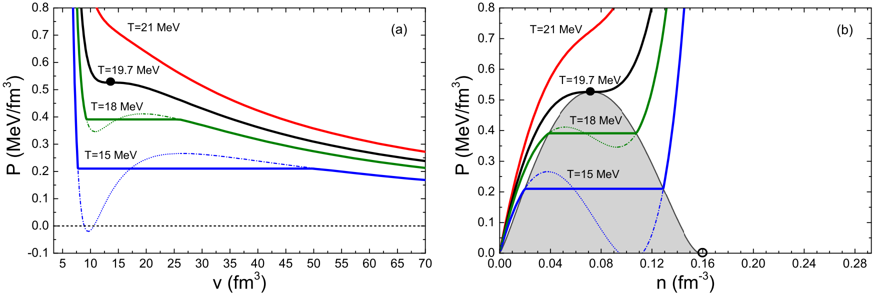
- Repulsive interactions suppress N-fluctuations
- Attractive interactions enhance N-fluctuations

V.V., Anchishkin, Gorenstein, J. Phys. A 48, 305001 (2015)

QvdW gas of nucleons: pressure isotherms

a and *b* fixed to reproduce saturation density and binding energy:

$n_0 = 0.16\ \text{fm}^{-3}$, $E/A = -16$ MeV $\Rightarrow$ $a \cong 329\ \text{MeV fm}^3$ and $b \cong 3.42\ \text{fm}^3$



Behavior qualitatively same as for Boltzmann case

Mixed phase results from Maxwell construction

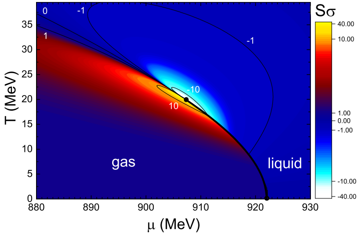
Critical point at $T_c \cong 19.7$ MeV and $n_c \cong 0.07\ \text{fm}^{-3}$

Experimental estimate[1]: $T_c = 17.9 \pm 0.4$ MeV, $n_c = 0.06 \pm 0.01\ \text{fm}^{-3}$

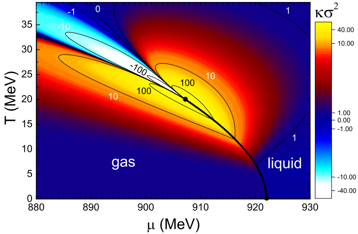
[1] J.B. Elliot, P.T. Lake, L.G. Moretto, L. Phair, Phys. Rev. C 87, 054622 (2013)

QvdW gas of nucleons: skewness and kurtosis

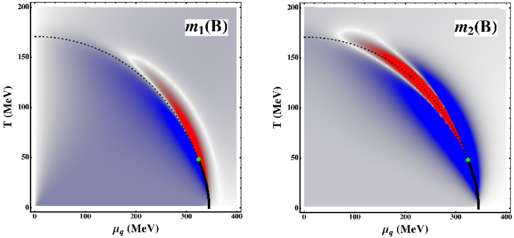
QvdW Skewness



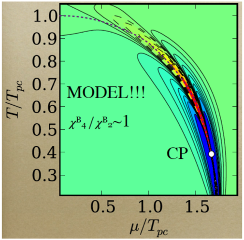
QvdW Kurtosis



NJL, J.W. Chen et al., PRD 93, 034037 (2016)



PQM, V. Skokov, QM2012



Fluctuation patterns in vdW very similar to effective QCD models

Fluctuation signals from nuclear matter critical point and from QCD critical point may very well look alike

QvdW-HRG model

Further applications require treatment of full hadron spectrum

QvdW-HRG model [**V.V.**, M.I. Gorenstein, H. Stoecker, PRL 118, 182301 (2017)]

- Hadron Resonance Gas (HRG) with attractive and repulsive vdW interactions between baryons
- vdW parameters $a = 329$ MeV fm^3 and $b = 3.42$ fm^3 tuned to nuclear ground state properties
- Critical point of nuclear matter at $T_c \simeq 19.7$ MeV, $\mu_c \simeq 908$ MeV

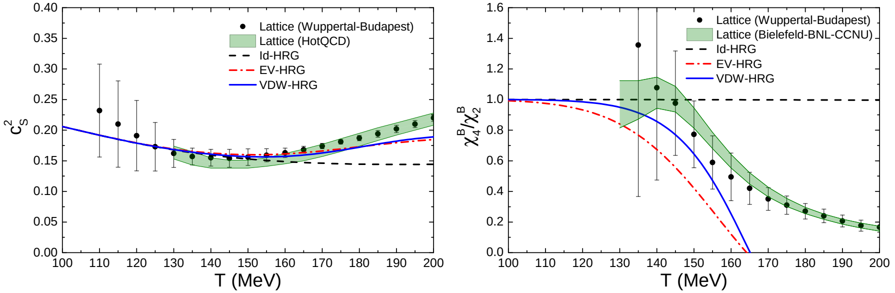
Three independent subsystems: mesons + baryons + antibaryons

$$p(T, \boldsymbol{\mu}) = P_M(T, \boldsymbol{\mu}) + P_B(T, \boldsymbol{\mu}) + P_{\bar{B}}(T, \boldsymbol{\mu}),$$

$$P_M(T, \boldsymbol{\mu}) = \sum_{j \in M} p_j^{\rm id}(T, \mu_j) \quad \text{and} \quad P_B(T, \boldsymbol{\mu}) = \sum_{j \in B} p_j^{\rm id}(T, \mu_j^{B*}) - a\, n_B^2$$

QvdW-HRG model at $\mu_B = 0$

At $\mu = 0$ the model can be confronted with the lattice data



Strong effects even at $\mu_B = 0$!

What about intermediate collision energies?