Nucleosynthesis in heavy-ion collisions at the LHC via the Saha equation

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Based on arXiv:1903.10024

in collaboration with K. Gallmeister, J. Schaffner-Bielich, C. Greiner

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FIAS Frankfurt Institute for Advanced Studies

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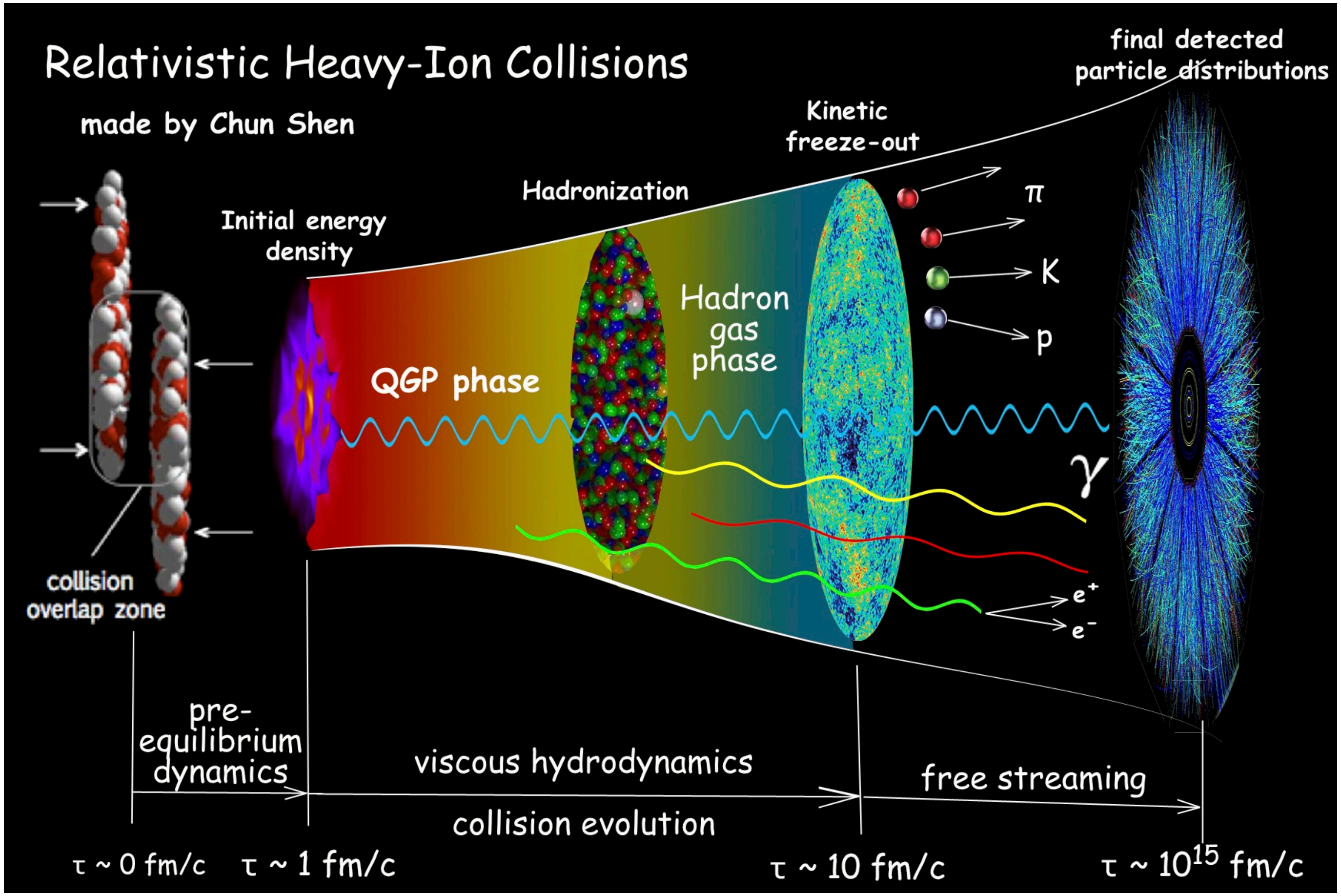
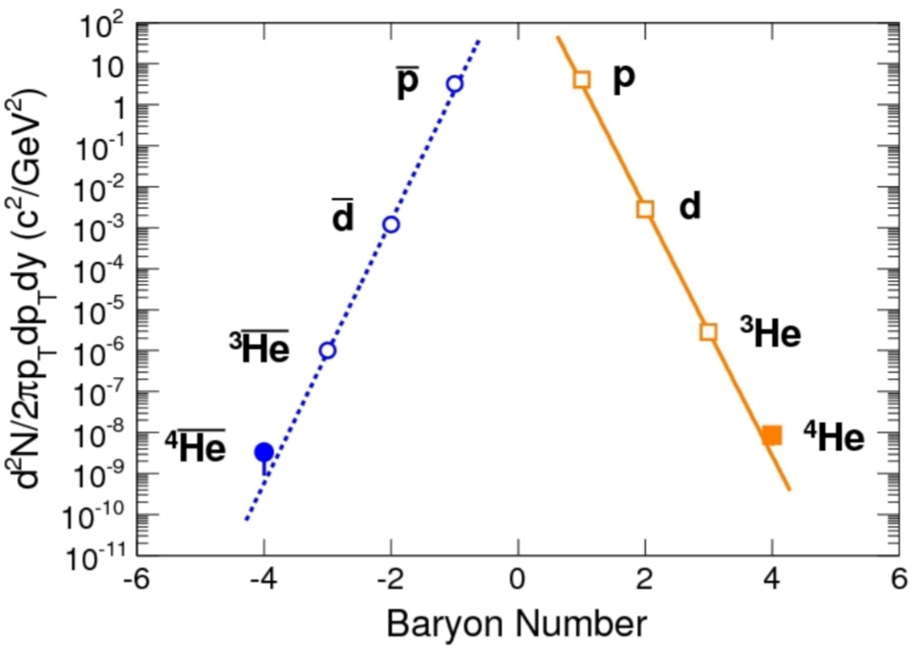
Heavy-ion collisions

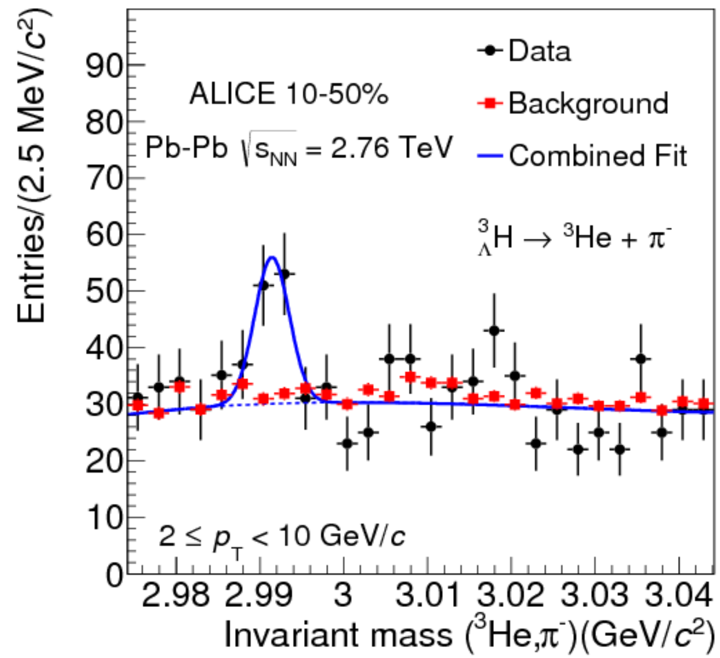


Heavy-ion collision experiments study properties of strongly interacting matter at extreme temperatures and densities, recreate conditions present in Early Universe

Loosely-bound objects in heavy-ion collisions



[STAR collaboration, Nature **473**, 353 (2011)]



[ALICE collaboration, PLB **754**, 360 (2016)]

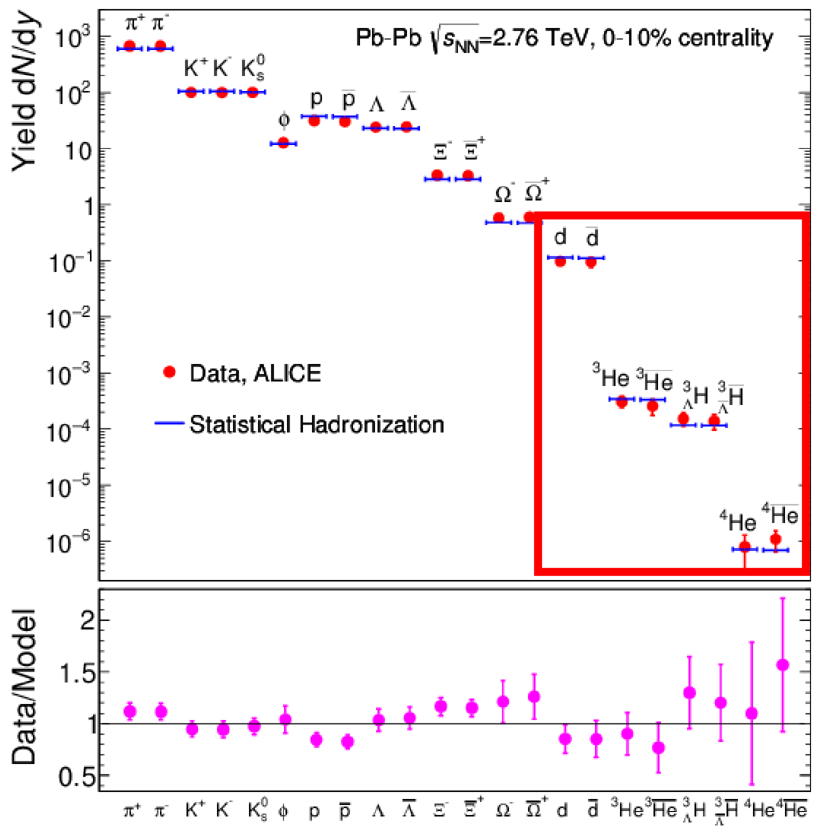
binding energies: ^{2}H, ^{3}He, ^{4}He, $^3_\Lambda$H: 2.22, 7.72, 28.3, 0.130 MeV $\ll T \sim 150$ MeV

"snowballs in hell"

The production mechanism is not established. Common approaches include **thermal** nuclei emission together with hadrons [Andronic et al., PLB '11;...] or final-state **coalescence** of nucleons close in phase-space [Butler, Pearson, PRL '61; Scheibl, Heinz, PRC '99;...]

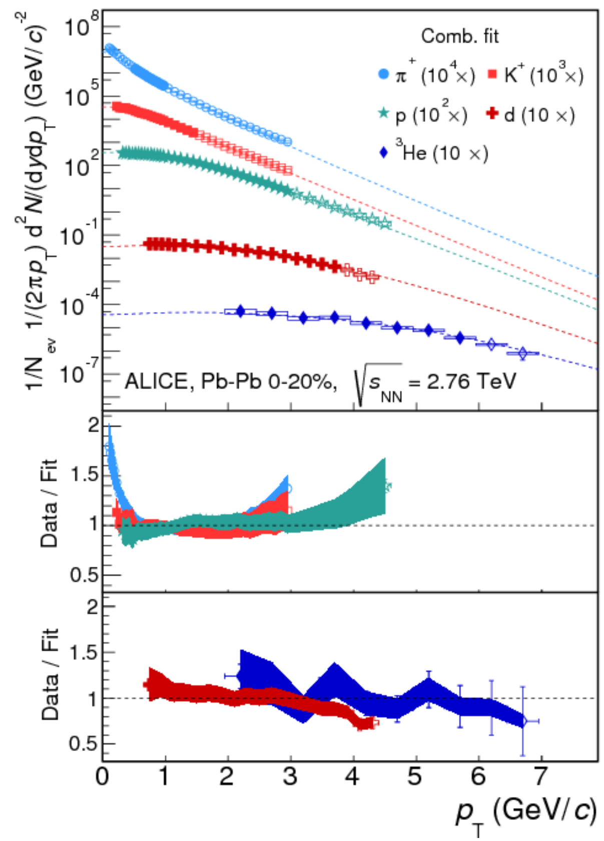
Two experimental observations at the LHC

1. Measured yields are described by thermal model at $T_{ch} \approx 155$ MeV*



[A. Andronic et al., Nature **561**, 321 (2018)]

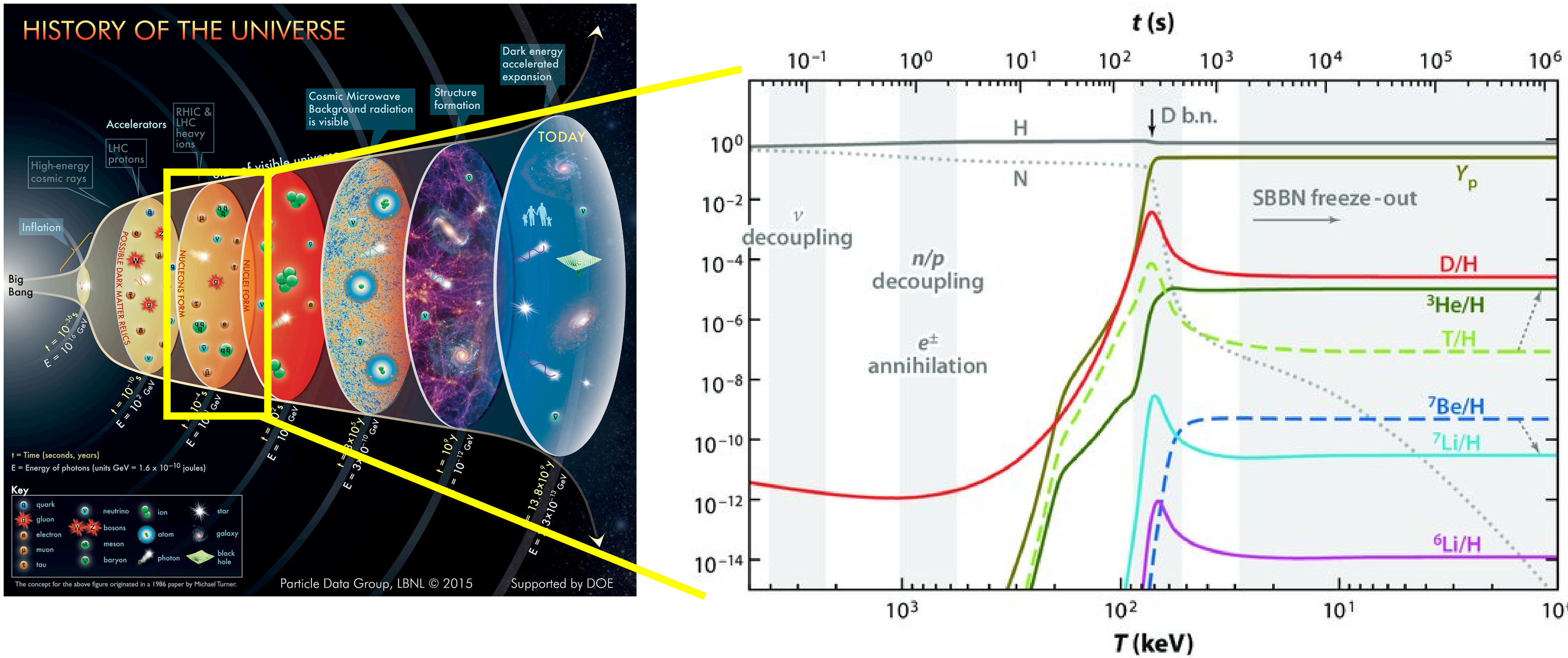
2. Spectra described by blast-wave model at $T_{kin} \approx 100 - 120$ MeV*

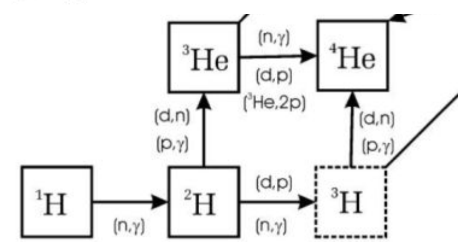


[ALICE collaboration, PRC **93**, 024917 (2016)]

What happens between T_{ch} and T_{kin}?

Big Bang nucleosynthesis





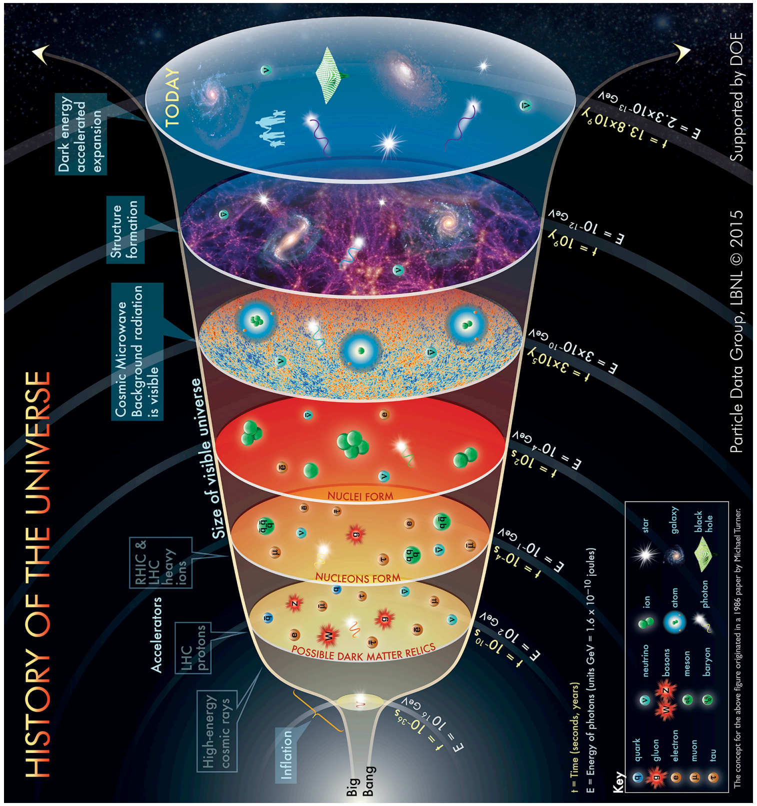
- Nuclei start to form after proton-neutron ratio freeze-out (T < 1 MeV)
- Early stage of Big Bang nucleosynthesis described by Nuclear Statistical Equilibrium (**Saha equation**)

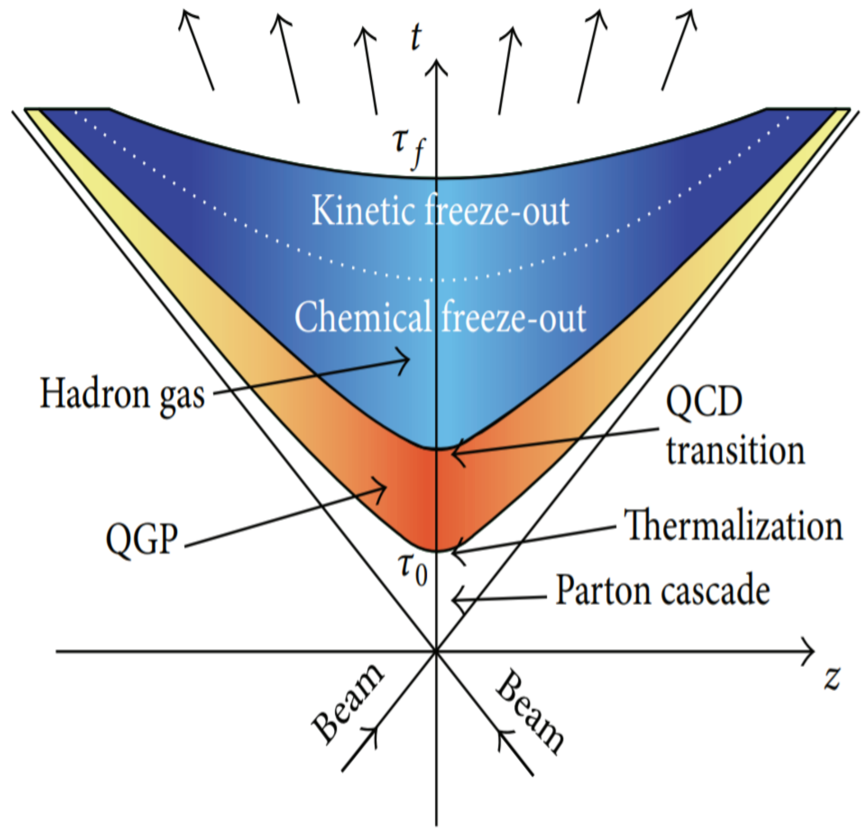
$$X_A = d_A \left[\zeta(3)^{A-1}\, \pi^{\frac{1-A}{2}}\, 2^{\frac{3A-5}{2}}\right] A^{\frac{5}{2}} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta^{A-1} X_p^Z X_n^{A-Z} \exp\left(\frac{B_A}{T}\right)$$

$\eta \sim 10^{-10}$ – baryon-to-photon ratio

[E. Kolb, M. Turner, "The Early Universe" (1990)]

Big Bang vs "Little Bangs"





- Hadrons (nucleons) form and "freeze-out" chemically before nuclei
- Bosons (photons or pions) catalyse nucleosynthesis

e.g. $p + n \leftrightarrow d + \gamma$ vs $p + n + \pi \leftrightarrow d + \pi$

Big Bang vs LHC nucleosynthesis

Similarities:

- Inelastic nucleonic reactions freeze-out before nuclei formation
- Isentropic expansion of boson-dominated matter (photons in BBN vs mesons in HIC), baryon-to-boson ratio: $\eta_{BBN} \sim 10^{-10}$, $\eta_{LHC} \sim 0.05$
- Strong nuclear formation and regeneration reactions → **Saha equation**

Differences:

- Time scales: 1-100 s in BBN vs ~10^{-22} s in HIC
- Temperatures: $T_{BBN} < 1$ MeV vs $T_{HIC} \sim 100$ MeV
- Binding energies, proton-neutron mass difference, and neutron lifetime important in BBN, less so in HICs
- $\mu_B \approx 0$ at the LHC, $\mu_B \neq 0$ in BBN
- Resonance feeddown important at LHC, irrelevant in BBN

LHC nucleosynthesis: simplified setup

- Chemical equilibrium lost at $T_{ch} = 155$ MeV, abundances of nucleons are frozen and acquire effective fugacity factors: $n_i = n_i^{eq} e^{\mu_N/T}$
- Isentropic expansion driven by effectively massless mesonic d.o.f.

$$\frac{V}{V_{\rm ch}} = \left(\frac{T_{\rm ch}}{T}\right)^3, \qquad \mu_N \simeq \frac{3}{2}\, T \ln\left(\frac{T}{T_{\rm ch}}\right) + m_N \left(1 - \frac{T}{T_{\rm ch}}\right)$$

- Detailed balance for nuclear reactions, $X + A \leftrightarrow X + \sum_i A_i$, X is e.g. a pion

$$\frac{n_A}{\prod_i n_{A_i}} = \frac{n_A^{\rm eq}}{\prod_i n_{A_i}^{\rm eq}}, \quad \Leftrightarrow \quad \mu_A = \sum_i \mu_{A_i}, \qquad \text{e.g.} \;\; \mu_d = \mu_p + \mu_n, \;\; \mu_{^3{\rm He}} = 2\mu_p + \mu_n, \;\; \dots$$

Saha equation

$$X_A = d_A \left[(d_M)^{A-1}\, \zeta(3)^{A-1}\, \pi^{\frac{1-A}{2}}\, 2^{-\frac{3+A}{2}}\right] A^{5/2} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta_B^{A-1} \exp\left(\frac{B_A}{T}\right)$$

$d_M \sim 11 - 13, \quad \eta_B \simeq 0.03$

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BBN: $$X_A = d_A \left[\zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{\frac{3A-5}{2}}\right] A^{\frac{5}{2}} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta^{A-1} X_p^Z X_n^{A-Z} \exp\left(\frac{B_A}{T}\right)$$

[E. Kolb, M. Turner, "The Early Universe" (1990)]

(Simplified) Saha equation vs thermal model

Saha equation:

$$\frac{N_A(T)}{N_A(T_{\rm ch})} \simeq \left(\frac{T}{T_{\rm ch}}\right)^{\frac{3}{2}(A-1)} \exp\left[B_A\left(\frac{1}{T}-\frac{1}{T_{\rm ch}}\right)\right] \qquad B_A \ll T$$

Thermal model:

$$\left[\frac{N_A(T)}{N_A(T_{\rm ch})}\right]_{\rm eq.} \simeq \left(\frac{T}{T_{\rm ch}}\right)^{\frac{3}{2}} \exp\left[-m_A\left(\frac{1}{T}-\frac{1}{T_{\rm ch}}\right)\right] \qquad m_A \gg T$$

Strong exponential dependence on the temperature is eliminated in the Saha equation approach

Further, quantitative applications require numerical treatment of full spectrum of *massive* mesonic and baryonic resonances

Full numerical implementation

Expansion of hadron resonance gas in partial chemical equilibrium at $T < T_{ch}$

[H. Bebie, P. Gerber, J.L. Goity, H. Leutwyler, Nucl. Phys. B '92]

Chemical composition of stable hadrons is fixed, kinetic equilibrium maintained through quasi-elastic resonance reactions $\pi\pi \leftrightarrow \rho,\ \pi\mathrm{K} \leftrightarrow \mathrm{K}^*, \pi N \leftrightarrow \Delta$, etc.

Effective chemical potentials:

$$\tilde{\mu}_j = \sum_{i \in \text{stable}} \langle n_i \rangle_j \, \mu_i , \qquad \langle n_i \rangle_j \text{ -- mean number of hadron } i \text{ from decays of hadron } j, \qquad j \in \text{HRG}$$

Conservation laws:

$$\sum_{j \in \text{hrg}} \langle n_i \rangle_j \, n_j(T, \tilde{\mu}_j) \, V = N_i(T_{\text{ch}}), \quad i \in \text{stable}$$

$$\sum_{j \in \text{hrg}} s_j(T, \tilde{\mu}_j) \, V = S(T_{\text{ch}})$$

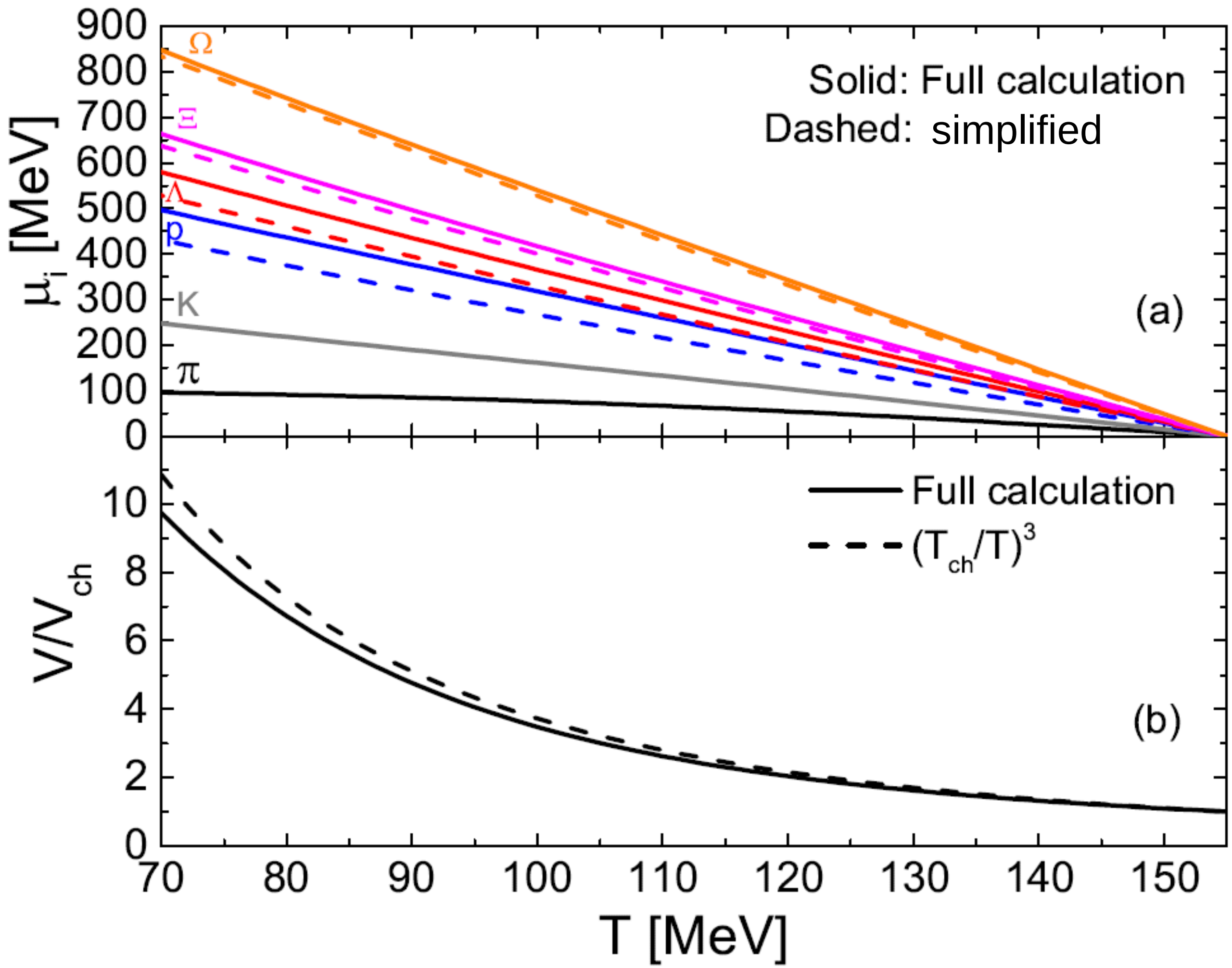
numerical solution $\longrightarrow$ $\{\mu_i(T)\}, V(T)$

Numerical implementation within (extended) **Thermal-FIST** package

[**V.V.**, H. Stoecker, arXiv:1901.05249, *Computer Physics Communications*, in print]

open source: https://github.com/vlvovch/Thermal-FIST

Full calculation: parameters

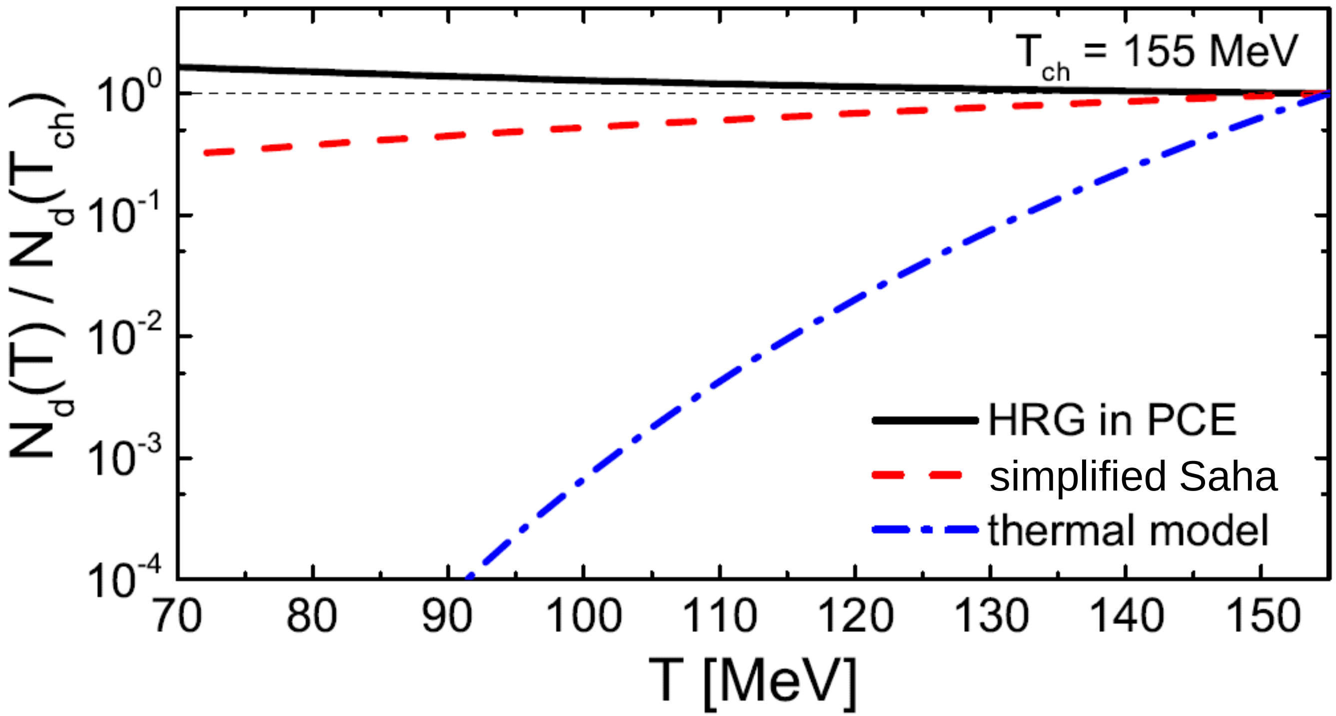


"Initial conditions" from thermal fits with Thermal-FIST to 0-10% ALICE hadron yields

$T_{ch} = 155$ MeV, $V_{ch} = 4700$ fm^3

[**V.V.**, Gorenstein, Stoecker, 1807.02079]

Full calculation: deuteron yield



Resonance feed-down is important in precision studies

LHC deuteron-synthesis

PHYSICAL REVIEW C **99**, 044907 (2019)

Editors' Suggestion Featured in Physics

Microscopic study of deuteron production in PbPb collisions at $\sqrt{s} = 2.76$ TeV via hydrodynamics and a hadronic afterburner

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[2]*Physics Department, University of California, Berkeley, California 94720, USA*

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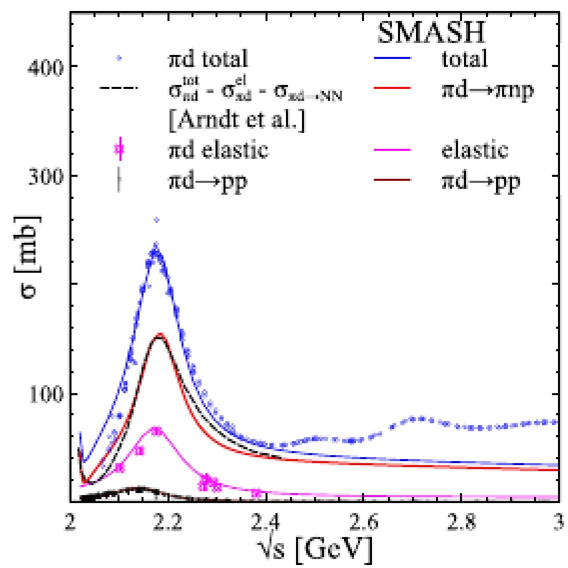


FIG. 1. Deuteron-pion interaction cross sections from SAID database [40] and partial wave analysis [41] are compared to our parametrizations (Tables II and III in the Appendix). Inelastic $d\pi \leftrightarrow$

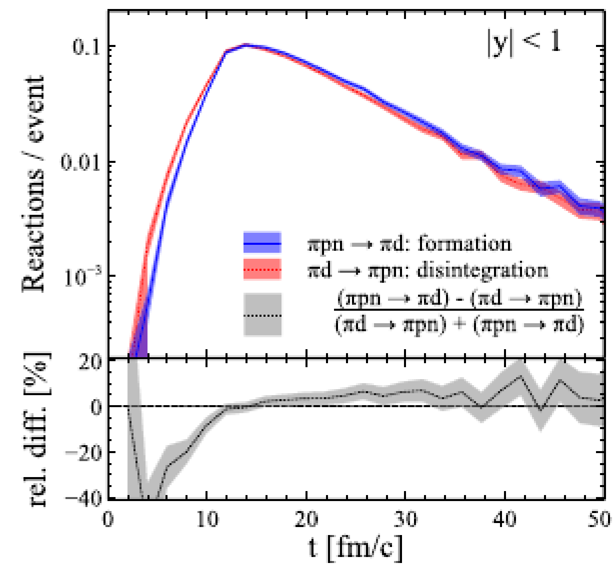
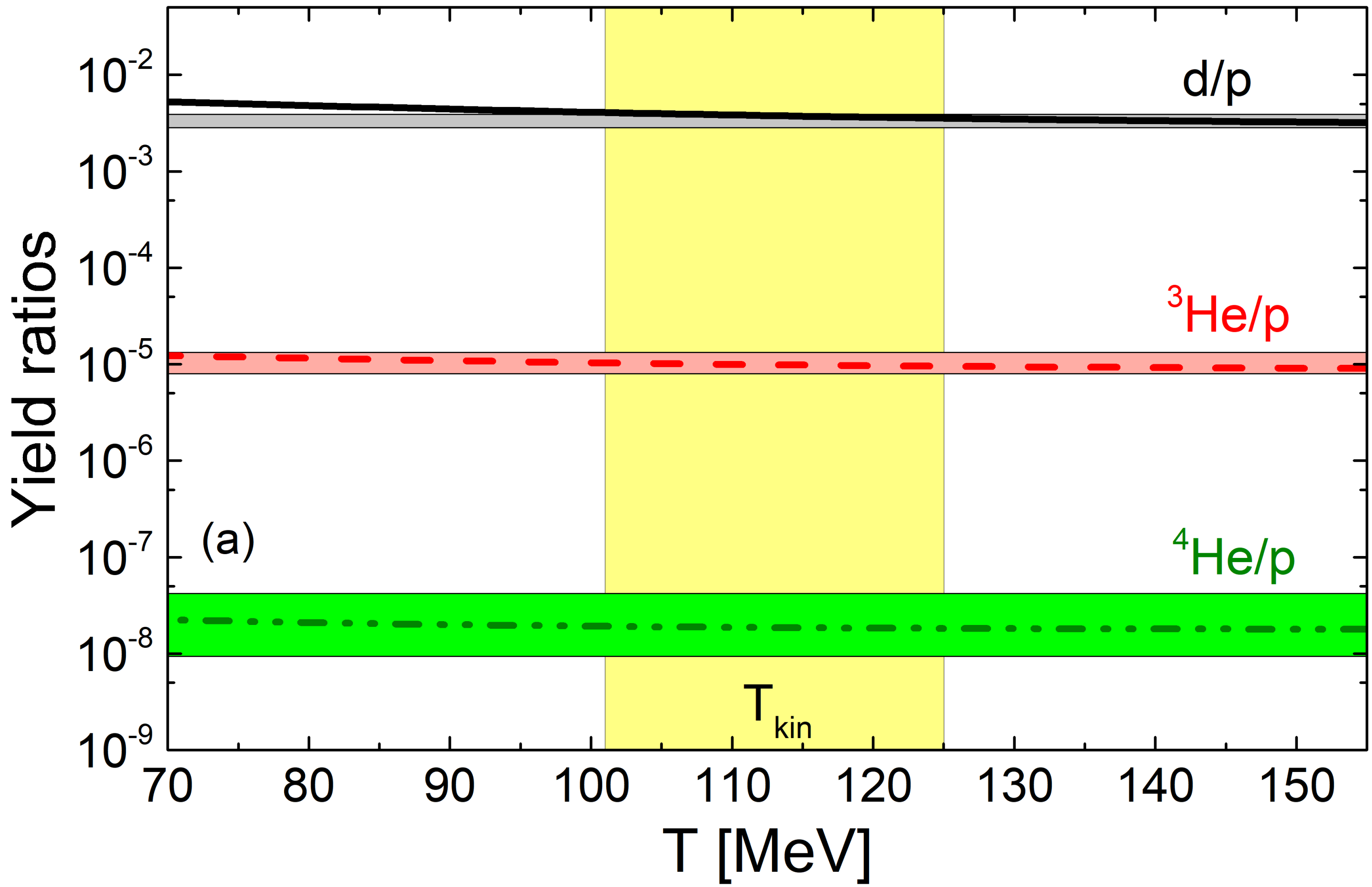


FIG. 5. Reaction rates of the most important $\pi d \leftrightarrow \pi pn$ reaction in forward and reverse direction.

- **Law of mass action at work**

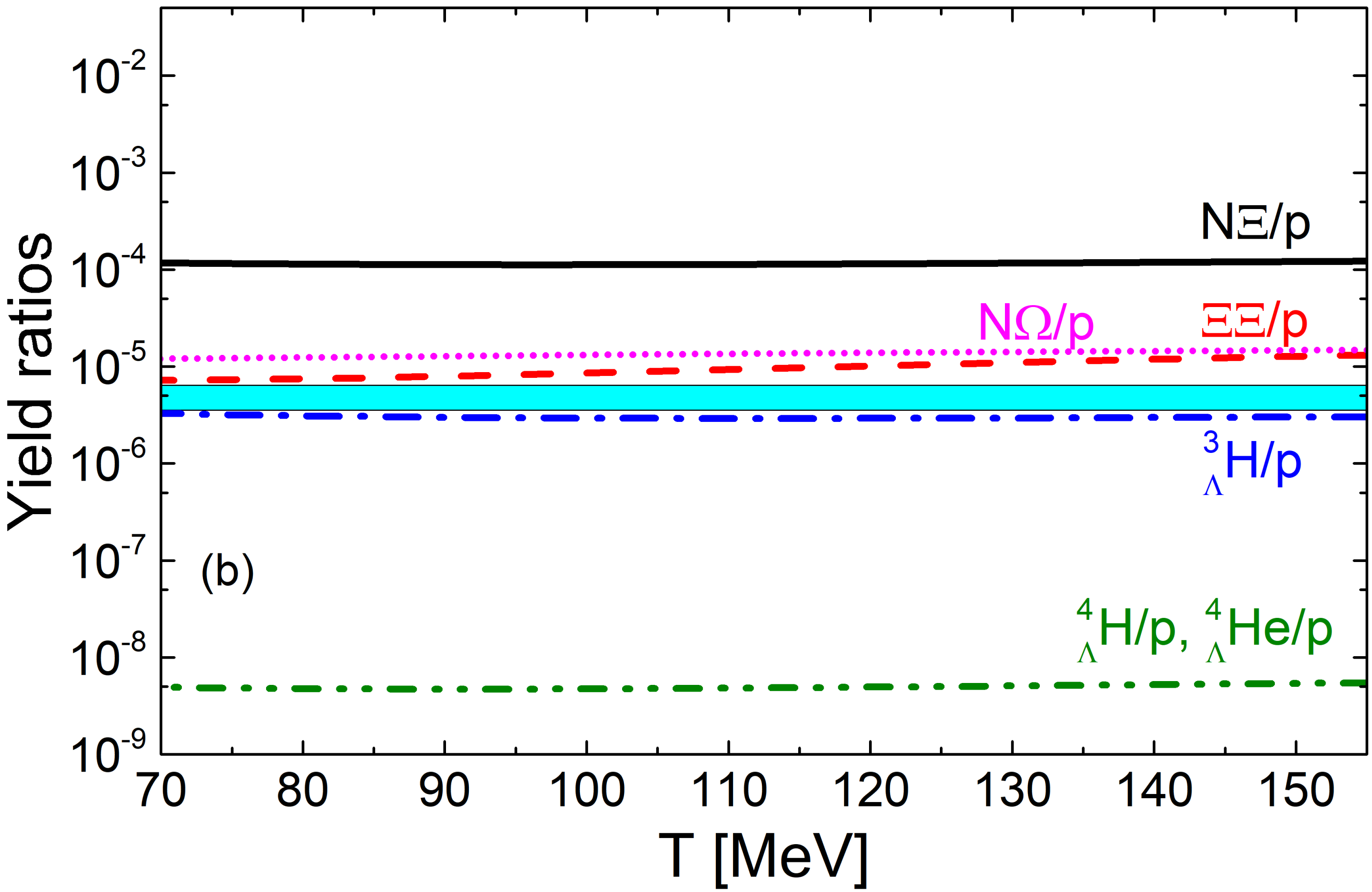
Full calculation: nuclei



Deviations from thermal model predictions are moderate despite significant cooling and dilution. Is this the reason for why thermal model works so well?

For $T = T_{kin}$ similar results reported in [X. Xu, R. Rapp, EPJA 55, 68 (2019)]

Full calculation: hypernuclei

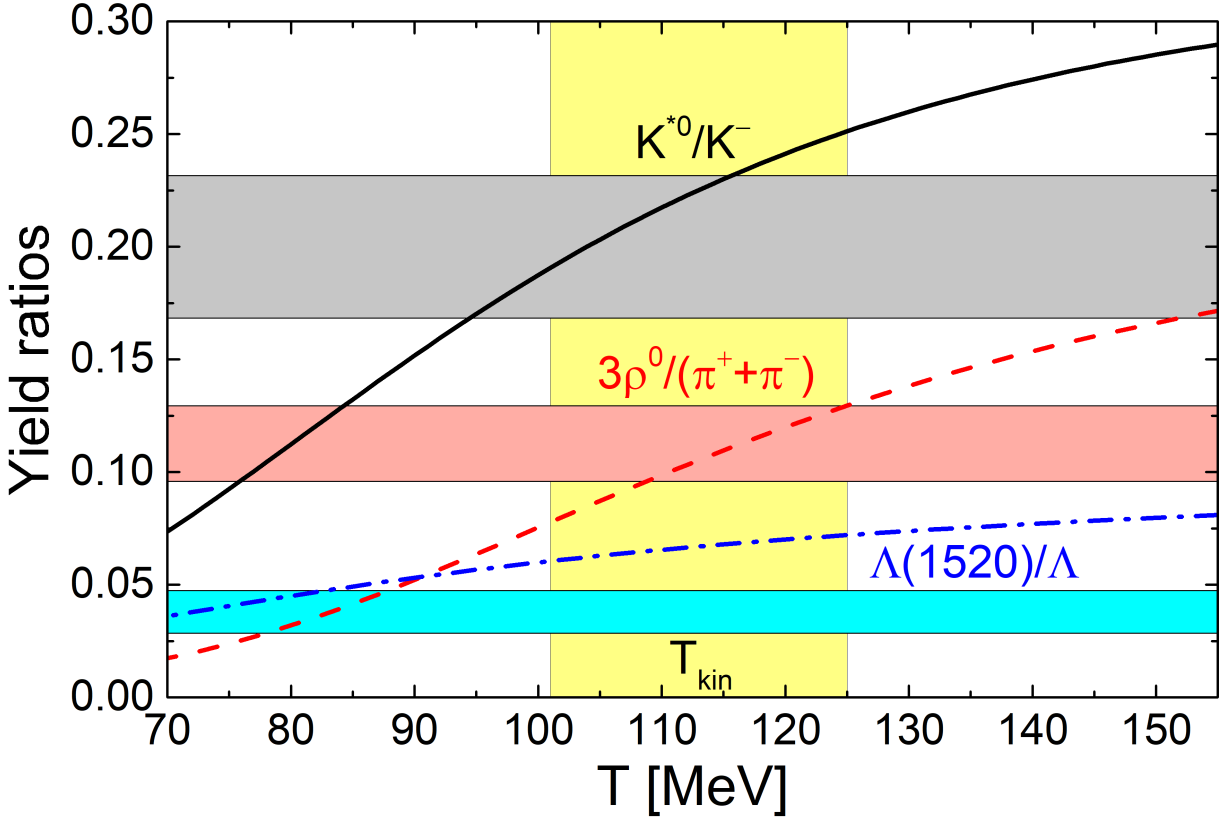


Hypernuclei stay close to the thermal model prediction. An exception is a hypothetical ΞΞ state ← *planned measurement in Runs 3 & 4 at the LHC*

[LHC Yellow Report, 1812.06772]

Full calculation: resonances

Yields of resonances are *not* conserved in partial chemical equilibrium



At $T \approx T_{kin}$ the suppressed resonance yields agree quite well with ALICE data for 0-20% central Pb+Pb collisions [ALICE, 1404.0495; 1805.04361; 1805.04365]

This implies significant resonance regeneration in the hadronic phase

Summary and outlook

- Nucleosynthesis in HICs at LHC via the Saha equation is in analogy to initial stages of big bang nucleosynthesis in the early universe.

- This would naturally explain why thermal model works for light (anti-)(hyper-)nuclei yields. It does *not* establish where exactly they are formed though, *any* $T < T_{ch}$ permitted!

- Who can give the answer?
  - Building of clusters (Hagedorn states)? [K. Gallmeister et al.]
  - Reconciliation with coalescence?
  - Reaction rates vs expansion rate and binding energies?
  - Transport description? [D. Oliinychenko et al.]
  - Internal consistency of model assumptions? [Cai, Cohen, Gelman, Yamaguchi, 1905.02753]
- Quantum mechanical treatment of bound systems in medium needed

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Thanks for your attention!