Statistical-thermal model: Applications using Thermal-FIST

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3rd EMMI Workshop: Anti-matter, hyper-matter and exotica production at the LHC

December 2, 2019

FIAS Frankfurt Institute for Advanced Studies

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Outline

1. Short description of **Thermal-FIST**

2. Recent applications to light nuclei and exotica
   - Canonical suppression
   - The Saha equation approach
   - Feeddown contributions from excited nuclear states

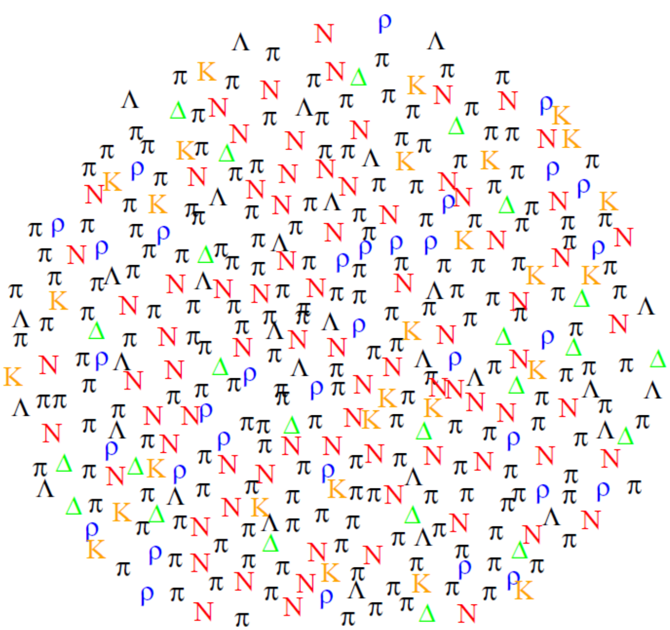
3. Summary

Hadron resonance gas (HRG) at freeze-out

HRG: Equation of state of hadronic matter as a multi-component (non-)interacting gas of known hadrons, resonances, and *light nuclei*

$$\ln Z \approx \sum_{i\in M,B} \ln Z_i^{id} = \sum_{i\in M,B} \frac{d_i V}{2\pi^2} \int_0^\infty \pm p^2 dp \ln\left[1 \pm \exp\left(\frac{\mu_i - E_i}{T}\right)\right]$$

Grand-canonical ensemble: $\mu_i = b_i \mu_B + q_i \mu_Q + s_i \mu_S$ ***chemical equilibrium***



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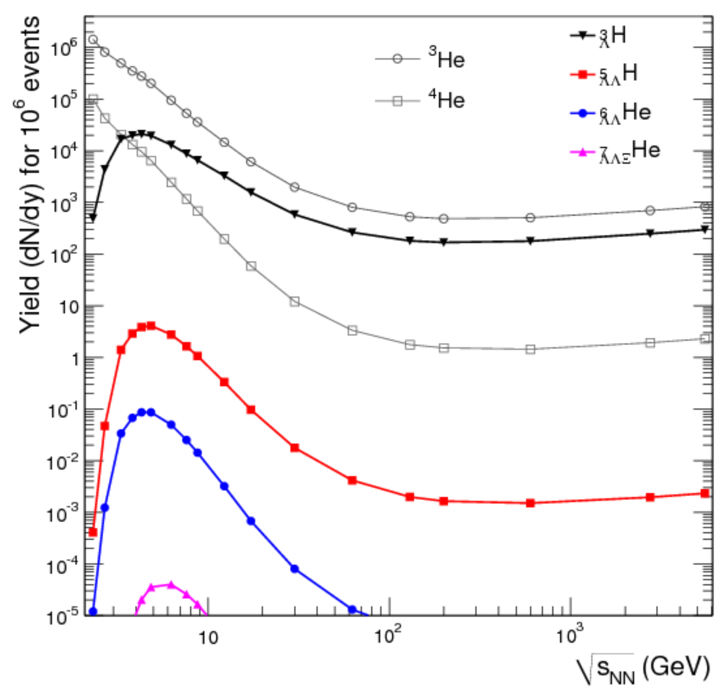


[A. Andronic et al., PLB '12]

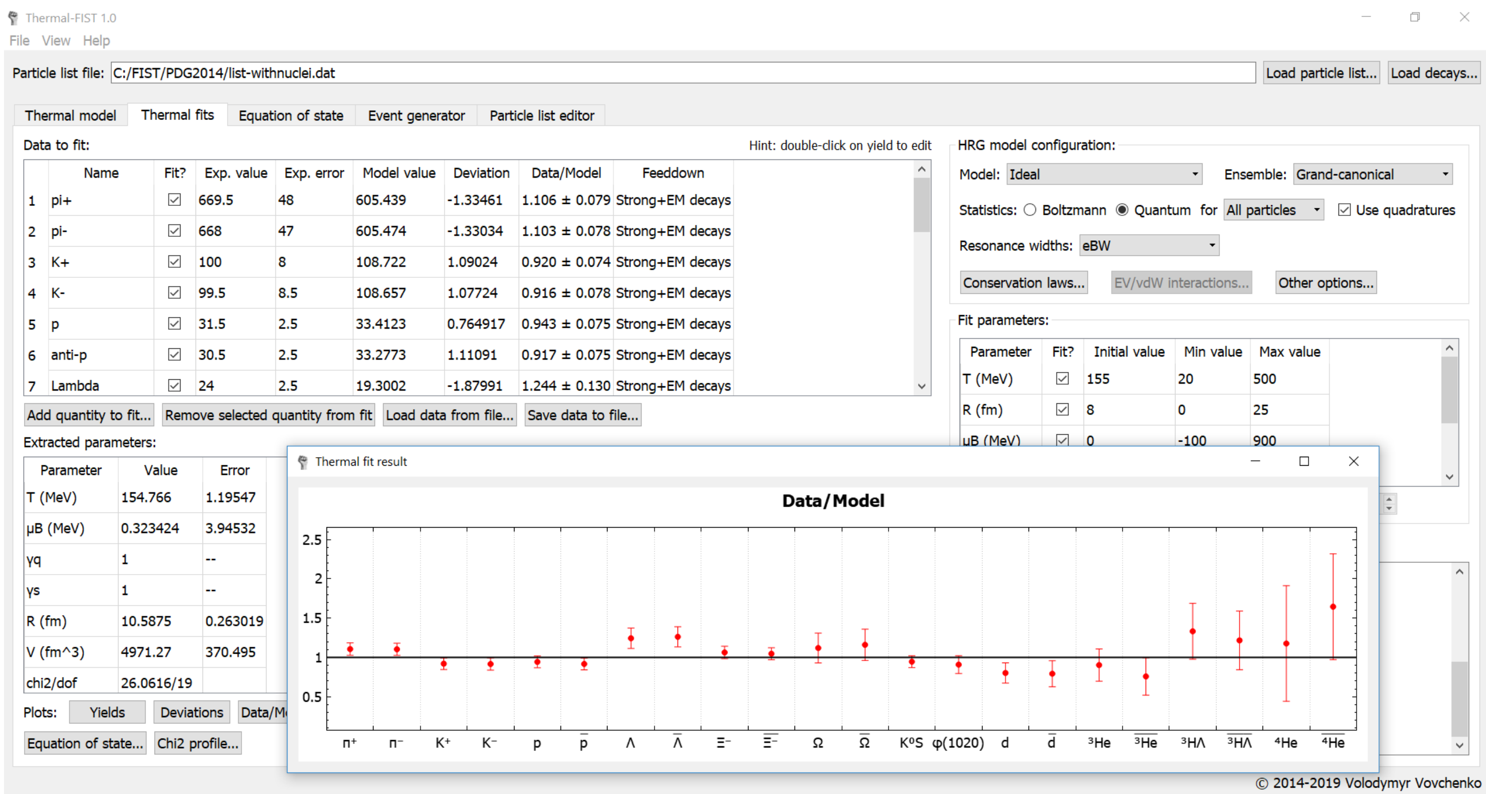
Thermal-FIST

Thermal-FIST* (a.k.a. **FIST** or **FAUST**)

C++/Qt/Jupyter

open source: https://github.com/vlvovch/Thermal-FIST

reference: **V.V.**, H. Stoecker, *Computer Physics Communications* ***244,*** *295 (2019)*



A framework for general-purpose statistical-thermal model applications

**<u>T</u>hermal, <u>F</u>ast and <u>I</u>nteractive <u>S</u>tatistical <u>T</u>oolkit*

Using Thermal-FIST

The package is ***cross-platform*** (Linux, Mac, Windows, Android)
Installation using **git** and **cmake**

```
# Clone the repository from GitHub
git clone https://github.com/vlvovch/Thermal-FIST.git
cd Thermal-FIST

# Create a build directory, configure the project with cmake
# and build with make
mkdir build
cd build
cmake ../
make

# Run the GUI frontend
./bin/QtThermalFIST

# Run the test calculations from the paper
./bin/examples/cpc1HRGTDep
./bin/examples/cpc2chi2
./bin/examples/cpc3chi2NEQ
./bin/examples/cpc4mcHRG
```

GUI requires free Qt5 framework, the rest of the package has no external dependencies

Quick start guide *Documentation* *Physics manual*

Statistical-thermal model aspects in FIST

- Extensions of the HRG model
  - finite resonance widths
  - repulsive (excluded volume) and van der Waals (*criticality*) interactions
  - particle number fluctuations and correlations
  - chemical non-equilibrium (γ_q , γ_s) a la Rafelski
  - unstable nuclear fragments
- Equation of state
- Canonical statistical model (CSM)
  - (local) (selective) exact conservation of conserved charges
  - canonical suppression of light nuclei
- Monte Carlo generator (Blast-wave, canonical ensemble,...)
- Hadronic phase and dynamical freeze-out
  - partial chemical equilibrium
  - suppression of resonance yields
  - evolution of light nuclei abundances via the Saha equation

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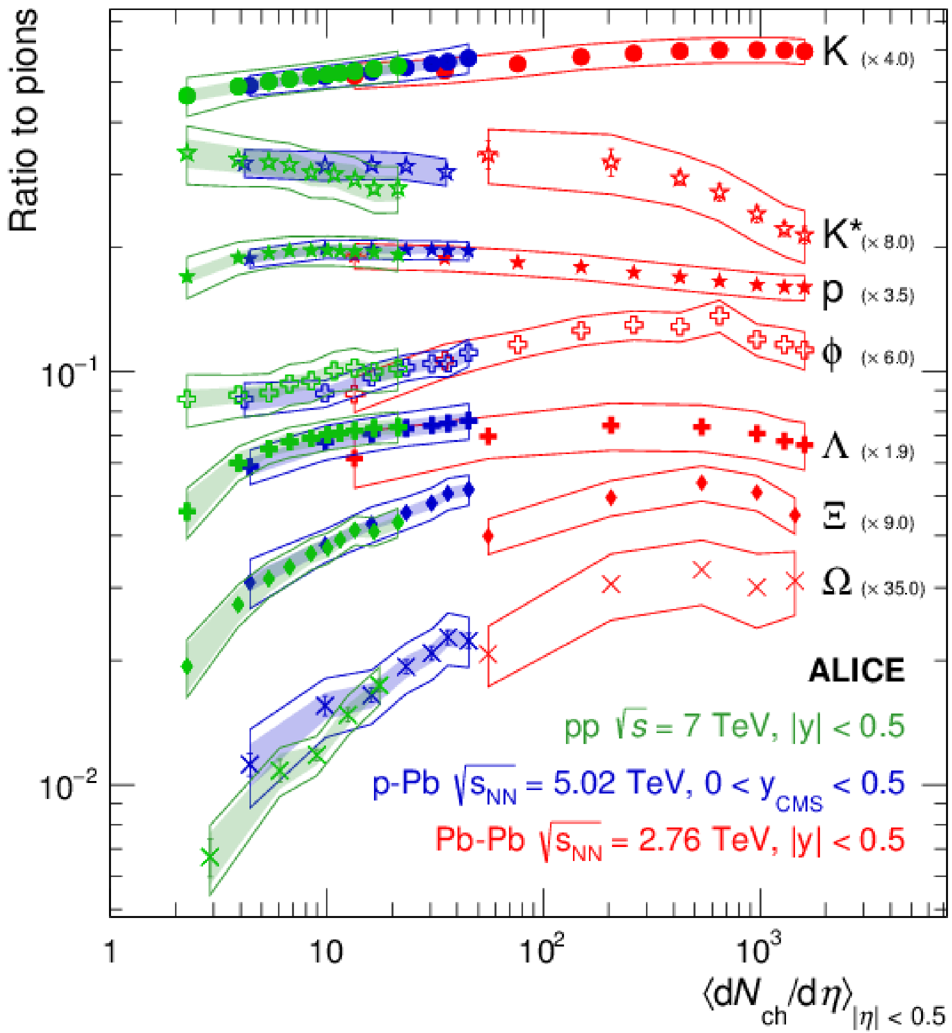
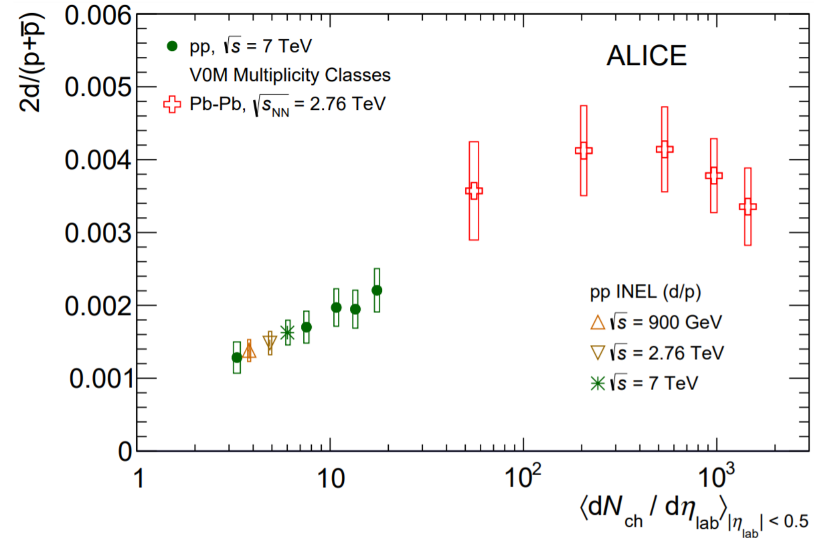
Canonical suppression of light nuclei at the LHC

Multiplicity dependence of hadrochemistry



[ALICE collaboration, 1807.11321]



[ALICE collaboration, 1902.09290]

- Grand-canonical thermal picture predicts no multiplicity dependence
- Apply **canonical** statistical model?

Canonical statistical model (CSM)

Exact conservation of B, Q, S in a correlation volume V_C

[Rafelski, Danos, et al., PLB '80; Hagedorn, Redlich, ZPC '85]

$$\mathcal{Z}(B,Q,S) = \int_{-\pi}^{\pi} \frac{d\phi_B}{2\pi} \int_{-\pi}^{\pi} \frac{d\phi_Q}{2\pi} \int_{-\pi}^{\pi} \frac{d\phi_S}{2\pi} \, e^{-i\,(B\phi_B + Q\phi_Q + S\phi_S)} \, \exp\left[\sum_j z_j^1 \, e^{i\,(B_j\phi_B + Q_j\phi_Q + S_j\phi_S)}\right]$$

$$z_j^1 = V_c \int dm \, \rho_j(m) \, d_j \frac{m^2 T}{2\pi^2} K_2(m/T)$$

$$\langle N_j^{\rm prim} \rangle^{\rm ce} = \frac{Z(B-B_j, Q-Q_j, S-S_j)}{Z(B,Q,S)} \langle N_j^{\rm prim} \rangle^{\rm gce}$$

[Becattini et al., ZPC '95, ZPC '97]

Implemented in Thermal-FIST for a full HRG

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Net-proton fluctuations affected by baryon number conservation

[Braun-Munzinger, Rustamov, Stachel, 1612.00702]

$$\frac{\kappa_2(\mathrm{p} - \bar{\mathrm{p}})}{\langle \mathrm{p} \rangle + \langle \bar{\mathrm{p}} \rangle} \simeq 1 - \frac{\langle \mathrm{p} \rangle}{k \, dN_B/dy}$$

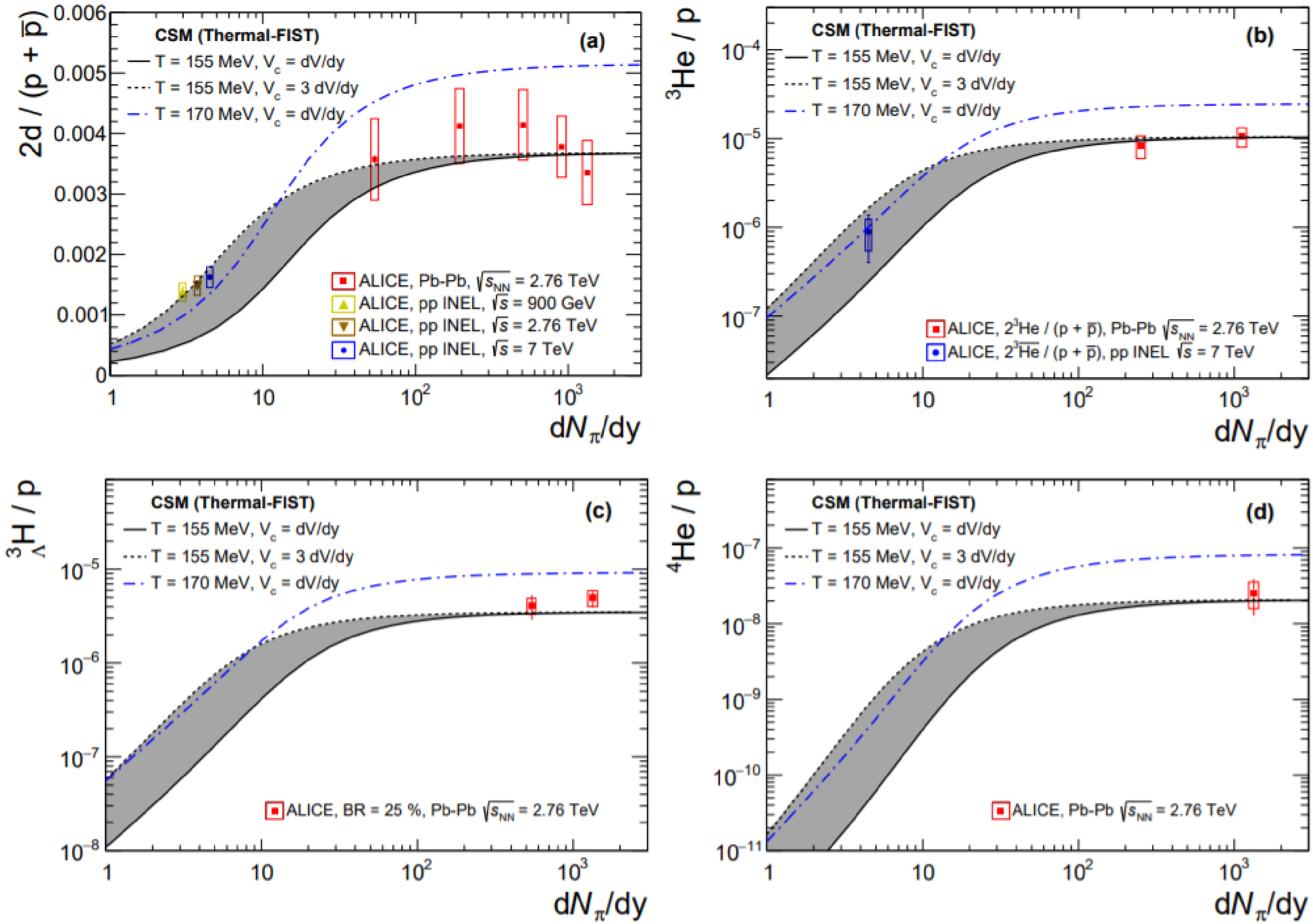
Using ALICE data for net-p fluctuations [1910.14396] one obtains $k \sim 3$-4 for most centrality bins in Pb-Pb collisions

[**V.V.**, Dönigus, Stoecker, 1906.03145, PRC '19]

“Vanilla” CSM

$T_{ch} = 155$ MeV, $V_C = 3dV/dy$, multiplicity dependence driven by V_C only

[**V.V.**, Dönigus, Stoecker, 1808.05245, PLB ‘18]



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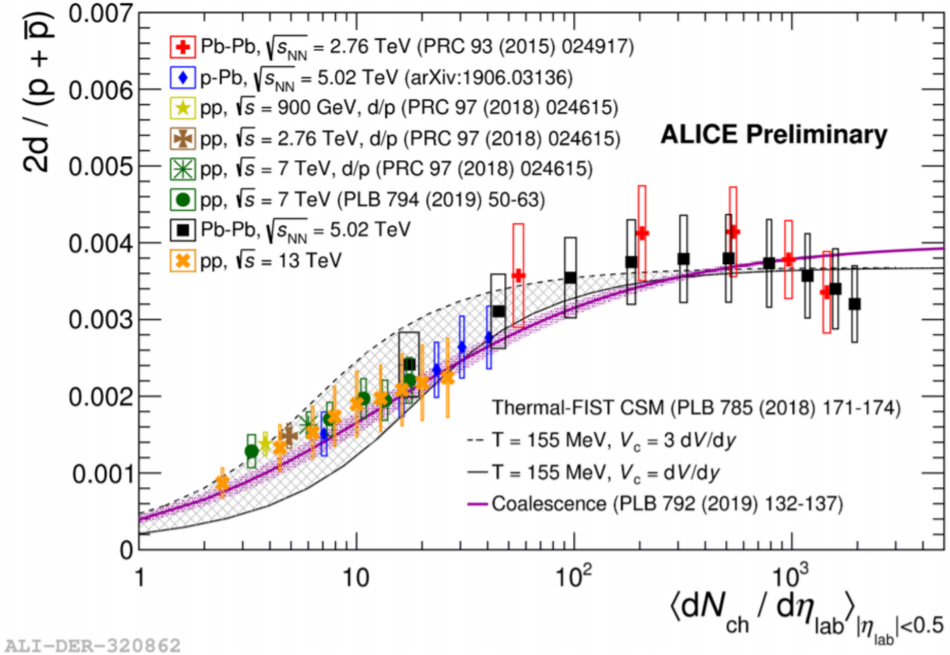
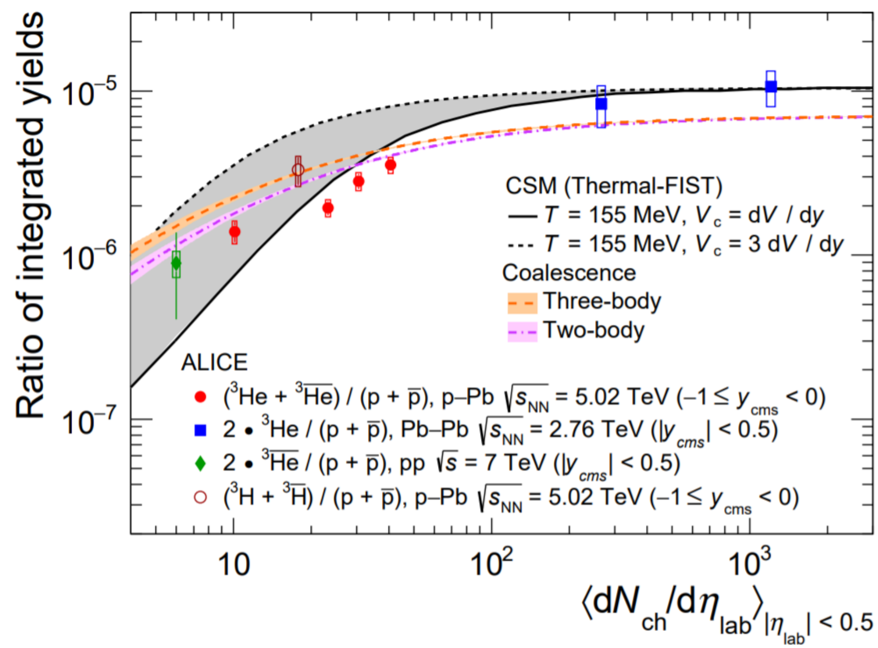




[L. Barioglio, QM19]

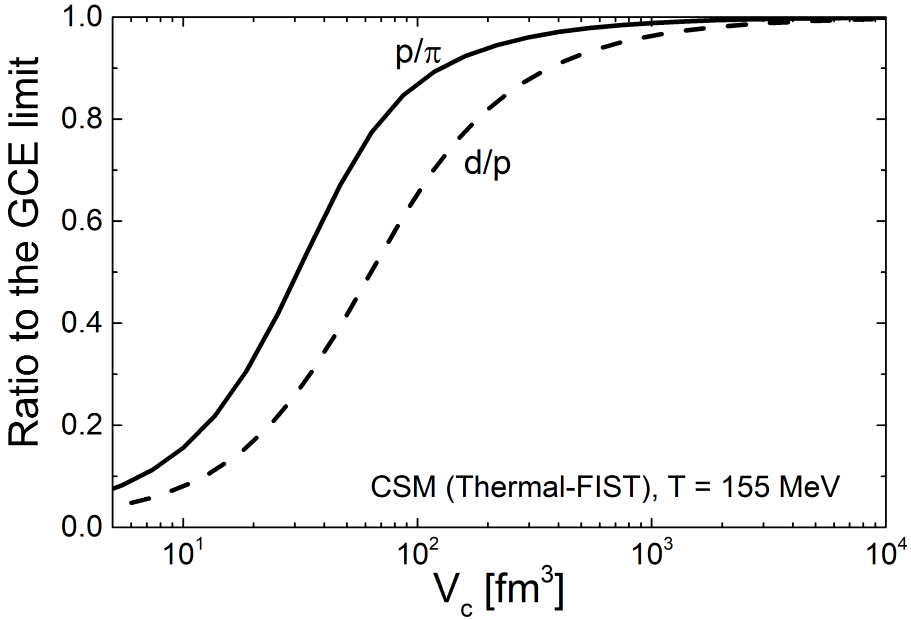
[ALICE collaboration, 1910.14401]

Basic CSM appears to capture trends seen in light nuclei production data

“Vanilla” CSM: nuclei vs p/π ratio

Canonical suppression affects not only nuclei, but also the p/π ratio

The effect for p/π is generally milder than d/p, but not insignificant



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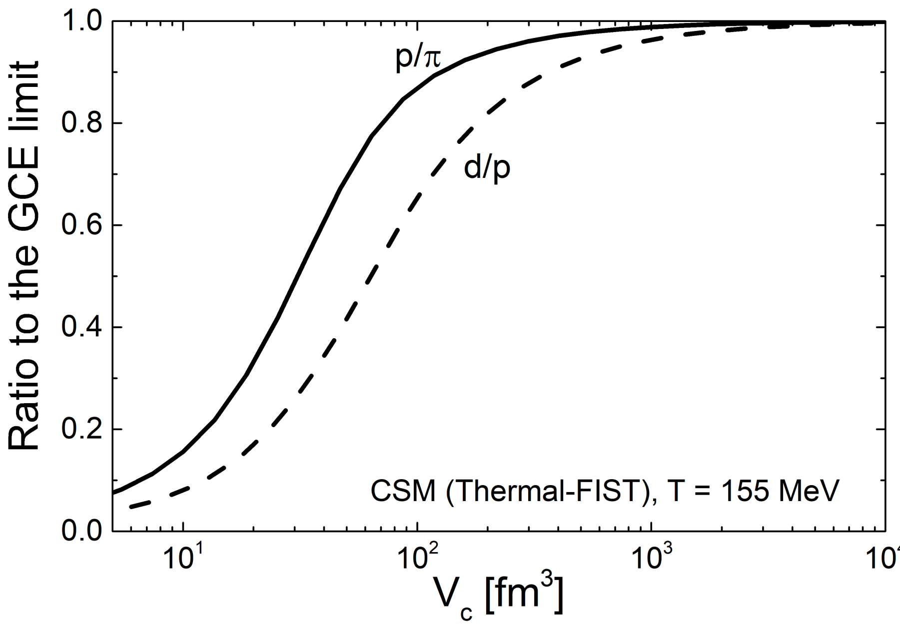
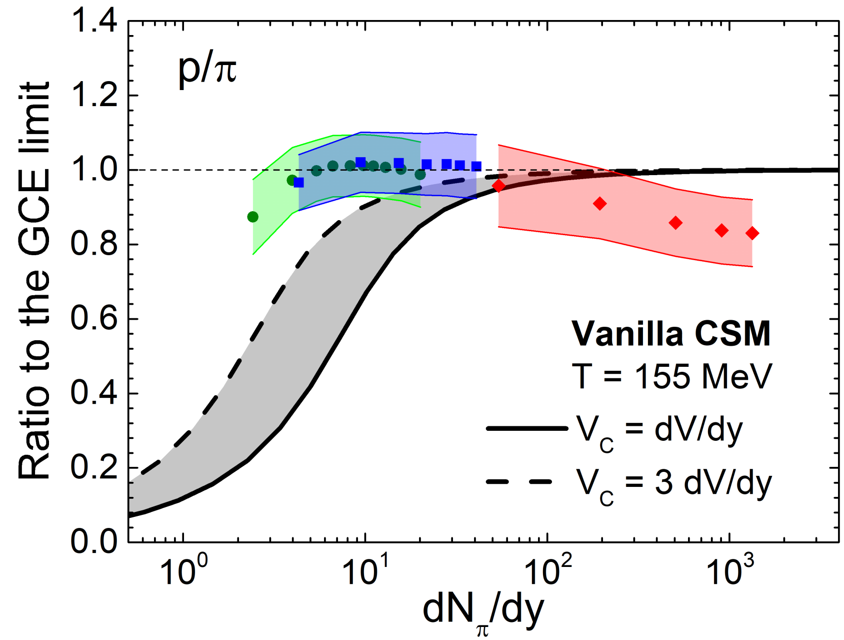
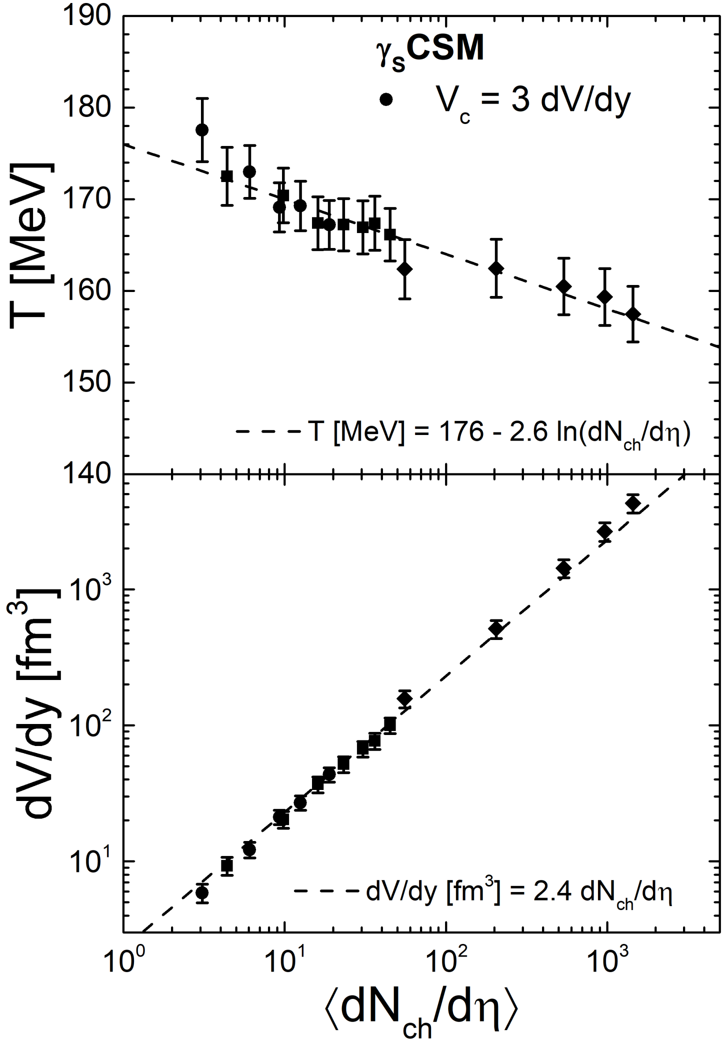




p/π suppression predicted by vanilla CSM not supported by the data

Full CSM

Full CSM: allow for multiplicity-dependent T_{ch} [**V.V.**, Dönigus, Stoecker, 1906.03145, PRC '19]



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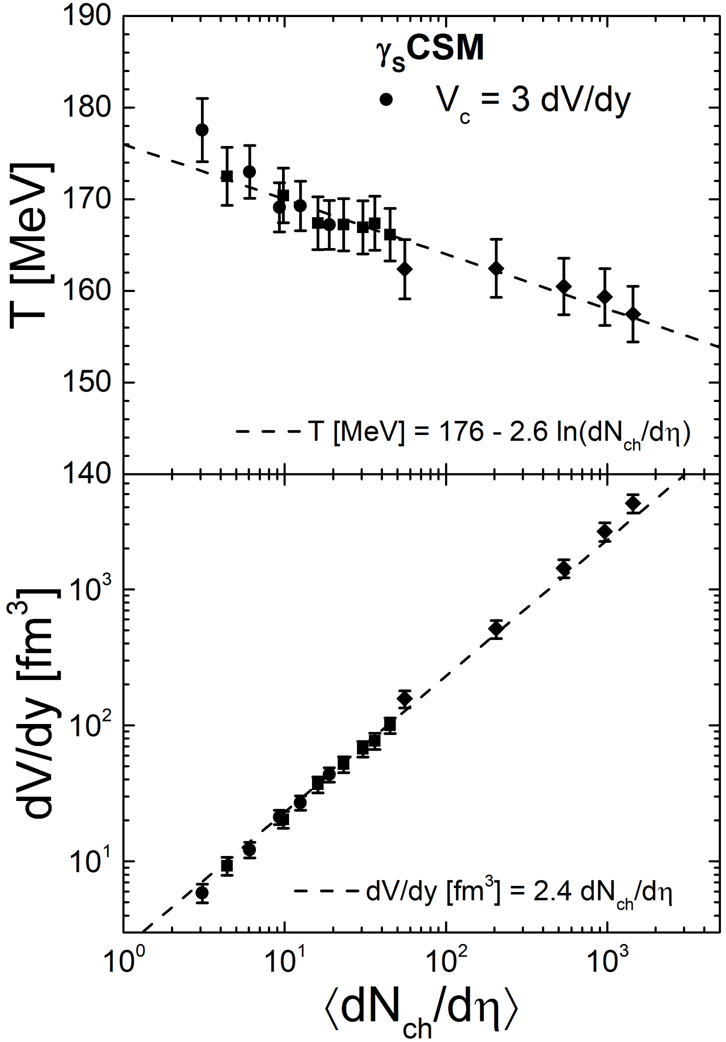

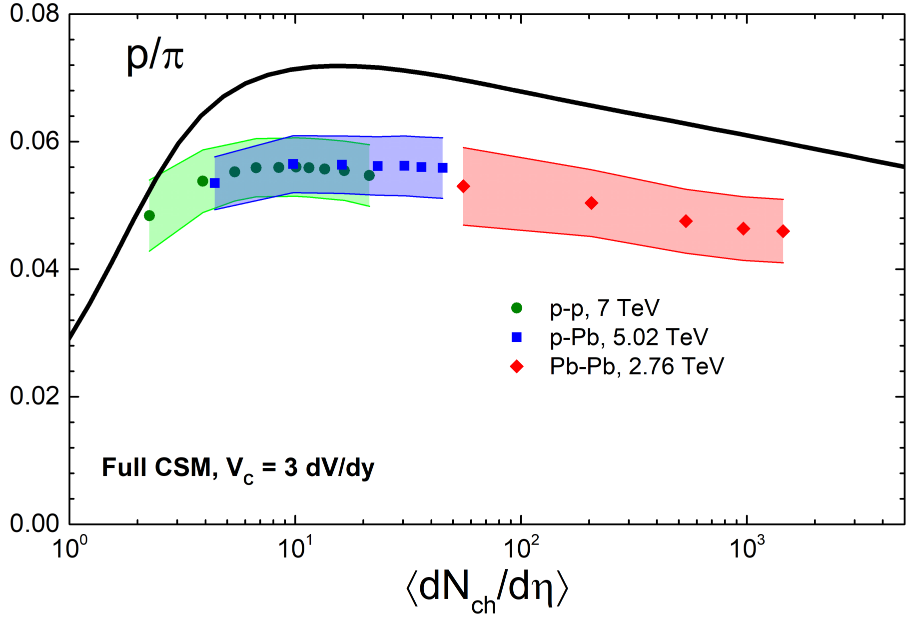

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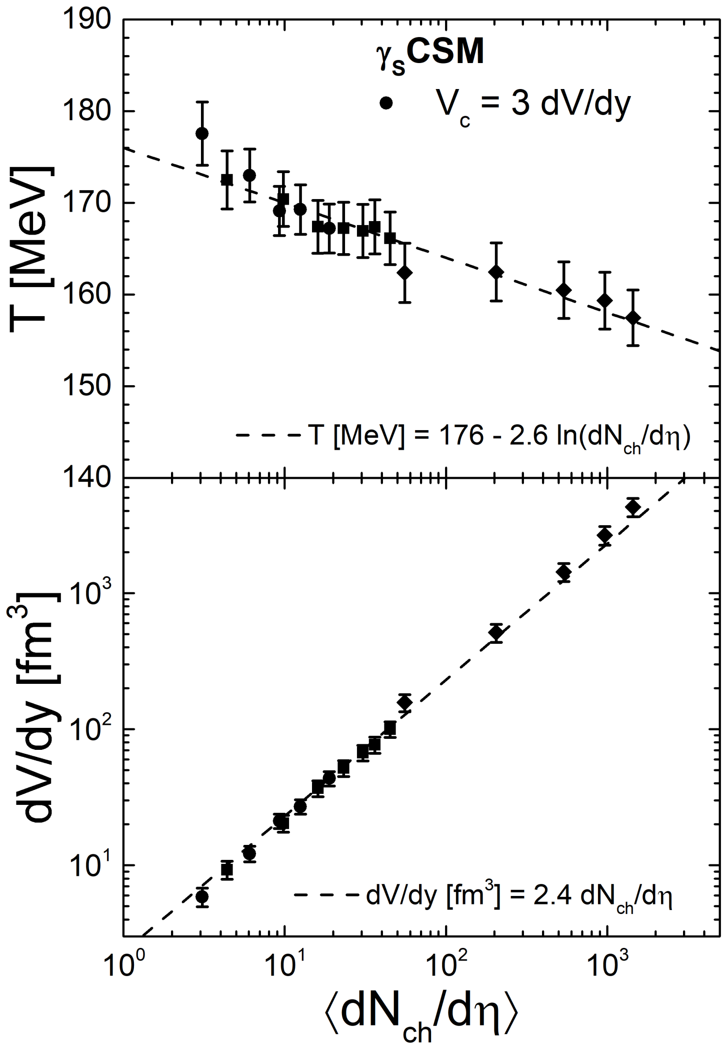
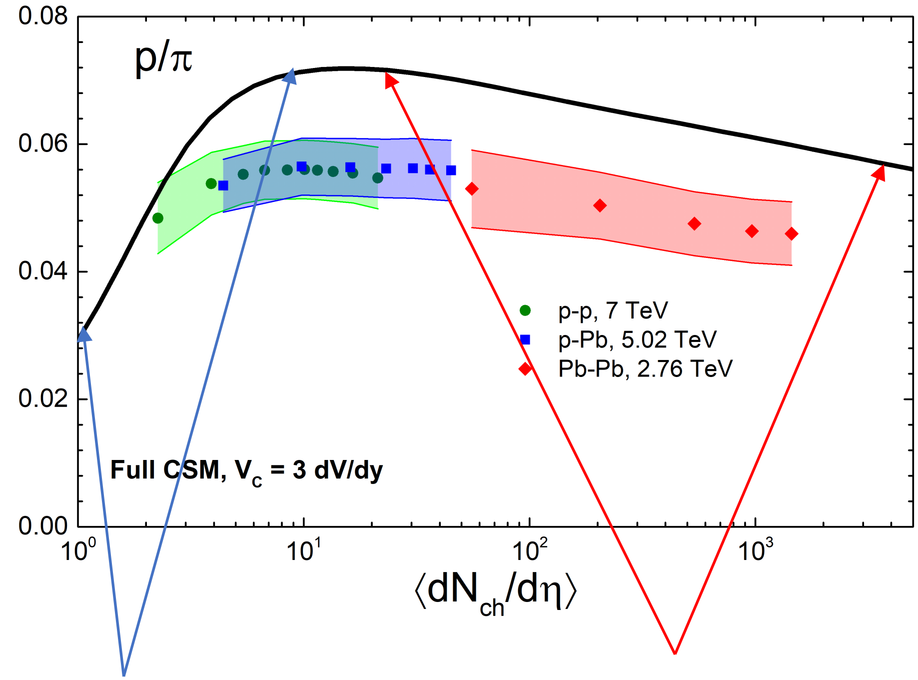
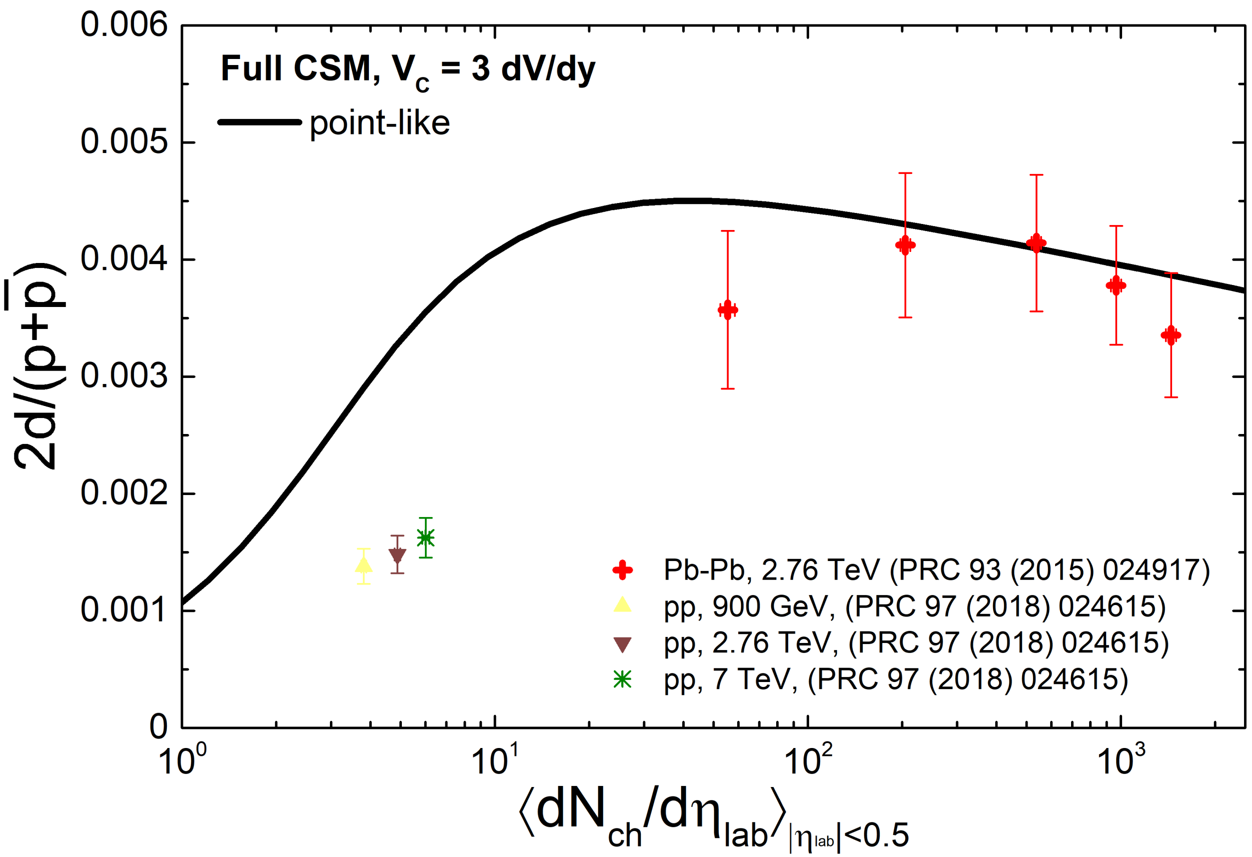




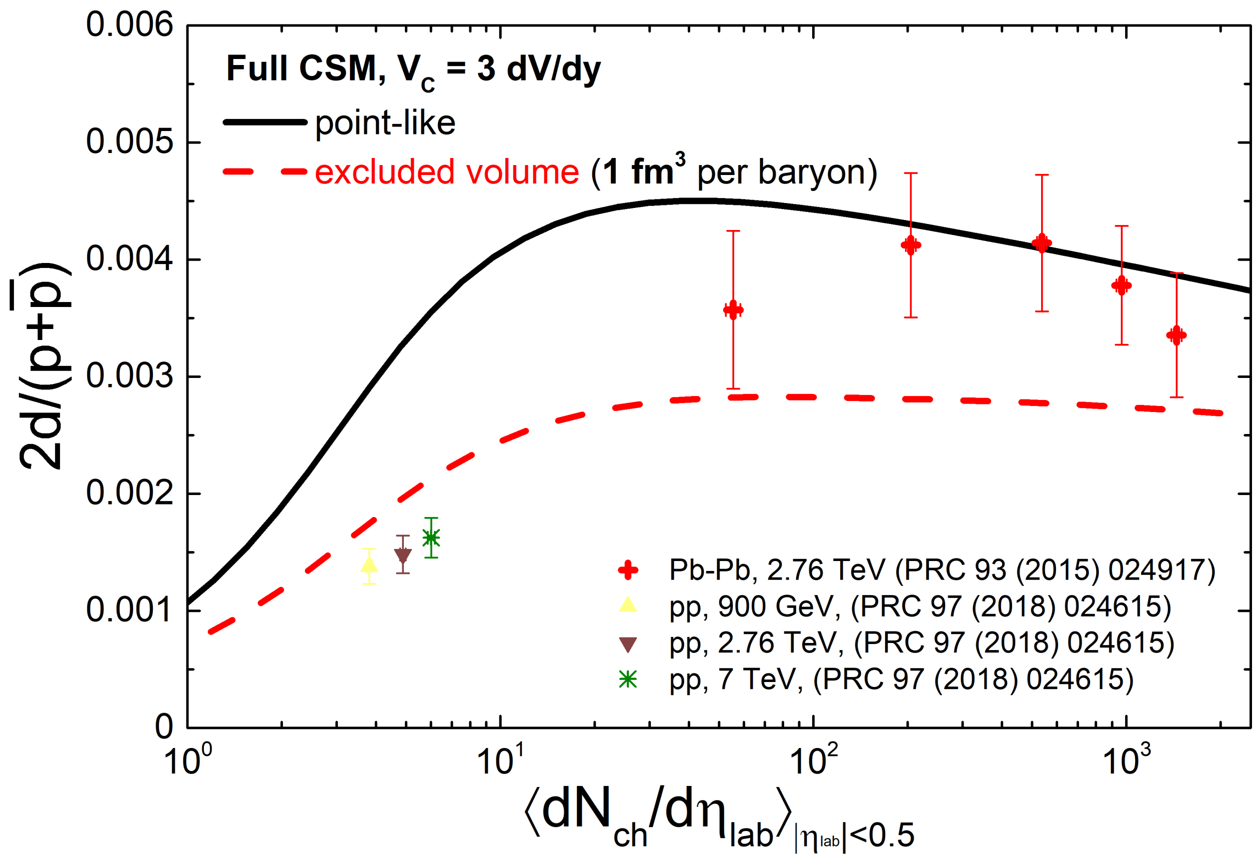
$V_c \searrow \quad \Longrightarrow \quad p/\pi \searrow$ $\qquad$ $T_{ch} \nearrow \quad \Longrightarrow \quad p/\pi \nearrow$

Full CSM: d/p



$$T_{ch} \nearrow \quad \Longrightarrow \quad d/p \nearrow$$

Full CSM: d/p



$T_{ch} \nearrow \quad \Longrightarrow \quad d/p \nearrow$

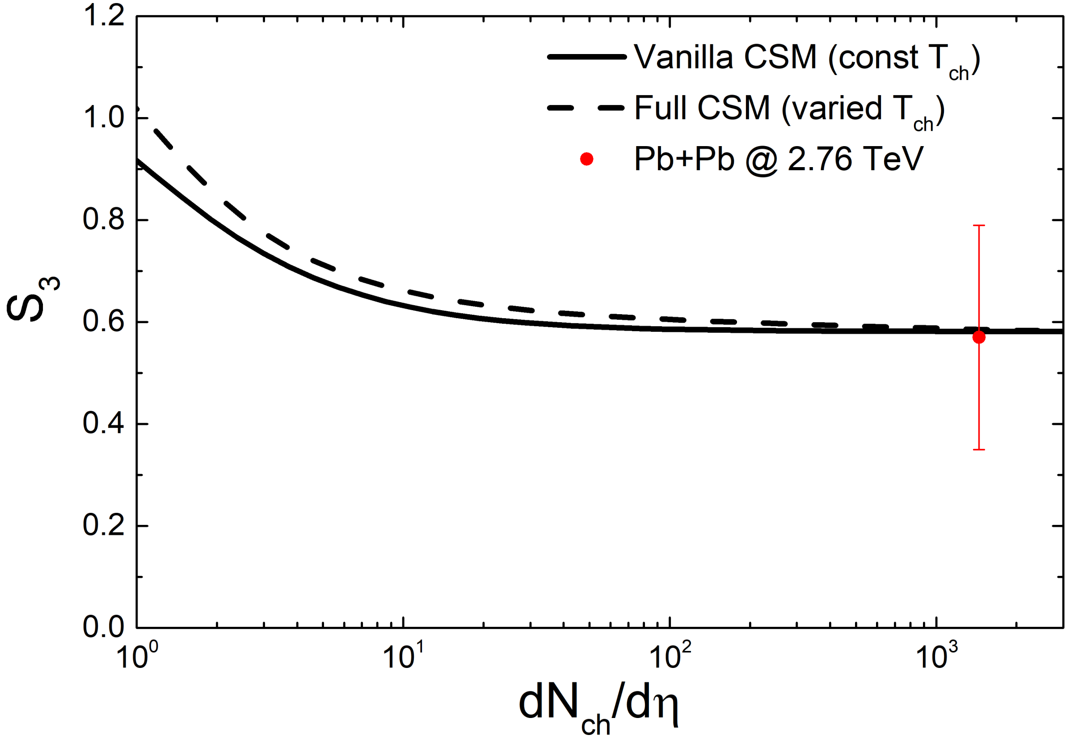
Excluded volume (schematic): $N_i \to N_i \exp\left(-\frac{v_i\, p}{T}\right) \quad \Longrightarrow \quad d/p \searrow$

Simultaneous description of light nuclei and p/π ratio remains challenging

CSM: S_3

$$S_3 = ({}^3_\Lambda H/{}^3He) / (\Lambda/p)$$

[E864 collab., PRC '04; Zhang et al., PLB '10]



Different versions of CSM give similar predictions, mild increase of S_3 due to baryon and strangeness conservation

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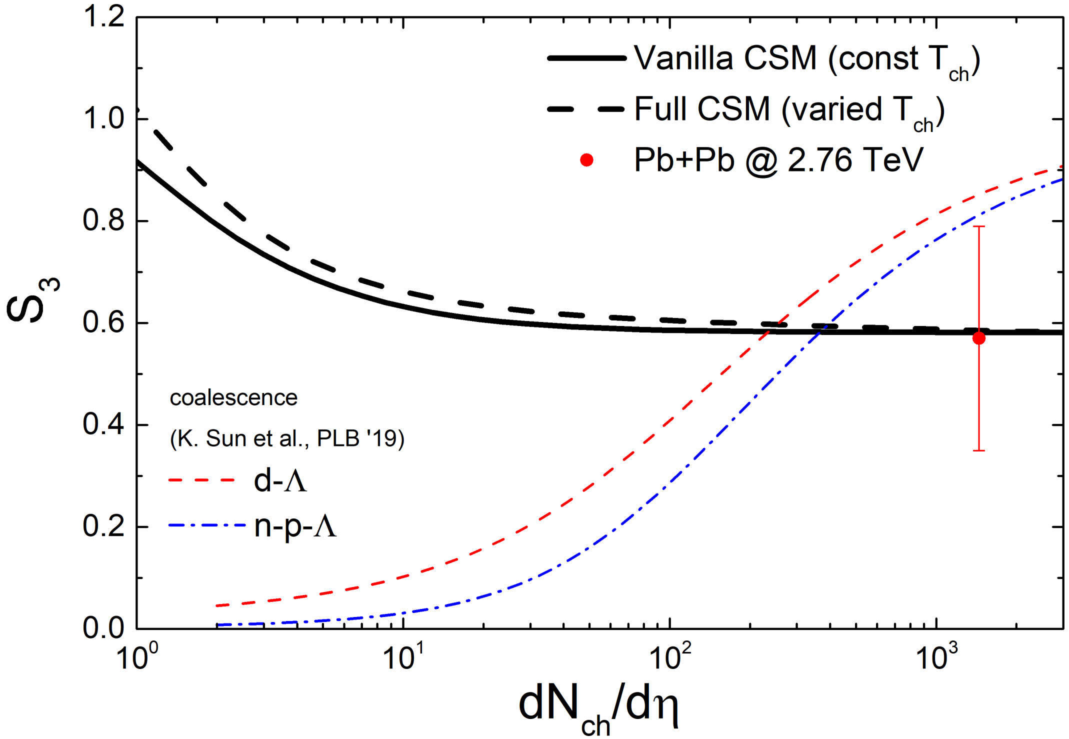


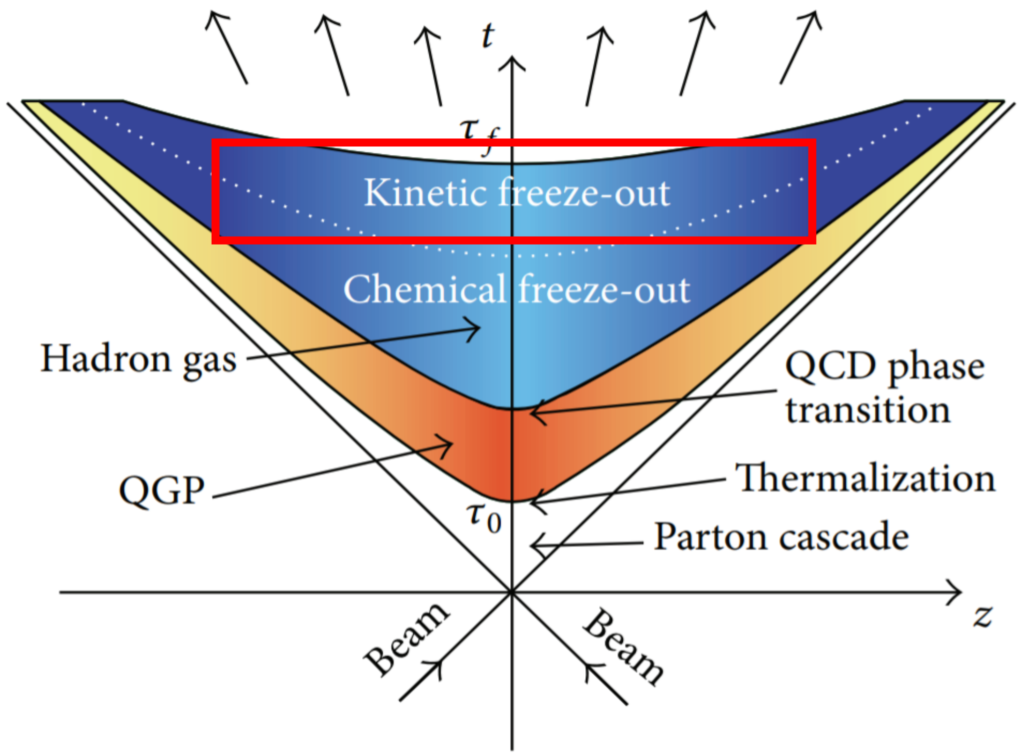
Different versions of CSM give similar predictions, mild increase of S_3 due to baryon and strangeness conservation

Coalescence [Sun, Dönigus, Ko, PLB '19] predicts opposite trend

Hadronic phase and the Saha equation approach to light nuclei production

V.V., *K. Gallmeister, J. Schaffner-Bielich, C. Greiner, 1903.10024, PLB (in print)*

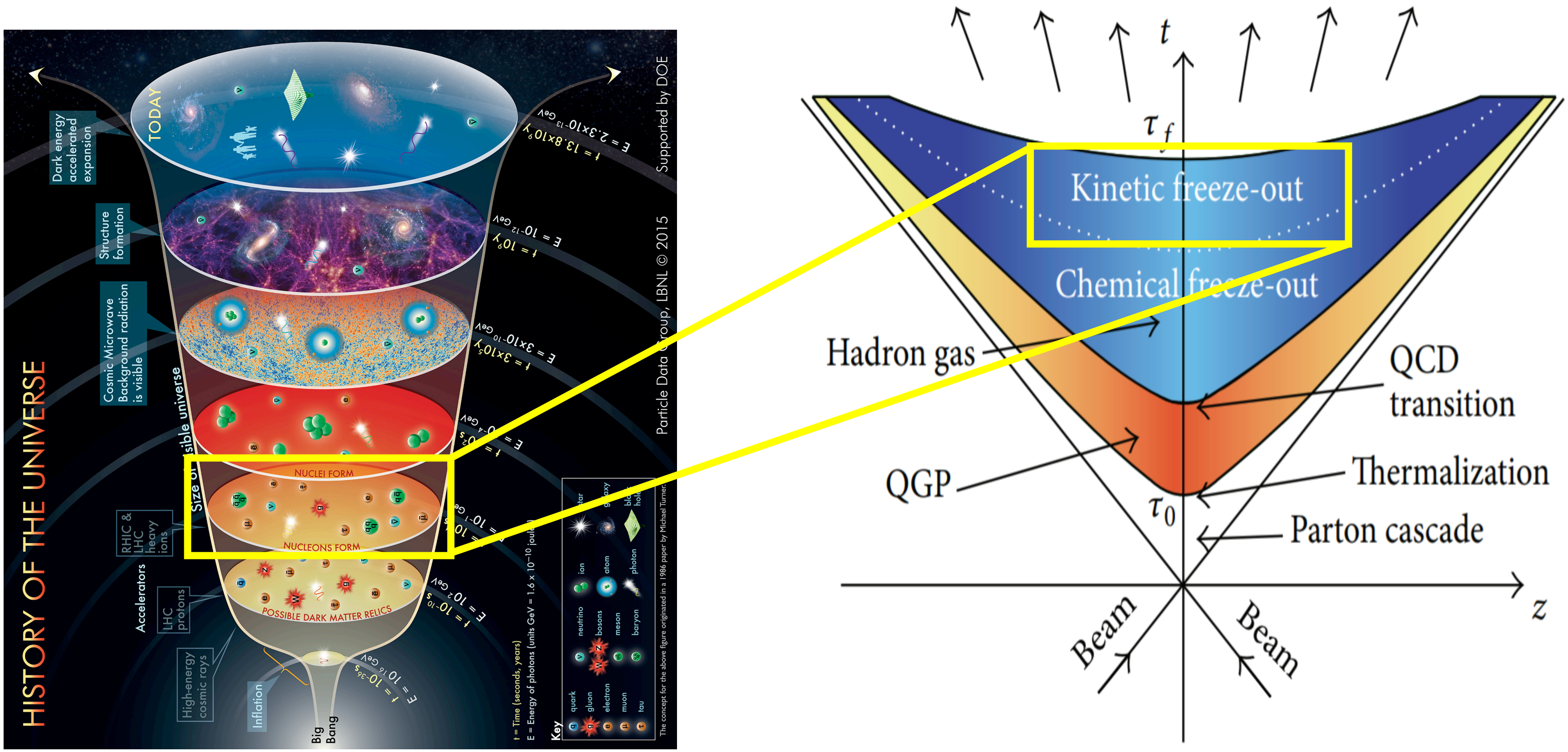
Hadronic phase in central HICs



- At $T_{ch} \approx 150 - 160$ MeV inelastic collisions cease, yields of hadrons frozen
- Kinetic equilibrium maintained down to $T_{kin} \approx 100 - 120$ MeV through (pseudo-)elastic scatterings

[e.g., E. Shuryak, Rev. Mod. Phys. 89, 035001 (2017)]

Big Bang vs "Little Bangs"



- Hadrons (nucleons) form and "freeze-out" chemically before nuclei
- Bosons (photons or pions) catalyse nucleosynthesis

e.g. $p + n \leftrightarrow d + \gamma$ vs $p + n + \pi \leftrightarrow d + \pi$

Saha equation: $\mu_A = \sum_i \mu_{A_i}$, e.g. $\mu_d = \mu_p + \mu_n$, $\mu_{^3\text{He}} = 2\mu_p + \mu_n$, ...

Partial chemical equilibrium (PCE)

Expansion of hadron resonance gas in partial chemical equilibrium at $T < T_{ch}$

[H. Bebie, P. Gerber, J.L. Goity, H. Leutwyler, Nucl. Phys. B '92; C.M. Hung, E. Shuryak, PRC '98]

Chemical composition of stable hadrons is fixed, kinetic equilibrium maintained through pseudo-elastic resonance reactions $\pi\pi \leftrightarrow \rho$, $\pi K \leftrightarrow K^*, \pi N \leftrightarrow \Delta$, etc.

Effective chemical potentials:

$$\tilde{\mu}_j = \sum_{i \in \text{stable}} \langle n_i \rangle_j \, \mu_i, \qquad \langle n_i \rangle_j \text{ – mean number of hadron } i \text{ from decays of hadron } j, \qquad j \in \text{HRG}$$

Conservation laws:

$$\sum_{j \in \text{hrg}} \langle n_i \rangle_j \, n_j(T, \tilde{\mu}_j) \, V = N_i(T_{\text{ch}}), \quad i \in \text{stable}$$

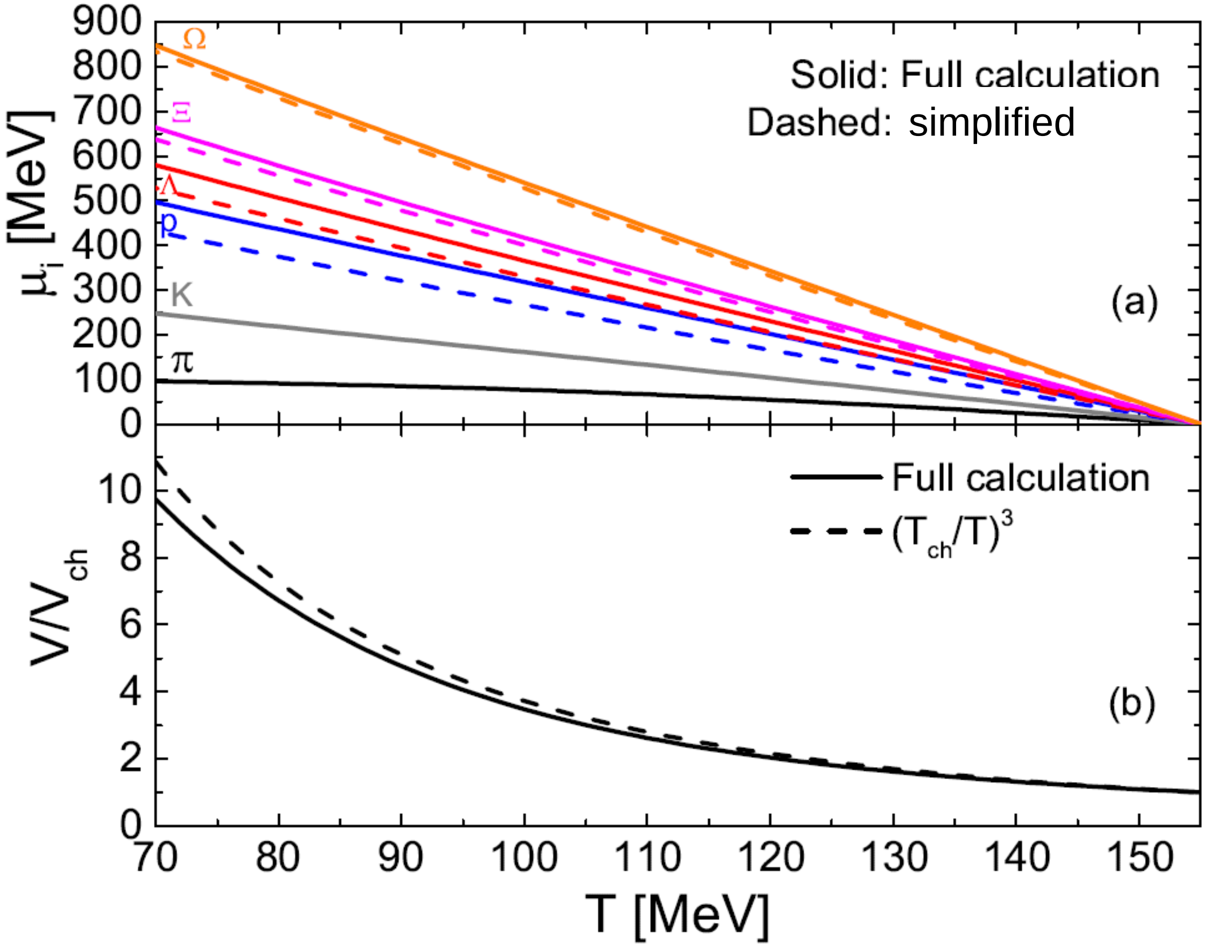
$$\sum_{j \in \text{hrg}} s_j(T, \tilde{\mu}_j) \, V = S(T_{\text{ch}})$$

numerical solution → $\{\mu_i(T)\}, V(T)$

E.g.: $\pi + 2\rho + 3\omega + \dots = const$, $\quad N + \Delta + N^* + \dots = const$, $\quad K + K^* + \dots = const$

Numerical implementation of PCE in **Thermal-FIST**

Partial chemical equilibrium at the LHC



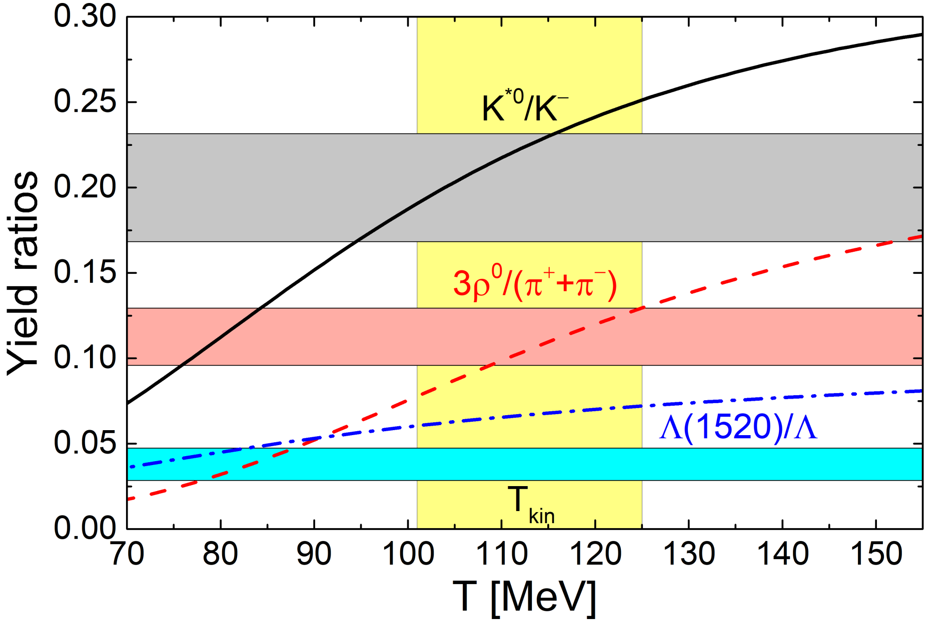
[**V.V.**, K. Gallmeister, J. Schaffner-Bielich, C. Greiner, 1903.10024]

"Initial conditions": $T_{ch} = 155$ MeV, $V_{ch} = 4700$ fm^3 (0-10% Pb-Pb 2.76 TeV)

Resonance suppression in hadronic phase

Yields of resonances are *not* conserved in partial chemical equilibrium

E.g. K^* yield dilutes during the cooling through reactions $\pi K \leftrightarrow K^*$



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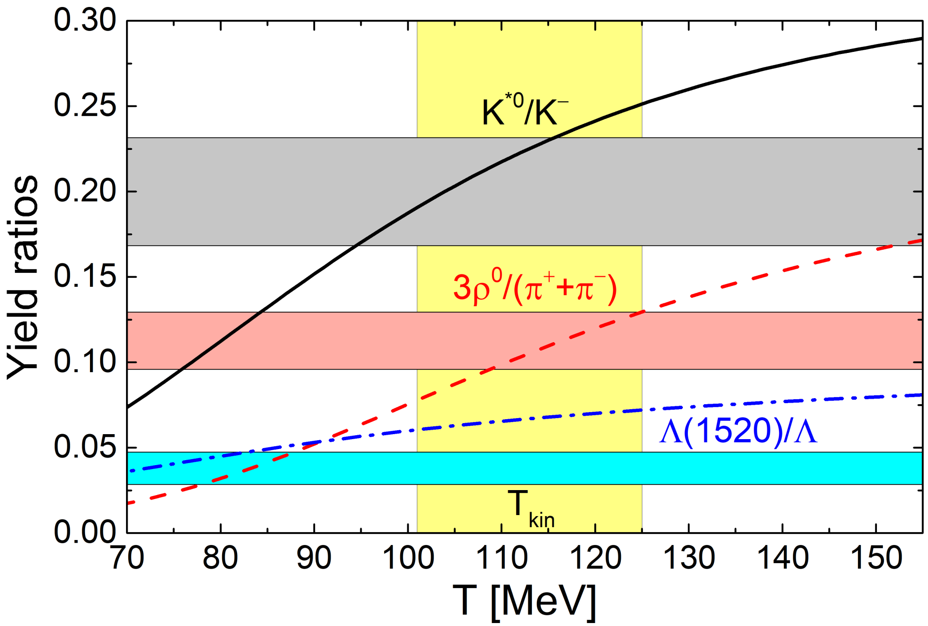

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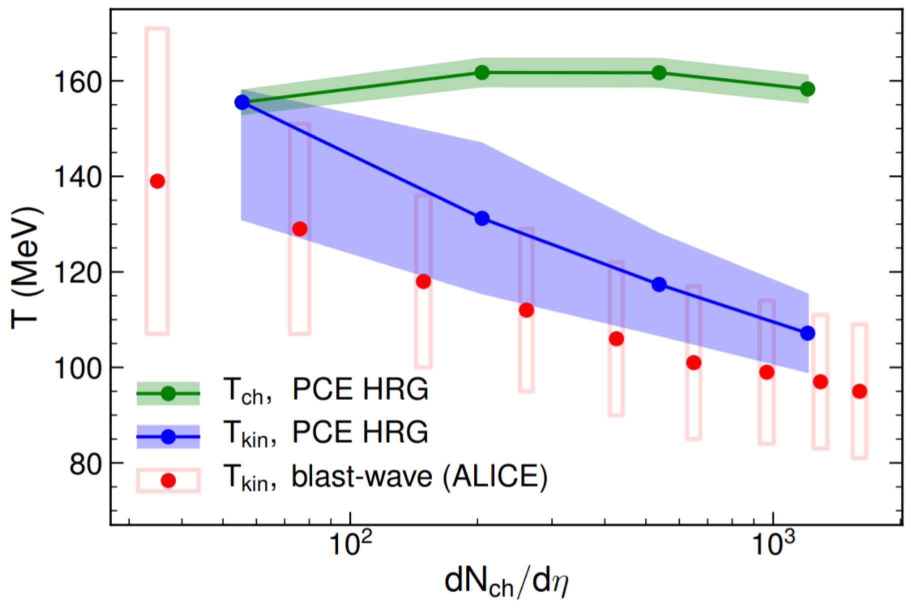


[Motornenko, **V.V.**, Greiner, Stoecker, 1908.11730]

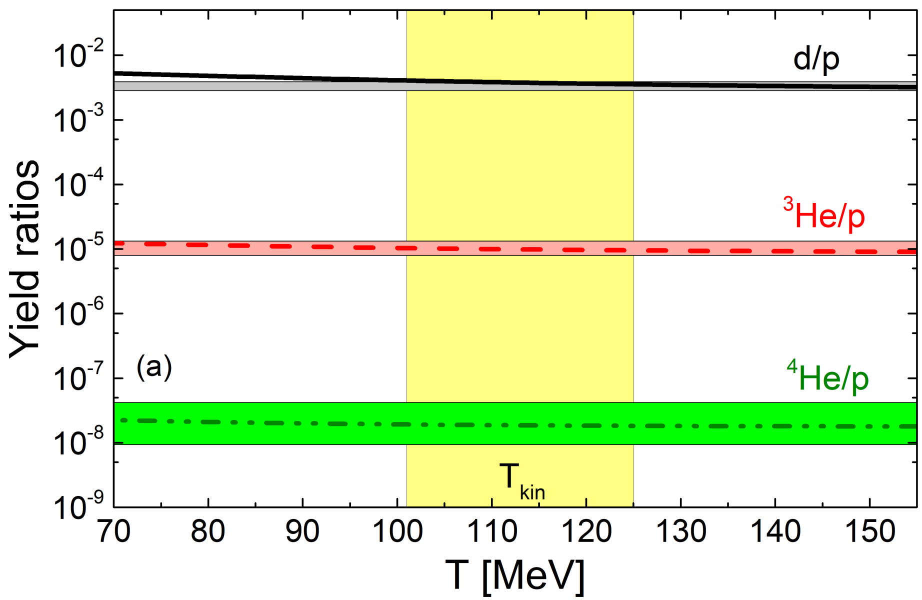
Fitting the yields of short-lived resonances is a new way to extract the kinetic freeze-out temperature

Saha equation: Light nuclei

Detailed balance for nuclear reactions, $X + A \leftrightarrow X + \sum_i A_i$, X is e.g. a pion

$$\frac{n_A}{\prod_i n_{A_i}} = \frac{n_A^{\text{eq}}}{\prod_i n_{A_i}^{\text{eq}}}, \quad \Leftrightarrow \quad \mu_A = \sum_i \mu_{A_i}, \qquad \text{e.g.} \;\; \mu_d = \mu_p + \mu_n, \;\; \mu_{^3\text{He}} = 2\mu_p + \mu_n, \;\; \ldots$$

Saha equation



Deviations from thermal model predictions are moderate despite significant cooling and dilution. *Is this the reason for why thermal model works so well?*

For $T = T_{kin}$ similar results reported in [X. Xu, R. Rapp, EPJA 55, 68 (2019)]

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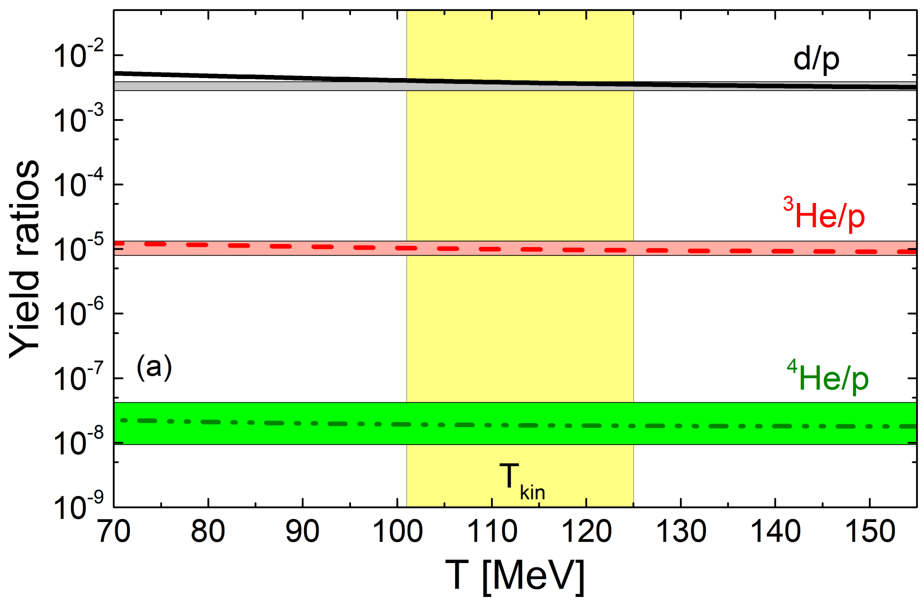
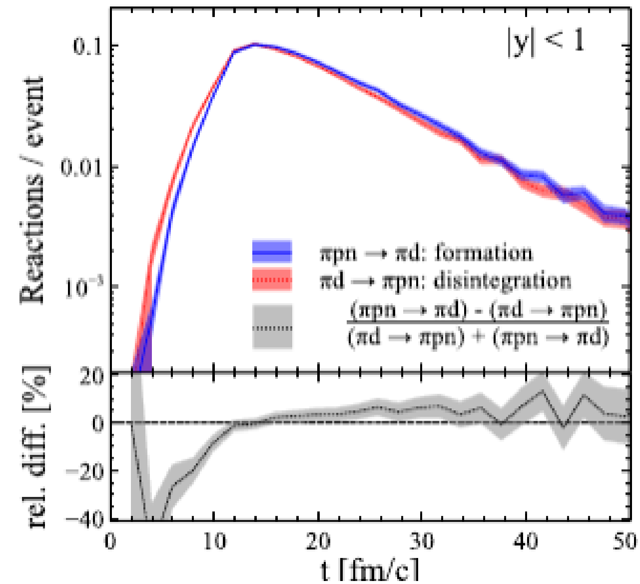


Transport [Oliinychenko et al., 1809.03071]



- **Law of mass action at work**

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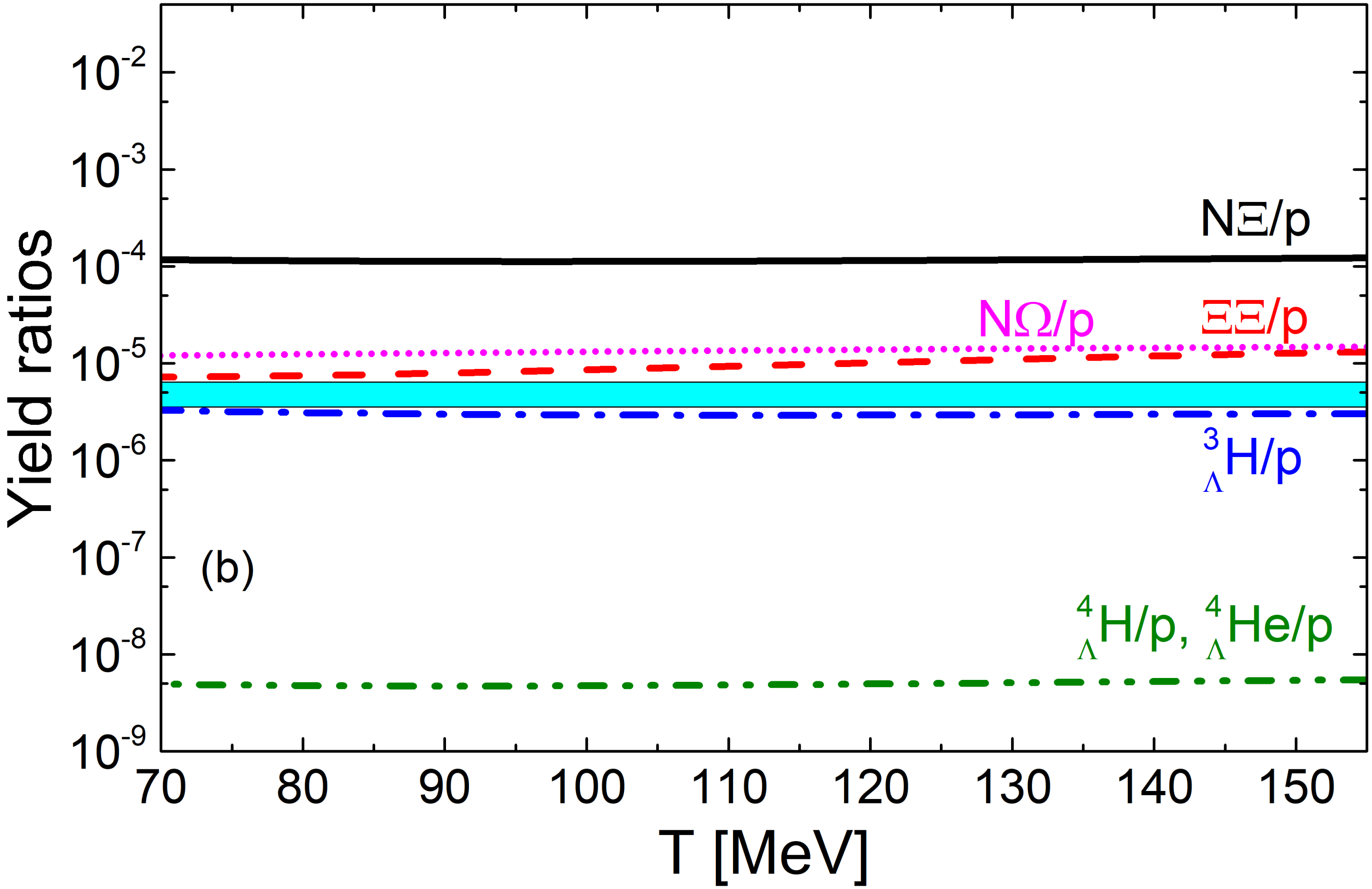
Saha equation: Hypernuclei

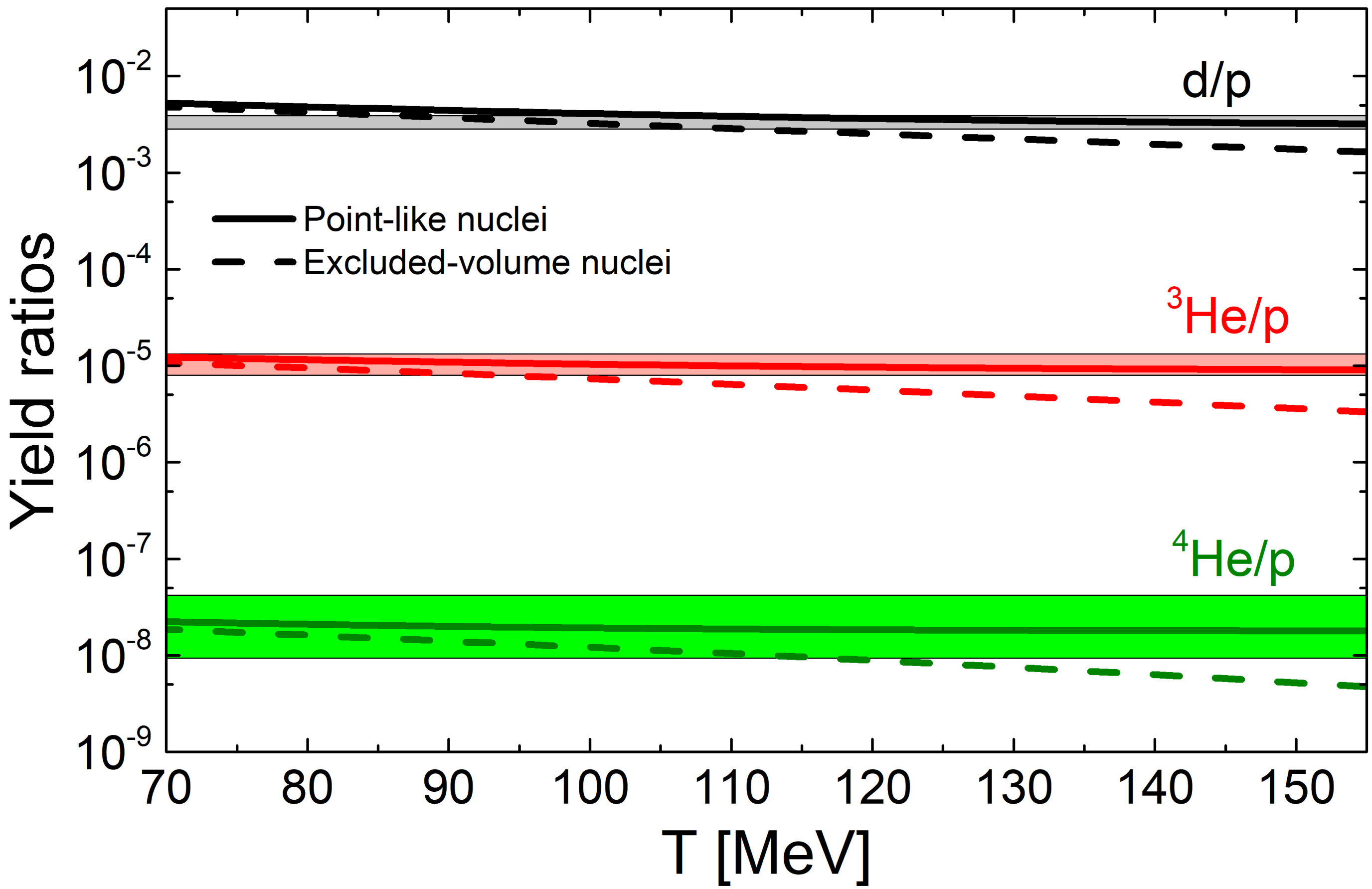


Hypernuclei stay close to the thermal model prediction. An exception is a hypothetical ΞΞ state ← *planned measurement in Runs 3 & 4 at the LHC*

[LHC Yellow Report, 1812.06772]

Saha equation and excluded volume effects

Eigenvolumes: effective mechanism for nuclei suppression at large densities



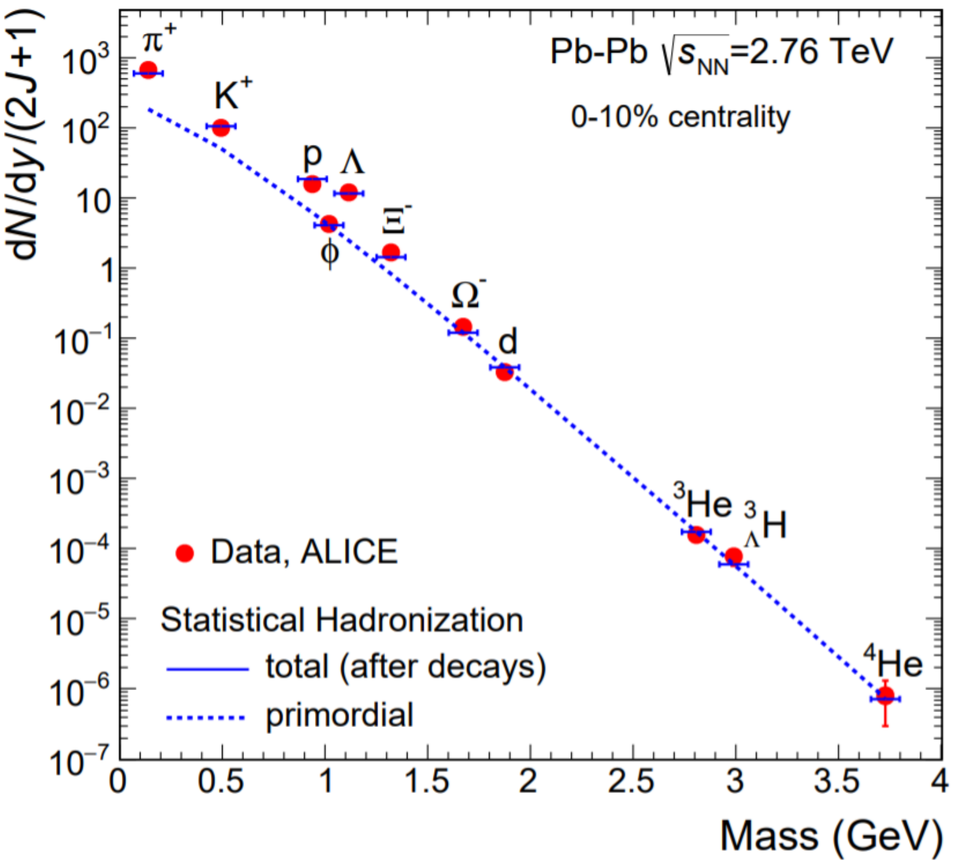
Excluded-volume effects go away as the system dilutes. At $T \cong 100$ MeV agrees with the point-particle model. Does not describe data for $T = T_{ch}$

On the feeddown contributions from decays of unstable fragments

V.V., *B. Dönigus, H. Stoecker, et al., in preparation*

Feeddown in thermal model

Is well-known to be important for hadron yields



[Andronic et al., Nature (2018)]

Production of p

Primordial density = 0.0028648 fm^{-3}

$T = 155$ MeV

Primordial yield = 11.4594

Total yield = 31.4347

Primordial + strong decays = 31.4347

Primordial + strong + EM decays = 31.4347

Primordial + strong + EM + weak decays = 48.5857

Source	Multiplicity	Fraction (%)
Primordial	11.4594	36.4545
Decays from primordial Delta(1232)++	4.86466	15.4755
Decays from primordial Delta(1232)+	3.24327	10.3175
Decays from primordial Delta(1232)0	1.62139	5.15797
Decays from primordial N(1520)0	0.5628	1.79038
Decays from primordial Delta(1600)++	0.540859	1.72058
Decays from primordial N(1520)+	0.436374	1.38819
Decays from primordial N(1440)0	0.412215	1.31134
Decays from primordial Delta(1600)+	0.3931	1.25053
Decays from primordial N(1440)+	0.367071	1.16773
Decays from primordial N(1675)+	0.362324	1.15263
Decays from primordial N(1680)0	0.352206	1.12044

[**V.V.**, Stoecker, CPC (2019)]

Feeddown to yields of light nuclei seldom considered in HICs

Feeddown from nuclear fragments

^{4}He

E_x (MeV)	J^π	Decay
g.s.	0^+	
20.21	0^+	p
21.01	0^-	p, n
21.84	2^-	p, n
23.33	2^-	p, n
23.64	1^-	p, n, (γ)
24.25	1^-	p, n, d
25.28	0^-	p, n
25.95	1^-	p, n, γ
27.42	2^+	p, n, d
28.31	1^+	p, n, d
28.37	1^-	(p, n), d
28.39	2^-	(p, n), d
28.64	0^-	d
28.67	2^+	d, γ
29.89	2^+	(p, n), d

^{4}H

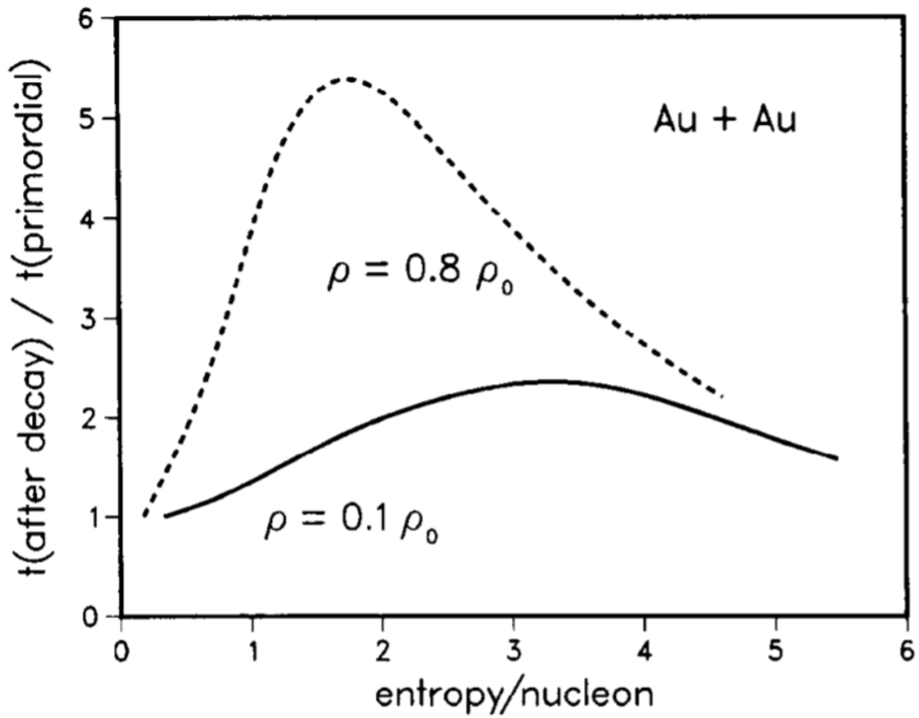
E_x (MeV)	J^π	Decay
g.s. [a]	2^-	n, ^{3}H
0.31	1^-	n, ^{3}H
2.08	0^-	n, ^{3}H
2.83	1^-	n, ^{3}H

^{4}Li

E_x (MeV)	J^π	Decay
g.s. [a]	2^-	p, ^{3}He
0.32	1^-	p, ^{3}He
2.08	0^-	p, ^{3}He
2.85	1^-	p, ^{3}He

[Tilley, Weller, Hale, NPA '92]

See also https://www.nndc.bnl.gov/

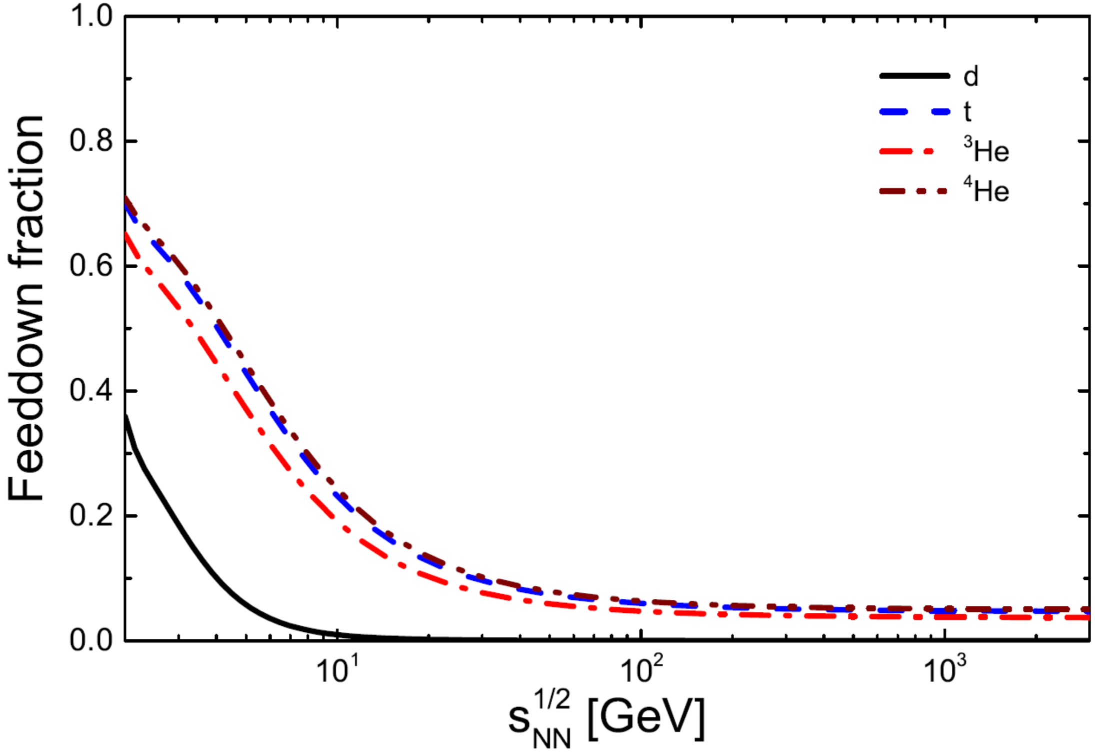


[Hahn, Stöcker, NPA '88]

In what follows feeddown from known $A = 4$ and significant $A = 5$ unstable fragments included. Fragments modeled as point particles.

Feeddown from nuclear fragments

Feeddown fraction along the phenomenological freeze-out curve

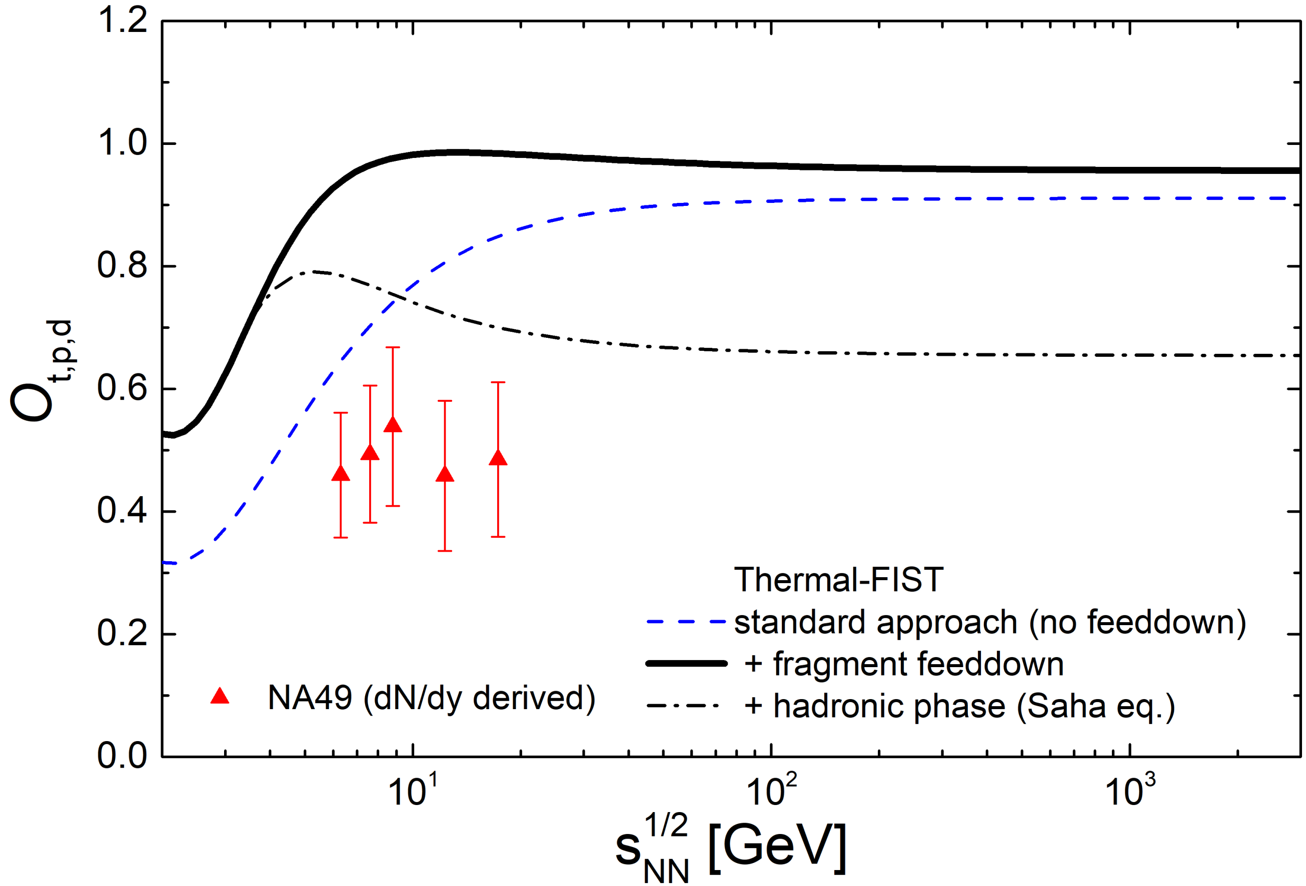


- **LHC:** 5% effect. Can be measured through p-^{3}He, p-^{4}He correlation?
- **RHIC/SPS:** 10-40% effect
- **GSI-HADES/FAIR:** Feeddown accounts for more than half of t, ^{3}He, ^{4}He

Feeddown from nuclear fragments: $O_{t,p,d}$

$O_{t,p,d} = N_t N_p/(N_d)^2$ suggested as a possible probe of critical behavior

[K.J. Sun et al., PLB '17, PLB '18]



Possible to obtain a non-monotonic behavior of $O_{t,p,d}$ in ideal gas picture

Relevance of excited ^{4}He states also pointed out in baryon preclustering study [Torres-Rincon, Shuryak, 1910.08119]

Summary and outlook

- **Thermal-FIST** is a user-friendly open source package for general purpose statistical-thermal model applications, in particular nuclei.
- Multiplicity dependence of light nuclei abundances at the LHC consistent with basic canonical suppression considerations, but no simultaneous description of the p/π ratio
- Saha equation extends the statistical approach down to the kinetic freeze-out, offers possible explanation why the thermal model for point-like nuclei works so well.
- Feeddown from unstable fragments is sizable for yields of t, ^{3}He, ^{4}He

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Thanks for your attention!

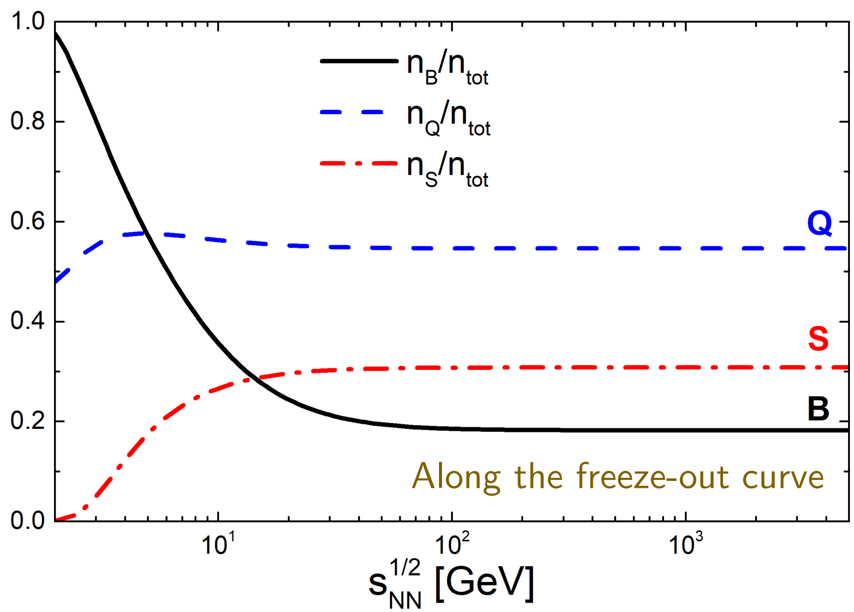
Backup slides

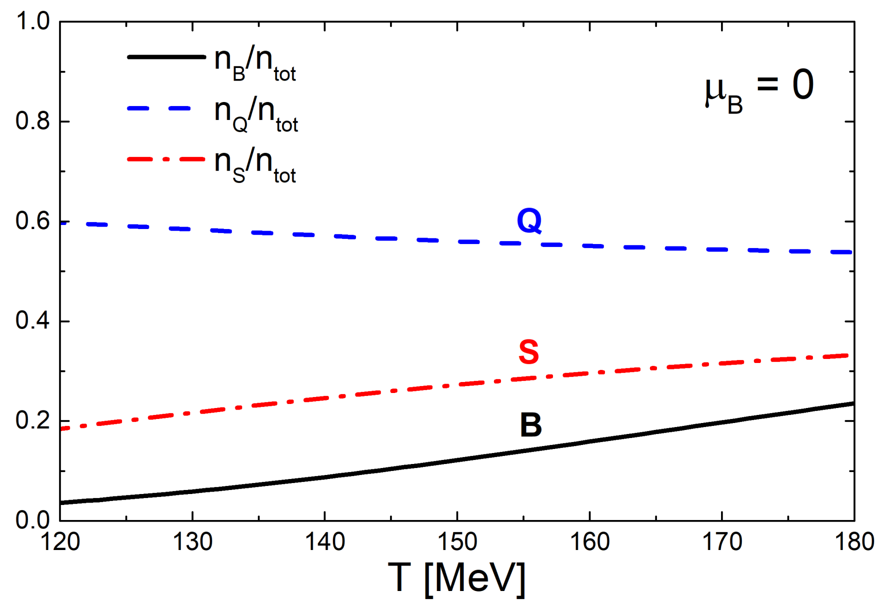
When is the canonical treatment necessary?

Normally, when the total number of particles carrying a conserved charge is smaller or of the order of unity

The canonical treatment is often restricted to strangeness only (SCE)

[STAR collaboration, 1701.07065; ALICE collaboration, 1807.11321]





- **Strangeness** conservation is most important at low energies (**HADES**, **CBM**)
- *Small systems at **RHIC** and **LHC**:* exact **baryon** conservation at least as important as **strangeness**

CSM at LHC

Enforce exact conservation of charges, $B = Q = S = 0$, in a *correlation volume* V_C around midrapidity

In general, $\boldsymbol{V_C \neq dV/dy}$

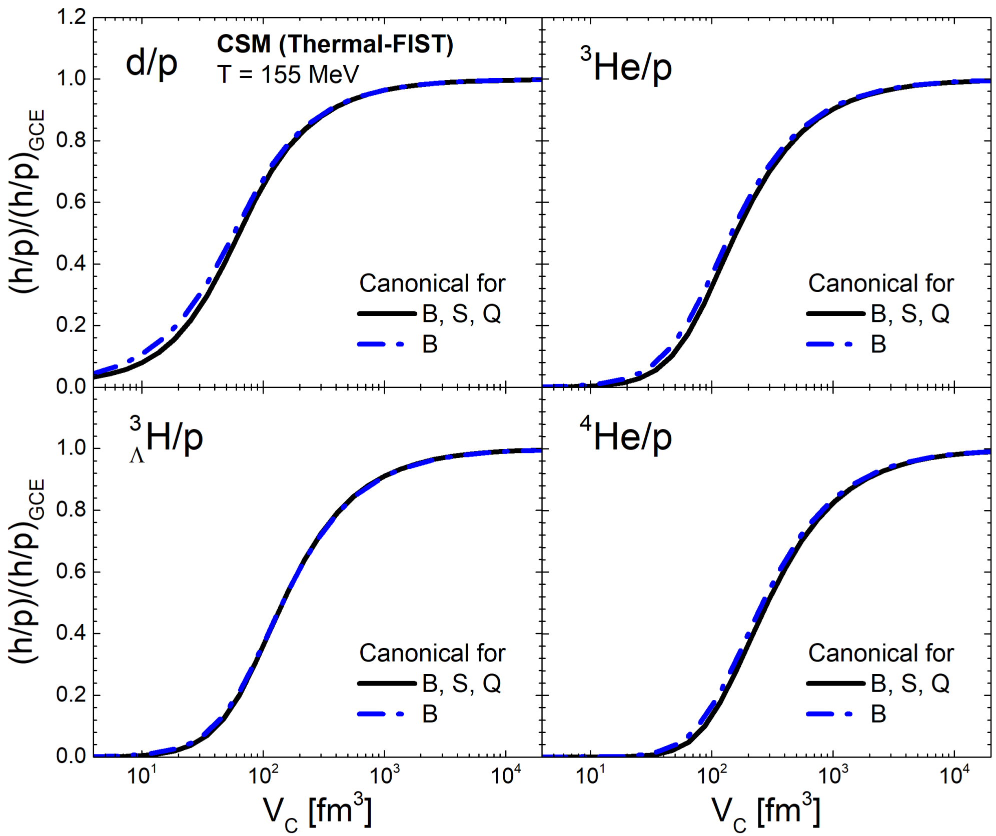
Causality argument: exact conservation across a few units of rapidity?

[Castorina, Satz, 1310.6932]

New application:

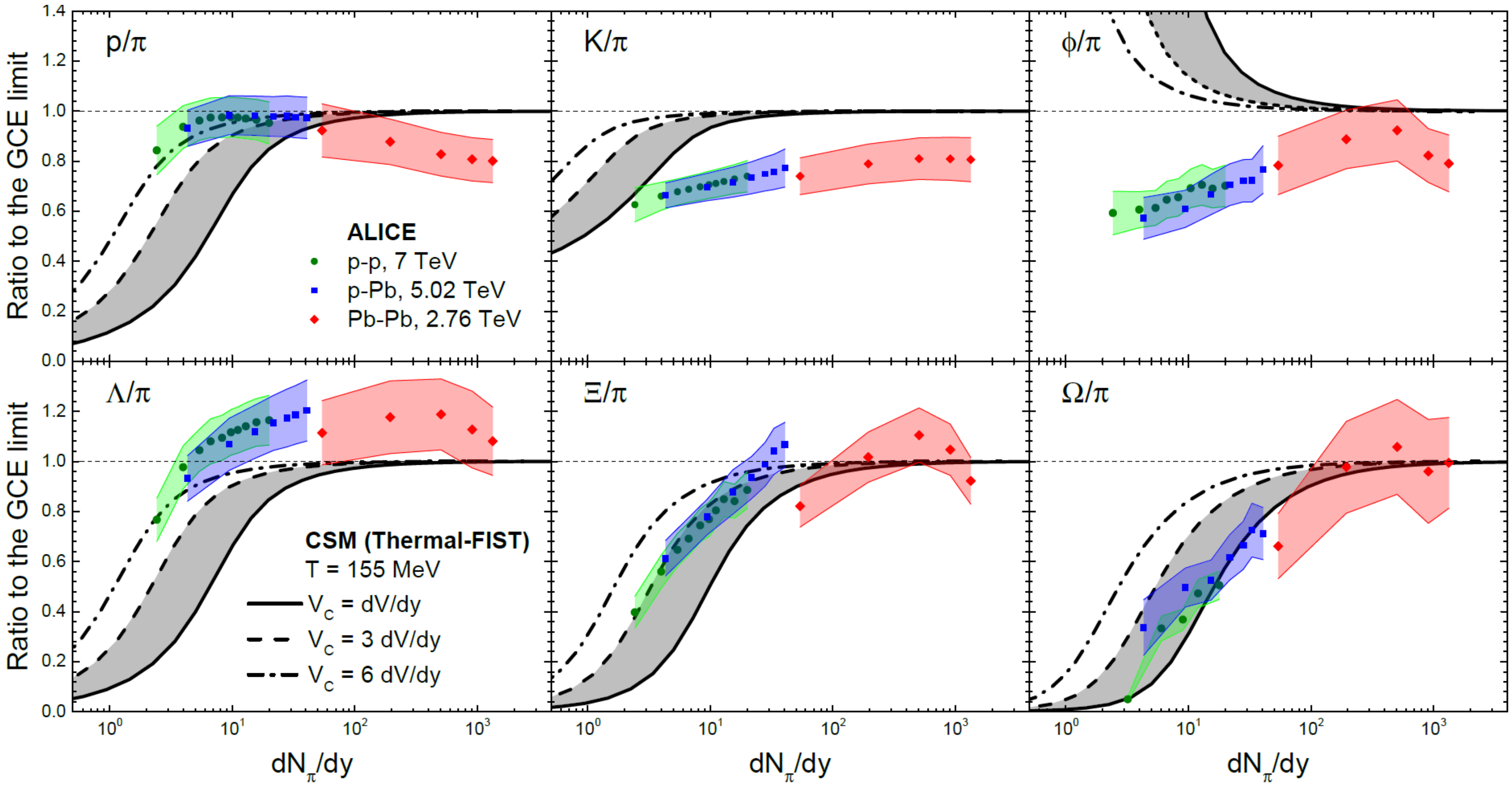
CSM for **light nuclei**

- Suppression of nuclei-to-proton ratios at low multiplicities
- For these observables sufficient to enforce exact baryon conservation only



"Vanilla" CSM at LHC

$T_{ch} = 155$ MeV, $V_C = 3dV/dy$, multiplicity dependence driven by V_C only

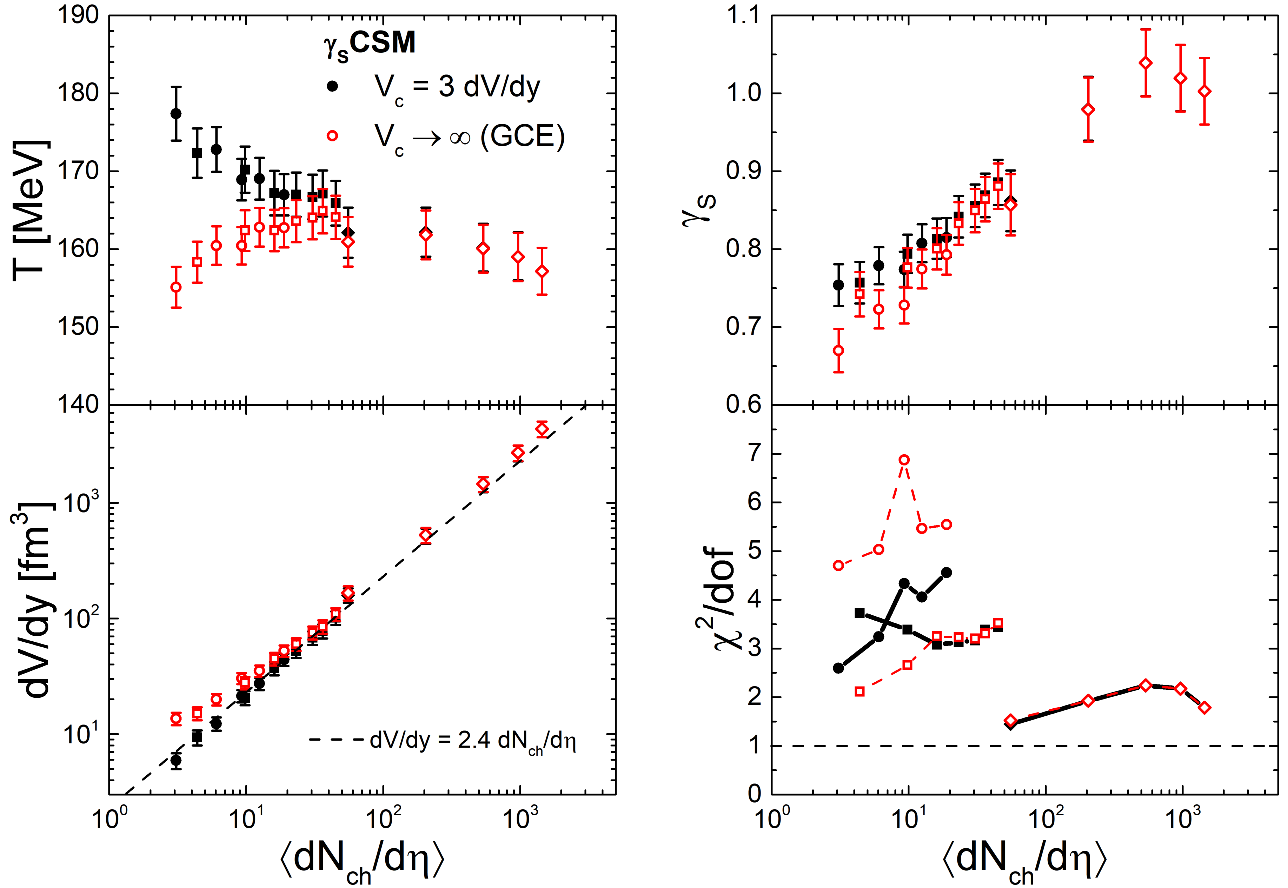


Fair for hyperons, protons and kaons worse, ϕ goes in the opposite direction

[**V.V.**, Dönigus, Stoecker, 1808.05245, PLB '18]

Full CSM analysis

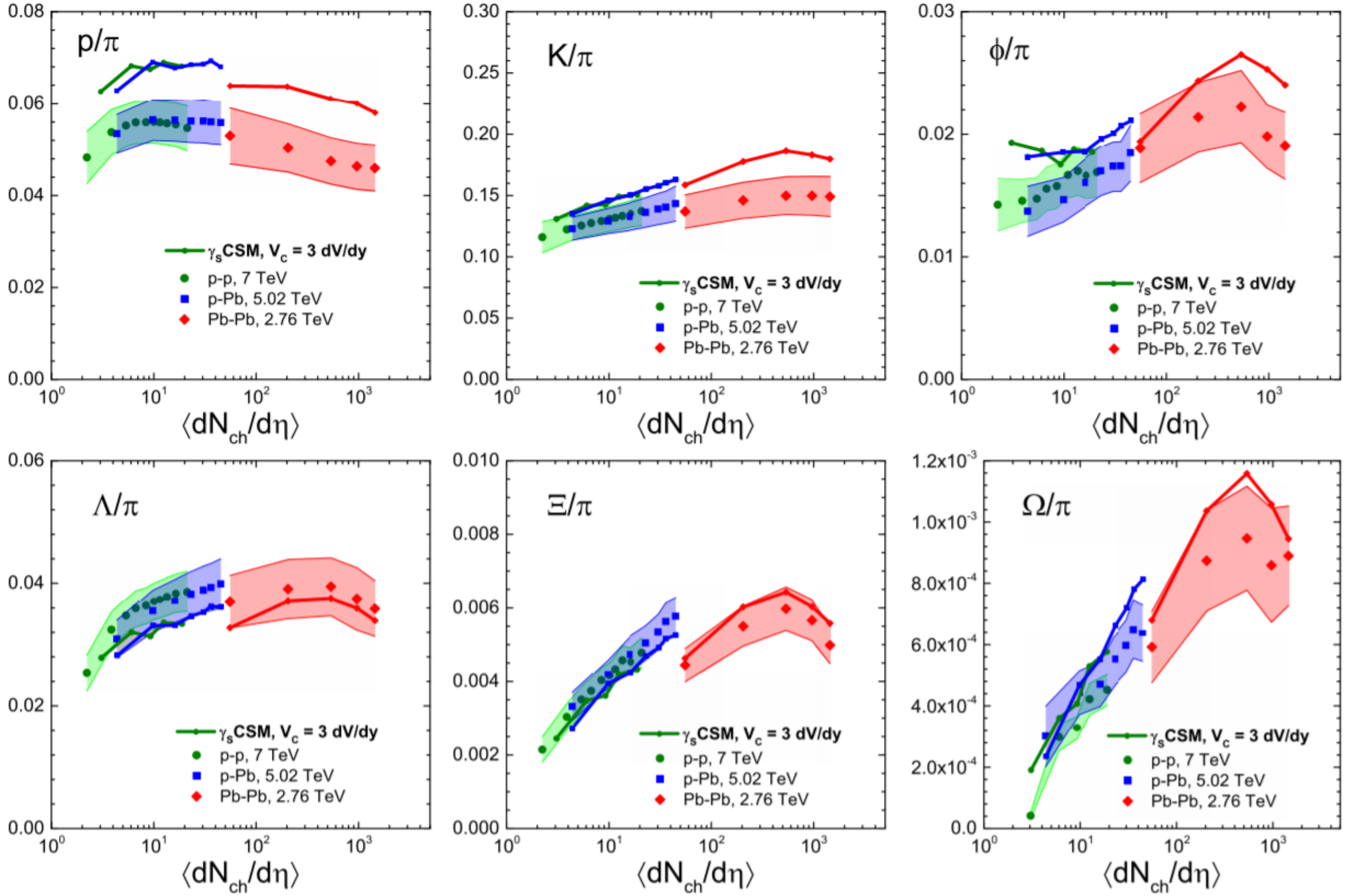
γ_S**CSM:** $V_C = 3dV/dy$, fit T_{ch} and γ_S at each centrality in p-p, p-Pb, Pb-Pb



Canonical suppression and strangeness saturation important below $dN_{ch}/d\eta \cong 100$

[**V.V.**, Dönigus, Stoecker, 1906.03145]

Full CSM analysis: yields

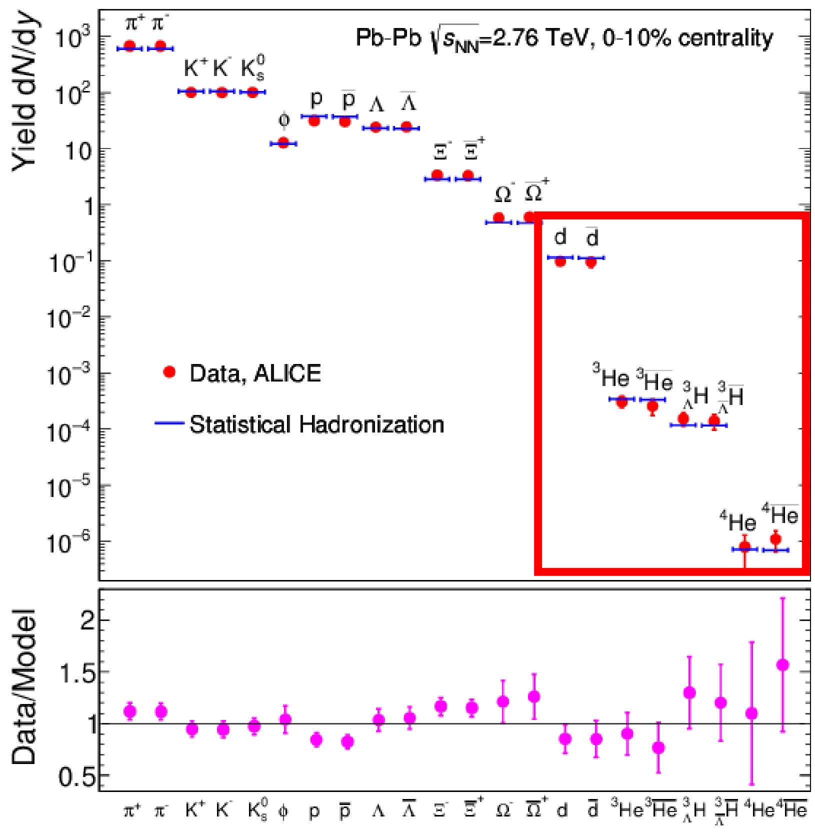


Relative accuracy of γ_SCSM is ∼15% across all multiplicity bins

[**V.V.**, Dönigus, Stoecker, 1906.03145]

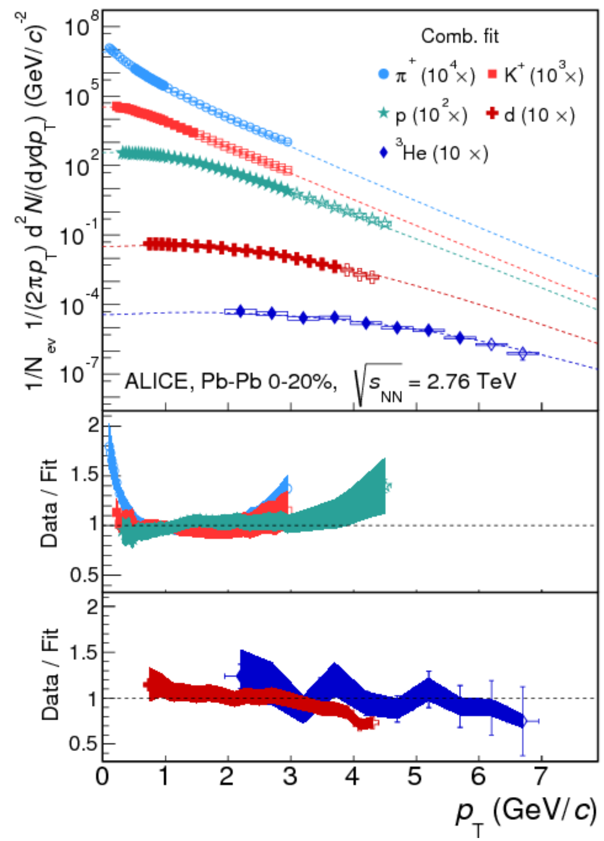
Two experimental observations at the LHC

1. Measured yields are described by thermal model at $T_{ch} \approx 155$ MeV*



[A. Andronic et al., Nature **561**, 321 (2018)]

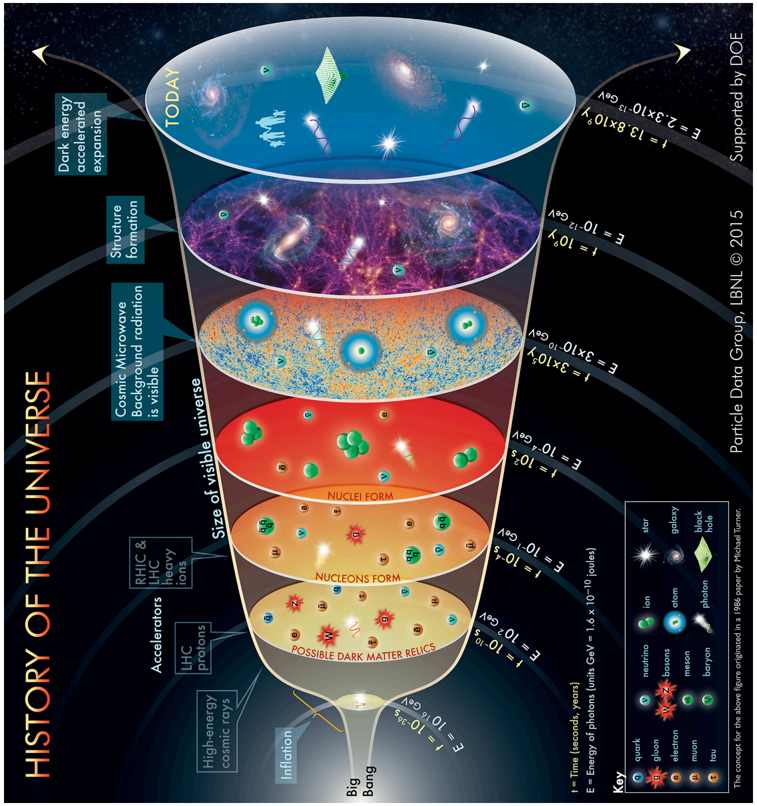
2. Spectra described by blast-wave model at $T_{kin} \approx 100 - 120$ MeV*

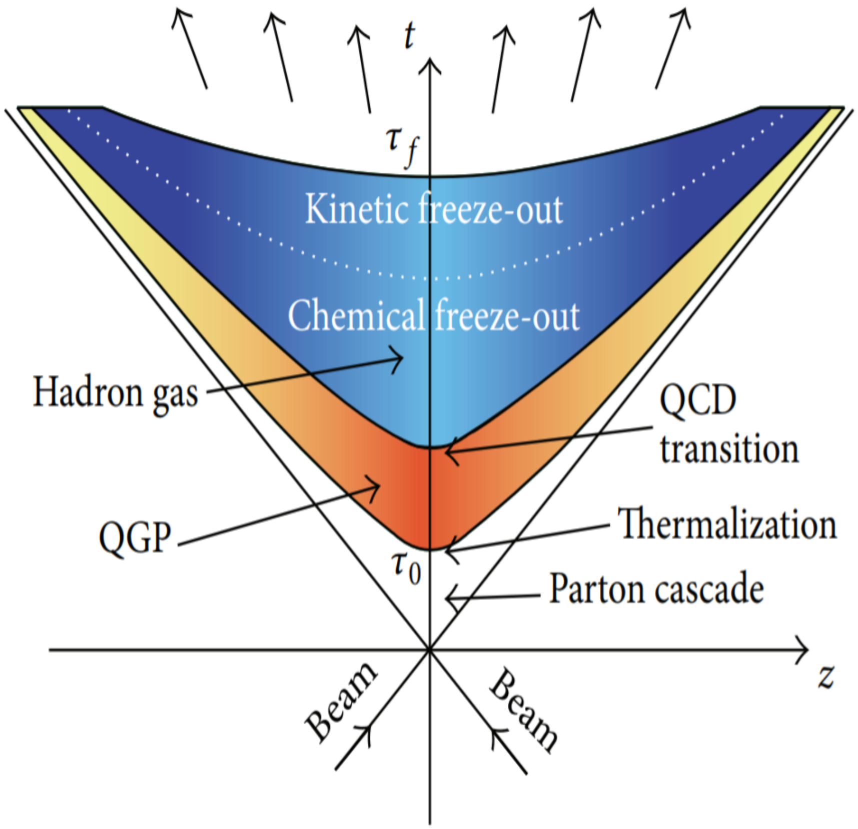


[ALICE collaboration, PRC **93**, 024917 (2016)]

What happens between $T_{\rm ch}$ and $T_{\rm kin}$?

Big Bang vs "Little Bangs"





- Hadrons (nucleons) form and "freeze-out" chemically before nuclei
- Bosons (photons or pions) catalyse nucleosynthesis

e.g. $p + n \leftrightarrow d + \gamma$ vs $p + n + \pi \leftrightarrow d + \pi$

Big Bang vs LHC nucleosynthesis

Similarities:

- Inelastic nucleonic reactions freeze-out before nuclei formation
- Isentropic expansion of boson-dominated matter (photons in BBN vs mesons in HIC), baryon-to-boson ratio: $\eta_{BBN} \sim 10^{-10}$, $\eta_{LHC} \sim 0.05$
- Strong nuclear formation and regeneration reactions → **Saha equation**

Differences:

- Time scales: 1-100 s in BBN vs $\sim 10^{-22}$ s in HIC
- Temperatures: $T_{BBN} < 1$ MeV vs $T_{HIC} \sim 100$ MeV
- Binding energies, proton-neutron mass difference, and neutron lifetime important in BBN, less so in HICs
- $\mu_B \approx 0$ at the LHC, $\mu_B \neq 0$ in BBN
- Resonance feeddown important at LHC, irrelevant in BBN

LHC nucleosynthesis: simplified setup

- Chemical equilibrium lost at $T_{ch} = 155$ MeV, abundances of nucleons are frozen and acquire effective fugacity factors: $n_i = n_i^{eq} e^{\mu_N/T}$
- Isentropic expansion driven by effectively massless mesonic d.o.f.

$$\frac{V}{V_{\rm ch}} = \left(\frac{T_{\rm ch}}{T}\right)^3, \qquad \mu_N \simeq \frac{3}{2}\, T \ln\left(\frac{T}{T_{\rm ch}}\right) + m_N \left(1 - \frac{T}{T_{\rm ch}}\right)$$

- Detailed balance for nuclear reactions, $X + A \leftrightarrow X + \sum_i A_i$, X is e.g. a pion

$$\frac{n_A}{\prod_i n_{A_i}} = \frac{n_A^{\rm eq}}{\prod_i n_{A_i}^{\rm eq}}, \quad \Leftrightarrow \quad \mu_A = \sum_i \mu_{A_i}, \qquad \text{e.g.}\ \ \mu_d = \mu_p + \mu_n,\ \ \mu_{^3{\rm He}} = 2\mu_p + \mu_n,\ \ \ldots$$

Saha equation

$$X_A = d_A \left[(d_M)^{A-1} \zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{-\frac{3+A}{2}}\right] A^{5/2} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta_B^{A-1} \exp\left(\frac{B_A}{T}\right)$$

$d_M \sim 11 - 13, \quad \eta_B \simeq 0.03$

BBN: $$X_A = d_A \left[\zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{\frac{3A-5}{2}}\right] A^{\frac{5}{2}} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta^{A-1} X_p^Z X_n^{A-Z} \exp\left(\frac{B_A}{T}\right)$$

[E. Kolb, M. Turner, "The Early Universe" (1990)]

(Simplified) Saha equation vs thermal model

Saha equation:

$$\frac{N_A(T)}{N_A(T_{\rm ch})} \simeq \left(\frac{T}{T_{\rm ch}}\right)^{\frac{3}{2}(A-1)} \exp\left[B_A\left(\frac{1}{T}-\frac{1}{T_{\rm ch}}\right)\right] \qquad B_A \ll T$$

$$\updownarrow$$

Thermal model:

$$\left[\frac{N_A(T)}{N_A(T_{\rm ch})}\right]_{\rm eq.} \simeq \left(\frac{T}{T_{\rm ch}}\right)^{\frac{3}{2}} \exp\left[-m_A\left(\frac{1}{T}-\frac{1}{T_{\rm ch}}\right)\right] \qquad m_A \gg T$$

Strong exponential dependence on the temperature is eliminated in the Saha equation approach

Further, quantitative applications require numerical treatment of full spectrum of *massive* mesonic and baryonic resonances

Saha equation: Entropy production effect