QCD phase structure from fluctuations in heavy-ion collisions: Connecting theory to experiment

Volodymyr Vovchenko (LBNL)

STAR Collaboration Meeting

September 22, 2021

Unterstützt von / Supported by

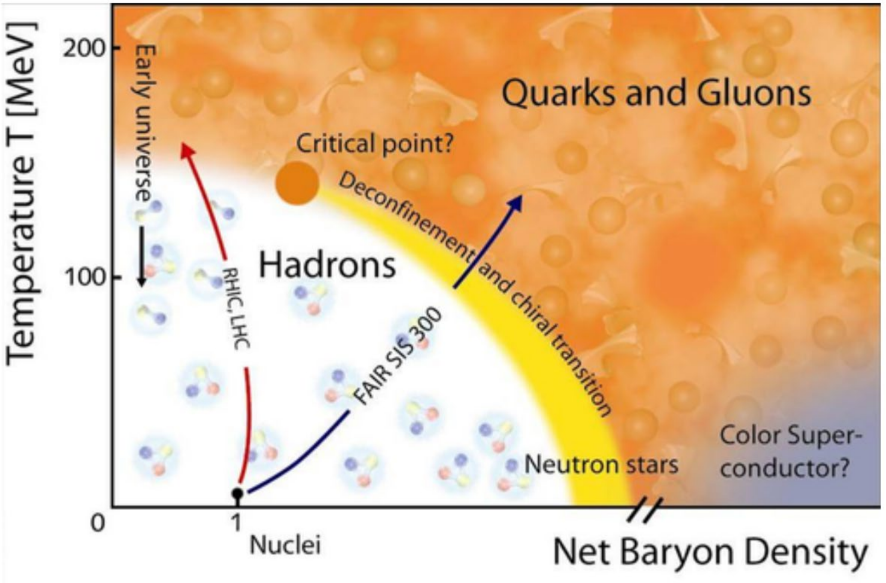
Alexander von Humboldt Stiftung/Foundation

Strongly interacting matter

- Theory of strong interactions: *Quantum Chromodynamics* (QCD)

$$\mathcal{L} = \sum_{q=u,d,s,\dots} \bar{q}\left[i\gamma^{\mu}(\partial_{\mu} - igA^{a}_{\mu}\lambda_{a}) - m_{q}\right] q - \frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a}$$

- Basic degrees of freedom: quarks and gluons
- At smaller energies confined into baryons (qqq) and mesons $(q\bar{q})$



Where is it relevant?

- Early universe
- Neutron star (mergers)
- **Heavy-ion collisions** (lab!)

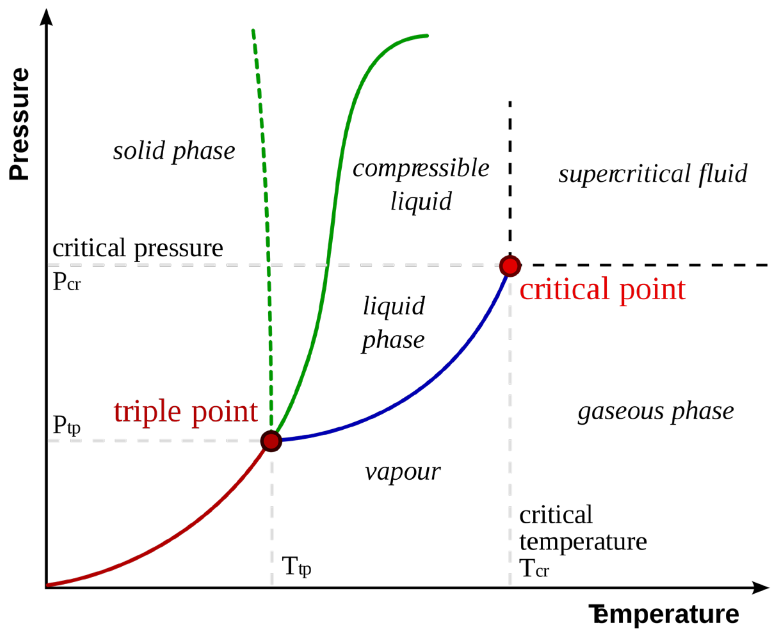
Length scale: 1 fm $= 10^{-15}$ m

Energy scale: 100 MeV $= 10^{12}$ K

$$\hbar = c = k_B = 1$$

QCD phase diagram

Ordinary fluid (e.g. water)



QCD

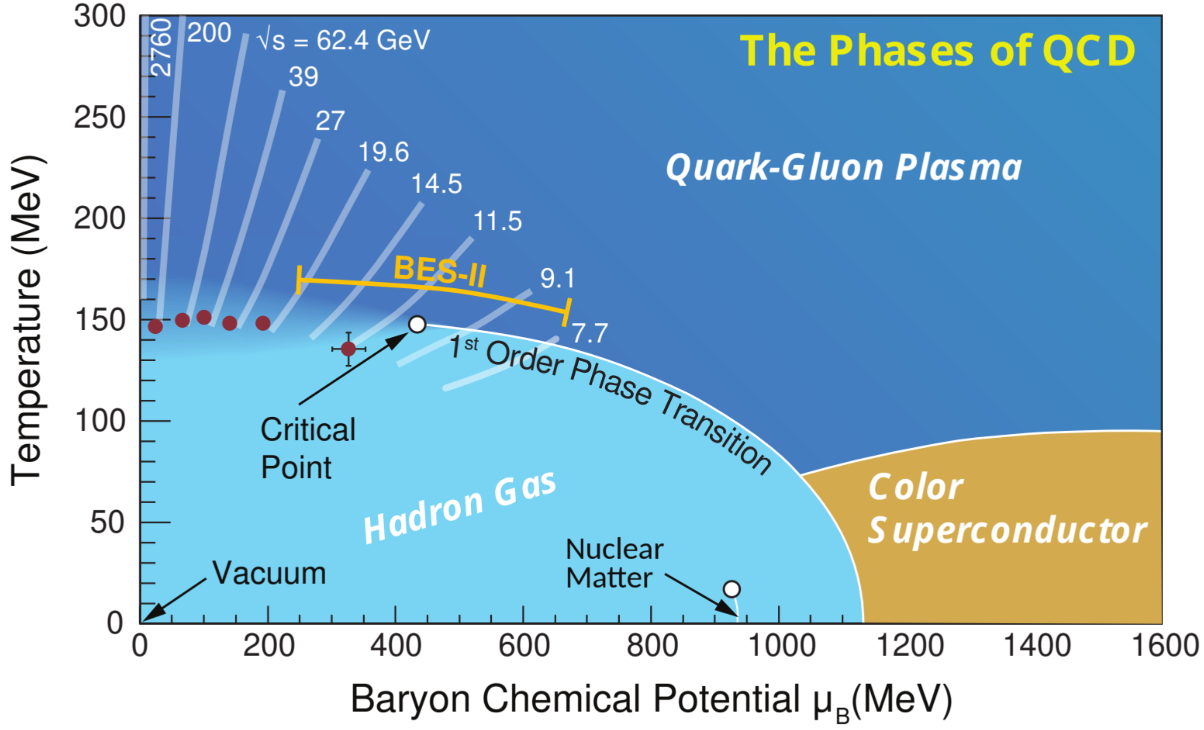


Figure from Bzdak et al., Phys. Rept. '20

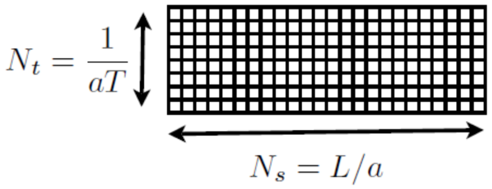
- Dilute hadron gas at low T/n_B due to confinement, quark-gluon plasma high T/n_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured

What is the nature of the quark-hadron transition?

QCD transition from lattice QCD

First-principle tool: Lattice QCD

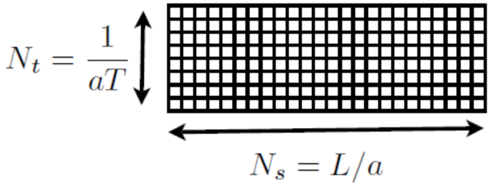
$$Z = \mathrm{Tr}(e^{-(\hat{H}-\mu\hat{N})/T}) = \int DU \det M[U,\mu]\, e^{-S_{YM}}$$



QCD transition from lattice QCD

First-principle tool: Lattice QCD

$$Z = \mathrm{Tr}(e^{-(\hat{H}-\mu\hat{N})/T}) = \int DU \det M[U,\mu]\, e^{-S_{YM}}$$



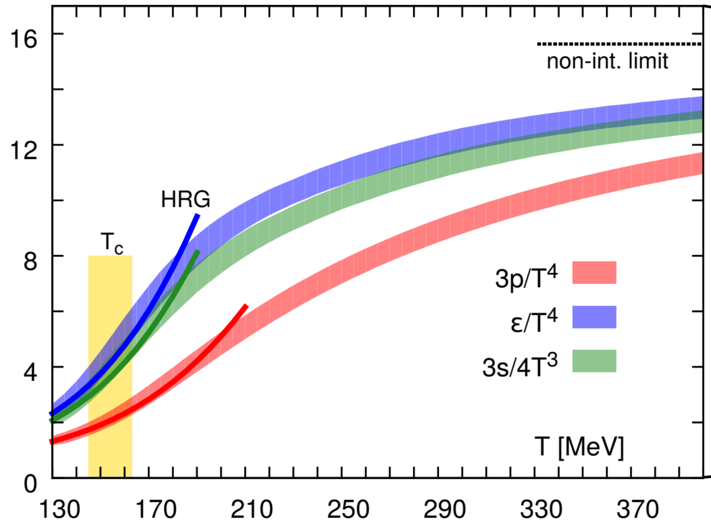


Figure from HotQCD coll., PRD '14

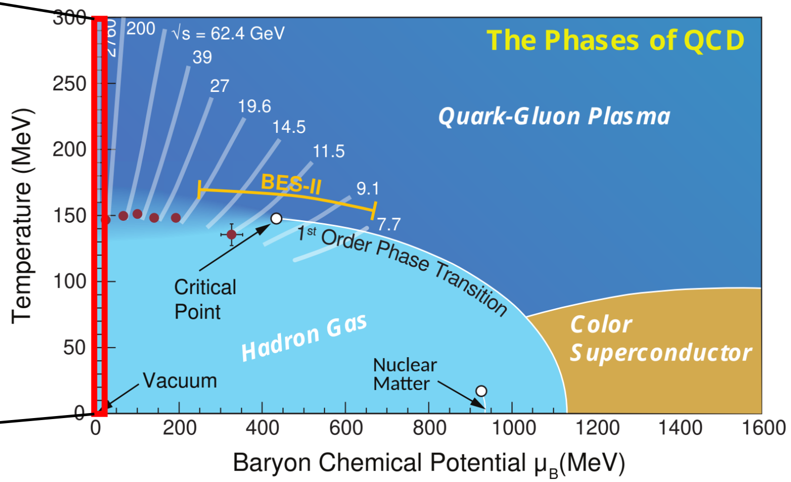


Figure from Bzdak et al., Phys. Rept. '20

- Analytic crossover at vanishing net baryon density – a first-principle result
  [Y. Aoki et al., Nature 443, 675 (2006)]

- Finite densities inaccessible due to **sign problem**, but many effective theories predict first-order phase transition and the **QCD critical point**

First-principle constraints on the QCD critical point

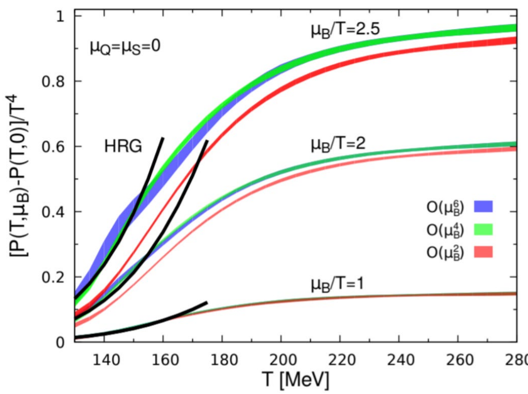
Indirect lattice QCD methods offer glimpse into small μ_B/T

- Taylor expansion around $\mu_B/T{=}0$

$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \frac{\chi_2^B(T,0)}{2!}(\mu_B/T)^2 + \frac{\chi_4^B(T,0)}{4!}(\mu_B/T)^4 + \dots$$

No hints for the critical point at T $>$ 135 MeV

Critical point $\mu_B/T < 3$ disfavored



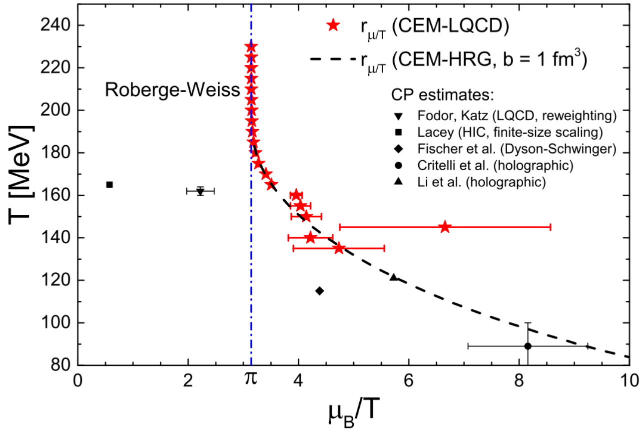
[HotQCD Collaboration, PRD 95, 054504 (2017)]

- Relativistic virial expansion in fugacities via analytic continuation from imaginary μ_B/T

$$\frac{p(T,\mu_B)}{T^4} = \sum_{k=0}^{\infty} p_k(T)\cosh\left(\frac{k\,\mu_B}{T}\right)$$

Expansion sees singularity in the complex plane, Im $[\mu_B/T] = \pi$

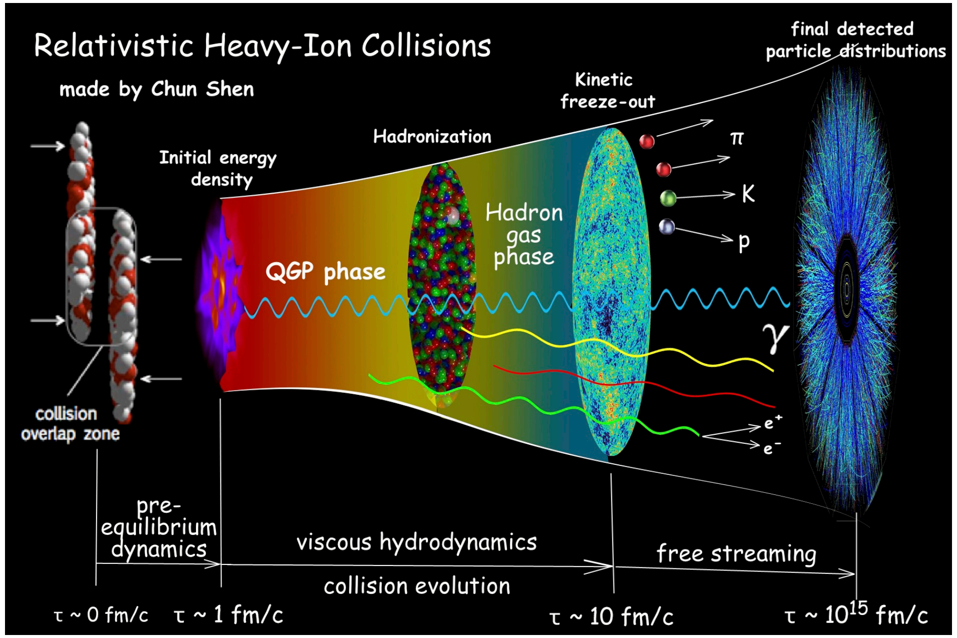
Critical point at $\mu_B/T < \pi$ disfavored

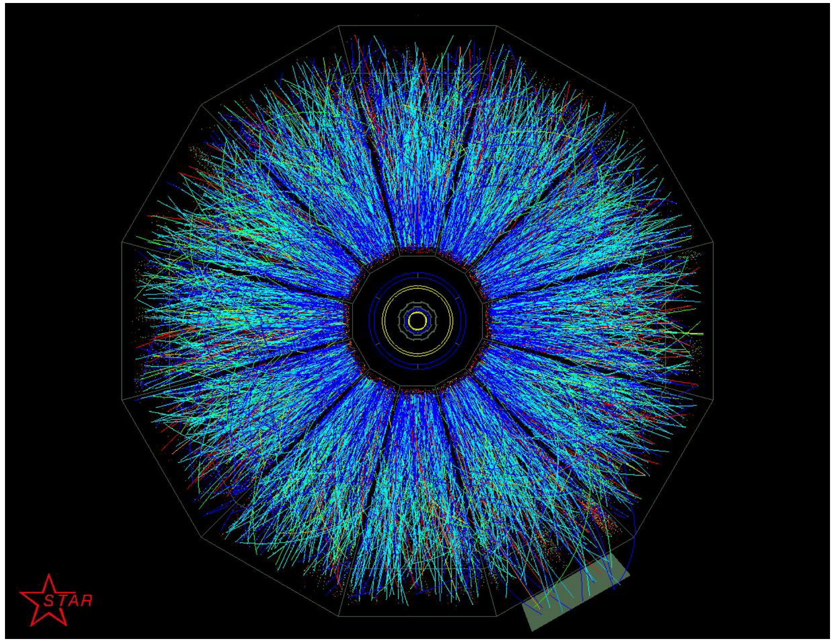


[V.V., Steinheimer, Philipsen, Stoecker, PRD 97, 114030 (2018)]

Critical point, if it exists, likely located beyond the reach of lattice methods

QCD phase diagram with heavy-ion collisions





STAR event display

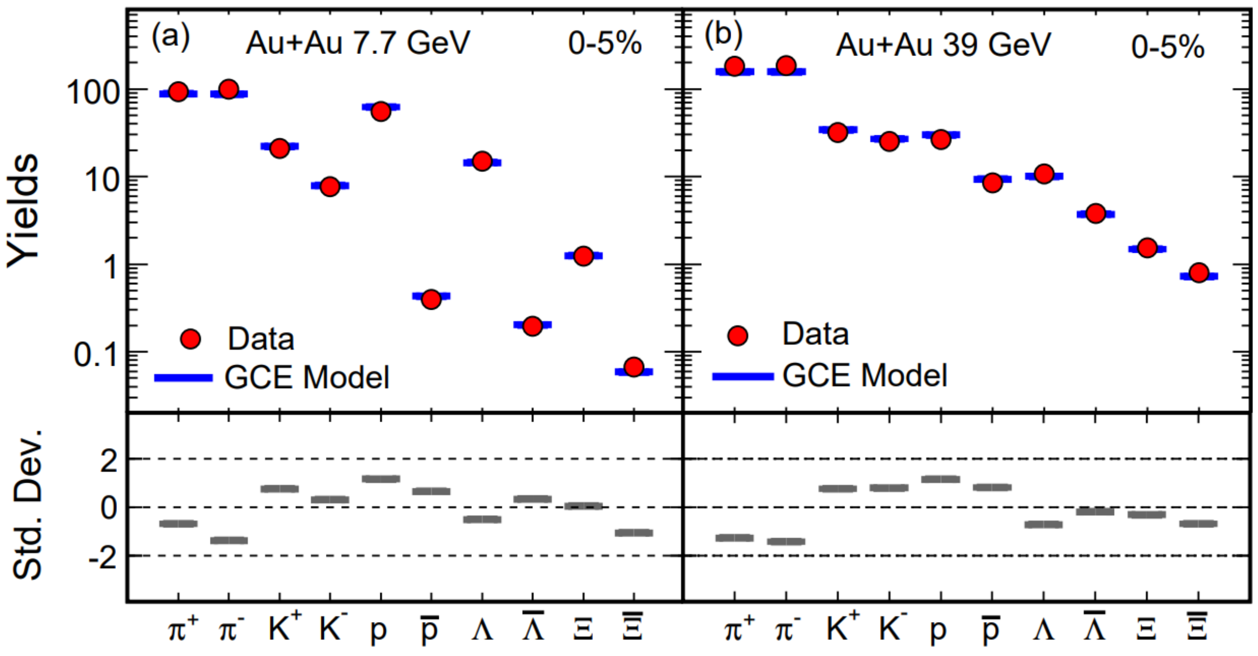
Figure from Bzdak et al., Phys. Rept. '20

Thousands of particles created in relativistic heavy-ion collisions

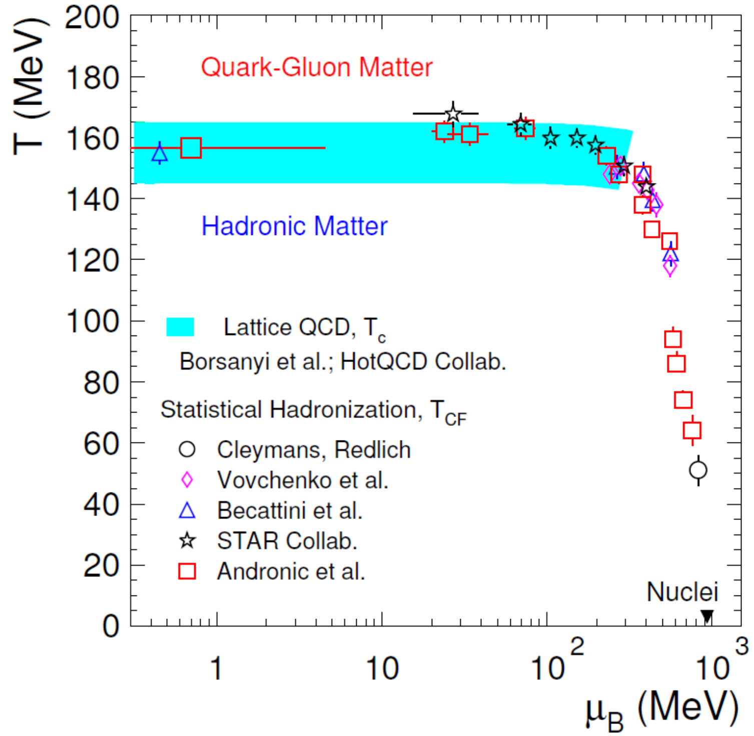
Apply concepts of statistical mechanics

Mapping heavy-ion collisions onto the QCD phase diagram

Fit hadron yields with the thermal model: $N_i = V \frac{d_i m_i^2 T}{2\pi^2} K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}} + \text{feedown}, \qquad \mu_i = b_i \mu_B + q_i \mu_Q + s_i \mu_S$



STAR Collaboration, PRC 96, 044904 (2017)



A. Andronic et al., Nature 561, 321 (2018)

$\sqrt{s_{NN}} \searrow \quad \Longrightarrow \quad \mu_B \nearrow$

For differential observables (spectra, flow, …) use relativistic hydrodynamics

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

Grand partition function

$$\ln Z^{\mathrm{gce}}(T, V, \mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\mathrm{ce}}(T, V, N)\right]$$

$$\kappa_n \propto \frac{\partial^n (\ln Z^{\mathrm{gce}})}{\partial \mu^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

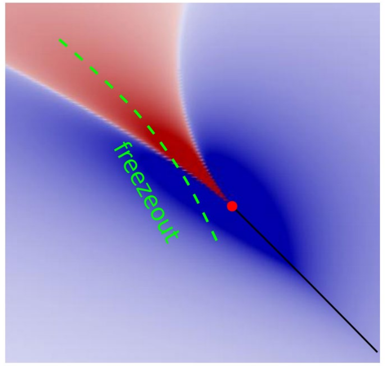
Grand partition function

$$\ln Z^{\rm gce}(T,V,\mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\rm ce}(T,V,N)\right]$$

$$\kappa_n \propto \frac{\partial^n(\ln Z^{\rm gce})}{\partial\mu^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- **(QCD) critical point** – large correlation length, critical fluctuations of baryon number



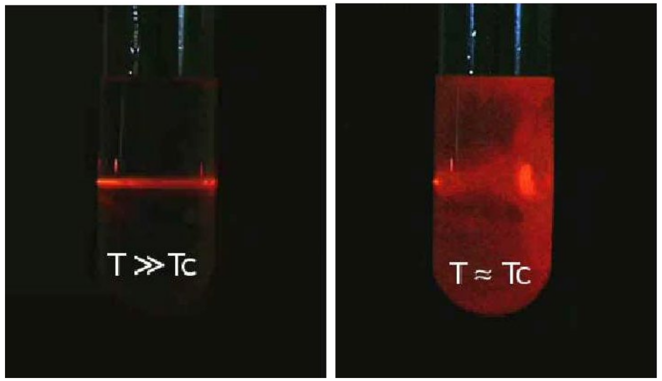
M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

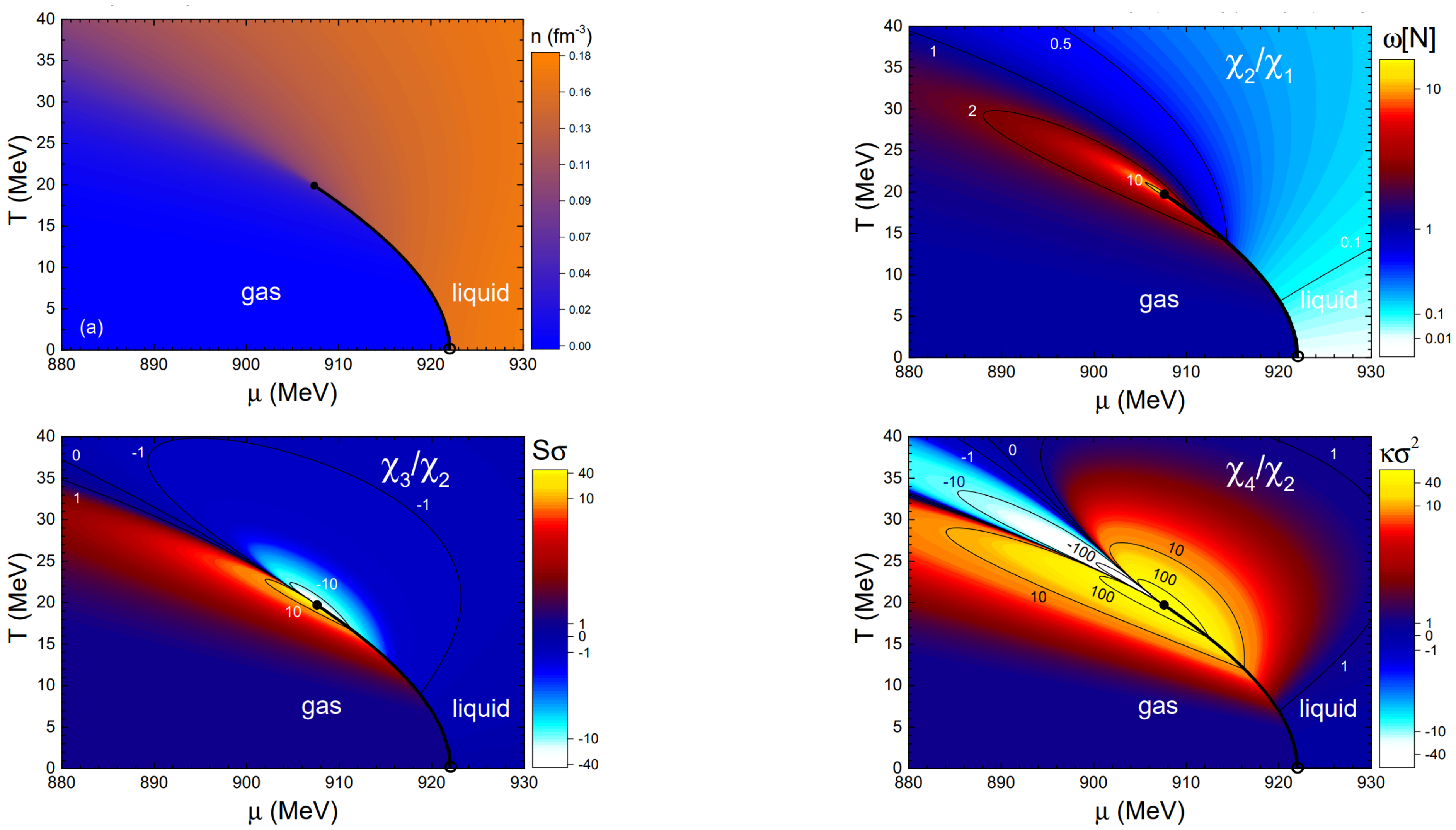
$$\xi \to \infty$$

Looking for non-monotonic dependence of κ_4

Critical opalescence



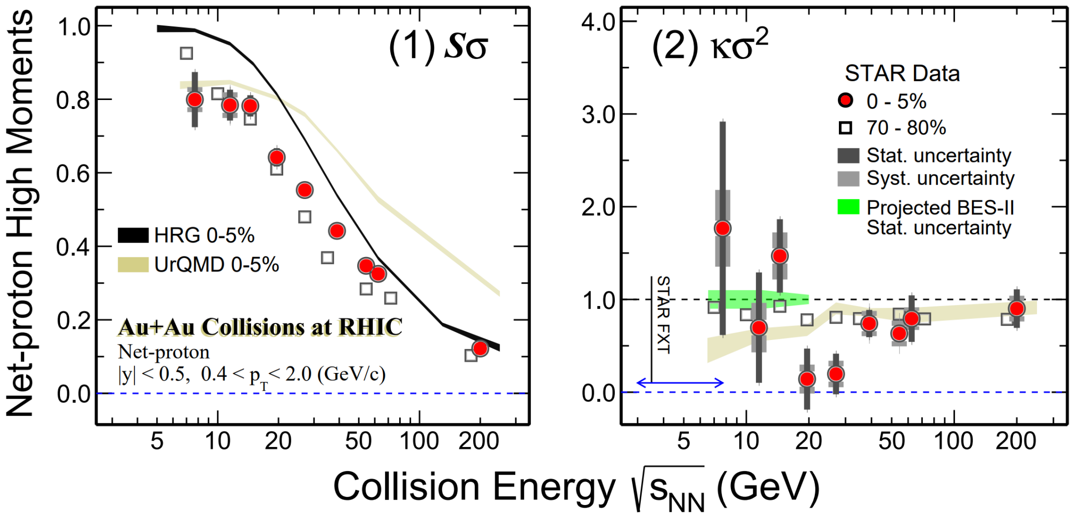
Example: Nuclear liquid-gas transition



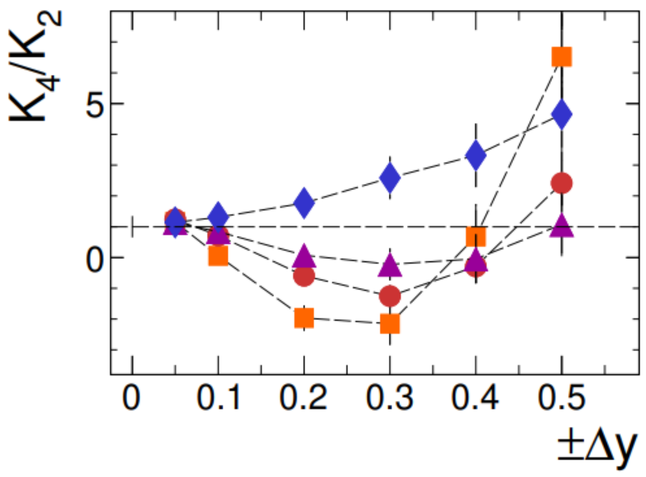
[VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)]

Experimental measurements

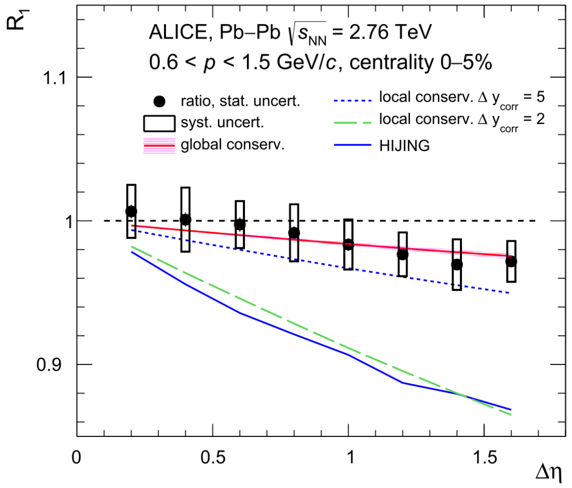
STAR Collaboration, PRL 126, 092301 (2021)



HADES Collaboration, PRC 102, 024914 (2020)



ALICE Collaboration, PLB 807, 135564 (2020)

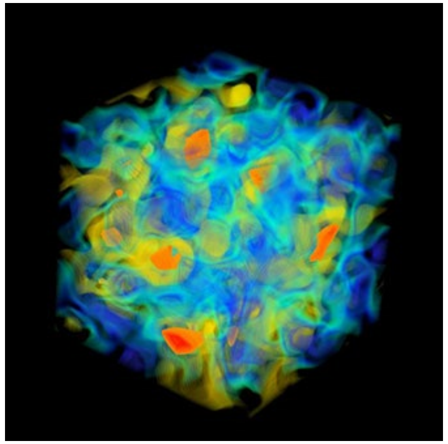


Deviations from Poisson statistics and indications for a non-monotonic collision energy dependence of net-proton $\kappa\sigma^2$

But how to interpret the measurements?

Theory vs experiment

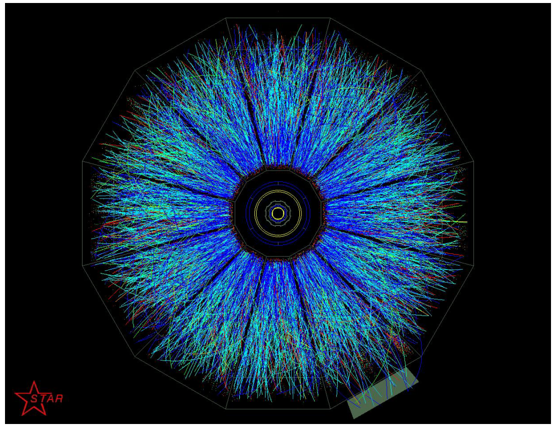
Theory



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment

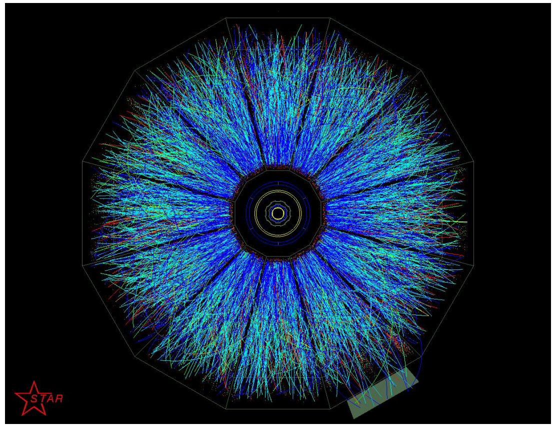


STAR event display

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

When are the measured fluctuations grand-canonical?

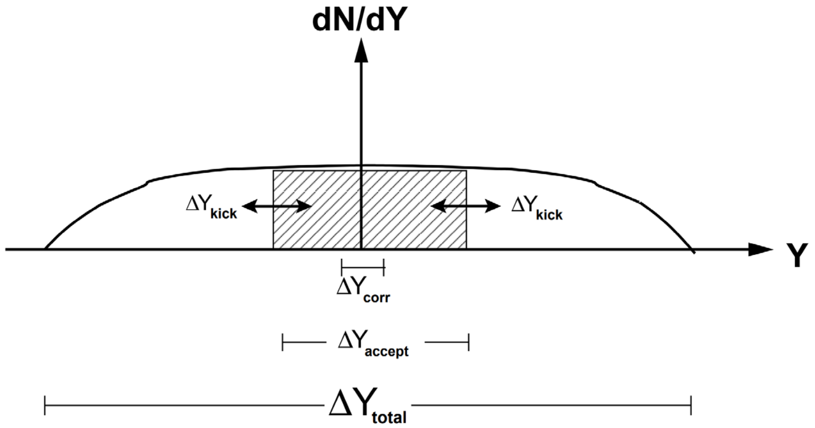
- Consider event-by-event fluctuations of particle number in acceptance ΔY_{accept} around midrapidity
- Scales
  - ΔY_{accept} – acceptance
  - ΔY_{total} – full space
  - ΔY_{corr} – rapidity correlation length (thermal smearing)
  - ΔY_{kick} – diffusion in the hadronic phase



STAR event display

- **GCE applies if $\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}, \Delta Y_{corr}$**

- In practice $\Delta Y_{total} \gg \Delta Y_{accept}$ and $\Delta Y_{accept} \gg \Delta Y_{corr}$ are not simultaneously satisfied
  - Corrections from baryon conservation are large [Bzdak et al., PRC '13]
  - $\Delta Y_{corr} \sim 1 \sim \Delta Y_{accept}$ [Ling, Stephanov, PRC '16]



V. Koch, arXiv:0810.2520

Need dynamical description

How to probe (critical) fluctuations?

1. Dynamical model calculations of critical fluctuations
   - Fluctuating hydrodynamics
   - Equation of state with tunable critical point
   - Predict CP signatures dependent on its location

   Under development within the Beam Energy Scan Theory (BEST) Collaboration

   [X. An et al., arXiv:2108.13867]

2. **Study deviations from the non-critical baseline**
   - Include essential non-critical contributions to (net-)proton number cumulants
   - Exact baryon conservation + baryon excluded volume
   - Based on realistic hydrodynamic simulations

   [VV, C. Shen, V. Koch, arXiv:2107.00163]

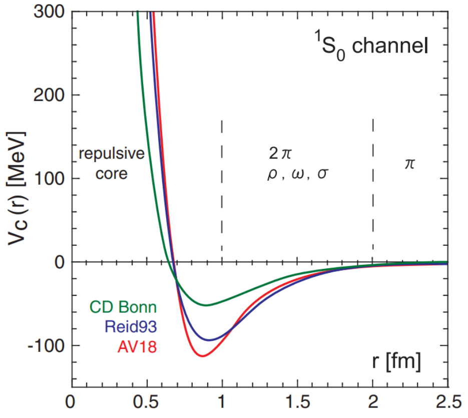


Figure from Ishii et al., PRL '07

The non-critical baseline from hydrodynamics

- Collision geometry based 3D initial state [Shen, Alzhrani, PRC '20]
  - Constrained to net proton distributions

- Viscous hydrodynamics evolution – MUSIC-3.0
  - Energy-momentum and baryon number conservation
  - NEOS-BQS crossover equation of state [Monnai, Schenke, Shen, PRC '19]
  - Shear viscosity via IS-type equation

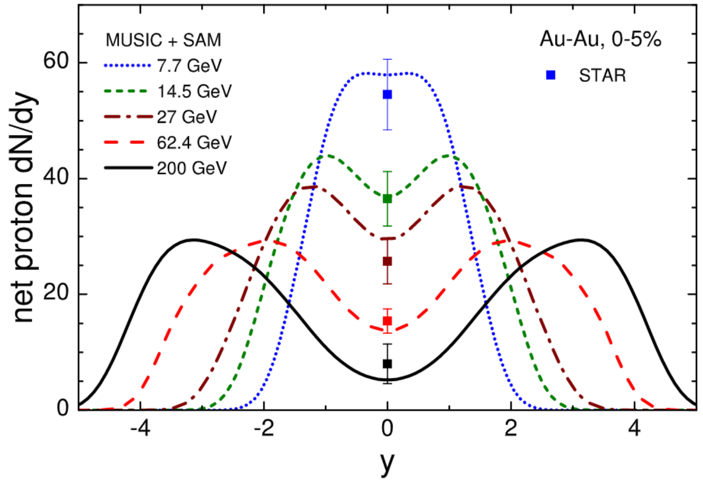
- Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3

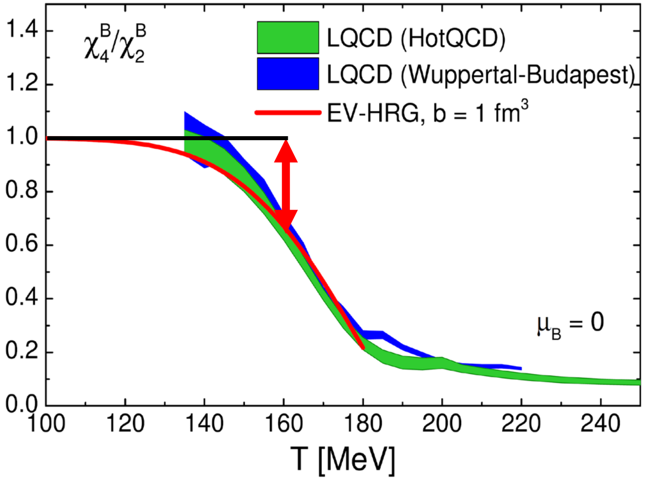
$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x)\, p^\mu \, \frac{d_j\, \lambda_j^{\rm ev}(x)}{(2\pi)^3} \, \exp\left[\frac{\mu_j(x) - u^\mu(x) p_\mu}{T(x)}\right].$$

- Particlization includes QCD-based baryon number distribution
  - Here incorporated via baryon excluded volume

[VV, Pasztor, Fodor, Katz, Stoecker, PLB 775, 71 (2017)]

VV, C. Shen, V. Koch, arXiv:2107.00163





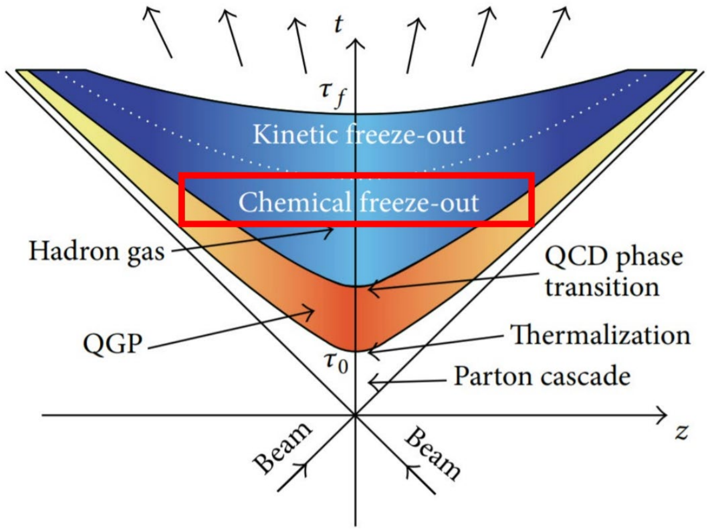
Calculating cumulants from hydrodynamics

- Strategy:
  1. Calculate proton cumulants in the experimental acceptance in the grand-canonical limit
  2. Apply correction for the exact global baryon number conservation

First step:

- Sum contributions from each hypersurface element x_i at freeze-out
  - Cumulants of joint (anti)proton/(anti)baryon distribution

$$\kappa_{n,m}^{B^\pm,p^\pm,\mathrm{gce}}(\Delta p_{\mathrm{acc}}) = \sum_{i\in\sigma} \delta\kappa_{n,m}^{B^\pm,p^\pm,\mathrm{gce}}(x_i;\Delta p_{\mathrm{acc}})$$

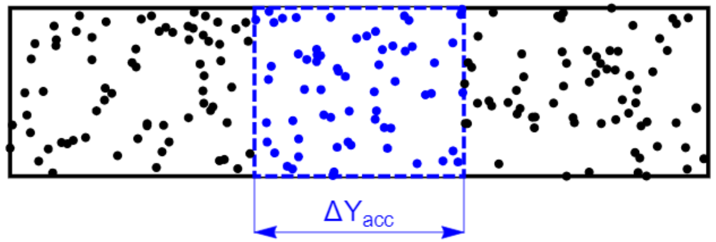


- To compute each contribution
  - GCE susceptibilities $\chi^{B^\pm}(x_i)$ define the distribution of the emitted (anti)baryons
  - Each baryon ends up in acceptance Δp_{acc} with binomial probability via Cooper-Frye formula
  - Each baryon is a proton with probability $q(x_i) = \langle N_p(x_i)\rangle / \langle N_B(x_i)\rangle$

*Cross-check with the Monte Carlo results of a canonical Cooper-Frye sampler from https://github.com/vlvovch/CooperFryeSampler

Correcting for baryon number conservation

$$P_1^{\text{ce}}(B_1) \propto \sum_{B_1,B_2} P_1^{\text{gce}}(B_1) P_2^{\text{gce}}(B_2) \times \delta_{B,B_1+B_2}$$



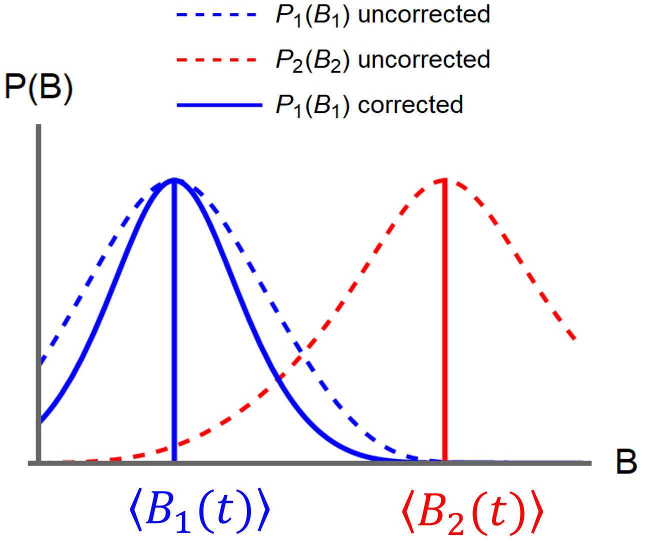
- Subensemble acceptance method (SAM)
  - Corrects *any* equation of state for global BQS-charge conservation
  - Canonical ensemble cumulants in terms of grand-canonical ones

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, Phys. Lett. B 811, 135868 (2020) [arXiv:2003.13905]

VV, Poberezhnyuk, Koch, JHEP 10, 089 (2020) [arXiv:2007.03850]

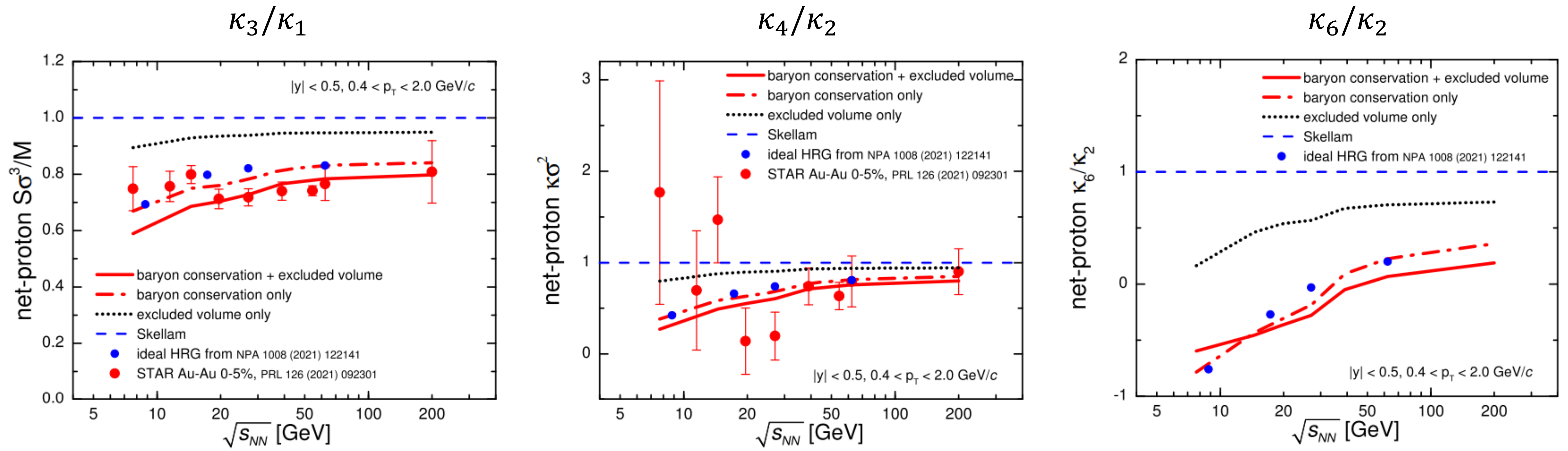
- SAM-2.0 VV, arXiv:2106.13775
  - Non-conserved quantities (e.g. proton number)
  - Spatially inhomogeneous systems
  - Momentum space
  - Maps "grand-canonical" cumulants inside and outside the acceptance to the "canonical" cumulants inside the acceptance:*

$$\kappa_{p,B}^{\text{in,ce}} = \text{SAM}\left[\kappa_{p,B}^{\text{in,gce}}, \kappa_{p,B}^{\text{out,gce}}\right]$$



*Explicit expressions for any cumulant available via a Mathematica notebook at https://github.com/vlvovch/SAM

Net proton cumulant ratios



- Both the baryon conservation and repulsion needed to describe data at $\sqrt{s_{NN}} \geq 20$ GeV quantitatively
- Effect from baryon conservation is larger than from repulsion
- Canonical ideal HRG limit is consistent with the data-driven study of [Braun-Munzinger et al., NPA 1008 (2021) 122141]
- κ_6/κ_2 turns negative at $\sqrt{s_{NN}}$~50 GeV

Cumulants vs Correlation Functions

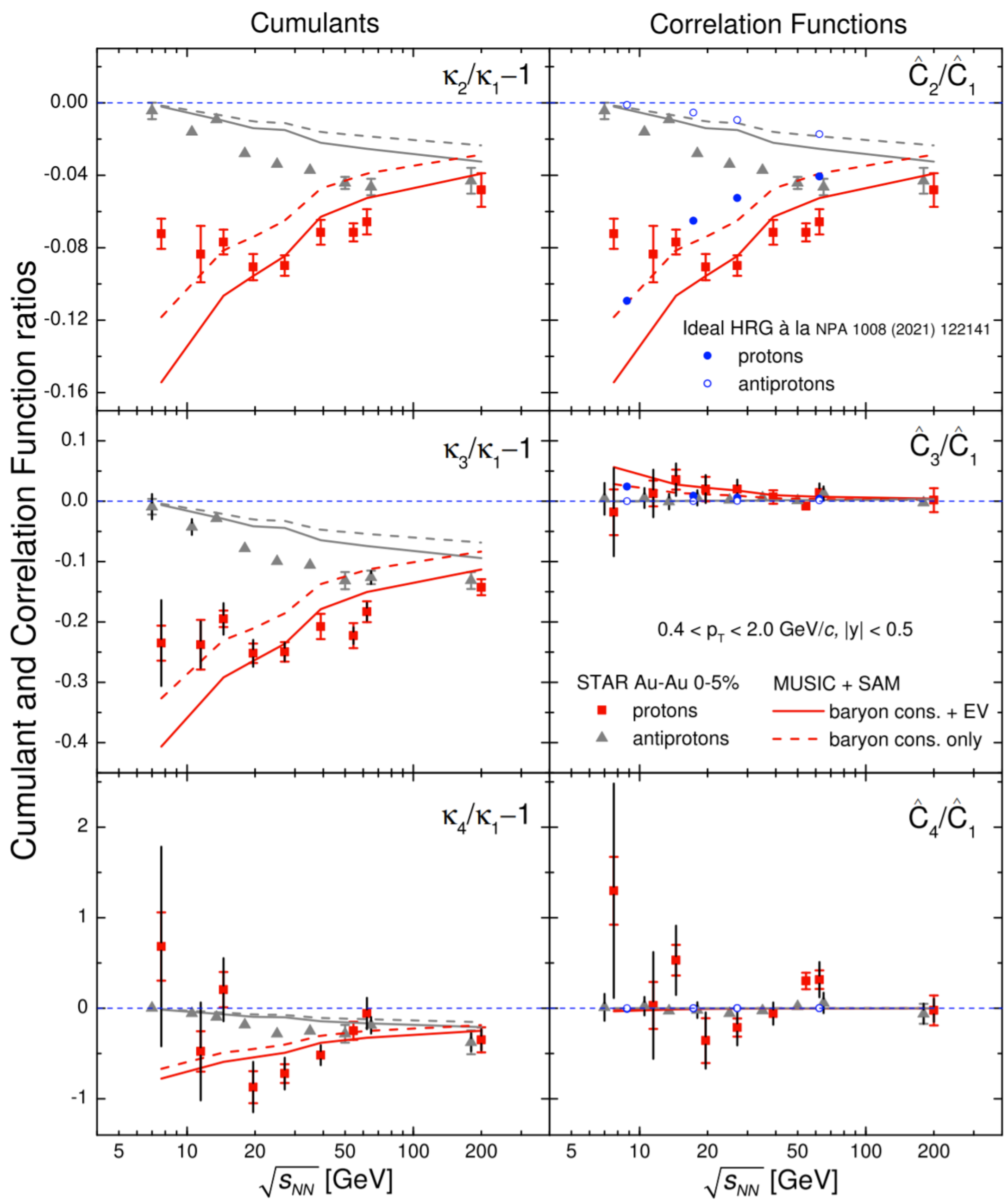
- Analyze genuine multi-particle correlations via **factorial cumulants** [Bzdak, Koch, Strodthoff, PRC '17]

$$\hat{C}_1 = \kappa_1, \qquad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$
$$\hat{C}_2 = -\kappa_1 + \kappa_2, \qquad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$

$$\hat{C}_n^{\text{cons}} \propto \alpha^n, \qquad \hat{C}_n^{\text{EV}} \propto b^n$$

[Bzdak, Koch, Skokov, EPJC '17] [VV et al, PLB '17]

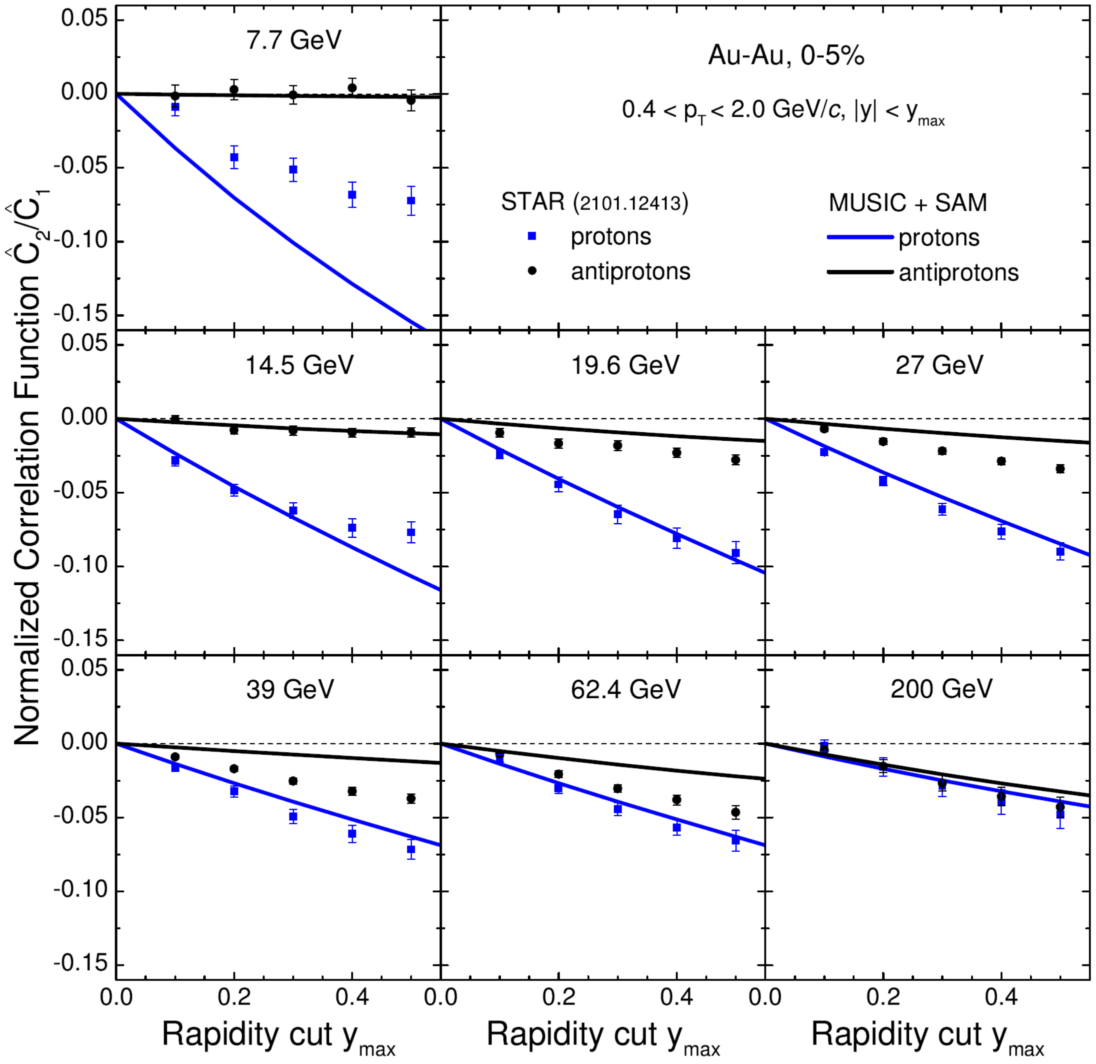
- Three- and four-particle correlations are small
  - Small positive $\hat{C}_3/\hat{C}_1$ in the data is explained by baryon conservation + excluded volume
  - Strong multi-particle correlations would be expected near the critical point [Ling, Stephanov, 1512.09125]
- Two-particle correlations are negative
  - Protons at $\sqrt{s_{NN}} \leq 14.5$ GeV overestimated
  - Antiprotons at $19.6 \leq \sqrt{s_{NN}} \leq 62.4$ GeV underestimated



*We use the notation for (factorial) cumulants from Bzdak et al., Phys. Rept. '20

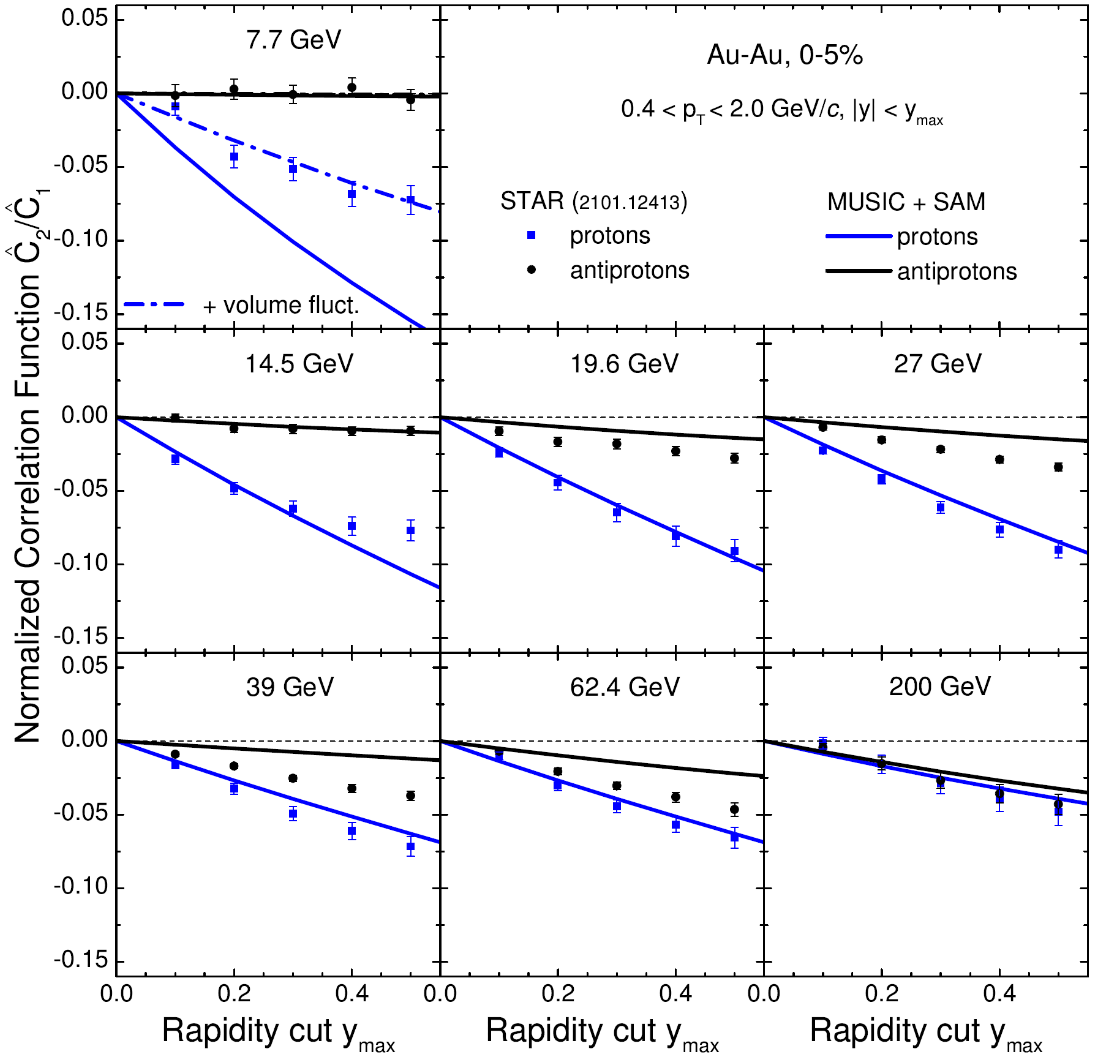
Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?



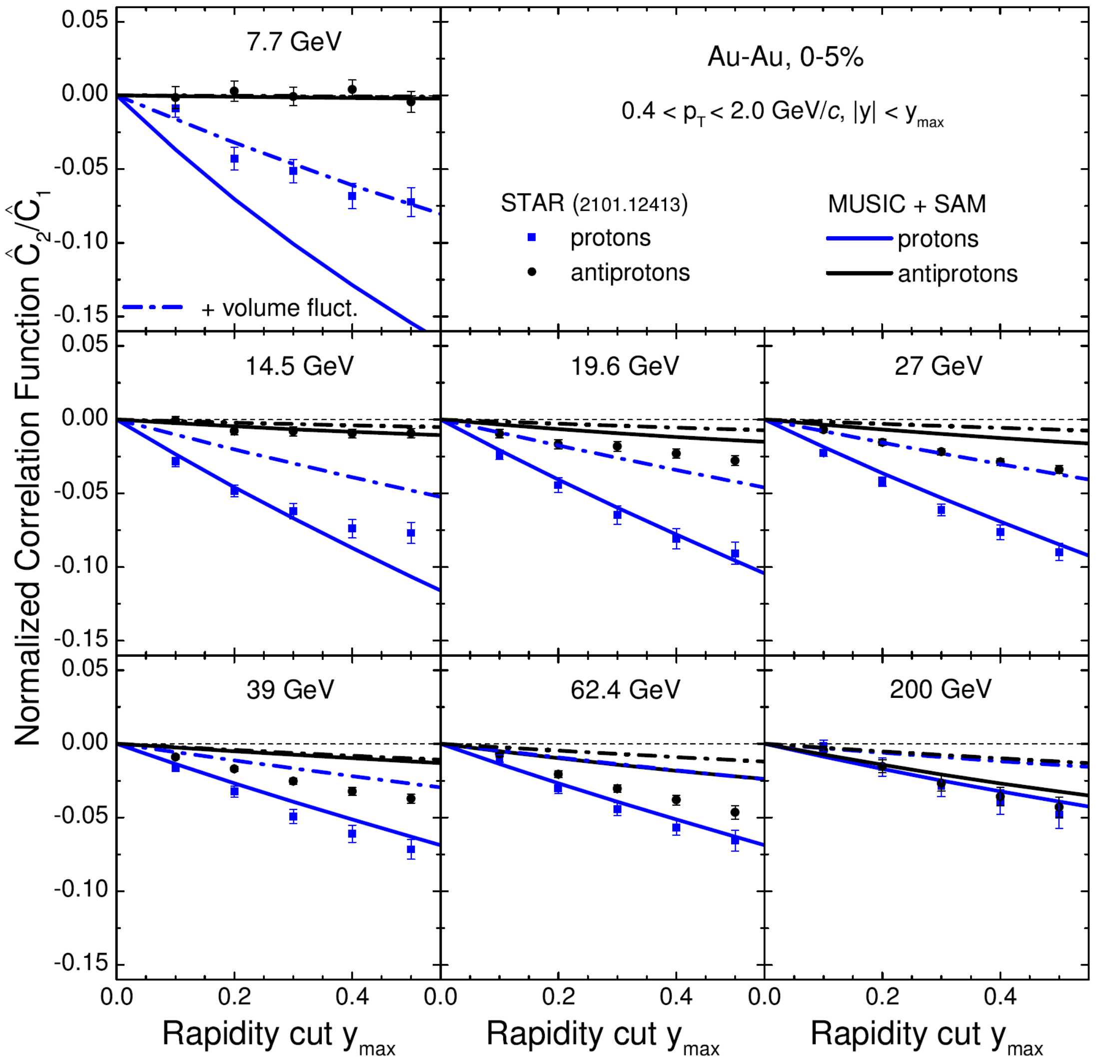
Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?
- **Volume fluctuations?** [Skokov, Friman, Redlich, PRC '13]
  - $C_2/C_1 \mathrel{+}= C_1 * v_2$



Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?
- **Volume fluctuations?** [Skokov, Friman, Redlich, PRC '13]
  - $C_2/C_1 += C_1 * v_2$
  - Can improve low energies but spoil high energies?

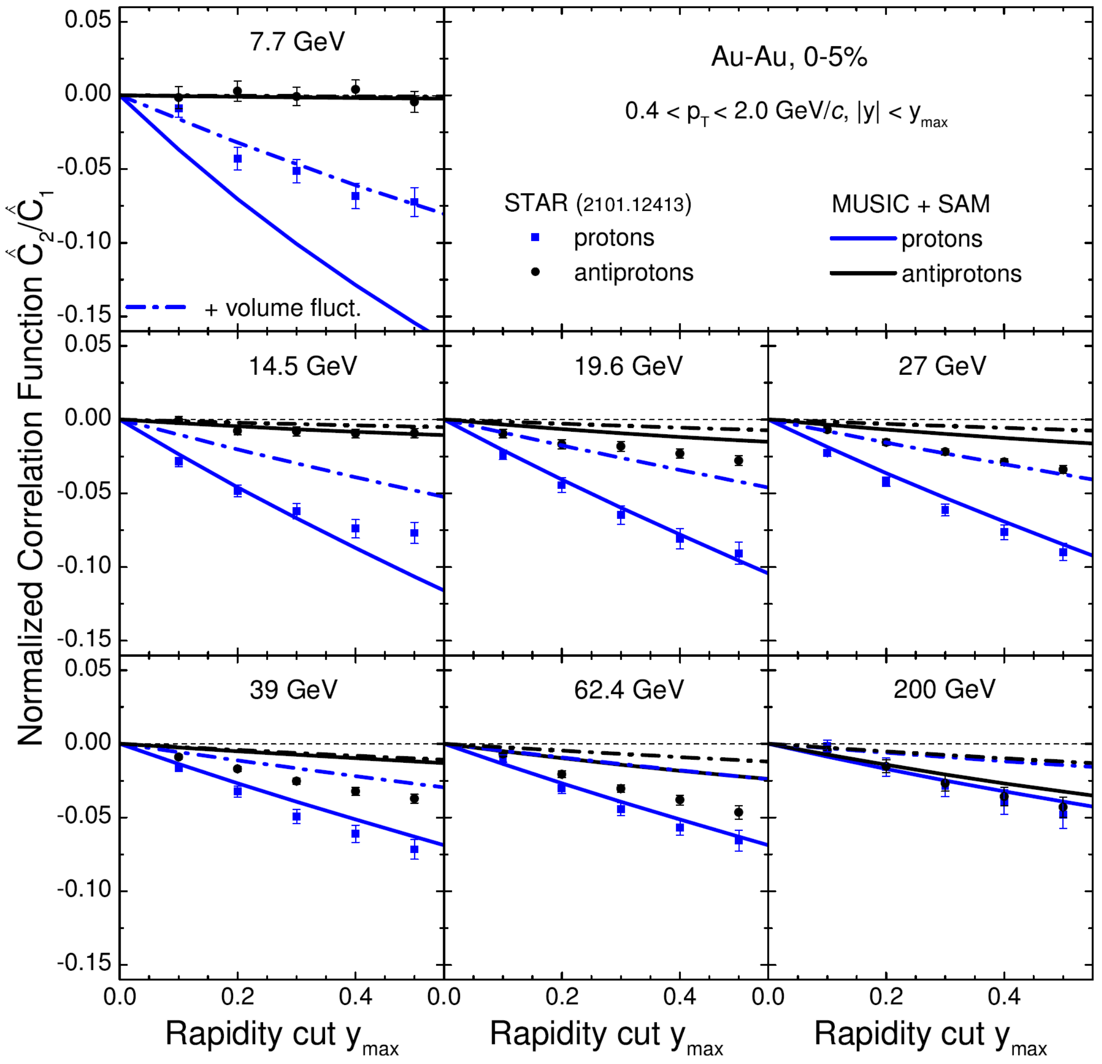


Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?

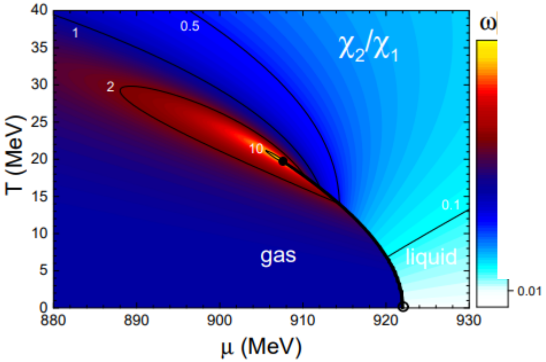
- Volume fluctuations? [Skokov, Friman, Redlich, PRC '13]
  - $C_2/C_1 \mathrel{+}= C_1 * v_2$
  - Can improve low energies but spoil high energies?

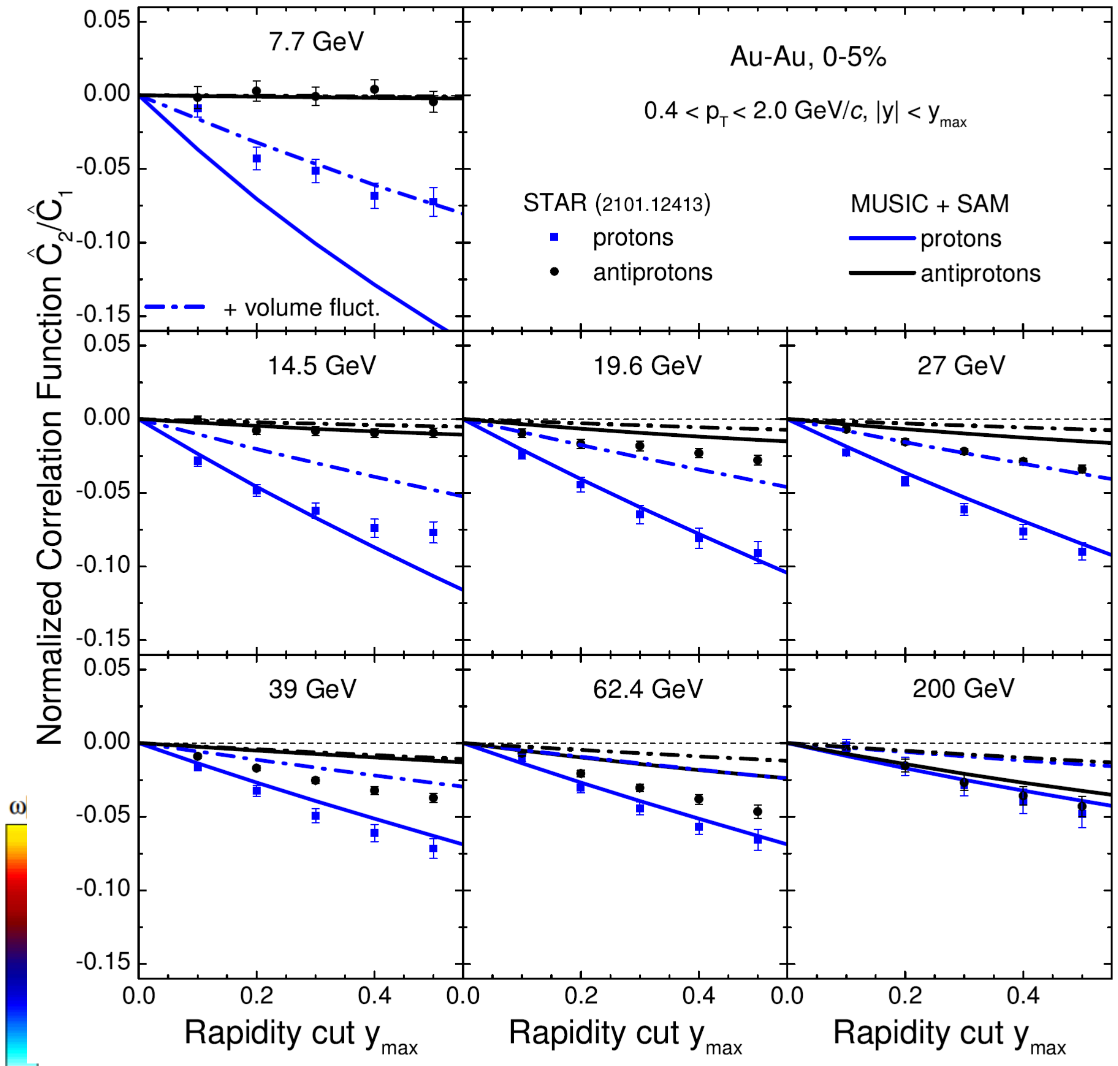
- **Exact electric charge conservation?**
  - Worsens the agreement at $\sqrt{s_{NN}} \leq 14.5$, higher energies virtually unaffected



Acceptance dependence of two-particle correlations

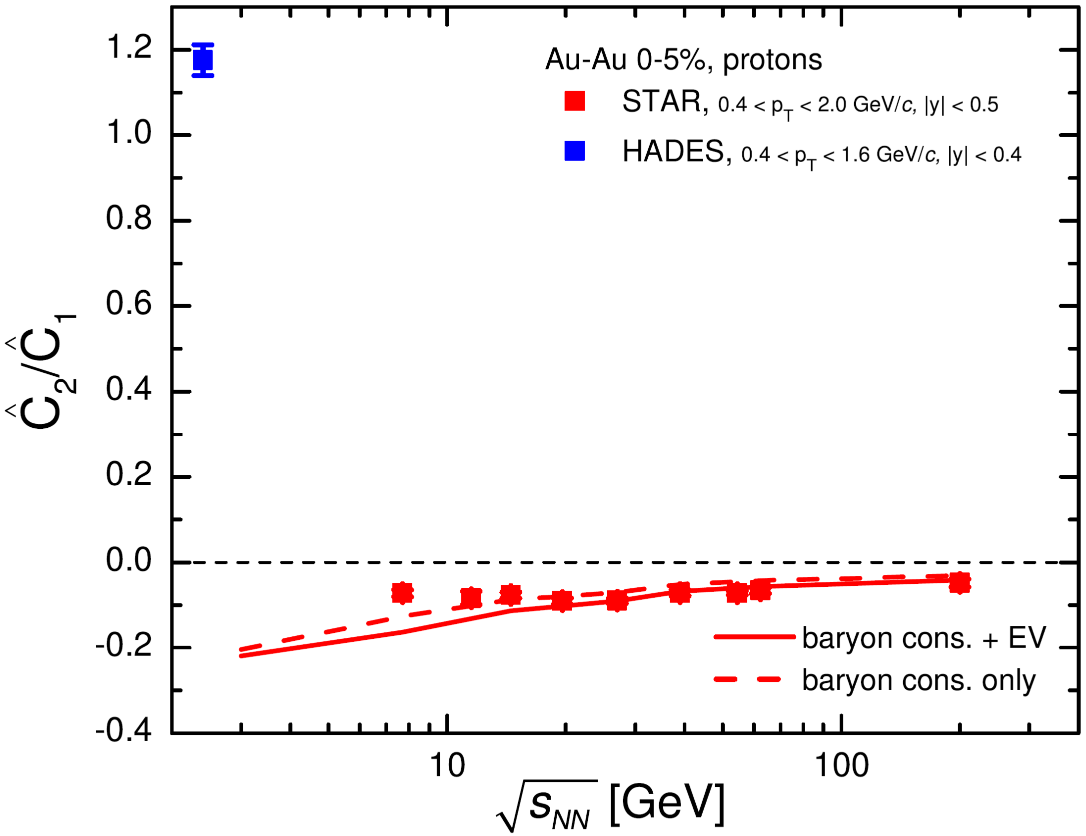
- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?
- Volume fluctuations? [Skokov, Friman, Redlich, PRC '13]
  - $C_2/C_1 \mathrel{+}= C_1 * v_2$
  - Can improve low energies but spoil high energies?
- Exact electric charge conservation?
  - Worsens the agreement at $\sqrt{s_{NN}} \leq 14.5$, higher energies virtually unaffected
- **Attractive interactions?**
  - Could work if baryon repulsion turns into attraction in the high-μ_B regime
  - **Critical point?**





Outlook: lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV

- Intriguing hint from HADES @ $\sqrt{s_{NN}} = 2.4$ GeV: huge two-particle correlations!
- Extend the calculations down to $\sqrt{s_{NN}} = 3$ GeV by means of the blast-wave model
- No change of trend in the non-critical baseline
- Other important effects to consider
  - Light nuclei formation
  - Nuclear liquid-gas transition

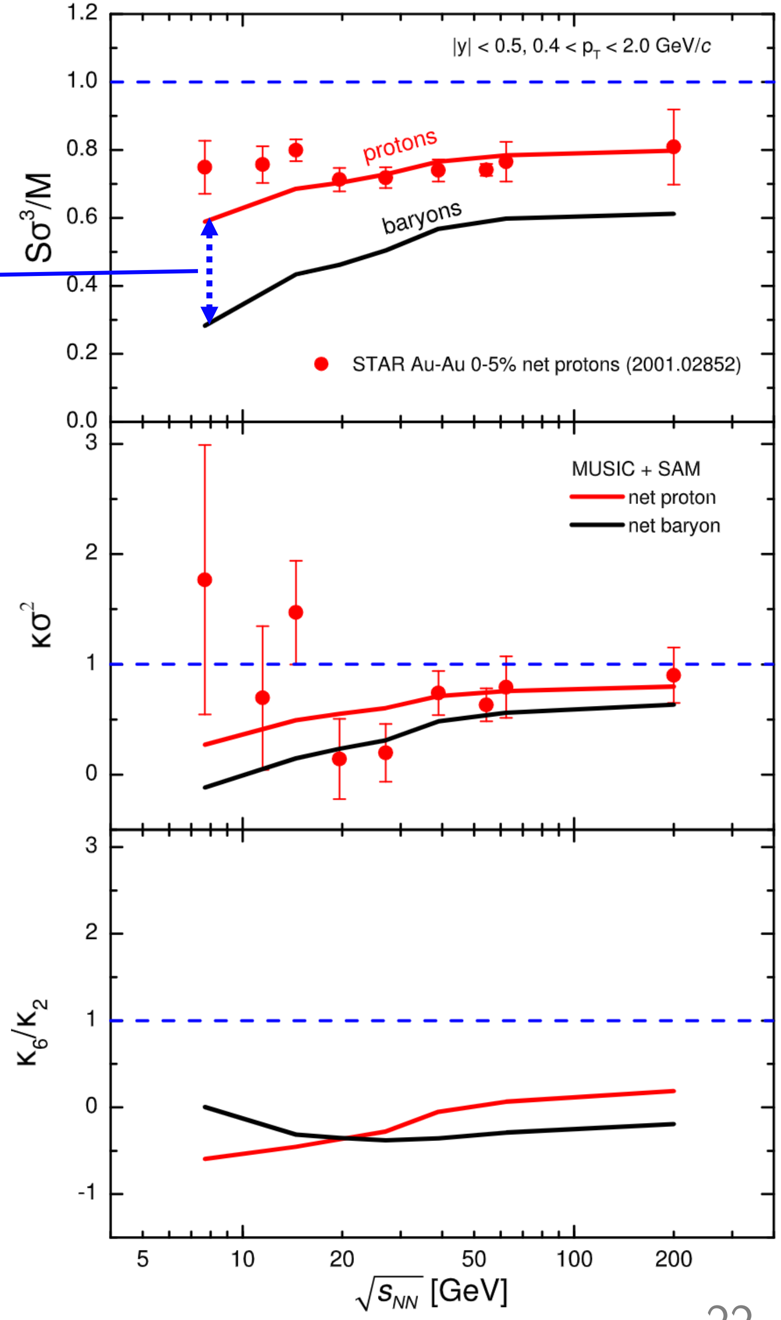


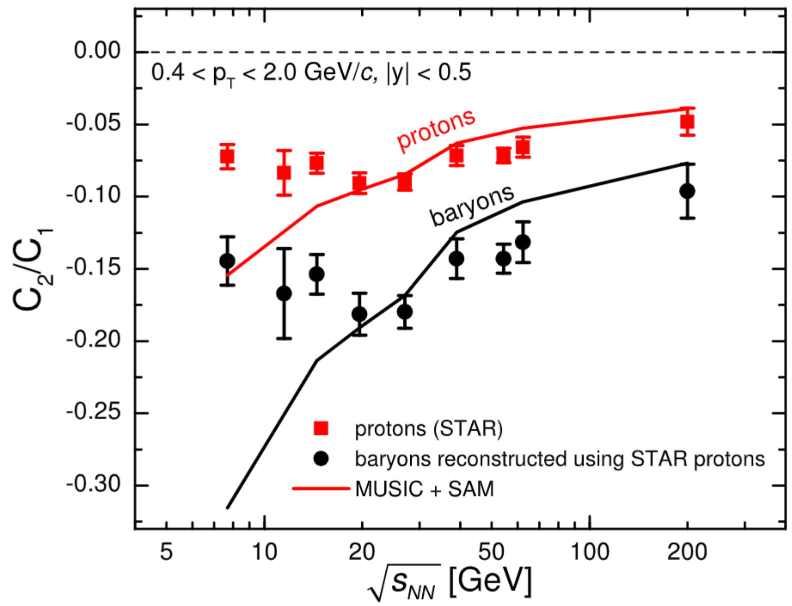
Data from STAR-FXT eagerly awaited!

Outlook: baryon cumulants from protons

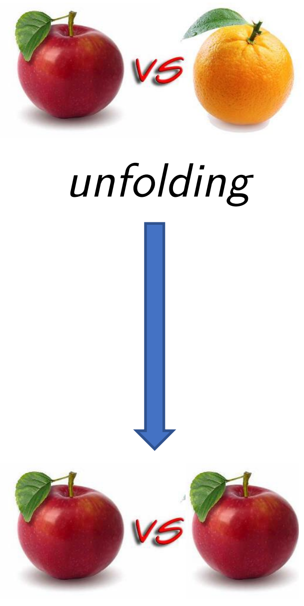
- **net baryon $\neq$ net proton**

- Baryon cumulants can be reconstructed from proton cumulants via binomial (un)folding based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]
  - Amounts to an additional "efficiency correction" and requires the use of joint factorial moments, only experiment can do it model-independently



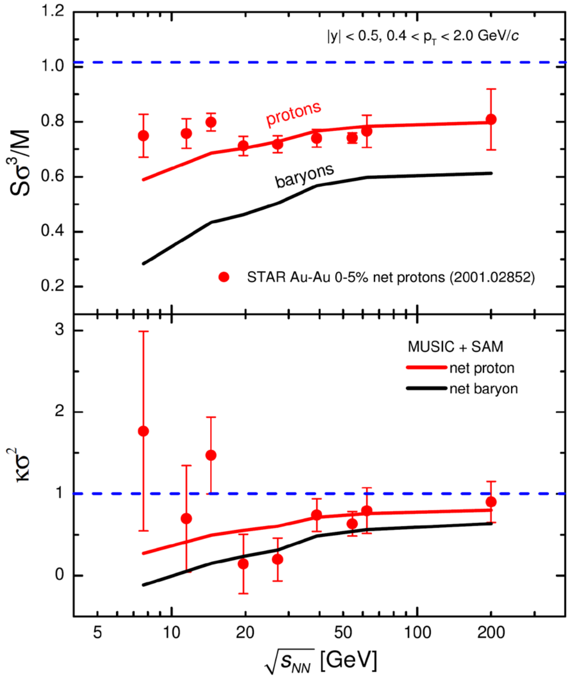


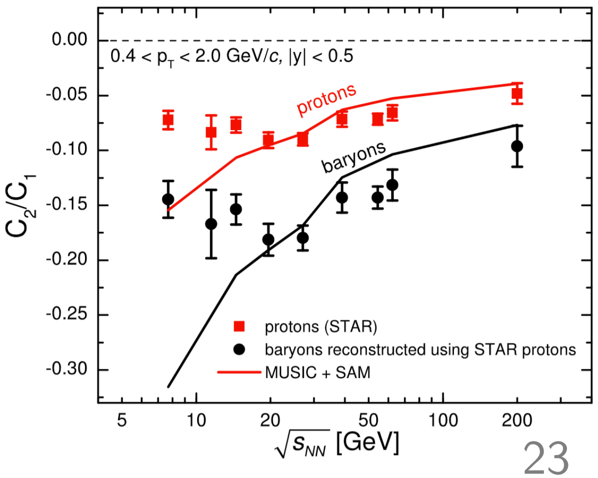
$$\frac{\hat{C}_2^B}{\hat{C}_1^B} \approx 2\,\frac{\hat{C}_2^p}{\hat{C}_1^p}$$



Summary

- Fluctuations are a powerful tool to explore the QCD phase diagram
  - test of lattice QCD and equilibration
  - probe the QCD critical point
- Quantitative analysis of Au-Au collisions at $\sqrt{s_{NN}}$=7.7-200 GeV
  - STAR protons are described quantitatively at $\sqrt{s_{NN}} \geq 20$ GeV
  - Significant difference between protons and baryons
- Factorial cumulants carry rich information
  - Small three- and four-particle correlations in absence of critical point effects
  - Possible evidence for attractive proton interactions at $\sqrt{s_{NN}} \leq 14.5$ GeV
- Outlook: Lower collision energies $\sqrt{s_{NN}} \leq 7.7$ GeV





Thanks for your attention!