

Sensitivity of thermal fits to heavy-ion yield data to the modeling of eigenvolume interactions

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In collaboration with Horst Stoecker
based on arXiv:1512.08046

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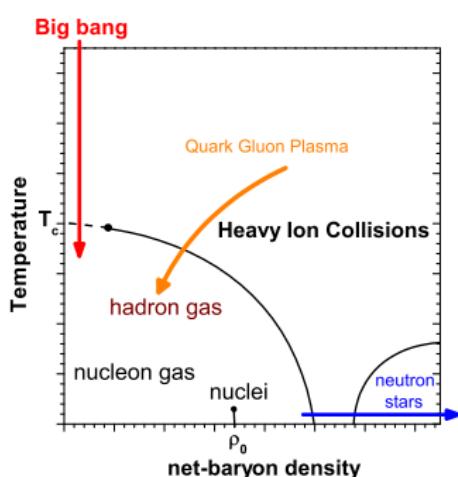
FIAS Frankfurt Institute
for Advanced Studies

GOETHE
UNIVERSITÄT
FRANKFURT AM MAIN

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

Strongly interacting matter

- Theory of strong interactions: Quantum Chromodynamics (QCD)
- Basic degrees of freedom: quarks and gluons
- At smaller energies confined into hadrons (baryons and mesons)



Scales

- Length: $1 \text{ fm} = 10^{-15} \text{ m} = 10^{-5} \text{ \AA}$
- Energy: $100 \text{ MeV} = 10^{10} \text{ times room } T$

Where is it relevant?

- Early universe
- Neutron stars
- Heavy-ion collisions (laboratory!)

First principles of QCD are rather established,
but direct calculations are problematic

Phenomenological tools are very useful

Experiment: heavy-ion collisions

Heavy-ion collisions

High energy collision of heavy ions creates excited QCD matter

Running experiments at CERN, RHIC, future at FAIR and NICA

Pb + Pb, $E_{\text{K}} = 158.0$ GeV
 $b = 0.0$ fm
Time: -20.00 fm/c



- Detectors measure yields and momenta of created hadrons
- Is equilibration reached in the collision?
- Can heavy-ion collisions be mapped on QCD phase diagram?

Thermal model

Thermal model - simplest model to describe hadron abundances

Key assumptions

- Matter is thermalized at chemical freeze-out
- Described as non-interacting gas of hadrons and resonances
- Characterized by few thermal parameters (T , μ_B , V)

Density of thermal hadrons

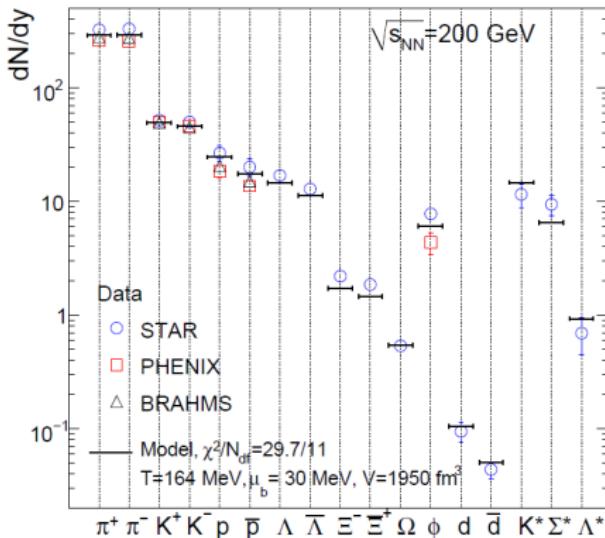
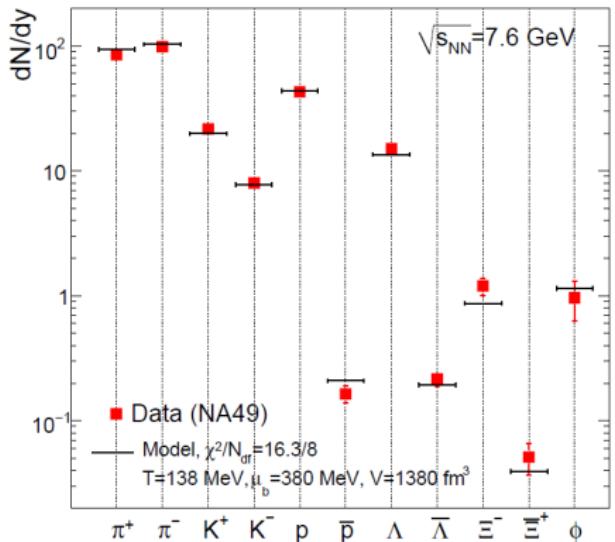
$$n_i = \frac{g_i}{(2\pi)^3} \int d^3p \left\{ \exp[(\omega_p^i - \mu_i)/T] \pm 1 \right\}^{-1}$$
$$\mu_i = B^i \mu_B + Q^i \mu_Q + S^i \mu_S, \quad \omega_p^i = \sqrt{m_i^2 + p^2}.$$

Total hadron density – thermal + resonance decay feed-down

$$n_i^{tot} = n_i + \sum_{j \neq i} Br(j \rightarrow i) n_j.$$

Absolute yields are defined by volume V : $N_i = V n_i$.

Thermal model

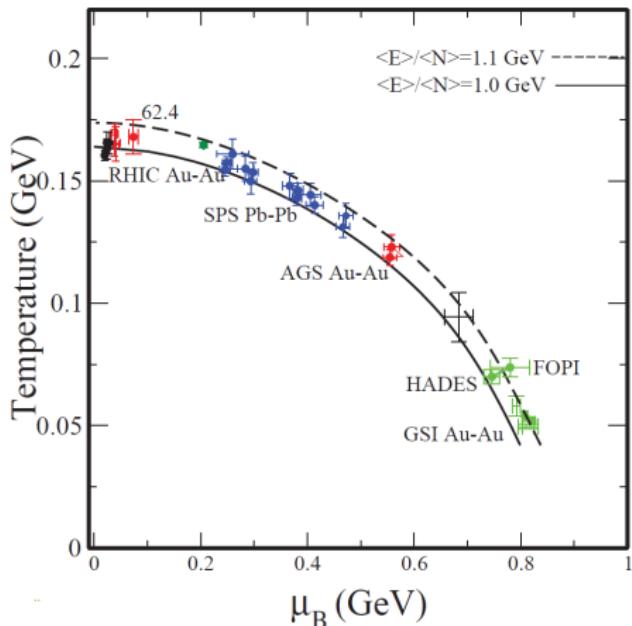


A. Andronic, P. Braun-Munzinger, J. Stachel, Phys. Lett. B 673, 142 (2009)

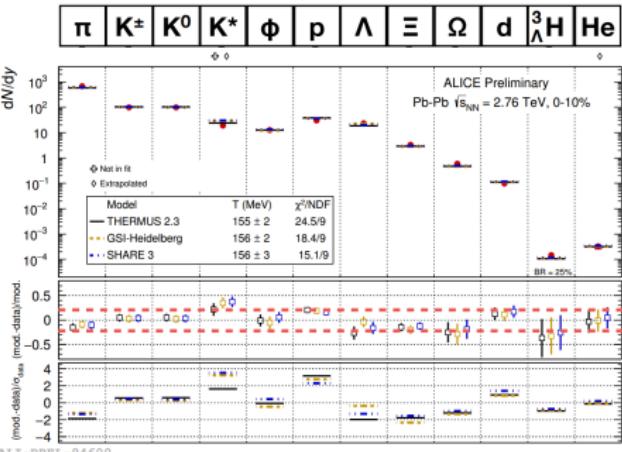
- Good qualitative description across several orders of magnitude
- Evidence for thermodynamic equilibrium

Chemical freeze-out curve

Minimizing the χ^2 of measured yields allows to extract thermal parameters at freeze-out



ALICE Pb+Pb $\sqrt{s_{NN}} = 2.76$ TeV



Chemical freeze-out in HIC mapped on QCD phase diagram but ...

- How **robust** are the conclusions based on **ideal gas**?
- Is there really a **sharp** freeze-out with well-defined temperature?

Interacting hadron gas

- In realistic hadron gas there are attractive and **repulsive** interactions
- Attraction already included by resonances
- Model repulsive interactions by **eigenvolume** correction
- **Van der Waals** procedure: $V \rightarrow V - vN$

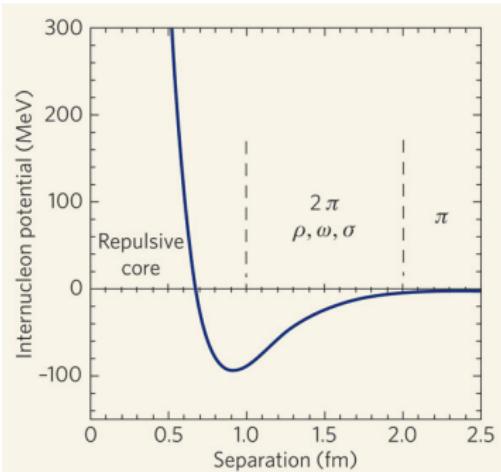
$$P = \frac{NT}{V - vN}$$

In GCE: transcendental equation for pressure¹

$$P(T, \mu) = P^{\text{id}}(T, \mu - vP), \quad n(T, \mu) = n^{\text{id}}(T, \mu^*) / (1 + v n^{\text{id}}(T, \mu^*))$$

In multi-component system $V \rightarrow V - \sum_i v_i N_i$ and²

$$P(T, \mu) = \sum_i P_i^{\text{id}}(T, \mu_i - v_i P), \quad n_i(T, \mu) = n_i^{\text{id}}(T, \mu_i^*) / (1 + \sum_i v_i n_i^{\text{id}}(T, \mu_i^*))$$



¹D.H. Rischke, M.I. Gorenstein, H. Stoecker, W. Greiner, Z.Phys. C 51, 485 (1991)

²G.D. Yen, M.I. Gorenstein, W. Greiner, S.N. Yan, Phys. Rev. C 56, 2210 (1997)

Some details about implementation

- Own implementation of eigenvolume HRG written in C++ is used
- Solves eigenvolume models, also many other features
- Auxiliary tool: GUI written within Qt framework
- Where possible cross-checked with THERMUS package

Thermal model calculations

Database file: D:/THERMUS23/Database.dat Load database...

Thermal model Fit to experiment Energy dependence Contour plots Event generator

	Name	PDG ID	Mass	Stable?	Baryon?	Neutral?	Charge	Strangeness	Charm	Prim. density	Prim. multiplicity	Total
1	pi0	111	0,13498	*	*					0,0221724	92,8755	228
2	pi+	211	0,13957	*			+1			0,0200586	84,0212	196
3	pi-	-211	0,13957	*			-1			0,023665	99,1277	222
4	K+	321	0,49368	*			+1	+1		0,00701418	29,3809	40,4
5	K-	-321	0,49368	*			-1	-1		0,00201722	8,44971	13,5
6	K0S	310	0,49767	*				(hidden)	0	0	27,1	
7	K0L	130	0,49767	*				(hidden)	0	0	27,1	
8	K0	311	0,49767						+1	0,00739406	30,9721	41,7
9	Klbar	-311	0,49767									
10	eta	221	0,5473									
11	rho+	213	0,7711									
12	rho-	-213	0,7711									
13	rho0	113	0,7711									
14	omega(782)	223	0,78257									
15	f0(600)	9000221	0,8									
16	Kstar(892)+	323	0,89166									
17	Kstar(892)-	-323	0,89166									
18	Kstar(892)0	313	0,8961									
19	Kstar(892)0bar	-313	0,8961									
20	p	2212	0,93827									
21	pbar	-2212	0,93827									
22	n	2112	0,93956									
23	nbar	-2112	0,93956									
24	eta'(958)	331	0,95778	*		0.66667 (hidden)				0,000203562	0,852678	0,93
25	f0(980)	9010221	0,98			*				0,000184701	0,773676	0,78

Model:
 Ideal HRG EV-HRG CE-HRG S-CE-HRG C-CE-HRG

Statistics:
 Boltzmann Quantum

Hadron radius (fm): 0,30

Parameters:
T (MeV): 125,00
Yc: 1,00
R (fm): 10,000
mu_c(MeV): -450,00
mu_s(MeV): 86,81
mu_q(MeV): -9,39
B: 2
S: 0
Q: 2

ratio: 0,400

life resonance width Renormalize branching ratios

lization:

perMP

ulate Write to file...

mark

aryon density = 0,0805138 fm^-3
aryon number = 337,256
lectric charge = 134,902
trangeness = 2,97647e-09
harm = 0
= 0,4
= 1,65152e-11
= nan

Total scaled variance = 1

Calculation time = 81 ms

Show only stable particles Show calculation results...

Eigenvalues: Scenario 0

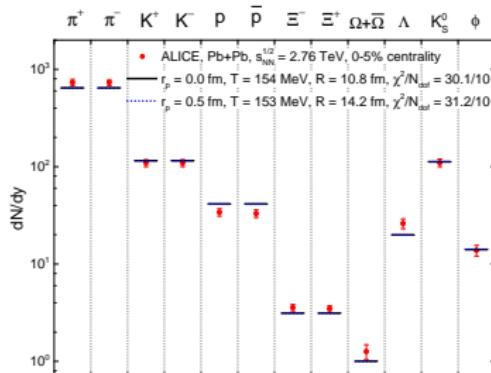
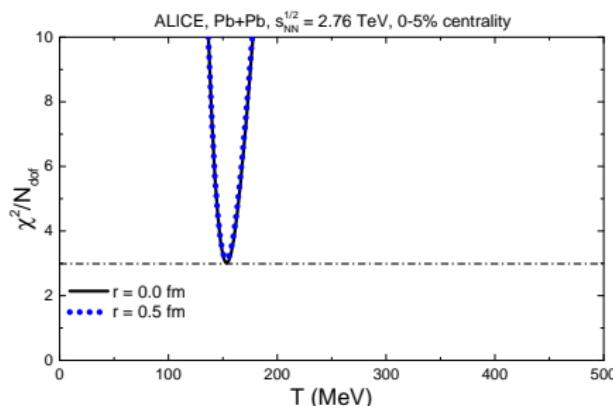
How to choose eigenvalues for different hadrons?

Not many constraints \Rightarrow consider different scenarios

Scenario 0: Constant eigenvolume for all hadrons ($v_i \equiv v$)

In this case in Boltzmann approximation

$$n_i(T, \mu) = \frac{n_i^{\text{id}}(T, \mu_i) e^{-vP/T}}{1 + \sum_i v n_i^{\text{id}}(T, \mu_i) e^{-vP/T}} \quad \text{and} \quad \frac{n_i(T, \mu)}{n_j(T, \mu)} = \frac{n_i^{\text{id}}(T, \mu)}{n_j^{\text{id}}(T, \mu)}$$

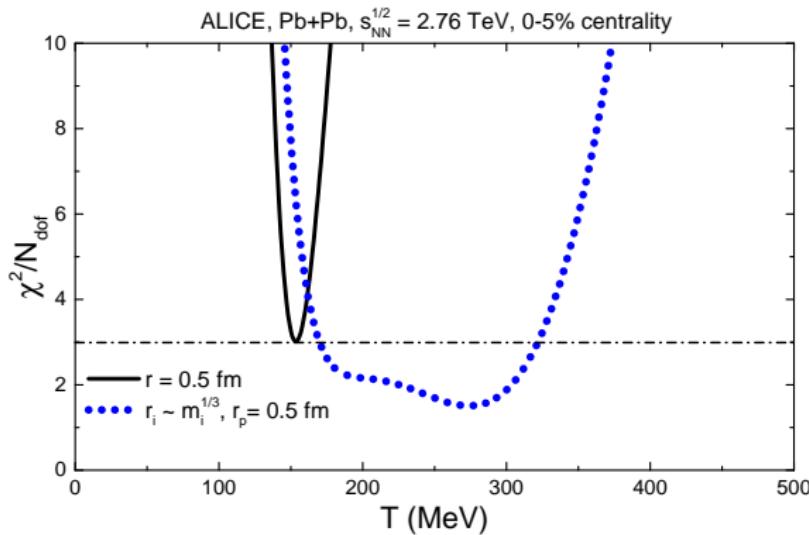


- Simplest and most **commonly used** parametrization
- Eigenvolume effects essentially **cancel out** in yield ratios
- **No change** in T or μ_B compared to point-particle case

Eigenvalues: Scenario 1

Scenario 1: Mass-proportional eigenvalues ($v_i = m_i/\varepsilon_0$ or $r_i \sim m_i^{1/3}$)³

- Bag model inspired
- Obtained originally for heavy Hagedorn states
- Results in stronger suppression of heavier hadrons



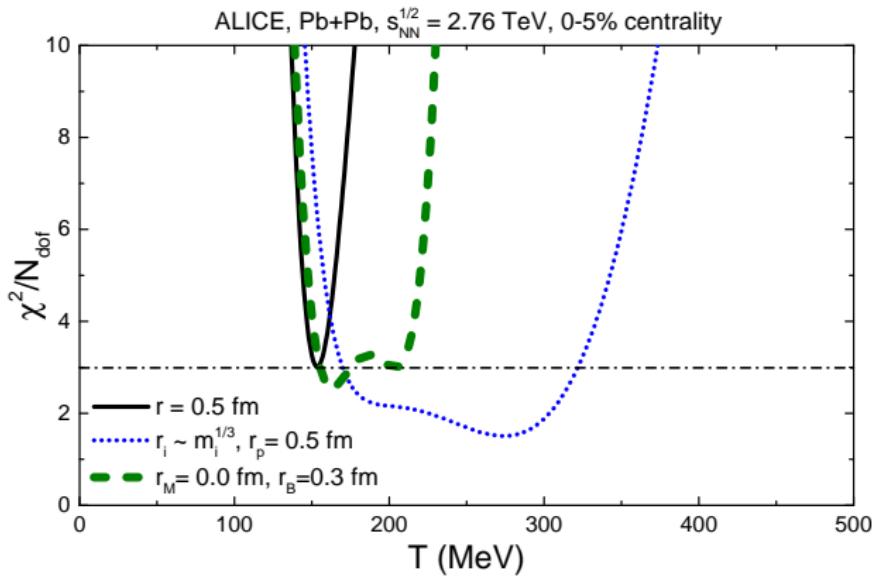
Drastic changes in ALICE χ^2 profile, also high sensitivity on ε_0
For $r_p = 0.5 \text{ fm}$ global minimum at $T \simeq 270 \text{ MeV}$

³Hagedorn, Rafelski, Phys. Lett. B (1980); Kapusta, Olive, Nucl. Phys. A (1983)

Eigenvalues: Scenario 2

Scenario 2: Two-component model: different volumes for mesons and baryons

We consider particular case $r_M = 0$ and $r_B = 0.3$ fm,
has been compared to lattice successfully⁴



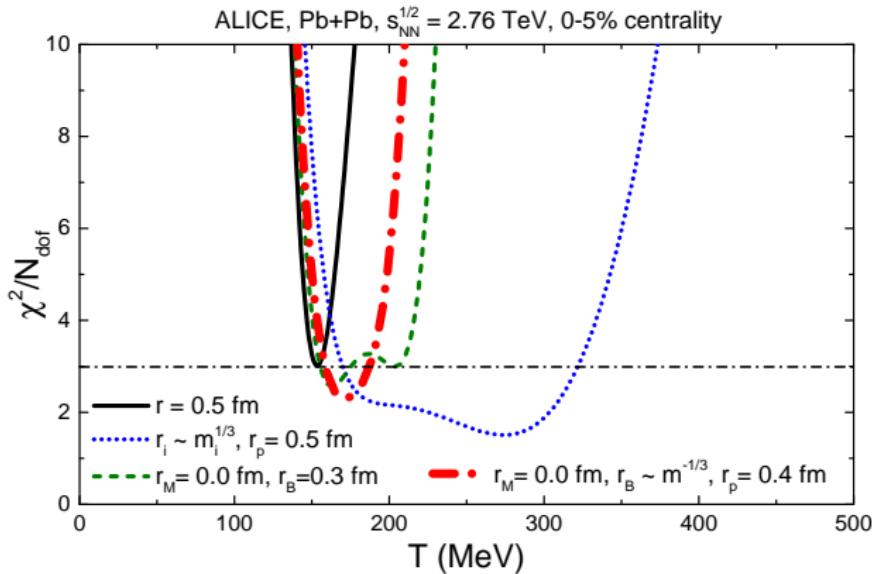
Wide irregular minimum in $T = 155 - 210$ MeV range

⁴A. Andronic, P. Braun-Munzinger, J. Stachel, M. Winn, Phys. Lett. B 718, 80 (2012).

Eigenvalues: Scenario 3

Scenario 3: Point-like mesons and reverse bag model for baryons $v_B \sim 1/m$

Strange baryons have generally smaller volumes than non-strange ones



Result: $T_{\text{ch}} = 175 \pm 20 \text{ MeV}$

Many other options possible...

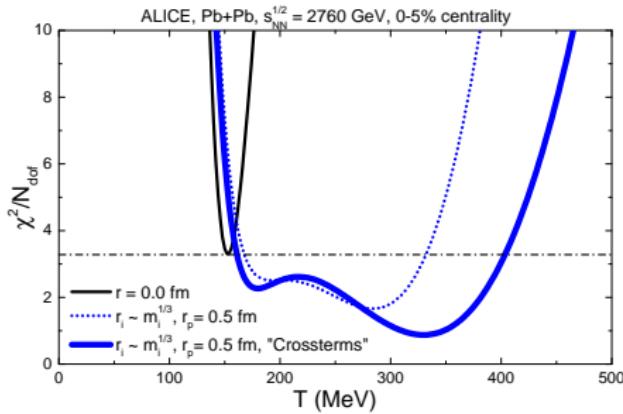
Crossterms eigenvolume model

The eigenvolume model we used is not perfectly consistent with **virial expansion** for multi-component system of hard spheres

$$P(T, \{n_i\}) = T \sum_i n_i + \sum_{ij} b_{ij} n_i n_j + \dots \quad \text{with} \quad b_{ij} = \frac{2\pi}{3} (r_i + r_j)^3$$

On the other hand, the “**Crossterms**” eigenvolume model is⁵

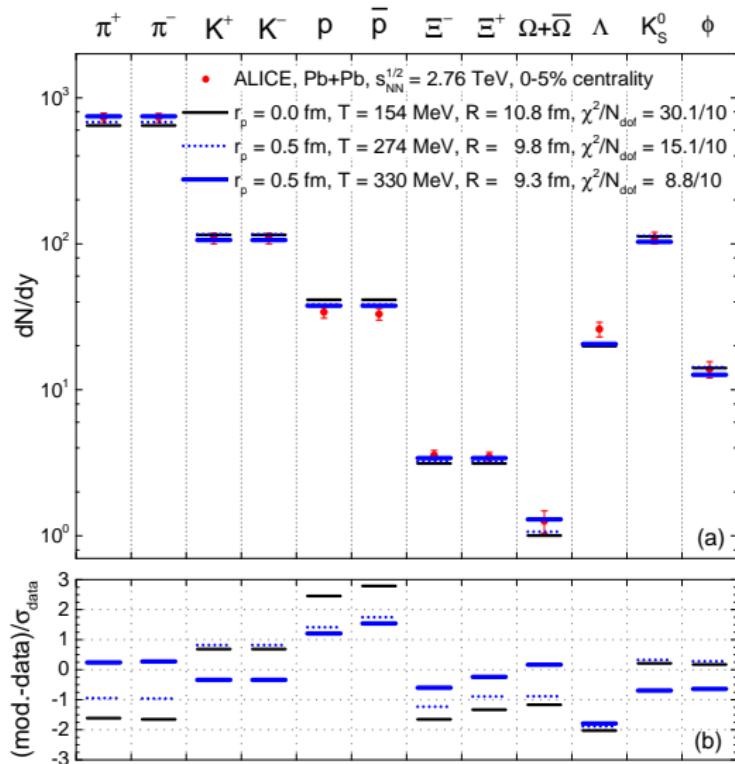
$$P(T, \{n_i\}) = T \sum_i \frac{n_i}{1 - \sum_j v_j n_j}, \quad \Rightarrow \quad P(T, \{n_i\}) = T \sum_i \frac{n_i}{1 - \sum_j \tilde{b}_{ji} n_j},$$



- Scenario 1: $r_i \sim m_i^{1/3}$
- “Crossterms” give even stronger effect
- $\chi^2/N_{\text{df}} : 30/10 \rightarrow 15/10 \rightarrow 9/10$
- $T_{\text{ch}} : 155 \rightarrow 270 \rightarrow 320$ MeV

⁵M.I. Gorenstein, A.P. Kostyuk, Ya.D. Krivenko, J. Phys. G 25, L75 (1999)

ALICE yields within bag-like eigenvolume parametrization



χ^2 profile at lower energies

So what about other experiments at lower collision energies?

Finite net-baryon density \Rightarrow additional fit parameter μ_B

Fits to NA49 Pb+Pb 4π data at $\sqrt{s_{NN}} = 6.3, 7.6, 8.8, 12.3$, and 17.3 GeV,
and STAR Au+Au dN/dy data at $\sqrt{s_{NN}} = 200$ GeV

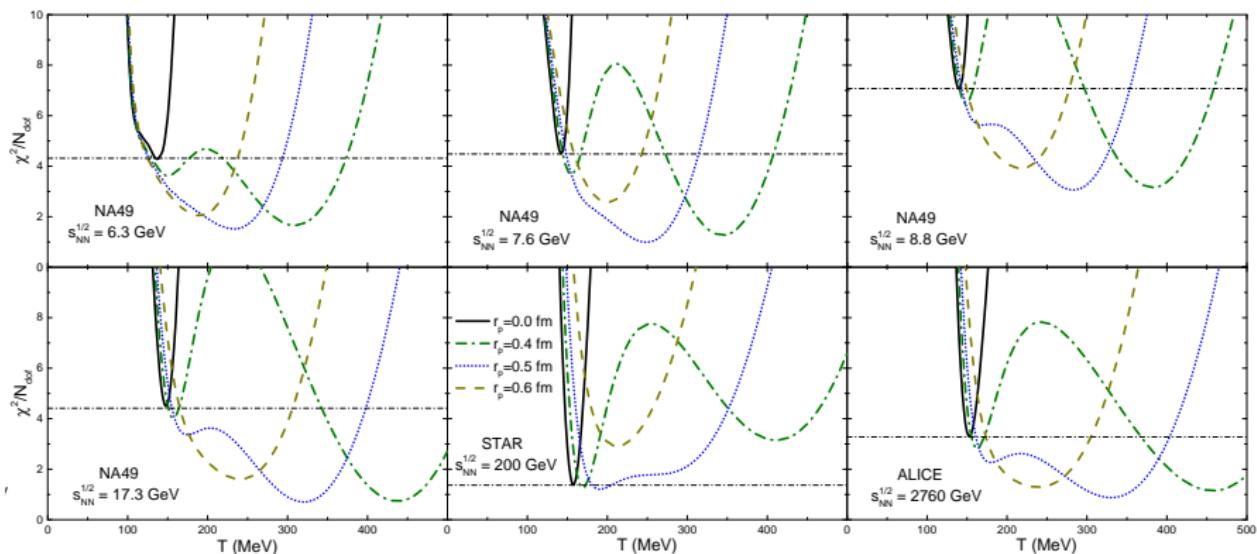
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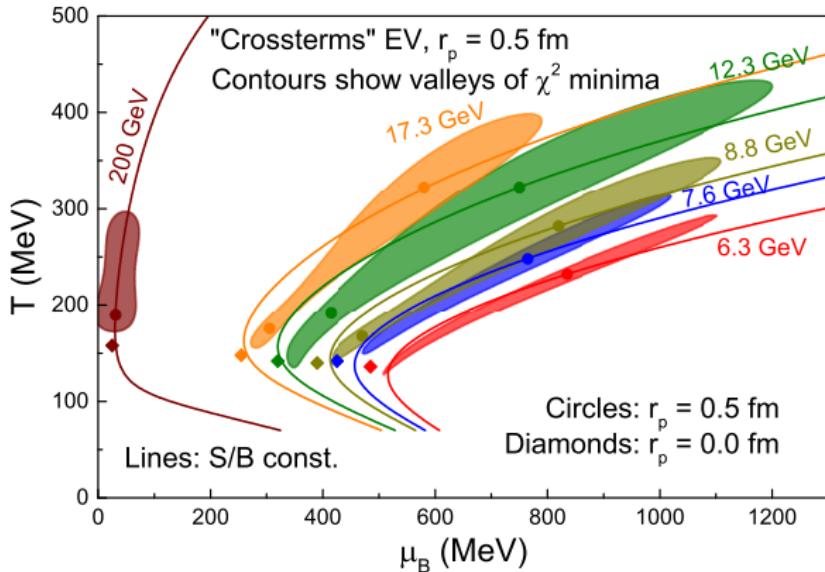
“Crossterms” model with $r_i = r_p (m_i/m_p)^{1/3}$ and $r_p = 0.4, 0.5, 0.6$ fm



All the same effect, improved χ^2 , huge sensitivity

χ^2 in T - μ_B plane

The T - μ_B dependence gives a more complete picture



- Conclusions based on point-particle HRG are not robust
- T and μ_B are clearly correlated
- Entropy per baryon S/B approx. constant along valleys of χ^2 minima
- Compatible with **isentropic** expansion and **continuous** freeze-out?

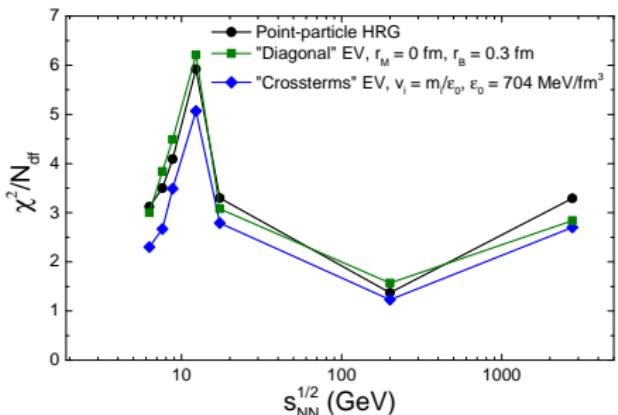
Constrained fit

High-temperature fit is problematic

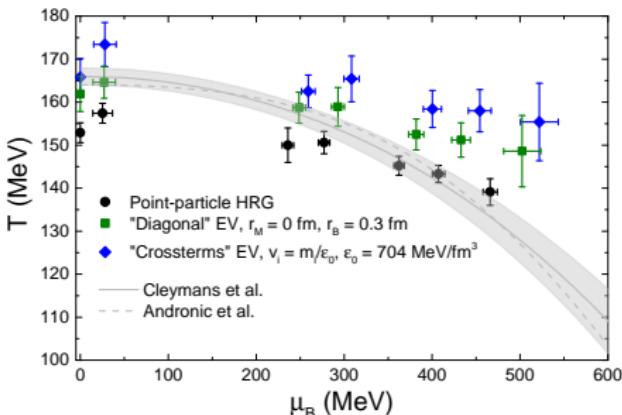
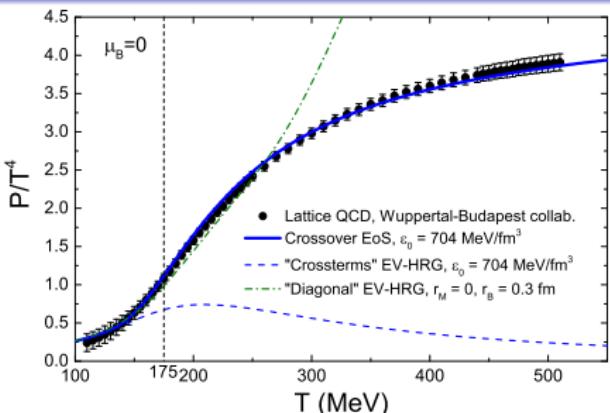
- VDW approximation breaks down at high densities
- Cannot properly be reconciled with lattice data

Constrained fit: limit $T \lesssim 180$ MeV

Scenario 1 ($v_i \sim m_i$) can be constrained to lattice data within crossover model for $r_p = 0.43$ fm⁶

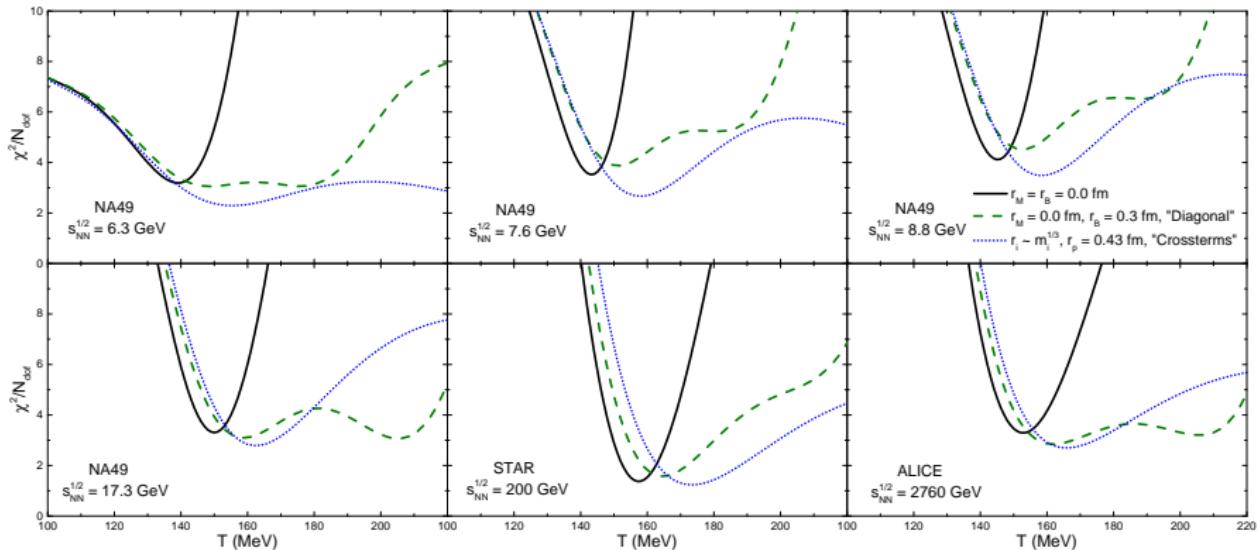


T and μ_B increase notably for Scen. 1 and 2, χ^2 improved in Scen. 1



⁶M. Albright, J. Kapusta, C. Young, Phys. Rev. C 90, 024915 (2014)

χ^2 profile in constrained fit



χ^2 still has a rather complicated non-parabolic structure
Standard statistical methods of extracting the uncertainties become
inapplicable

Summary

- ➊ Modeling of eigenvolume interactions plays crucial role in thermal analysis of heavy-ion yield data.
- ➋ Chemical freeze-out criteria based on point-particle-like HRG are not robust with regard to eigenvolume interactions. Permitting different eigenvolumes for different hadrons changes the results drastically.
- ➌ Standard eigenvolume model not consistent with virial expansion for multi-component system of hard spheres. In this case “Crossterms” model might be preferable.
- ➍ Mass-proportional eigenvolumes improve agreement with data and lead to generally wider and irregular χ^2 minima. Obtained results hint on isentropic expansion and continuous chemical freeze-out.

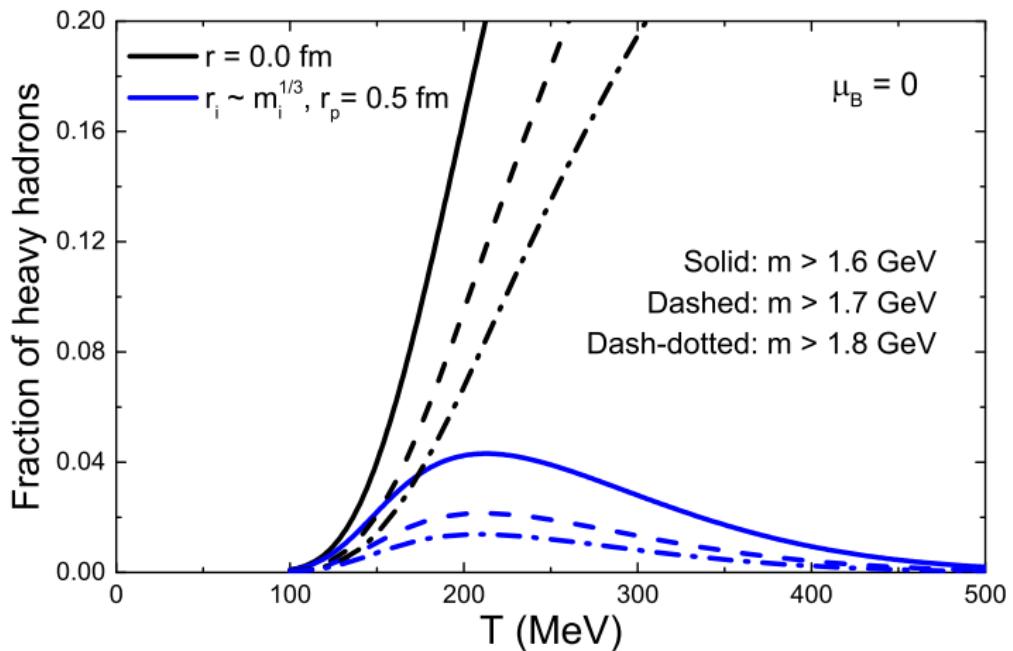
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Thanks for your attention!

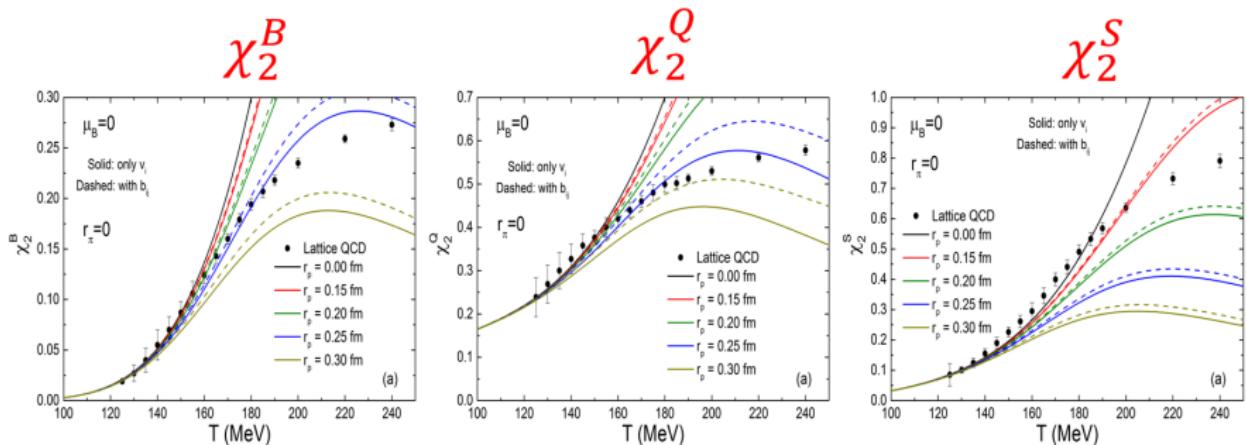
Backup slides

Heavy baryons contribution



Hagedorn divergences are tamed within eigenvolume model
Limiting temperature may be artefact of using point-particle gas

Susceptibilities in eigenvolume HRG



Strangeness susceptibility behave differently from baryon and electric charge
Hint at flavor dependence of eigenvolumes?