Sensitivity of thermal fits to heavy-ion yield data to the modeling of eigenvolume interactions

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based on arXiv:1512.08046

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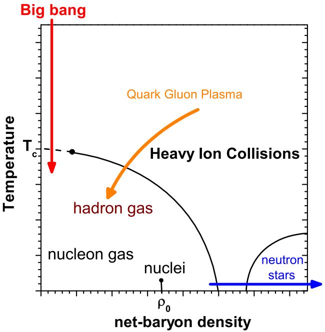
Strongly interacting matter

- Theory of strong interactions: Quantum Chromodynamics (QCD)
- Basic degrees of freedom: quarks and gluons
- At smaller energies confined into hadrons (baryons and mesons)



Scales

- Length: 1 fm $= 10^{-15}$ m $= 10^{-5}$ Å
- Energy: 100 MeV $= 10^{10}$ times room T

Where is it relevant?

- Early universe
- Neutron stars
- Heavy-ion collisions (laboratory!)

First principles of QCD are rather established,
but direct calculations are problematic

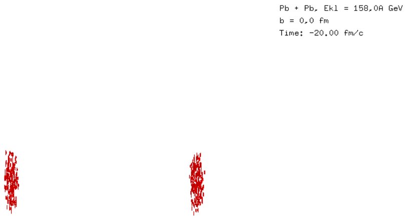
Phenomenological tools are very useful

Experiment: heavy-ion collisions

Heavy-ion collisions

High energy collision of heavy ions creates excited QCD matter

Running experiments at CERN, RHIC, future at FAIR and NICA



- Detectors measure yields and momenta of created hadrons
- Is equilibration reached in the collision?
- Can heavy-ion collisions be mapped on QCD phase diagram?

Thermal model

Thermal model - simplest model to describe hadron abundances

Key assumptions

- Matter is thermalized at chemical freeze-out
- Described as non-interacting gas of hadrons and resonances
- Characterized by few thermal parameters (T, μ_B, V)

Density of thermal hadrons

$$n_i = \frac{g_i}{(2\pi)^3} \int d^3p \left\{\exp[(\omega_p^i - \mu_i)/T] \pm 1\right\}^{-1}$$

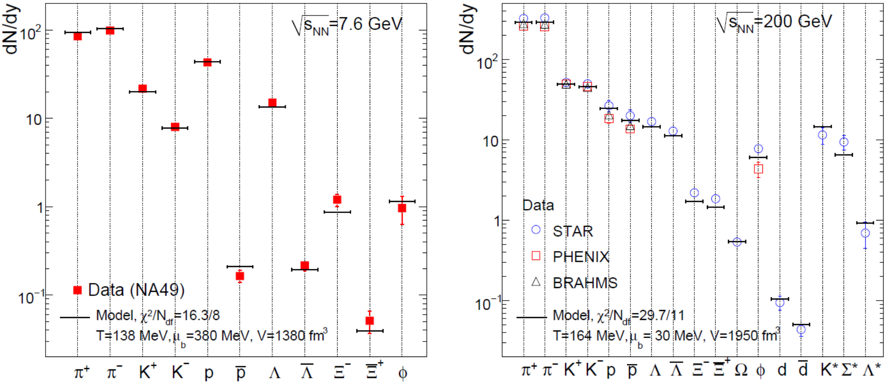
$$\mu_i = B^i \mu_B + Q^i \mu_Q + S^i \mu_S, \quad \omega_p^i = \sqrt{m_i^2 + p^2}.$$

Total hadron density – thermal + resonance decay feed-down

$$n_i^{tot} = n_i + \sum_{j \neq i} Br(j \to i) n_j.$$

Absolute yields are defined by volume V: $N_i = V n_i$.

Thermal model

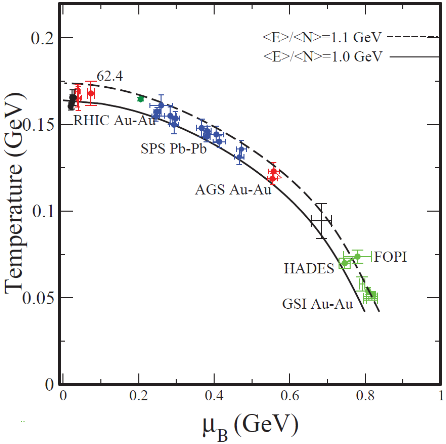


A. Andronic, P. Braun-Munzinger, J. Stachel, Phys. Lett. B 673, 142 (2009)

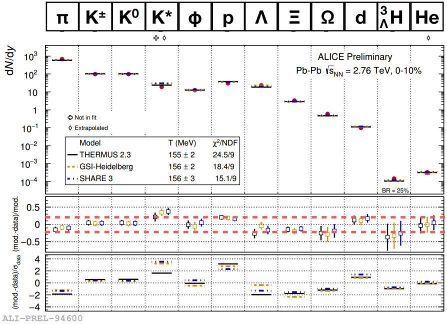
- Good qualitative description across several orders of magnitude
- Evidence for thermodynamic equilibrium

Chemical freeze-out curve

Minimizing the χ^2 of measured yields allows to extract thermal parameters at freeze-out



ALICE Pb+Pb $\sqrt{s_{NN}} = 2.76$ TeV



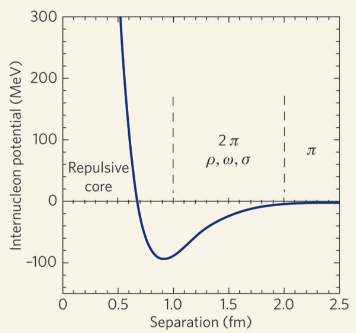
Chemical freeze-out in HIC mapped on QCD phase diagram but ...

- How robust are the conclusions based on ideal gas?
- Is there really a sharp freeze-out with well-defined temperature?

Interacting hadron gas

- In realistic hadron gas there are attractive and repulsive interactions
- Attraction already included by resonances
- Model repulsive interactions by eigenvolume correction
- Van der Waals procedure: $V \to V - vN$

$$P = \frac{NT}{V - vN}$$



In GCE: transcendental equation for pressure[1]

$$P(T,\mu) = P^{\rm id}(T, \mu - v\,P), \qquad n(T,\mu) = n^{\rm id}(T,\mu^*)/(1 + v\,n^{\rm id}(T,\mu^*))$$

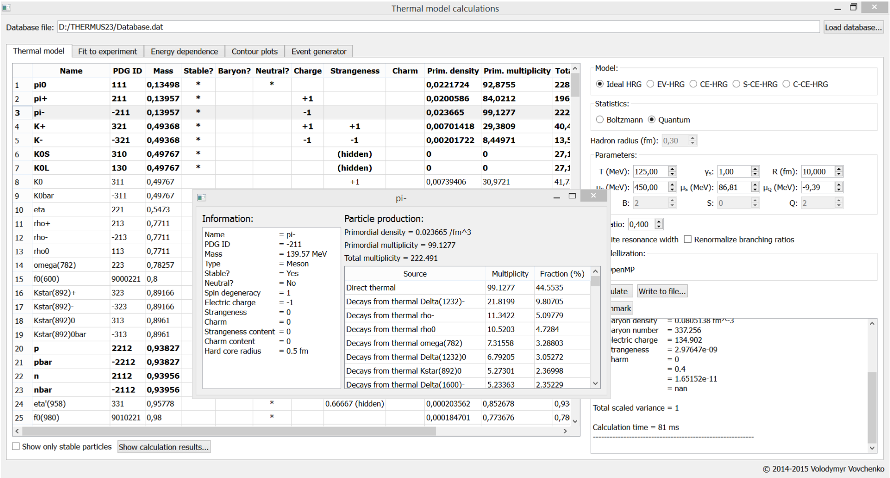
In multi-component system $V \to V - \sum_i v_i N_i$ and[2]

$$P(T,\mu) = \sum_i P_i^{\rm id}(T, \mu_i - v_i\,P), \qquad n_i(T,\mu) = n_i^{\rm id}(T,\mu_i^*)/(1 + \sum_i v_i\, n_i^{\rm id}(T,\mu_i^*))$$

[1] D.H. Rischke, M.I. Gorenstein, H. Stoecker, W. Greiner, Z.Phys. C 51, 485 (1991)
[2] G.D. Yen, M.I. Gorenstein, W. Greiner, S.N. Yan, Phys. Rev. C 56, 2210 (1997)

Some details about implementation

- Own implementation of eigenvolume HRG written in C++ is used
- Solves eigenvolume models, also many other features
- Auxiliary tool: GUI written within Qt framework
- Where possible cross-checked with THERMUS package



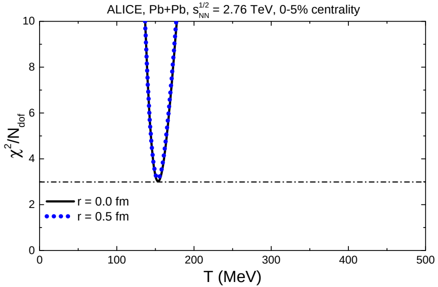
Eigenvolumes: Scenario 0

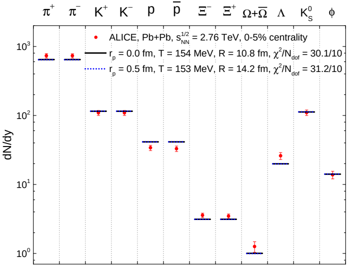
How to choose eigenvolumes for different hadrons?
Not many constraints $\Rightarrow$ consider different scenarios

Scenario 0: Constant eigenvolume for all hadrons ($v_i \equiv v$)

In this case in Boltzmann approximation

$$n_i(T,\mu) = \frac{n_i^{\rm id}(T,\mu_i)\, e^{-vP/T}}{1 + \sum_i v\, n_i^{\rm id}(T,\mu_i)\, e^{-vP/T}} \qquad \text{and} \qquad \frac{n_i(T,\mu)}{n_j(T,\mu)} = \frac{n_i^{\rm id}(T,\mu)}{n_j^{\rm id}(T,\mu)}$$



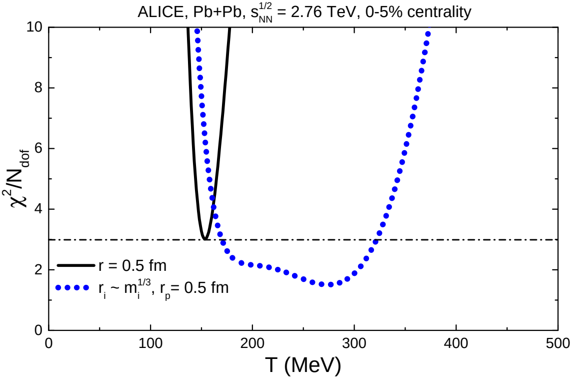


- Simplest and most commonly used parametrization
- Eigenvolume effects essentially cancel out in yield ratios
- No change in T or μ_B compared to point-particle case

Eigenvolumes: Scenario 1

Scenario 1: Mass-proportional eigenvolumes ($v_i = m_i/\varepsilon_0$ or $r_i \sim m_i^{1/3}$)[3]

- Bag model inspired
- Obtained originally for heavy Hagedorn states
- Results in stronger suppression of heavier hadrons



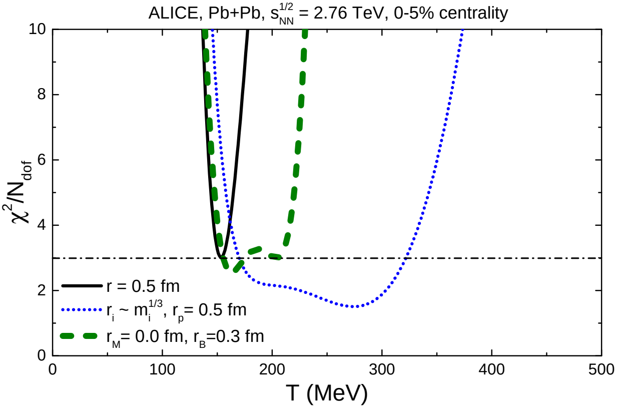
Drastic changes in ALICE χ^2 profile, also high sensitivity on ε_0
For $r_p = 0.5$ fm global minimum at $T \simeq 270$ MeV

[3] Hagedorn, Rafelski, Phys. Lett. B (1980); Kapusta, Olive, Nucl. Phys. A (1983)

Eigenvolumes: Scenario 2

Scenario 2: Two-component model: different volumes for mesons and baryons

We consider particular case $r_M = 0$ and $r_B = 0.3$ fm, has been compared to lattice successfully[4]



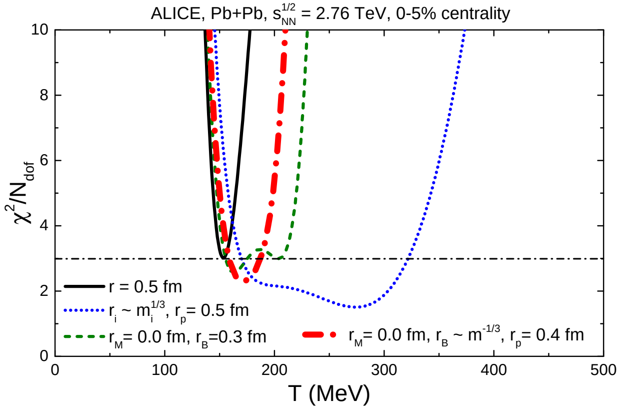
Wide irregular minimum in $T = 155 - 210$ MeV range

[4] A. Andronic, P. Braun-Munzinger, J. Stachel, M. Winn, Phys. Lett. B 718, 80 (2012).

Eigenvolumes: Scenario 3

Scenario 3: Point-like mesons and reverse bag model for baryons $v_B \sim 1/m$

Strange baryons have generally smaller volumes than non-strange ones



Result: $T_{ch} = 175 \pm 20$ MeV

Many other options possible...

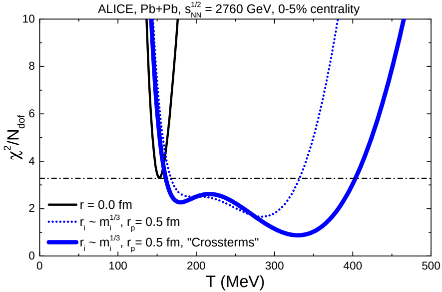
Crossterms eigenvolume model

The eigenvolume model we used is not perfectly consistent with virial expansion for multi-component system of hard spheres

$$P(T,\{n_i\}) = T\sum_i n_i + \sum_{ij} b_{ij} n_i n_j + \ldots \quad \text{with} \quad b_{ij} = \frac{2\pi}{3}(r_i + r_j)^3$$

On the other hand, the "Crossterms" eigenvolume model is[5]

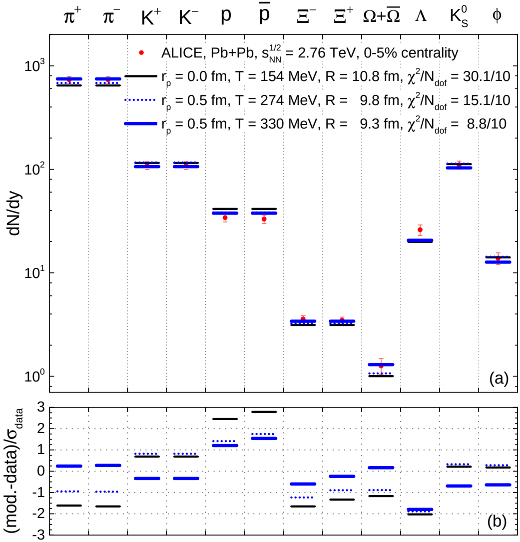
$$P(T,\{n_i\}) = T\sum_i \frac{n_i}{1-\sum_j v_j n_j}, \qquad \Rightarrow \qquad P(T,\{n_i\}) = T\sum_i \frac{n_i}{1-\sum_j \tilde{b}_{ji} n_j},$$



- Scenario 1: $r_i \sim m_i^{1/3}$
- "Crossterms" give even stronger effect
- $\chi^2/N_{\rm df}$: $30/10 \to 15/10 \to 9/10$
- $T_{\rm ch}$: $155 \to 270 \to 320$ MeV

[5] M.I. Gorenstein, A.P. Kostyuk, Ya.D. Krivenko, J. Phys. G 25, L75 (1999)

ALICE yields within bag-like eigenvolume parametrization



χ^2 profile at lower energies

So what about other experiments at lower collision energies?

Finite net-baryon density $\Rightarrow$ additional fit parameter μ_B

Fits to NA49 Pb+Pb 4π data at $\sqrt{s_{\rm NN}} = 6.3, 7.6, 8.8, 12.3$, and 17.3 GeV, and STAR Au+Au dN/dy data at $\sqrt{s_{\rm NN}} = 200$ GeV

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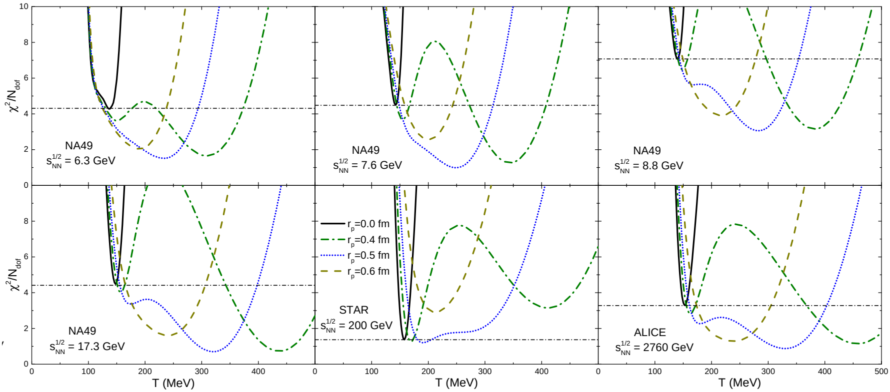
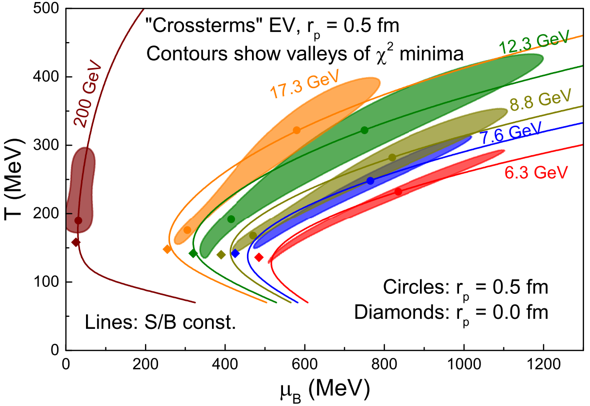
"Crossterms" model with $r_i = r_p\,(m_i/m_p)^{1/3}$ and $r_p = 0.4,\ 0.5,\ 0.6$ fm



All the same effect, improved χ^2, huge sensitivity

χ^2 in T-μ_B plane

The T-μ_B dependence gives a more complete picture



- Conclusions based on point-particle HRG are not robust
- T and μ_B are clearly correlated
- Entropy per baryon S/B approx. constant along valleys of χ^2 minima
- Compatible with isentropic expansion and continuous freeze-out?

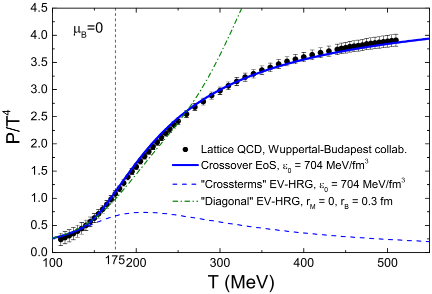
Constrained fit

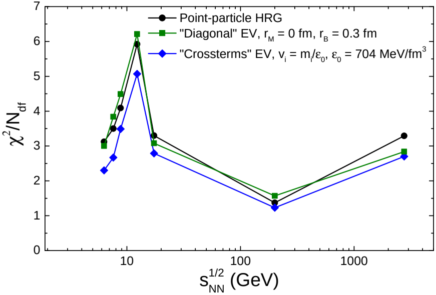
High-temperature fit is problematic

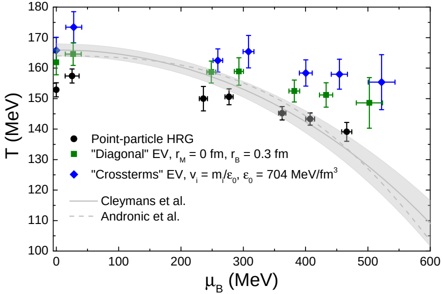
- VDW approximation breaks down at high densities
- Cannot properly be reconciled with lattice data

Constrained fit: limit $T \lesssim 180$ MeV

Scenario 1 ($v_i \sim m_i$) can be constrained to lattice data within crossover model for $r_p = 0.43$ fm[6]



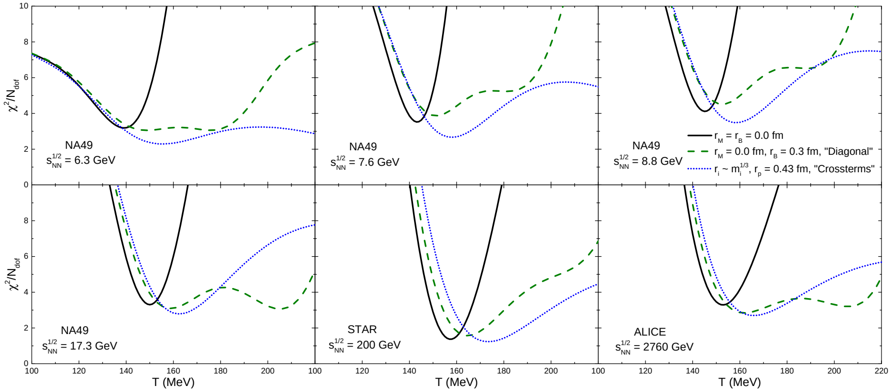




T and μ_B increase notably for Scen. 1 and 2, χ^2 improved in Scen. 1

[6] M. Albright, J. Kapusta, C. Young, Phys. Rev. C 90, 024915 (2014)

χ^2 profile in constrained fit



χ^2 still has a rather complicated non-parabolic structure
Standard statistical methods of extracting the uncertainties become inapplicable

Summary

1. Modeling of eigenvolume interactions plays crucial role in thermal analysis of heavy-ion yield data.
2. Chemical freeze-out criteria based on point-particle-like HRG are not robust with regard to eigenvolume interactions. Permitting different eigenvolumes for different hadrons changes the results drastically.
3. Standard eigenvolume model not consistent with virial expansion for multi-component system of hard spheres. In this case “Crossterms” model might be preferrable.
4. Mass-proportional eigenvolumes improve agreement with data and lead to generally wider and irregular χ^2 minima. Obtained results hint on isentropic expansion and continuous chemical freeze-out.

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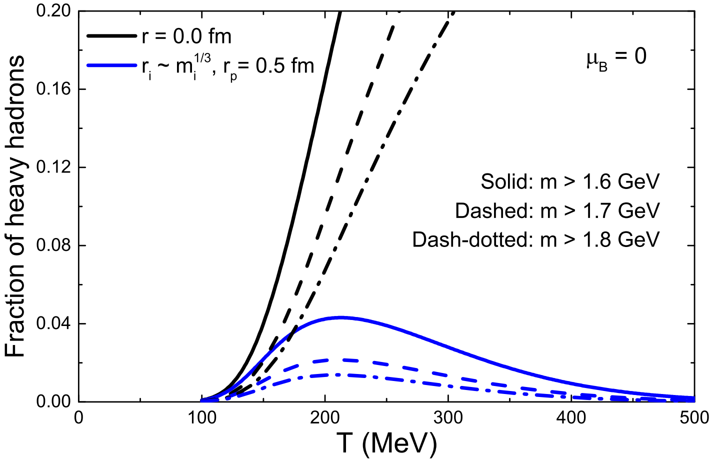
Thanks for your attention!

Backup slides

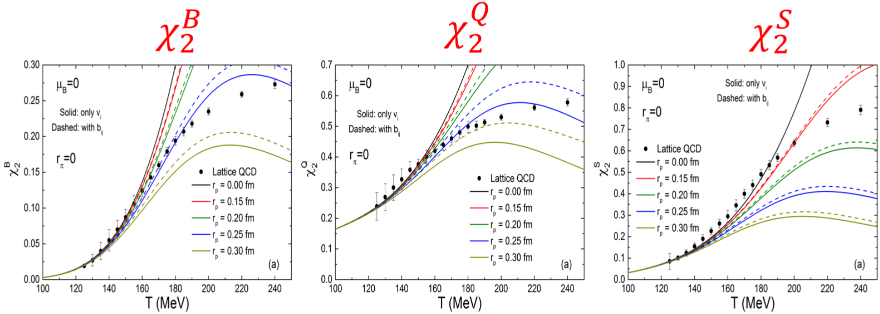
Heavy baryons contribution



Hagedorn divergences are tamed within eigenvolume model

Limiting temperature may be artefact of using point-particle gas

Susceptibilities in eigenvolume HRG



Strangeness susceptibility behave differently from baryon and electric charge

Hint at flavor dependence of eigenvolumes?