van der Waals Interactions in Hadron Resonance Gas:

From Nuclear Matter to Lattice QCD

Volodymyr Vovchenko

Based on:

- V.V, Anchishkin, Gorenstein, Phys. Rev. C 91, 064314 (2015)
- V.V, Anchishkin, Gorenstein, Poberezhnyuk, Phys. Rev. C 92, 054901 (2015)
- V.V, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

and ongoing work

NA61-Theory Seminar

June 1, 2017

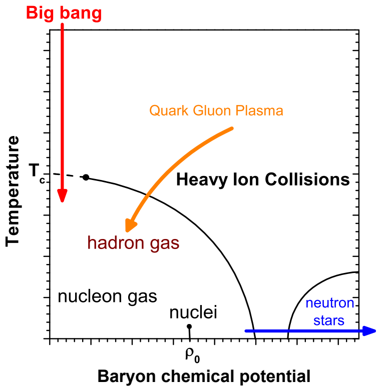
FIAS Frankfurt Institute for Advanced Studies
GOETHE UNIVERSITÄT FRANKFURT AM MAIN
HGS-HIRe for FAIR Helmholtz Graduate School for Hadron and Ion Research

Outline

- Motivation
- van der Waals equation
- Nuclear matter as van der Waals system
- van der Waals in hadron resonance gas and lattice QCD
- Extensions
- Summary

Strongly interacting matter

- Theory of strong interactions: Quantum Chromodynamics (QCD)
- Basic degrees of freedom: quarks and gluons
- At smaller energies confined into hadrons: baryons (qqq) and mesons ($q\bar{q}$)



Where is it relevant?

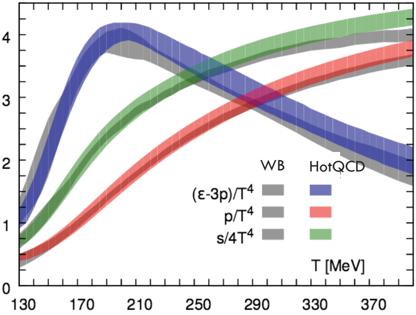
- Early universe
- Neutron stars
- Heavy-ion collisions

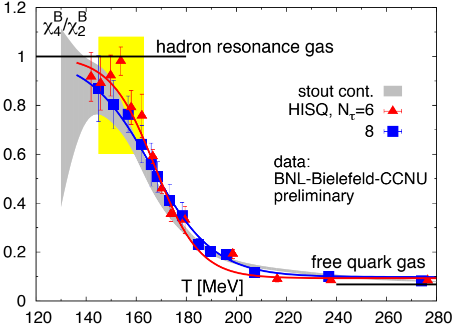
First principles of QCD are rather established,
but direct calculations are problematic

Phenomenological tools are very useful

QCD equation of state at $\mu = 0$

Lattice simulations provide equation of state at $\mu_B = 0$[1]





Common model for confined phase is ideal HRG: non-interacting gas of known hadrons and resonances

- Good description of thermodynamic functions up to 180 MeV
- Rapid breakdown in crossover region for description of susceptibilities[2]
- Often interpreted as clear signal of deconfinement...
- But what is the role of hadronic interactions beyond those in ideal HRG?

[1]Bazavov et al., PRD 90, 094503 (2014); Borsanyi et al., PLB 730, 99 (2014)
[2]Ding, Karsch, Mukherjee, IJMPE 24, 1530007 (2015)

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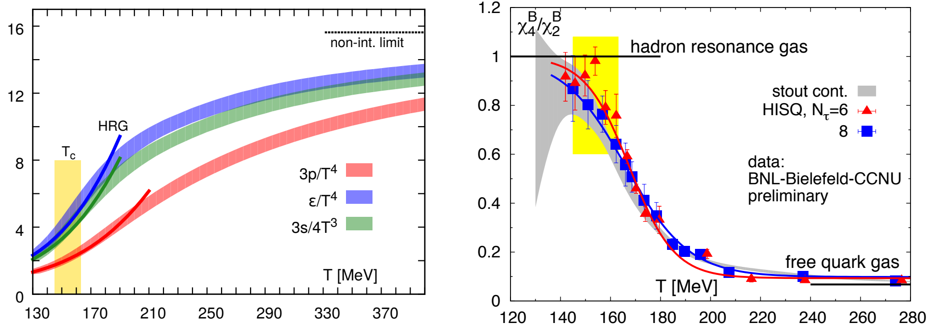

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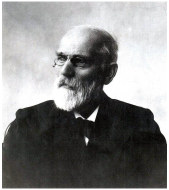
van der Waals (VDW) equation

$$P(T,V,N) \;=\; \frac{NT}{V-bN} \;-\; a\frac{N^2}{V^2}$$



Formulated in 1873.

Simplest model which contains attractive and repulsive interactions

Contains 1st order phase transition and critical point

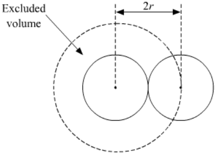
Can elucidate role of fluctuations in phase transitions

Nobel Prize in 1910.

Two ingredients:

1) Short-range repulsion: excluded volume (EV) procedure,

$$V \to V - bN, \qquad b = 4\frac{4\pi r^3}{3}$$



2) Intermediate range attraction in mean-field approximation,
$P \to P - a\,n^2$

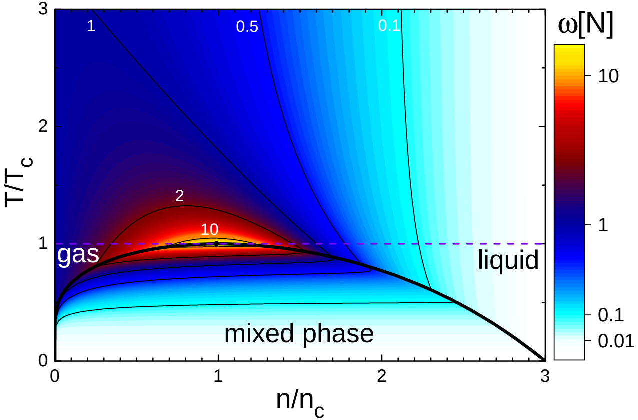
Motivation:

- Toy model to study fluctuations near critical point
- Include essential features of nuclear matter physics

Scaled variance for classical VDW equation

Particle number fluctuations in classical VDW gas within GCE

$$\omega[N] = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} = \frac{\chi_2}{\chi_1} = \left[\frac{1}{(1-bn)^2} - \frac{2an}{T} \right]^{-1}$$



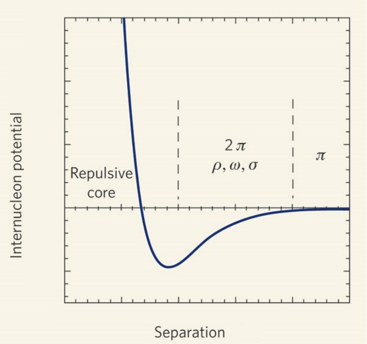
- Repulsive interactions suppress N-fluctuations
- Attractive interactions enhance N-fluctuations

V.V., Anchishkin, Gorenstein, J. Phys. A 48, 305001 (2015)

Nucleon-nucleon interaction

Nucleon-nucleon potential:

- Repulsive core at small distances
- Attraction at intermediate distances
- Suggestive similarity to VDW interactions
- Could nuclear matter be described by VDW equation?



Standard VDW equation is for Boltzmann statistics

Nucleons are fermions, obey Pauli exclusion principle

Unlike for classical fluids, quantum statistics is important

Quantum statistical van der Waals fluid

Free energy of classical VDW fluid:

$$F(T, V, N) = F^{\rm id}(T, V - bN, N) - a\,\frac{N^2}{V}$$

Ansatz: $F^{\rm id}(T, V - bN, N)$ is free energy of ideal *quantum* gas

$$\text{Pressure:} \qquad p = -\left(\frac{\partial F}{\partial V}\right)_{T,N} = p^{\rm id}(T, \mu^*) - a\,n^2$$

$$\text{Particle density:} \qquad n = \left(\frac{\partial p}{\partial \mu}\right)_{T} = \frac{n^{\rm id}(T, \mu^*)}{1 + b\,n^{\rm id}(T, \mu^*)}$$

$$\text{Shifted chemical potential:} \qquad \mu^* = \mu - b\,p - a\,b\,n^2 + 2\,a\,n$$

Model properties:

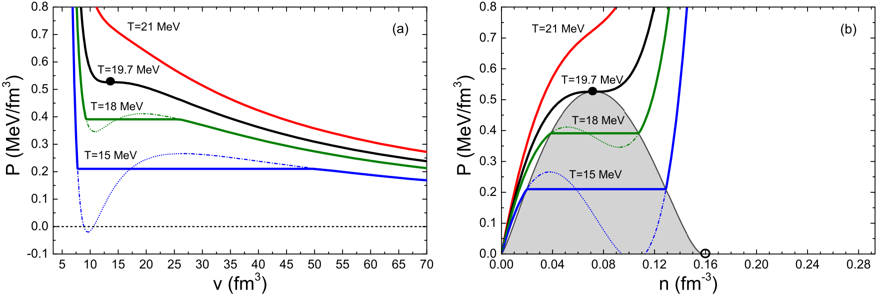
- Reduces to classical VDW equation when quantum statistics are negligible
- Reduces to ideal quantum gas for $a = 0$ and $b = 0$
- Entropy density non-negative and $s \to 0$ with $T \to 0$

V.V., Anchishkin, Gorenstein, JPA '15 and PRC '15; Redlich, Zalewski, APPB '16.
a=0 $\Rightarrow$ excluded-volume model, D. Rischke et al., ZPC '91

VDW gas of nucleons: pressure isotherms

a and b fixed to reproduce saturation density and binding energy:

$$n_0 = 0.16 \text{ fm}^{-3},\ E/A = -16 \text{ MeV} \Rightarrow a \cong 329 \text{ MeV fm}^3 \text{ and } b \cong 3.42 \text{ fm}^3$$



Behavior qualitatively same as for Boltzmann case

Mixed phase results from Maxwell construction

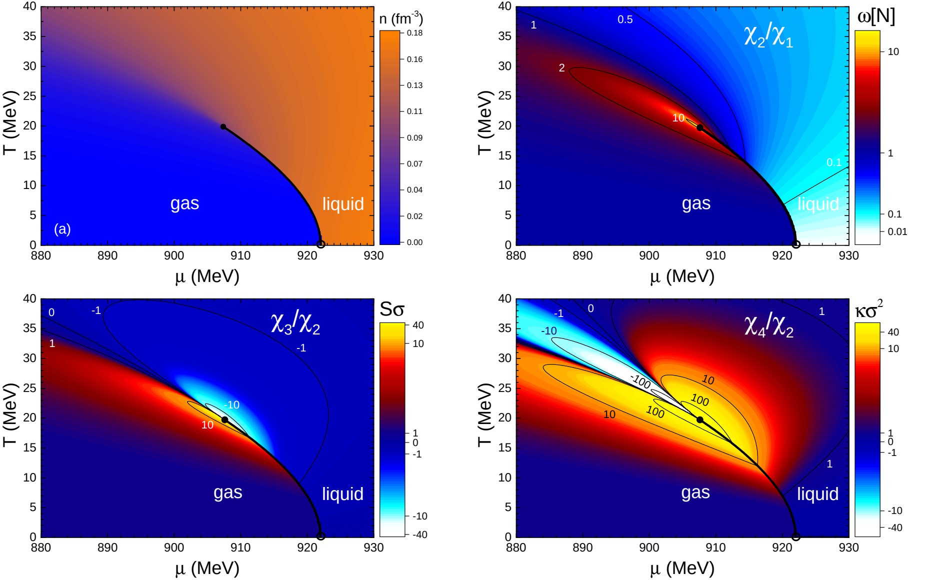
Critical point at $T_c \cong 19.7$ MeV and $n_c \cong 0.07$ fm^{-3}

Experimental estimate[1]: $T_c = 17.9 \pm 0.4$ MeV, $n_c = 0.06 \pm 0.01$ fm^{-3}

[1] J.B. Elliot, P.T. Lake, L.G. Moretto, L. Phair, Phys. Rev. C 87, 054622 (2013)

VDW gas of nucleons: (T, μ) plane

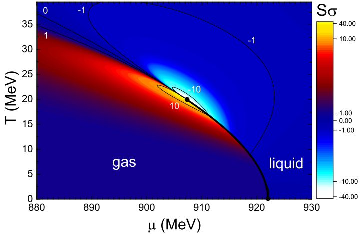
(T, μ) plane: structure of critical fluctuations $\chi_i = \partial^i (p/T^4)/\partial(\mu/T)^i$



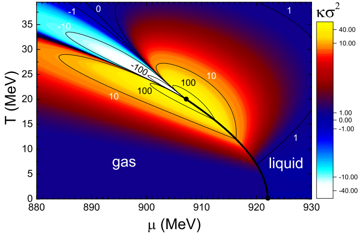
V.V., D. Anchishkin, M. Gorenstein, R. Poberezhnyuk, PRC 91, 064314 (2015)

VDW gas of nucleons: skewness and kurtosis

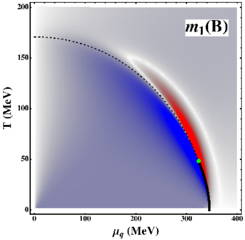
VDW Skewness

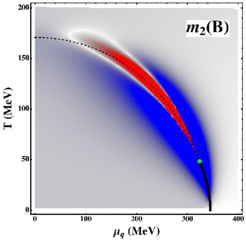


VDW Kurtosis

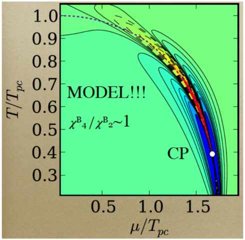


NJL, J.W. Chen et al., PRD 93, 034037 (2016)





PQM, V. Skokov, QM2012



Fluctuation patterns in VDW very similar to effective QCD models

Strongly intensive measures near CP

Strongly intensive (SI) measures: Gorenstein, Gazdzicki, PRC 84, 014904 (2011)

- Independent of volume fluctuations, mitigate impact parameter fluctuations
- Can be constructed from moments of two extensive quantities

$$\Delta[A,B] = C_{\Delta}^{-1}\left[\langle A\rangle\omega[B] - \langle B\rangle\omega[A]\right]$$
$$\Sigma[A,B] = C_{\Sigma}^{-1}\left[\langle A\rangle\omega[B] + \langle B\rangle\omega[A] - 2(\langle AB\rangle - \langle A\rangle\langle B\rangle)\right]$$

- For most models without PT and CP equal/close to unity
- Supposedly show critical behavior, but no model calculation
- Used in search for CP, e.g. NA61/SHINE program[1]

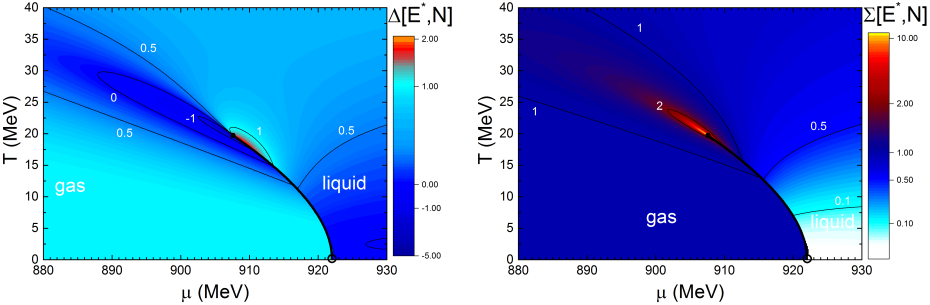
SI measures of excitation energy and particle number fluctuations in cl. VDW

$$\Delta[E^*,N] \;=\; 1 \;-\; \frac{an(2\overline{\epsilon}_{\rm id} - 3an)}{\overline{\epsilon^2_{\rm id}} - \overline{\epsilon}^2_{\rm id}}\,\omega[N], \quad \Sigma[E^*,N] \;=\; 1 + \frac{a^2 n^2}{\overline{\epsilon^2_{\rm id}} - \overline{\epsilon}^2_{\rm id}}\,\omega[N].$$

- Critical behavior is present due to criticality of $\omega[N]$ term[2]
- If a=0 then no signal at all! Deviations really stem from criticality.

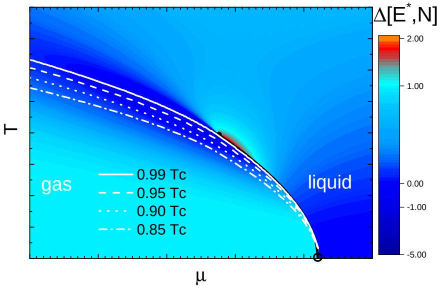
[1] Gazdzicki, Seyboth, APP '15; E. Andronov, 1610.05569; A. Seryakov, 1704.00751
[2] V.V., Poberezhnyuk, Anchishkin, Gorenstein, J. Phys. A 49, 015003 (2016)

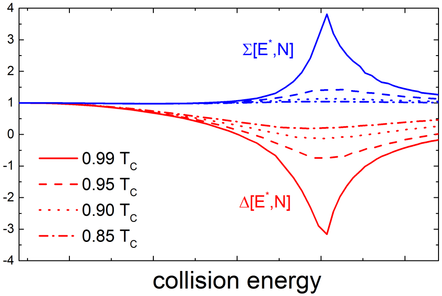
Strongly intensive measures in T-μ plane: Nuclear matter



- Both $\Delta[E^*, N]$ and $\Sigma[E^*, N]$ signal nuclear liquid-gas criticality
- $\Sigma[E^*, N] > 0$ always. However, $\Delta[E^*, N]$ can be both positive and negative

Strongly intensive measures in T-μ plane: Nuclear matter



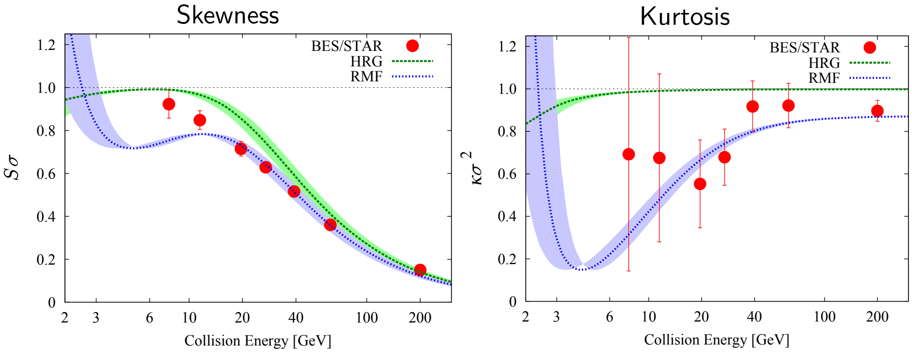


- Both $\Delta[E^*, N]$ and $\Sigma[E^*, N]$ signal nuclear liquid-gas criticality
- $\Sigma[E^*, N] > 0$ always. However, $\Delta[E^*, N]$ can be both positive and negative
- Non-monotonous energy/system-size dependence of $\Delta[E^*, N]$ and $\Sigma[E^*, N]$ in a scenario with CP
- $\Delta[E^*, N]$ is more sensitive than $\Sigma[E^*, N]$ to proximity of CP

Net-baryon fluctuations and nuclear matter

Are NN interactions relevant for heavy-ion collisions?

Net-nucleon fluctuations within RMF (σ-ω model) of nuclear matter along line of "chemical freeze-out"



A notable effect in fluctuations even at $\mu_B \simeq 0$

Reconciliation of HRG with nuclear matter can be interesting

K. Fukushima, PRC 91, 044910 (2015)

van der Waals interactions in hadron resonance gas

Let us now include nuclear matter physics into HRG...

VDW-HRG model

- Identical VDW interactions between all baryons
- The baryon-antibaryon, meson-meson, and meson-baryon VDW interactions are neglected
- Baryon VDW parameters extracted from ground state of nuclear matter ($a = 329$ MeV fm^3, $b = 3.42$ fm^3)

Three independent subsystems: mesons + baryons + antibaryons

$$p(T, \boldsymbol{\mu}) = P_M(T, \boldsymbol{\mu}) + P_B(T, \boldsymbol{\mu}) + P_{\bar{B}}(T, \boldsymbol{\mu}),$$

$$P_M(T, \boldsymbol{\mu}) = \sum_{j \in M} p_j^{\rm id}(T, \mu_j) \quad \text{and} \quad P_B(T, \boldsymbol{\mu}) = \sum_{j \in B} p_j^{\rm id}(T, \mu_j^{B*}) - a\, n_B^2$$

$$n_B(T, \boldsymbol{\mu}) = (1 - b\, n_B) \sum_{j \in B} n_j^{\rm id}(T, \mu_j^{B*}).$$

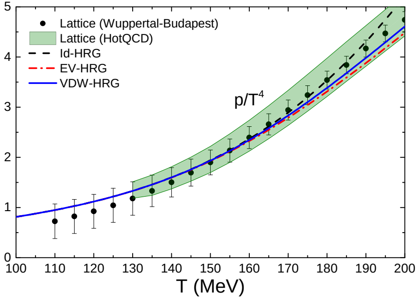
In this simplest setup model is essentially "parameter-free"

V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

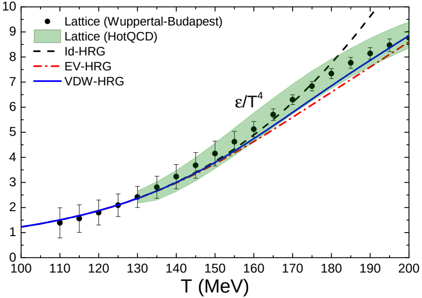
VDW-HRG at $\mu_B = 0$: thermodynamic functions

Comparison of VDW-HRG with lattice QCD at $\mu_B = 0$

Pressure p/T^4



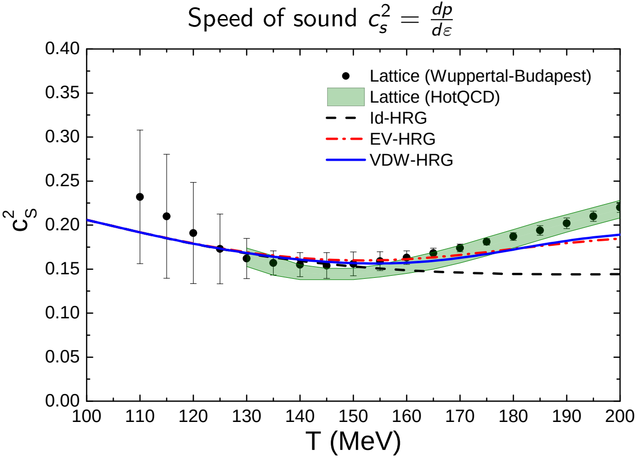
Energy density ε/T^4



- VDW-HRG does not spoil existing agreement of Id-HRG with LQCD despite significant excluded-volume interactions between baryons
- Not surprising: matter meson-dominated at $\mu_B = 0$

V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

VDW-HRG at $\mu_B = 0$: speed of sound



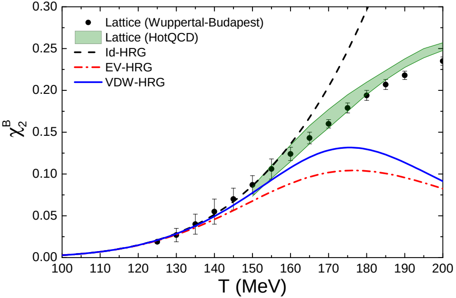
- Monotonic decrease in Id-HRG, at odds with lattice
- Minimum for EV-HRG/VDW-HRG at 150-160 MeV
- No acausal behavior, often an issue in models with eigenvolumes

V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

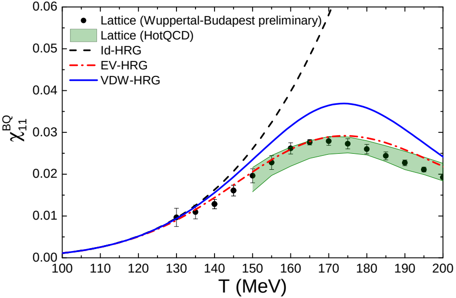
VDW-HRG at $\mu_B = 0$: baryon number fluctuations

Susceptibilities: $$\chi^{BSQ}_{lmn} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \, \partial(\mu_S/T)^m \, \partial(\mu_Q/T)^n}$$

Net-baryon χ_2^B



Baryon-charge correlator χ_{11}^{BQ}



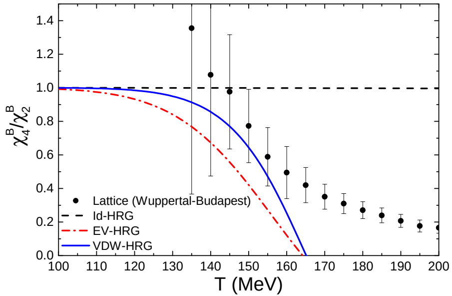
- Very different qualitative behavior between Id-HRG and VDW-HRG
- For χ_2^B lattice data is between Id-HRG and VDW-HRG at high T
- For χ_{11}^{BQ} lattice data is below all models, closer to EV-HRG

V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

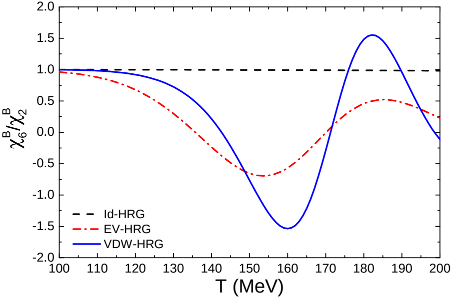
VDW-HRG at $\mu_B = 0$: baryon number fluctuations

Higher-order of fluctuations are expected to be even more sensitive

χ_4^B/χ_2^B



χ_6^B/χ_2^B



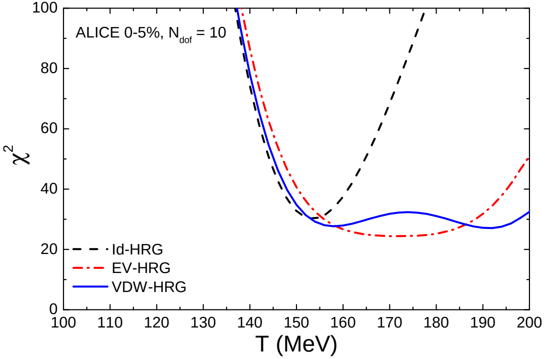
- χ_4^B deviates from χ_2^B at high enough T, they stay equal in Id-HRG
- Cannot be related only to onset of deconfinement
- VDW-HRG predicts strong non-monotonic behavior for χ_6^B/χ_2^B

V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

VDW-HRG: influence on hadron ratios

VDW interactions change relative hadron yields in HRG

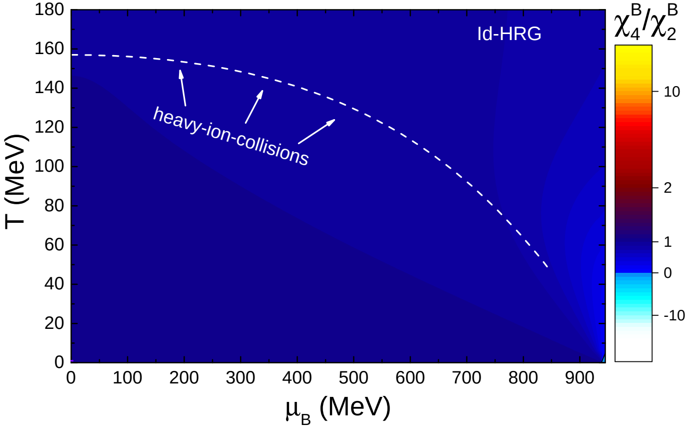
Thermal model fit to ALICE (Pb+Pb @ 2.76 TeV) yields: from π to Ω



- Fit quality slightly better in EV-HRG/VDW-HRG vs Id-HRG but very different picture!
- All temperatures between 150 and 200 MeV yield similarly fair data description in VDW-HRG
- Results likely to be sensitive to further modifications, e.g for strangeness

VDW-HRG at finite μ_B

Net-baryon fluctuations in T-μ_B plane: χ_4^B/χ_2^B



- Almost no effect in Id-HRG, only Fermi statistics...

V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

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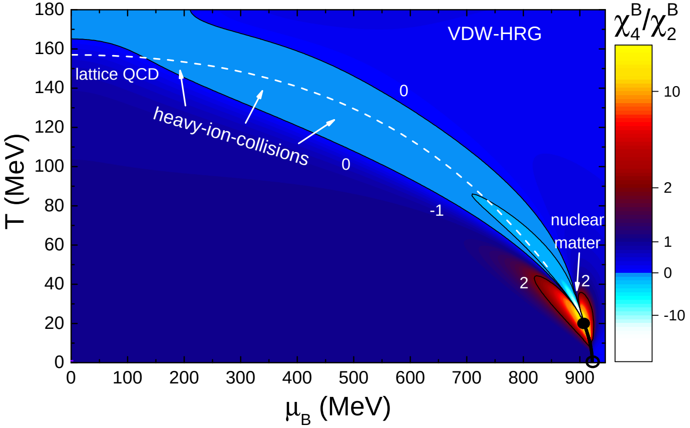


- Almost no effect in Id-HRG, only Fermi statistics...
- Rather rich structure for VDW-HRG, huge effect of VDW interactions!
- Fluctuations seen at RHIC are remnants of nuclear liquid-gas PT?

V.V., M. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

Beyond van der Waals

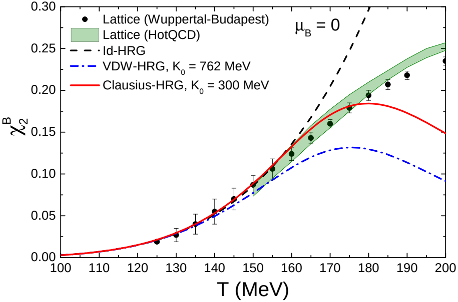
From van der Waals equation to Clausius equation:

$$p = \frac{nT}{1-bn} - a\,n^2 \qquad \Rightarrow \qquad p = \frac{nT}{1-bn} - \frac{a\,n^2}{1+cn}$$

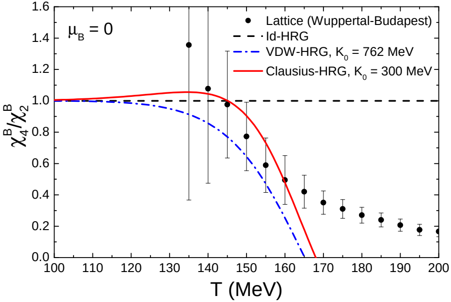
Nuclear incompressibility K_0: from 762 MeV in VDW to 300 MeV in Clausius

Clausius-HRG: baryon-baryon interactions in HRG with Clausius equation

net baryon χ_2^B



net baryon χ_4^B/χ_2^B



- Clausius-HRG yields improved K_0 and improved description of LQCD
- Behavior of LQCD observables correlates with nuclear matter properties

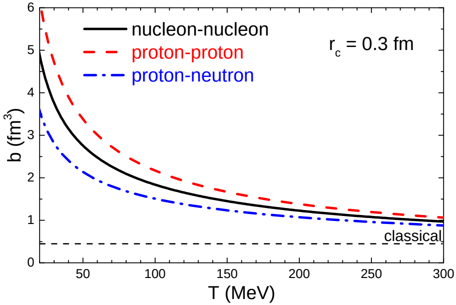
V.V., arXiv:1701.06524

Hard-core repulsion: classical vs Beth-Uhlenbeck

QM approach to NN hard-core repulsion: Beth-Uhlenbeck (BU) formula

$$p(T,\mu) \simeq T\,\phi(T)\,\lambda + T\,B_2(T)\,\lambda^2, \qquad \lambda = e^{\mu/T}, \quad \phi(T) = n^{\rm id}(T,\mu=0)$$

$$B_2(T) = \frac{T}{2\pi^3}\int_{2m_N}^{\infty} d\varepsilon\,\varepsilon^2\,K_2(\varepsilon/T)\sum_{J,T}(2J+1)\,(2T+1)\,\frac{\partial\delta_{J,T}(\varepsilon)}{\partial\varepsilon}$$



$$\delta_{J,T}(\varepsilon) = \arctan\left\{\frac{j_L[2r_c q(\varepsilon)]}{y_L[2r_c q(\varepsilon)]}\right\}$$

Classical eigenvolume:

$$p = p^{\rm id}(T,\mu - bp), \quad b = \frac{16\pi r_c^3}{3}$$

Beth-Uhlenbeck eigenvolume:

$$b = b(T) = -B_2(T)/[\phi(T)]^2$$

NN-scattering data[1]: $r_c \simeq 0.25$-0.3 fm

- EV of nucleon-nucleon interaction is strongly T-dependent
- Classical approach underestimates EV by factor 2-3 at $T \sim 100 - 200$ MeV

[1] R. B. Wiringa et al., Phys. Rev. C **51**, 38 (1995)

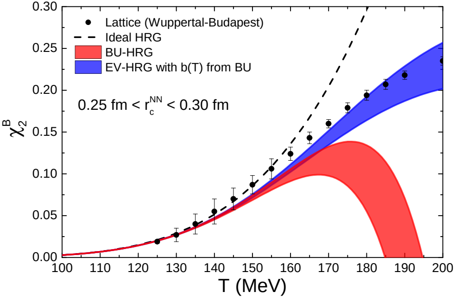
Hard-core repulsion: classical vs Beth-Uhlenbeck

Now use this T-dependent eigenvolume $b(T)$ to model BB repulsion in HRG

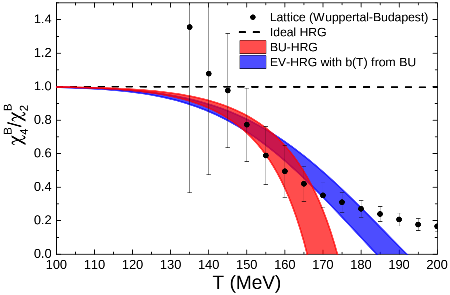
$$\text{BU-HRG:} \quad P_B(T,\mu) = \sum_{i \in B} p_i^{\rm id}(T,\mu_B) - T\, b(T) \sum_{i,j} \phi_i(T)\, \phi_j(T)\, \lambda_B^2$$

$$\text{EV-HRG:} \quad P_B(T,\mu) = \sum_{i \in B} p_i^{\rm id}[T,\mu_B - b(T)\, P_B]$$

net baryon χ_2^B



net baryon χ_4^B/χ_2^B



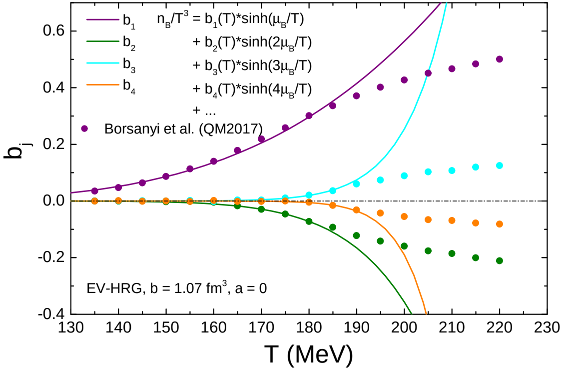
- Pure BU approach breaks down at $T \sim 160 - 170$ MeV, higher orders matter!
- EV-HRG with BU-motivated $b \sim 1 - 1.5$ fm^3 describes LQCD fairly well

Repulsive baryonic interactions and imaginary μ_B

Lattice QCD is problematic at real μ but tractable at imaginary μ

E.g., net-baryon density is imaginary and has trigonometric series form

$$\mu_B \to i\tilde{\mu}_B \quad \Rightarrow \quad \frac{n_B(T, i\tilde{\mu}_B)}{T^3} = -i \sum_{j=1}^{\infty} b_j(T) \sin(j\,\tilde{\mu}_B/T)$$



- Non-zero $b_j(T)$ for $j \geq 2$ signal deviations from ideal HRG
- Addition of EV interactions between baryons reproduces lattice trend

Summary

- Nuclear matter can be described as VDW equation with Fermi statistics
- Strongly intensive measures of energy and particle number fluctuations are suitable probes for critical behavior
- VDW interactions between baryons have strong influence on fluctuations of conserved charges in the crossover region within HRG
- Nuclear liquid-gas transition manifests itself into non-trivial net-baryon fluctuations in regions of phase diagram probed by HIC
- Interpretation of results obtained within standard ideal HRG should be done with extreme care

Summary

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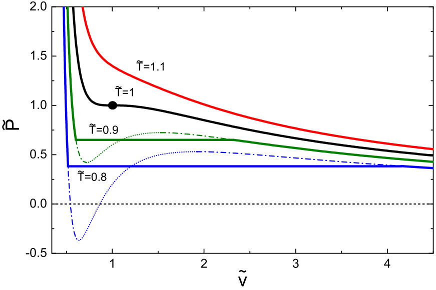
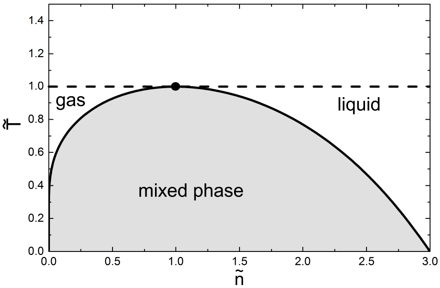
Thanks for your attention!

Backup slides

Van der Waals equation

- VDW isotherms show irregular behavior below certain temperature T_C
- Below T_C isotherms are corrected by Maxwell's rule of equal areas
- Results in appearance of mixed phase





Critical point

$$\frac{\partial p}{\partial v} = 0, \quad \frac{\partial^2 p}{\partial v^2} = 0, \quad v = V/N$$

$$p_C = \frac{a}{27b^2},\ n_C = \frac{1}{3b},\ T_C = \frac{8a}{27b}$$

Reduced variables

$$\tilde{p} = \frac{p}{p_C},\ \tilde{n} = \frac{n}{n_C},\ \tilde{T} = \frac{T}{T_C}$$

VDW equation originally formulated in canonical ensemble

How to transform CE pressure $P(T,n)$ into GCE pressure $P(T,\mu)$?

- Calculate $\mu(T,V,N)$ from standard TD relations
- Invert the relation to get $N(T,V,\mu)$ and put it back into $P(T,V,N)$
- Consistency due to thermodynamic equivalence of ensembles

Result: transcendental equation for $n(T,\mu)$

$$\frac{N}{V} \equiv n(T,\mu) = \frac{n_{\rm id}(T,\mu^*)}{1 + b\, n_{\rm id}(T,\mu^*)}, \qquad \mu^* = \mu - b\,\frac{n\,T}{1-b\,n} + 2a\,n$$

- Implicit equation in GCE, in CE it was explicit
- May have multiple solutions below T_C
- Choose one with largest pressure – equivalent to Maxwell rule in CE

Advantages of the GCE formulation

1) Hadronic physics applications: number of hadrons usually not conserved.

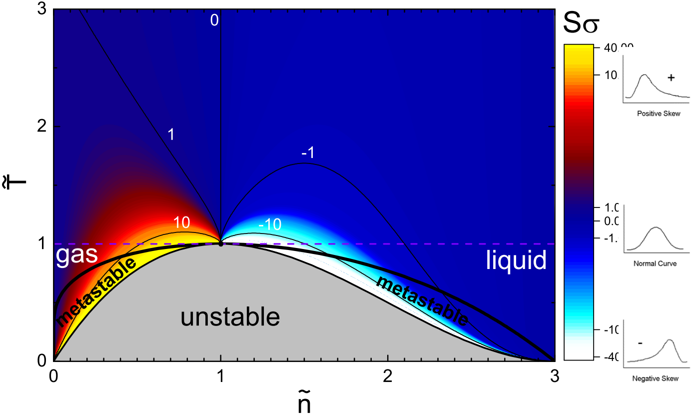
2) CE cannot describe particle number fluctuations. N-fluctuations in a small ($V \ll V_0$) subsystem follow GCE results.

3) Good starting point to include effects of quantum statistics.

Skewness

Higher-order (non-gaussian) fluctuations are even more sensitive

Skewness: $$S\sigma = \frac{\langle(\Delta N)^3\rangle}{\sigma^2} = \omega[N] + \frac{T}{\omega[N]}\left(\frac{\partial\omega[N]}{\partial\mu}\right)_T$$ asymmetry

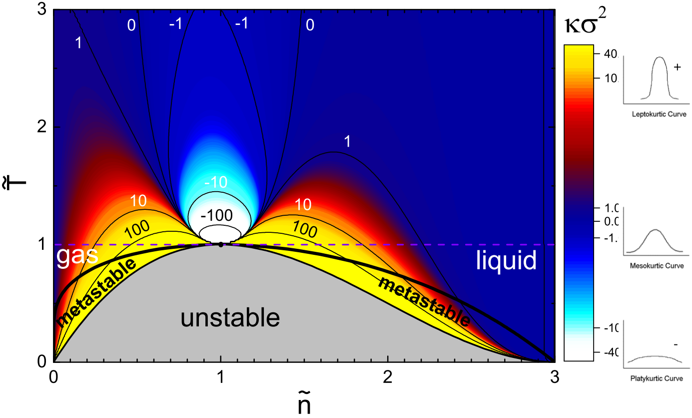


Skewness is

- Positive (right-tailed) in gaseous phase
- Negative (left-tailed) in liquid phase

Kurtosis

Kurtosis: $\kappa\sigma^2 = \dfrac{\langle(\Delta N)^4\rangle - 3\langle(\Delta N)^2\rangle^2}{\sigma^2}$ peakedness



Kurtosis is negative (flat) above critical point (crossover), positive (peaked) elsewhere and very sensitive to the proximity of the critical point

V. Vovchenko et al., J. Phys. A 015003, 49 (2016)

VDW equation with quantum statistics in GCE

Requirements for VDW equation with quantum statistics

1) Reduce to ideal quantum gas at $a = 0$ and $b = 0$
2) Reduce to classical VDW when quantum statistics are negligible
3) $s \geq 0$ and $s \to 0$ as $T \to 0$

Ansatz: Take pressure in the following form[1,2]

$$p(T,\mu) = p^{\rm id}(T,\mu^*) - an^2, \quad \mu^* = \mu - b\,p - a\,b\,n^2 + 2an$$

where $p^{\rm id}(T,\mu^*)$ is pressure of ideal quantum gas.

$$n(T,\mu) = \left(\frac{\partial p}{\partial \mu}\right)_T = \frac{n^{\rm id}(T,\mu^*)}{1 + b\,n^{\rm id}(T,\mu^*)}$$

Algorithm for GCE

1) Solve system of eqs. for p and n at given (T,μ)
2) Choose the solution with largest pressure

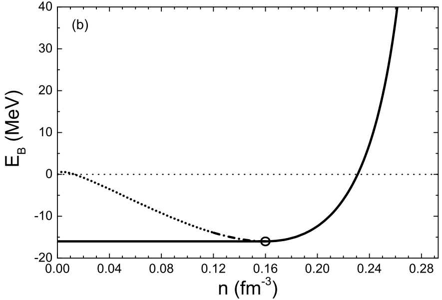
[1] V. Vovchenko, D. Anchishkin, M. Gorenstein, Phys. Rev. C 91, 064314 (2015)
[2] Alternative derivation: K. Redlich, K. Zalewski, arXiv:1605.09686 (2016)
[3] a=0 ⇒ excluded-volume model, D. Rischke et al., Z.Phys. C51, 485 (1991)

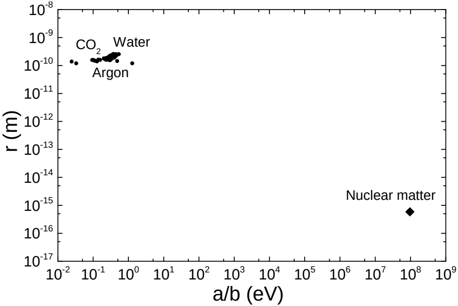
VDW gas of nucleons: zero temperature

How to fix a and b? For classical fluid usually tied to CP location.
Different approach: Reproduce saturation density and binding energy

From $E_B = E/A \cong -16$ MeV and $n = n_0 \cong 0.16$ fm^{-3} at $T = 0$ and $p = 0$

$a \cong 329$ MeV fm^3 and $b \cong 3.42$ fm^3





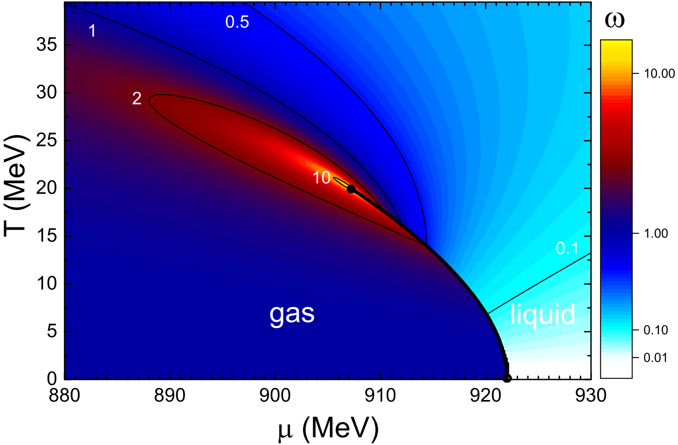
Mixed phase at $T = 0$ is specific:
A mix of vacuum ($n = 0$) and liquid at $n = n_0$

VDW eq. now at very different scale!

VDW gas of nucleons: scaled variance

Scaled variance in quantum VDW:

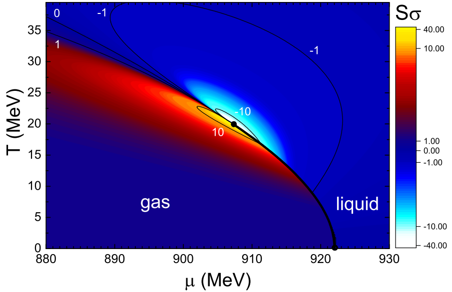
$$\omega[N] = \omega_{\rm id}(T,\mu^*) \left[\frac{1}{(1-bn)^2} - \frac{2an}{T}\,\omega_{\rm id}(T,\mu^*)\right]^{-1}$$



VDW gas of nucleons: skewness and kurtosis

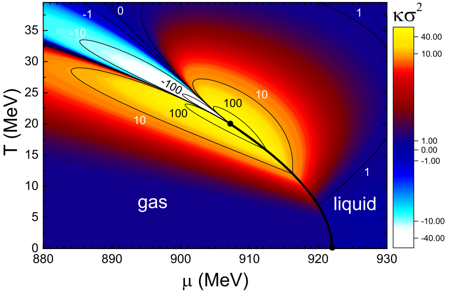
Skewness

$$S\sigma = \omega[N] + \frac{T}{\omega[N]} \left(\frac{\partial \omega[N]}{\partial \mu} \right)_T$$



Kurtosis

$$\kappa\sigma^2 = (S\sigma)^2 + T \left(\frac{\partial [S\sigma]}{\partial \mu} \right)_T$$



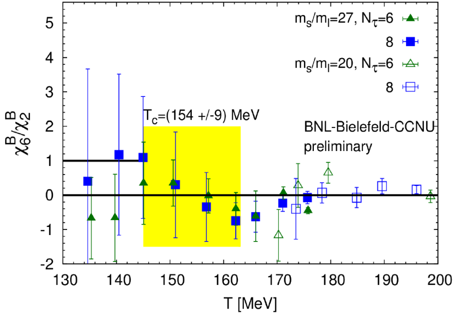
For skewness and kurtosis singularity is rather specific: sign depends on the path of approach

V. Vovchenko et al., Phys. Rev. C 92, 054901 (2015)

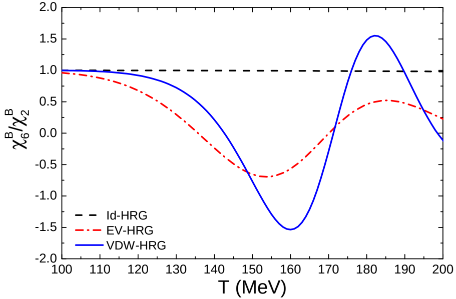
VDW-HRG at $\mu = 0$: baryon number fluctuations

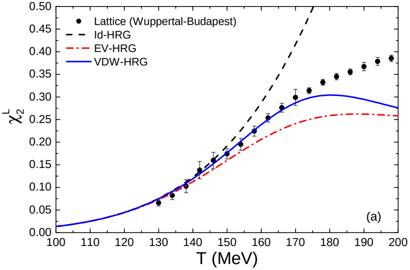
χ_6^B/χ_2^B

BNL-Bielefeld-CCNU



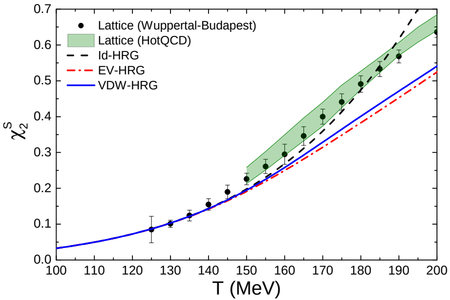
VDW-HRG





- Net number of light quarks χ_2^L
- $L = (u + d)/2 = (3B + S)/2$
- Improved description in VDW-HRG

- Net-strangeness χ_2^S
- Underestimated by HRG models, similar for χ_{11}^{BS}
- Extra strange states?[1]
- Weaker VDW interactions for strange baryons?[2]



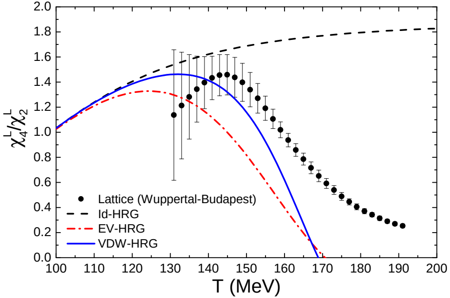
[1] Bazavov et al., PRL 113, 072001 (2014)

[2] Alba, Vovchenko, Gorenstein, Stoecker, arXiv:1606.06542

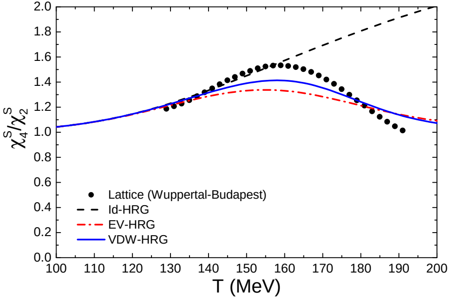
VDW-HRG at $\mu_B = 0$: net-light and net-strangeness

Fluctuations of net-light $L = (u+d)/2 = (3B+S)/2$ and net-strangeness

χ_4^L/χ_2^L



χ_4^S/χ_2^S



- Lattice shows peaked structures in crossover regions
- Not at all reproduced by Id-HRG, signal for deconfinement?[1]
- Peaks at different T for net-L and net-S $\Rightarrow$ flavor hierarchy?[2]
- VDW-HRG also shows peaks and flavor hierarchy $\Rightarrow$ cannot be traced back directly to deconfinement

[1] S. Ejiri, F. Karsch, K. Redlich, PLB 633, 275 (2006)
[2] Bellwied et al., PRL 111, 202302 (2013)

VDW-HRG: extensions

Effect of reducing VDW interactions involving strange hadrons

- 3 times smaller EV for strange baryons
- Small EV for mesons
- Illustrative calculation
- Most observables improved