# Off-equilibrium production of light nuclei in heavy-ion collisions

Volodymyr Vovchenko (LBNL)

*2022 MIAPP Program "Antinuclei"*

**February 11, 2022**

Based on:

VV, K. Gallmeister, J. Schaffner-Bielich, C. Greiner, *Phys. Lett. B 800, 135131 (2020)*

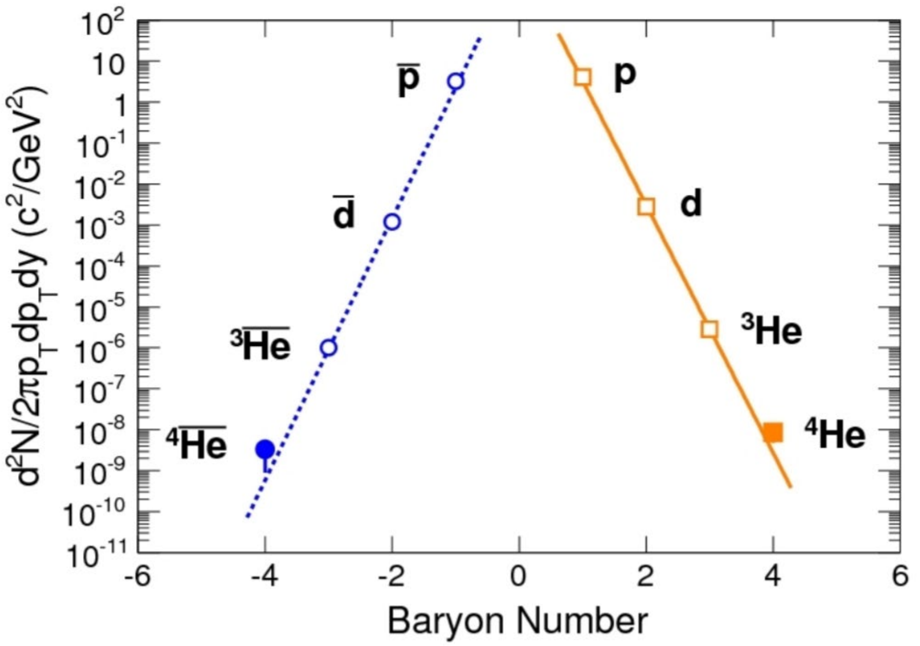
T. Neidig, K. Gallmeister, C. Greiner, M. Bleicher, VV, *Phys. Lett. B 827, 136891 (2022)*

BEST COLLABORATION

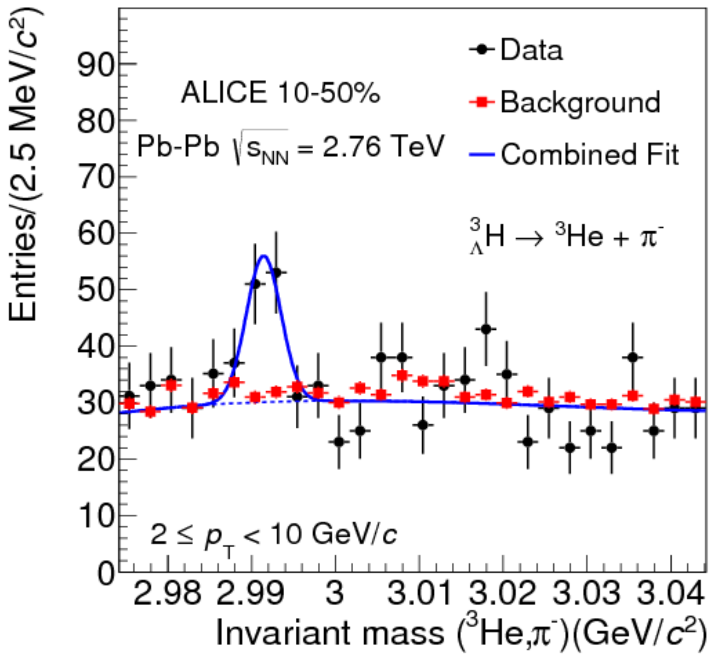
BERKELEY LAB

Unterstützt von / Supported by
Alexander von Humboldt
Stiftung/Foundation

# Loosely-bound objects in heavy-ion collisions




[STAR collaboration, Nature **473**, 353 (2011)]




[ALICE Collaboration, PLB **754**, 360 (2016)]

binding energies: $^2$H, $^3$He, $^4$He, $^3_\Lambda$H: 2.22, 7.72, 28.3, 0.130 MeV $\ll T \sim 150$ MeV

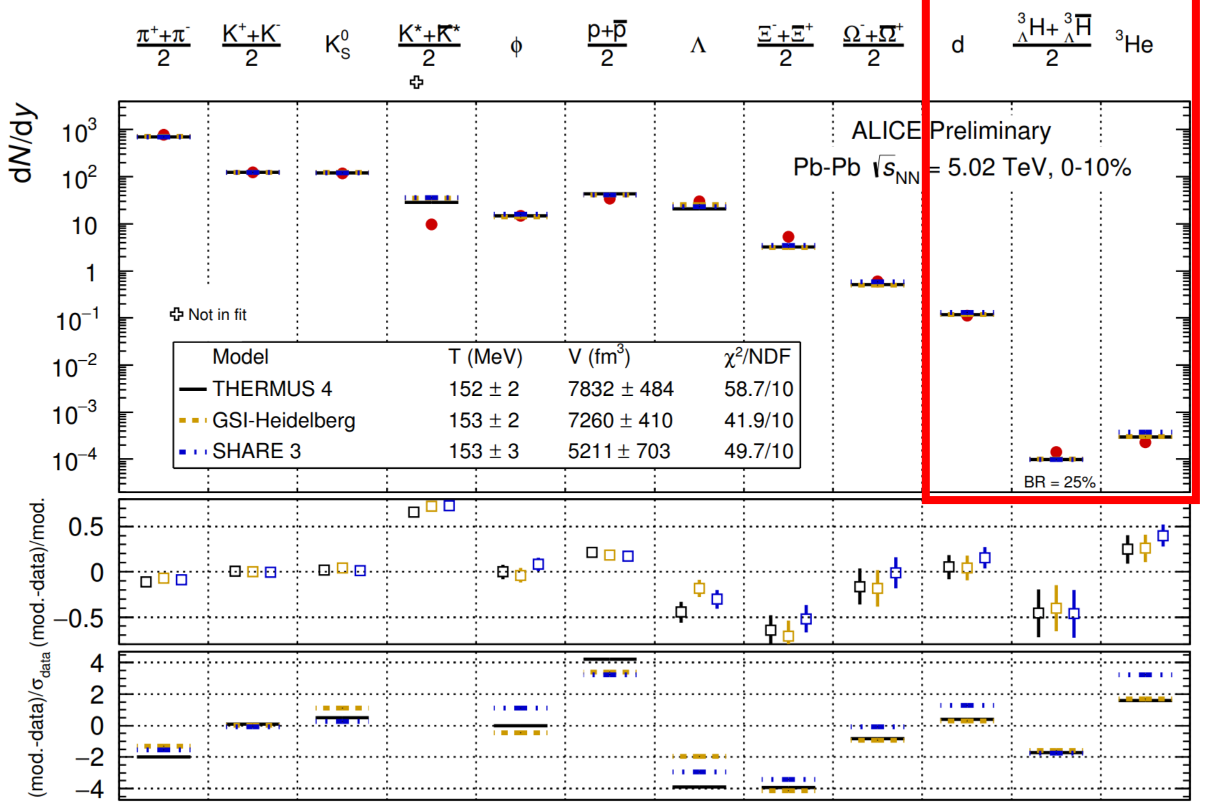
"**snowballs in hell**"

The production mechanism is not established. Common approaches:

- **thermal** nuclei emission together with hadrons [Andronic et al., PLB '11;...]
- final-state **coalescence** of nucleons close in phase-space [Butler, Pearson, PRL '61; Scheibl, Heinz, PRC '99;...]
  A. Caliva's talk

# Two experimental observations at the LHC

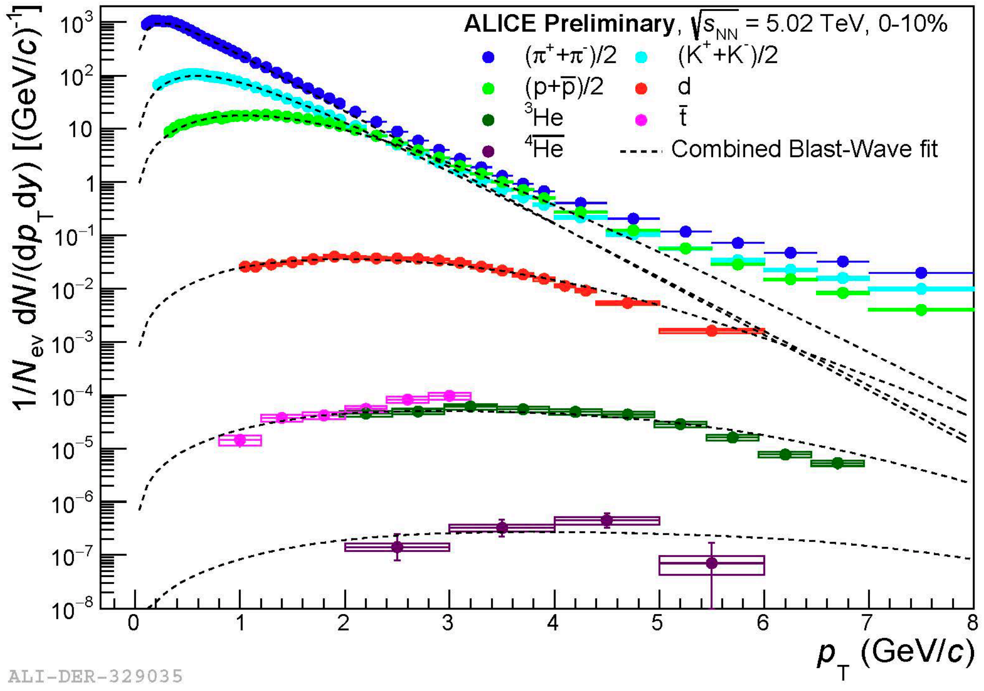
1. Measured hadron yields are described by thermal model at $T_{ch} \approx 155$ MeV




ALI-PREL-148739 ALICE Collaboration, arXiv:1910.07678

A. Kalweit's talk

2. Momentum spectra described by the blast-wave model at $T_{kin} \approx 100 - 120$ MeV

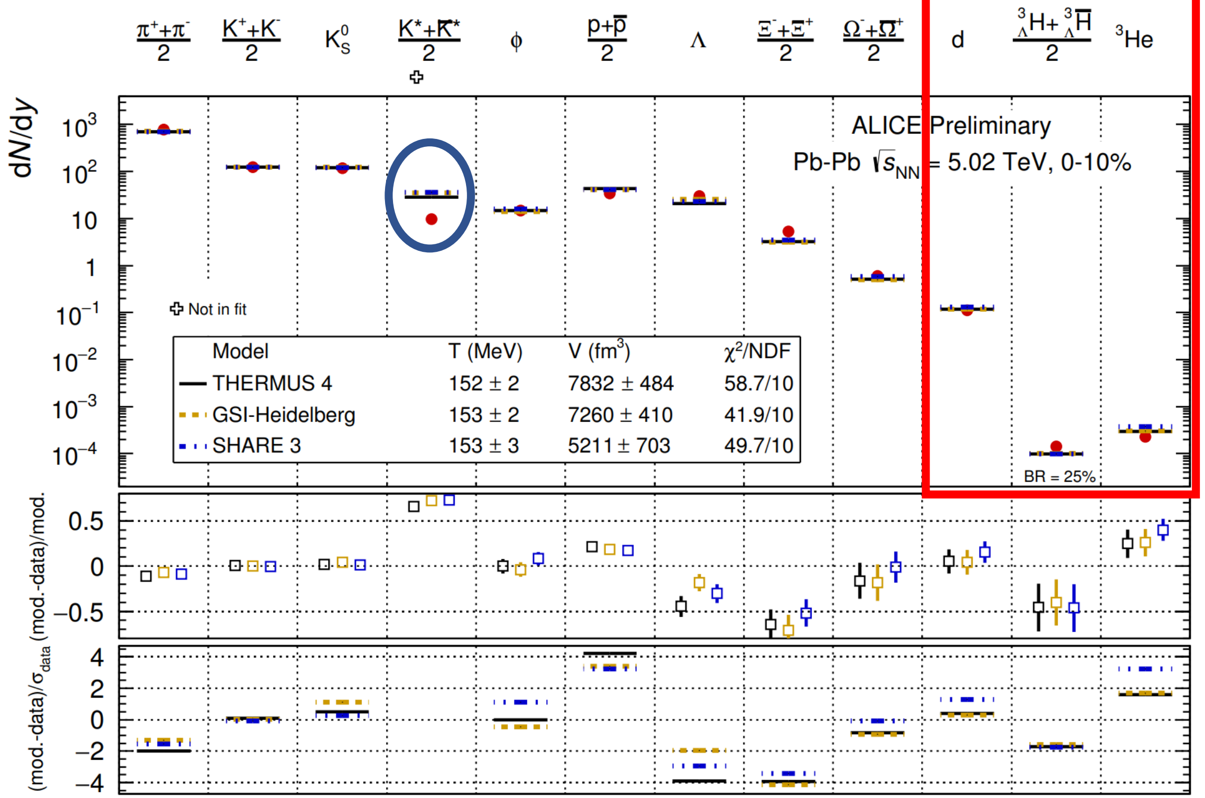



[E. Bartsch (ALICE Collaboration), QM2019]

What happens between $T_{\mathrm{ch}}$ and $T_{\mathrm{kin}}$?

# Two experimental observations at the LHC

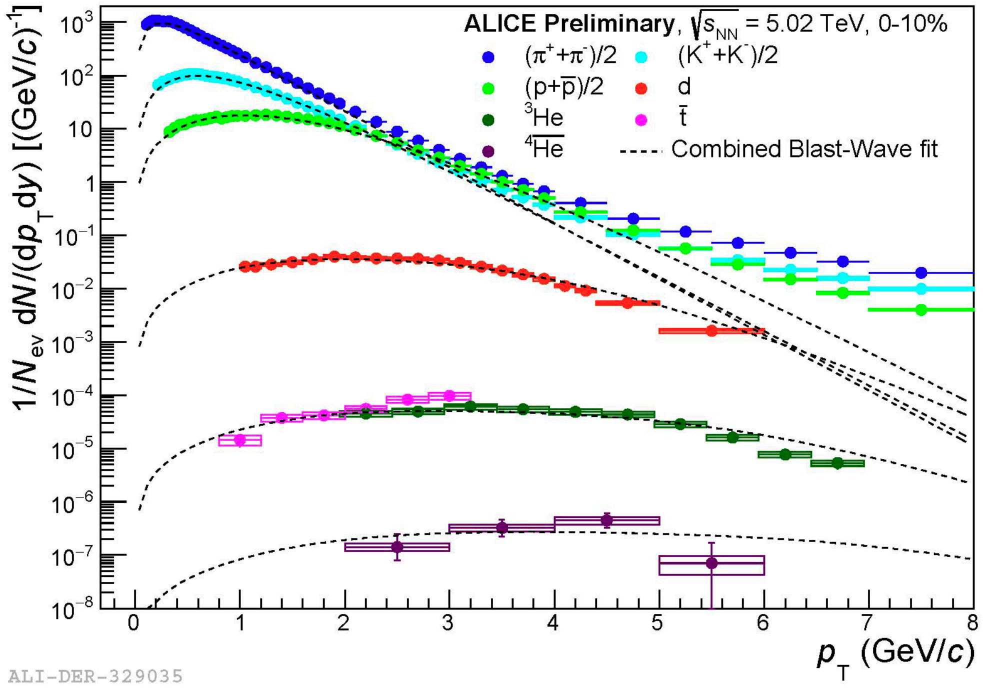
1. Measured hadron yields are described by thermal model at $T_{ch} \approx 155$ MeV




ALI-PREL-148739 ALICE Collaboration, arXiv:1910.07678

A. Kalweit's talk

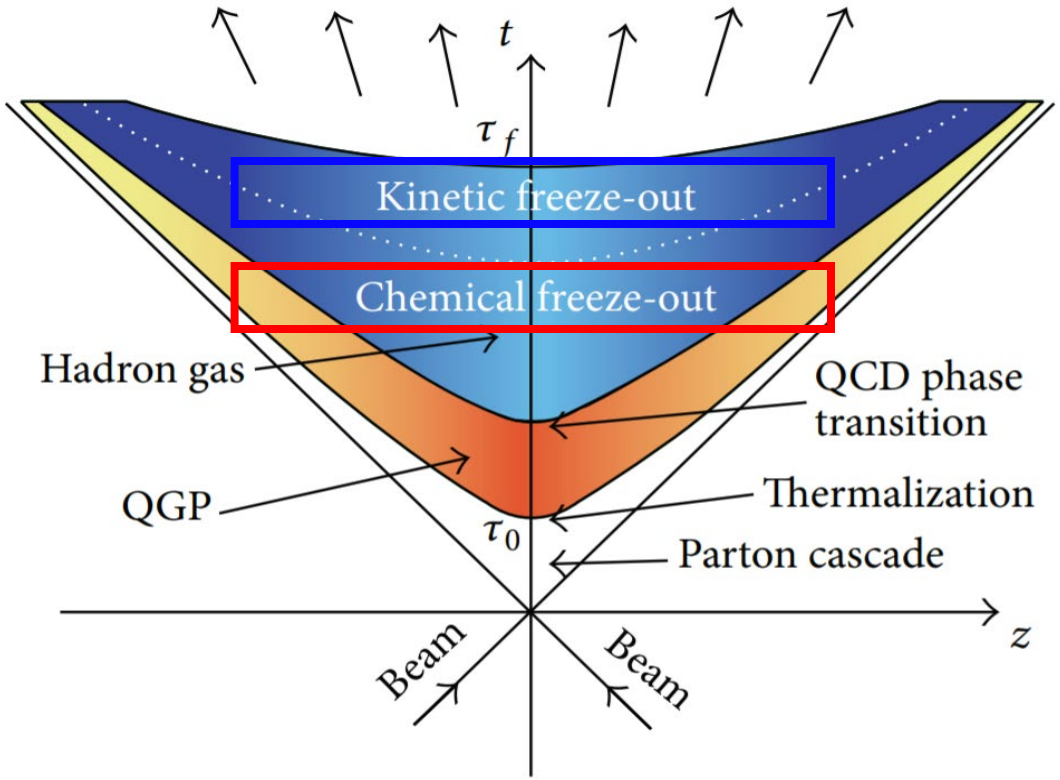
2. Momentum spectra described by the blast-wave model at $T_{kin} \approx 100 - 120$ MeV




[E. Bartsch (ALICE Collaboration), QM2019]

What happens between $T_{\rm ch}$ and $T_{\rm kin}$?

# Hadronic phase in heavy-ion collisions




- At $T_{ch} \approx 150 - 160$ MeV inelastic collisions cease, yields of *stable* hadrons frozen
- Kinetic equilibrium maintained down to $T_{kin} \approx 100 - 120$ MeV through (pseudo)elastic scatterings

# Partial chemical equilibrium (PCE)

Expansion of hadron resonance gas in partial chemical equilibrium at $T < T_{ch}$

[H. Bebie, P. Gerber, J.L. Goity, H. Leutwyler, Nucl. Phys. B '92; C.M. Hung, E. Shuryak, PRC '98]

Chemical composition of stable hadrons is fixed, kinetic equilibrium maintained through pseudo-elastic resonance reactions $\pi\pi \leftrightarrow \rho$, $\pi \mathrm{K} \leftrightarrow \mathrm{K}^*, \pi N \leftrightarrow \Delta$, etc.

E.g.: $\pi + 2\rho + 3\omega + \cdots = const$, $\quad \mathrm{K} + \mathrm{K}^* + \cdots = const$, $\quad N + \Delta + N^* + \cdots = const$,

## Effective chemical potentials:

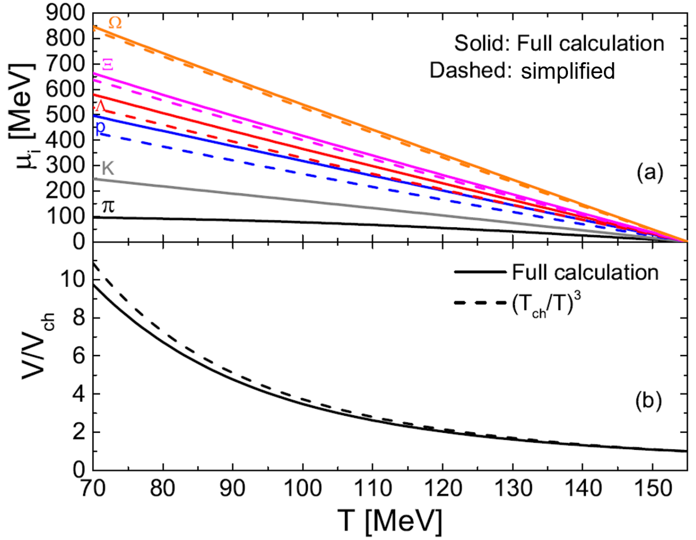
$$\tilde{\mu}_j = \sum_{i \in \text{stable}} \langle n_i \rangle_j \, \mu_i, \qquad \langle n_i \rangle_j - \text{mean number of hadron } i \text{ from decays of hadron } j, \qquad j \in \text{HRG}$$

## Conservation laws:

$$\sum_{j \in \text{hrg}} \langle n_i \rangle_j \, n_j(T, \tilde{\mu}_j) \, V = N_i(T_{\text{ch}}), \quad i \in \text{stable}$$

$$\sum_{j \in \text{hrg}} s_j(T, \tilde{\mu}_j) \, V = S(T_{\text{ch}})$$

*numerical solution* $\Longrightarrow$ $\{\mu_i(T)\}, V(T)$




Numerical implementation within **Thermal-FIST** package (since v1.3)

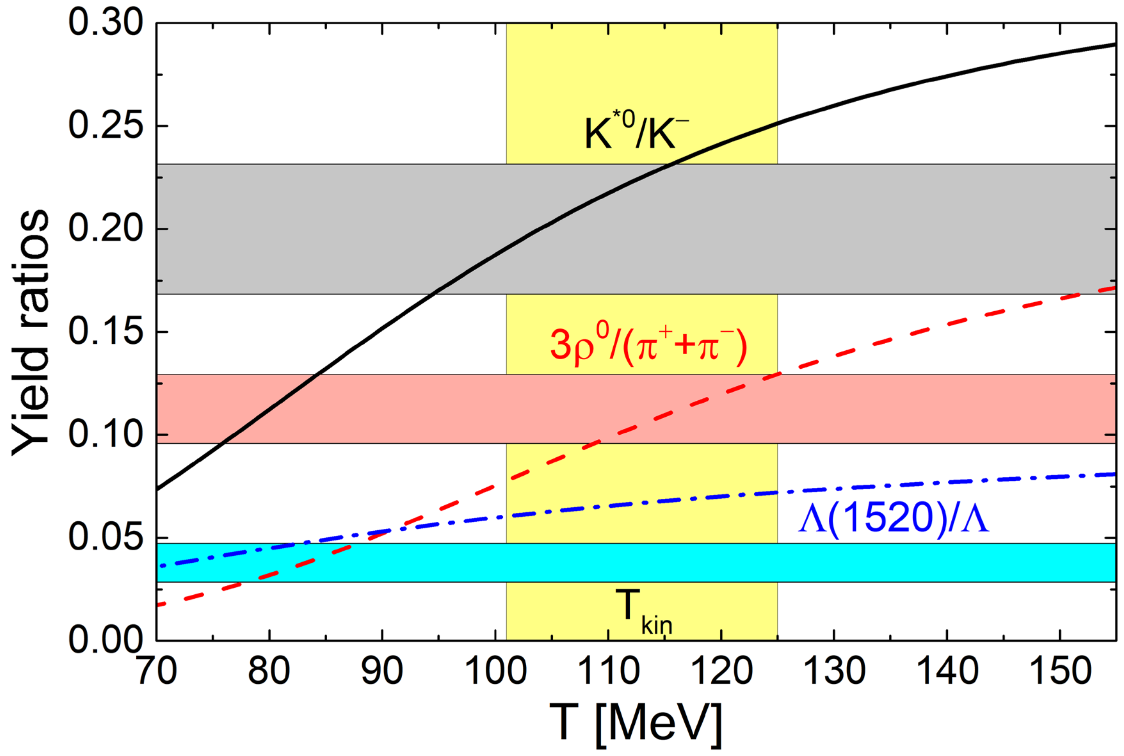
[**VV**, H. Stoecker, *Comput. Phys. Commun.* ***244,*** *295 (2019)*]

**open source:** https://github.com/vlvovch/Thermal-FIST

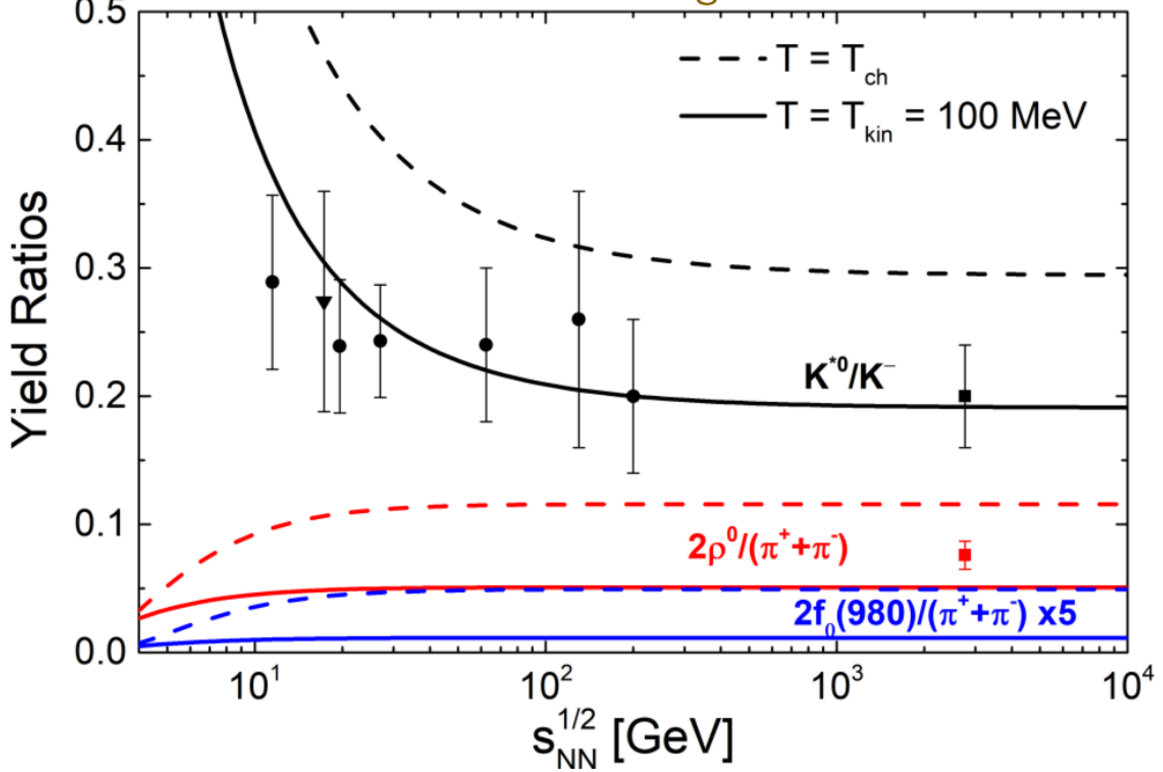
# Resonance suppression in hadronic phase

Yields of resonances are *not* conserved in partial chemical equilibrium

E.g. $\mathrm{K}^*$ yield dilutes during the cooling through reactions $\pi\mathrm{K} \leftrightarrow \mathrm{K}^*$




Along the freeze-out curve

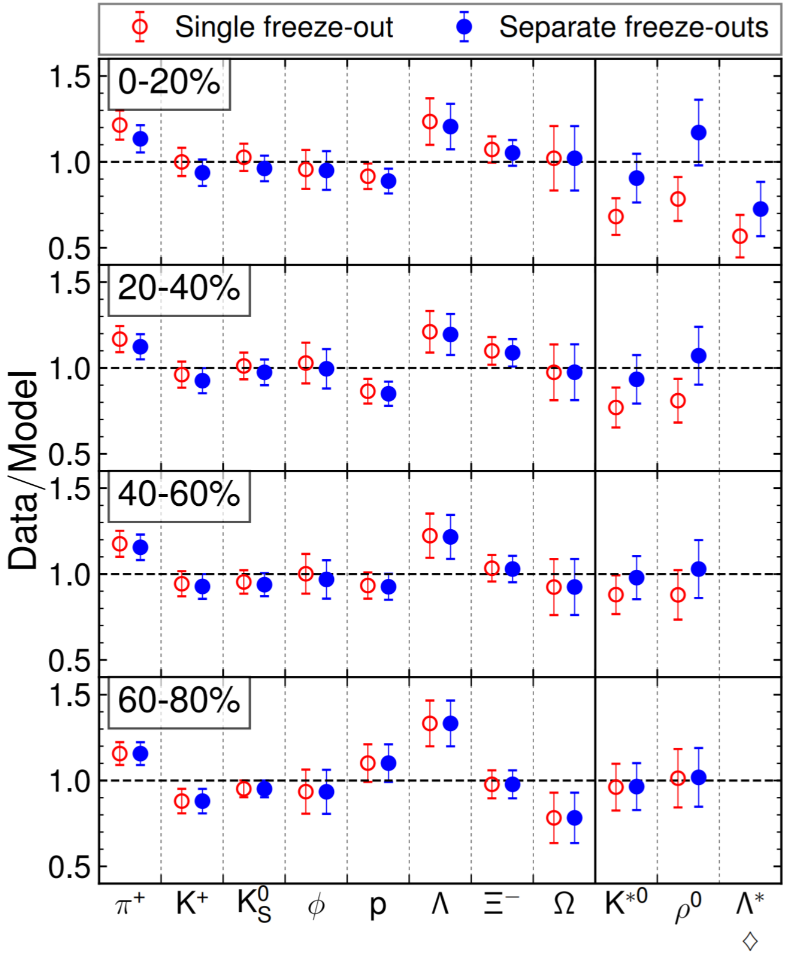


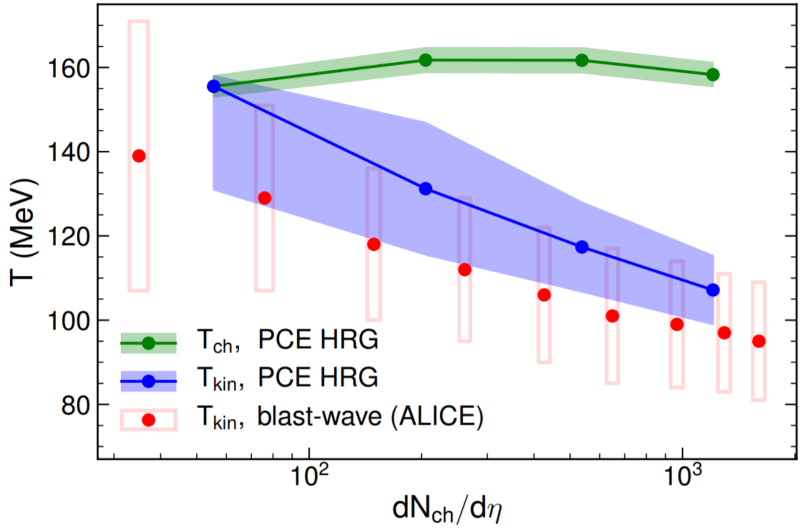

Use the sensitivity of short-lived resonance yields to $T_{kin}$ extract the kinetic freeze-out temperature

A. Motornenko, **VV**, C. Greiner, H. Stoecker, Phys. Rev. C 102, 024909 (2020)

# Kinetic freeze-out temperature from resonances

**Thermal fits 2.0:** Fit $T_{ch}$ and $T_{kin}$ simultaneously to yields of both stable and short-lived hadrons




| Centrality | $T_{ch}$ (MeV) | $T_{kin}$ (MeV) | $\chi^2$/dof |
|---|---|---|---|
| 0-20% | $160.2 \pm 3.1$ | – | 23.6/8 |
| | $158.3 \pm 2.8$ | $107.1 \pm 8.2$ | 10.5/7 |
| 20-40% | $162.9 \pm 3.1$ | – | 19.5/8 |
| | $161.7 \pm 2.9$ | $117.3 \pm 10.8$ | 12.8/7 |
| 40-60% | $162.3 \pm 3.0$ | – | 12.5/8 |
| | $161.8 \pm 2.9$ | $131.2 \pm 15.9$ | 10.6/7 |
| 60-80% | $155.5 \pm 2.5$ | – | 19.1/8 |
| | $155.5 \pm 2.5$ | $155.5^{+2.5}_{-24.5}$ | 19.1/7 |

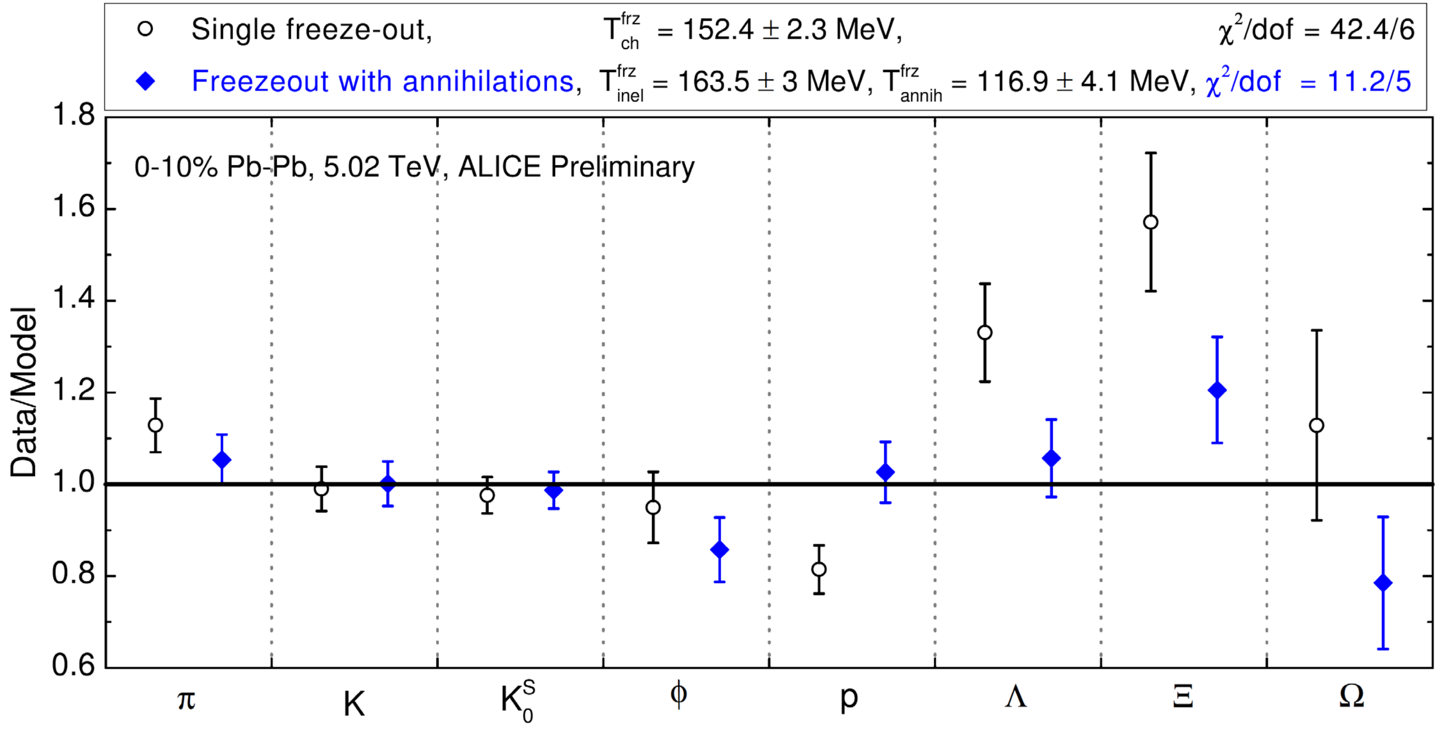
Solves the $T_{kin}$-vs-$\langle\beta_T\rangle$ anticorrelation problem of blast-wave fits

A. Motornenko, **VV**, C. Greiner, H. Stoecker, Phys. Rev. C 102, 024909 (2020)

# Hadronic phase with annihilations

Add nucleon annihilations $N\bar{N} \leftrightarrow 5\pi$ into the PCE framework

(Anti)nucleon and pions numbers no longer conserved, $N_N, N_{\bar{N}}, N_\pi \neq$ const. but $\frac{N_N + N_{\bar{N}}}{2} + \frac{N_\pi}{5} = \text{const}$




$$\mu_N = \mu_{\bar{N}} = 2.5\mu_\pi$$

Including annihilations in the hadronic phase leads to a much nicer fit

NB: hyperon annihilations not allowed here

# Light nuclei: Saha equation

Detailed balance for nuclear reactions

$$\frac{n_A}{\prod_i n_{A_i}} = \frac{n_A^{\rm eq}}{\prod_i n_{A_i}^{\rm eq}}\,, \quad \Leftrightarrow \quad \mu_A = \sum_i \mu_{A_i}\,, \qquad \text{e.g.}\ \ \mu_d = \mu_p + \mu_n,\ \ \mu_{^3\mathrm{He}} = 2\mu_p + \mu_n,\ \ \ldots$$

*Saha equation*

Kinetic theory example: deuteron number evolution through $p + n + X \leftrightarrow d + X$ reactions

**gain** **loss**

$$\frac{dN_d}{d\tau} = \langle \sigma_{dX} v_{rel} \rangle\, N_d^0\, n_x^0\, e^{\mu_p/T}\, e^{\mu_n/T}\, e^{\mu_X/T} - \langle \sigma_{dX} v_{rel} \rangle\, N_d^0\, n_x^0\, e^{\mu_d/T}\, e^{\mu_X/T}$$

***small*** ***big*** ***big***

**Saha equation**

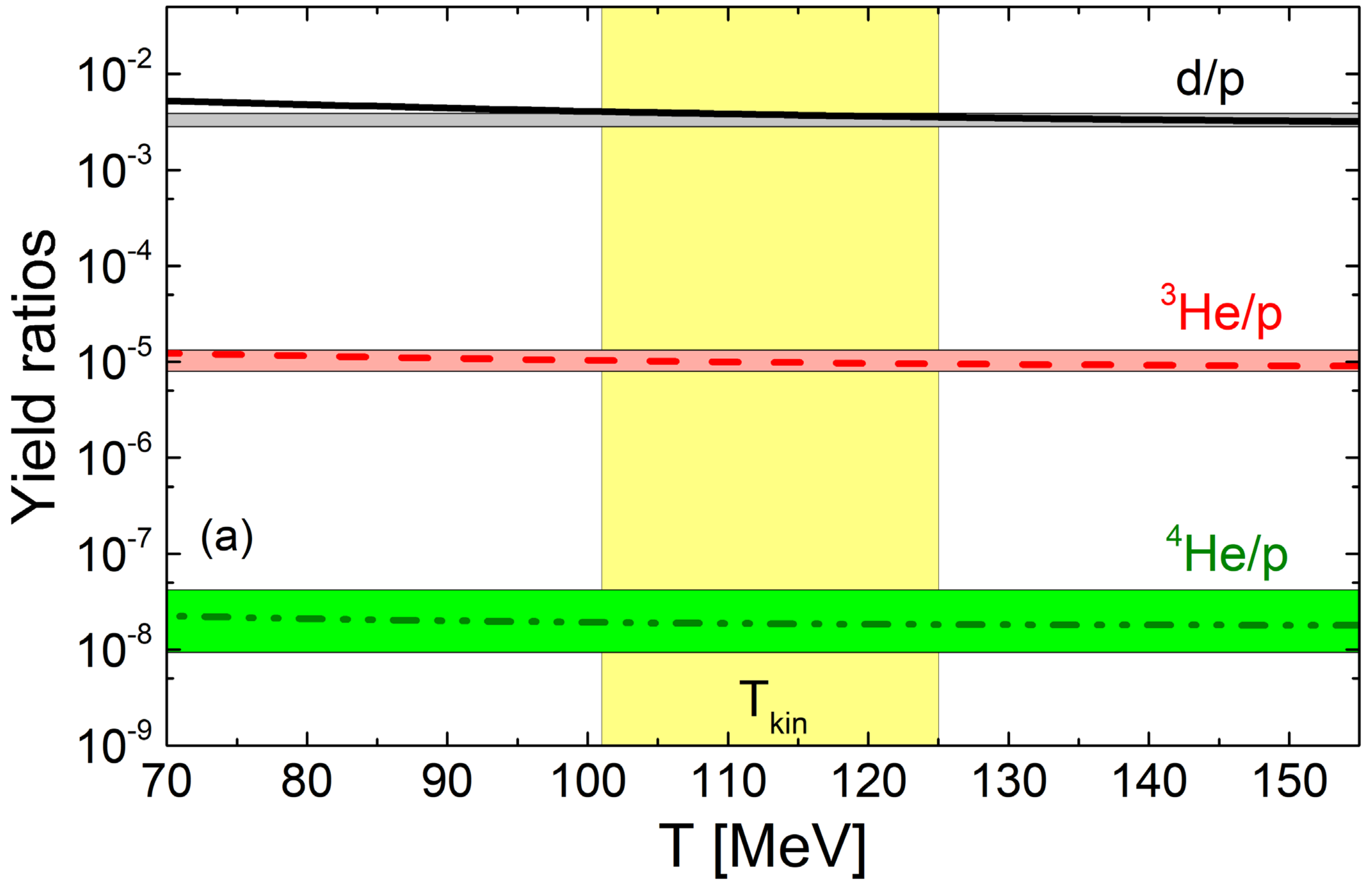
gain $\approx$ loss $\quad \rightarrow \quad \mu_d \approx \mu_p + \mu_n$ $\quad$ = detailed balance

= law of mass action

**Early Universe:** $X_A = d_A \left[\zeta(3)^{A-1}\, \pi^{\frac{1-A}{2}}\, 2^{\frac{3A-5}{2}}\right] A^{\frac{5}{2}} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta^{A-1}\, X_p^Z\, X_n^{A-Z}\, \exp\left(\frac{B_A}{T}\right)$

E. Kolb, M. Turner, "The Early Universe" (1990)

# Saha equation at LHC

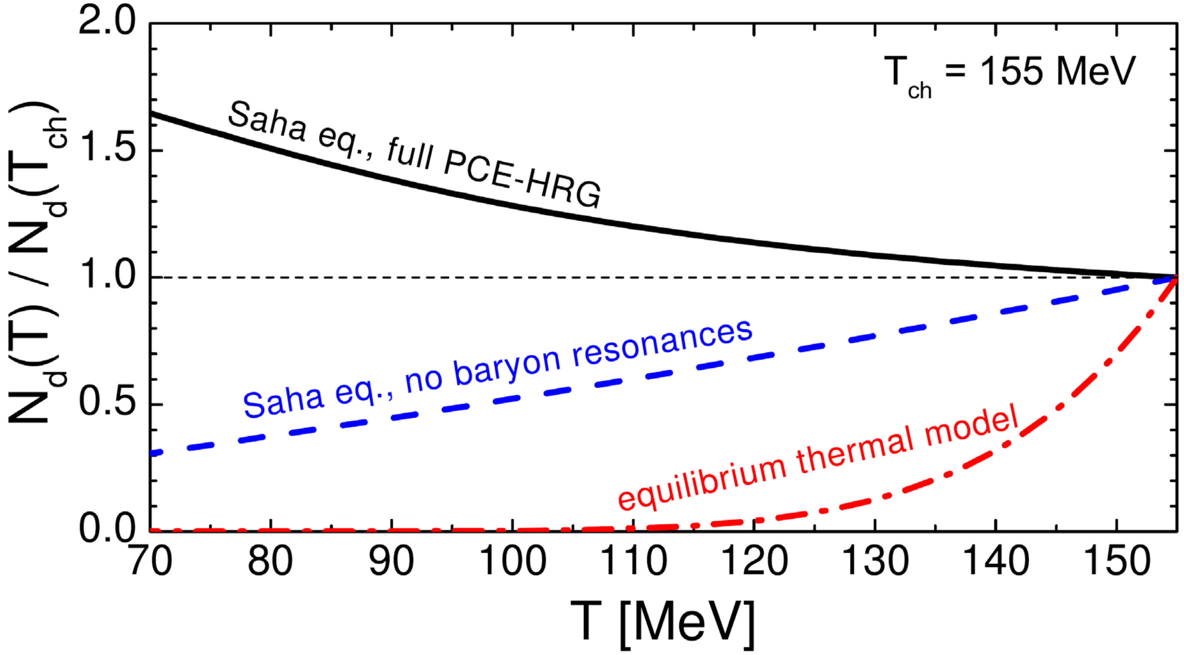



Deviations from thermal model predictions are moderate despite significant cooling and dilution. Is this the reason for why thermal model works so well?

Echoes earlier transport model conclusions for d [D. Oliinychenko, et al., PRC 99, 044907 (2019)]

For $T = T_{kin}$ similar results reported in [X. Xu, R. Rapp, EPJA 55, 68 (2019)]

# Closer look at the yields




**Saha equation (no resonances):**

$$\frac{N_A(T)}{N_A(T_{\mathrm{ch}})} \simeq \left(\frac{T}{T_{\mathrm{ch}}}\right)^{\frac{3}{2}(A-1)} \exp\left[B_A\left(\frac{1}{T}-\frac{1}{T_{\mathrm{ch}}}\right)\right]$$

$B_A \ll T$

**Thermal model:**

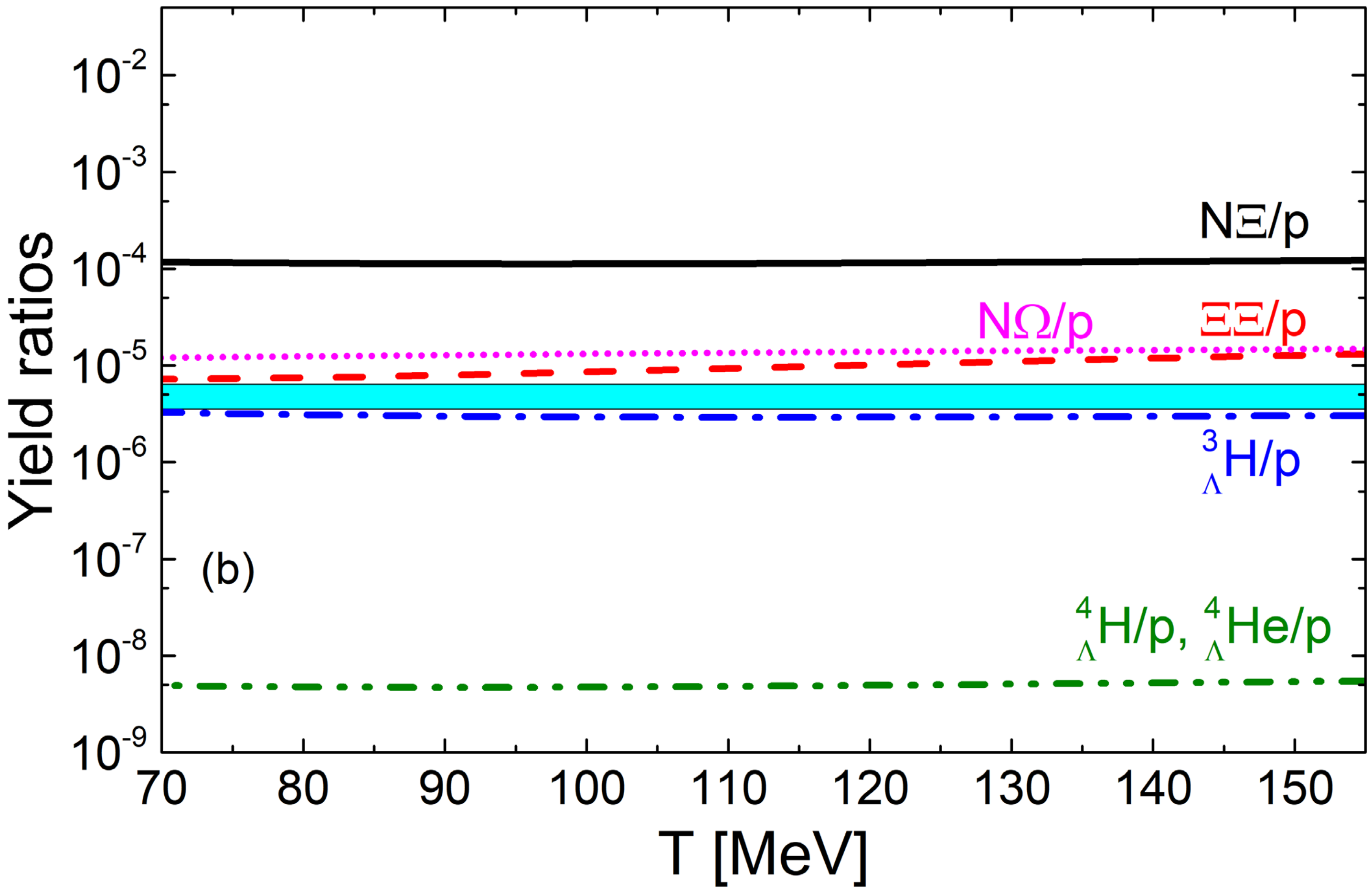
$$\left[\frac{N_A(T)}{N_A(T_{\mathrm{ch}})}\right]_{\mathrm{eq.}} \simeq \left(\frac{T}{T_{\mathrm{ch}}}\right)^{-\frac{3}{2}} \exp\left[-m_A\left(\frac{1}{T}-\frac{1}{T_{\mathrm{ch}}}\right)\right]$$

$m_A \gg T$

Nuclei yields are *not constant* in the Saha equation approach but the strong exponential dependence on the temperature is eliminated

Quantitative outcome is sensitive to the feeding from baryonic resonances

# Saha equation: hypernuclei




Hypernuclei stay close to the thermal model prediction. An exception is a hypothetical $\Xi\Xi$ state $\leftarrow$ *planned measurement in Runs 3 & 4 at the LHC*

[LHC Yellow Report, 1812.06772]

# Light nuclei production with rate equations

T. Neidig, K. Gallmeister, C. Greiner, M. Bleicher, VV, *Phys. Lett. B 827, 136891 (2022)*

**gain** **loss**

$$\frac{dN_d}{d\tau} = \langle \sigma_{dX} v_{rel} \rangle N_d^0 \, n_x^0 \, e^{\mu_p/T} \, e^{\mu_n/T} \, e^{\mu_X/T} - \langle \sigma_{dX} v_{rel} \rangle N_d^0 \, n_x^0 \, e^{\mu_d/T} \, e^{\mu_X/T}$$

~~small~~ ~~big~~ ~~big~~

~~**Saha equation**~~

~~gain ≈ loss → $\mu_d = \mu_p + \mu_n$ = detailed balance~~

~~= law of mass action~~

Relax the assumption of equilibrium for $AX \leftrightarrow \sum_i A_i X$ reactions

# Light nuclei production with rate equations

Catalyzed light nuclei reactions. Destruction through $AX \to \sum_i A_i X$ and creation through $\sum_i A_i X \ \to AX$. Detailed balance principle respected but *relative chemical equilibrium not enforced*

$$\frac{dN_A}{d\tau} = \langle \sigma^{\text{in}}_{AX} v_{\text{rel}} \rangle \, n_X \, (N_A^{\text{saha}} - N_A)$$

Static fireball: $n_X \, , N_A^{\text{saha}}, \langle \sigma^{\text{in}}_{AX} v_{rel} \rangle = const$

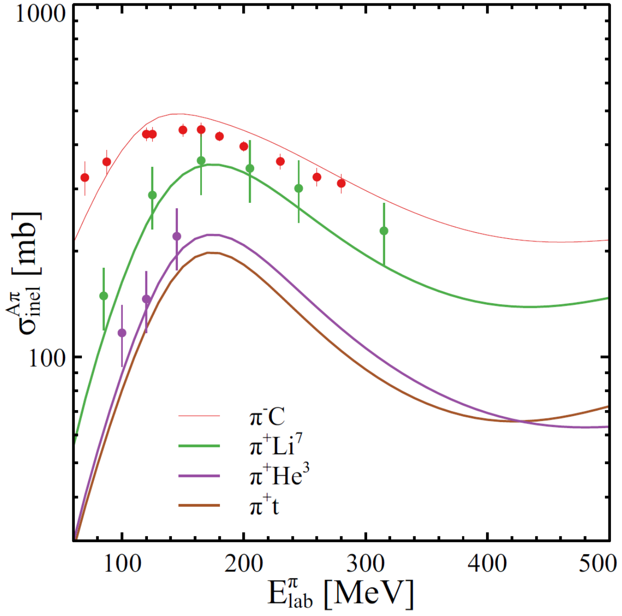
$$N_A(\tau) = N_A^{\text{saha}} + (N_A(\tau_0) - N_A^{\text{saha}}) \, e^{-\frac{\tau - \tau_0}{\tau_{\text{eq}}}} \, , \qquad \tau_{\text{eq}} = \frac{1}{\langle \sigma^{\text{in}}_{AX} v_{\text{rel}} \rangle \, n_X}$$

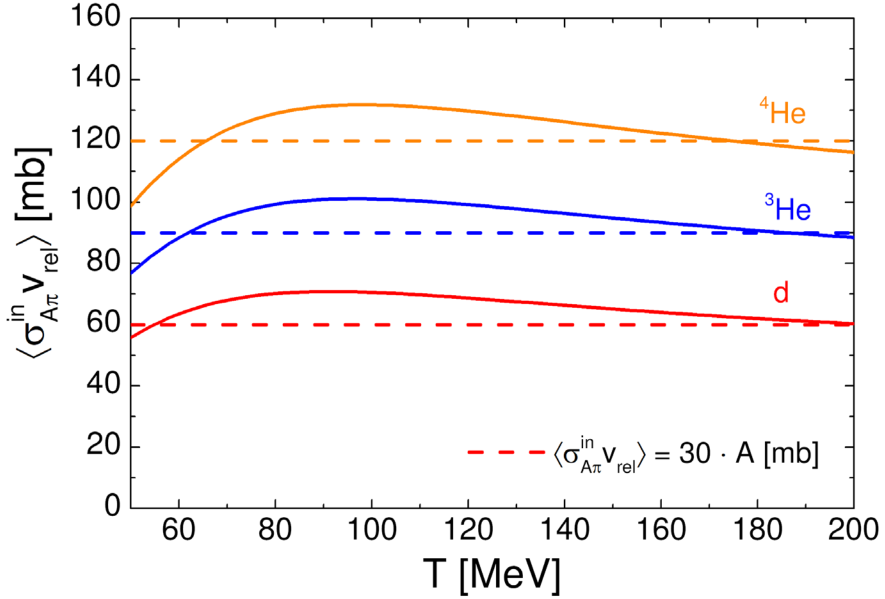
Saha limit: $\tau_{eq} \to 0 \ (\sigma^{\text{in}}_{AX} \ \to \infty)$

# Model input

- **Rates:** Use guidance from kinetic theory

Optical model for $\sigma_{A\pi}^{\text{in}}$ [J. Eisenberg, D.S. Koltun, '80]




Being implemented in SMASH transport [D. Oliinychenko's talk]

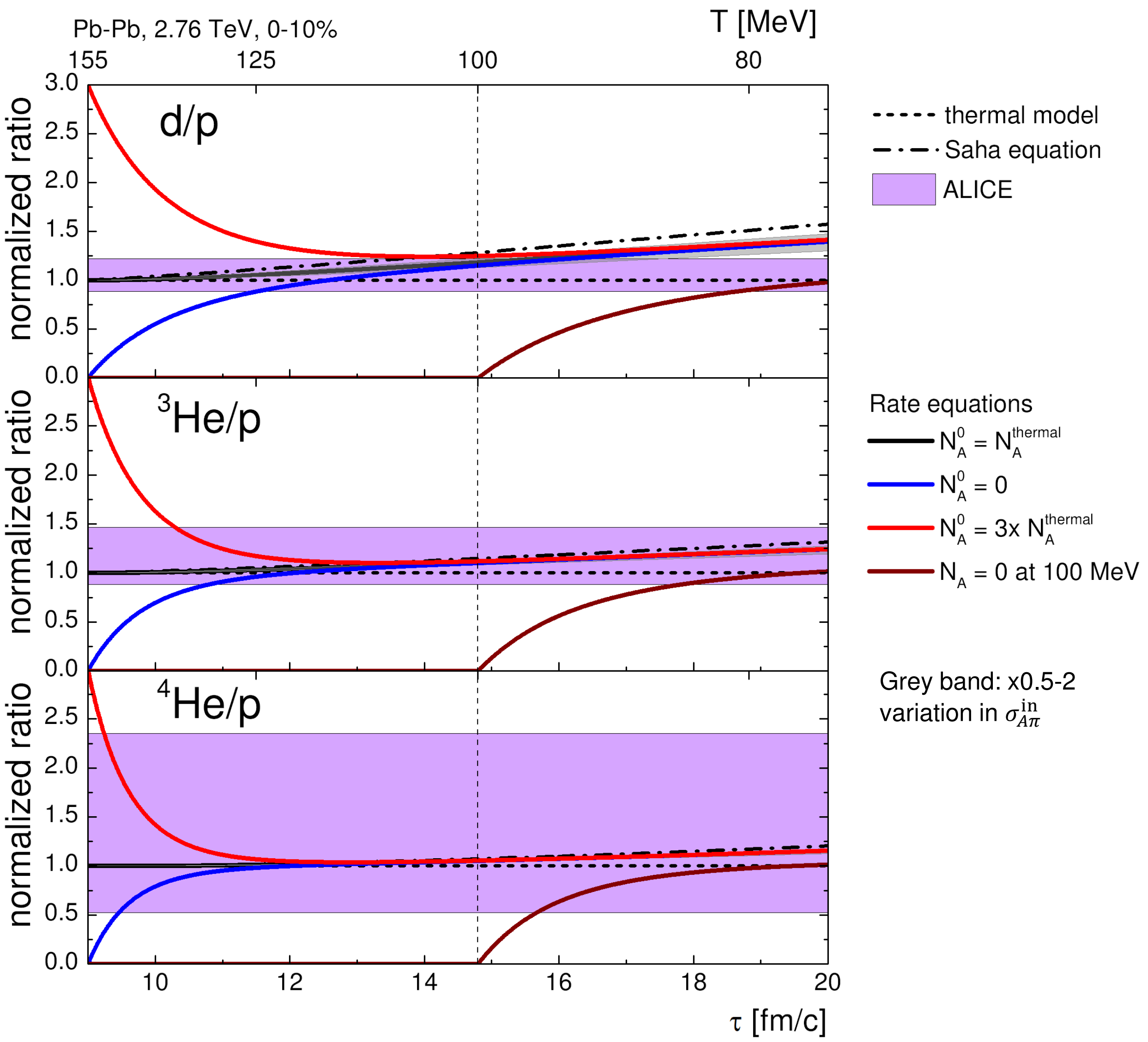
In practice $\langle\sigma_{A\pi}^{\text{in}} v_{rel}\rangle \gtrsim 30 \cdot A$ [mb]

- **Expansion** (both transverse and longitudinal)

$$\frac{V}{V_{\text{ch}}} = \frac{\tau}{\tau_{\text{ch}}} \frac{\tau_\perp^2 + \tau^2}{\tau_\perp^2 + \tau_{\text{ch}}^2}, \qquad \tau_{\text{ch}} = 9 \text{ fm}, \qquad \tau_\perp = 6.5 \text{ fm}$$

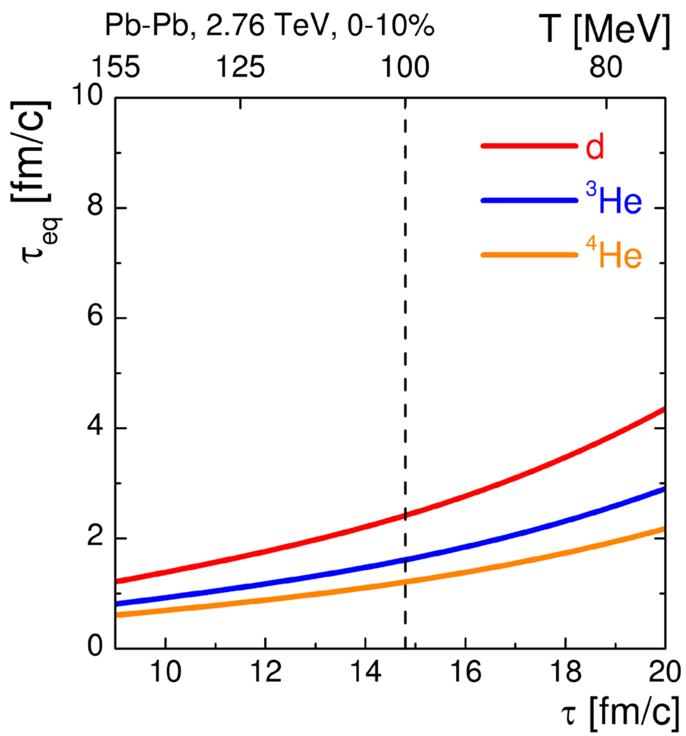
[Y. Pan, S. Pratt, PRC 89, 044911 (2014)]

# Rate equations at LHC

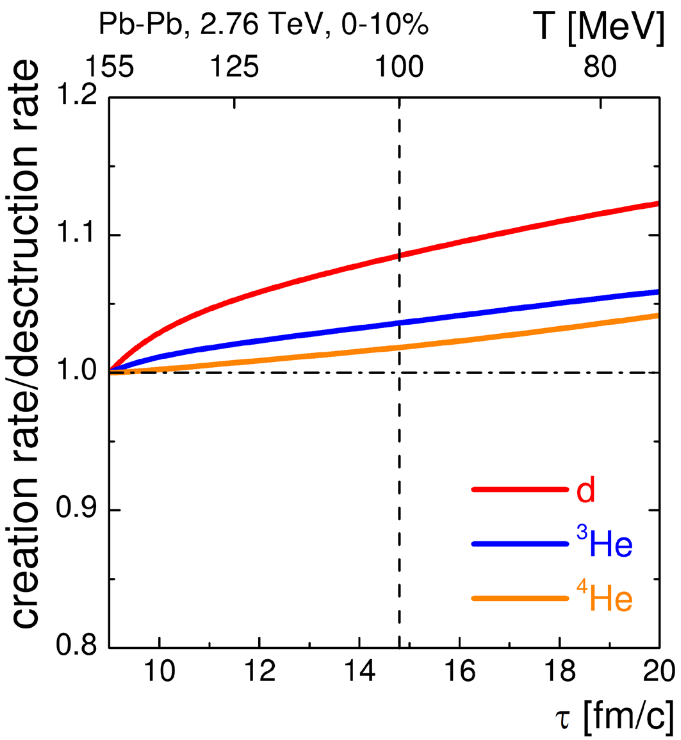


# Rate equations at LHC

$$\tau_{\mathrm{eq}}^{-1} = \langle \sigma_{A\pi}^{\mathrm{in}} v_{rel} \rangle \, n_{\pi}^{\mathrm{pce}}$$




gain vs loss

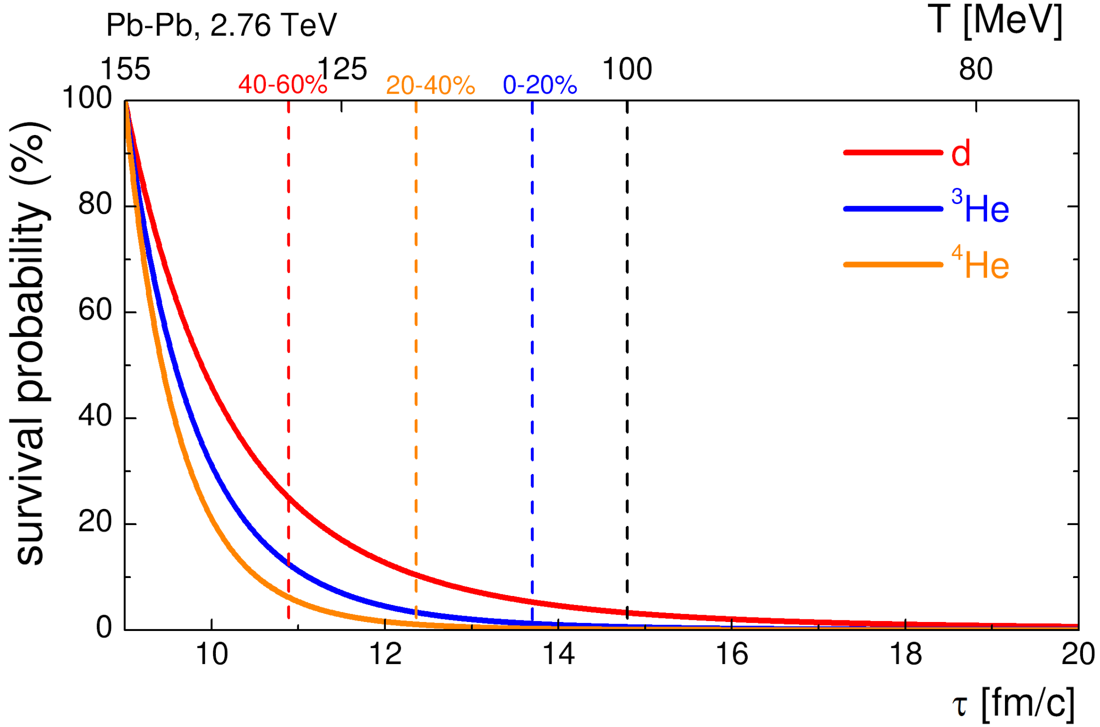



- Local equilibration times remain small
- $\tau_A^{\mathrm{eq}} \ll B_A^{-1}$ meaning light nuclei are not fully formed
- (**gain** + **loss**) ≫ |**gain** – **loss**| → **Saha equation at work**

# Can snowballs survive hell?

Count only the nuclei produced at "QGP hadronization" (at $T_{ch}$)

$$\frac{dN_A^{\text{qgp}}}{d\tau} = -\langle \sigma_{A\pi} v_{rel} \rangle n_\pi^{\text{pce}} N_A^{\text{qgp}}, \qquad \text{survival probability} = N_A^{\text{qgp}}(\tau)/N_A^{\text{qgp}}(\tau_{\text{ch}})$$




The observed nuclei are unlikely to be (pre-)formed at the "QCD phase boundary" even the "thermal" production mechanism is correct

# Rate equation for nuclei and resonances

Treat both the nuclear reactions and resonances decays and regenerations with rate equations, i.e. partial chemical equilibrium is not enforced

**Rate equations**

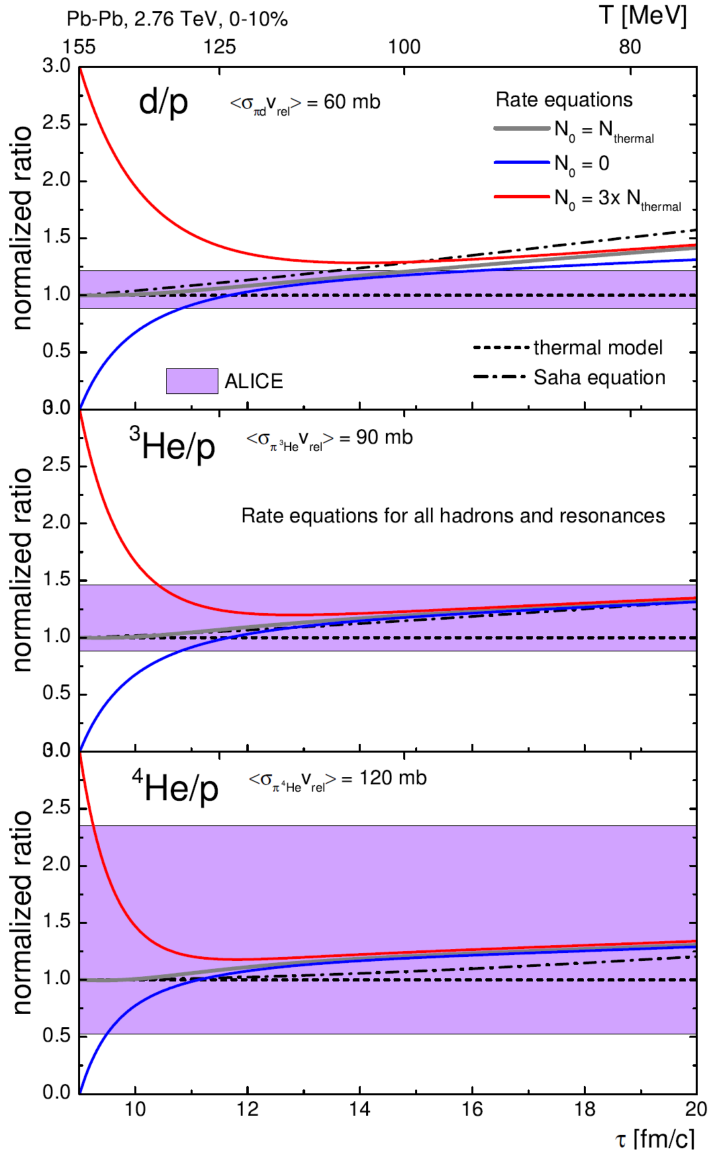
*nuclei* $$\frac{dN_A}{d\tau} = \langle \sigma_{A\pi} v_{rel} \rangle N_\pi \, n_A^{\rm eq} (e^{A\mu_N/T} - e^{\mu_A/T})$$

*resonances* $$\frac{dN_R}{d\tau} = \langle \Gamma_{R \to \sum_i a_i} \rangle N_R^{\rm eq} (e^{\sum_{i \in R} \mu_i / T} - e^{\mu_R/T})$$

**Entropy production:**

Increases by 0.6% between $T_{\rm ch} = 155$ MeV and $T_{\rm kin} = 100$ MeV

Results remain very close to the Saha equation



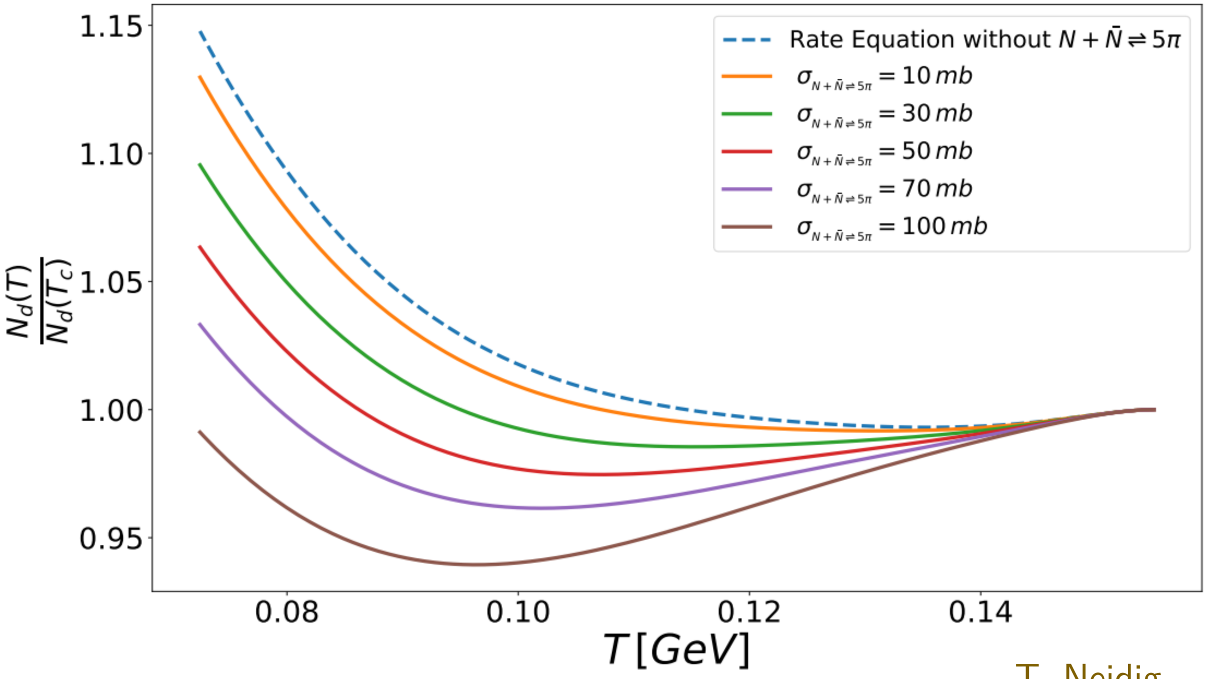
# Effect of baryon annihilations

Use rate equations to implement $N\bar{N} \leftrightarrow 5\pi$ reactions

*nucleons* $$\frac{\mathrm{d}N_N}{\mathrm{d}t} \mathrel{+}= \tilde{\alpha}_{N+\overline{N}\rightleftharpoons 5\pi}(-N_N N_{\overline{N}} + c^{\pi^5}_{N\overline{N}} N_\pi^5)$$

*nucleons* $$\frac{\mathrm{d}N_{\overline{N}}}{\mathrm{d}t} \mathrel{+}= \tilde{\alpha}_{N+\overline{N}\rightleftharpoons 5\pi}(-N_N N_{\overline{N}} + c^{\pi^5}_{N\overline{N}} N_\pi^5)$$

*pions* $$\frac{\mathrm{d}N_\pi}{\mathrm{d}t} \mathrel{+}= 5\tilde{\alpha}_{N+\overline{N}\rightleftharpoons 5\pi}(N_N N_{\overline{N}} - c^{\pi^5}_{N\overline{N}} N_\pi^5)$$




T. Neidig

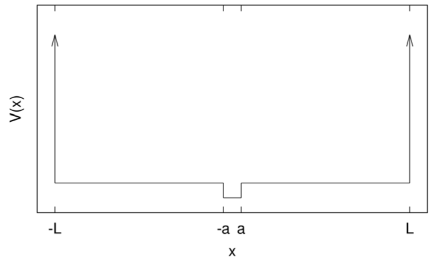
Annihilations decrease light nuclei yields

Stronger effect (up to 25%) for heavier nuclei $^3$He, $^4$He

# Summary

- Saha equation is an extended thermal model framework for light nuclei production
  - results agree with the thermal model but essentially *any* $T < T_{ch}$ permitted
  - quantitative predictions are sensitive to baryon resonance feeddowns
- Rate equations validate the framework when using nuclei break-up cross sections based on kinetic theory
  - nuclei (pre-)formed at "QCD phase boundary" do not survive the hadronic phase
  - baryon annihilations may suppress the nuclei yields on ~5-20% level
- Outlook: quantum mechanical formation of bound states
  - Schrödinger equation J. Rais, C. Greiner
  - Open quantum systems

$$i\hbar\partial_t\psi(x,t) = \left[-\frac{\hbar^2}{2m}\partial_x^2 + \hat{V}(x,t)\right]\psi(x,t)$$




**Thanks for your attention!**