Nucleosynthesis and resonance production via the Saha equation

Volodymyr Vovchenko

(Virtual) Nuclear Theory Lunch Seminar at LBNL

March 25, 2020

V.V., K. Gallmeister, J. Schaffner-Bielich, C. Greiner, *Phys. Lett. B **800,** 135131 (2020)*

A. Motornenko, V.V., C. Greiner, H. Stoecker, *1908.11730*

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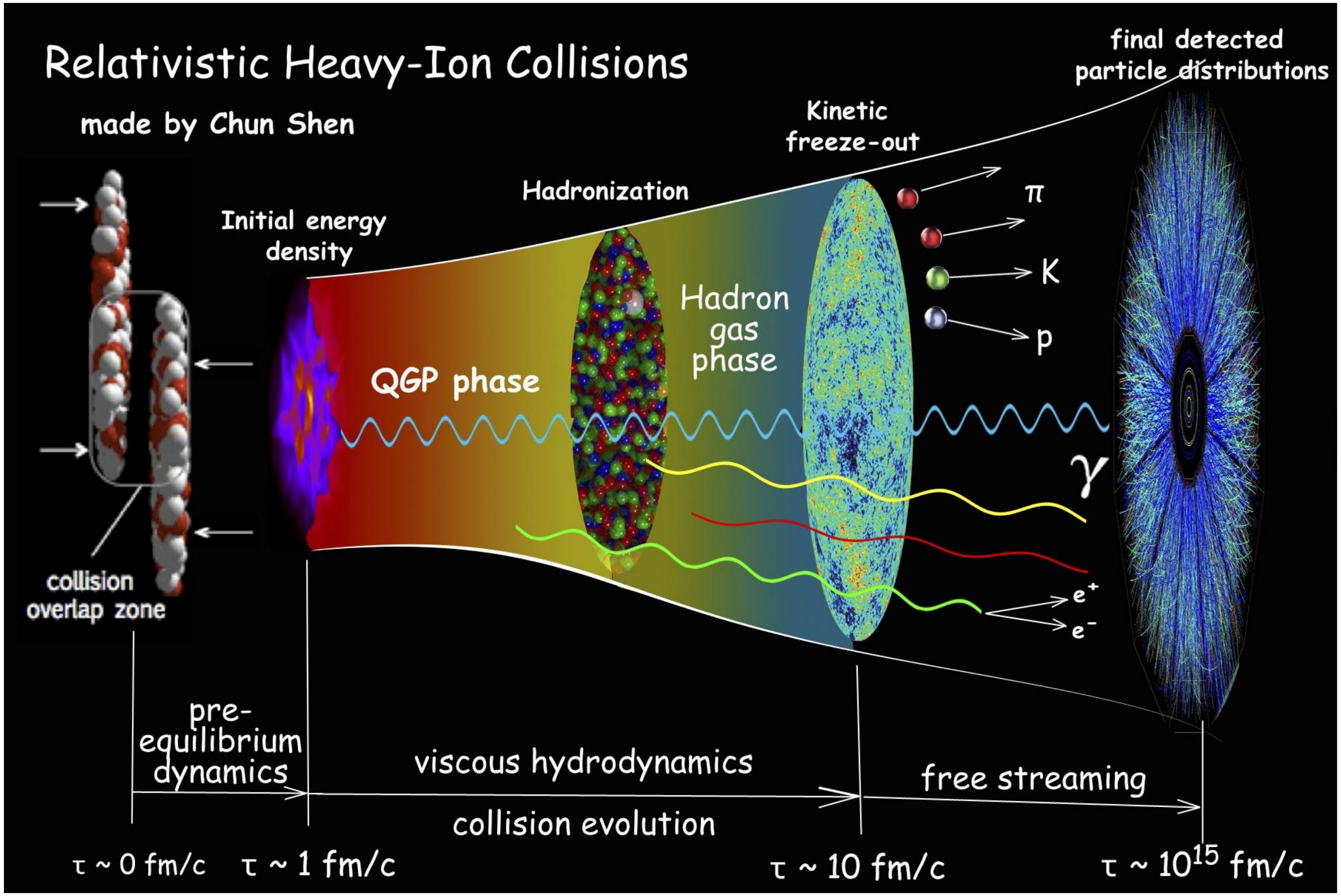
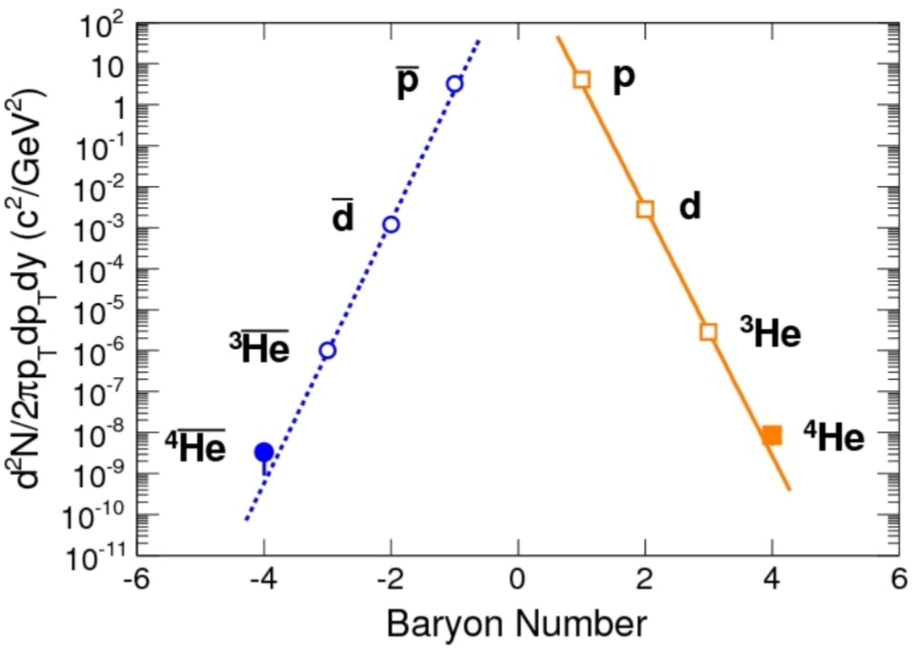
Heavy-ion collisions

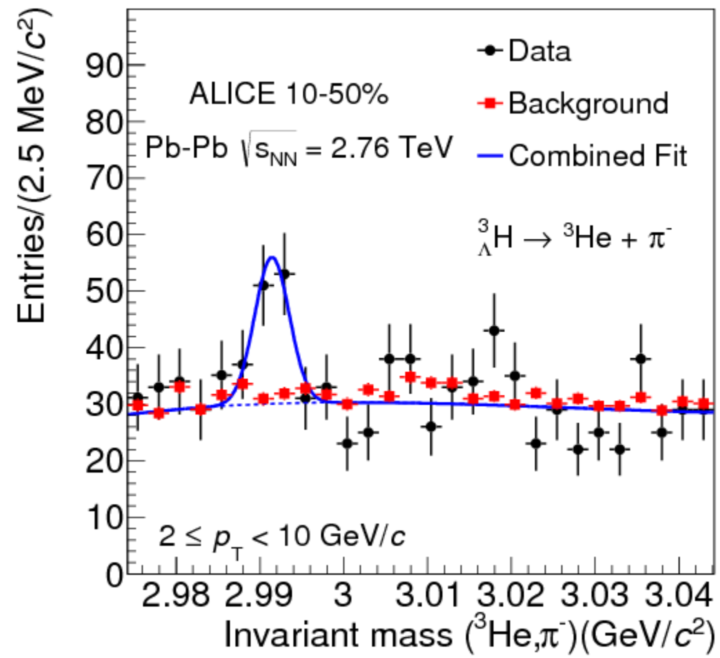


Heavy-ion collision experiments study properties of strongly interacting matter at extreme temperatures and densities, recreate conditions present in Early Universe

Loosely-bound objects in heavy-ion collisions



[STAR collaboration, Nature **473**, 353 (2011)]



[ALICE collaboration, PLB **754**, 360 (2016)]

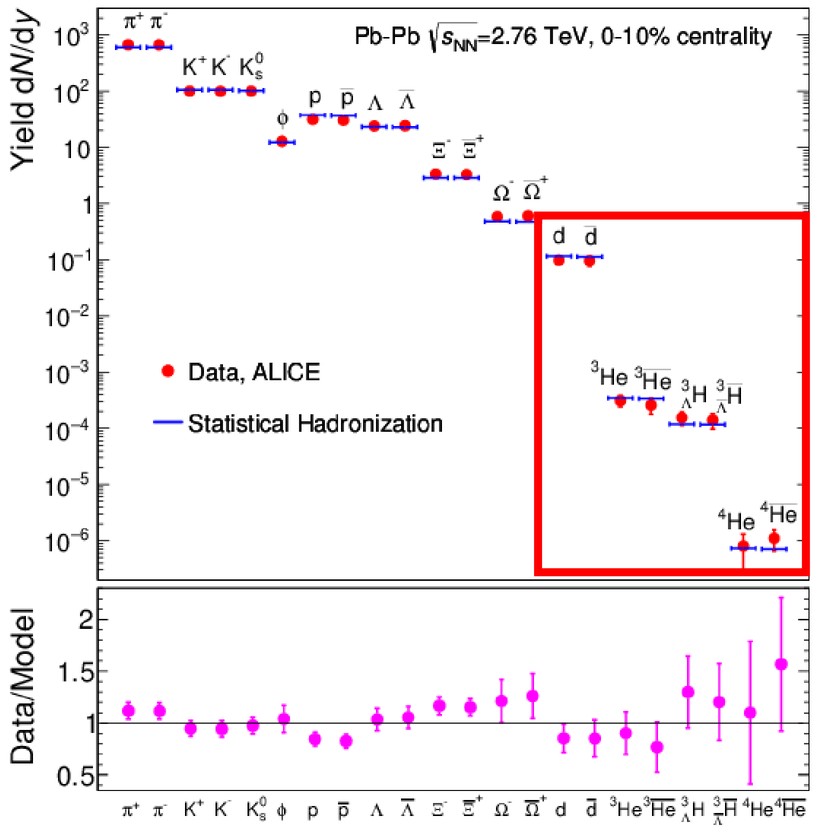
binding energies: ^{2}H, ^{3}He, ^{4}He, $^3_\Lambda$H: 2.22, 7.72, 28.3, 0.130 MeV $\ll T \sim 150$ MeV

"snowballs in hell"

The production mechanism is not established. Common approaches include **thermal** nuclei emission together with hadrons [Andronic et al., PLB '11;...] or final-state **coalescence** of nucleons close in phase-space [Butler, Pearson, PRL '61; Scheibl, Heinz, PRC '99;...]

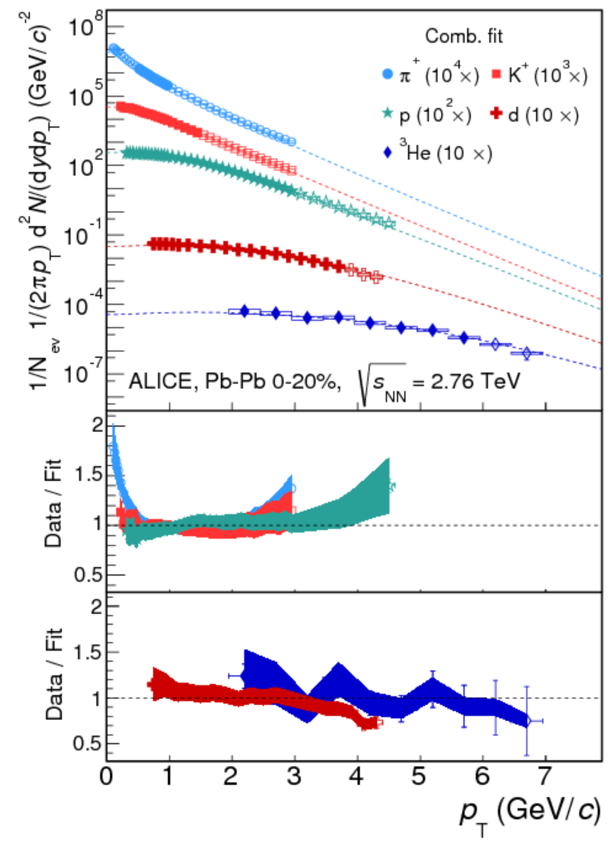
Two experimental observations at the LHC

1. Measured yields are described by thermal model at $T_{ch} \approx 155$ MeV*



[A. Andronic et al., Nature **561**, 321 (2018)]

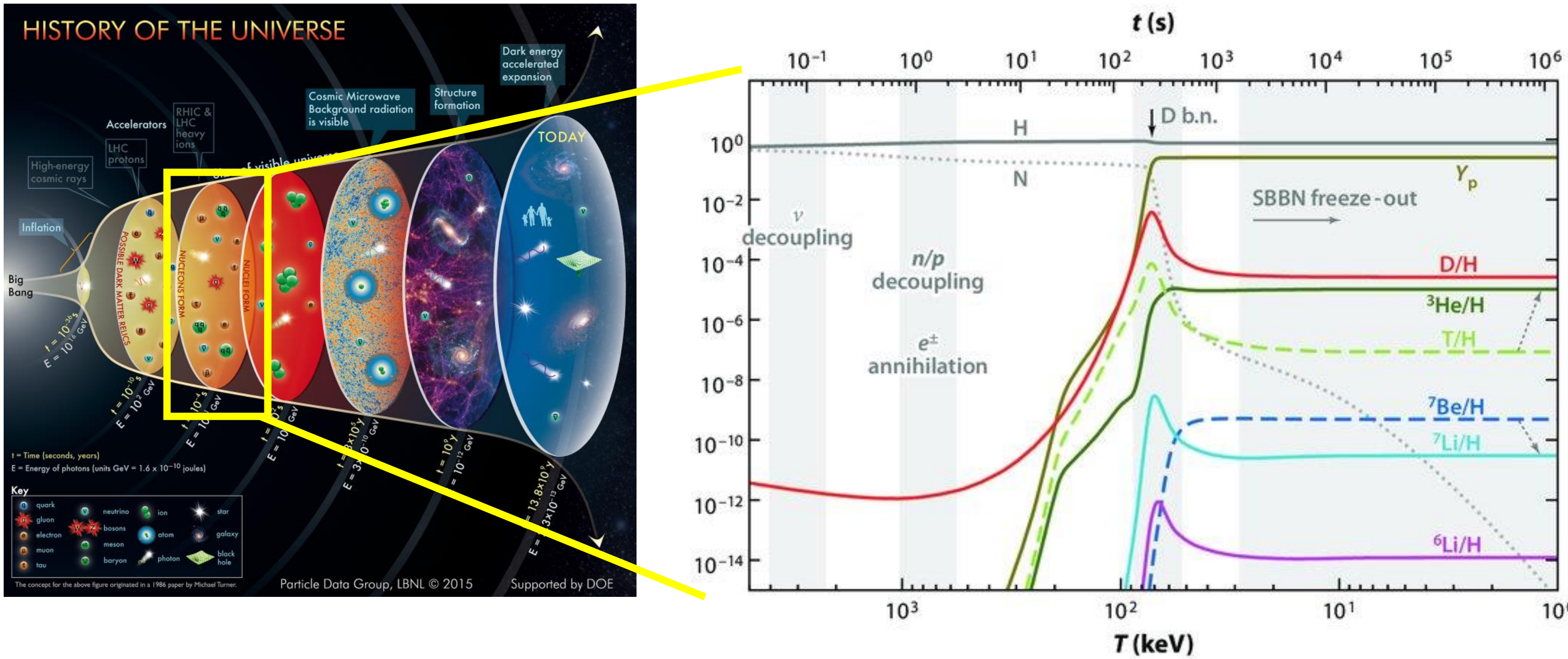
2. Spectra described by blast-wave model at $T_{kin} \approx 100 - 120$ MeV*



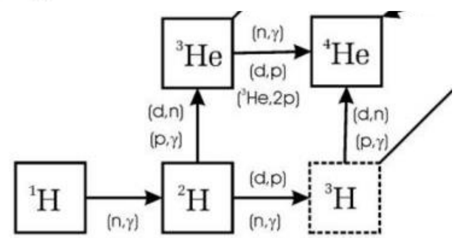
[ALICE collaboration, PRC **93**, 024917 (2016)]

What happens between $T_{\rm ch}$ and $T_{\rm kin}$?

Big Bang nucleosynthesis



- Nuclei start to form after proton-neutron ratio freeze-out (T $<$ 1 MeV)



- Early stage of Big Bang nucleosynthesis described by Nuclear Statistical Equilibrium, $\mu_A = A\mu_N$ **(Saha equation)**

$$X_A = d_A \left[\zeta(3)^{A-1}\, \pi^{\frac{1-A}{2}}\, 2^{\frac{3A-5}{2}}\right] A^{\frac{5}{2}} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta^{A-1} X_p^Z X_n^{A-Z} \exp\left(\frac{B_A}{T}\right)$$

$\eta \sim 10^{-10}$ – baryon-to-photon ratio

[E. Kolb, M. Turner, "The Early Universe" (1990)]

Saha equation (1920)

- ionization of a gas (one level)

$$X \longleftrightarrow X^+ + e^-$$

$$\frac{n_e^2}{n_0} = \frac{2}{\lambda_e^3}\frac{g_1}{g_0}\exp(-\epsilon/T) \qquad n_1 = n_e \qquad \lambda_e : \text{ deBroglie}$$

Megh Nad Saha, Phil. Mag. Series 6 40:238 (1920) 472

- equivalently, chemical potentials: $\mu_0 = \mu_1 + \mu_e$

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Nuclear equivalent: detailed balance in an expanding system (early universe/HIC)

Deuteron number evolution through $pnX \leftrightarrow dX$, in kinetic equilibrium

gain **loss**

$$\frac{dN_d}{d\tau} = \langle \sigma_{dX} v_{rel} \rangle \, n_d^0 n_X^0 e^{\mu_p/T} e^{\mu_n/T} e^{\mu_X/T} - \langle \sigma_{dX} v_{rel} \rangle \, n_d^0 n_X^0 e^{\mu_d/T} e^{\mu_X/T}$$

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small ***big*** ***big***

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small ***big*** ***big***

gain ≈ loss → $\mu_d \approx \mu_p + \mu_n$

Saha equation
= detailed balance
= law of mass action

Nucleosynthesis at "Bevalac"

VOLUME 43, NUMBER 20 PHYSICAL REVIEW LETTERS 12 NOVEMBER 1979

Evidence for a Soft Nuclear-Matter Equation of State

Philip J. Siemens[(a)] and Joseph I. Kapusta

Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

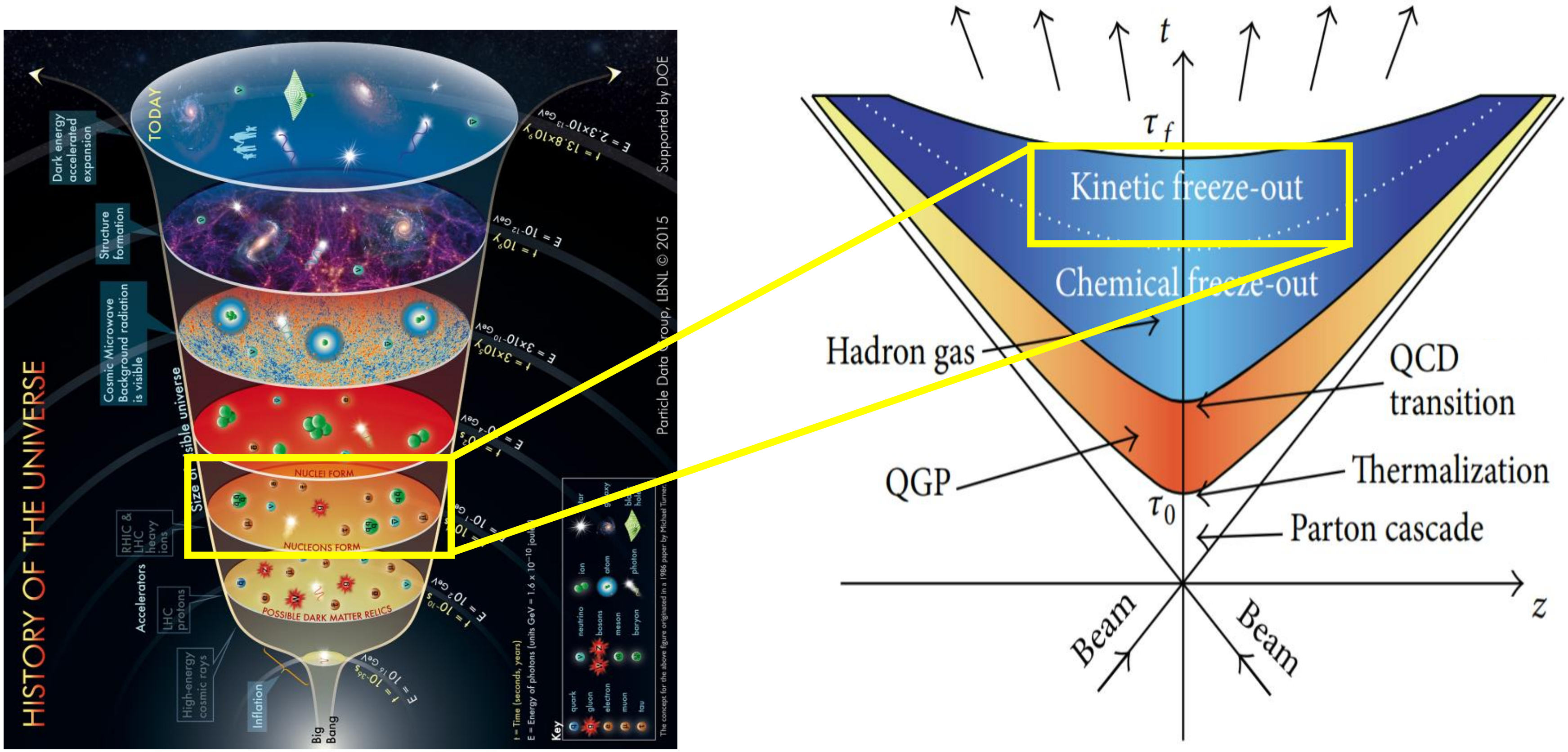
(Received 3 August 1979)

The entropy of the fireball formed in central collisions of heavy nuclei at center-of-mass kinetic energies of a few hundred MeV per nucleon is estimated from the ratio of deuterons to protons at large transverse momentum. The observed paucity of deuterons suggests that strong attractive forces are present in hot, dense nuclear matter, or that degrees of freedom beyond the nucleon and pion may already be realized at an excitation energy of 100 MeV per baryon.

Because of the reaction $d + N \leftrightarrow p + n + N$, where N is a spectator nucleon or cluster, deuterons will be constantly breaking up and reforming. If collisions are frequent enough, the deuterons will quickly reach an equilibrium concentration determined by detailed balancing[4]:

$$\exp(-\mu_d/T)d_d(\vec{R},\vec{P},S_z) = \sum_{s_z} d_p(\vec{R},\vec{P}/2,s_z)d_n(\vec{R},\vec{P}/2,S_z-s_z)\exp[-(\mu_n+\mu_p)/T],$$

Big Bang vs LHC "Little Bangs"



- Hadrons (nucleons) form and "freeze-out" chemically before nuclei
- Bosons (photons or pions) catalyse nucleosynthesis

e.g. $p + n \leftrightarrow d + \gamma$ vs $p + n + \pi \leftrightarrow d + \pi$

Big Bang vs LHC nucleosynthesis

Similarities:

- Inelastic nucleonic reactions freeze-out before nuclei formation
- Isentropic expansion of boson-dominated matter (photons in BBN vs mesons in HIC), baryon-to-boson ratio: $\eta_{BBN} \sim 10^{-10}$, $\eta_{LHC} \sim 0.05$
- Strong nuclear formation and regeneration reactions → **Saha equation**

Differences:

- Time scales: 1-100 s in BBN vs $\sim 10^{-22}$ s in HIC
- Temperatures: $T_{BBN} < 1$ MeV vs $T_{HIC} \sim 100$ MeV
- Binding energies, proton-neutron mass difference, and neutron lifetime important in BBN, less so in HICs
- $\mu_B \approx 0$ at the LHC, $\mu_B \neq 0$ in BBN
- Resonance feeddown important at LHC, irrelevant in BBN

LHC nucleosynthesis: simplified setup

- Chemical equilibrium lost at $T_{ch} = 155$ MeV, abundances of nucleons are frozen and acquire effective fugacity factors: $n_i = n_i^{eq} e^{\mu_N/T}$
- Isentropic expansion driven by effectively massless mesonic d.o.f.

$$\frac{V}{V_{\rm ch}} = \left(\frac{T_{\rm ch}}{T}\right)^3, \qquad \mu_N \simeq \frac{3}{2} T \ln\left(\frac{T}{T_{\rm ch}}\right) + m_N \left(1 - \frac{T}{T_{\rm ch}}\right)$$

- Detailed balance for nuclear reactions, $X + A \leftrightarrow X + \sum_i A_i$, X is e.g. a pion

$$\frac{n_A}{\prod_i n_{A_i}} = \frac{n_A^{\rm eq}}{\prod_i n_{A_i}^{\rm eq}}, \quad \Leftrightarrow \quad \mu_A = \sum_i \mu_{A_i}, \qquad \text{e.g.} \;\; \mu_d = \mu_p + \mu_n, \;\; \mu_{^3{\rm He}} = 2\mu_p + \mu_n, \;\; \ldots$$

Saha equation

$$X_A = d_A \left[(d_M)^{A-1} \zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{-\frac{3+A}{2}}\right] A^{5/2} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta_B^{A-1} \exp\left(\frac{B_A}{T}\right)$$

$d_M \sim 11 - 13, \quad \eta_B \simeq 0.03$

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$$d_M \sim 11 - 13, \quad \eta_B \simeq 0.03$$

BBN: $$X_A = d_A \left[\zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{\frac{3A-5}{2}}\right] A^{\frac{5}{2}} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta^{A-1} X_p^Z X_n^{A-Z} \exp\left(\frac{B_A}{T}\right)$$

[E. Kolb, M. Turner, "The Early Universe" (1990)]

(Simplified) Saha equation vs thermal model

Saha equation:
$$\frac{N_A(T)}{N_A(T_{\rm ch})} \simeq \left(\frac{T}{T_{\rm ch}}\right)^{\frac{3}{2}(A-1)} \exp\left[B_A\left(\frac{1}{T}-\frac{1}{T_{\rm ch}}\right)\right] \qquad B_A \ll T$$

Thermal model:
$$\left[\frac{N_A(T)}{N_A(T_{\rm ch})}\right]_{\rm eq.} \simeq \left(\frac{T}{T_{\rm ch}}\right)^{-\frac{3}{2}} \exp\left[-m_A\left(\frac{1}{T}-\frac{1}{T_{\rm ch}}\right)\right] \qquad m_A \gg T$$

Strong exponential dependence on the temperature is eliminated in the Saha equation approach

Further, quantitative applications require numerical treatment of full spectrum of *massive* mesonic and baryonic resonances

Full numerical implementation

Expansion of hadron resonance gas in partial chemical equilibrium at $T < T_{ch}$

[H. Bebie, P. Gerber, J.L. Goity, H. Leutwyler, Nucl. Phys. B '92]

Chemical composition of stable hadrons is fixed, kinetic equilibrium maintained through quasi-elastic resonance reactions $\pi\pi \leftrightarrow \rho,\ \pi \mathrm{K} \leftrightarrow \mathrm{K}^*, \pi N \leftrightarrow \Delta$, etc.

Effective chemical potentials:

$$\tilde{\mu}_j = \sum_{i \in \text{stable}} \langle n_i \rangle_j \, \mu_i , \qquad \langle n_i \rangle_j \text{ – mean number of hadron } i \text{ from decays of hadron } j, \qquad j \in \text{HRG}$$

Conservation laws:

$$\sum_{j \in \text{hrg}} \langle n_i \rangle_j \, n_j(T, \tilde{\mu}_j) \, V = N_i(T_{\text{ch}}), \quad i \in \text{stable}$$

$$\sum_{j \in \text{hrg}} s_j(T, \tilde{\mu}_j) \, V = S(T_{\text{ch}})$$

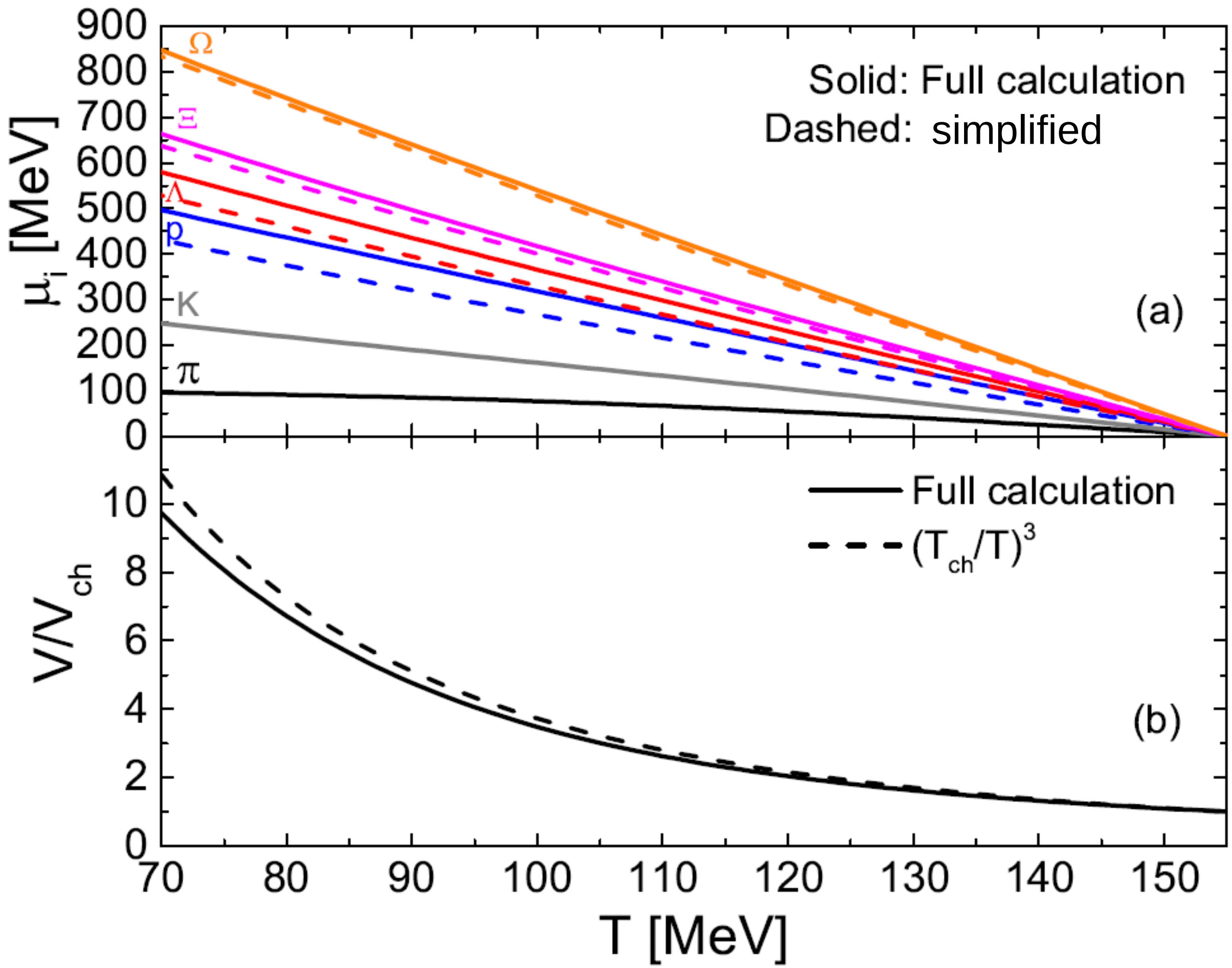
numerical solution → $\{\mu_i(T)\}, V(T)$

Numerical implementation within (extended) **Thermal-FIST** package

[**V.V.**, H. Stoecker, *Computer Physics Communications* ***244,*** *295 (2019)*]

open source: https://github.com/vlvovch/Thermal-FIST

Full calculation: parameters

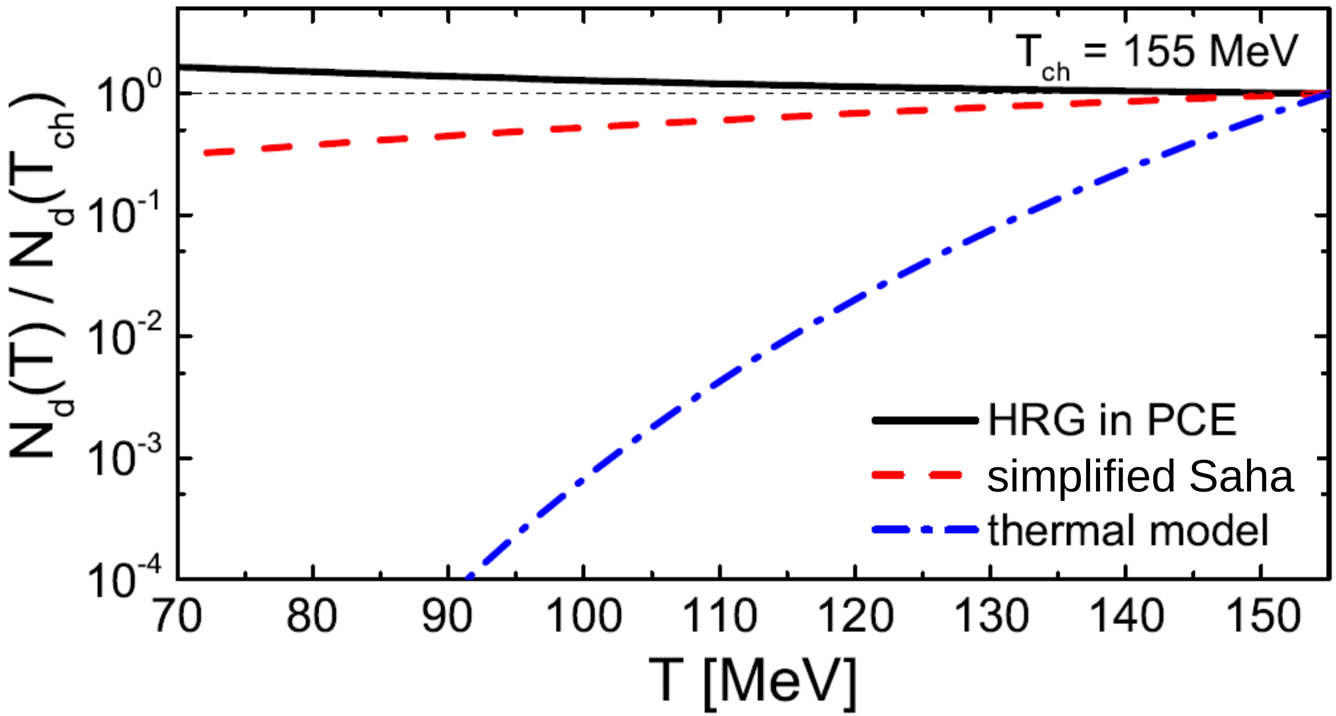


"Initial conditions" from thermal fits with Thermal-FIST to 0-10% ALICE hadron yields

[**V.V.**, Gorenstein, Stoecker, 1807.02079]

$T_{ch} = 155$ MeV, $V_{ch} = 4700$ fm^3

Full calculation: deuteron yield



Resonance feed-down is important in precision studies

LHC deuteron-synthesis

PHYSICAL REVIEW C **99**, 044907 (2019)

Editors' Suggestion | Featured in Physics

Microscopic study of deuteron production in PbPb collisions at $\sqrt{s} = 2.76$ TeV via hydrodynamics and a hadronic afterburner

Dmytro Oliinychenko,[1] Long-Gang Pang,[1,2] Hannah Elfner,[3,4,5] and Volker Koch[1]

[1] *Lawrence Berkeley National Laboratory, 1 Cyclotron Rd, Berkeley, California 94720, USA*

[2] *Physics Department, University of California, Berkeley, California 94720, USA*

[3] *Frankfurt Institute for Advanced Studies, Ruth-Moufang-Strasse 1, 60438 Frankfurt am Main, Germany*

[4] *Institute for Theoretical Physics, Goethe University, Max-von-Laue-Strasse 1, 60438 Frankfurt am Main, Germany*

[5] *GSI Helmholtzzentrum für Schwerionenforschung, Planckstr. 1, 64291 Darmstadt, Germany*

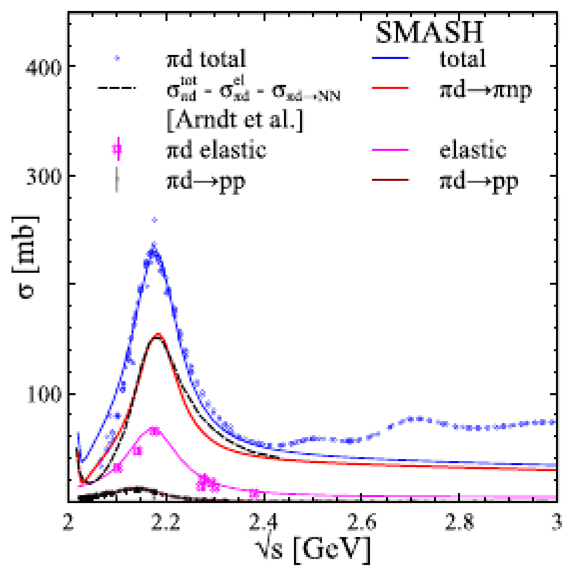


FIG. 1. Deuteron-pion interaction cross sections from SAID database [40] and partial wave analysis [41] are compared to our parametrizations (Tables II and III in the Appendix). Inelastic $d\pi \leftrightarrow$

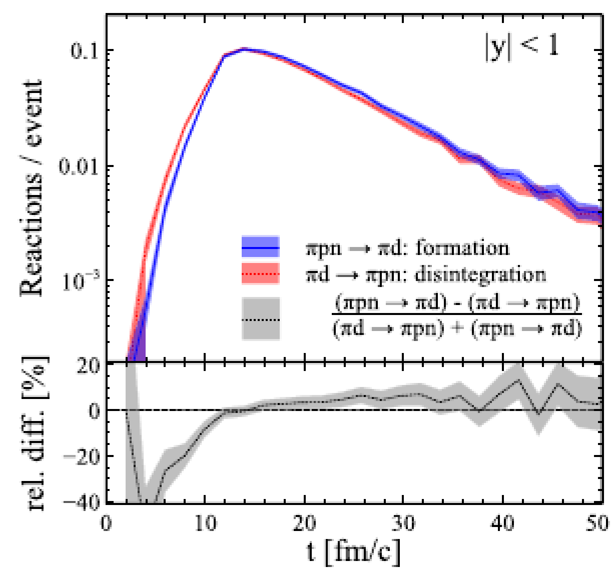
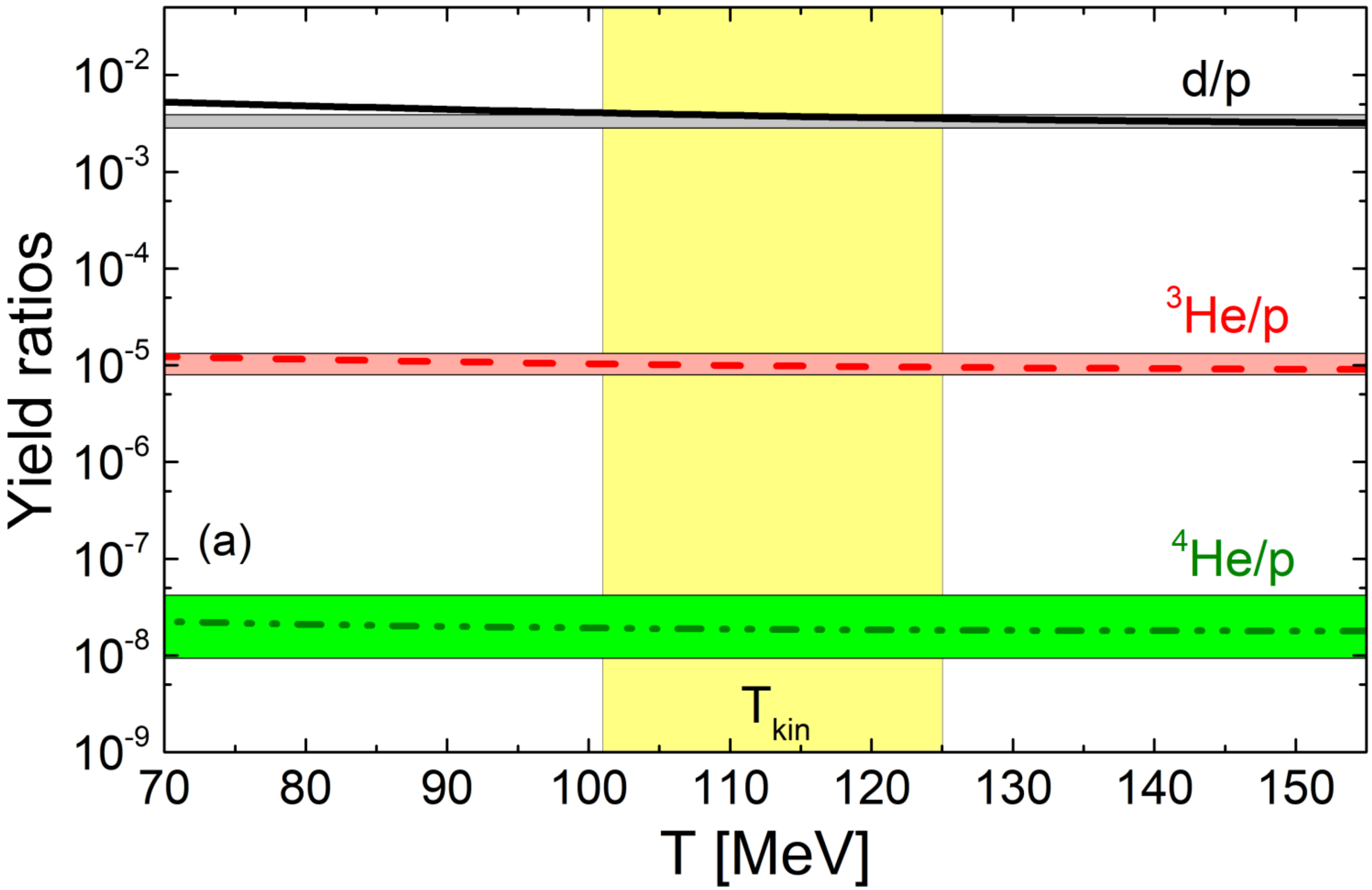


FIG. 5. Reaction rates of the most important $\pi d \leftrightarrow \pi pn$ reaction in forward and reverse direction.

- Law of mass action at work

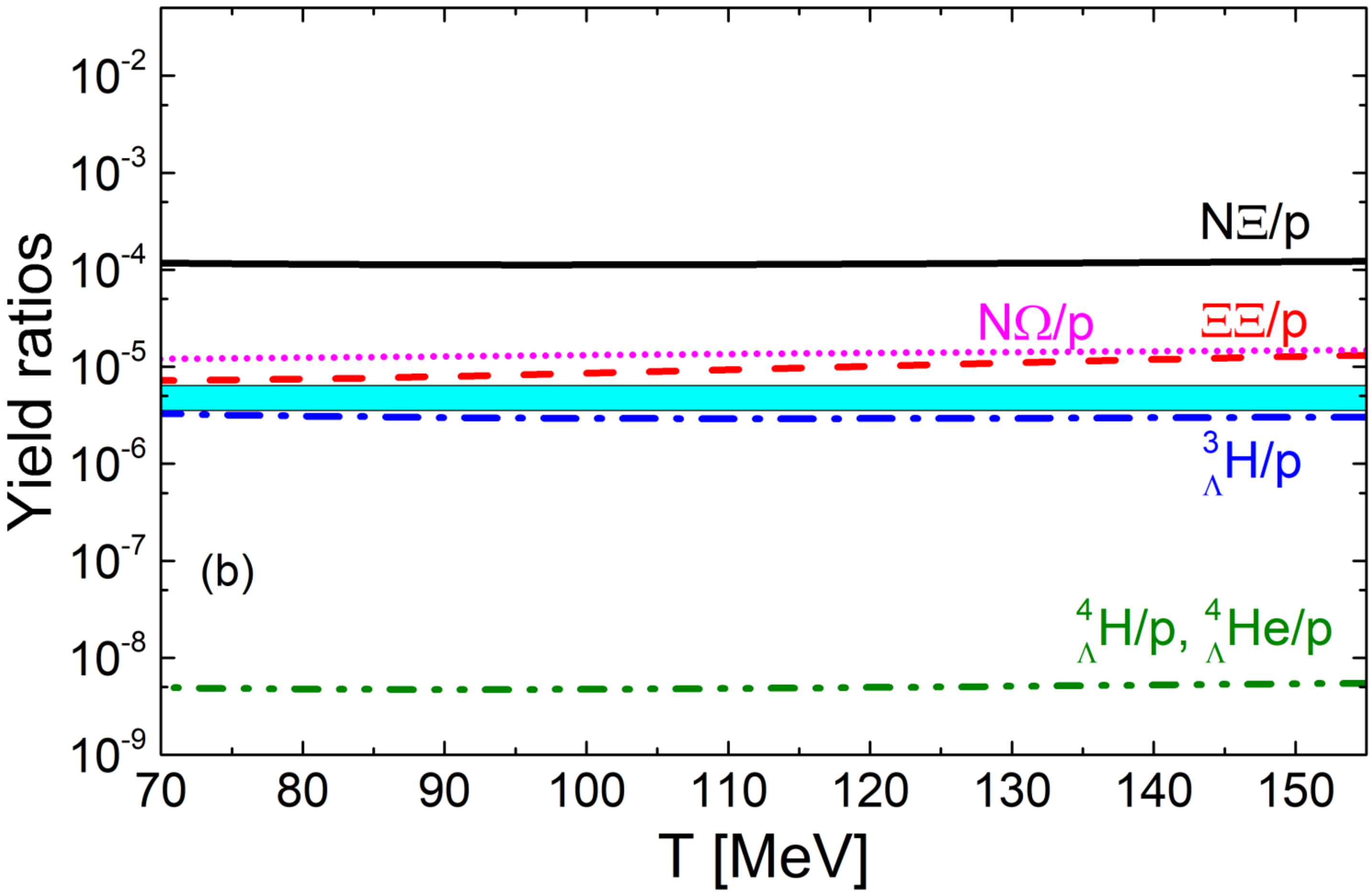
Full calculation: nuclei



Deviations from thermal model predictions are moderate despite significant cooling and dilution. Is this the reason for why thermal model works so well?

For $T = T_{kin}$ similar results reported in [X. Xu, R. Rapp, EPJA 55, 68 (2019)]

Full calculation: hypernuclei



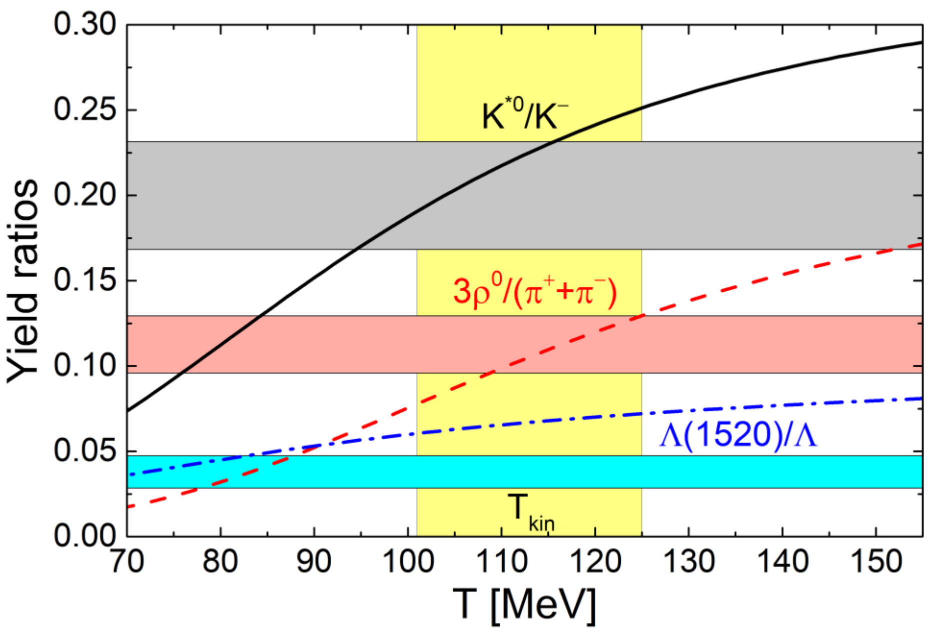
Hypernuclei stay close to the thermal model prediction. An exception is a hypothetical ΞΞ state ← *planned measurement in Runs 3 & 4 at the LHC*

[LHC Yellow Report, 1812.06772]

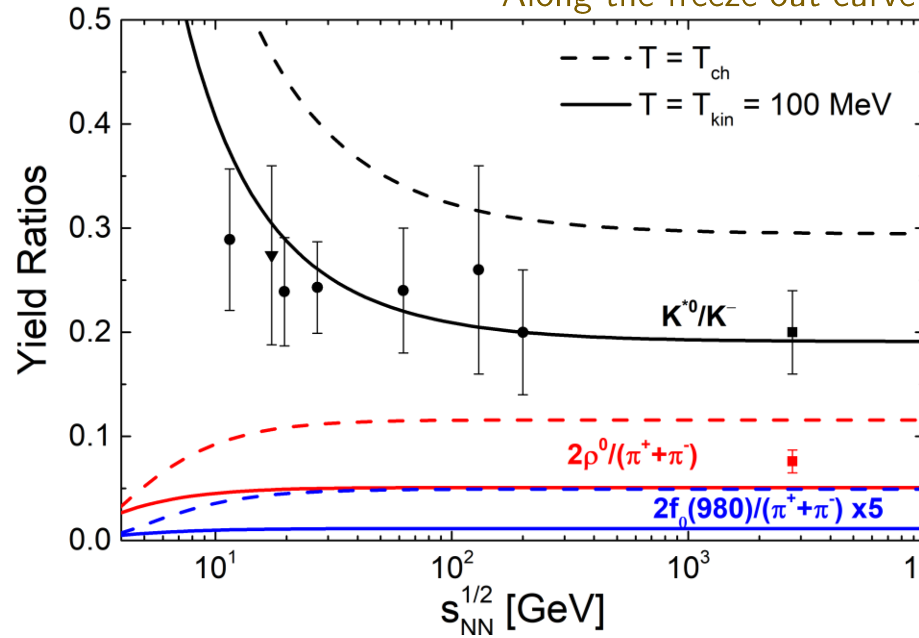
Resonance suppression in hadronic phase

Yields of resonances are *not* conserved in partial chemical equilibrium

E.g. K^* yield dilutes during the cooling through $\pi\mathrm{K} \leftrightarrow \mathrm{K}^*$, $\mu_{K^*} = \mu_K + \mu_\pi$



Along the freeze-out curve



with A. Motornenko

At $T \approx T_{kin}$ the suppressed resonance yields agree quite well with ALICE/RHIS/SPS data for central A+A collisions

This implies significant resonance regeneration in the hadronic phase

Kinetic freeze-out temperature

The final decoupling temperature, T_{kin}, typically estimated by fitting π, K, p p_T spectra with the Blast-Wave model [E. Schnedermann, J. Sollfrank, U. Heinz, PRC '93]

$$\frac{1}{p_T}\frac{dN}{dp_T} \propto \int_0^R r\mathrm{d}r\, m_T I_0\left(\frac{p_T \sinh\rho}{T_{kin}}\right) K_1\left(\frac{m_T \cosh\rho}{T_{kin}}\right), \qquad \rho = \tanh^{-1}\beta_T = \tanh^{-1}\left(\left(\frac{r}{R}\right)^n \beta_s\right)$$

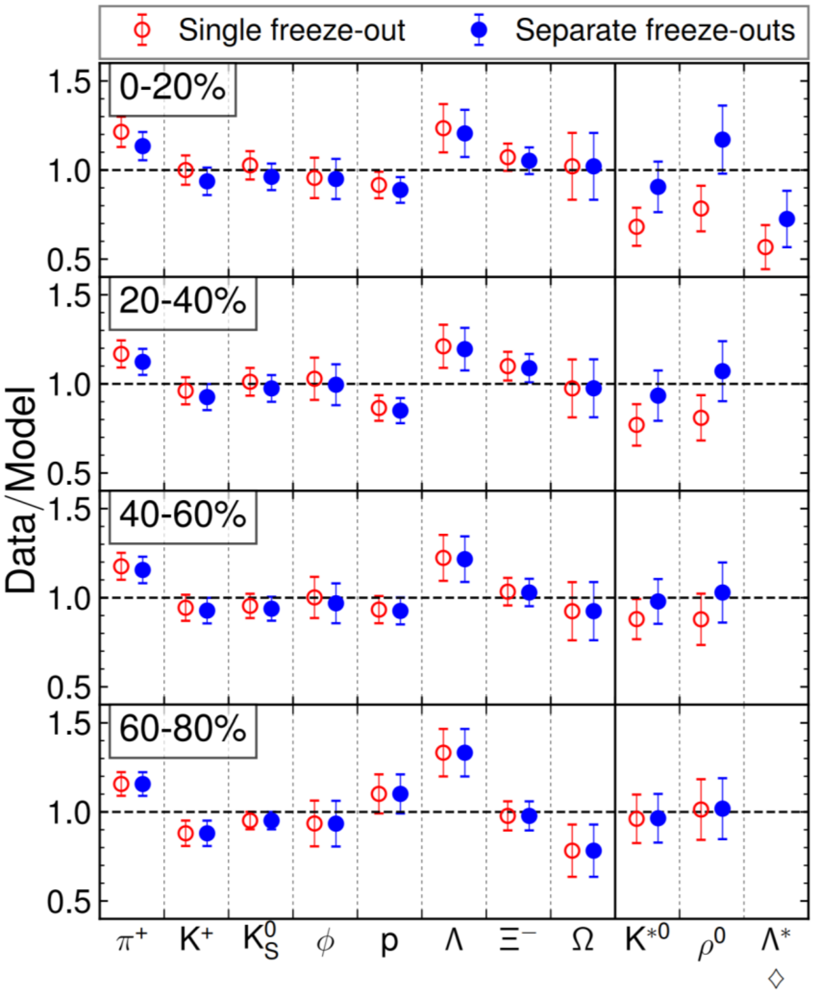
Blast-wave fits, 0-10% Pb+Pb collisions at 2.76 TeV:

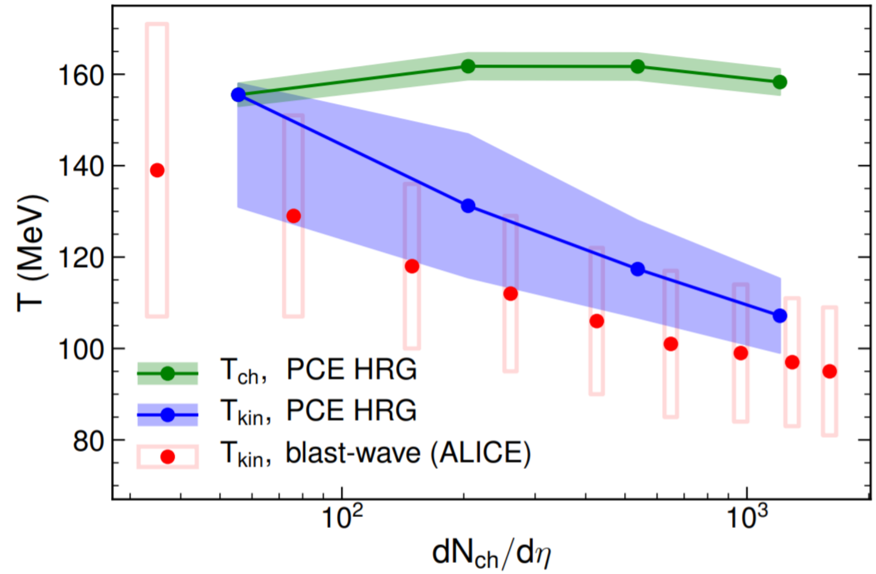
1. ALICE collaboration [1303.0737] $T_{kin} = 96 \pm 11$ MeV
2. Fit with resonances [A. Mazeliauskas, V. Vislavicius, 1907.11059] $T_{kin} = 126 \pm 2$ MeV
3. Another fit with resonances [I. Melo, B. Tomasik, 1908.03023] $T_{kin} = 78 \pm 2$ MeV

No agreement between different blast-wave fits

Kinetic freeze-out temperature from resonances

Fit of K^{*0} and ρ^0 abundances extracts the kinetic freeze-out temperature

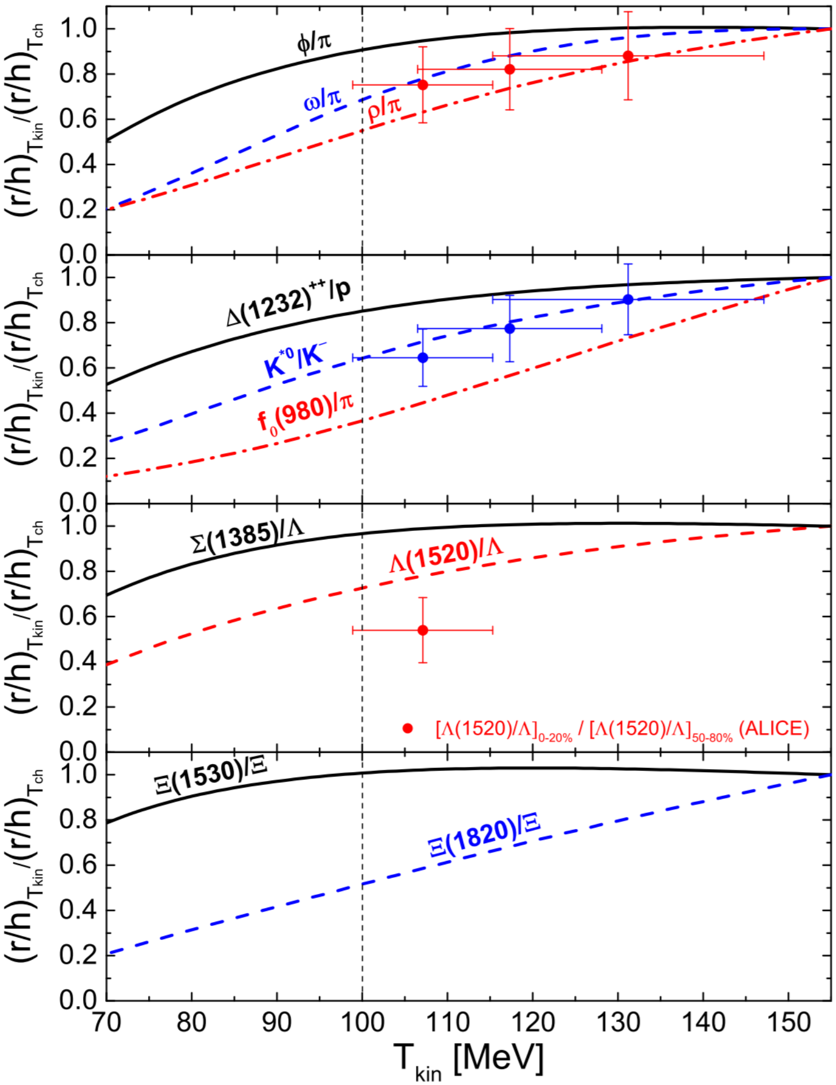




Centrality	$T_{\rm ch}$ (MeV)	$T_{\rm kin}$ (MeV)	χ^2/dof
0-20%	160.2 ± 3.1	–	23.6/8
	158.3 ± 2.8	107.1 ± 8.2	10.5/7
20-40%	162.9 ± 3.1	–	19.5/8
	161.7 ± 2.9	117.3 ± 10.8	12.8/7
40-60%	162.3 ± 3.0	–	12.5/8
	161.8 ± 2.9	131.2 ± 15.9	10.6/7
60-80%	155.5 ± 2.5	–	19.1/8
	155.5 ± 2.5	$155.5^{+2.5}_{-24.5}$	19.1/7

Solves the T_{kin}-*vs*-$\langle\beta_T\rangle$ anticorrelation problem of blast-wave fits

Predictions for other resonances



- Short-lived Δ(1232) and Σ(1385) not suppressed

- $f_0(980)$ suppressed if it interacts in the hadronic phase

  → if confirmed experimentally, will imply a short $f_0(980)$ lifetime

Summary and outlook

- Nucleosynthesis in HICs at LHC via the Saha equation is in analogy to initial stages of big bang nucleosynthesis in the early universe. Results agree with the thermal model, but *any* $T < T_{ch}$ permitted!

- Kinetic freeze-out temperature extracted directly from yields of short-lived resonances. Result: $T_{kin} = 107 \pm 8$ MeV at LHC (0-20%).

- Outlook: Beyond Saha with rate equations and/or transport

$$\frac{d(Vn_A)}{d\tau} = \langle \sigma_{A\pi} v_{rel} \rangle \, n_A^0 n_\pi \left(e^{A\mu_N/T} - e^{\mu_A/T} \right)$$

$$\frac{d(Vn_R)}{d\tau} = \langle \Gamma_{R \to \sum_i a_i} \rangle \, n_R^0 \left(e^{\sum_i \mu_{a_i}/T} - e^{\mu_R/T} \right)$$

[D. Oliinychenko, in progress]

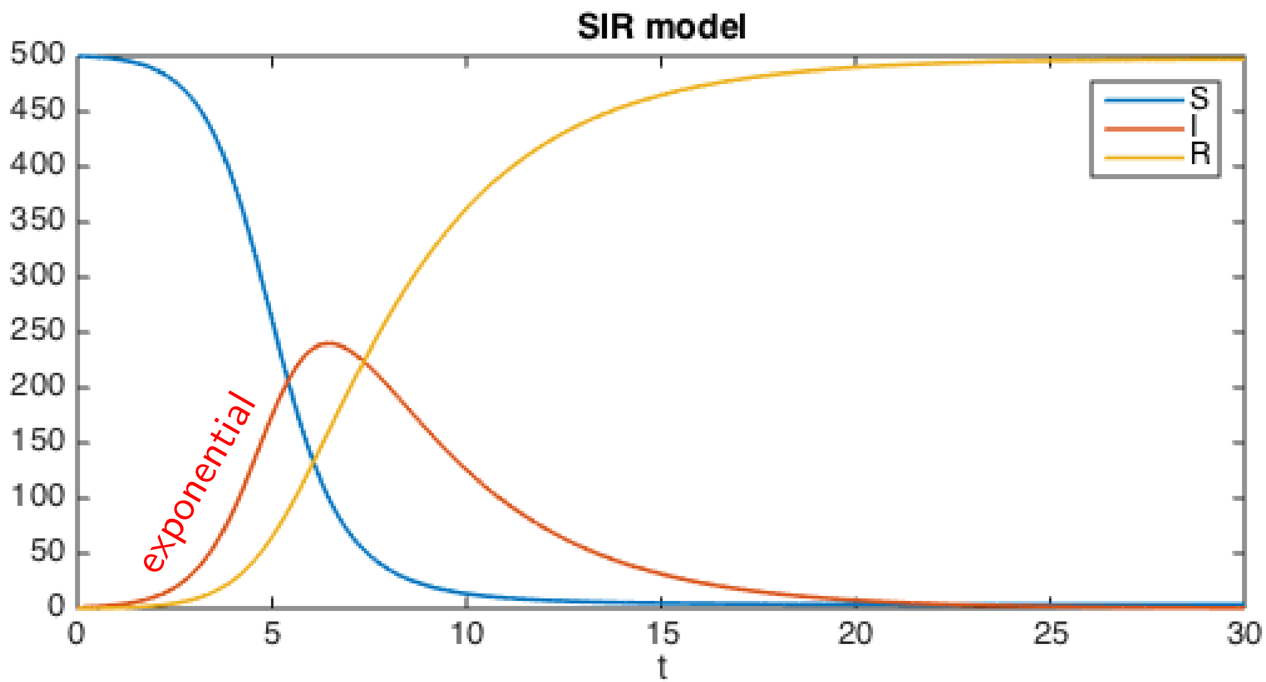
Rate equations and COVID-19

The SIR (Susceptible-Infected-Recovered) model:

$$\frac{dN_s}{dt} = -\frac{R_0}{\tau_I} N_s N_I$$

$$\frac{dN_I}{dt} = \frac{R_0}{\tau_I} N_s N_I - \frac{N_I}{\tau_I}$$

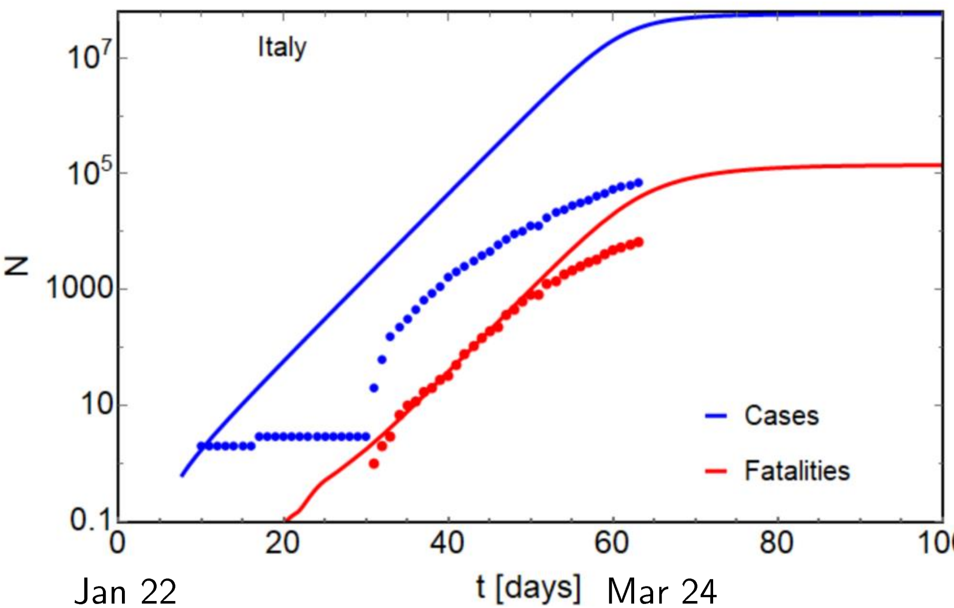
For $N_I \ll N_S$: $\quad N_I(t) \propto \exp\left(\frac{R_0 - 1}{\tau_I} t\right)$

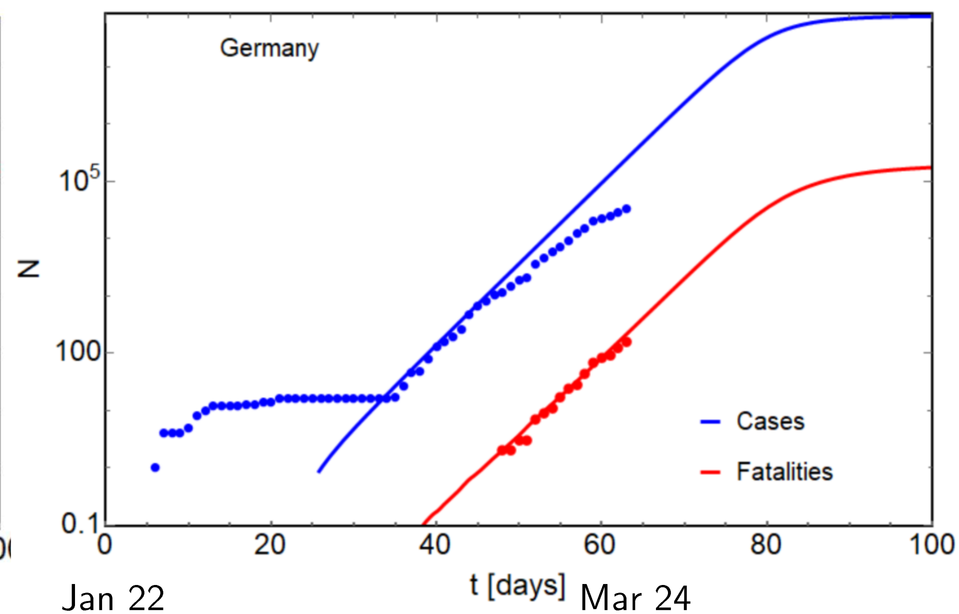


Rate equations and COVID-19

Some SIR model fits to the data

https://github.com/CSSEGISandData/COVID-19/tree/master/csse_covid_19_data



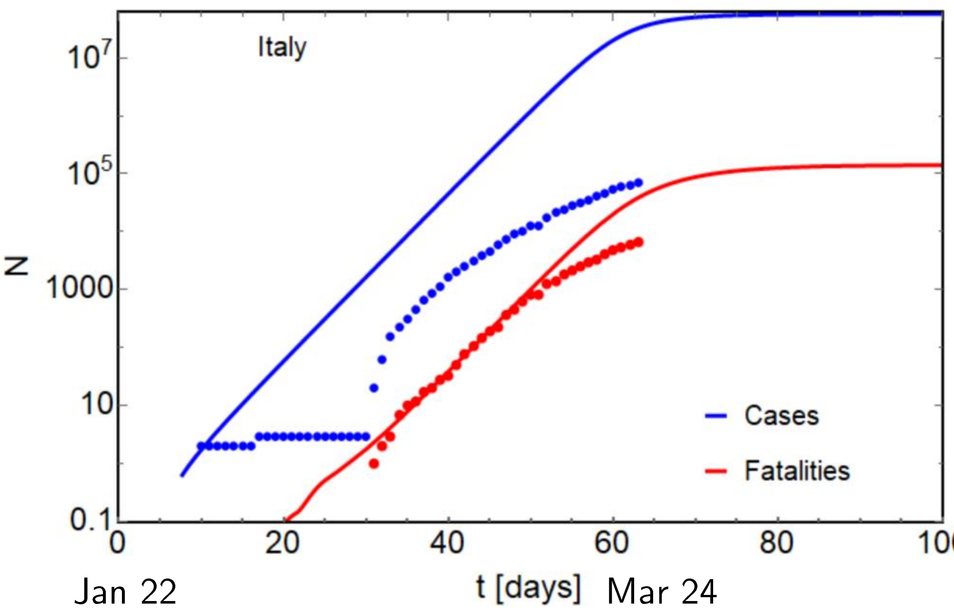


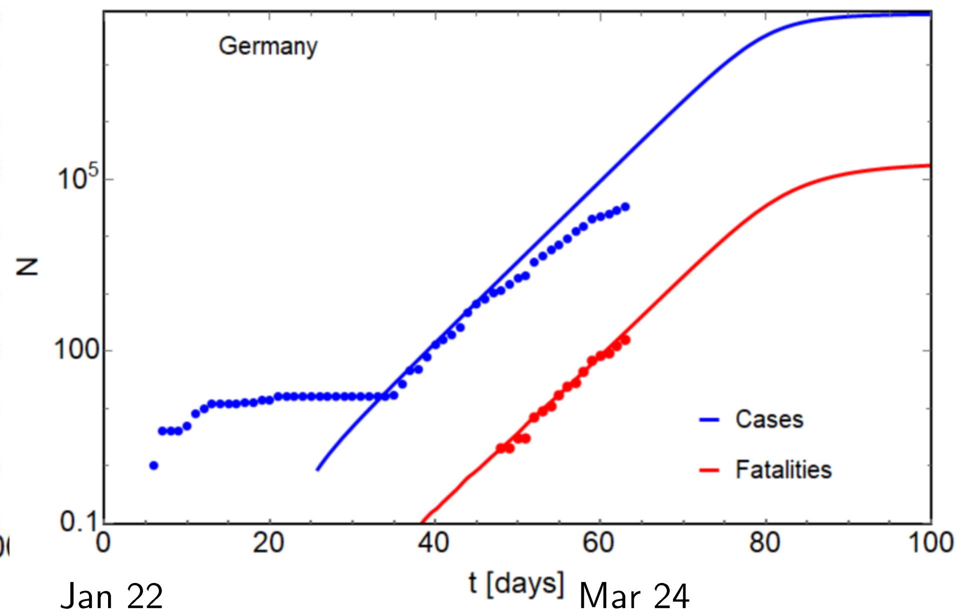
$$R_0 \approx 3$$

Rate equations and COVID-19

Some SIR model fits to the data

https://github.com/CSSEGISandData/COVID-19/tree/master/csse_covid_19_data





$$R_0 \approx 3$$

Thank you and stay healthy!