Probing the QCD equation of state with fluctuations of conserved charges

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XXXII International Workshop on High Energy Physics "Hot problems of Strong Interactions"

November 12, 2020

Acknowledgements:

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QCD phase diagram

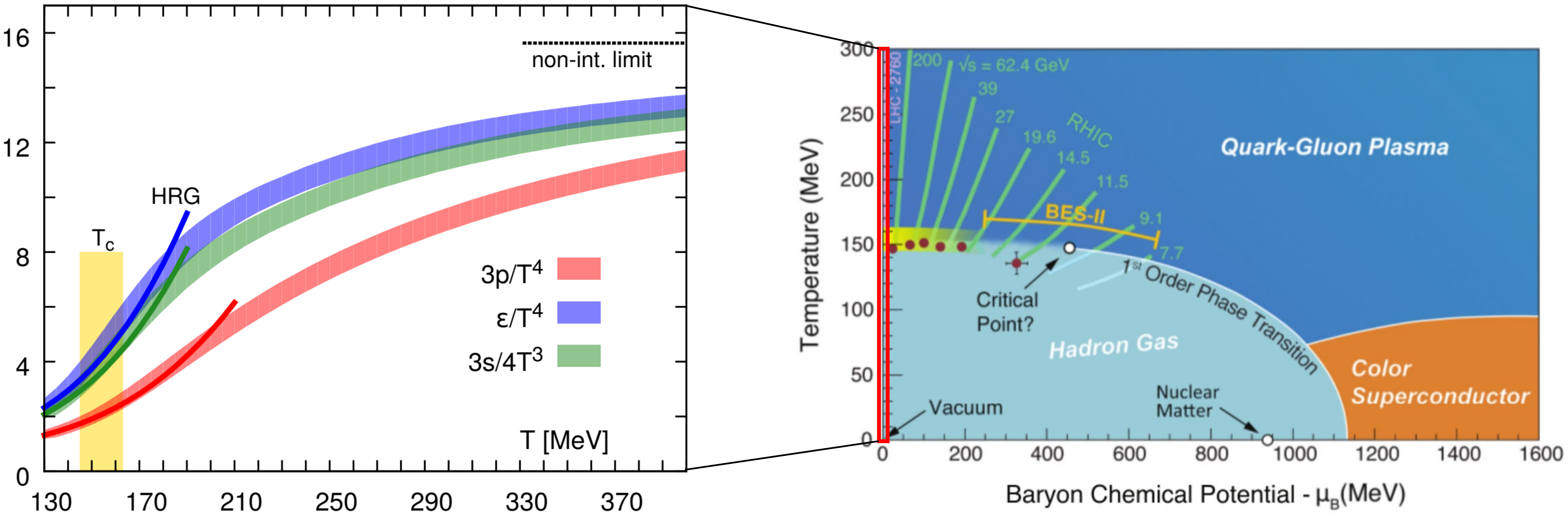


Figure from HotQCD coll., PRD '14

- Analytic crossover at vanishing net baryon density – a first-principle result from lattice QCD
- Phase structure at finite density is largely unknown
- Probed by heavy-ion collisions

Fluctuations of conserved charges

Consider a random variable N

Cumulants: $K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$

variance $\kappa_2 = \langle(\Delta N)^2\rangle = \sigma^2$  width

skewness $\kappa_3 = \langle(\Delta N)^3\rangle$  asymmetry

kurtosis $\kappa_4 = \langle(\Delta N)^4\rangle - 3\langle(\Delta N^2)\rangle^2$  peak shape

Statistical mechanics:

Grand partition function $$\ln Z^{\mathrm{gce}}(T, V, \mu) = \ln\left[\sum_N e^{\mu N} Z^{\mathrm{ce}}(T, V, N)\right],$$ $$\boxed{\kappa_n \propto \frac{\partial^n(\ln Z^{\mathrm{gce}})}{\partial(\mu_N)^n}}$$

Fluctuations probe finer details of the (QCD) equation of state

Common uses of thermal fluctuations

- Taylor expansion of the equation of state

$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \frac{\chi_2^B(T,0)}{2!}(\mu_B/T)^2 + \frac{\chi_4^B(T,0)}{4!}(\mu_B/T)^4 + \ldots$$

$$\chi_n(T,\mu_B) = \frac{\partial^n(p/T^4)}{\partial(\mu_B/T)^n} \qquad \text{– susceptibilities}$$

- Fluctuation signals of the QCD critical point

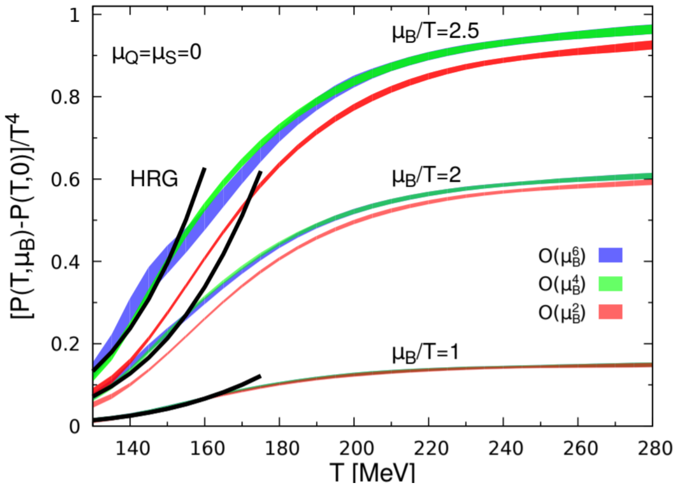
$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7, \quad \xi \to \infty$$

[M. Stephanov, PRL '09]

- Chiral criticality at $\mu_B = 0$

[Friman, Karsch, Redlich, Skokov, EPJC '11]

Figure from HotQCD coll., PRD '17



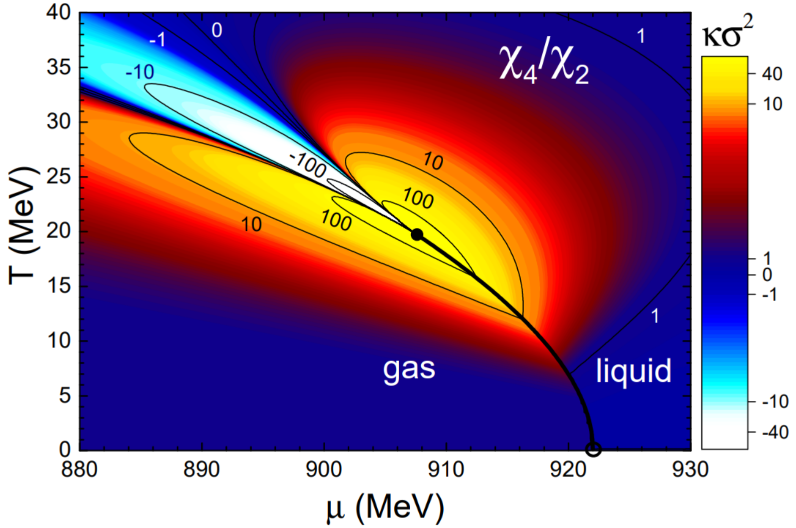


Figure from VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC '15

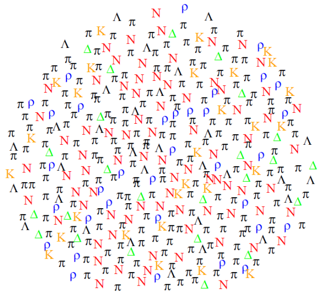
Constraining the excluded volume interactions in hadron resonance gas model

- Matching HRG to lattice QCD at heavy-ion freeze-out stage
- First step toward connecting thermal (grand-canonical) QCD fluctuations with experimental measurements

VV, M.I. Gorenstein, H. Stoecker, *Phys. Rev. Lett. 118, 182301 (2017)*
VV, A. Pasztor, S.D. Katz, Z. Fodor, H. Stoecker, *Phys. Lett. B 775, 71 (2017)*

Hadron resonance gas (HRG) model

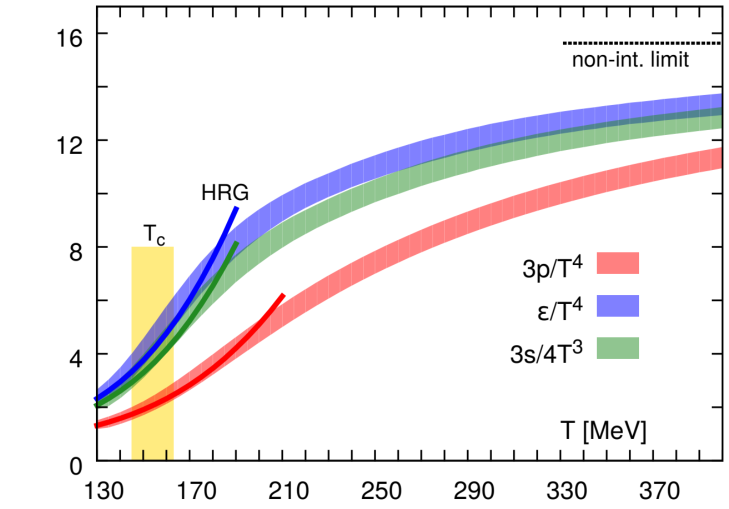
- **HRG model:** free gas of known hadrons and resonances

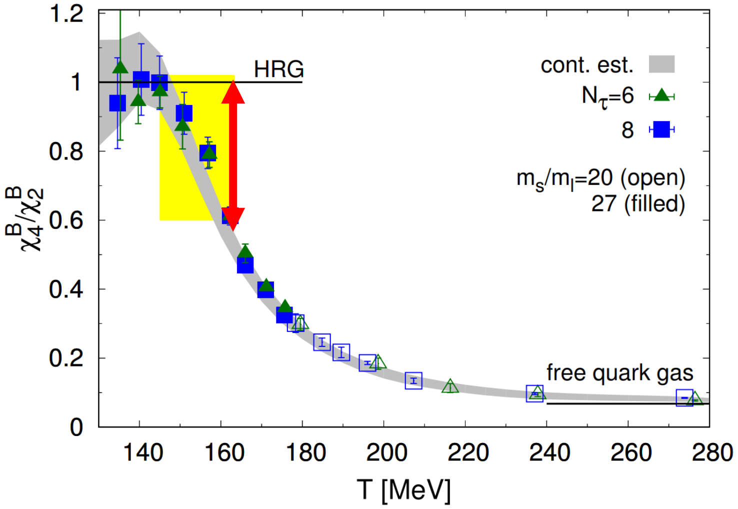


$$p(T,\mu_B) = \underbrace{T\,\phi_M(T)}_{\text{mesons}} + 2\,\underbrace{T\,\phi_B(T)}_{\text{baryons}}\cosh(\mu_B/T)$$

$$\phi_{M(B)}(T) = \sum_{i\in M(B)} \frac{d_i\, m_i^2\, T}{2\pi^2} K_2(m_i/T)$$

  - Hadronic interactions dominated by resonance formation*
  - Single term in relativistic virial expansion
  - Rich history in describing hadron yields in heavy-ion collisions
  - Matches well with lattice QCD below T_{pc}





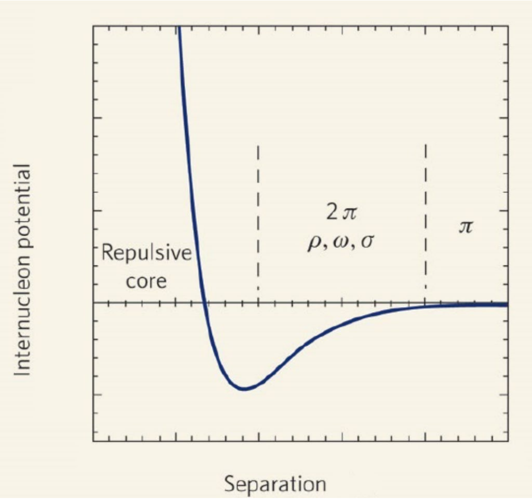
[HotQCD collaboration, 1701.04325]

- Net baryon fluctuations: *Skellam distribution*
  - $\chi^B_{2n} \propto \langle N_B\rangle + \langle N_{\bar{B}}\rangle$, $\chi^B_{2n-1} \propto \langle N_B\rangle - \langle N_{\bar{B}}\rangle$
  - LQCD suggests breakdown of the model close to $T_{pc} \approx 155$ MeV

* *Dashen, Ma, Bernstein, "S-matrix formulation of statistical mechanics", Phys. Rev. (1969); Prakash, Venugopalan, NPA '92*

Excluded volume HRG model

EV-HRG model: incorporate repulsive baryon-baryon, antibaryon-antibaryon interactions [VV, Gorenstein, Stoecker, PRL '17; VV, Pasztor, Katz, Fodor, Stoecker, PLB '17]



$$p(T,\mu_B) = p_M^{\text{id}}(T) + p_B^{\text{ev}}(T,\mu_B) + p_{\bar{B}}^{\text{ev}}(T,\mu_B)$$

$$p_{B(\bar{B})}^{\text{ev}} = p_{B(\bar{B})}^{\text{id}}\, e^{-b p_{B(\bar{B})}^{\text{ev}}/T} \qquad \text{or} \qquad p_{B(\bar{B})}^{\text{ev}} = \frac{T}{b} W[b\,\phi_B(T)\,e^{\pm\mu_B/T}]$$

Lambert W function

Net baryon fluctuations no longer Skellam

$$\frac{\chi_4^B}{\chi_2^B} = \frac{1 - 8W(b\phi_B) + 6[W(b\phi_B)]^2}{[1+W(b\phi_B)^4]} \simeq 1 - 12\,b\,\phi_B(T) + O(b^2\phi_B^2)$$

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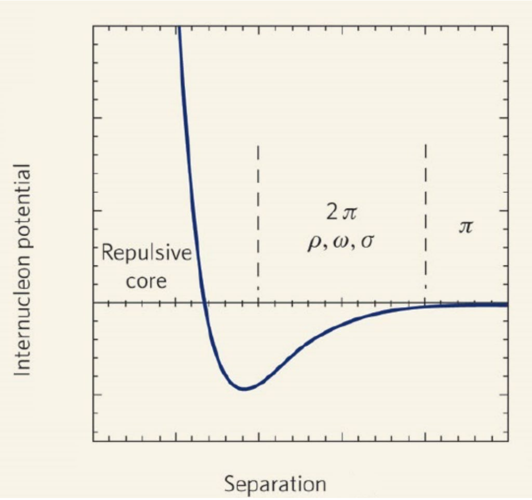

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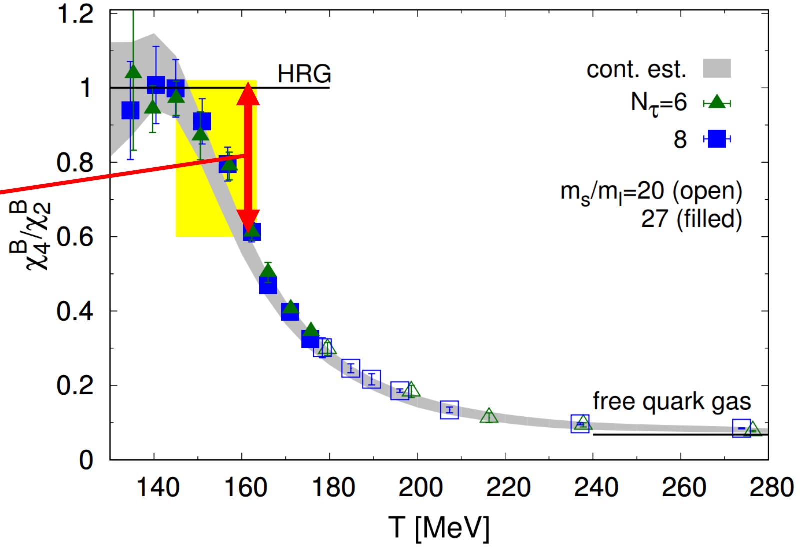
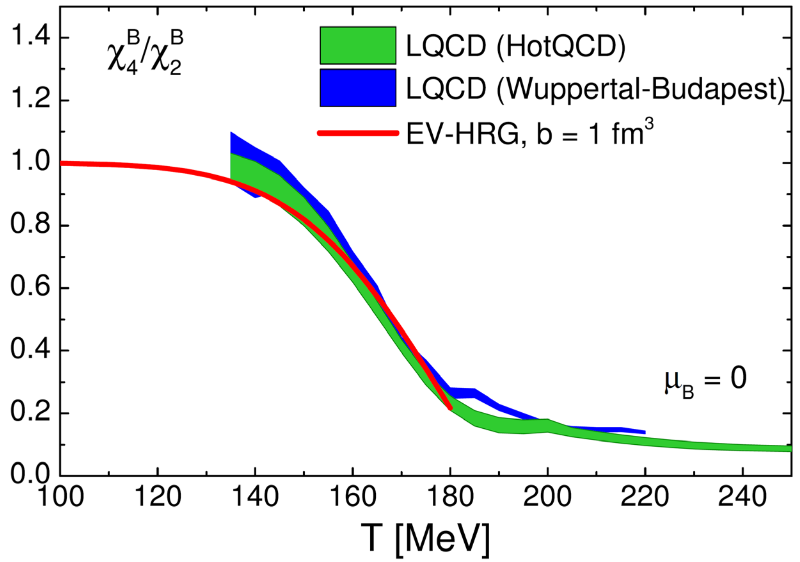
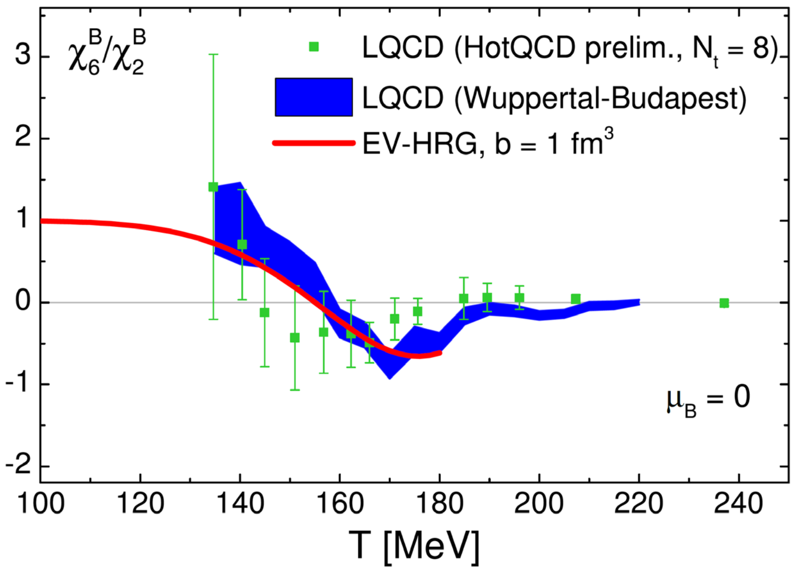


EV-HRG vs lattice

WB: 1805.04445; HotQCD: 1708.04897



$b = 1\ \mathrm{fm}^3$



EV-HRG $\approx$ QCD at $T \approx T_{pc}$

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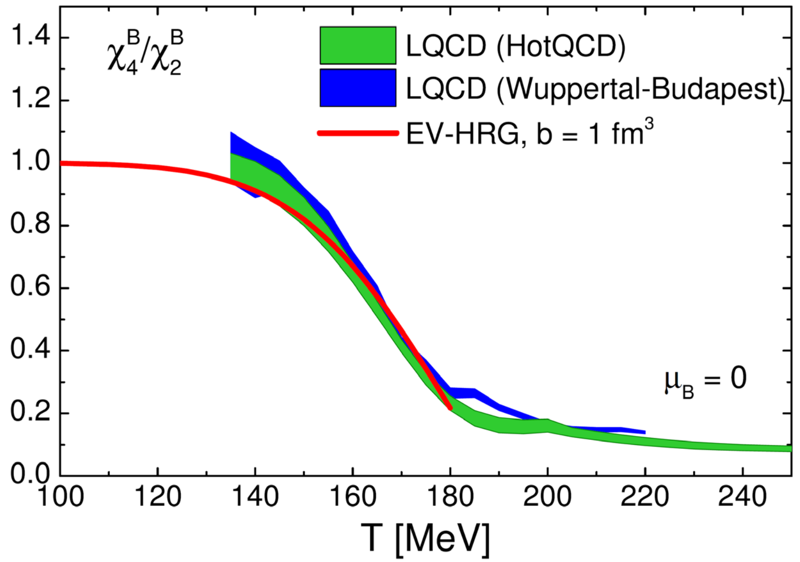

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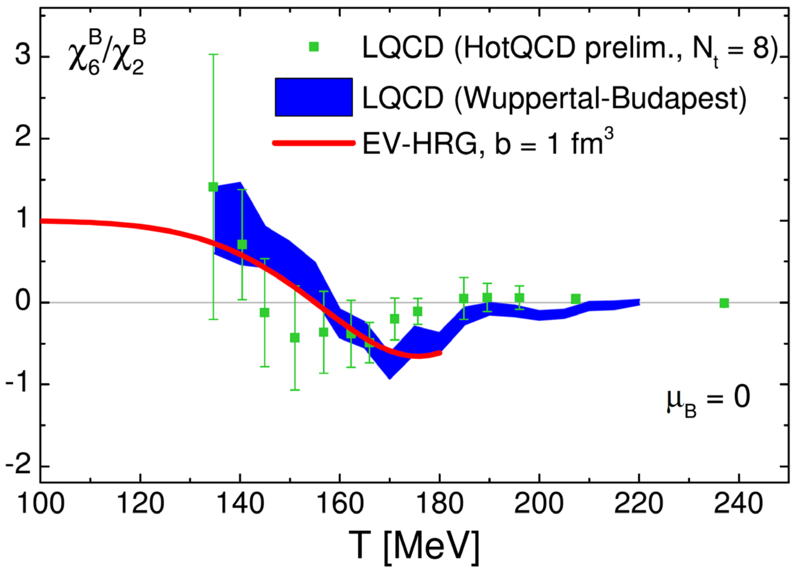
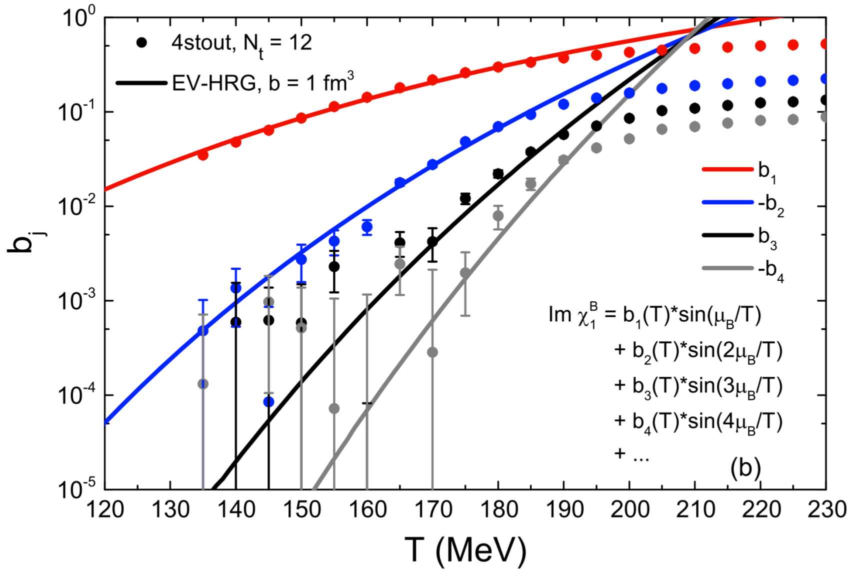


EV-HRG ≈ QCD at $T \approx T_{pc}$

BONUS: Describes imaginary μ_B lattice data for the leading four virial coefficients

$$\rho_B(T,\mu_B) = \sum_{k=1}^{\infty} b_k(T)\,\sinh(k\mu_B/T)$$

[VV, Pasztor, Katz, Fodor, Stoecker, PLB '17]



Connecting fluctuations in heavy-ion collisions with grand-canonical susceptibilities

- Measurements affected by global conservation laws, thermal smearing, resonance decays, etc.
- Tackle these with a novel *particlization* routine for hydro

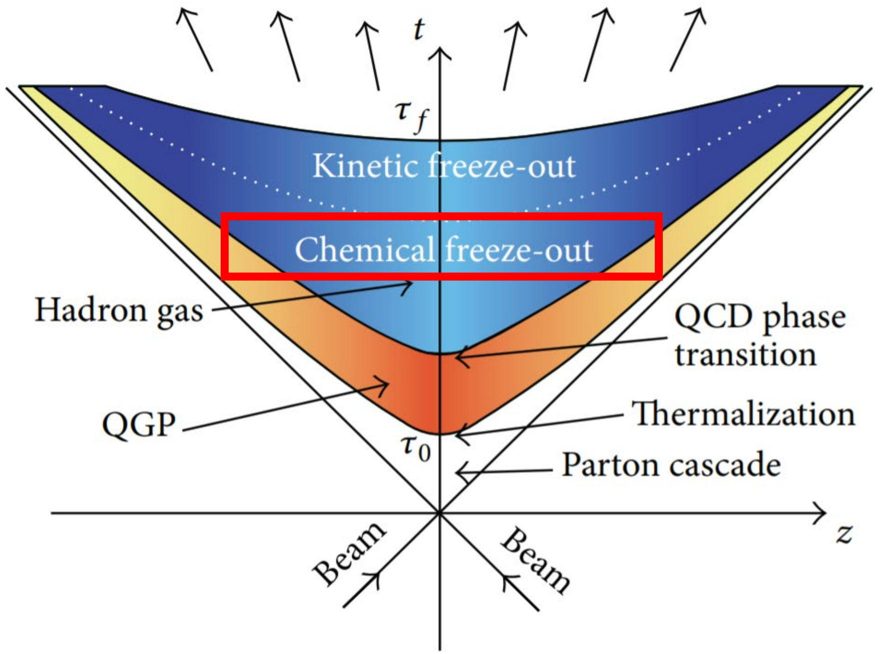
VV, O. Savchuk, R.V. Poberezhnyuk, M.I. Gorenstein, V. Koch, *Phys. Lett. B 811, 135868 (2020); JHEP 10, 089 (2020)*
VV, V. Koch, *to appear*

Heavy-ion collisions

- Heavy-ion collisions are commonly described with relativistic fluid dynamics

- Hydro expansion ends with a particlization of locally equilibrated QCD matter, this happens roughly at "chemical freeze-out", $T \approx 160$ MeV at highest energies

- Fairly successful in describing bulk observables like hadrochemistry, p_T spectra, flow etc.



- What about event-by-event fluctuations?
  - *A (too) common approach:* directly compare cumulant ratios measured in experiment with grand-canonical (lattice QCD) susceptibility ratios and hope for the best see e.g. [HotQCD coll., 2001.08530] and others

Theory vs experiment: Caveats

- proxy observables in experiment (net-proton, net-kaon) vs actual conserved charges in QCD (net-baryon, net-strangeness)

  Asakawa, Kitazawa, PRC '12; **VV**, Jiang, Gorenstein, Stoecker, PRC '18

- volume fluctuations

  Gorenstein, Gazdzicki, PRC '11; Skokov, Friman, Redlich, PRC '13; Braun-Munzinger, Rustamov, Stachel, NPA '17

- non-equilibrium (memory) effects

  Mukherjee, Venugopalan, Yin, PRC '15

- final-state interactions in the hadronic phase

  Steinheimer, **VV**, Aichelin, Bleicher, Stoecker, PLB '18

- accuracy of the grand-canonical ensemble (global conservation laws)

  Jeon, Koch, PRL '00; Bzdak, Skokov, Koch, PRC '13; Braun-Munzinger, Rustamov, Stachel, NPA '17

- coordinate vs momentum space (thermal smearing)

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When are the measured fluctuations grand-canonical?

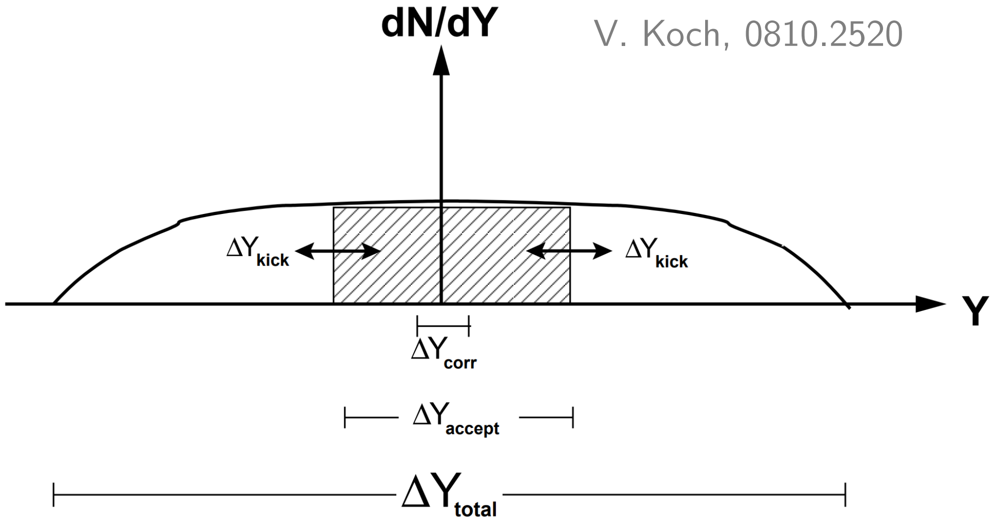
- Consider event-by-event fluctuations of particle number in acceptance ΔY_{accept} around midrapidity

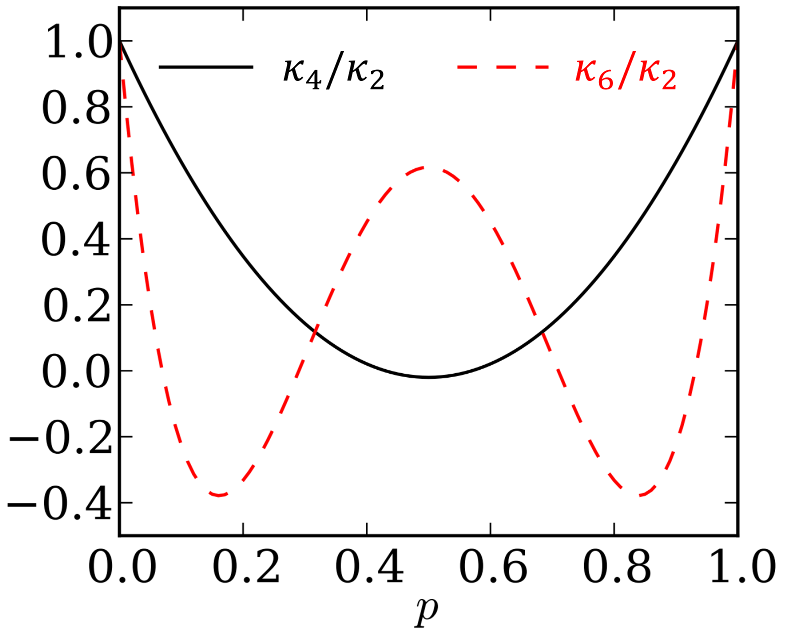
- Scales
  - ΔY_{accept} – acceptance
  - ΔY_{total} – full space
  - ΔY_{corr} – rapidity correlation length (thermal smearing)
  - ΔY_{kick} – diffusion in hadronic phase

- **GCE applies if $\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}, \Delta Y_{corr}$**

- In practice neither $\Delta Y_{total} \gg \Delta Y_{accept}$ nor $\Delta Y_{accept} \gg \Delta Y_{corr}$ is satisfied
  - Corrections from global conservation are large [Bzdak et al., PRC '13]
  - $\Delta Y_{corr} \sim 1 \sim \Delta Y_{accept}$ [Ling, Stephanov, PRC '16]

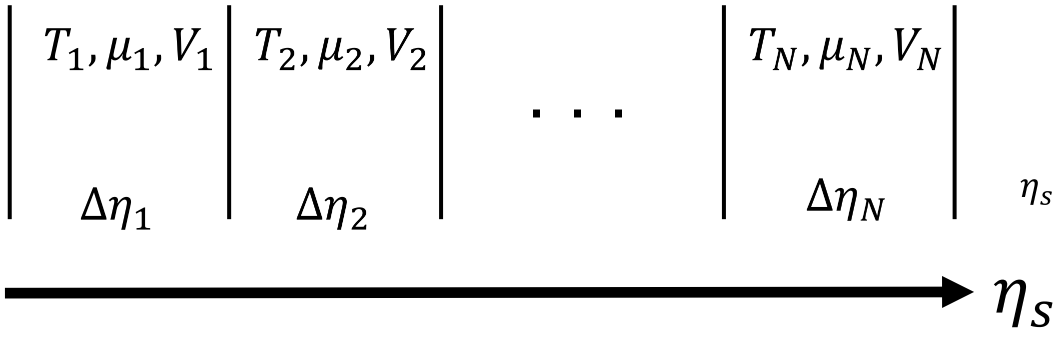
V. Koch, 0810.2520





Subensemble sampler

A novel **particlization** routine



$\eta_s = \frac{1}{2} \ln \frac{t+z}{t-z}$

1. Partition the hydro (blast-wave) particlization hypersurface into subvolumes along the space-time rapidity axis
2. Sample each subvolume grand-canonically, using the partition function of an *interacting* HRG (e.g. EV-HRG)
3. Reject the event if global conservation is violated
4. Sample the momenta of particles
5. Do resonance decays or plug into hadronic afterburner

Partition into subensembles*

$$Z^{\rm tot} = \prod_{i=1}^{N} \sum_{B_i} e^{\mu_i B_i / T} Z^{\rm ce}(T_i, B_i, V_i) \times \delta\left(B_{\rm tot} - \sum_{i=1}^{N} B_i\right)$$

*Following the idea put forward in
[**VV**, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB '20]

✓ **(event-by-event) hydro**

✓ **locally grand-canonical fluctuations**

✓ **global conservation**

✓ **thermal smearing**

✓ **resonance decays**

VV, Koch, *to appear*

A case study: net proton/baryon fluctuations at LHC

Central Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV

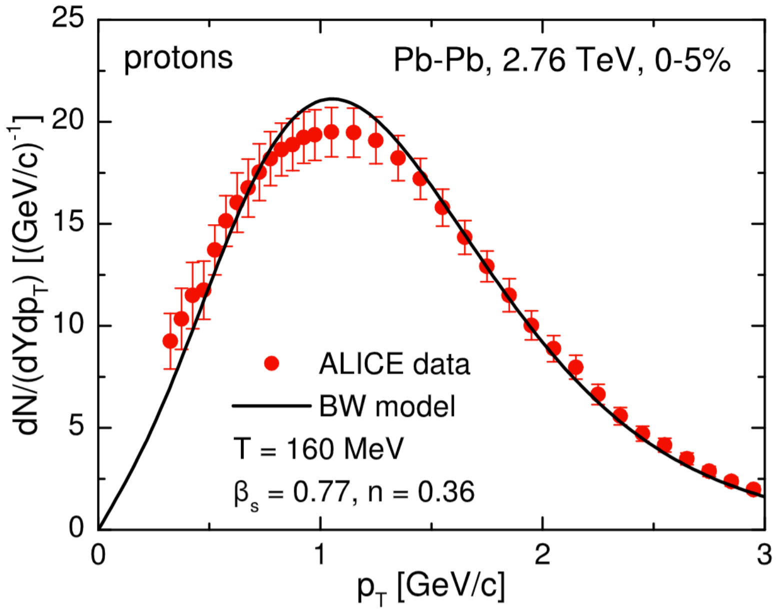
- Particlization at $T = 160$ MeV, $\mu_B = 0$
- Rapidity axis partitioned into 96 slices, $\Delta\eta_s = 0.1$, $|\eta_s| < 4.8$
- Boost-invariant blast-wave hypersurface and flow profile
- Sampling of (anti)baryons from the lattice-based EV-HRG model with global baryon conservation, 10^{10} events

$$P(N) \sim \frac{(V - bN)^N}{N!}\,\theta(V - bN)$$

Poisson + rejection sampling

details in VV, Gorenstein, Stoecker, 1805.01402

BW parameters from Mazeliauskas, Vislavicius, 1907.11059



GCE baseline: $\frac{\kappa_2^B}{\langle B+\bar{B}\rangle} = 0.94$, $\frac{\chi_4^B}{\chi_2^B} = 0.69$, $\frac{\chi_6^B}{\chi_2^B} = -0.18$ $\leftarrow$ compatible with lattice

Net baryon fluctuations at LHC

- Global baryon conservation distorts the cumulant ratios already for one unit of rapidity acceptance

- Neglecting thermal smearing, effects of global conservation can be described analytically via SAM*

$$\frac{\kappa_2}{\langle B+\bar{B}\rangle} = (1-\alpha)\frac{\kappa_2^{\rm gce}}{\langle B+\bar{B}\rangle}, \qquad \alpha = \frac{\Delta Y_{\rm acc}}{9.6}, \quad \beta \equiv 1-\alpha$$

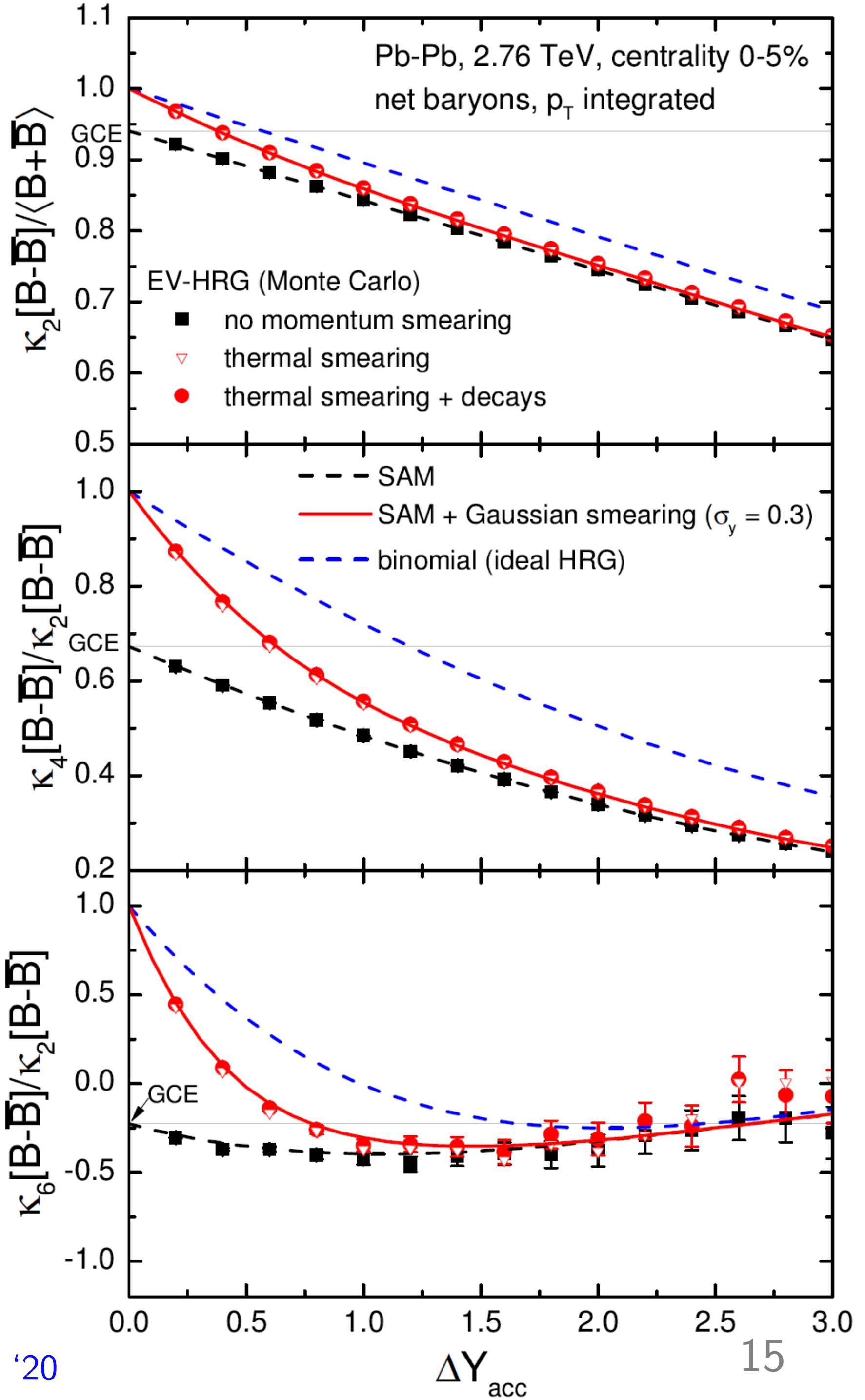
$$\frac{\kappa_4}{\kappa_2} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B},$$

$$\frac{\kappa_6}{\kappa_2} = [1-5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$

- Thermal smearing distorts the signal at $\Delta Y_{accept} \leq 1$

- Effect of resonance decays is negligible

*Subensemble acceptance method (SAM): **VV**, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB ‘20



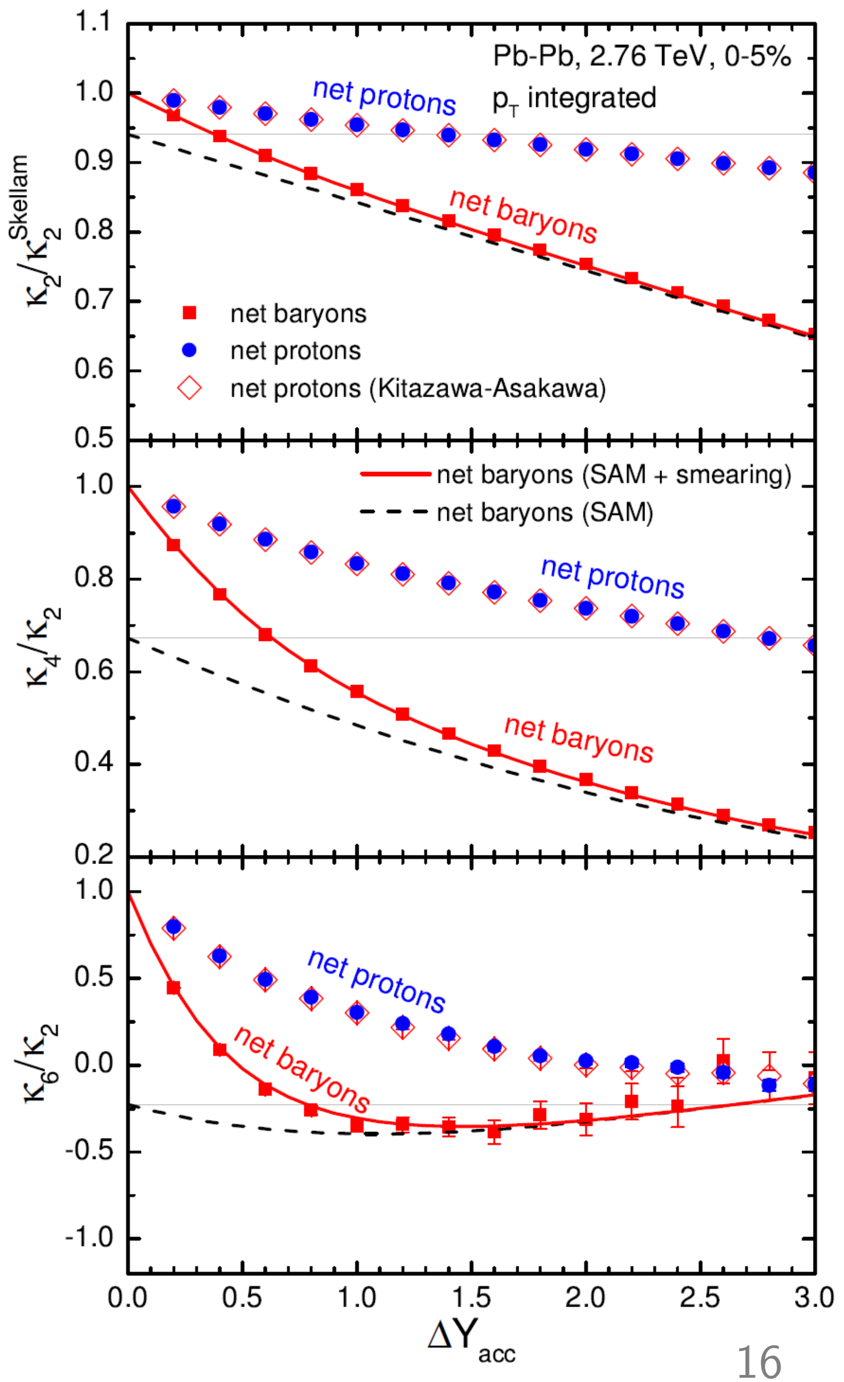
Net baryon vs net proton

- Experiments measure protons as a proxy for baryons
- Protons form a subset of all baryons – this dilutes the signal
- For example

$$\left.\frac{\chi_4^B}{\chi_2^B}\right|_{T=160MeV}^{\mathrm{GCE}} \simeq 0.67 \neq \left.\frac{\chi_4^B}{\chi_2^B}\right|_{\Delta Y_{acc}=1}^{\mathrm{HIC}} \simeq 0.56 \neq \left.\frac{\chi_4^p}{\chi_2^p}\right|_{\Delta Y_{acc}=1}^{\mathrm{HIC}} \simeq 0.83$$

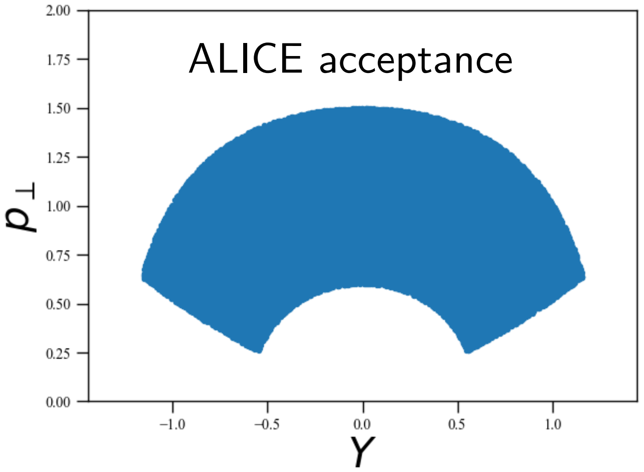
"lattice QCD" *experiment*

- **HIC net proton ≠ HIC net baryon ≠ LQCD net baryon**
- Baryon cumulants can be reconstructed from proton cumulants via binomial (un)folding method of Kitazawa and Asakawa [Phys. Rev. C 85 (2012) 021901]

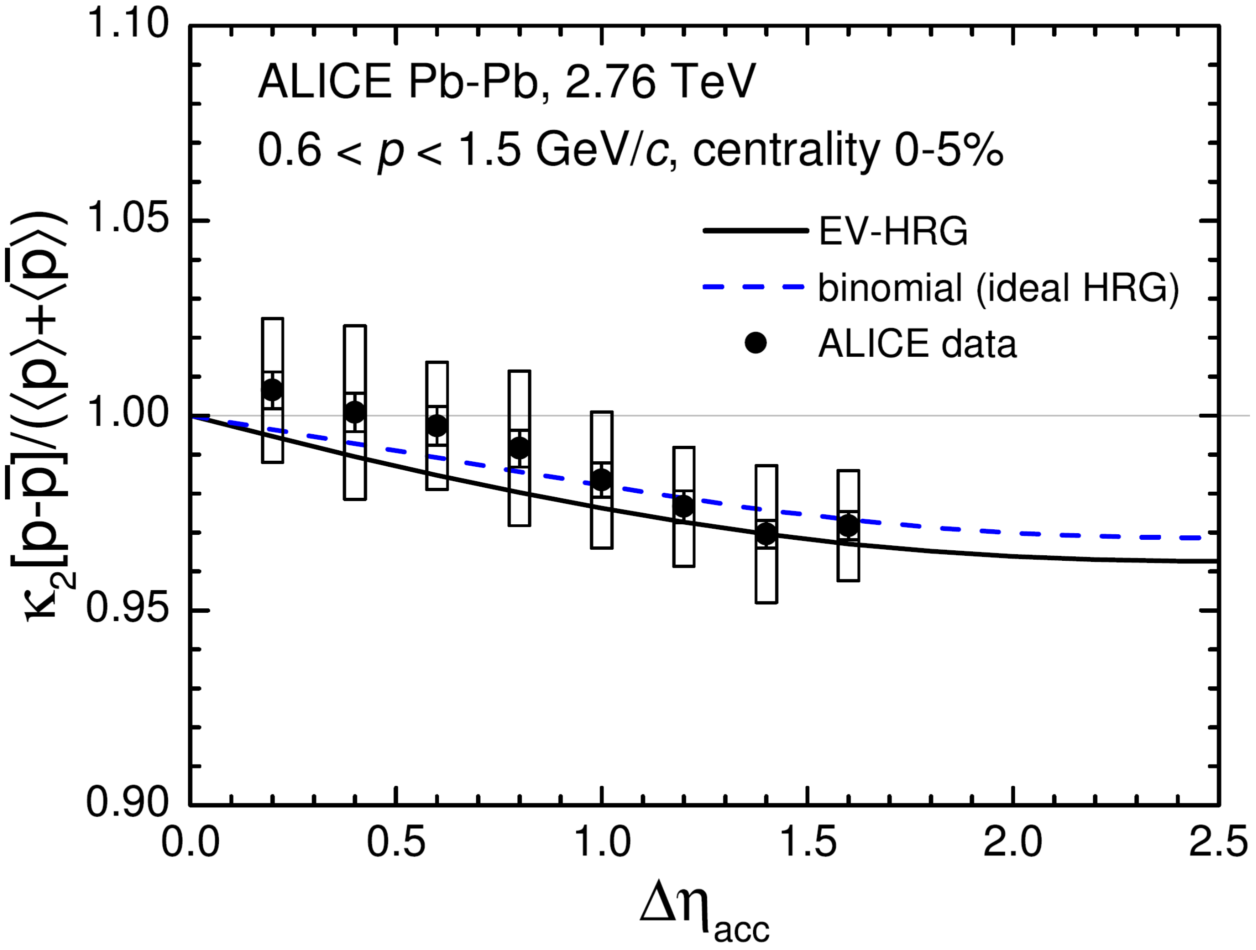


Comparison to ALICE data

- Model describes data within errors
- "Large" error bars and "small" p_T coverage do allow to distinguish the equation of state
- Future measurements will require larger acceptance

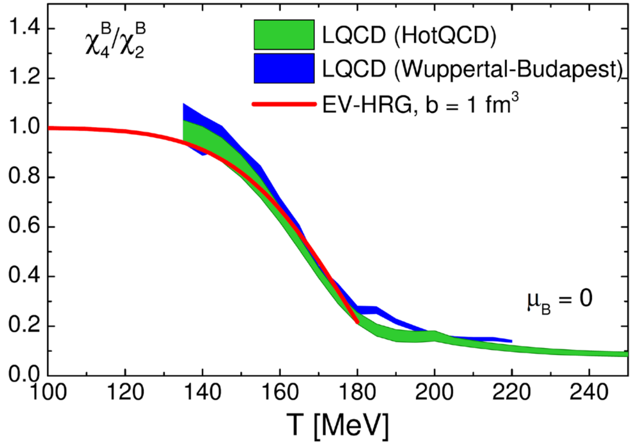


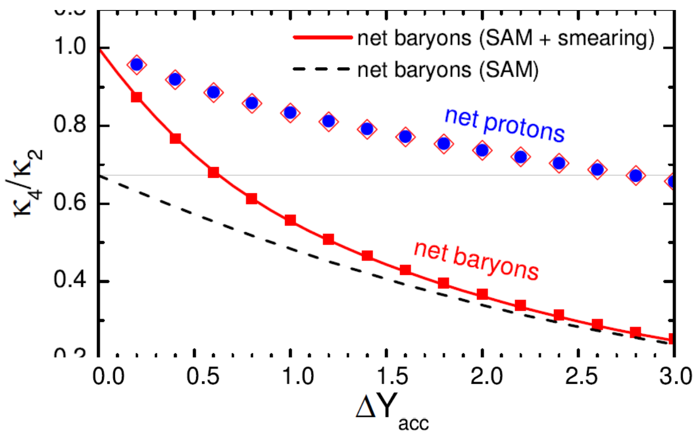
$0.6 < p < 1.5$ GeV$/c$, $\Delta\eta_{acc} = 1.6$



Summary

- HRG with a baryonic excluded volume matches lattice QCD susceptibilities in vicinity of the pseudocritical temperature
    - excluded-volume parameter $b = 1$ fm^3
    - fluctuations at freeze-out in heavy-ion collisions
- Subensemble sampler is a novel particlization routine
    - event-by-event fluctuations in a fluid dynamical picture
    - global charge conservation, thermal smearing, resonance decays
- Quantitative analysis of fluctuations in Pb-Pb collisions at LHC
    - HIC net proton $\neq$ HIC net baryon $\neq$ LQCD/GCE net baryon
    - EV-HRG agrees with available experimental data





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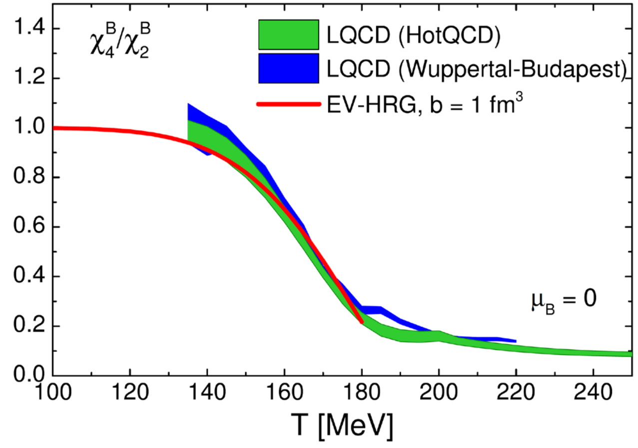

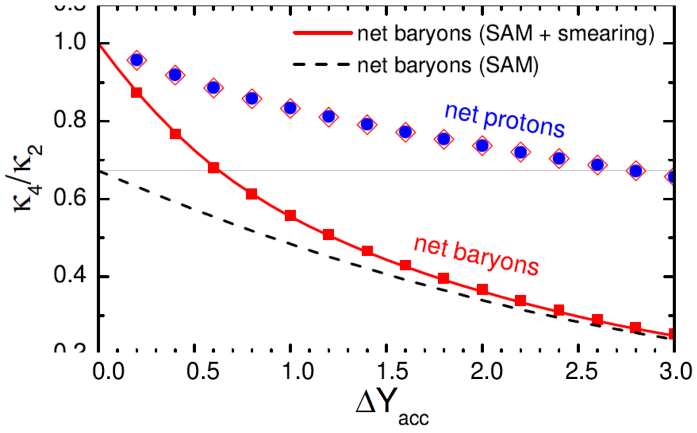

Thanks for your attention!