Quantum statistical van der Waals equation and its QCD applications

Volodymyr Vovchenko

PhD Defense

$$p(T, n) = p_q^{\mathrm{id}}\left(T, \frac{n}{1 - bn}\right) - an^2$$

FIAS, Frankfurt
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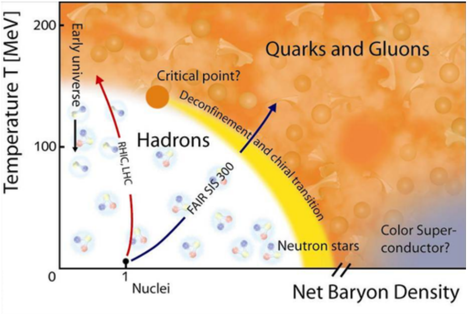
Strongly interacting matter

- Theory of strong interactions: *Quantum Chromodynamics* (QCD)

$$\mathcal{L} = \sum_{\mathrm{q=u,d,s,\ldots}} \bar{q}[i\gamma^\mu(\partial_\mu - igA^a_\mu\lambda_a) - m_q]q - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$

- Basic degrees of freedom: quarks and gluons
- At smaller energies confined into baryons (qqq) and mesons ($q\bar{q}$)



Where is it relevant?

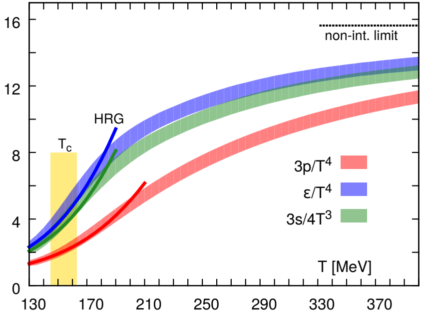
- Early universe
- Neutron star (mergers)
- Heavy-ion collisions (lab!)

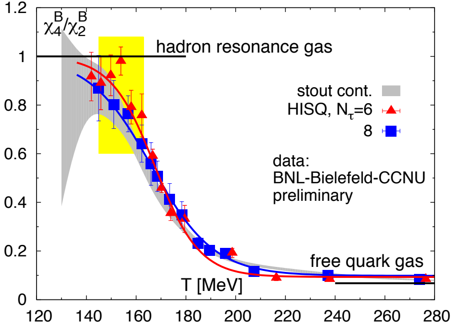
Length scale: 1 fm $= 10^{-15}$ m   Energy scale: 100 MeV $= 10^{12}$ K

$$\hbar = c = k_B = 1$$

QCD equation of state at $\mu = 0$

Lattice QCD simulations provide the equation of state at $\mu_B = 0$



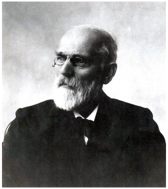


- A crossover-type transition
- Common model for confined phase is ideal HRG: non-interacting gas of known hadrons and resonances, clearly breaks down as T increases
- What is the role of hadronic interactions beyond those in ideal HRG?

HotQCD Collaboration: 1407.6387, 1701.04325, 1708.04897
Wuppertal-Budapest Collaboration: 1112.4416, 1309.5258, 1507.04627

van der Waals equation

$$P(T,V,N) = \frac{NT}{V-bN} - a\frac{N^2}{V^2}$$



Formulated in 1873.

Simplest model which contains attractive and repulsive interactions

Contains 1st order phase transition and critical point

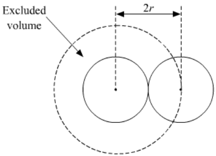
Nobel Prize in 1910.

1. Short-range repulsion: excluded volume (EV) procedure

   $V \to V - bN, \qquad b = 4\frac{4\pi r_c^3}{3}$

2. Intermediate range attraction in mean-field approx.

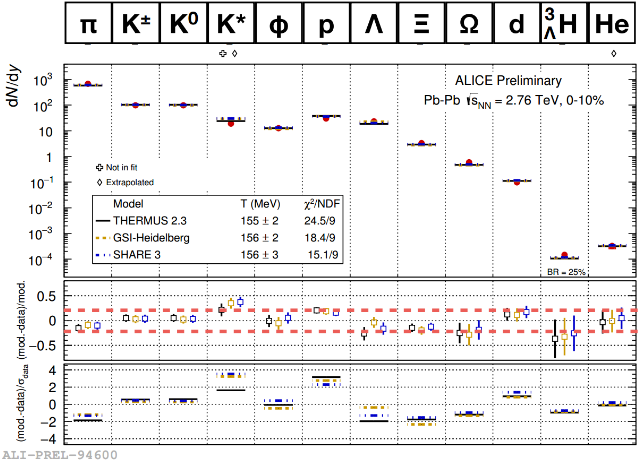
   $P \to P - a\,n^2, \qquad a = \pi \int_{2r_c}^{\infty} |U_{12}(r)| r^2 dr$



A fairly generic extension of ideal gas for interactions

Excluded volume effects on thermal fits

Ideal HRG: $N_i \propto \exp\left(-\frac{m_i}{T}\right) \exp\left(\frac{b_i \, \mu_B}{T}\right) \quad + \quad \text{feeddown}$



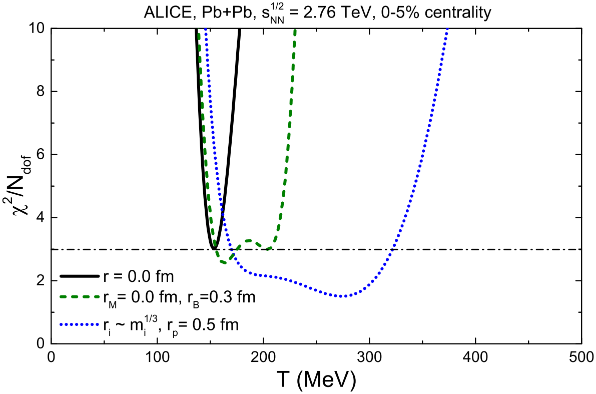
ALICE Collaboration, SQM2015, QM2015

How robust are conclusions based on ideal gas?

Excluded volume effects on thermal fits

Excluded-volume: $N_i \propto \exp\left(-v_i \frac{p}{T}\right)$ ← larger hadrons suppressed

- Bag model: $v_i \propto m_i$ [Chodos et al., PRD '74; Kapusta, et al., NPA '83, PRC '15]
- Two-component model: $r_M = 0$, $r_B = 0.3$ fm [Andronic et al., 1201.0693]

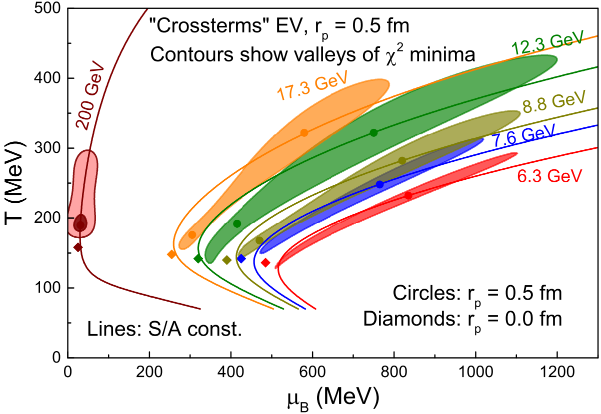


V.V., H. Stoecker, J. Phys. G 44, 055103 (2017)

Excluded-volume model: Rischke, Gorenstein, Stoecker, Greiner, Z. Phys. C (1991)
D. Anchishkin, Sov. JETP (1992)

Excluded volume effects on thermal fits: finite μ_B

Fits to NA49 and STAR data



V.V., H. Stoecker, Phys. Rev. C 95, 044904 (2017)

Entropy per baryon, S/A, is a *robust observable*

Full van der Waals equation: GCE formulation

Full vdW equation opens new applications, such as *criticality*

$$p = \frac{Tn}{1 - bn} - an^2$$

First step: transform the CE pressure $p(T, V, N)$ into the GCE

Result: van der Waals equation in the grand canonical ensemble

$$\frac{N}{V} \equiv n(T,\mu) = \frac{n_{\rm id}(T,\mu^*)}{1 + b\, n_{\rm id}(T,\mu^*)}, \qquad \mu^* = \mu - b\frac{n\,T}{1 - b\,n} + 2a\,n$$

Never published in 100+ years!

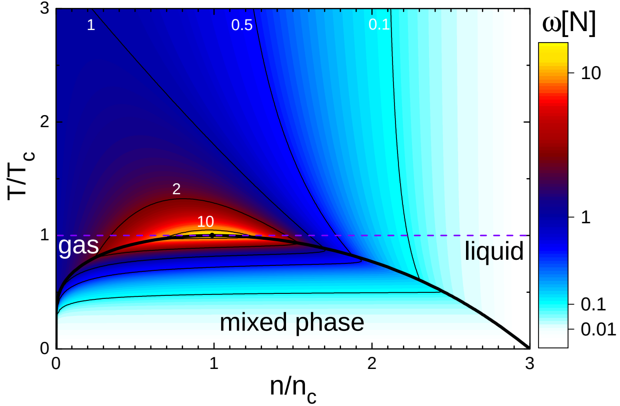
- Implicit equation in the GCE, now depends on mass and degeneracy
- May have multiple solutions below T_C
- Choose one with largest pressure – equivalent to the Maxwell rule in CE

V.V, Anchishkin, Gorenstein, J. Phys. A 48, 305001 (2015)

Scaled variance for classical VDW equation

Particle number fluctuations in a classical vdW gas within the GCE

$$\omega[N] = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} = \left[\frac{1}{(1-bn)^2} - \frac{2an}{T} \right]^{-1}$$



- Repulsive interactions suppress N-fluctuations
- Attractive interactions enhance N-fluctuations

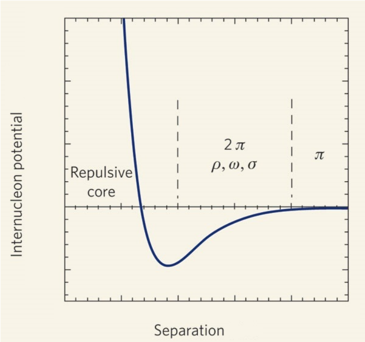
V.V., Anchishkin, Gorenstein, J. Phys. A 48, 305001 (2015)

Nucleon-nucleon interaction

Nuclear liquid-gas transition appears due to the vdW type structure of the nucleon-nucleon interaction

Nucleon-nucleon potential:

- Repulsive core at small distances
- Attraction at intermediate distances
- Suggestive similarity to vdW interactions



Could nuclear matter be described by the van der Waals equation?

Yes! But we need Fermi statistics...

Quantum statistical van der Waals fluid

Free energy of classical vdW fluid:

$$F(T,V,N) = F^{\mathrm{id}}(T, V - bN, N) - a\,\frac{N^2}{V}$$

Ansatz: $F^{\mathrm{id}} \Rightarrow F_q^{\mathrm{id}}(T, V - bN, N)$ is free energy of ideal *quantum* gas

Quantum van der Waals equation:

$$p(T,n) = p_q^{\mathrm{id}}\left(T, \frac{n}{1-bn}\right) - an^2$$

$p_q^{\mathrm{id}}(T,n)$ corresponds to Fermi-Dirac or Bose-Einstein distribution

Model properties:

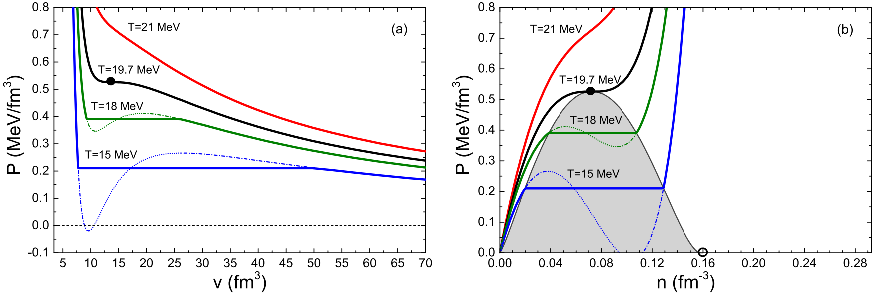
- Reduces to the classical vdW equation when quantum statistics are negligible
- Reduces to ideal quantum gas for $a = 0$ and $b = 0$
- Entropy density non-negative and $s \to 0$ with $T \to 0$

V.V., Anchishkin, Gorenstein, Phys. Rev. C 91, 064314 (2015)

QvdW gas of nucleons: pressure isotherms

a and *b* fixed to reproduce saturation density and binding energy:

$n_0 = 0.16\ \text{fm}^{-3}$, $E/A = -16$ MeV $\Rightarrow$ $a \cong 329$ MeV fm^3 and $b \cong 3.42$ fm^3



Behavior qualitatively same as for Boltzmann case

Mixed phase results from Maxwell construction

Critical point at $T_c \cong 19.7$ MeV and $n_c \cong 0.07\ \text{fm}^{-3}$

Experimental estimate[1]: $T_c = 17.9 \pm 0.4$ MeV, $n_c = 0.06 \pm 0.01\ \text{fm}^{-3}$

[1] J.B. Elliot, P.T. Lake, L.G. Moretto, L. Phair, Phys. Rev. C 87, 054622 (2013)
Pioneering measurements: J. Pochodzalla et al. [ALADIN collaboration], PRL '95

QvdW-HRG model

We are now ready to include QvdW interactions into HRG model

QvdW-HRG model [**V.V.**, M.I. Gorenstein, H. Stoecker, PRL 118, 182301 (2017)]

- Hadron Resonance Gas (HRG) with attractive and repulsive vdW interactions between baryons
- vdW parameters $a = 329$ MeV fm^3 and $b = 3.42$ fm^3 tuned to nuclear ground state properties
- Critical point of nuclear matter at $T_c \simeq 19.7$ MeV, $\mu_c \simeq 908$ MeV

3 subsystems: non-int. mesons + QvdW baryons + QvdW antibaryons

$$p(T, \boldsymbol{\mu}) = P_M(T, \boldsymbol{\mu}) + P_B(T, \boldsymbol{\mu}) + P_{\bar{B}}(T, \boldsymbol{\mu}),$$

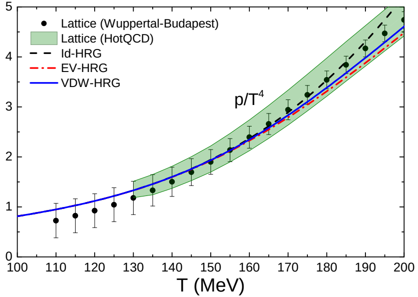
$$P_M(T, \boldsymbol{\mu}) = \sum_{j \in M} p_j^{\rm id}(T, \mu_j) \quad \text{and} \quad P_B(T, \boldsymbol{\mu}) = \sum_{j \in B} p_j^{\rm id}(T, \mu_j^{B*}) - a\, n_B^2$$

A "parameter-free" minimal-interaction extension

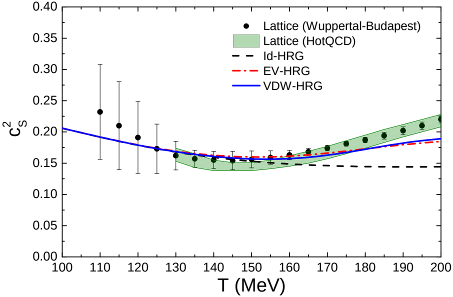
QvdW-HRG at $\mu_B = 0$: thermodynamics

Comparison of QvdW-HRG with lattice QCD at $\mu_B = 0$

Pressure p/T^4

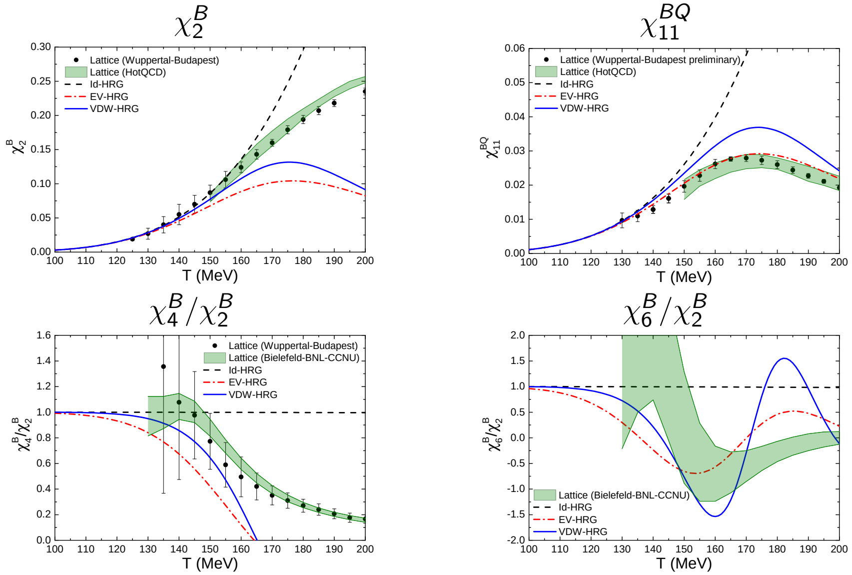


Speed of sound $c_s^2 = \frac{dp}{d\varepsilon}$



- QvdW-HRG does not spoil existing agreement of Id-HRG with LQCD despite significant excluded-volume interactions between baryons
- No acausal behavior

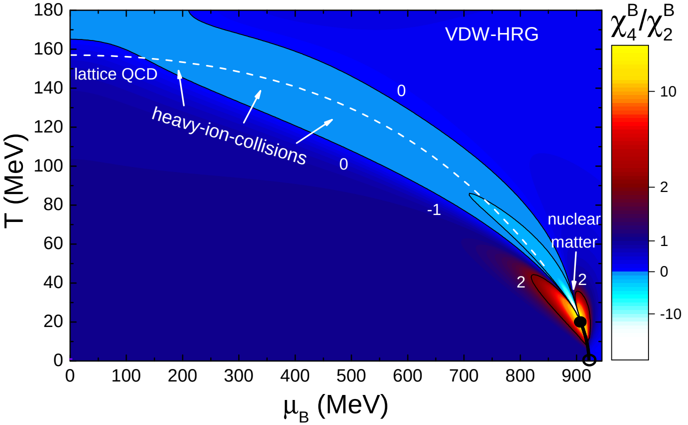
QvdW-HRG at $\mu_B = 0$: susceptibilities



- Quantitative features of QCD captured by QvdW-HRG

QvdW-HRG at finite μ_B

Net-baryon fluctuations in T-μ_B plane: net baryon kurtosis χ_4^B/χ_2^B

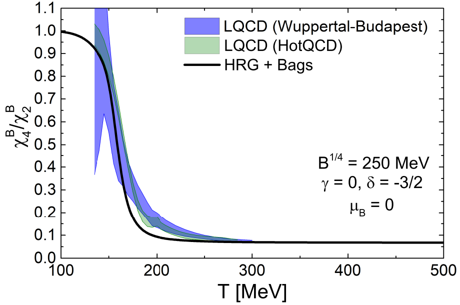


Critical point of nuclear matter shines brightly in fluctuation observables, across the whole region of phase diagram probed by heavy-ion collisions

V.V., M.I. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

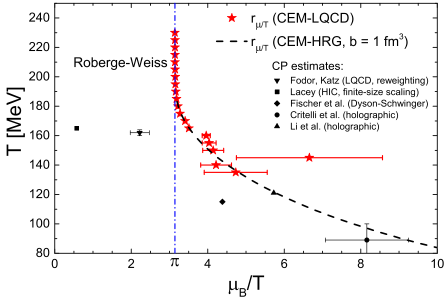
Subsequent work: outlook

LQCD crossover transition:
Hagedorns + excluded volume?



With C. Greiner

Radius of convergence sees
Roberge-Weiss transition?



With O. Philipsen

A *lot* of Lattice QCD data requiring *careful* interpretation...

Summary I: Main results

- Thermal fits are very sensitive to excluded volume effects. Entropy per baryon is a robust observable
- Quantum statistical van der Waals equation in the grand canonical ensemble opens new applications in high energy nuclear physics
- Enhanced, non-monotonic behavior of high-order fluctuations is a promising signal of critical behavior
- van der Waals like interactions between nucleons/baryons are surprisingly important for observables in high-temperature QCD

Summary II: Selected Publications

Excluded volume effects

Equation of state

- V.V., D. Anchishkin, M.I. Gorenstein, Phys. Rev. C 91, 024905 (2015)
- V.V., A. Pasztor, Z. Fodor, S.D. Katz, H. Stoecker, Phys. Lett. B 775, 71 (2017)

Thermal fits

- V.V., H. Stoecker, J. Phys. G 44, 055103 (2017); Phys. Rev. C 95, 044904 (2017)

Textbook extensions of van der Waals equation

Grand canonical ensemble

- V.V., D. Anchishkin, M.I. Gorenstein, J. Phys. A 48, 30, 305001 (2015)
- V.V., R. Poberezhnyuk, D. Anchishkin, M.I. Gorenstein, J. Phys. A 49, 015003 (2016)

Quantum statistics

- V.V., D. Anchishkin, M.I. Gorenstein, Phys. Rev. C 91, 064314 (2015)
- V.V., Phys. Rev. C 96, 015206 (2017)

Mixtures (multi-component)

- V.V., M.I. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)
- V.V., Motornenko, Alba, Gorenstein, Satarov, Stoecker, Phys. Rev. C 96, 045202 (2017)

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Thanks for your attention!

Backup slides

Excluded volume procedure

Let us start with the excluded volume only:
substitute volume by the available volume $V \to V - bN$

$$\text{CE:} \quad p(T,n) = \frac{Tn}{1-bn}$$

$$\text{GCE:} \quad p(T,\mu) = p^{\rm id}[T,\mu - b\,p(T,\mu)], \quad n(T,\mu) = \frac{n^{\rm id}(T,\mu^*)}{1 + b\,n^{\rm id}(T,\mu^*)},$$

$$\mu^* = \mu - b\,p, \qquad b = \frac{16\pi}{3} r^3$$

CE: Excluded volume effects increase CE pressure $p(T, n)$

GCE: Excluded volume effects decrease GCE pressure $p(T, \mu)$

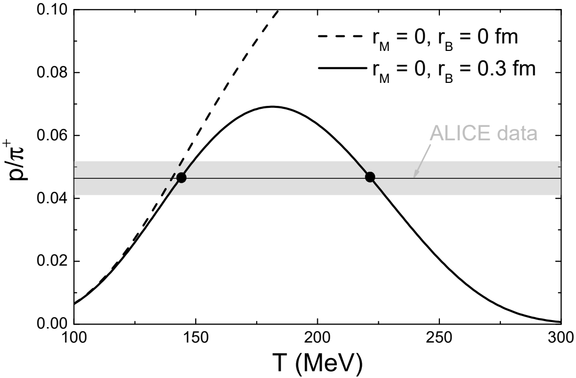
Early ideas by Hagedorn, Rafelski, Phys. Lett. B (1980)
Thermodynamically consistent procedure in D.H. Rischke et al., Z. Phys. C (1991)
D. Anchishkin, JETP (1992): Identical formulation with mean-field approach

Origin of the two minima

Where does the 2nd minimum come from?

Consider p/π ratio in the EV model

$$\frac{n_p^{ev}}{n_\pi^{ev}} = \frac{n_p^{id}}{n_\pi^{id}} \, e^{(v_\pi - v_p)P/T}$$

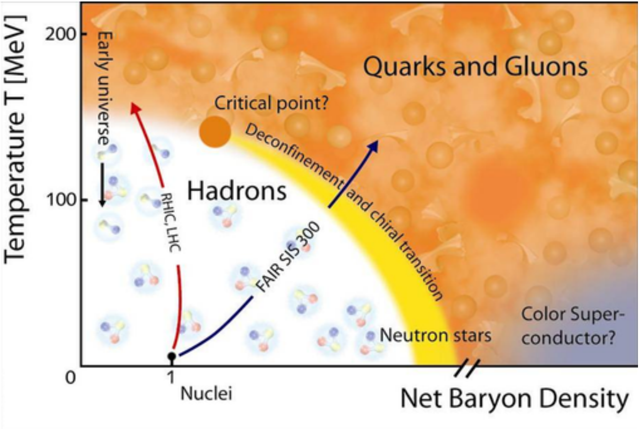


Non-monotonic behavior when $v_\pi < v_p$, yielding two solutions

L.M. Satarov, V.V., P. Alba, M.I. Gorenstein, H. Stoecker, 1610.08753

Exploring the QCD phase diagram with fluctuations

The QCD phase diagram has many unknowns at finite density



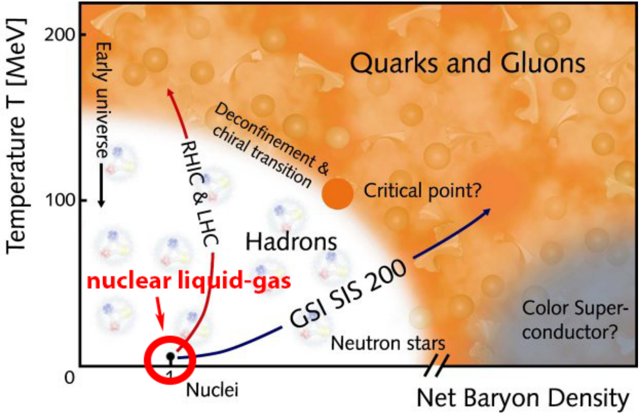
Does the critical point exists? Use *fluctuations* as its signal!

Theory: $\chi^{(n)} = \partial^n(p/T^4)/\partial(\mu/T)^n \sim \xi^k$, $\xi \to \infty$ at the CP

In heavy-ion collisions $\xi \lesssim 2-3$ fm [M. Stephanov, PRL '09]

Critical point of nuclear matter

The QCD phase diagram



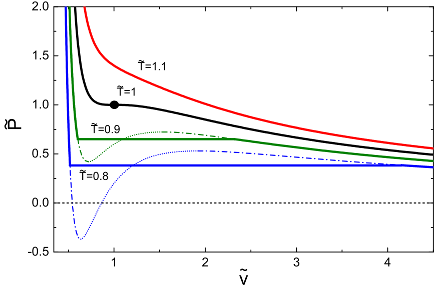
is known to contain the critical point of nuclear matter at $T_c \sim 15$ MeV and $(\mu_B/T)_c \sim 40$ $\quad\Rightarrow\quad$ *way beyond current lattice methods*

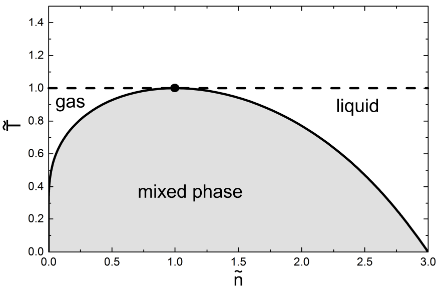
It is the only QCD critical point we know is there...

How does it influence the fluctuation observables in heavy-ion collisions?

van der Waals isotherms

- vdW isotherms show irregular behavior below certain temperature T_C
- Below T_C isotherms are corrected by Maxwell's rule of equal areas
- Results in appearance of mixed phase





Critical point

$$\frac{\partial p}{\partial v} = 0, \quad \frac{\partial^2 p}{\partial v^2} = 0, \quad v = V/N$$

$$p_C = \frac{a}{27b^2},\ n_C = \frac{1}{3b},\ T_C = \frac{8a}{27b}$$

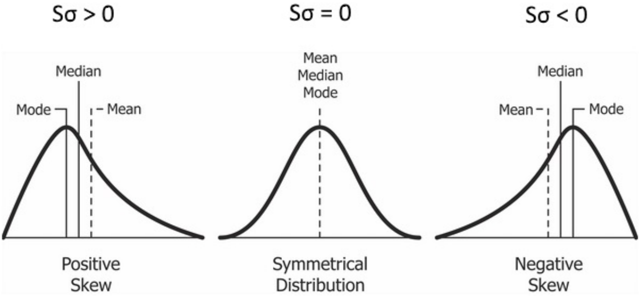
Reduced variables

$$\tilde{p} = \frac{p}{p_C},\ \tilde{n} = \frac{n}{n_C},\ \tilde{T} = \frac{T}{T_C}$$

Non-Gaussian fluctuations: Skewness

(Normalized) skewness measures the degree of asymmetry of distribution

$$S\sigma \;=\; \frac{\kappa_3}{\kappa_2} = \frac{\langle(\Delta N)^3\rangle}{\sigma^2} \;.$$



Baselines:

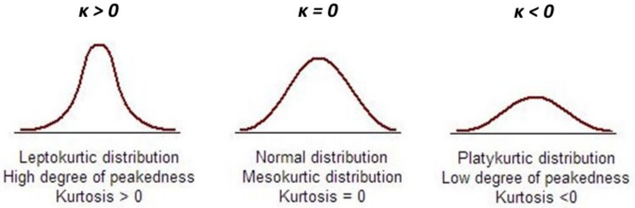
- Gaussian: $S\sigma = 0$
- Poisson: $S\sigma = 1$ $\leftarrow$ ideal gas in grand canonical ensemble

At CP: $S\sigma \sim \xi^{4.5}$

Non-Gaussian fluctuations: Kurtosis

(Normalized) kurtosis measures "peakedness" of distribution

$$\kappa\sigma^2 \; = \; \frac{\kappa_4}{\kappa_2} = \frac{\langle(\Delta N)^4\rangle \; - \; 3\,\langle(\Delta N)^2\rangle^2}{\sigma^2} \; .$$



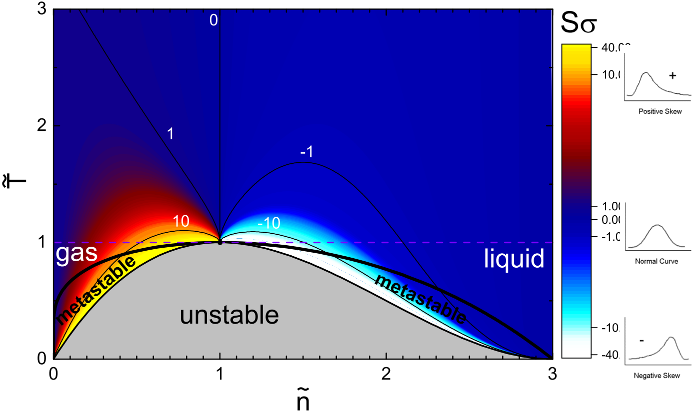
Baselines:

- Gaussian: $\kappa\sigma^2 = 0$
- Poisson: $\kappa\sigma^2 = 1$ $\leftarrow$ ideal gas in grand canonical ensemble

At CP: $\kappa\sigma^2 \sim \xi^7$

Classical vdW equation: Skewness

Skewness: $$S\sigma = \frac{\langle(\Delta N)^3\rangle}{\sigma^2} = \omega[N] + \frac{T}{\omega[N]}\left(\frac{\partial\omega[N]}{\partial\mu}\right)_T$$ asymmetry

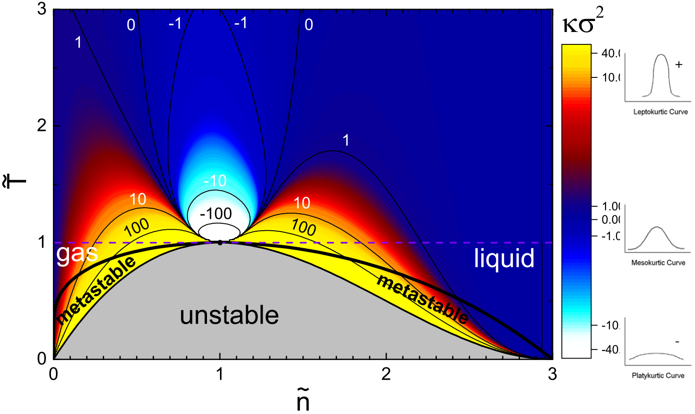


- Positive (right-tailed) in gaseous phase
- Negative (left-tailed) in liquid phase

V.V., R. Poberezhnyuk, D. Anchishkin, M. Gorenstein, J. Phys. A 49, 015003 (2016)

Classical vdW equation: Kurtosis

Kurtosis: $\kappa\sigma^2 = \frac{\langle(\Delta N)^4\rangle - 3\langle(\Delta N)^2\rangle^2}{\sigma^2}$ peakedness

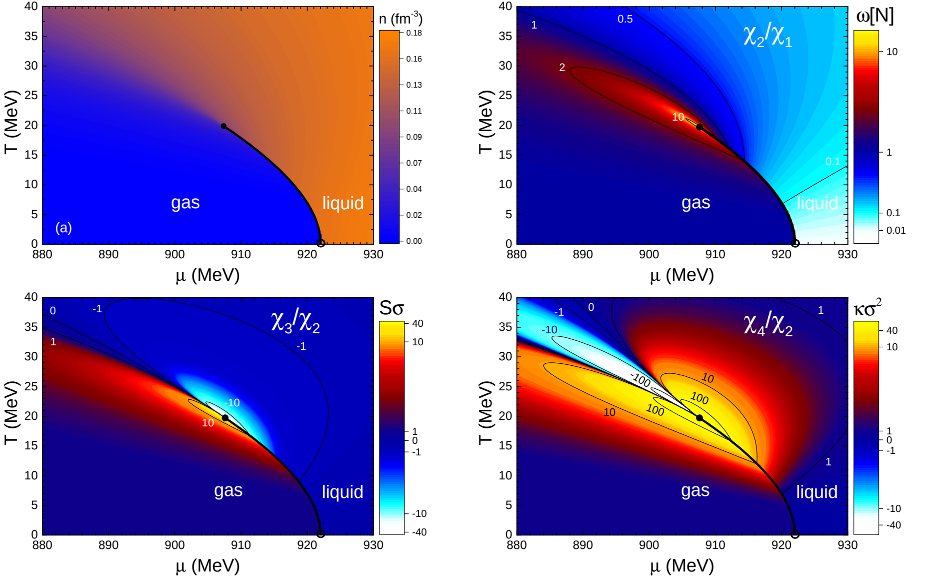


Kurtosis is negative (flat) above critical point (crossover), positive (peaked) elsewhere and very sensitive to the proximity of the critical point

V.V., R. Poberezhnyuk, D. Anchishkin, M. Gorenstein, J. Phys. A 49, 015003 (2016)

QvdW fluid of nucleons: (T,μ) plane

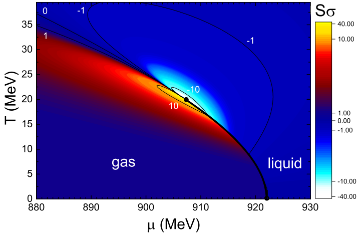
(T,μ) plane: structure of critical fluctuations $\chi_i = \partial^i(p/T^4)/\partial(\mu/T)^i$



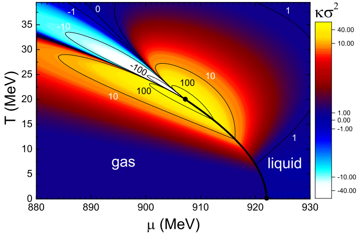
V.V., D. Anchishkin, M. Gorenstein, R. Poberezhnyuk, PRC 91, 064314 (2015)

QvdW gas of nucleons: skewness and kurtosis

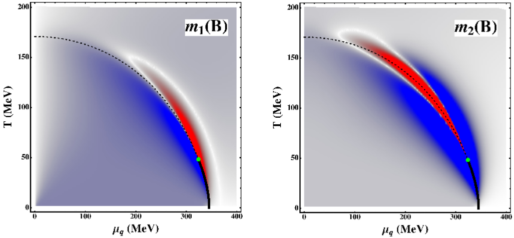
QvdW Skewness



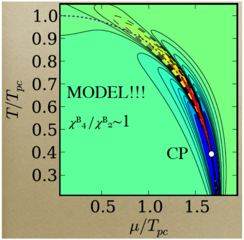
QvdW Kurtosis



NJL, J.W. Chen et al., PRD 93, 034037 (2016)



PQM, V. Skokov, QM2012



Fluctuation patterns in vdW very similar to effective QCD models

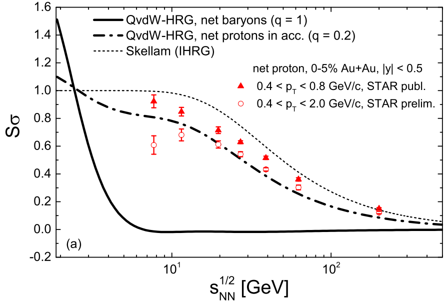
Fluctuation signals from nuclear matter critical point and from QCD critical point may very well look alike

QvdW-HRG model: collision energy dependence

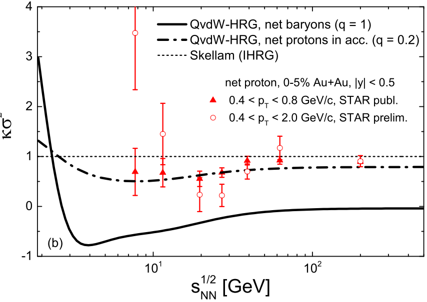
Calculating fluctuations along the "freeze-out" curve

Acceptance effects (protons instead of baryons, momentum cut) modeled *schematically*, by applying the *binomial filter* [M. Kitazawa, M. Asakawa, PRC '12; A. Bzdak, V. Koch, PRC '12]

χ_3^B/χ_2^B



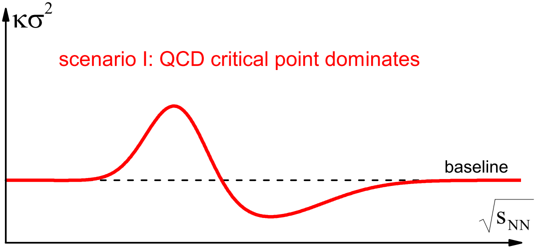
χ_4^B/χ_2^B



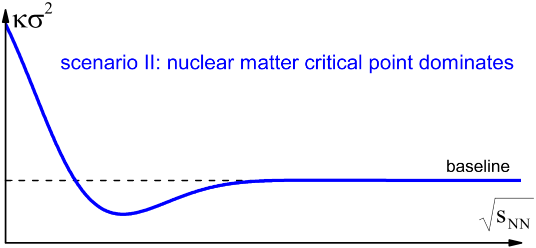
Effects of nuclear liquid-gas criticality:

- Non-monotonic collision energy dependence
- Net proton quite different from net baryon

Scenarios for collision energy dependence



M. Stephanov, JPG "11



V.V., *L. Jiang, M. Gorenstein, H. Stoecker, 1711.07260*

Can the scenarios be distinguished? Need data at lower energies...

Opportunities for HADES, CBM, NA61/SHINE, STAR!

Strongly intensive measures near CP

Strongly intensive (SI) measures [M.I. Gorenstein, M. Gazdzicki, PRC '11]

- Independent of volume fluctuations, mitigate impact parameter fluctuations
- Can be constructed from moments of two extensive quantities

$$\Delta[A,B] = C_\Delta^{-1}\left[\langle A\rangle \omega[B] - \langle B\rangle \omega[A]\right]$$
$$\Sigma[A,B] = C_\Sigma^{-1}\left[\langle A\rangle \omega[B] + \langle B\rangle \omega[A] - 2(\langle AB\rangle - \langle A\rangle\langle B\rangle)\right]$$

- For most models without PT and CP equal/close to unity
- Supposedly show critical behavior, but no model calculation
- Used in search for CP, e.g. NA61/SHINE program[1]

SI measures of excitation energy and particle number fluctuations in vdW

$$\Delta[E^*,N] \;=\; 1 \;-\; \frac{an(2\bar{\epsilon}_{\rm id} - 3an)}{\overline{\epsilon^2_{\rm id}} - \bar{\epsilon}^2_{\rm id}}\,\omega[N], \quad \Sigma[E^*,N] \;=\; 1 + \frac{a^2n^2}{\overline{\epsilon^2_{\rm id}} - \bar{\epsilon}^2_{\rm id}}\,\omega[N].$$

- Critical behavior is present due to criticality of $\omega[N]$ term[2]
- If a=0 then no signal at all! Deviations really stem from criticality.

[1]Gazdzicki, Seyboth, APP '15; E. Andronov, 1610.05569; A. Seryakov, 1704.00751
[2]V.V., Poberezhnyuk, Anchishkin, Gorenstein, J. Phys. A 49, 015003 (2016)