

Influence of Van der Waals interactions between hadrons on observables from heavy-ion collisions and lattice QCD

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In collaboration with P. Alba, M. Gorenstein, H. Stoecker
based on [arXiv:1609.03975](https://arxiv.org/abs/1609.03975)

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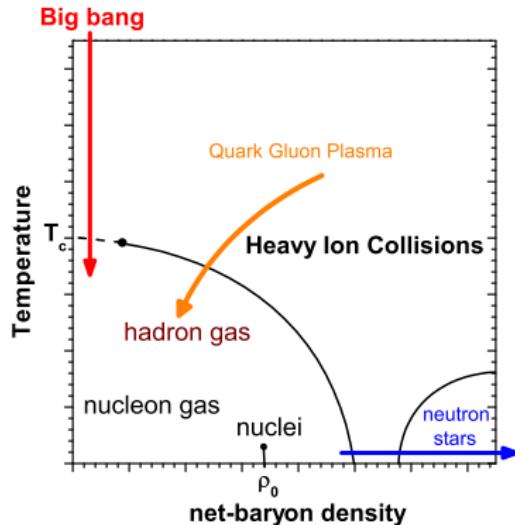
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UNIVERSITÄT
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Helmholtz Graduate School for Hadron and Ion Research

Strongly interacting matter

- Theory of strong interactions: **Quantum Chromodynamics** (QCD)
- Basic degrees of freedom: quarks and gluons
- At smaller energies confined into hadrons (baryons and mesons)



Where is it relevant?

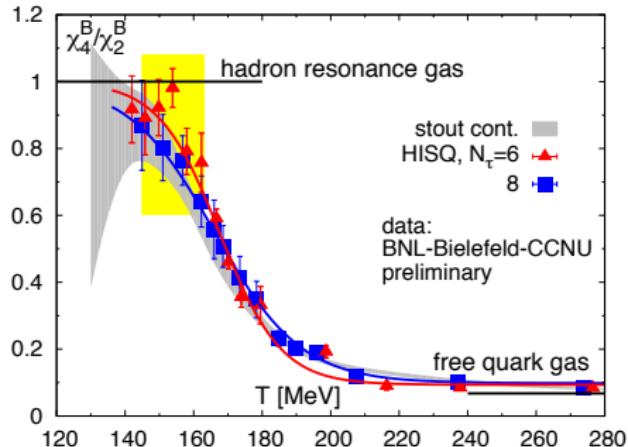
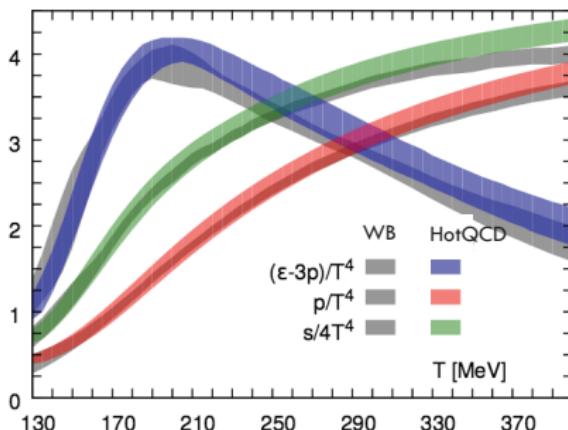
- Early universe
- Neutron stars
- Heavy-ion collisions

First principles of QCD are rather established,
but direct calculations are problematic

Phenomenological tools are very useful

QCD equation of state at $\mu = 0$

Lattice simulations provide EoS at $\mu_B = 0$ ¹



Common model for confined phase is **ideal HRG**: non-interacting gas of known hadrons and resonances

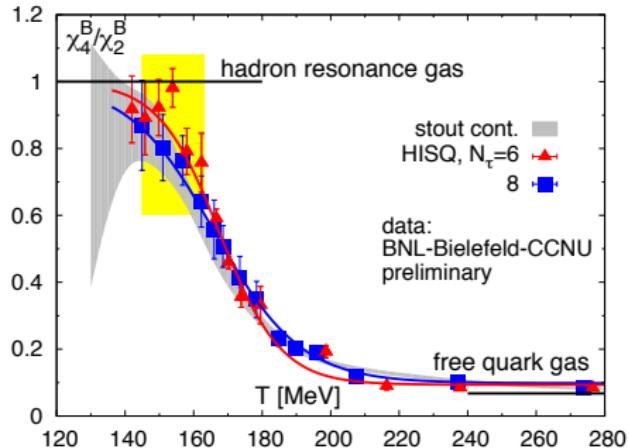
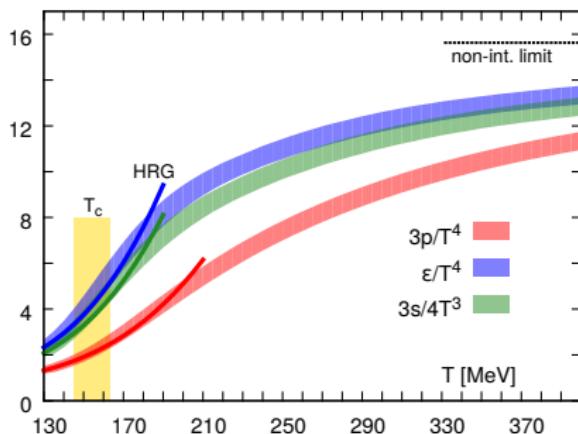
- Good description of thermodynamic functions up to 180 MeV
- Rapid **breakdown** in crossover region for description of **susceptibilities**²
- Often interpreted as clear signal of deconfinement...
- **But what is the role of hadronic interactions** beyond normal HRG?

¹Bazavov et al., PRD 90, 094503 (2014); Borsanyi et al., PLB 730, 99 (2014)

²Ding, Karsch, Mukherjee, IJMPE 24, 1530007 (2015)

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Van der Waals equation

Van der Waals equation

$$P(T, V, N) = \frac{NT}{V - bN} - a \frac{N^2}{V^2}$$



Formulated in
1873.

Simplest model which contains attractive and repulsive interactions

Contains 1st order phase transition and critical point

Can elucidate role of fluctuations in phase transitions



Nobel Prize in
1910.

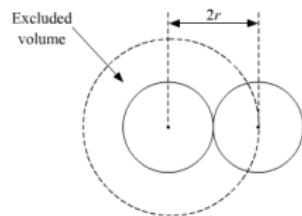
Two ingredients:

1) Short-range repulsion: excluded volume procedure,

$$V \rightarrow V - bN, \quad b = 4 \frac{4\pi r^3}{3}$$

2) Attractive interactions in mean-field approximation,

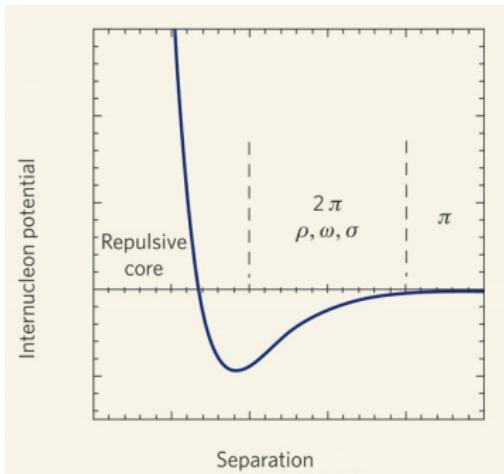
$$P \rightarrow P - a n^2$$



Nucleon-nucleon interaction

Nucleon-nucleon potential:

- Repulsive core at small distances
- Attraction at intermediate distances
- Suggestive similarity to VDW interactions
- Could nuclear matter described by VDW equation?



Standard VDW equation is for **canonical ensemble** and **Boltzmann** statistics

Nucleons are fermions, obey Pauli exclusion principle

Unlike for classical fluids, **quantum statistics** is important

Framework: **VDW equation with quantum statistics**¹

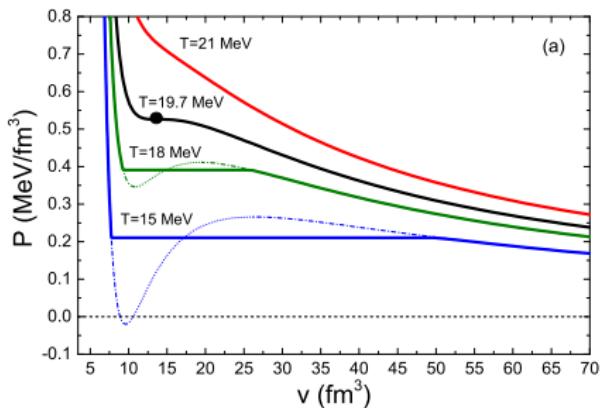
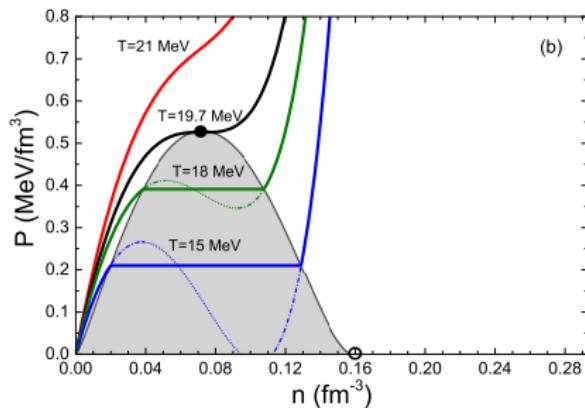
$$p = p^{\text{id}} \left[T, \mu^{\text{id}} \left(\frac{n}{1 - bn}, T \right) \right] - a n^2$$

¹V.V., Anchishkin, Gorenstein, JPA '15 and PRC '15; Redlich, Zalewski, APPB '16.

VDW gas of nucleons: pressure isotherms

a and b fixed to reproduce saturation density and binding energy:

$$n_0 = 0.16 \text{ fm}^{-3}, E/A = -16 \text{ MeV} \Rightarrow a \cong 329 \text{ MeV fm}^3 \text{ and } b \cong 3.42 \text{ fm}^3$$



Behavior qualitatively same as for Boltzmann case

Mixed phase results from Maxwell construction

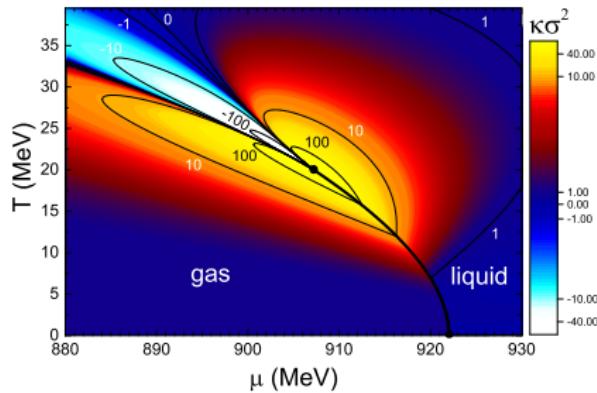
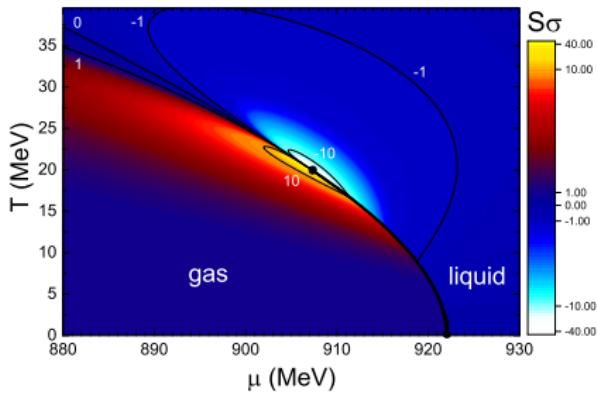
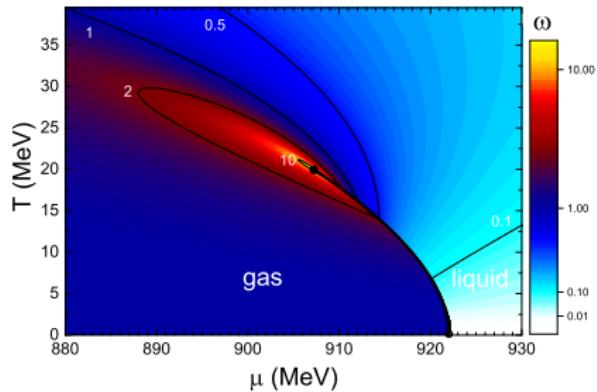
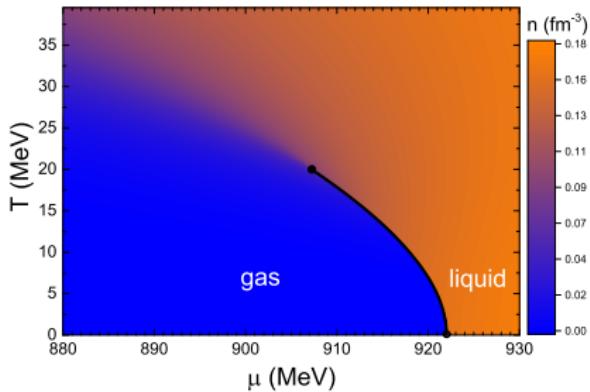
Critical point at $T_c \cong 19.7 \text{ MeV}$ and $n_c \cong 0.07 \text{ fm}^{-3}$

Experimental estimate¹: $T_c = 17.9 \pm 0.4 \text{ MeV}$, $n_c = 0.06 \pm 0.01 \text{ fm}^{-3}$

¹J.B. Elliot, P.T. Lake, L.G. Moretto, L. Phair, Phys. Rev. C 87, 054622 (2013)

VDW gas of nucleons: (T, μ) plane

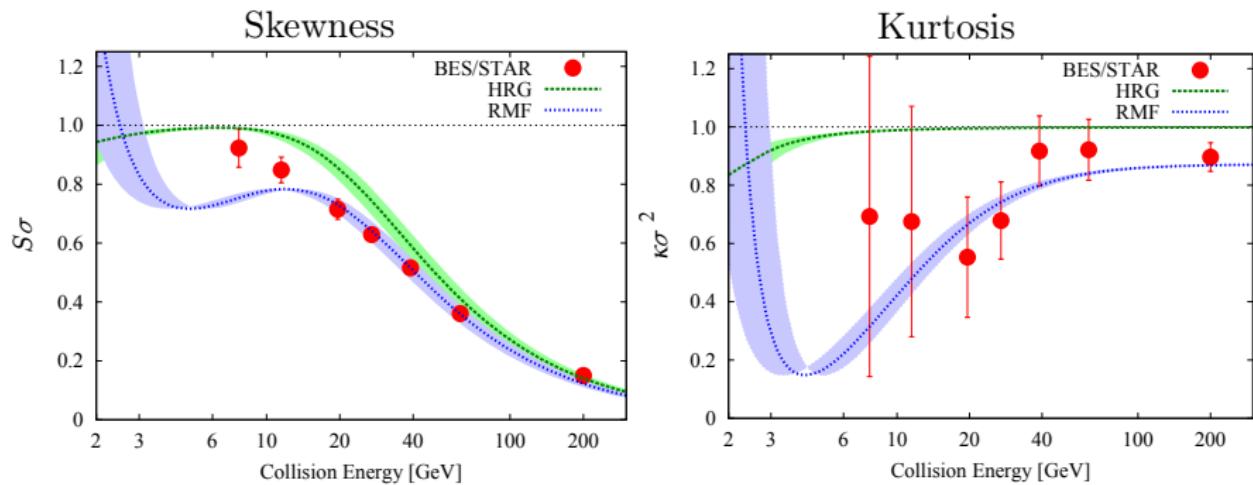
Nuclear matter in (T, μ) plane: structure of critical fluctuations



Net-baryon fluctuations and nuclear matter

Are NN interactions relevant for heavy-ion collisions?

Net-nucleon fluctuations within RMF ($\sigma\text{-}\omega$ model) of nuclear matter along line of “chemical freeze-out”



K. Fukushima, PRC 91, 044910 (2015)

A notable effect in fluctuations even at $\mu_B \simeq 0$

Reconciliation of HRG with nuclear matter can be interesting

Van der Waals interactions in HRG

Simplest generalization of VDW nuclear matter model to full HRG:

- Same VDW interactions between all baryons
- The baryon-antibaryon, meson-meson, and meson-baryon VDW interactions are neglected
- Baryon VDW parameters extracted from ground state of nuclear matter ($a = 329 \text{ MeV fm}^3$, $b = 3.42 \text{ fm}^3$)

Three independent subsystems: **mesons + baryons + antibaryons**

$$p(T, \mu) = P_M(T, \mu) + P_B(T, \mu) + P_{\bar{B}}(T, \mu),$$

$$P_M(T, \mu) = \sum_{j \in M} p_j^{\text{id}}(T, \mu_j) \quad \text{and} \quad P_B(T, \mu) = \sum_{j \in B} p_j^{\text{id}}(T, \mu_j^{B*}) - a n_B^2$$

$$n_B(T, \mu) = (1 - b n_B) \sum_{j \in B} n_j^{\text{id}}(T, \mu_j^{B*}).$$

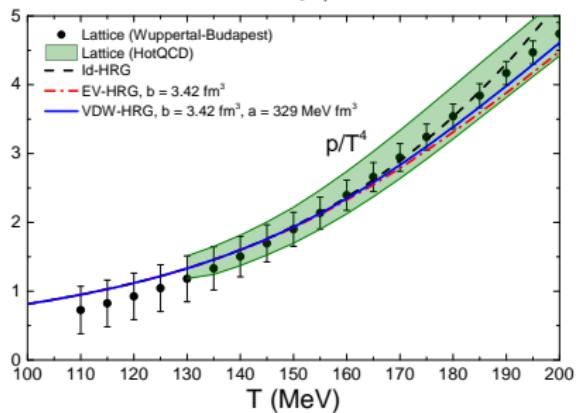
In this simplest setup model is essentially “**parameter-free**”

Transcendental equations for P_B and n_B

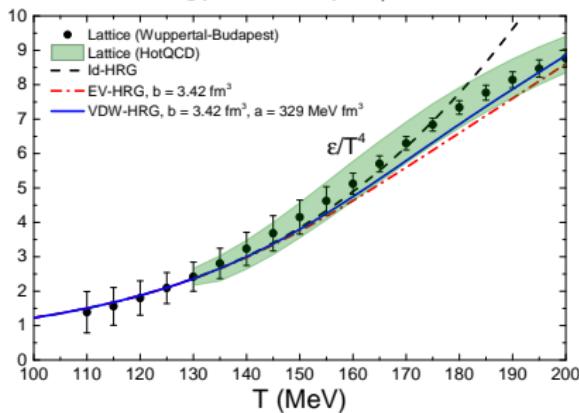
VDW-HRG at $\mu_B = 0$: thermodynamic functions

Comparison of VDW-HRG with lattice QCD at $\mu_B = 0$

Pressure p/T^4

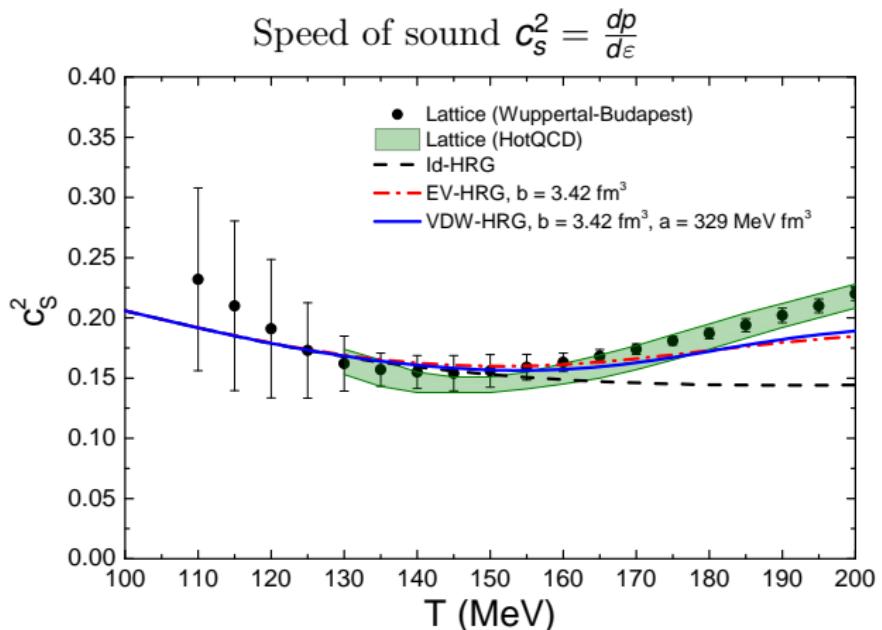


Energy density ε/T^4



- VDW-HRG **does not spoil** existing agreement of Id-HRG with lQCD despite significant EV interactions between baryons
- Not surprising: matter **meson-dominated** at $\mu_B = 0$

VDW-HRG at $\mu_B = 0$: speed of sound

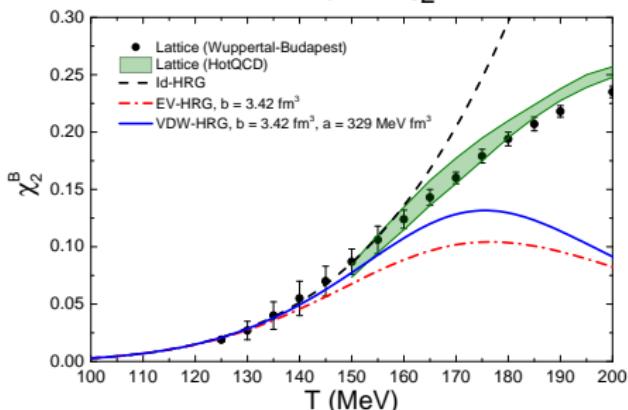


- Monotonic decrease in Id-HRG, at odds with lattice
- **Minimum** for EV-HRG/VDW-HRG at 150-160 MeV
- **No acausal behavior**, often an issue in models with eigenvalues

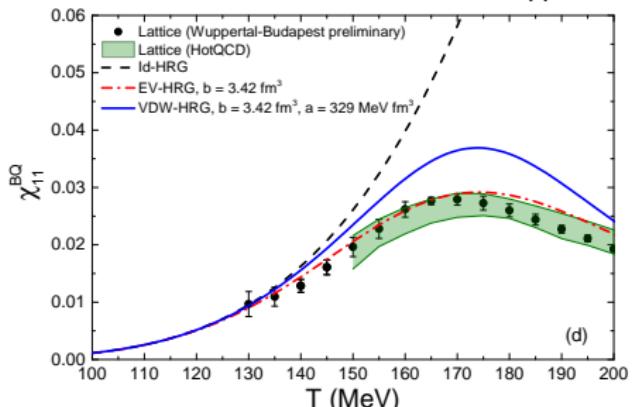
VDW-HRG at $\mu_B = 0$: baryon number fluctuations

Susceptibilities: $\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}$

Net-baryon χ_2^B



Baryon-charge correlator χ_{11}^{BQ}

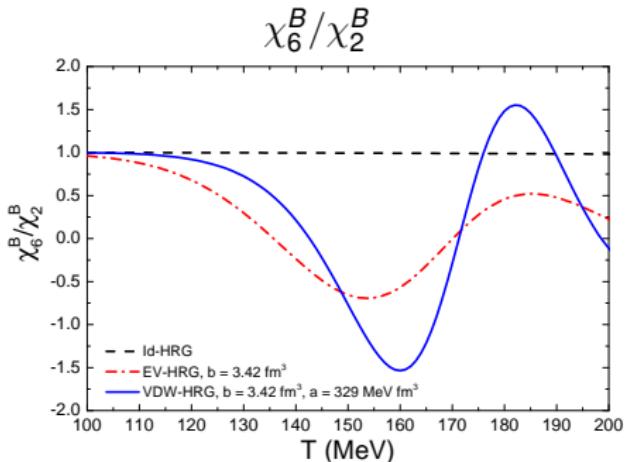
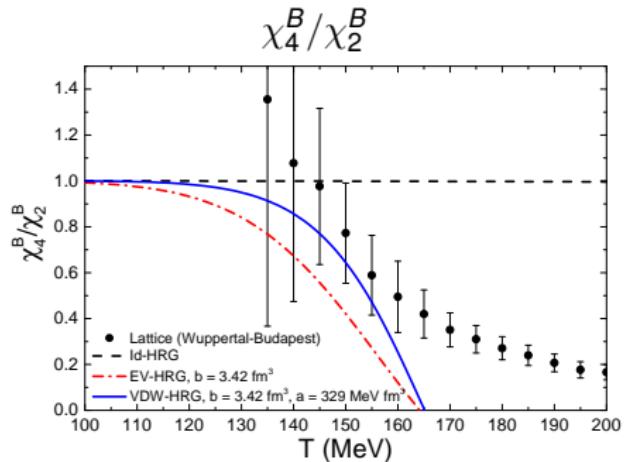


(d)

- Very **different qualitative** behavior between Id-HRG and VDW-HRG
- For χ_2^B lattice data is **between** Id-HRG and VDW-HRG at high T
- For χ_{11}^{BQ} lattice data is below all models, closer to EV-HRG

VDW-HRG at $\mu_B = 0$: baryon number fluctuations

Higher-order of fluctuations are expected to be even more sensitive



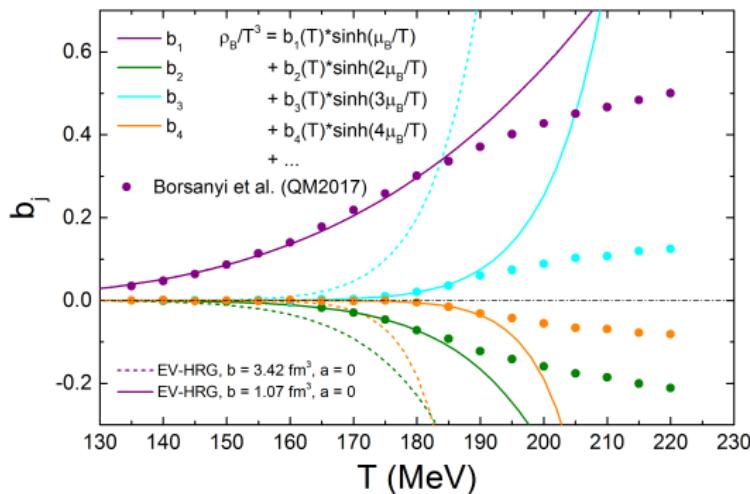
- χ_4^B deviates from χ_2^B at high enough T , stays equal in Id-HRG
- Cannot be related only to onset of deconfinement
- VDW-HRG predicts strong non-monotonic behavior for χ_6^B / χ_2^B

Repulsive baryonic interactions and imaginary μ_B

Lattice QCD is problematic at real μ but tractable at **imaginary** μ

E.g., net-baryon density is **imaginary** and has **trigonometric series** form

$$\mu_B \rightarrow \tilde{\mu}_B = i \mu_B \quad \Rightarrow \quad \frac{n_B}{T^3} = -i \sum_{j=1}^{\infty} b_j(T) \sin(j \mu_B / T)$$



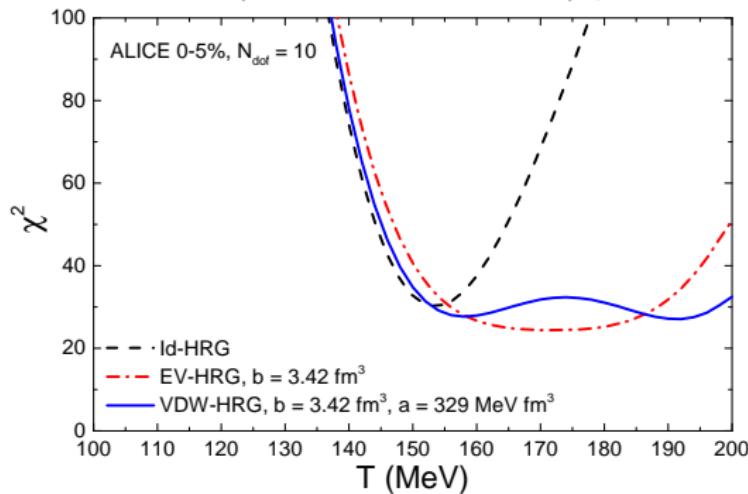
- Non-zero $b_j(T)$ for $j \geq 2$ signal deviations from ideal gas
- Addition of EV interactions between baryons **reproduces lattice trend**

V.V., H. Stoecker, and Wuppertal-Budapest collaboration, in preparation

VDW-HRG: influence on hadron ratios

VDW interactions **change** relative **hadron yields**

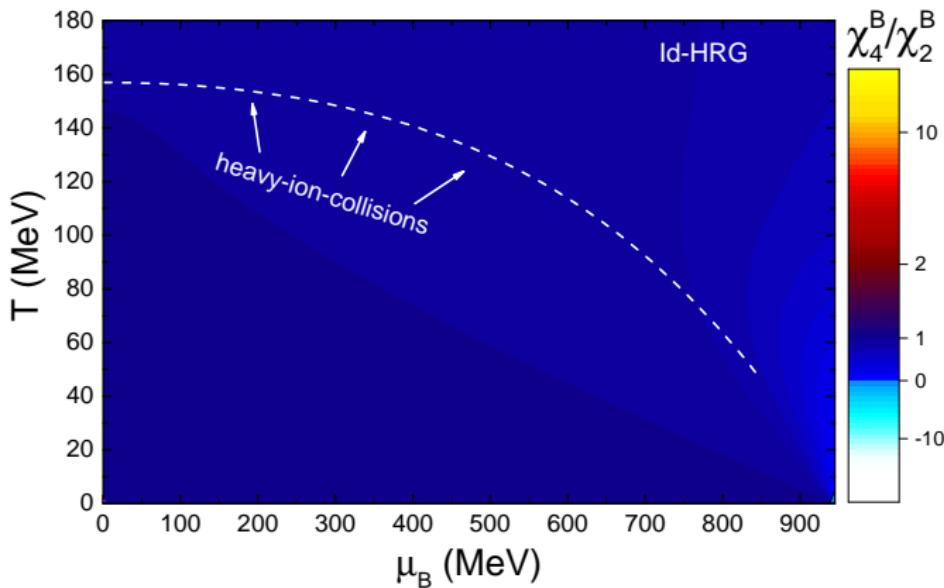
Thermal fit to **ALICE** (Pb+Pb @ 2.76 TeV) yields: from π to Ω



- Fit quality slightly **better** in EV-HRG/VDW-HRG vs Id-HRG but very **different picture!**
- All** temperatures between 150 and 200 MeV yield similarly **fair** data description in VDW-HRG
- Results likely to be **sensitive** to further **modifications**, e.g for strangeness

VDW-HRG at finite μ_B

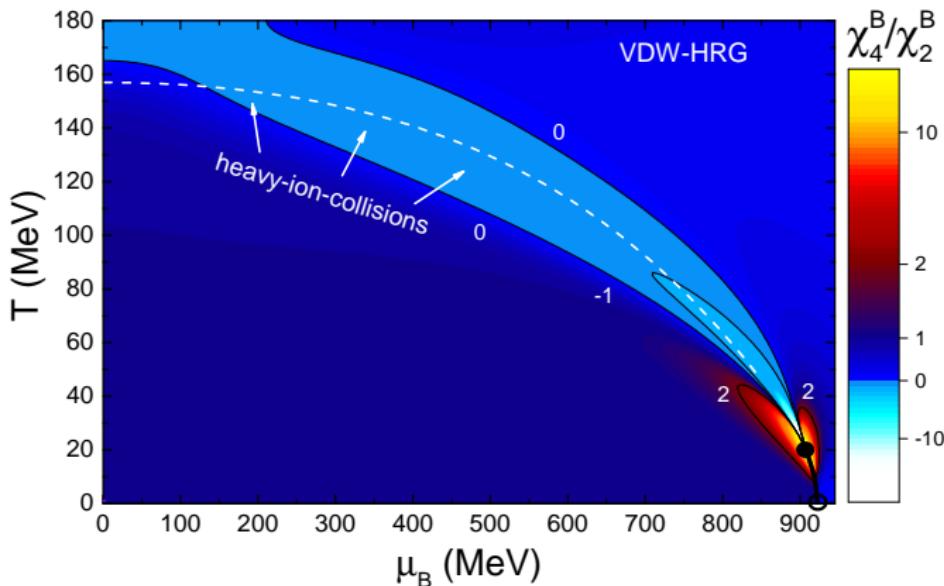
Net-baryon fluctuations in T - μ_B plane: χ_4^B/χ_2^B



- Almost no effect in Id-HRG, only Fermi statistics...

VDW-HRG at finite μ_B

Net-baryon fluctuations in T - μ_B plane: χ_4^B/χ_2^B



- Almost no effect in Id-HRG, only Fermi statistics...
- Rather rich structure for VDW-HRG, huge effect of VDW interactions!
- Fluctuations seen at RHIC are remnants of nuclear liquid-gas PT?

Summary

- Nuclear matter can be described as VDW equation with Fermi statistics
- VDW interactions between baryons have strong influence on fluctuations of conserved charges in the crossover region within HRG
- Freeze-out parameters extracted from thermal fits within ideal HRG are not unique, procedure sensitive to modeling of VDW interactions
- Nuclear liquid-gas transition manifests itself into non-trivial net-baryon fluctuations in regions of phase diagram probed by HIC
- Interpretation of results obtained within standard ideal HRG should be done with extreme care

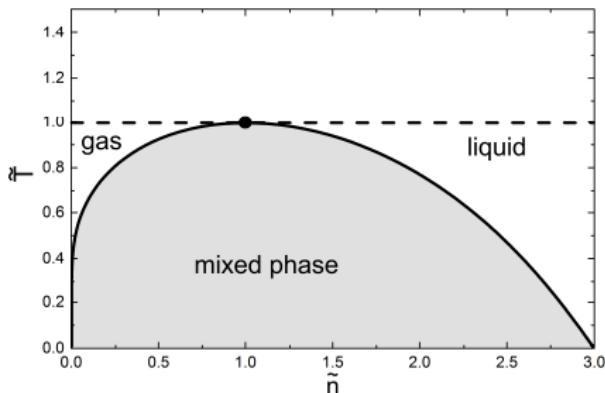
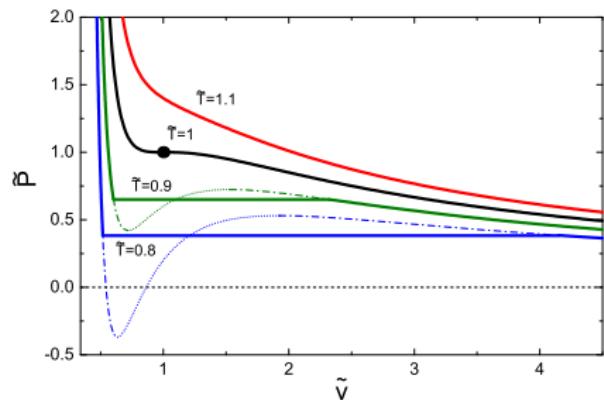
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Thanks for your attention!

Backup slides

Van der Waals equation

- VDW isotherms show irregular behavior below certain temperature T_C
- Below T_C isotherms are corrected by Maxwell's rule of equal areas
- Results in appearance of mixed phase



Critical point

$$\frac{\partial p}{\partial v} = 0, \quad \frac{\partial^2 p}{\partial v^2} = 0, \quad v = V/N$$

$$p_C = \frac{a}{27b^2}, \quad n_C = \frac{1}{3b}, \quad T_C = \frac{8a}{27b}$$

Reduced variables

$$\tilde{p} = \frac{p}{p_C}, \quad \tilde{n} = \frac{n}{n_C}, \quad \tilde{T} = \frac{T}{T_C}$$

VDW equation originally formulated in **canonical ensemble**

How to transform **CE** pressure $P(T, n)$ into **GCE** pressure $P(T, \mu)$?

- Calculate $\mu(T, V, N)$ from standard TD relations
- Invert the relation to get $N(T, V, \mu)$ and put it back into $P(T, V, N)$
- Consistency due to thermodynamic equivalence of ensembles

Result: transcendental equation for $n(T, \mu)$

$$\frac{N}{V} \equiv n(T, \mu) = \frac{n_{\text{id}}(T, \mu^*)}{1 + b n_{\text{id}}(T, \mu^*)}, \quad \mu^* = \mu - b \frac{n T}{1 - b n} + 2 a n$$

- Implicit equation in GCE, in CE it was explicit
- May have multiple solutions below T_C
- Choose one with largest pressure – equivalent to Maxwell rule in CE

Advantages of the GCE formulation

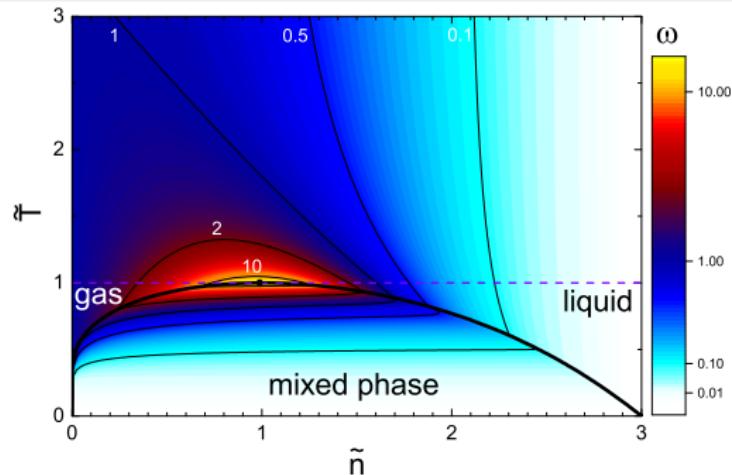
- 1) **Hadronic** physics applications: number of hadrons usually **not conserved**.
- 2) **CE** cannot describe particle number **fluctuations**. N-fluctuations in a small ($V \ll V_0$) subsystem follow **GCE** results.
- 3) Good starting point to include effects of **quantum statistics**.

Scaled variance for classical VDW equation

Transformation of VDW eq. to GCE leads to new physical applications¹

Scaled variance of particle number fluctuations in VDW gas (pure phases)

$$\omega[N] = \frac{\sigma^2}{\langle N \rangle} = \left[\frac{1}{(1 - bn)^2} - \frac{2an}{T} \right]^{-1}$$



- Repulsive interactions suppress N-fluctuations
- Attractive interactions enhance N-fluctuations

¹Vovchenko et al., J. Phys. A 305001, 48 (2015) and D. Anchishkin's talk on Monday

Scaled variance in VDW equation

New application from GCE formulation: **particle number fluctuations**

Scaled variance is an **intensive** measure of N-fluctuations

$$\frac{\sigma^2}{N} = \omega[N] \equiv \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} = \frac{T}{n} \left(\frac{\partial n}{\partial \mu} \right)_T = \frac{T}{n} \left(\frac{\partial^2 P}{\partial \mu^2} \right)_T$$

In **ideal** Boltzmann gas fluctuations are Poissonian and $\omega_{id}[N] = 1$.

$\omega[N]$ in VDW gas (pure phases)

$$\omega[N] = \left[\frac{1}{(1 - bn)^2} - \frac{2an}{T} \right]^{-1}$$

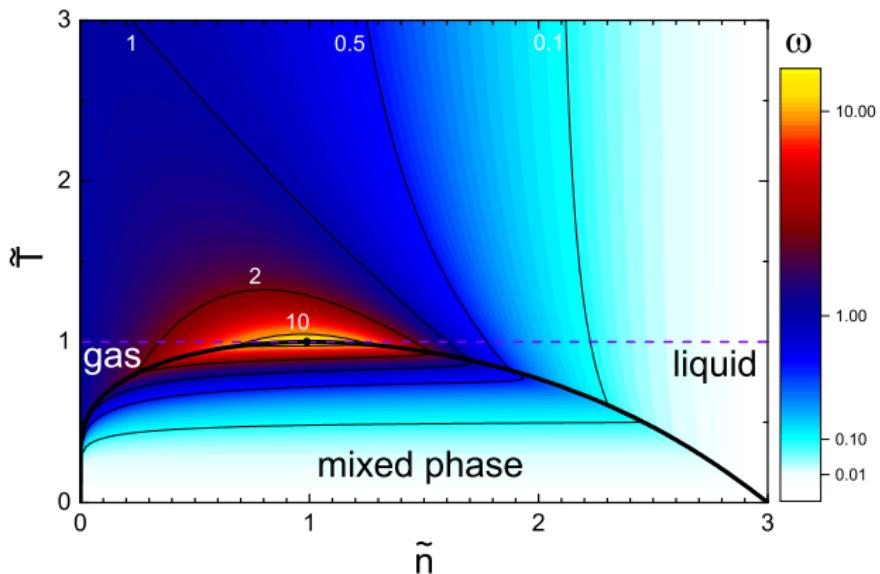
- Repulsive interactions **suppress** N-fluctuations
- Attractive interactions **enhance** N-fluctuations

N-fluctuations are useful because they

- Carry information about finer details of EoS, e.g. **phase transitions**
- Measurable **experimentally**

Scaled variance

$$\omega[N] = \frac{1}{9} \left[\frac{1}{(3 - \tilde{n})^2} - \frac{\tilde{n}}{4\tilde{T}} \right]^{-1}$$



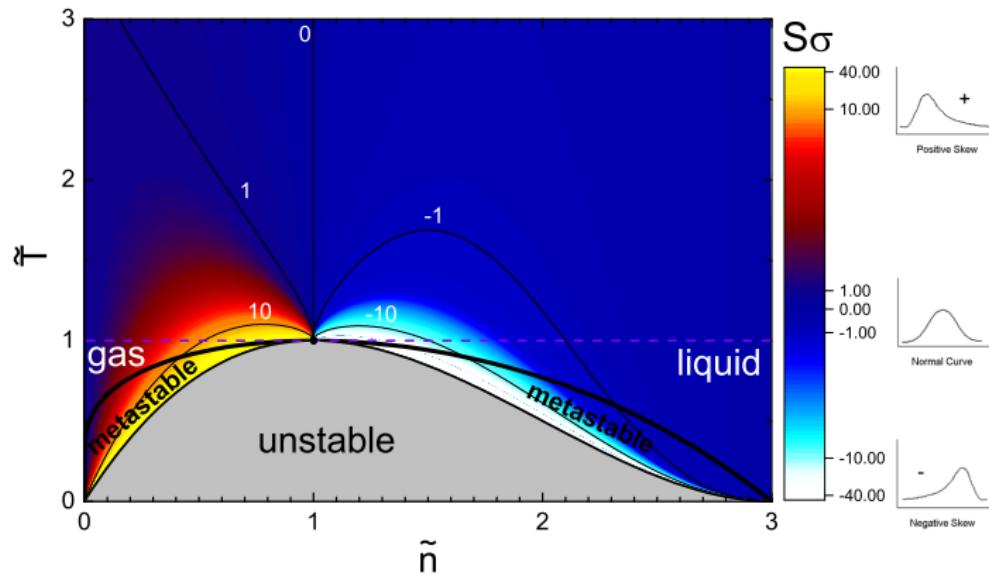
- Deviations from unity signal effects of interaction
- Fluctuations grow rapidly near critical point

V. Vovchenko et al., J. Phys. A 305001, 48 (2015)

Skewness

Higher-order (non-gaussian) fluctuations are even more sensitive

Skewness: $S\sigma = \frac{\langle(\Delta N)^3\rangle}{\sigma^2} = \omega[N] + \frac{T}{\omega[N]} \left(\frac{\partial \omega[N]}{\partial \mu} \right)_T$ asymmetry

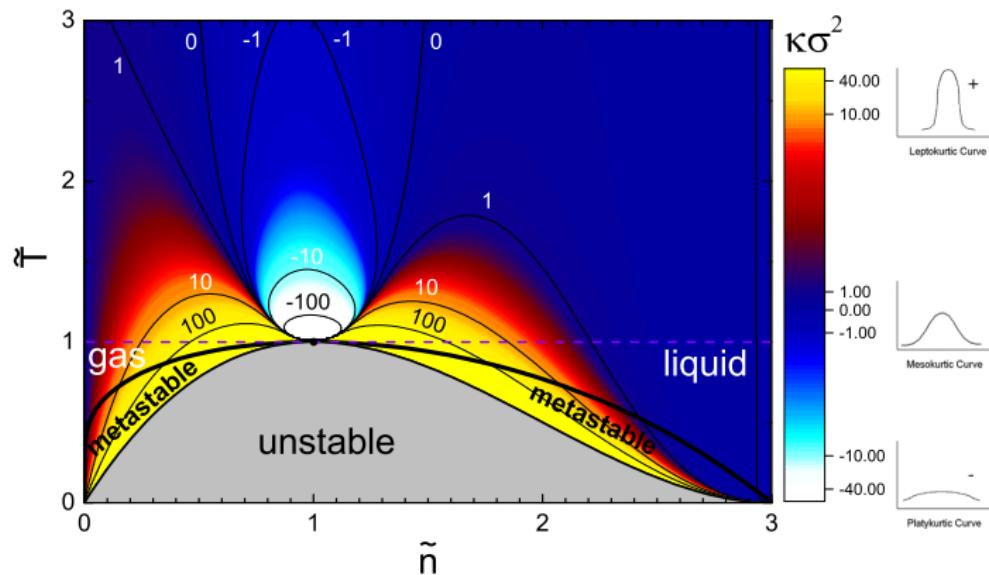


Skewness is

- Positive (right-tailed) in gaseous phase
- Negative (left-tailed) in liquid phase

Kurtosis

$$\text{Kurtosis: } \kappa\sigma^2 = \frac{\langle(\Delta N)^4\rangle - 3\langle(\Delta N)^2\rangle^2}{\sigma^2} \quad \text{peakedness}$$



Kurtosis is **negative** (flat) above critical point (crossover), **positive** (peaked) elsewhere and very **sensitive** to the **proximity** of the critical point

V. Vovchenko et al., J. Phys. A 015003, 49 (2016)

VDW equation with quantum statistics in GCE

Requirements for VDW equation with quantum statistics

- 1) Reduce to **ideal quantum gas** at $a = 0$ and $b = 0$
- 2) Reduce to **classical VDW** when quantum statistics are negligible
- 3) $s \geq 0$ and $s \rightarrow 0$ as $T \rightarrow 0$

Ansatz: Take pressure in the following form^{1,2}

$$p(T, \mu) = p^{\text{id}}(T, \mu^*) - an^2, \quad \mu^* = \mu - bp - abn^2 + 2an$$

where $p^{\text{id}}(T, \mu^*)$ is pressure of ideal **quantum** gas.

$$n(T, \mu) = \left(\frac{\partial p}{\partial \mu} \right)_T = \frac{n^{\text{id}}(T, \mu^*)}{1 + b n^{\text{id}}(T, \mu^*)}$$

Algorithm for GCE

- 1) Solve system of eqs. for p and n at given (T, μ)
- 2) Choose the solution with **largest** pressure

¹V. Vovchenko, D. Anchishkin, M. Gorenstein, Phys. Rev. C 91, 064314 (2015)

²**Alternative derivation:** K. Redlich, K. Zalewski, arXiv:1605.09686 (2016)

³ $a=0 \Rightarrow$ **excluded-volume** model, D. Rischke et al., Z.Phys. C51, 485 (1991)

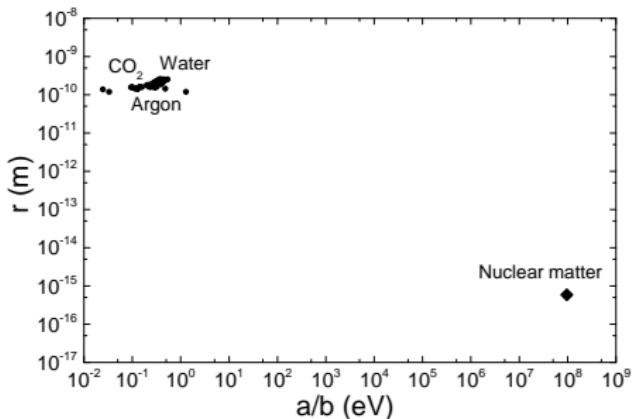
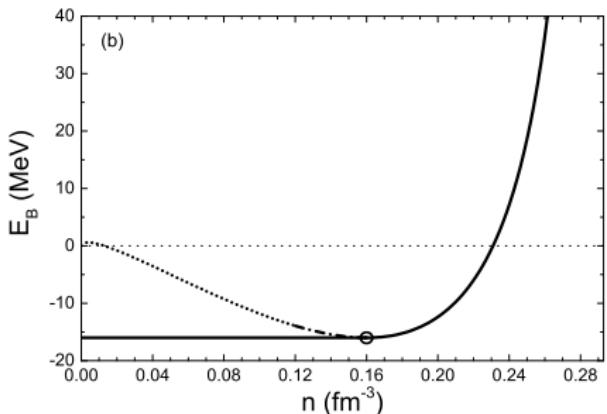
VDW gas of nucleons: zero temperature

How to fix a and b ? For classical fluid usually tied to CP location.

Different approach: Reproduce **saturation density** and **binding energy**

From $E_B = E/A \cong -16$ MeV and $n = n_0 \cong 0.16$ fm $^{-3}$ at $T = 0$ and $p = 0$

$$a \cong 329 \text{ MeV fm}^3 \text{ and } b \cong 3.42 \text{ fm}^3$$



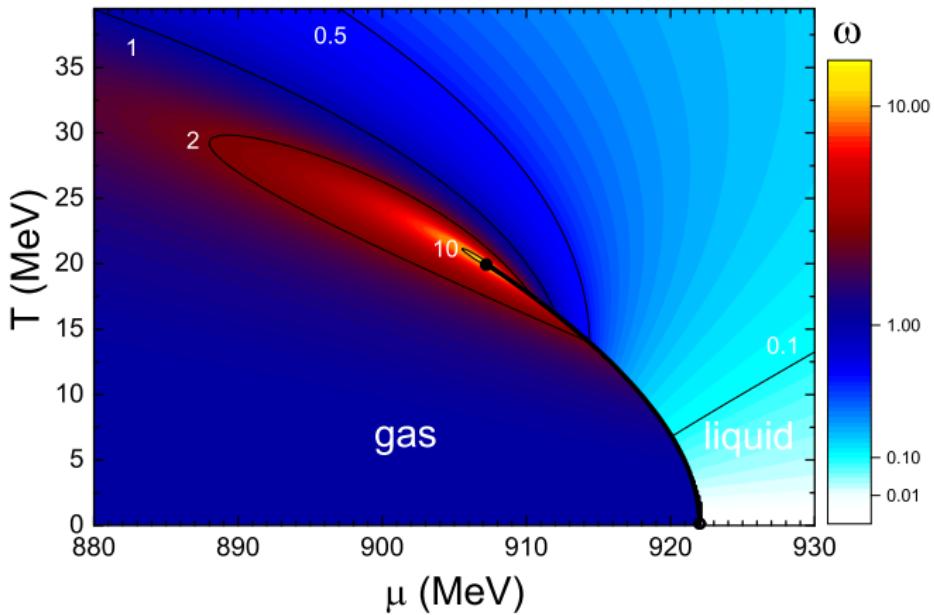
Mixed phase at $T = 0$ is specific:
A mix of vacuum ($n = 0$) and liquid
at $n = n_0$

VDW eq. now at very different scale!

VDW gas of nucleons: scaled variance

Scaled variance in quantum VDW:

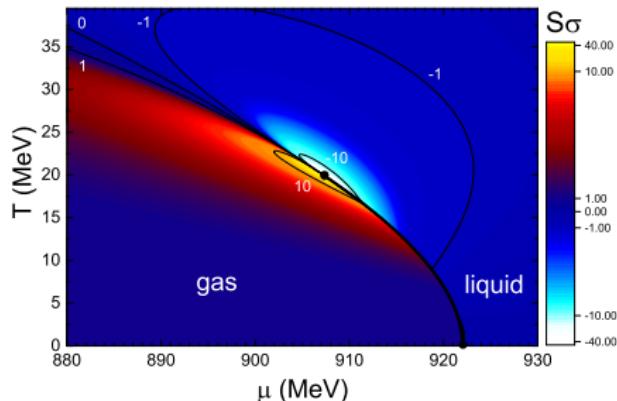
$$\omega[N] = \omega_{\text{id}}(T, \mu^*) \left[\frac{1}{(1 - bn)^2} - \frac{2an}{T} \omega_{\text{id}}(T, \mu^*) \right]^{-1}$$



VDW gas of nucleons: skewness and kurtosis

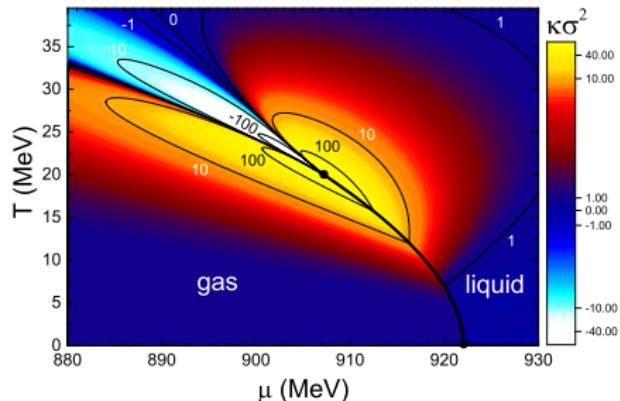
Skewness

$$S\sigma = \omega[N] + \frac{T}{\omega[N]} \left(\frac{\partial \omega[N]}{\partial \mu} \right)_T$$



Kurtosis

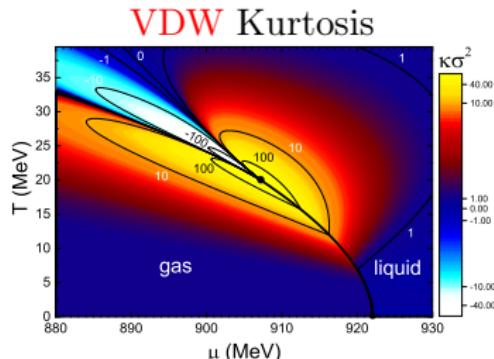
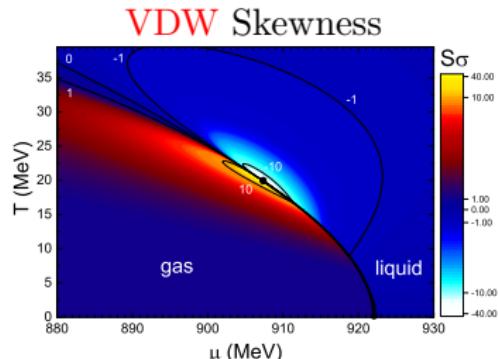
$$\kappa\sigma^2 = (S\sigma)^2 + T \left(\frac{\partial [S\sigma]}{\partial \mu} \right)_T$$



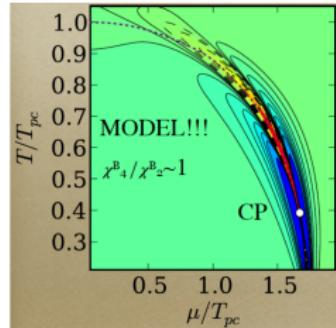
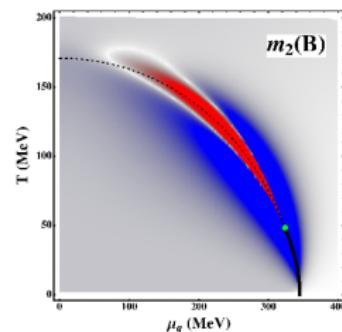
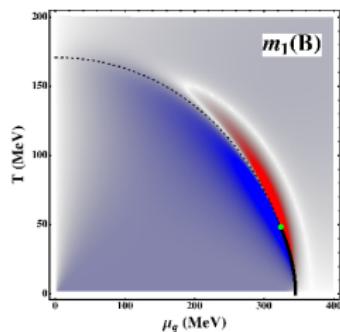
For skewness and kurtosis singularity is rather specific: sign depends on the path of approach

V. Vovchenko et al., Phys. Rev. C 92, 054901 (2015)

VDW gas of nucleons: skewness and kurtosis



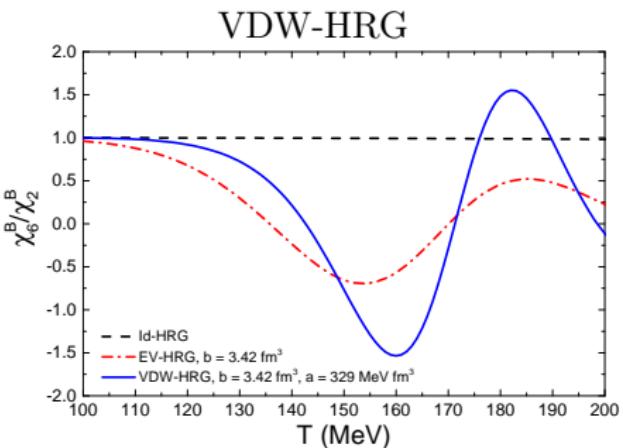
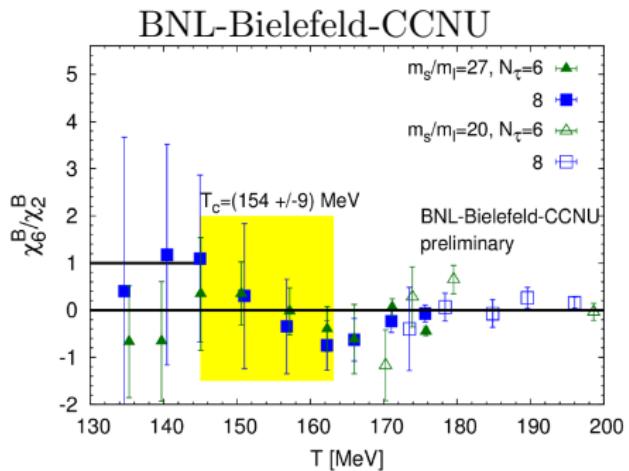
NJL, J.W. Chen et al., PRD 93, 034037 (2016) PQM, V. Skokov, QM2012



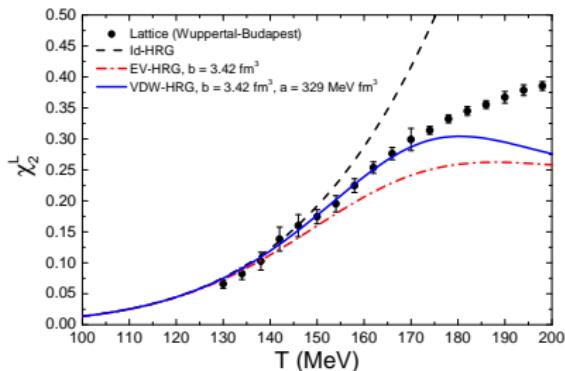
Fluctuation patterns in VDW very similar to effective QCD models

VDW-HRG at $\mu = 0$: baryon number fluctuations

$$\chi_6^B/\chi_2^B$$

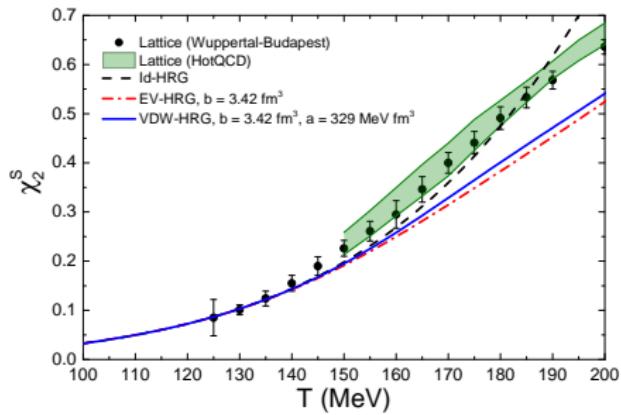


VDW-HRG at $\mu = 0$: net-light and net-strangeness



- Net number of light quarks χ_2^L
- $L = (u + d)/2 = (3B + S)/2$
- Improved description in VDW-HRG

- Net-strangeness χ_2^S
- Underestimated by HRG models, similar for χ_{11}^{BS}
- Extra strange states?¹
- Weaker VDW interactions for strange baryons?²



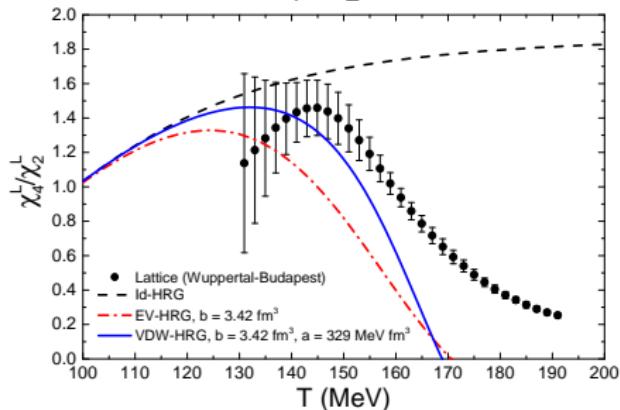
¹Bazavov et al., PRL 113, 072001 (2014)

²Alba, Vovchenko, Gorenstein, Stoecker, arXiv:1606.06542

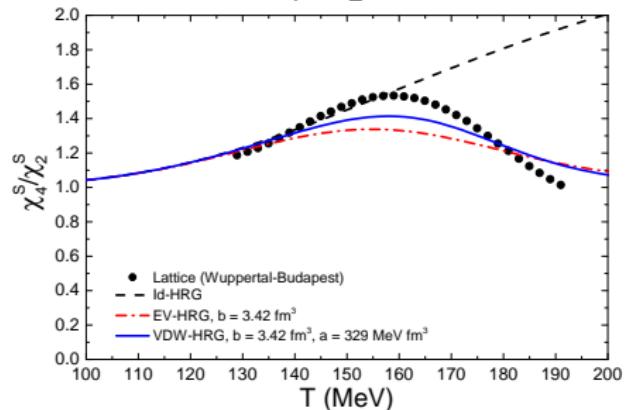
VDW-HRG at $\mu_B = 0$: net-light and net-strangeness

Fluctuations of **net-light** $L = (u + d)/2 = (3B + S)/2$ and **net-strangeness**

$$\chi_4^L/\chi_2^L$$



$$\chi_4^S/\chi_2^S$$



- Lattice shows **peaked structures** in crossover regions
- Not at all reproduced by Id-HRG, signal for deconfinement?¹
- **Peaks** at different T for net-L and net-S \Rightarrow **flavor hierarchy?**²
- VDW-HRG **also shows** peaks and flavor hierarchy \Rightarrow cannot be traced back directly to deconfinement

¹S. Ejiri, F. Karsch, K. Redlich, PLB 633, 275 (2006)

²Bellwied et al., PRL 111, 202302 (2013)

VDW-HRG: extensions

Effect of reducing VDW interactions involving strange hadrons

- 3 times smaller EV for strange baryons
- Small EV for mesons
- Illustrative calculation
- Most observables improved

