Charge fluctuations in isobar collisions and connections with lattice QCD

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RBRC Workshop: Physics Opportunities from the RHIC Isobar Run

January 27, 2021

- Conserved charge fluctuations in heavy-ion collisions and (lattice) QCD EoS
- Opportunities with isobar collisions

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QCD phase diagram with heavy-ion collisions

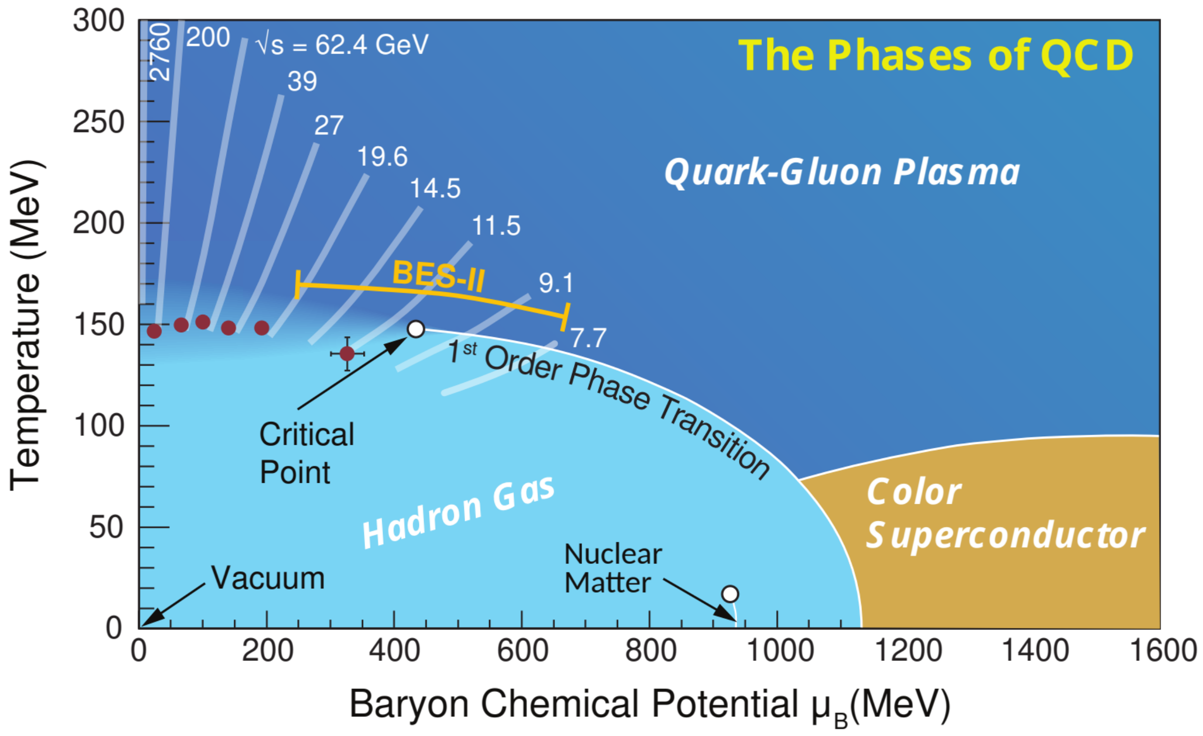
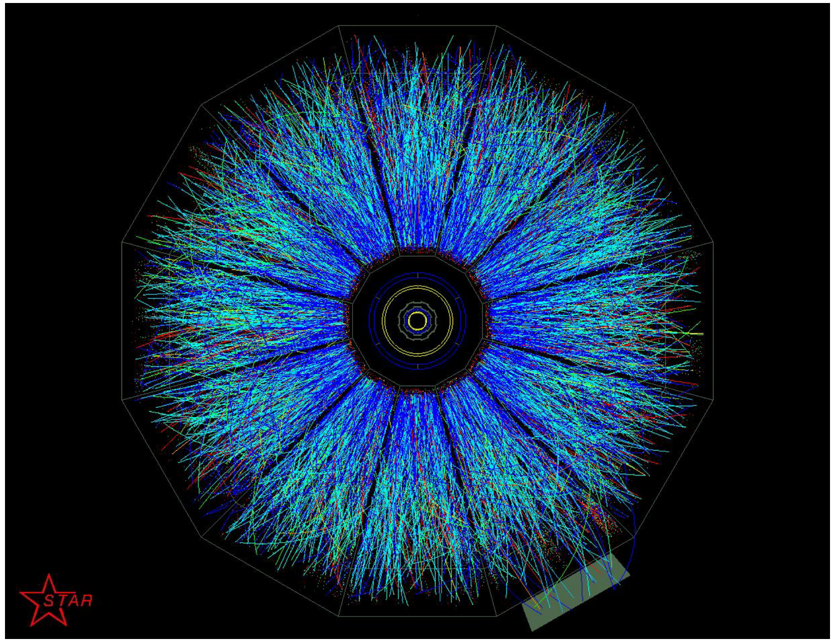


Figure from Bzdak et al., Phys. Rept. '20



STAR event display

Thousands of particles created in relativistic heavy-ion collisions

↓

Apply concepts of statistical mechanics

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

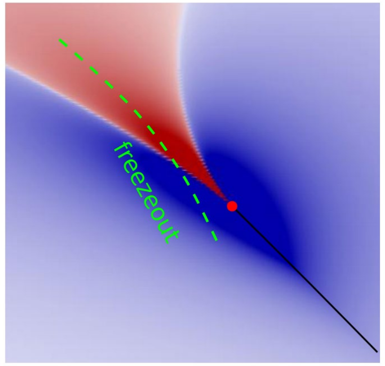
Grand partition function

$$\ln Z^{\mathrm{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\mathrm{ce}}(T,V,N)\right]$$

$$\boxed{\kappa_n \propto \frac{\partial^n(\ln Z^{\mathrm{gce}})}{\partial\mu^n}}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- QCD critical point
- Test of (lattice) QCD at $\mu_B \approx 0$
- Freeze-out from fluctuations



M. Stephanov, PRL '09
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

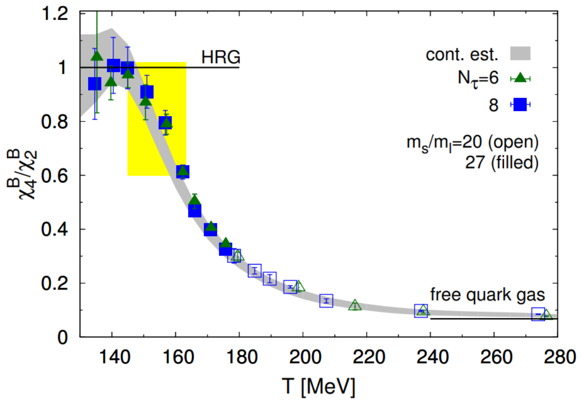
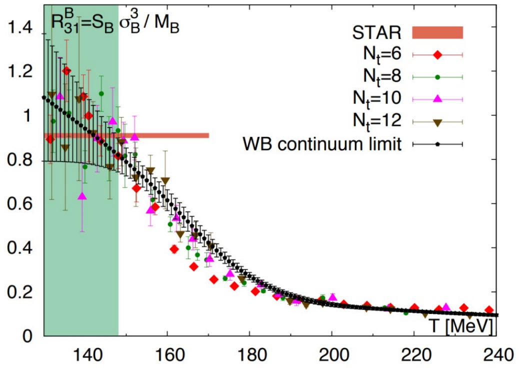


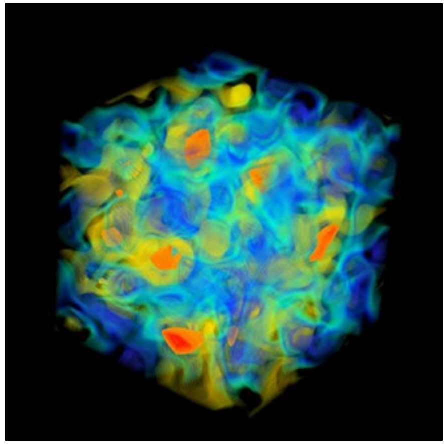
Figure from Bazavov et al. PRD 95, 054504 (2017)
Probed by LHC and top RHIC



Borsanyi et al. PRL 113, 052301 (2014)
Bazavov et al. PRL 109, 192302 (2012)
...

Theory vs experiment

Theory



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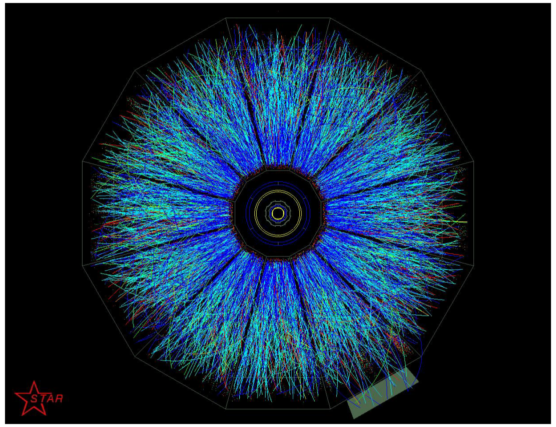
- Coordinate space
- Periodic box, grand-canonical
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

- Momentum space
- Expanding in vacuum, canonical
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

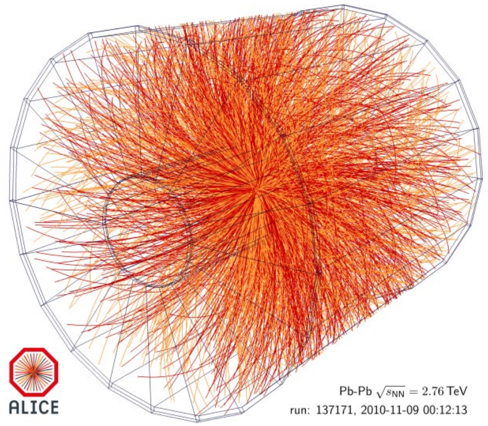
When are the measured fluctuations grand-canonical?

- Consider event-by-event fluctuations of particle number in acceptance ΔY_{accept} around midrapidity

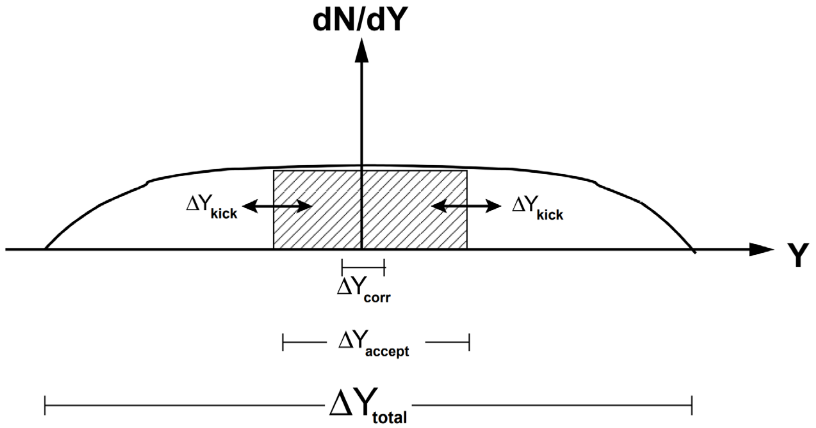
- Scales
  - ΔY_{accept} – acceptance
  - ΔY_{total} – full space
  - ΔY_{corr} – rapidity correlation length (thermal smearing)
  - ΔY_{kick} – diffusion in the hadronic phase

- **GCE applies if** $\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}, \Delta Y_{corr}$

- In practice $\Delta Y_{total} \gg \Delta Y_{accept}$ nor $\Delta Y_{accept} \gg \Delta Y_{corr}$ are not simultaneously satisfied
  - Corrections from global conservation are large [Bzdak et al., PRC '13]
  - $\Delta Y_{corr} \sim 1 \sim \Delta Y_{accept}$ [Ling, Stephanov, PRC '16]



ALICE event display

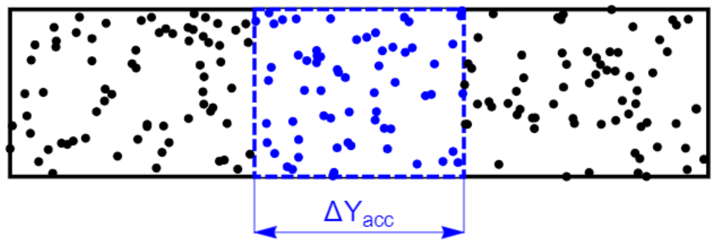


V. Koch, arXiv:0810.2520

Correcting for exact conservation of conserved charges

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, Phys. Lett. B 811, 135868 (2020)

$$P_1^{\rm ce}(B_1) \propto \sum_{B_1,B_2} P_1^{\rm gce}(B_1) P_2^{\rm gce}(B_2) \times \delta_{B,B_1+B_2}$$



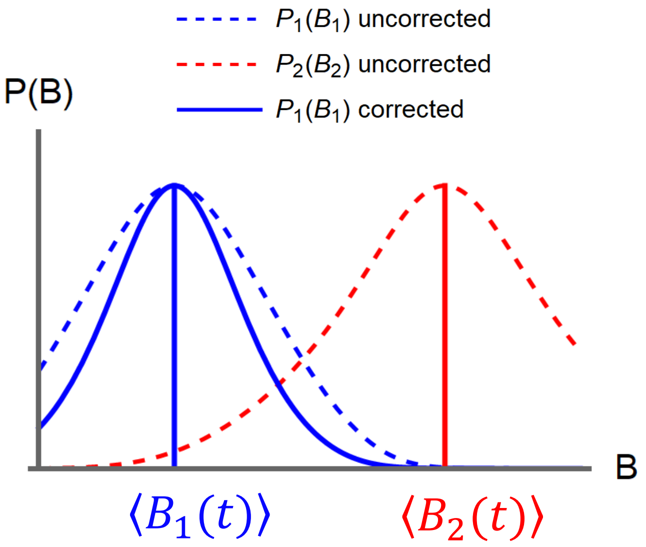
- Subensemble acceptance method (SAM)
  - Corrects *any* equation of state for global BQS-charge conservation
  - Canonical ensemble cumulants in terms of grand-canonical susceptibilities ← *connection to lattice QCD*

$$\kappa_2/\kappa_1 = (1-\alpha)\,\chi_2^B/\chi_1^B, \qquad \kappa_3/\kappa_2 = (1-2\alpha)\,\chi_3^B/\chi_2^B, \qquad \ldots$$

α – acceptance fraction

- SAM-2.0 VV, Phys. Rev. C 105, 014903 (2022)
  - Non-conserved quantities (e.g. proton number)
  - Spatially inhomogeneous systems
  - Momentum space
  - Maps "grand-canonical" cumulants inside and outside the acceptance to the "canonical" cumulants inside the acceptance:*

$$\boxed{\kappa_{p,B}^{\rm in,ce} = {\rm SAM}\left[\kappa_{p,B}^{\rm in,gce}, \kappa_{p,B}^{\rm out,gce}\right]}$$



*Explicit expressions for any cumulant available via a Mathematica notebook at https://github.com/vlvovch/SAM

Multiple conserved charges (B,Q,S), uniform systems

VV, Poberezhnyuk, Koch, JHEP 10, 089 (2020)

Key findings:

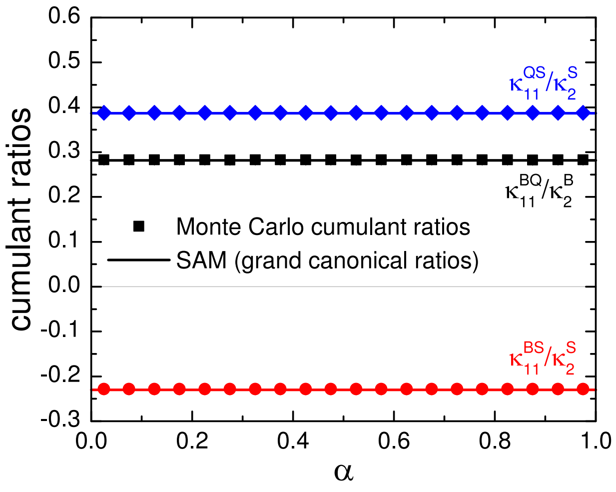
- Cumulants up to 3rd order factorize into product of binomial and grand-canonical cumulants

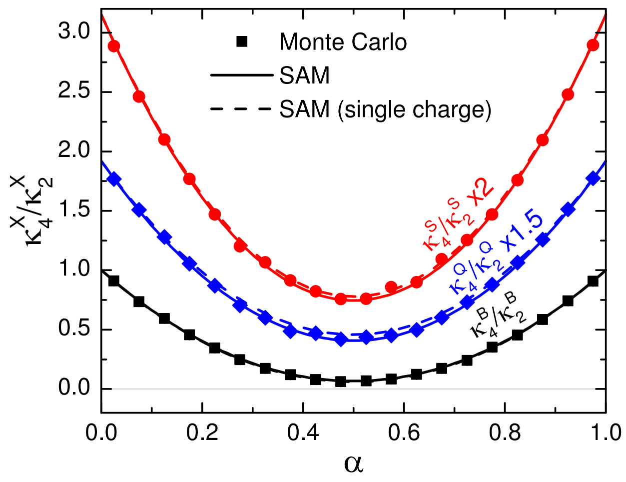
$$\kappa_{l,m,n} = \kappa^{\text{bino}}_{l+m+n}(\alpha) \times \kappa^{\text{gce}}_{l,m,n}\,, \qquad l+m+n \leq 3$$

  - Ratios of second and third order cumulants are NOT sensitive to charge conservation
  - Also true for the measurable ratios of covariances involving one non-conserved charge, such as κ_{pQ}/κ_{kQ}

- For order $n > 3$ charge cumulants “mix”. Effect in HRG is tiny

$$\kappa_4^B = \kappa_4^{B,\text{gce}}\,\beta\left[(1-3\alpha\beta)\,\chi_4^B - 3\,\alpha\,\beta\,\frac{(\chi_3^B)^2\chi_2^Q - 2\chi_{21}^{BQ}\chi_{11}^{BQ}\chi_3^B + (\chi_{21}^{BQ})^2\chi_2^B}{\chi_2^B\chi_2^Q - (\chi_{11}^{BQ})^2}\right]$$





Experiment: Measurements of the off-diagonal cumulants are in progress, e.g. [STAR Collaboration, arXiv:1903.05370]

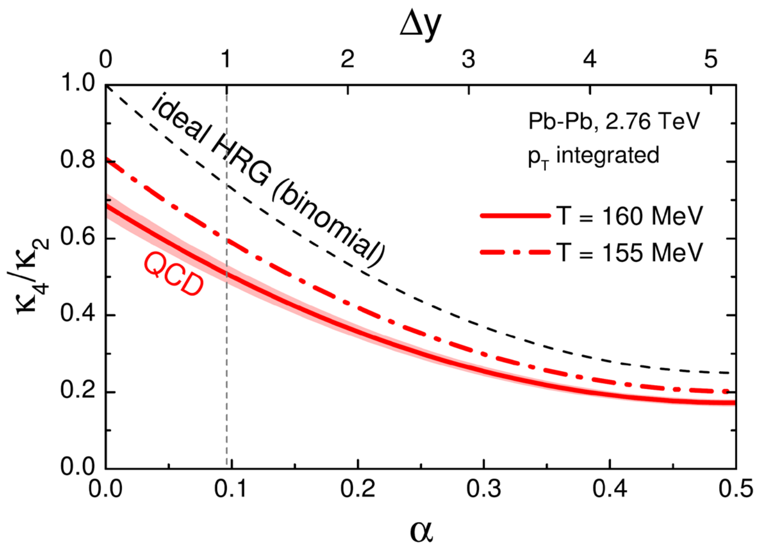
Mathematica notebook to express any B,Q,S-cumulant of order $n \leq 6$ in terms of grand-canonical susceptibilities available at https://github.com/vlvovch/SAM

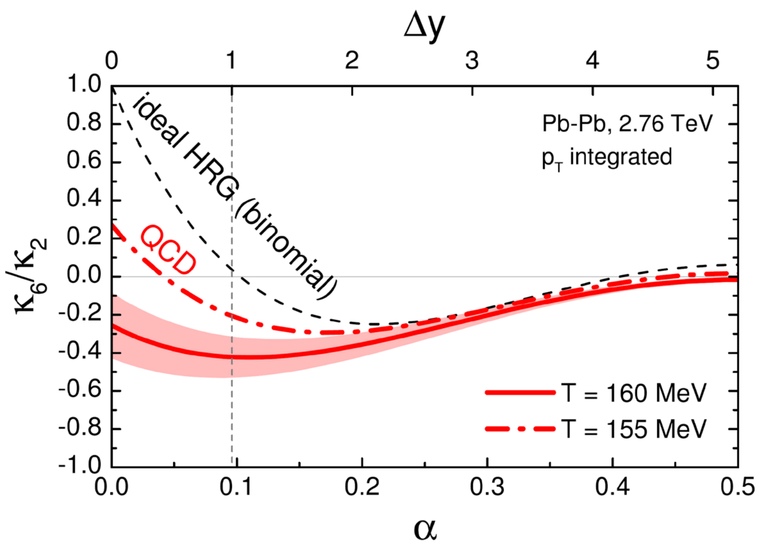
Net baryon fluctuations at LHC from lattice QCD ($\mu_B = 0$)

Momentum rapidity $y \approx$ space-time rapidity η_s

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

$$\left(\frac{\kappa_4}{\kappa_2}\right)_{LHC} = (1-3\alpha\beta)\frac{\chi_4^B}{\chi_2^B} \qquad \left(\frac{\kappa_6}{\kappa_2}\right)_{LHC} = [1-5\alpha\beta(1-\alpha\beta)]\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$





Lattice QCD input:
Borsanyi et al., 1805.04445

Theory: negative χ_6^B/χ_2^B is a possible signal of chiral criticality [Friman, Karsch, Redlich, Skokov, EPJC '11]

Experiment*: $\alpha \approx \frac{N_{ch}(\Delta y)}{N_{ch}(\infty)} \approx \mathrm{erf}\left(\frac{\Delta y}{2\sqrt{2}\sigma_y}\right)$, for $\Delta y \approx 1$ the κ_6/κ_2 is mainly sensitive to the EoS

For the effect of thermal smearing + other net-particle fluctuations see VV, Koch, PRC 103, 044903 (2021)

*$N_{ch}(\Delta y)$ measurement: ALICE Collaboration, PLB 726 (2013) 610-622

RHIC-BES: Hydrodynamic description in non-critical scenario

- Collision geometry based 3D initial state [Shen, Alzhrani, PRC '20]
  - Constrained to net proton distributions
- Viscous hydrodynamics evolution – MUSIC-3.0
  - Energy-momentum and baryon number conservation
  - NEOS-BQS crossover equation of state [Monnai, Schenke, Shen, PRC '19]

- Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3

$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x)\, p^\mu \, \frac{d_j \, \lambda_j^{\rm ev}(x)}{(2\pi)^3} \, \exp\left[\frac{\mu_j(x) - u^\mu(x) p_\mu}{T(x)}\right].$$

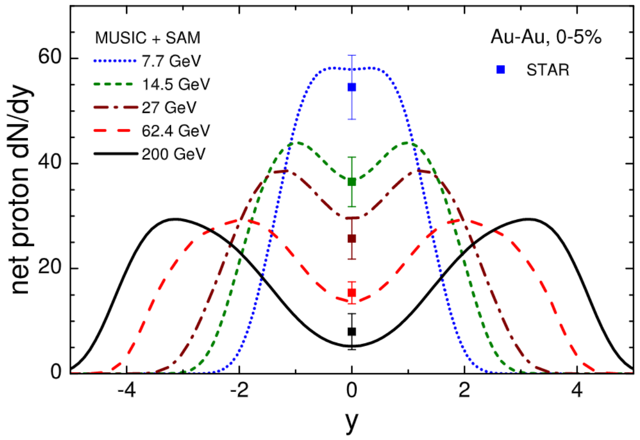
- Particlization includes QCD-based baryon number distribution
  - Here incorporated via baryon excluded volume

[VV, Pasztor, Fodor, Katz, Stoecker, PLB 775, 71 (2017)]

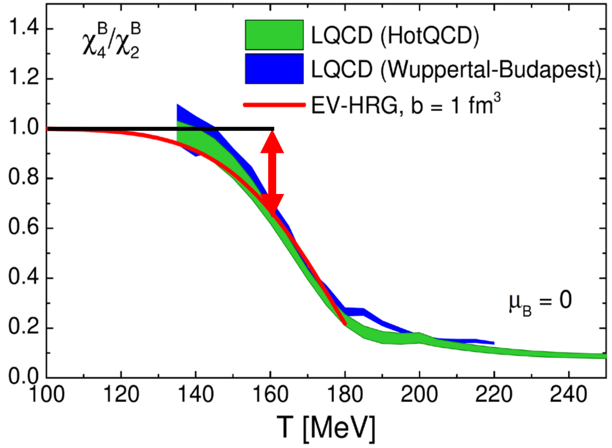
- Correction for baryon conservation via SAM-2.0

[VV, PRC 105, 014903 (2022)]

VV, C. Shen, V. Koch, arXiv:2107.00163

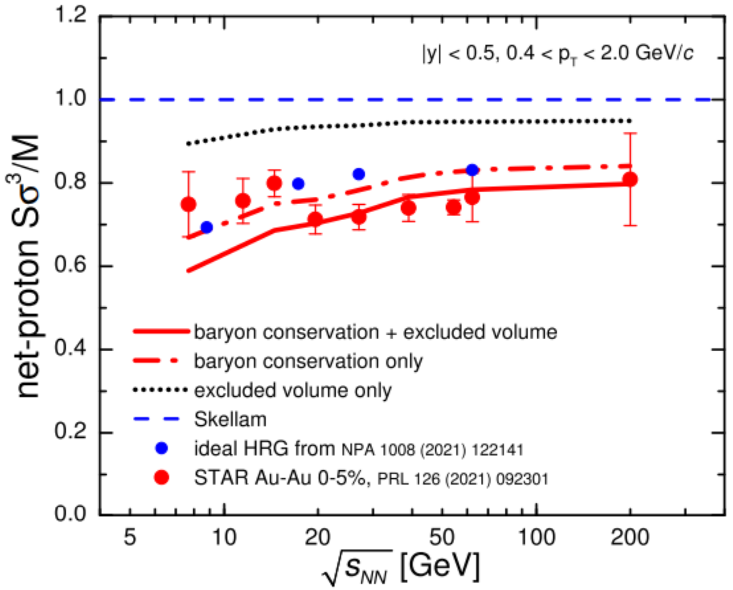


Lattice QCD input

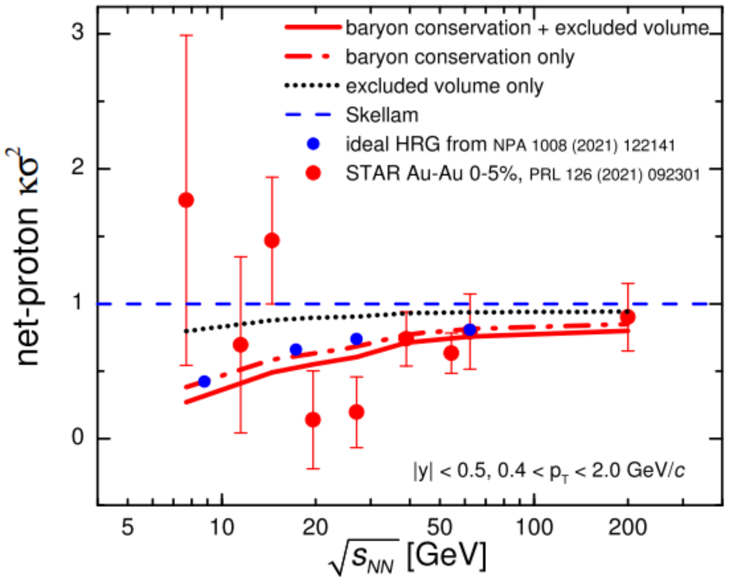


Net proton cumulant ratios

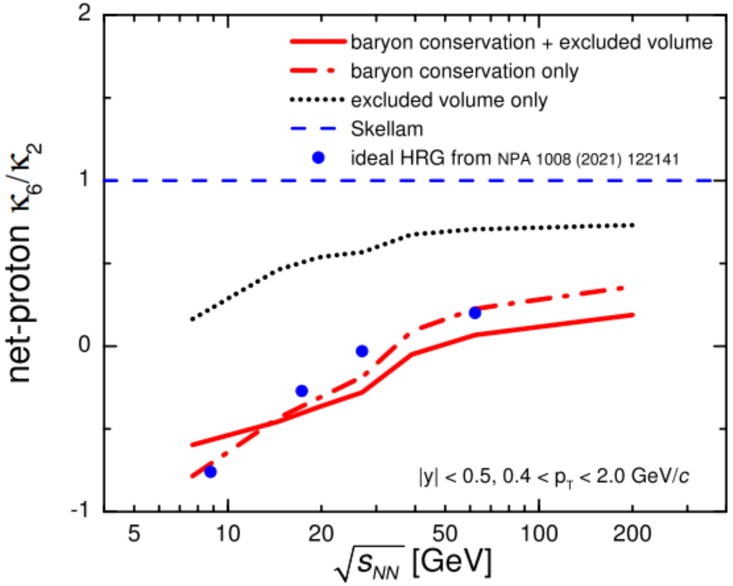
κ_3/κ_1



κ_4/κ_2

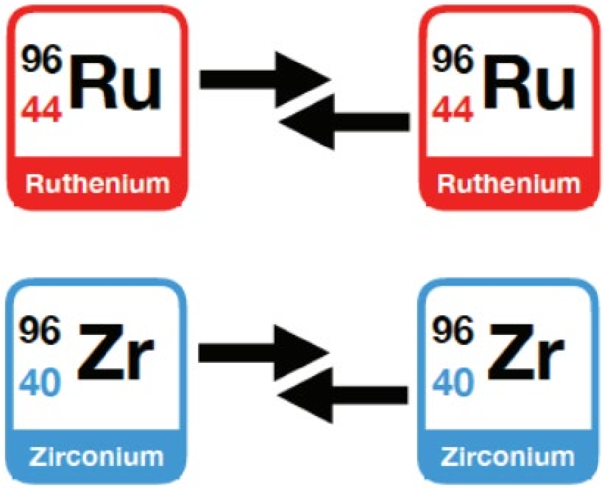


κ_6/κ_2



- Both the baryon conservation and repulsion needed to describe data at $\sqrt{s_{NN}} \geq 20$ GeV quantitatively
- Effect from baryon conservation is larger than from repulsion
- Canonical ideal HRG limit is consistent with the data-driven study of [Braun-Munzinger et al., NPA 1008 (2021) 122141]
- κ_6/κ_2 turns negative at $\sqrt{s_{NN}}$~50 GeV

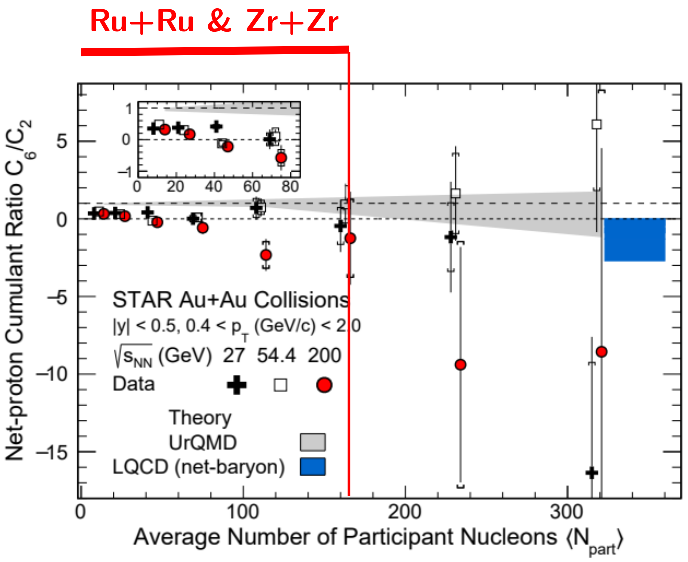
Opportunities from isobar collisions



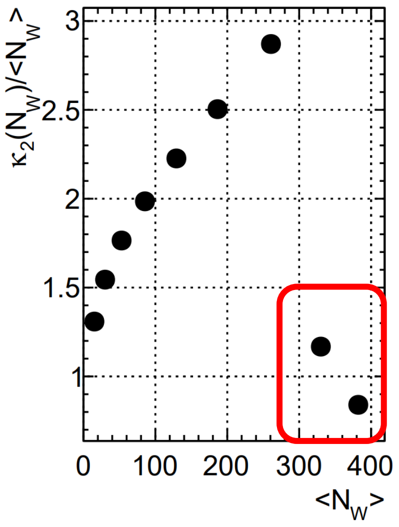
1. Tackle volume fluctuations for system-size dependence

2. A possible new way to measure *grand-canonical* charge cumulant ratios

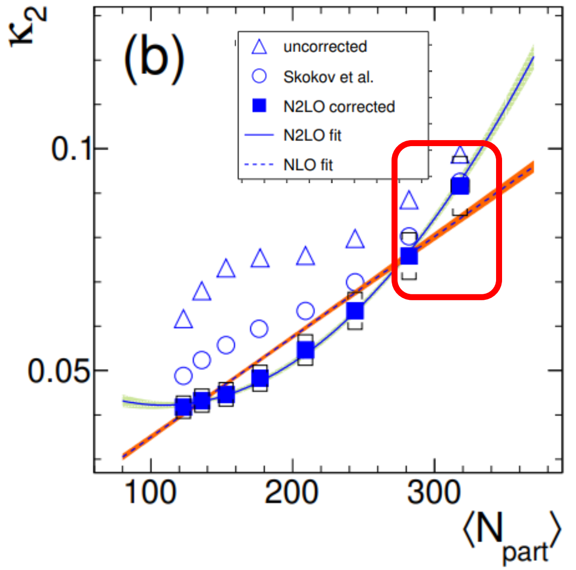
Controlling volume fluctuations in smaller systems



STAR Coll., PRL 127, 262301 (2021)



Braun-Munzinger et al., NPA 960, 114 (2017)



HADES Coll., PRC 102, 024914 (2020)

Volume fluctuations, like N_{part} fluctuations, affect cumulants and should be corrected for

$$\tilde{\kappa}_2 = \kappa_2 + \kappa_1^2\,\tilde{v}_2, \qquad \tilde{\kappa}_3 = \kappa_3 + 3\tilde{v}_2\kappa_1\kappa_2 + \tilde{v}_3\kappa_1^3, \qquad \dots$$

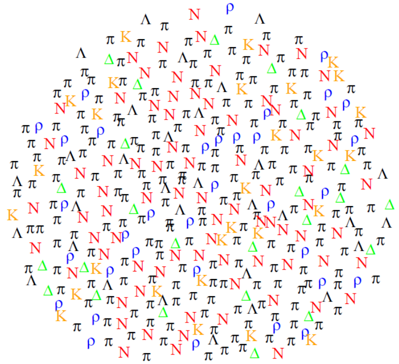
Skokov et al., PRC 88, 034911 (2013)

Volume fluctuations are smaller in central collisions of smaller nuclei (isobars) relative non-central collisions of heavier nuclei (Au-Au) at same N_{part}

Use isobar collisions as a complement to Au-Au to control volume fluctuations in smaller systems

Ru+Ru vs Zr+Zr: difference in the freeze-out conditions

Analyze freeze-out using thermal model, charge and strangeness conservation define μ_Q & μ_S



$$n_Q/n_B = Q/B, \qquad n_S = 0$$

$$(Q/B)_{RuRu} \simeq 0.458 \qquad vs \qquad (Q/B)_{ZrZr} \simeq 0.417$$

Use T = 160 MeV, μ_B = 25 MeV and Thermal-FIST* package to estimate the difference in μ_Q:

$$\boxed{\Delta\mu_Q = \mu_Q^{Ru} - \mu_Q^{Zr} \approx 0.3 \text{ MeV}}$$

The effect on hadrochemistry is very mild:

$$(\pi^+/\pi^-)_{RuRu} \;/\; (\pi^+/\pi^-)_{ZrZr} \simeq 1.002, \qquad (K^+/K^-)_{RuRu} \;/\; (K^+/K^-)_{ZrZr} \simeq 1.001$$

How about fluctuations?

*V.V., H. Stoecker, Comput. Phys. Commun. 244, 295 (2019)

Ru+Ru & Zr+Zr: access to grand-canonical cumulants?

Grand-canonical charge cumulants: $\kappa_n^Q(\mu_Q) = VT^3\,\chi_n^Q(\mu_Q)$ where χ_n^Q are grand-canonical susceptibilities

Taylor expand Ru+Ru around Zr+Zr:

$$\chi_n^Q(\mu_Q^{Ru}) \approx \chi_n^Q(\mu_Q^{Zr}) + \frac{\Delta\mu_Q}{T}\chi_{n+1}^Q(\mu_Q^{Zr})$$

Ratios of RuRu and ZrZr charge cumulants measured in the same acceptance:

$$\frac{(\kappa_1^Q)_{Ru}}{(\kappa_1^Q)_{Zr}} \approx \frac{V_{Ru}}{V_{Zr}}\left(1+\frac{\Delta\mu_Q}{T}\frac{\chi_2^Q}{\chi_1^Q}\right), \qquad \frac{(\kappa_2^Q)_{Ru}}{(\kappa_2^Q)_{Zr}} \approx \frac{V_{Ru}}{V_{Zr}}\left(1+\frac{\Delta\mu_Q}{T}\frac{\chi_3^Q}{\chi_2^Q}\right), \qquad \frac{(\kappa_3^Q)_{Ru}}{(\kappa_3^Q)_{Zr}} \approx \frac{V_{Ru}}{V_{Zr}}\left(1+\frac{\Delta\mu_Q}{T}\frac{\chi_4^Q}{\chi_3^Q}\right),$$

Advantages:

- Access to higher-order cumulants through measurements of lower order ones
- Corrections due to global charge conservation [factors $(1-\alpha)$ for κ_2^Q and $(1-2\alpha)$ for κ_3^Q] cancel out

Volume ratio $V_{Ru}/V_{Zr} \approx 1$ to leading order or use e.g. $\langle N_{Ru}^{ch}\rangle/\langle N_{Zr}^{ch}\rangle$ as an estimate

HRG model estimates: $(\kappa_1^Q)_{Ru}/(\kappa_1^Q)_{Zr},\ (\kappa_3^Q)_{Ru}/(\kappa_3^Q)_{Zr} \approx 1.1, \qquad (\kappa_2^Q)_{Ru}/(\kappa_2^Q)_{Zr} \approx 1$

Summary

- Fluctuations are a powerful tool to explore the QCD phase diagram
  - test of lattice QCD and equilibration, probe the QCD critical point
- Use the collisions of smaller isobar collisions to improve the control over volume fluctuations in small systems
- Exploit the difference between Ru+Ru and Zr+Zr for a new way to measure charge cumulant ratios
  - Measure lower-order cumulants to extract higher-order ones
  - Cancellation of global charge conservation corrections

Thanks for your attention!