Baryonic excluded volume and its role in QCD equation of state at imaginary chemical potential

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On the occasion of Mark Gorenstein's 70th birthday

Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine

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My collaboration with Mark and excluded-volume model

- My collaboration with Mark started in 2014
- According to INSPIRE, we have 14 common journal articles
- 1 PRL, 8 PRC, 2 JPA, 2 JPG, 1 NPA
- 8 of them are related to the excluded volume (EV) model

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Назва 1–20		Посилання	Рік
On the Early Stage of Nucleus--Nucleus Collisions M Gazdzicki, MI Gorenstein arXiv preprint hep-ph/9803462		530	1998
Excluded volume effect for the nuclear matter equation of state DH Rischke, MI Gorenstein, H Stoecker, W Greiner Zeitschrift für Physik C Particles and Fields 51 (3), 485-489	✓	252	1991
Statistical coalescence model with exact charm conservation MI Gorenstein, AP Kostyuk, H Stöcker, W Greiner Physics Letters B 509 (3), 277-282		186	2001
Analysis of particle multiplicities in Pb+ Pb collisions at 1 5 8 A GeV/c within hadron gas models GD Yen, MI Gorenstein Physical Review C 59 (5), 2788	✓	181	1999
Excluded volume hadron gas model for particle number ratios in A+ A collisions GD Yen, MI Gorenstein, W Greiner, SN Yang Physical Review C 56 (4), 2210	✓	179	1997

3 of top 5 most cited papers of Mark are related to excluded volume model

What is so special about the excluded volume?

Excluded volume procedure

- Particles (hadrons) exhibit repulsion at short range $\Rightarrow$ finite size of particles
- EV model: substitute volume by the available volume $V \to V - bN$

$$p = \frac{T\,N}{V} \quad \Rightarrow \quad p = \frac{T\,N}{V - bN}, \qquad \text{(van der Waals, 1873)}$$

Necessity of the EV effects for hadronic EoS was realized long time ago:
too high hadron densities at freeze-out, mechanism to suppress hadrons at high T

Hagedorn, Rafelski, Phys. Lett. B (1980)
Cleymans, Redlich, Satz, Suhonen, Z. Phys. C (1986)
Kouno, Takagi, Z. Phys. C (1989)
PBM, Stachel, Wessels, Nu Xu, Phys. Lett. B (1994)

These early studies suffered from thermodynamic inconsistency in the GCE:

$$n^{\rm ev}(T,\mu) \neq \frac{\partial p^{\rm ev}(T,\mu)}{\partial \mu}$$

A thermodynamically consistent procedure to treat EV effects was in order

Thermodynamically consistent formulation

CE: $$p(T,n) = \frac{Tn}{1-bn}$$

GCE: $$p(T,\mu) = p^{\rm id}[T,\mu - b\,p(T,\mu)], \quad n(T,\mu) = \frac{n^{\rm id}(T,\mu^*)}{1 + b\,n^{\rm id}(T,\mu^*)},$$

$$\mu^* = \mu - b\,p$$

CE: Excluded volume effects increase CE pressure $p(T,n)$

GCE: Excluded volume effects decrease GCE pressure $p(T,\mu)$

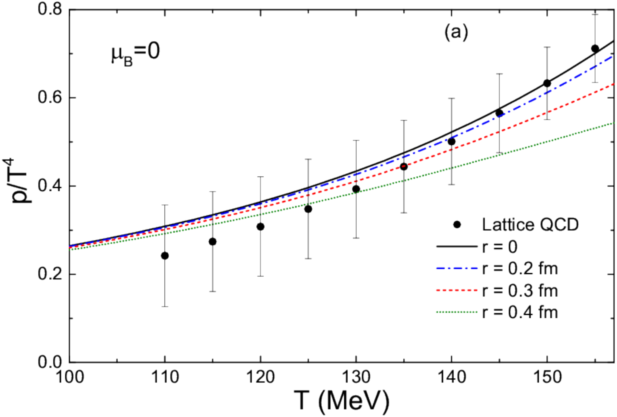
There is no paradox: In GCE particle density is not an independent variable, and it decreases itself due to EV effects, this leads to suppression of GCE pressure

M.I. Gorenstein, V.K. Petrov, G.M. Zinovjev, Phys. Lett. B **106**, 327 (1981)
D.H. Rischke, **M.I. Gorenstein**, H. Stoecker and W. Greiner, Z. Phys. C **51**, 485 (1991).
D. Anchishkin, Lett. JETP **75**, 195 (1992); Identical formulation with mean-field approach

Excluded volume and QCD thermodynamics

In spite of common use, not established that hadronic EV is needed at all

First-principles check: Lattice QCD EoS vs EV hadron resonance gas



- Lattice data do not exclude presence of a moderate hadronic EV
- But neither do it suggest it. . .

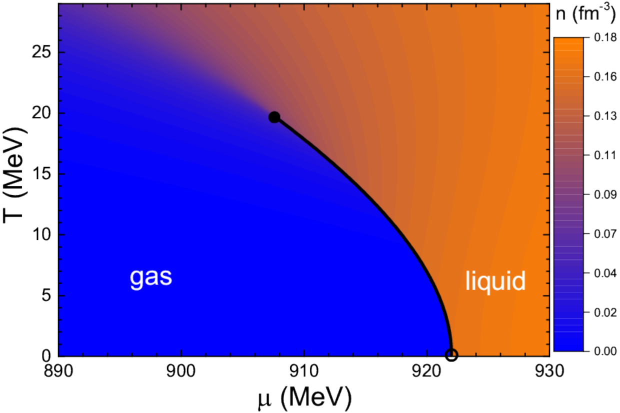
V.V., D. Anchishkin, **M.I. Gorenstein**, Phys. Rev. C 91, 024905 (2015)
My most cited paper to date

Excluded volume in cold nuclear matter

Many cold nuclear matter properties are known empirically

Nuclear matter with fermionic van der Waals equation for nucleons

$E/A = -16$ MeV, $n_0 = 0.16$ fm^{-3} $\Rightarrow$ $a_{NN} = 329$ MeV fm^3, $b_{NN} = 3.42$ fm^3



- Large EV effects in nuclear matter! Is it about baryonic EV, not mesonic?

V.V., D. Anchishkin, **M.I. Gorenstein**, Phys. Rev. C 91, 064314 (2015)

QCD observables at imaginary μ_B

QCD thermodynamics with relativistic cluster/virial expansion:

$$\text{Pressure:} \quad \frac{p(T,\mu_B)}{T^4} = \sum_{k=0}^{\infty} a_k(T) \cosh\left(\frac{k\,\mu_B}{T}\right),$$

$$\text{Net baryon density:} \quad \frac{\rho_B(T,\mu_B)}{T^3} = \sum_{k=1}^{\infty} b_k(T) \sinh\left(\frac{k\,\mu_B}{T}\right), \quad b_k(T) \equiv k\, a_k(T)$$

Lattice QCD is problematic at real μ but tractable at imaginary μ

$\mu_B \to \mathrm{i}\tilde{\mu}_B \quad \Rightarrow \quad$ QCD observables obtain trigonometric Fourier series form

$$\text{Pressure:} \quad \frac{p(T,\mathrm{i}\tilde{\mu}_B)}{T^4} = \sum_{k=0}^{\infty} a_k(T) \cos\left(\frac{k\,\tilde{\mu}_B}{T}\right),$$

$$\text{Net baryon density:} \quad \frac{\rho_B(T,\mathrm{i}\tilde{\mu}_B)}{T^3} = \mathrm{i}\sum_{k=1}^{\infty} b_k(T) \sin\left(\frac{k\,\tilde{\mu}_B}{T}\right), \quad b_k(T) \equiv k\, a_k(T)$$

Expected asymptotics

- At low T/densities QCD $\simeq$ ideal hadron resonance gas

$$\frac{p^{\rm hrg}(T,\mu_B)}{T^4} = \frac{\phi_M(T)}{T^3} + 2\,\frac{\phi_B(T)}{T^3}\cosh\left(\frac{\mu_B}{T}\right),$$

$$\phi_B(T) = \sum_{i\in B}\int dm\,\rho_i(m)\,\frac{d_i\,m^2\,T}{2\pi^2}\,K_2\left(\frac{m}{T}\right),$$

$$a_0^{hrg}(T) = \frac{\phi_M(T)}{T^3}, \quad a_1^{hrg}(T) = \frac{2\,\phi_B(T)}{T^3}, \quad a_k^{\rm hrg}(T) \equiv 0,\ k = 2,3,\ldots$$

- At high T QCD $\simeq$ ideal gas of massless quarks and gluons

$$\frac{p^{\rm SB}(T,\mu_B)}{T^4} = \frac{8\pi^2}{45} + \sum_{f=u,d,s}\left[\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_f}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_f}{T}\right)^4\right], \quad \mu_f = \frac{\mu_B}{3}\,*,$$

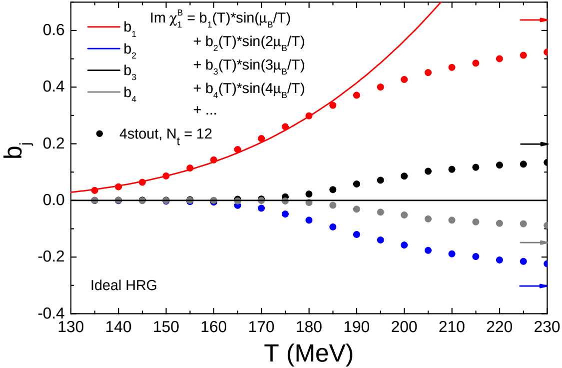
$$a_0^{\rm SB} = \frac{64\pi^2}{135}, \quad a_k^{\rm SB} = \frac{(-1)^{k+1}}{k^2}\,\frac{4\,[3+4\,(\pi k)^2]}{27\,(\pi k)^2}.$$

*In this study we assume that $\mu_S = \mu_Q = 0$

Lattice QCD results on imaginary μ_B observables

Coefficients $b_k(T)$ of net-baryon expansion are now calculated on the lattice

$$\frac{n_B(T, \mathrm{i}\tilde{\mu}_B)}{T^3} = \mathrm{i} \sum_{j=1}^{\infty} b_j(T) \sin\left(j\, \tilde{\mu}_B / T\right)$$



- Ideal HRG describes well $b_1(T)$ at small temperatures
- All four coefficients appear to converge slowly to Stefan-Boltzmann limit
- What is the mechanism of appearance of non-zero b_k for $k > 1$?

V.V., A. Pásztor, Z. Fodor, S.D. Katz, H. Stoecker, 1708.02852; S. Borsányi, QM2017

Baryonic excluded volume

Baryon-baryon interactions seem to exhibit a repulsive core

EV-HRG model

- Identical EV interactions for all baryon-baryon and antibaryon-antibaryon pairs
- Baryon-antibaryon, meson-meson, meson-baryon EV terms neglected
- A single parameter b characterizing interactions

Three independent subsystems: mesons + baryons + antibaryons

$$p(T,\boldsymbol{\mu}) = P_M(T,\boldsymbol{\mu}) + P_B(T,\boldsymbol{\mu}) + P_{\bar{B}}(T,\boldsymbol{\mu}),$$

$$P_M(T,\boldsymbol{\mu}) = \sum_{j\in M} p_j^{\mathrm{id}}(T,\mu_j) \quad \text{and} \quad P_B(T,\boldsymbol{\mu}) = \sum_{j\in B} p_j^{\mathrm{id}}(T,\mu_j - b\,P_B)$$

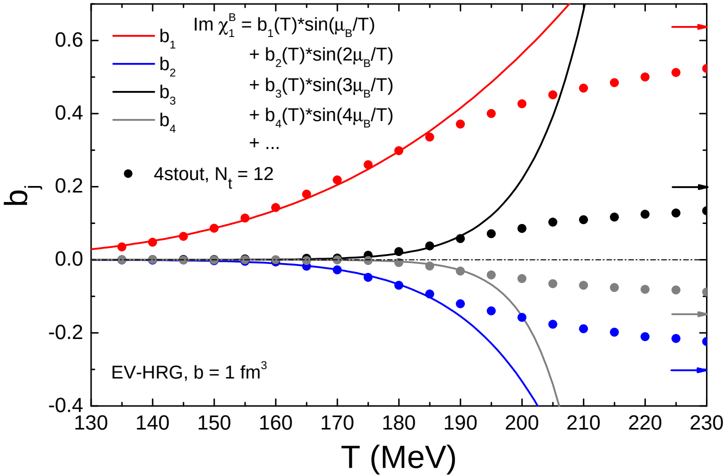
Total density of baryons: $n_B^{\mathrm{ev}} = (1 - b\,n_B^{\mathrm{ev}})\,\lambda_B\,\phi_B(T)\,\exp\left(-\frac{b\,n_B^{\mathrm{ev}}}{1-b\,n_B^{\mathrm{ev}}}\right).$

$$b_1^{\mathrm{ev}}(T) = 2\,\frac{\phi_B(T)}{T^3}, \qquad b_2^{\mathrm{ev}}(T) = -4\,[b\phi_B(T)]\,\frac{\phi_B(T)}{T^3},$$

$$b_3^{\mathrm{ev}}(T) = 9\,[b\,\phi_B(T)]^2\,\frac{\phi_B(T)}{T^3}, \qquad b_4^{\mathrm{ev}}(T) = -\frac{64}{3}\,[b\,\phi_B(T)]^3\,\frac{\phi_B(T)}{T^3}.$$

V.V., A. Pásztor, Z. Fodor, S.D. Katz, H. Stoecker, 1708.02852

Repulsive baryonic interactions and imaginary μ_B



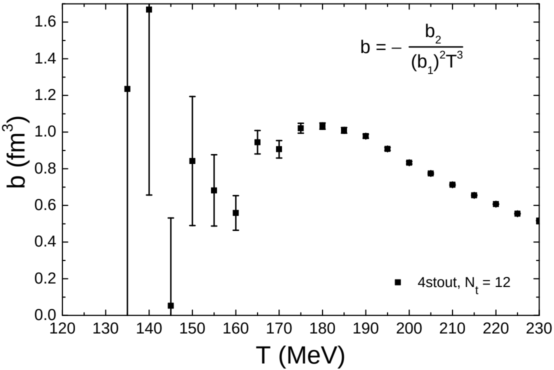
- Non-zero $b_j(T)$ for $j \geq 2$ signal deviations from ideal HRG
- Addition of EV interactions between baryons reproduces lattice trend
- Lattice data for all four coefficients described well up to $T \simeq 185$ MeV

V.V., A. Pásztor, Z. Fodor, S.D. Katz, H. Stoecker, 1708.02852

“Excluded volume” parameter from imaginary μ_B data

“Excluded volume” parameter of BB interactions can be estimated from lattice

$$b(T) = -\frac{b_2(T)}{[b_1(T)]^2 \, T^3}$$



- $b(T)$ mostly consistent with 1 fm^3 at $T < 190$ MeV
- $b \sim 1/T^3$ at high T: limiting SB-type behavior

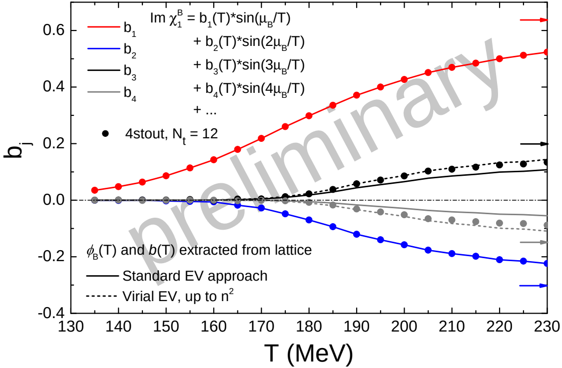
V.V., A. Pásztor, Z. Fodor, S.D. Katz, H. Stoecker, 1708.02852

Constraining model parameters from imaginary μ_B data

Constrain parameters of model with repulsive baryon interactions from lattice

$$P_B^{\rm EV} = \frac{T\,n_B}{1 - b(T)\,n_B} = T\,n_B + T\,b(T)\,n_B^2 + \ldots$$

- Fix "baryon spectrum" and BB EV parameter from b_1 and b_2
- Predict b_3 and b_4



- EV approach appears to have predictive power, even at high T
- Variations on EV mechanism should be considered
- Lattice data for b_3 and b_4 lies between two EV-type models predictions

Summary

- LQCD data at imaginary μ_B suggests presence of repulsive baryonic interactions with 2nd virial coefficient $b \sim 1$ fm^3 in the crossover region
- It provides a first-principle evidence for the baryonic "excluded-volume"
- LQCD observables at imaginary μ are promising in studying hadronic interactions
- Exciting times ahead!

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- Exciting times ahead!

Happy Birthday, Mark!