

QCD phase diagram at finite baryon density from heavy-ion collisions and new expansions

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XQCD 2026, PKNU, Busan, Korea

July 14, 2026

Thanks to: F. Di Clemente, Y. Fujimoto, T. Gyure, V. Kuznietsov, G. Pihan, R. Poberezhniuk, H. Shah



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Science

What we know

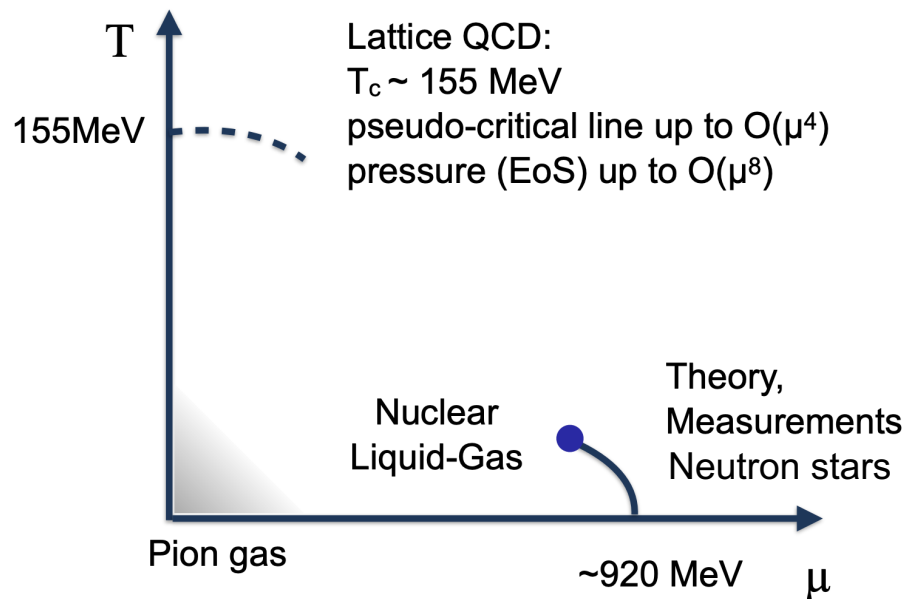


Figure courtesy of V. Koch

What we hope to know

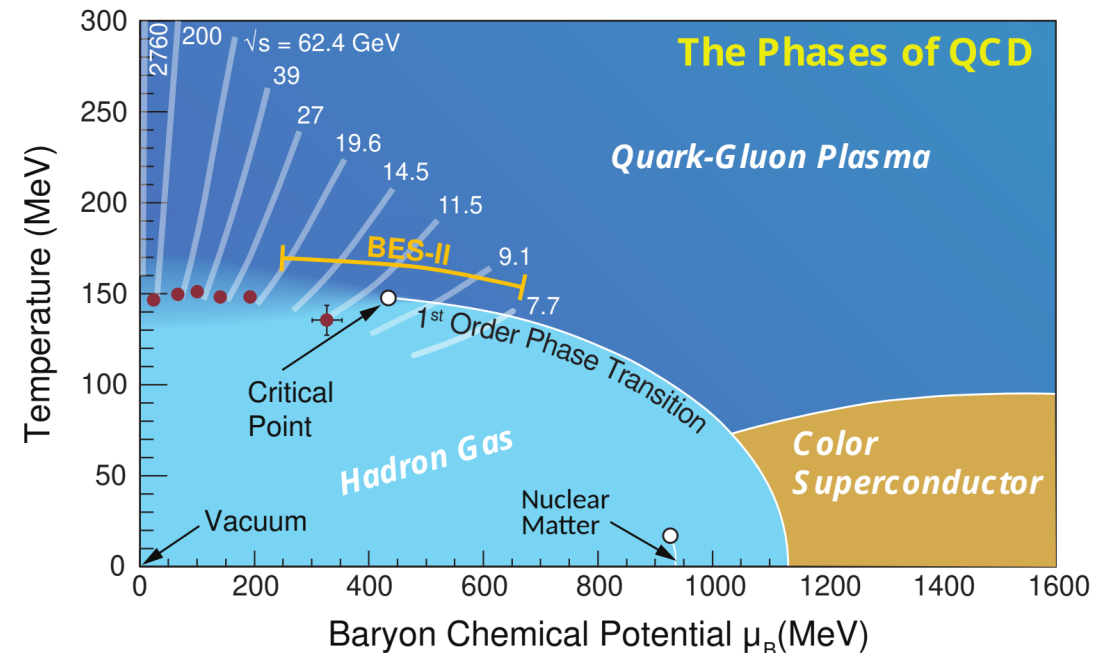


Figure from Bzdak et al., Phys. Rept. '20 & 2015 US Nuclear Long Range Plan

- Dilute hadron gas at low T & μ_B due to confinement, quark-gluon plasma high T & μ_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured
- Chiral crossover at $\mu_B = 0$ which may turn into a *first-order phase transition* at finite μ_B

What we know

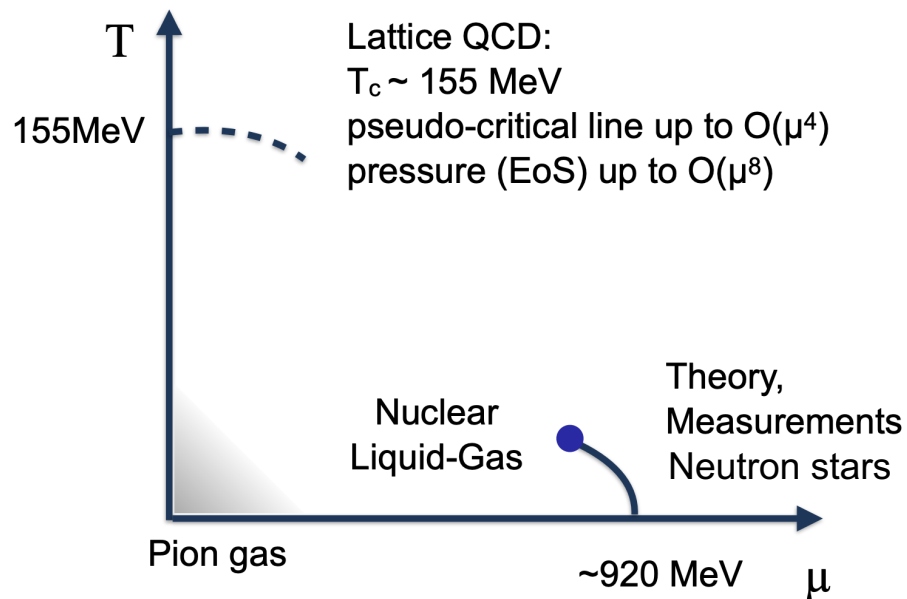
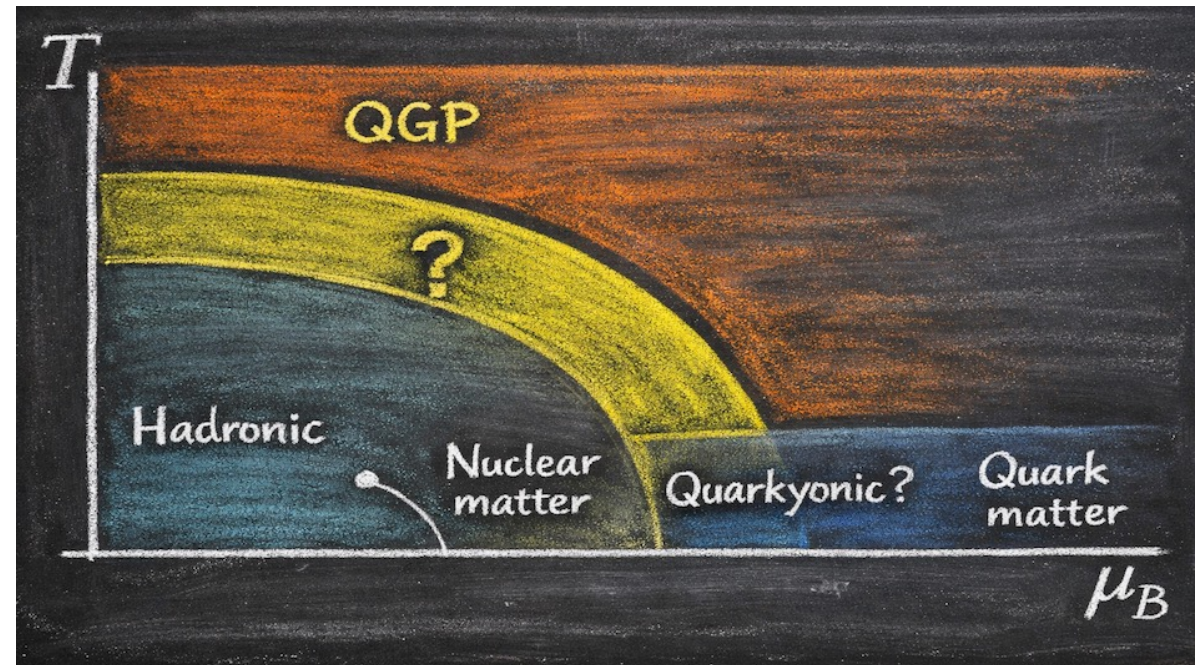


Figure courtesy of V. Koch

What we hope to know



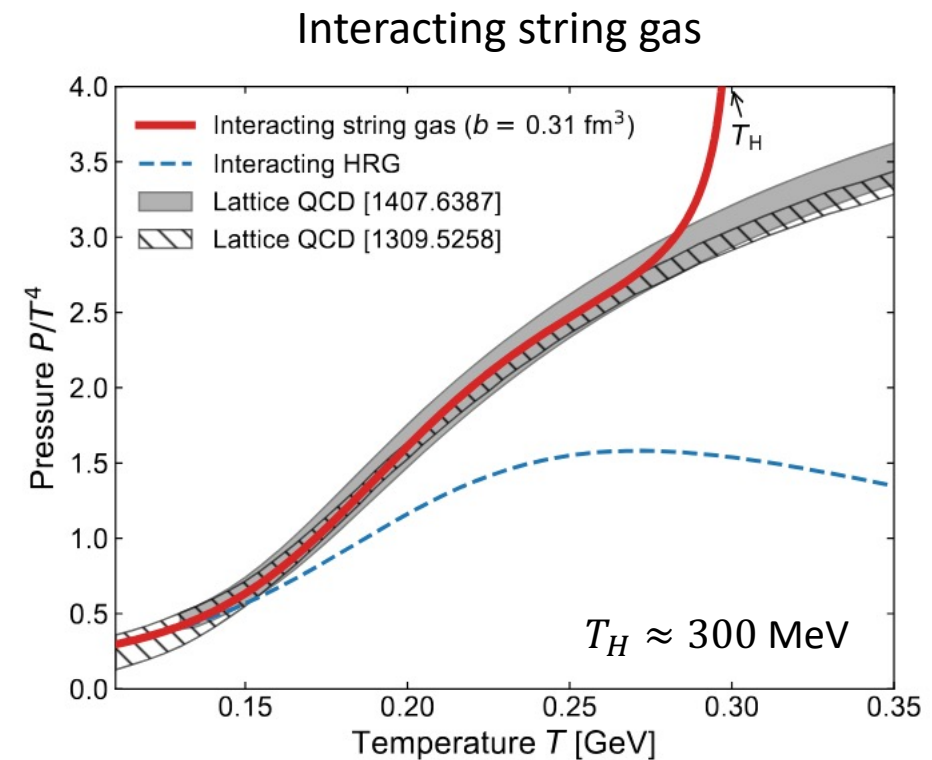
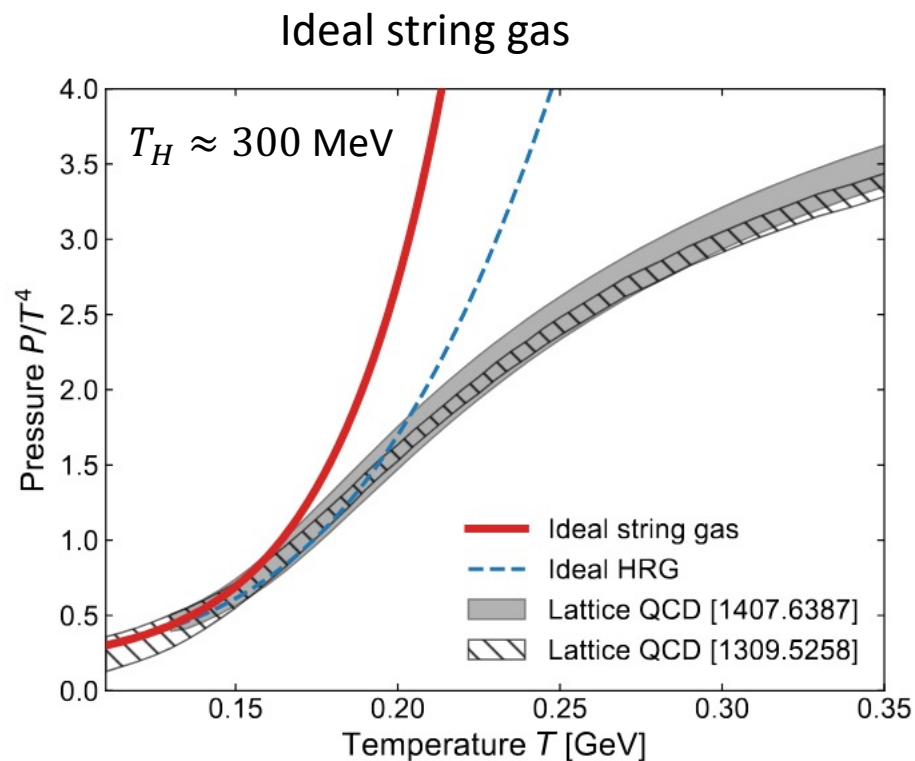
Conference poster "Buenas Ideas on the QCD Phase Diagram"

- Dilute hadron gas at low T & μ_B due to confinement, quark-gluon plasma high T & μ_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured
- Chiral crossover at $\mu_B = 0$ which may turn into a *first-order phase transition* at finite μ_B
- Or perhaps a different state of matter between T_c and $(2-3)T_c$?

HRG as a gas of confining strings with interaction

- Strings with excluded volume, a la EV/VDW-HRG model [VV, Gorenstein, Stoecker, PRL 118, 182301 (2017)]

$$P(T) = T n_{\text{id}}(T) \exp\left(-\frac{bP}{T}\right) \quad T\phi_i(T) = \int_{m_{\text{str},i}}^{\infty} dm \rho_{\text{str}}(m) d_{\text{str},i} P(m) \quad \rho_{\text{str}}(m) = \frac{\sqrt{2\pi}}{6T_H} \left(\frac{m}{T_H}\right)^{-3/2} e^{m/T_H}$$

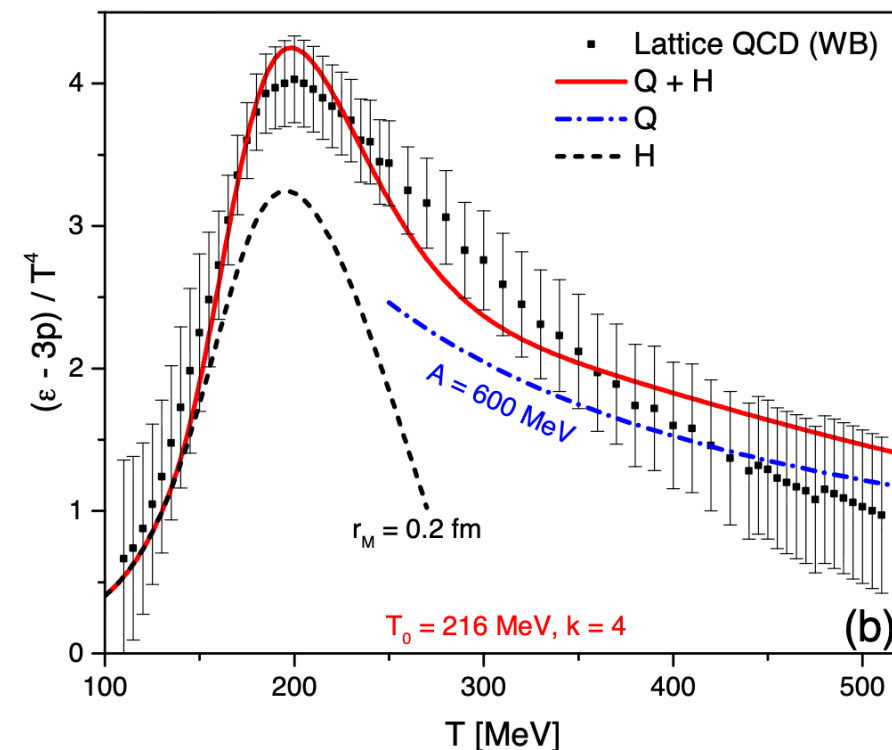
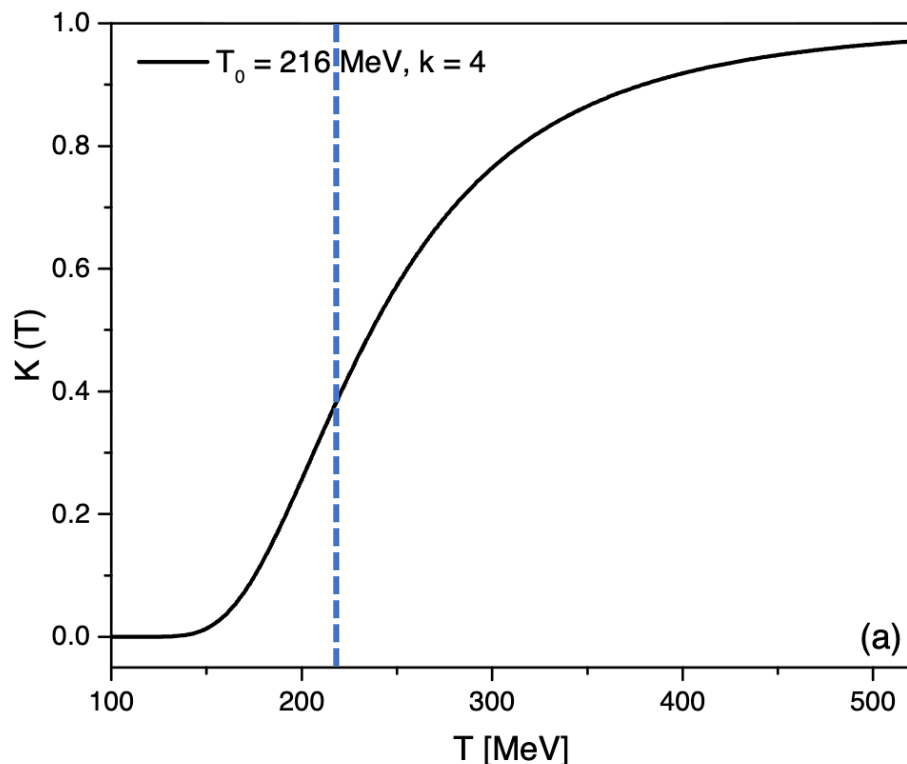


Description of thermodynamics lowers the Hagedorn temperature to $T_H \approx 300$ MeV relative to $T_H \approx 340$ MeV from PDG fits

Hadronic and partonic decomposition across the crossover

$$p(T) = \underbrace{\mathcal{K}(T)}_{\text{QGP}} p_Q(T) + [1 - \mathcal{K}(T)] \underbrace{p_H(T)}_{\text{EV-HRG}}$$

Switching function $\mathcal{K}(T)$ [a la Albright, Kapusta, Young, PRC 90, 024915 (2015)]



Switching temperature of $T_0 \simeq 216$ MeV substantially higher than T_c , hadronic contributions may remain important well above the chiral crossover

QCD phase diagram with constant entropy contours and search for critical point

H. Shah, M. Hippert, J. Noronha, C. Ratti, VV, PRC 113, L012201 (2026) & 2601.08823

H. Shah, **T. Gyure**, A. Leon, F. Di Clemente, M. Hippert, C. Ratti, VV, 2606.28282

Critical point predictions from some years ago

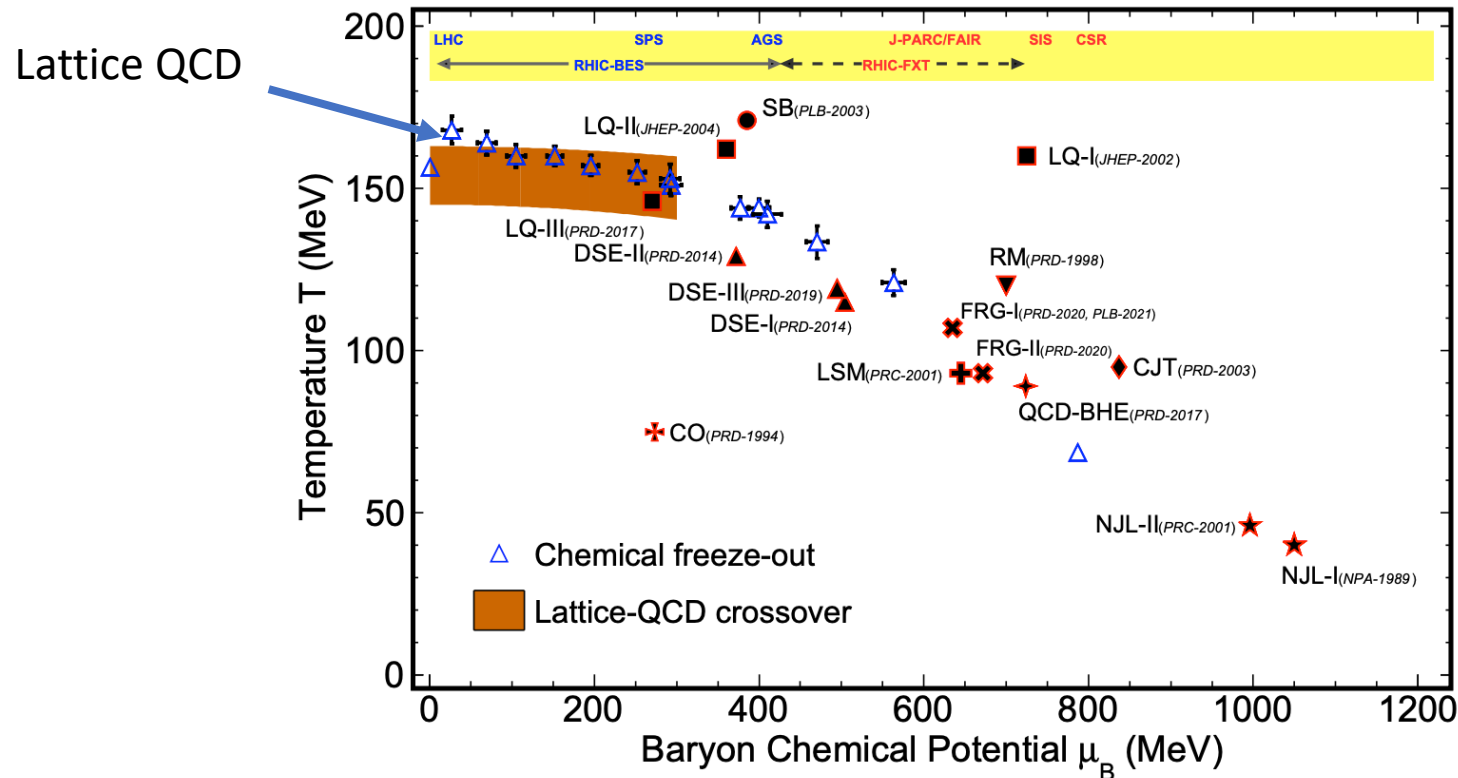


Figure adapted from A. Pandav, D. Mallick, B. Mohanty, Prog. Part. Nucl. Phys. 125 (2022)

- Including the possibility that the QCD critical point does not exist

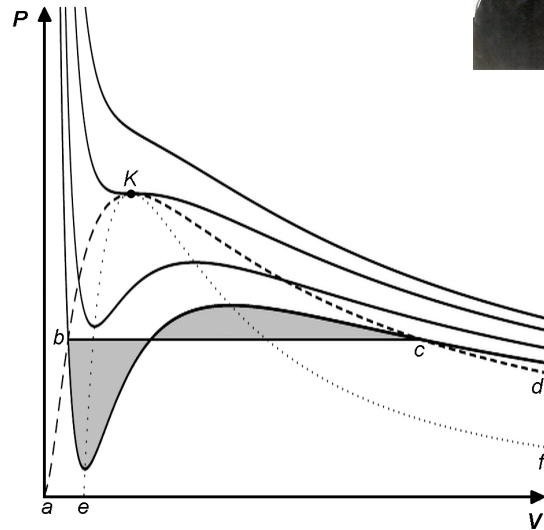
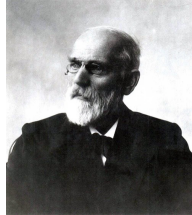
de Forcrand, Philipsen, JHEP 01, 077 (2007); VV, Steinheimer, Philipsen, Stoecker, PRD 97, 114030 (2018)

- Lattice QCD excludes the CP at $\mu_B < 450$ MeV on (one-sided) 2σ level

Borsanyi et al., PRD 112, L111505 (2025)

Extrapolating critical point from lattice QCD

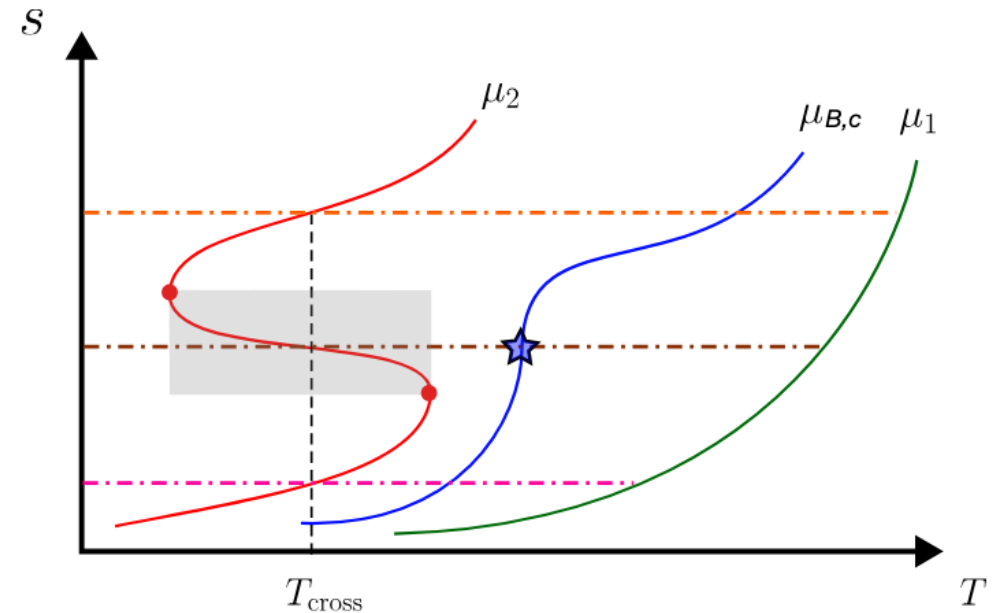
van der Waals (1873)



change of variables



Shah, Hippert, Noronha, Ratti, VV, PRC 113, L012201 (2026)



Critical Point:

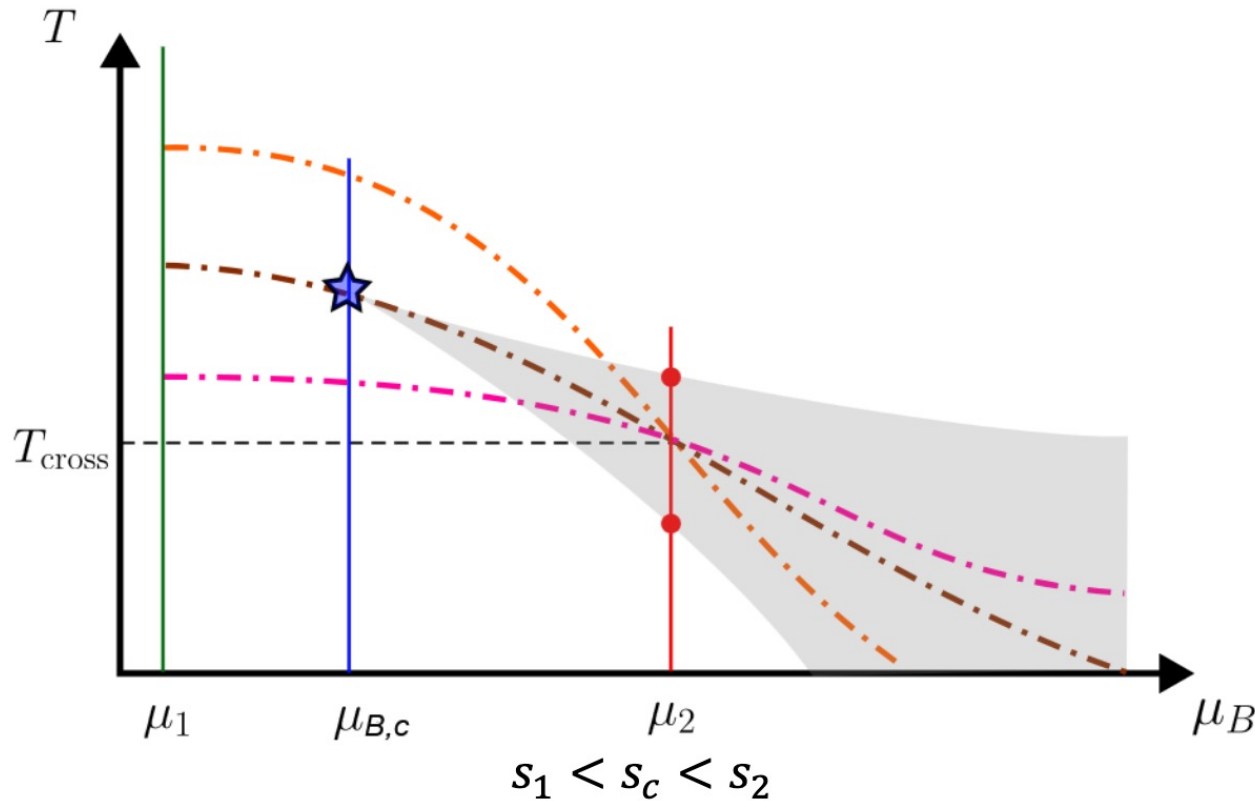
$$\left(\frac{\partial P}{\partial \rho_B}\right)_T = 0, \quad \left(\frac{\partial^2 P}{\partial \rho_B^2}\right)_T = 0.$$

$$\left(\frac{\partial T}{\partial s}\right)_{\mu_B} = 0, \quad \left(\frac{\partial^2 T}{\partial s^2}\right)_{\mu_B} = 0.$$

Strategy: Follow contours of constant entropy density away from $\mu_B = 0$ and look for crossings

Building the contours

Shah, Hippert, Noronha, Ratti, VV, PRC 113, L012201 (2026)



Define T -vs- μ_B **constant entropy line** through Taylor series

$$T_s(\mu_B; T_0) = T_0 + \alpha_2(T_0) \frac{\mu_B^2}{2} \quad \text{order } \mathcal{O}(\mu_B^2)$$

$$\alpha_2(T_0) = -\frac{2T_0\chi_2^B(T_0) + T_0^2\chi_2^{B'}(T_0)}{s'(T_0)}$$

- Allows **multiple T_0 values** to correspond to **same T_s at finite μ_B** (parabolas cross)
- In contrast to the standard Taylor expansion, permits a description of a first-order phase transition (mean-field-like behavior)

$$T_s(\mu_B; T_0) = T_0 + \alpha_2(T_0) \frac{\mu_B^2}{2}$$

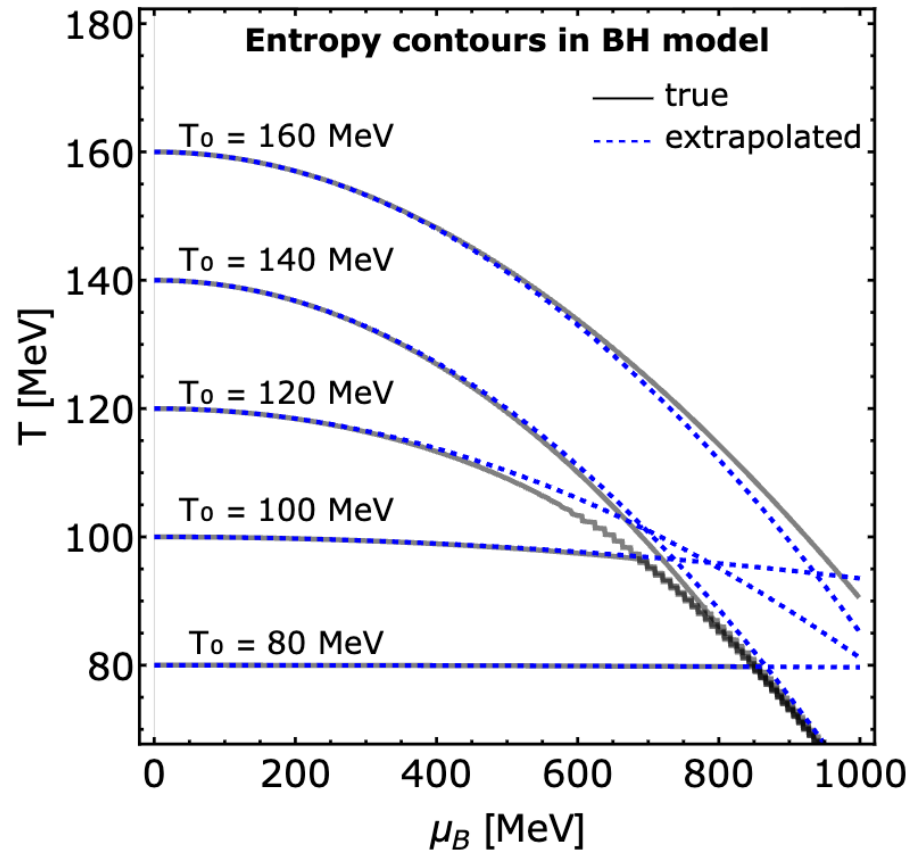
expansion

$$\left(\frac{\partial T}{\partial s}\right)_{\mu_B} = 0 \quad \Rightarrow \quad \mu_{B,c} = \sqrt{-\frac{2}{\alpha'_2(T_{0,c})}} \quad \left(\frac{\partial^2 T}{\partial s^2}\right)_{\mu_B} = 0 \quad \Rightarrow \quad \alpha''_2(T_{0,c}) = 0$$

spinodals **critical point**

Testing with effective theories

Shah, Hippert, Noronha, Ratti, VV, arXiv:2601.08823



true CP

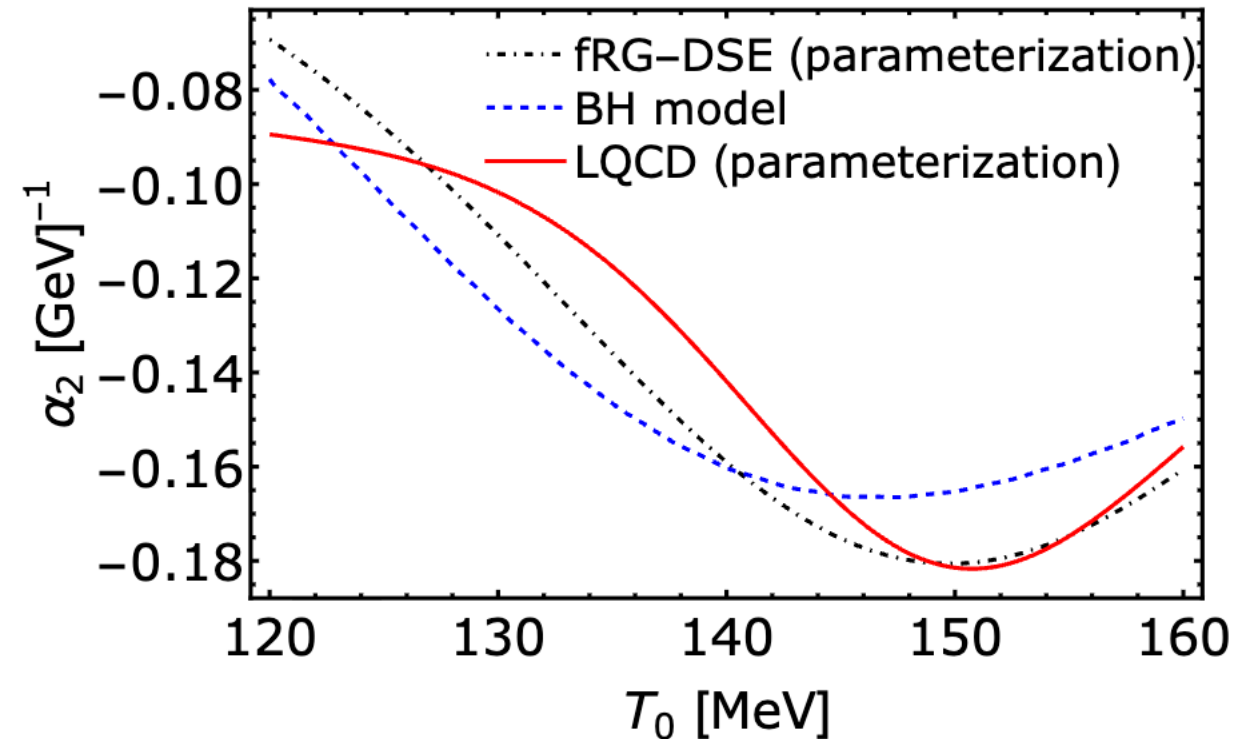
extrapolated CP

✓ **fRG-DSE:** (103,660) (108,629)

✓ **BH model:** (103,599) (104,637)

fRG-DSE: Lu, Gao, Liu, Pawłowski, PRD 113, 054019 (2026)

BH model: Hippert et al. PRD 110, 094006 (2024)



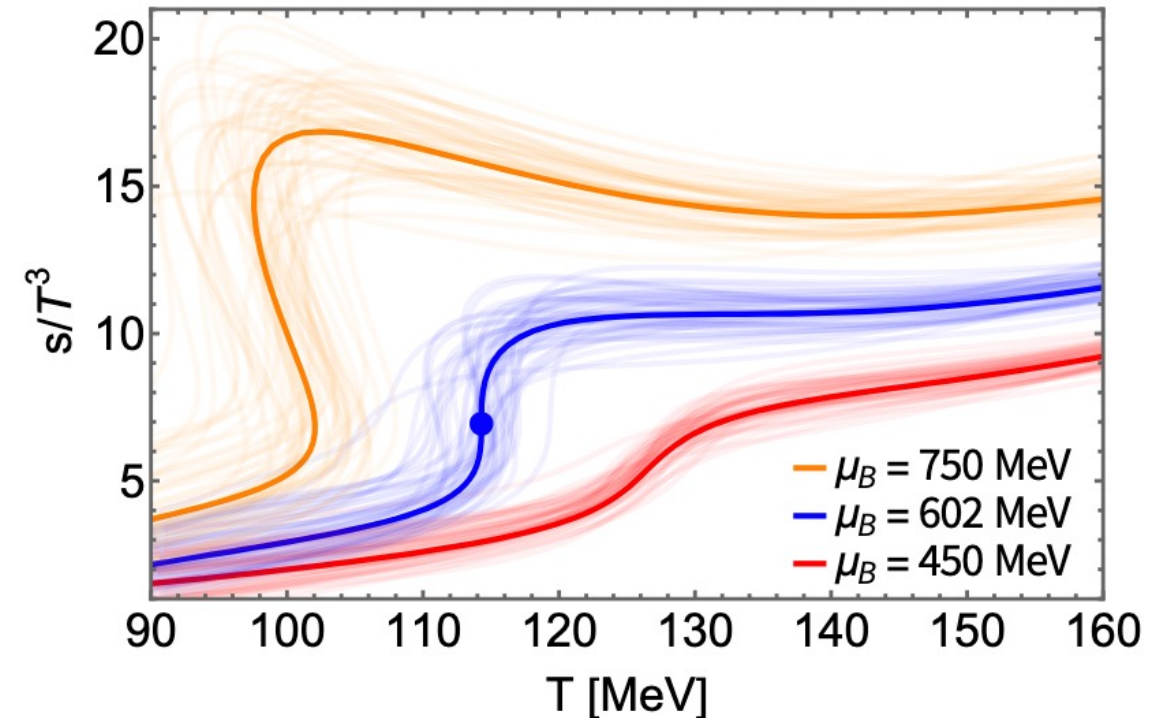
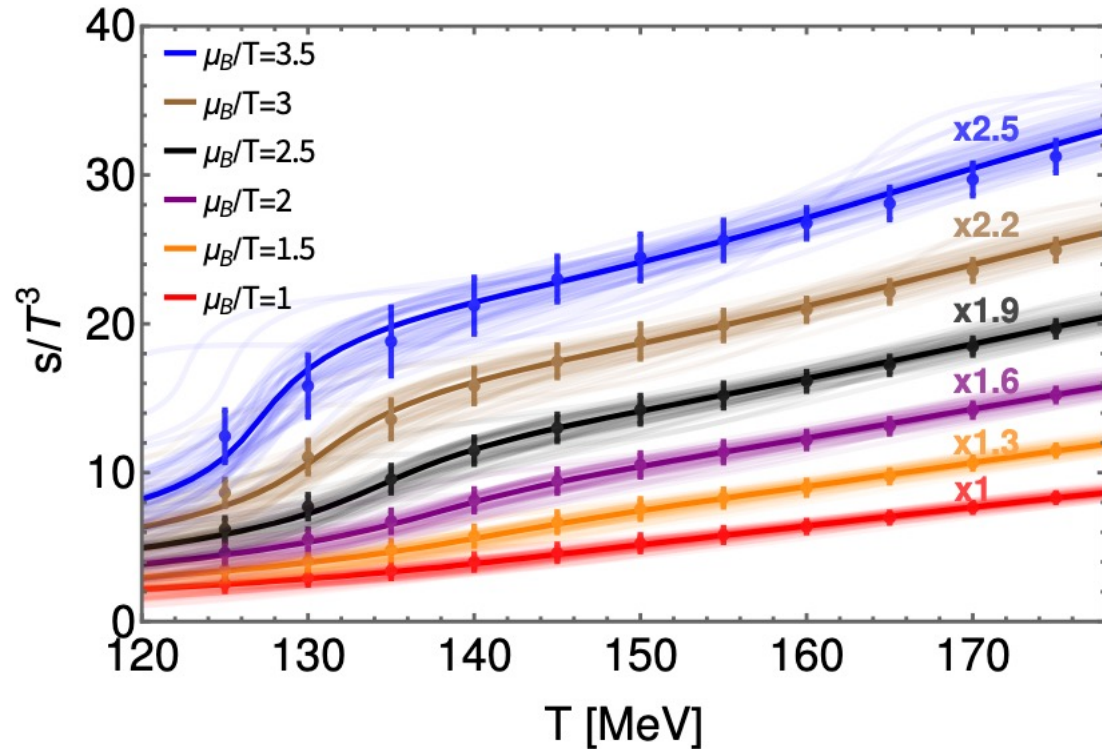
✓ **free QGP:** no true CP and no extrapolated CP

⚠ **HRG:** no true CP but fake extrapolated CP at $\sim(80,830)$ MeV

Marczenko et al., PRD 112, 034019 (2025)

Entropy density at finite μ_B at $O(\mu_B^2)$ from lattice QCD

- Parametrized lattice QCD input from $\mu_B = 0$ [$s(T)$ and $\chi_2^B(T)$]

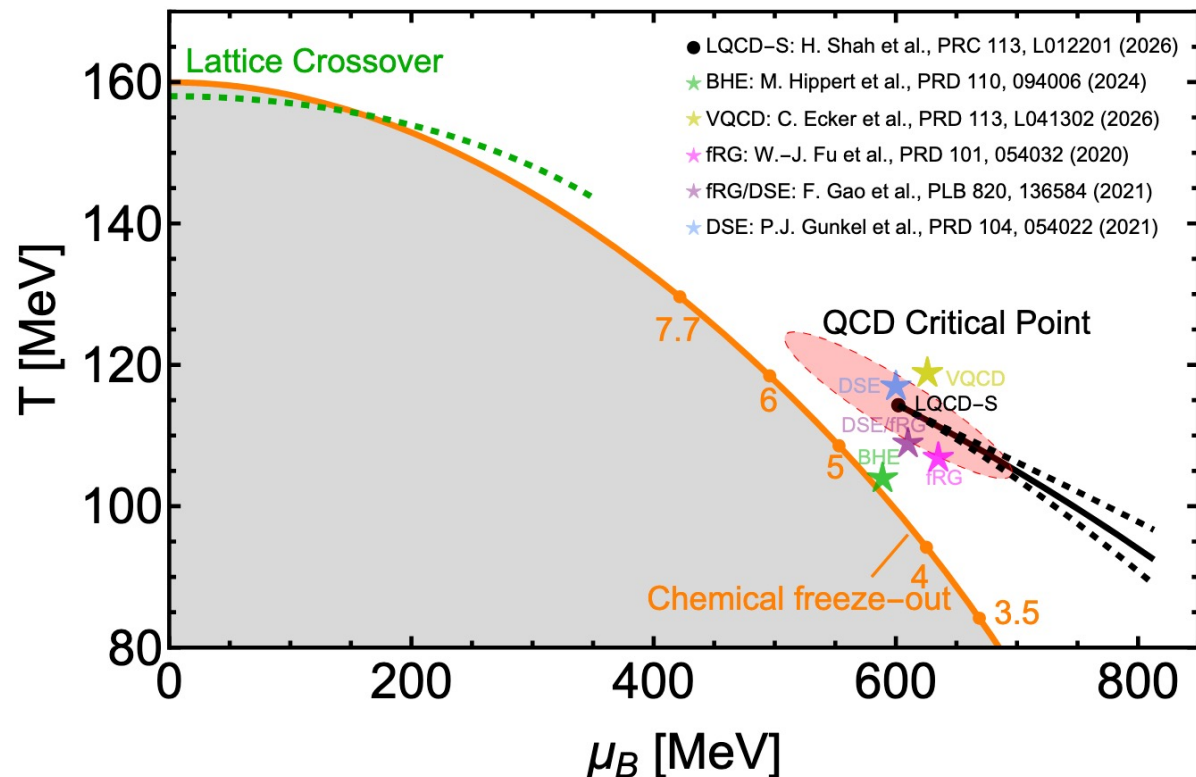


- Excellent agreement at low μ_B/T with available lattice QCD constraints from T' -expansion
- First-order phase transition emerges at $\mu_B > 600$ MeV

Borsanyi et al., PRL 126, 232001 (2021)

Shah, Hippert, Noronha, Ratti, VV, PRC 113, L012201 (2026)

Candidate critical region



Critical point estimate at $O(\mu_B^2)$:

$$T_c = 114 \pm 7 \text{ MeV}, \quad \mu_B = 602 \pm 62 \text{ MeV}$$

Estimates from recent literature:

*YLE-1: D.A. Clarke et al. (Bielefeld-Parma), PRD 112, L091504 (2025)

*YLE-2: G. Basar, PRC 110, 015203 (2024)

BHE: M. Hippert et al., PRD 110, 094006 (2024)

fRG: W-J. Fu et al., PRD 101, 054032 (2020)

DSE/fRG: Gao, Pawłowski., PLB 820, 136584 (2021)

DSE: P.J. Gunkel et al., PRD 104, 054022 (2021)

VQCD: C. Ecker et al., PRD 113, L041302 (2026)

Optimist's view: Different estimates converge onto the same region because QCD CP is likely there

Pessimist's view: Different estimates converge onto the same region because it's the closest not yet ruled out by LQCD

⚠ This is a candidate region subject to validity of the expansion (other estimates listed here have their own caveats). More rigorous constraints determine exclusion regions. More on phase diagram from lattice in **J. Guenther talk on Fri**

*More on Yang-Lee Edge singularities in talk by M. Kitazawa in the next session

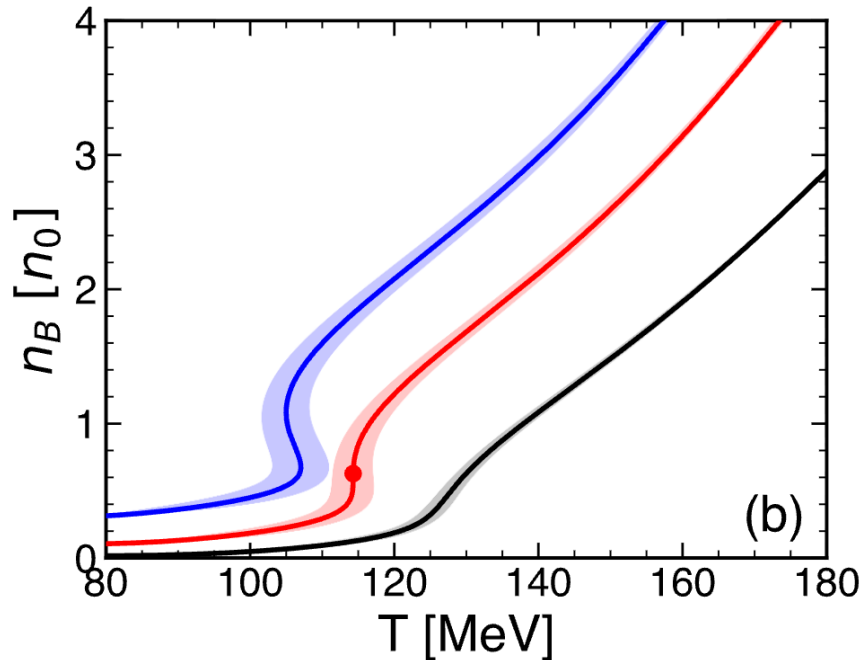
Equation of state

The method yields entropy density $s(T, \mu_B)$ across the (T, μ_B) phase diagram

→ Integrate over T to obtain the pressure $P(T, \mu_B)$ and the rest of the thermodynamics

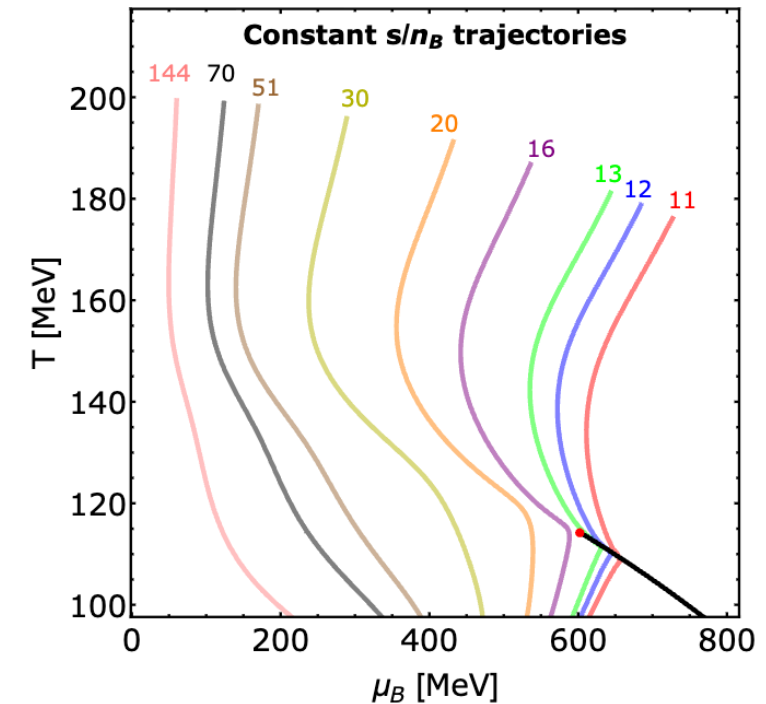
$$\int_{T_{\text{low}}}^T s(T', \mu_B) dT' = P(T, \mu_B) - P(T_{\text{low}}, \mu_B)$$

Integration constant at $T_{\text{low}} = 80$ MeV
matched to van der Waals HRG



Baryon density at the CP is $n_B \simeq 0.59 n_0 \simeq 0.09 \text{ fm}^{-3}$

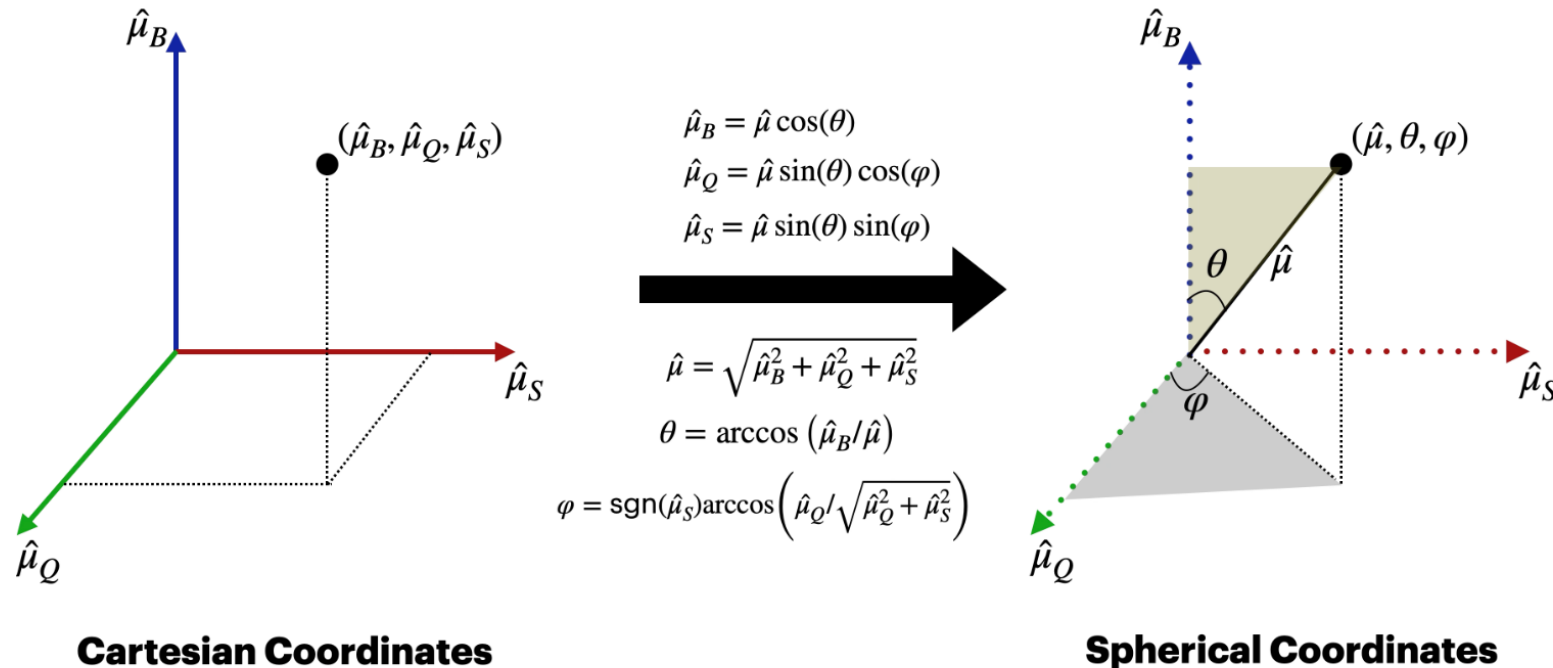
n_B in mixed phase region increases at lower T



So far, we restricted the analysis to $\mu_S = \mu_Q = 0$ conditions

General case: fix the direction in 3D space of (μ_B, μ_Q, μ_S) using spherical coordinates

A. Abuali, S. Borsanyi, et al., PRD 112, 054502 (2025)



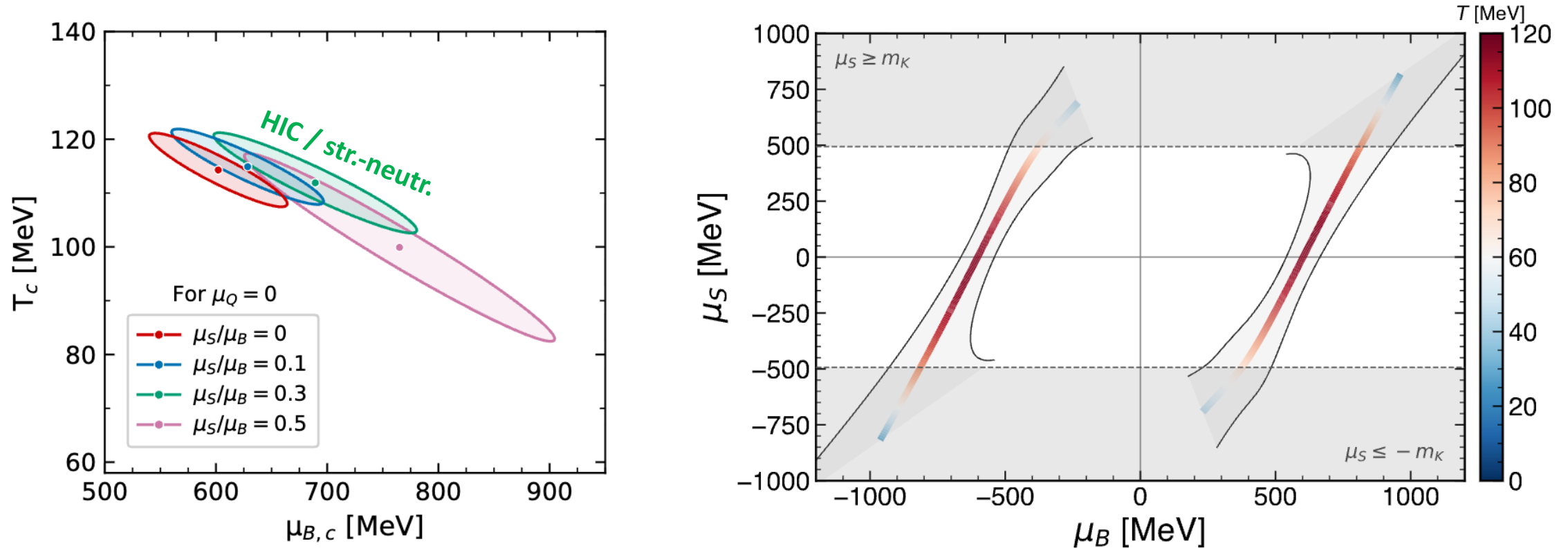
Every ray in (μ_B, μ_Q, μ_S) : use **directional susceptibility** constructed from the six second-order susceptibilities.

$$\chi_2^B(T) \longrightarrow X_2^{\theta, \varphi}(T) = c_\theta^2 \chi_2^B(T) + s_\theta^2 c_\varphi^2 \chi_2^Q(T) + s_\theta^2 s_\varphi^2 \chi_2^S(T) + 2c_\theta s_\theta c_\varphi \chi_{11}^{BQ}(T) + 2c_\theta s_\theta s_\varphi \chi_{11}^{BS}(T) + 2s_\theta^2 c_\varphi s_\varphi \chi_{11}^{QS}(T)$$

Critical point in 4D: $\mu_B - \mu_S$ plane

H. Shah, T. Gyure, F. Di Clemente, et al., 2606.28282

Using the lattice QCD input for susceptibilities from [A. Abuali, et al., PRD 112, 054502 (2025)]



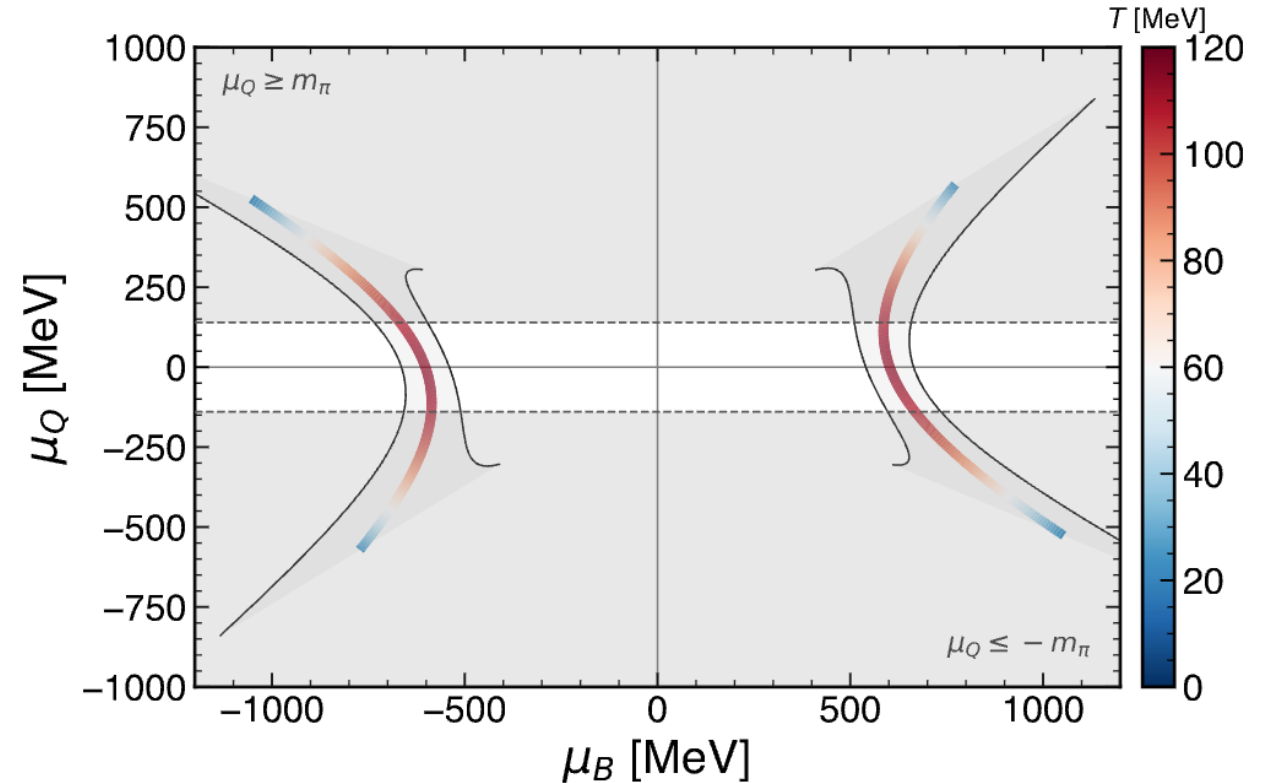
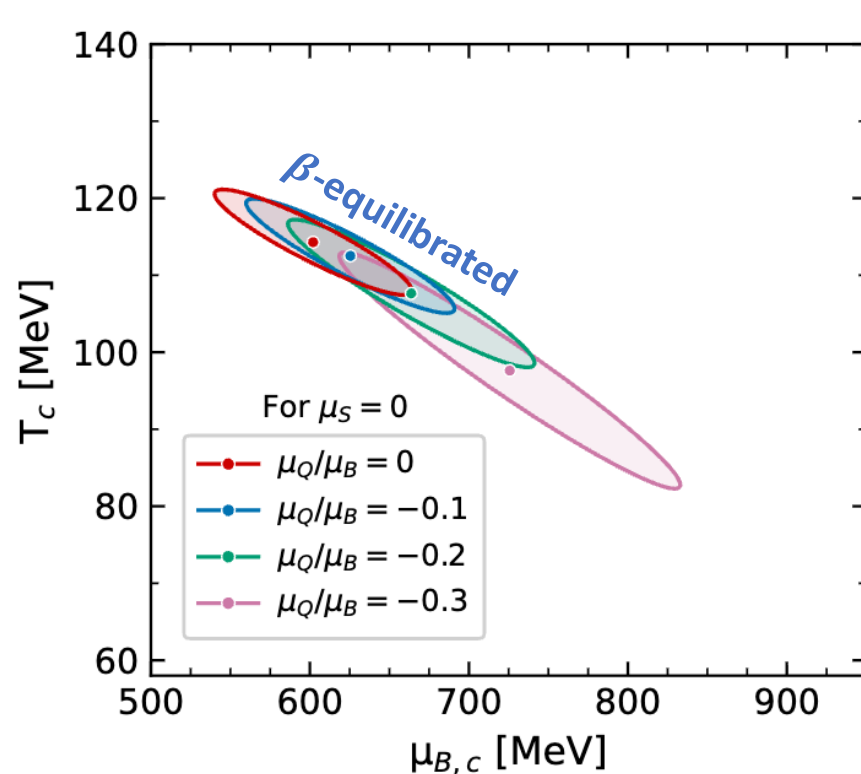
Heavy-ion collisions and strangeness neutrality ($n_S = 0 \leftrightarrow \mu_S \approx 0.15 - 0.30\mu_B$): $\mu_{B,c}$ shifts to the right.

From $(T, \mu_B) = (114,602)$ MeV at $\mu_S = 0$ to $(T, \mu_B) = (112,689)$ MeV at $\mu_S = 0.3\mu_B$

Similar effect of an increased $\mu_{B,c}$ in recent functional QCD analysis [W.J. Fu et al., 2603.13455]

Critical point in 4D: $\mu_B - \mu_Q$ plane

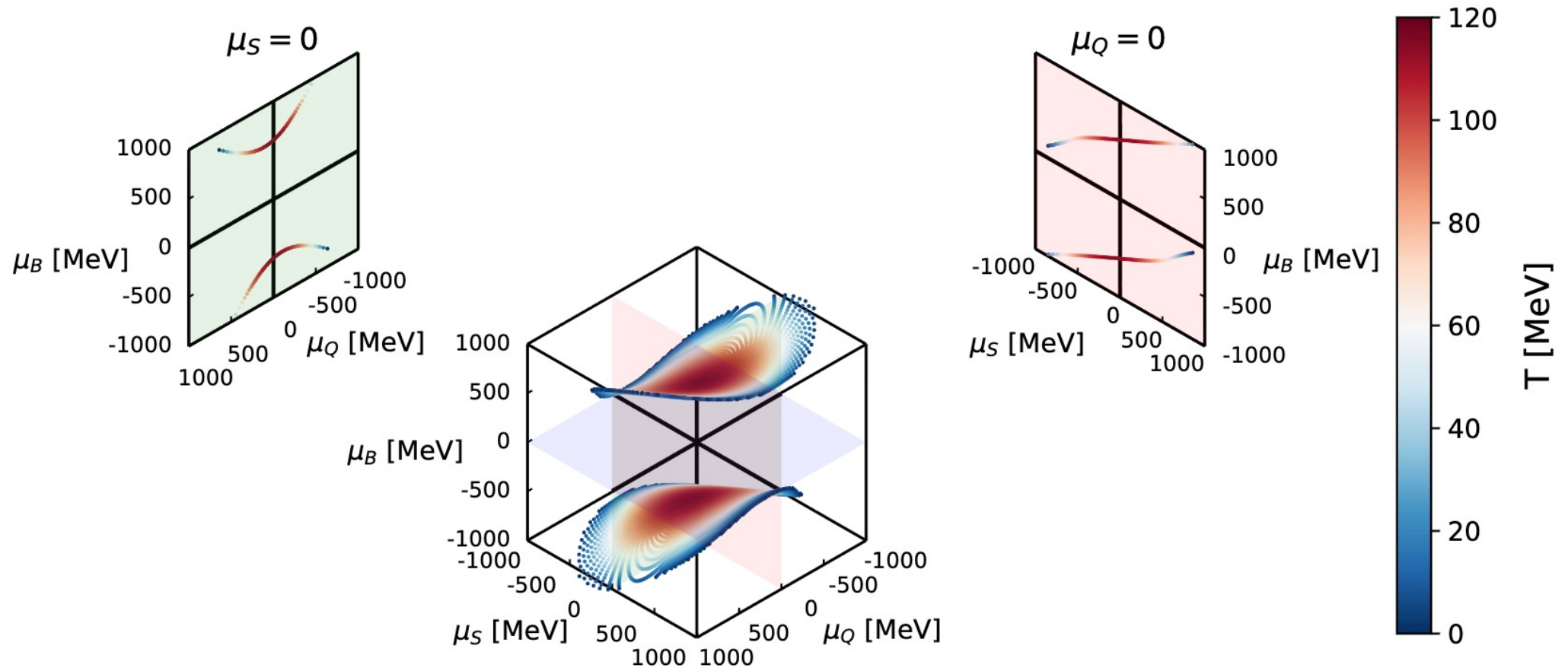
H. Shah, T. Gyure, F. Di Clemente, et al., 2606.28282



- **β -equilibrium matter** ($n_Q \approx 0 \leftrightarrow \mu_Q \approx -0.05 - 0.1\mu_B, \mu_S = 0$): **candidate CP is preserved and barely moves**
 - Suggests a possibility of a **first-order phase transition** in **neutron star mergers**
- **Large isospin asymmetry** ($|\mu_Q|/\mu_B \gtrsim 1$): **CP disappears**
 - Disfavors first-order phase transition in the Early Universe for standard scenarios

Critical point in 4D: critical surface

H. Shah, T. Gyure, F. Di Clemente, et al., 2606.28282



Caveats: Pion and kaon condensation, results are conditional on the accuracy of truncated expansion

Outlook: Full 4D EoS with a critical point in MUSES

MUSES — MODULAR UNIFIED SOLVER OF THE EQUATION OF STATE

An open-source cyberinfrastructure to support research in nuclear physics and astrophysics, computational relativistic fluid dynamics, gravitational-wave and computational astrophysics.

The latest *Calliope* version of the MUSES calculation engine focuses on heavy-ion workflows, in particular the open-source implementation of the 4D equations of state.

Studying the QCD Matter produced in Heavy-Ion Collisions
using the MUSES Calculation Engine

Johannes Jahan¹, Kevin P. Pala², Yumu Yang³, Isabella Danhoni³, Prachi Garella¹,
Jonathan Gonzales¹, Joaquin Grefa^{4,1}, Mauricio Hippert⁵, Surkhab Kaur Virk³, Micheal Kahangirwe⁴,
Musa R. Khan¹, Feyisola Nana³, Mateus Reinke Pelicer³, Tulio E. Restrepo¹,
Hitansh Shah¹, T. Andrew Manning⁶, Mark Alford⁷, Dekrayat Almaalol⁸, Ahmed Abuali¹,
Alexander Clevinger⁴, Nikolas Cruz-Camacho³, Carlos Conde-Ocazonez³, Francesco Di Clemente¹,
David Friedenberg^{9,10}, Hosein Gholami¹¹, Marco Hofmann¹¹, Jeremy W. Holt^{9,10}, Isaac Legred³,
Jamie M. Kartheim^{9,10}, Toru Kojo^{12,13,14}, Konstantin Maslov¹, Paolo Parotto¹⁵, Leonardo Pena³,
Grégoire Pihan¹, Christopher Plumberg¹⁶, Roman Poberezhniuk¹, Romulo Rougemont¹⁷,
Jordi Salinas San Martín³, Rajesh Kumar^{4,18}, Volodymyr Vovchenko¹, Veronica Dexheimer⁴,
Jorge Noronha³, Jacquelyn Noronha-Hostler³, Claudia Ratti¹ and Nicolás Yunes³

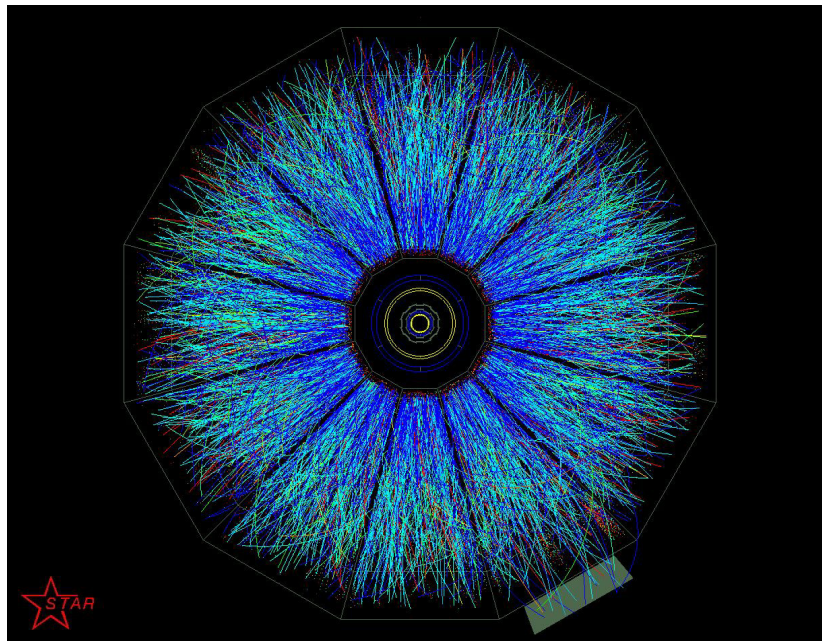
MUSES Webpage:

<https://muses.physics.illinois.edu/>

MUSES Calculation Engine:

<https://ce.musesframework.io/>

Fluctuations in heavy-ion collisions



Critical point and heavy-ion collisions

Control parameters

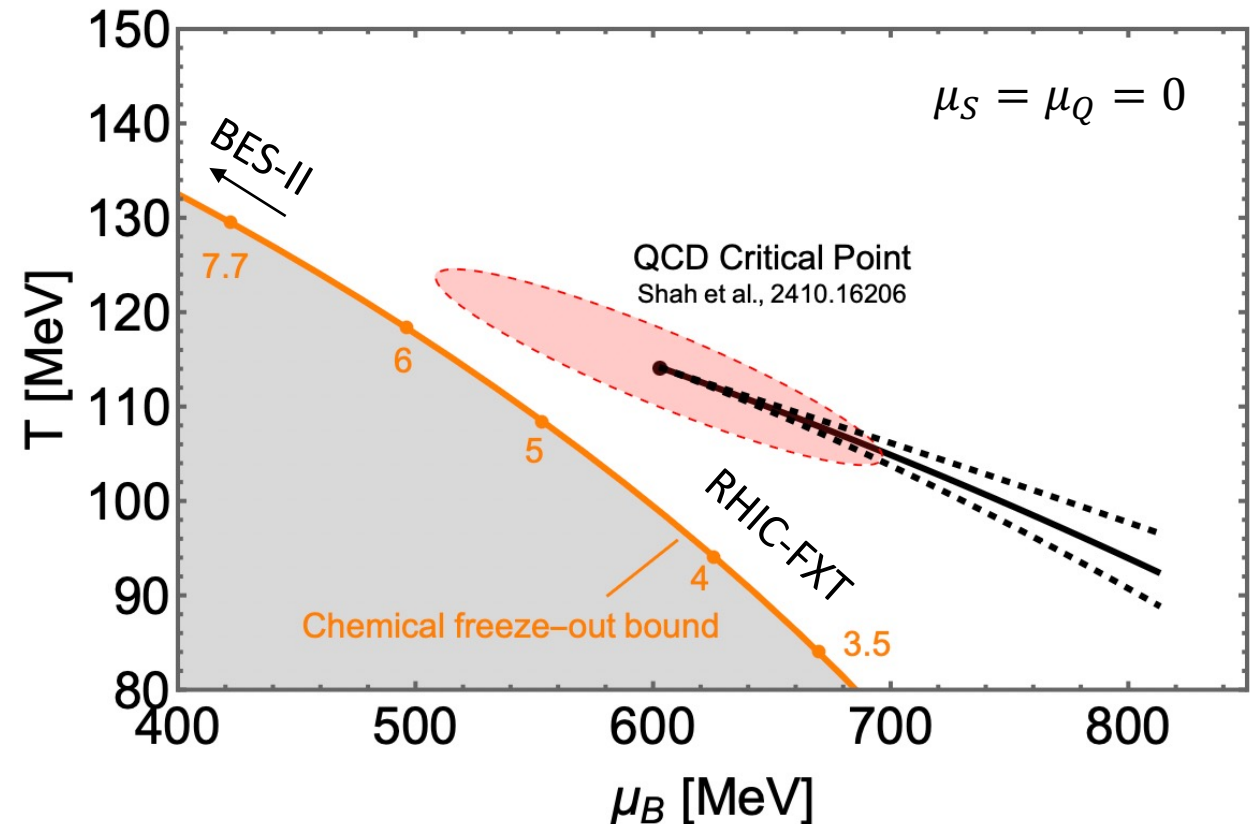
- Collision energy $\sqrt{s_{NN}} = 2.4 - 5020$ GeV
 - Scan the QCD phase diagram
- Size of the collision region
 - Expect stronger signal in larger systems

Measurements

- Final hadron abundances and momentum distributions **event-by-event**

Chemical freeze-out curve and CP

- Sets the **lower bound** on the temperature of the CP
- **Caveats:** strangeness neutrality ($\mu_S \neq 0$), uncertainty in the freeze-out curve
- If the $\mu_B \simeq 600$ MeV predictions are true, the CP may be close to freeze-out line at $\sqrt{s_{NN}} \sim 3.5 - 5$ GeV



[Lysenko, Poberezhniuk, Gorenstein, VV, PRC 111, 054903 (2025)]

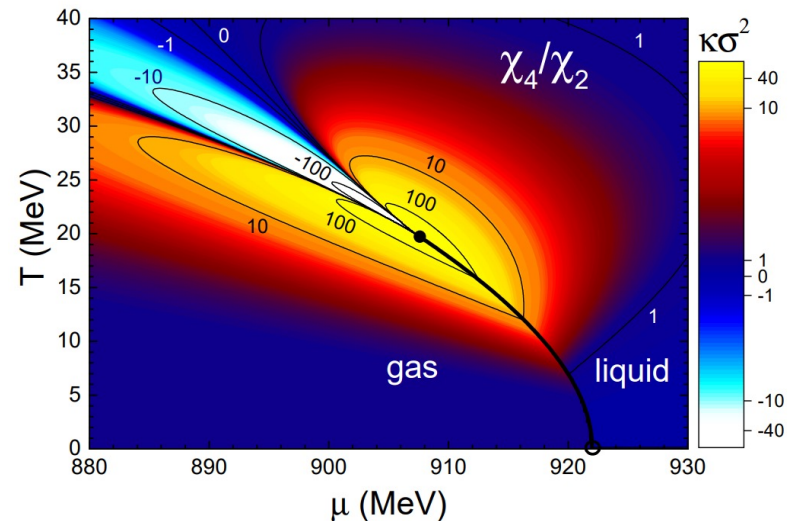
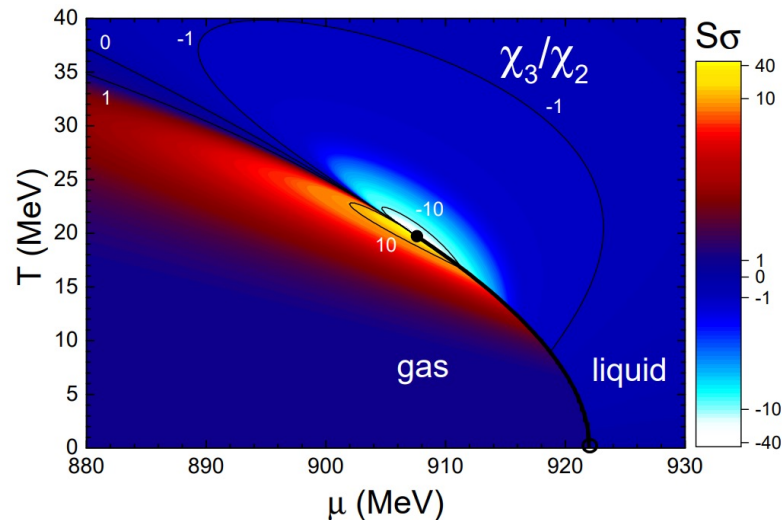
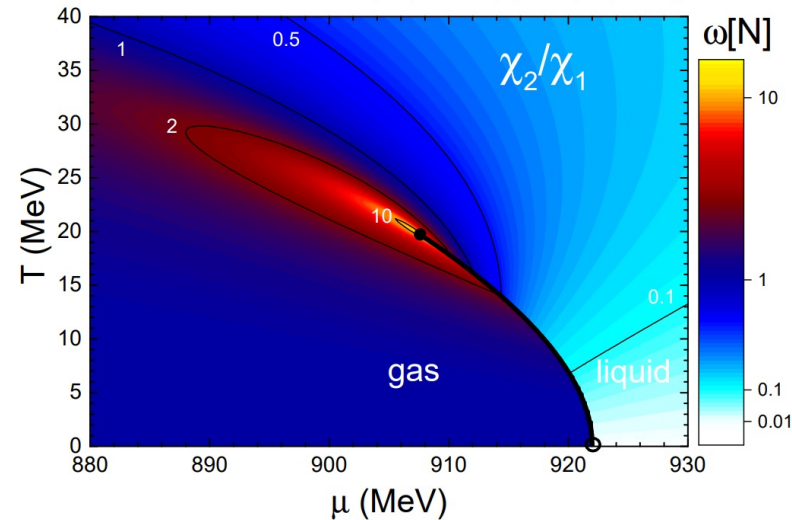
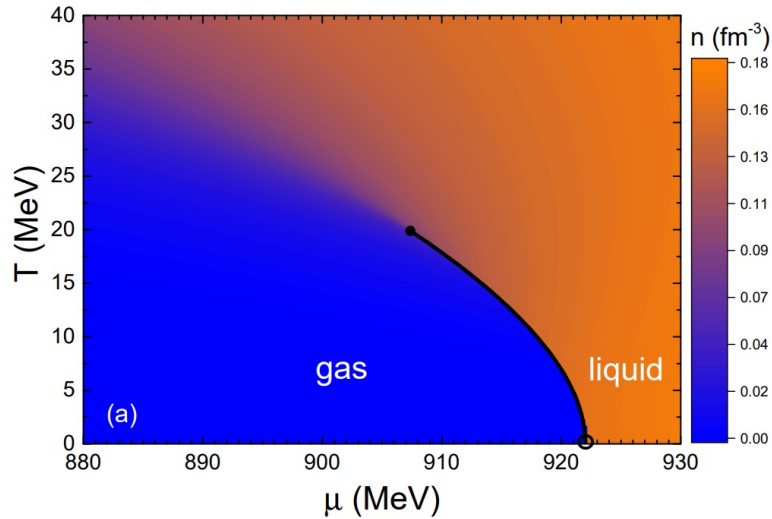
Example: (Nuclear) Liquid-gas transition

(QCD) critical point: large correlation length and fluctuations

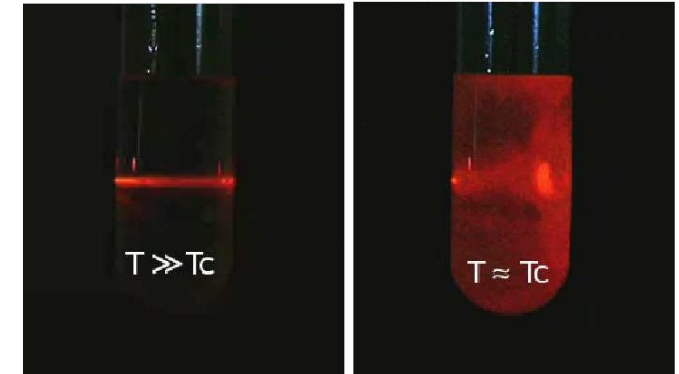
$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

$$\xi \rightarrow \infty$$

M. Stephanov, PRL '09, '11



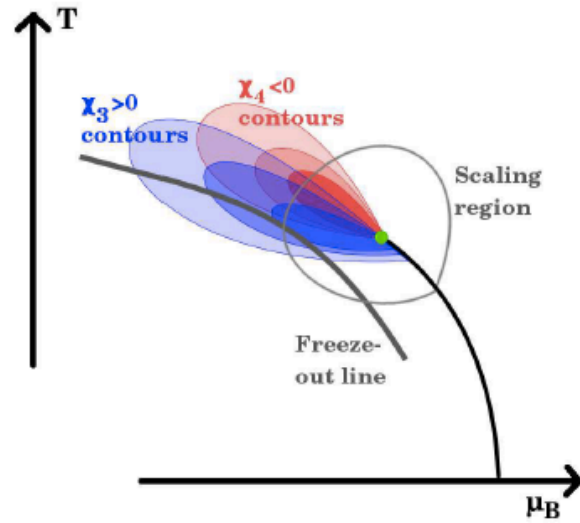
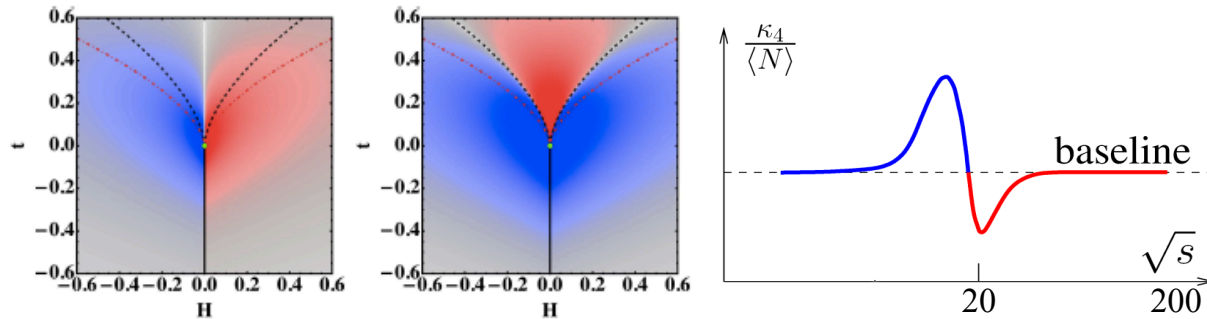
Critical opalescence



$$\langle N^2 \rangle - \langle N \rangle^2 \sim \langle N \rangle \sim 10^{23}$$

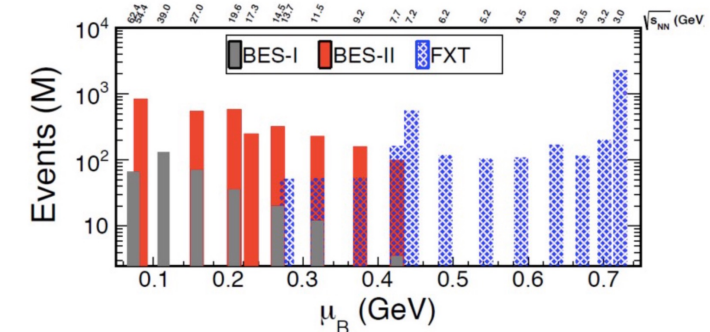
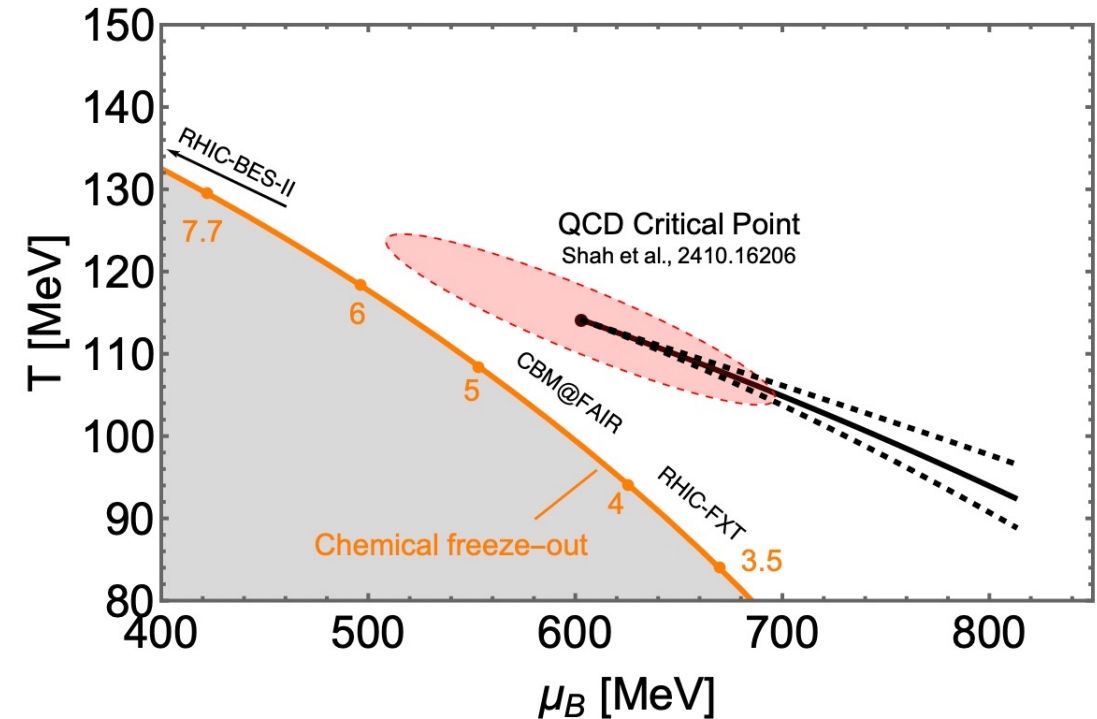
in equilibrium

Expectation from Calculations



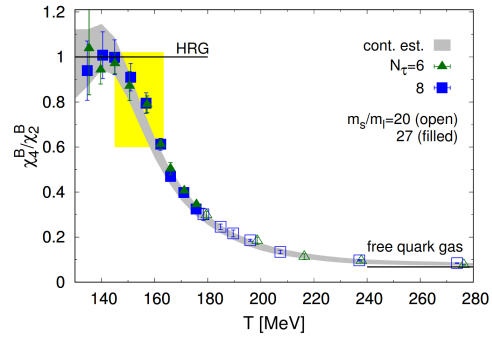
Characteristic “Oscillating pattern” is expected for the QCD critical point but *the exact shape depends on the location of freeze-out with respect to the location of CP*

- M. Stephanov, *PRL* **107**, 052301(2011)
- V. Skokov, Quark Matter 2012
- J.W. Chen, J. Deng, H. Kohyama, arXiv: 1603.05198, Phys. Rev. **D93** (2016) 034037

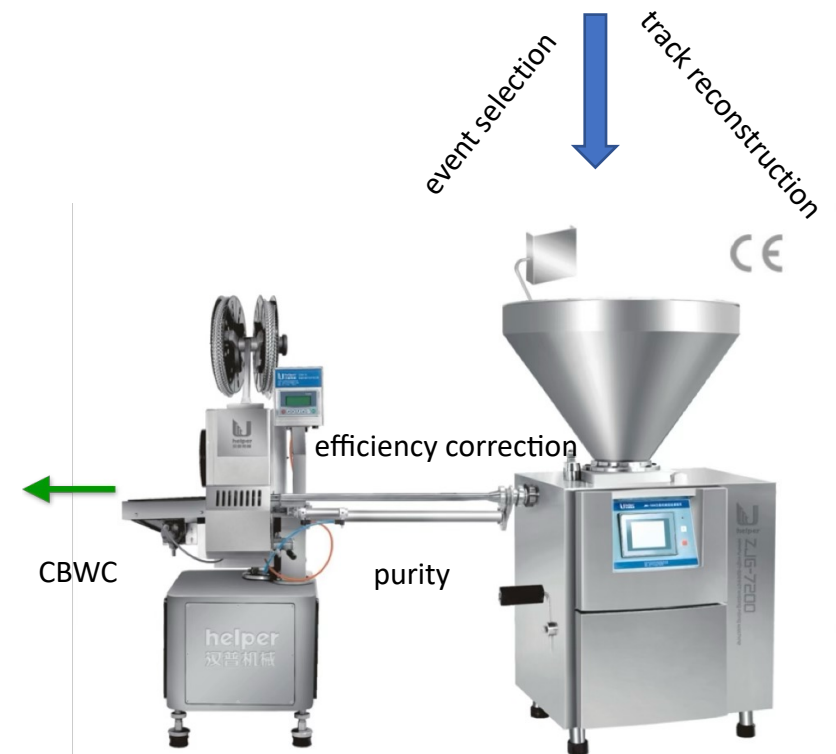
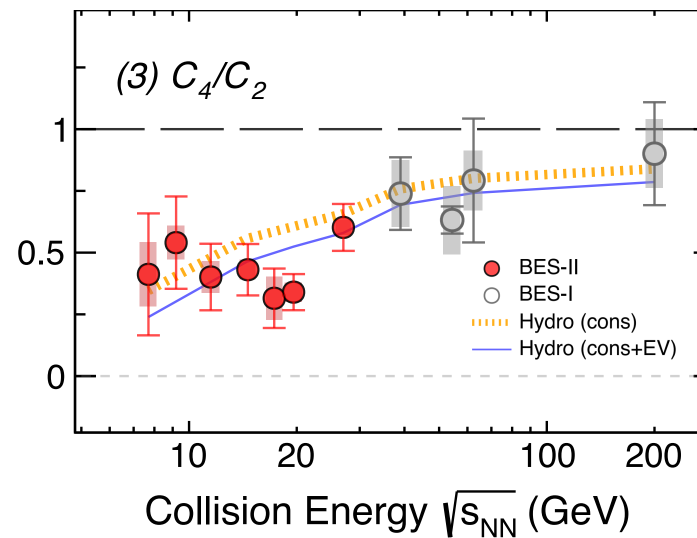
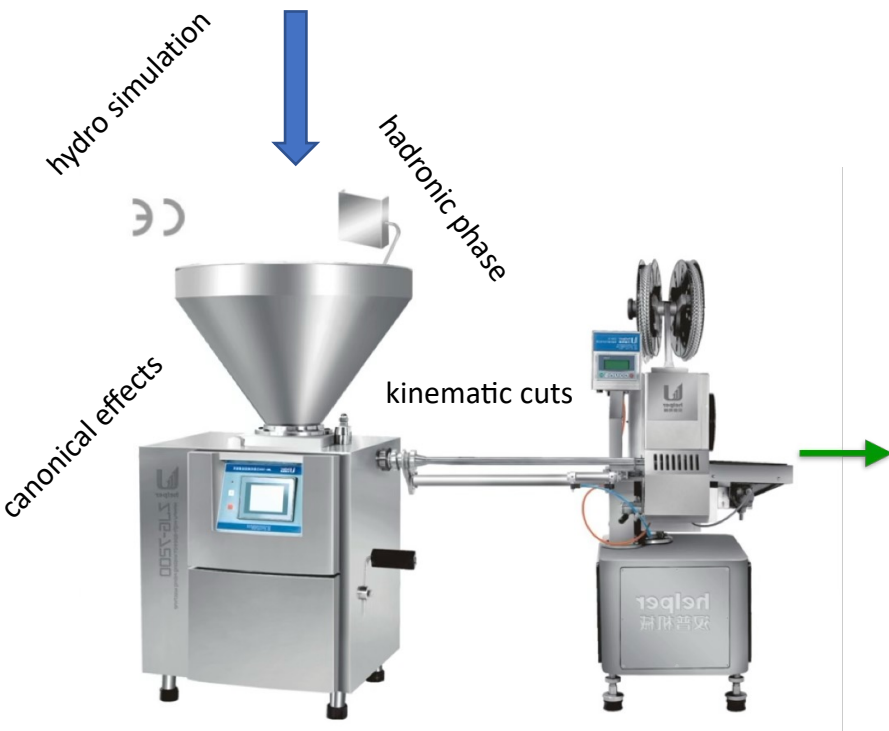
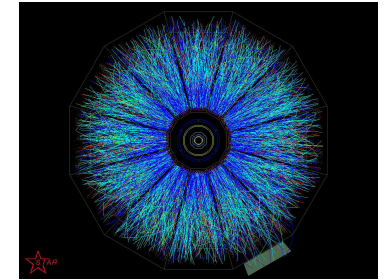


Theory vs experiment

guidance from theory (e.g. lattice)



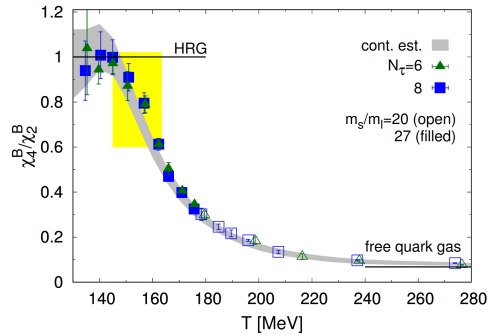
experiment (the real thing)



Theory vs experiment

guidance from theory (e.g. lattice)

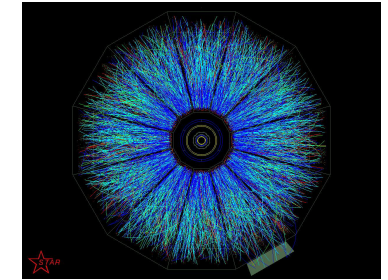
experiment (the real thing)



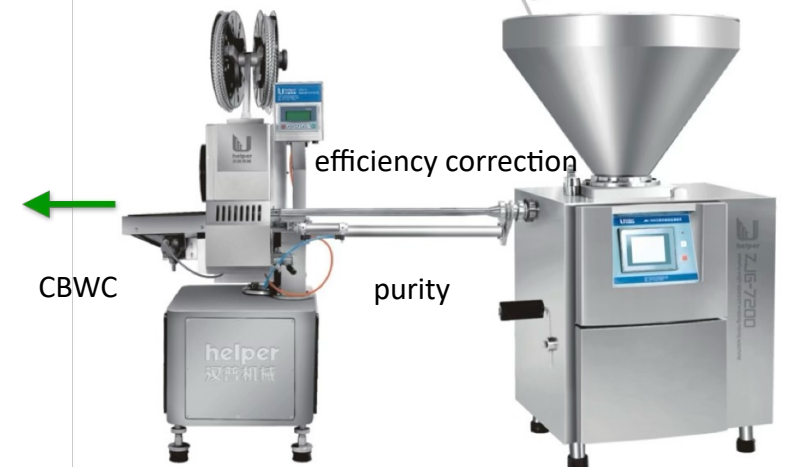
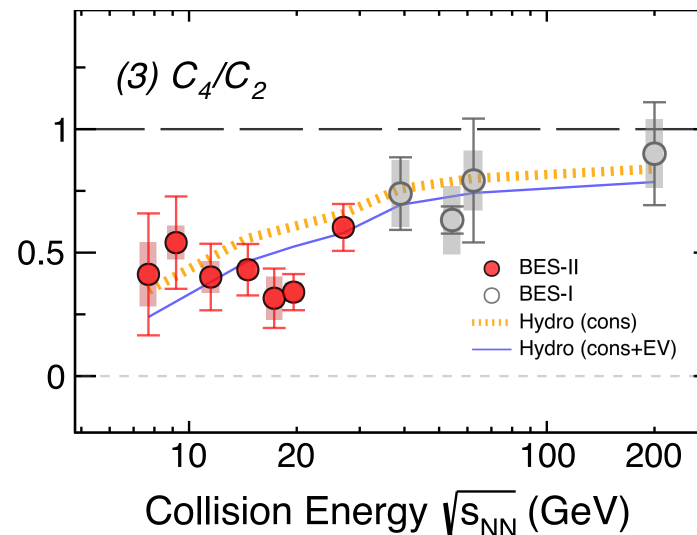
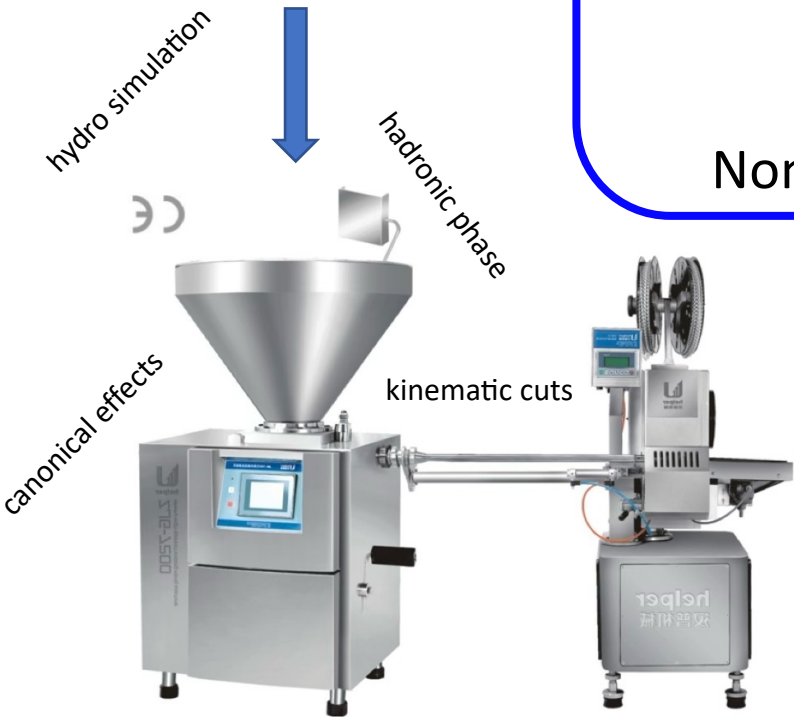
This was done in [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)]

- Full hydro simulation
- Lattice QCD-like baryon susceptibilities (interacting HRG)
- Experimental kinematic cuts
- Global baryon conservation: SAM correction

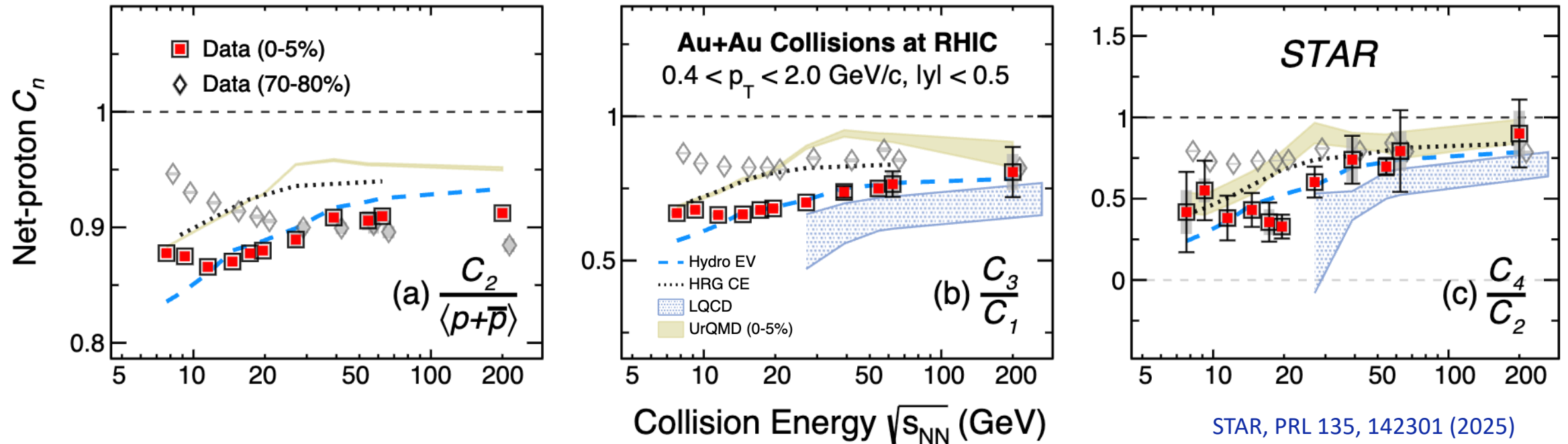
Non-critical baseline (hydro EV) prediction



event selection
track reconstruction



Net-proton cumulant ratios

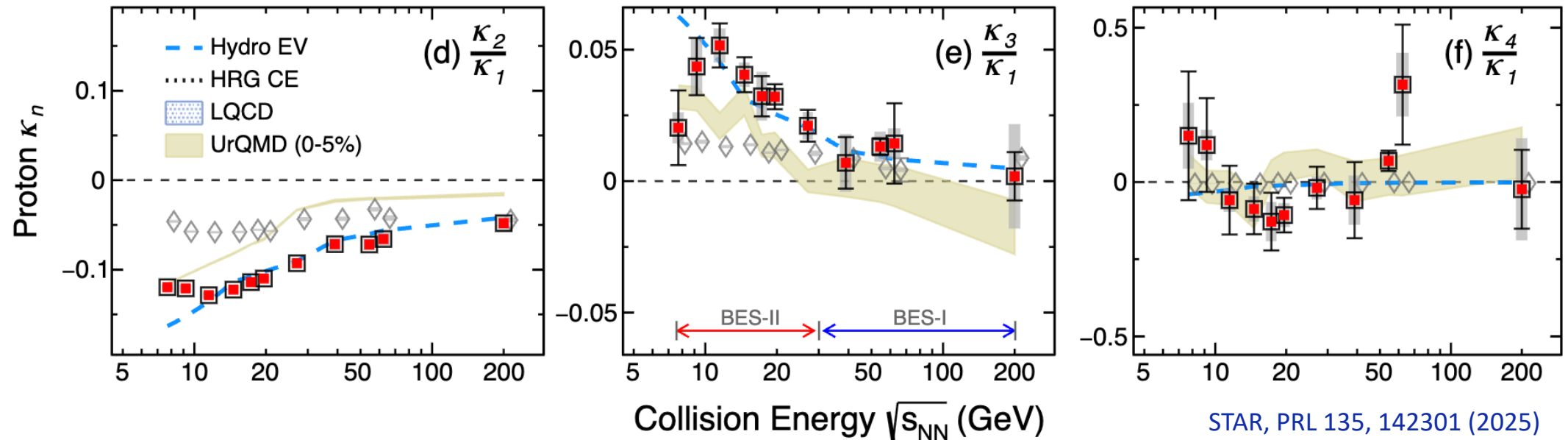


Hydro EV: [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 \(2022\)](#)

Agreement with the baseline above $\sqrt{s_{NN}} \sim 10 - 20$ GeV

Deviations at lower energies. More pronounced in factorial cumulants.

Proton factorial cumulant ratios



Hydro EV: [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 \(2022\)](#)

Factorial cumulants: linear combinations of ordinary cumulants.
Measure deviations from Poisson and isolate multi-particle correlations.

$$\hat{C}_1 = C_1$$

$$\hat{C}_2 = C_2 - C_1$$

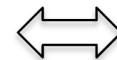
$$\hat{C}_3 = C_3 - 3C_2 + 2C_1$$

$$\hat{C}_4 = C_4 - 6C_3 + 11C_2 - 6C_1$$



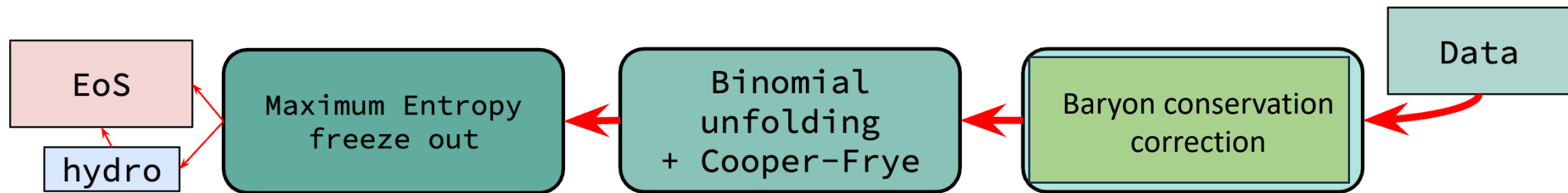
STAR
Cumulants (C)
Factorial cumulants (κ)

Others
Cumulants (κ)
Factorial cumulants (C)



Inference of baryon number susceptibilities from data

G. Pihan, R. Poberezhniuk, VV, to appear



Hydro EV [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)]

- Particlization at a constant energy density $\sim 0.26 \text{ GeV/fm}^3$
- Canonical (baryon conservation) statistical hadronization with **excluded volume repulsion**
- **Drawback:** Cannot incorporate arbitrary EoS, in particular one with a CP

Maximum entropy freeze-out [M. Pradeep et al., PRL 130, 162301 (2023); J.M. Kartheim, PRD 113, 074010 (2026)]

- Incorporate single hydrodynamic mode – **baryon density fluctuations**
- Maximum entropy method defines local **joint susceptibilities of particle numbers** G. Pihan et al., to appear
 - **Joint susceptibilities of baryons (+) and antibaryons (-)**
- Use the χ_n^B input from **arbitrary EoS** (LQCD expansion, s-contours, functional QCD, ...)

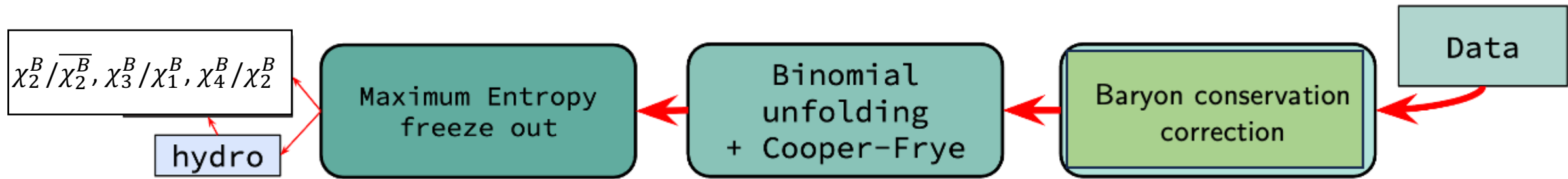
SAM-3.0: General correction for exact baryon conservation

- Based on saddle-point method [R. Poberezhniuk, V.A. Kuznietsov, G. Pihan, VV, 2607.01783]

Bayesian EoS inference

Instead of using χ_n^B as an input, extract χ_n^B 's that fit data directly at each energy

Specifically, the ratios $\chi_2^B / \overline{\chi_2^B}$, χ_3^B / χ_1^B , χ_4^B / χ_2^B where $\overline{\chi_2^B}$ is the HRG baseline



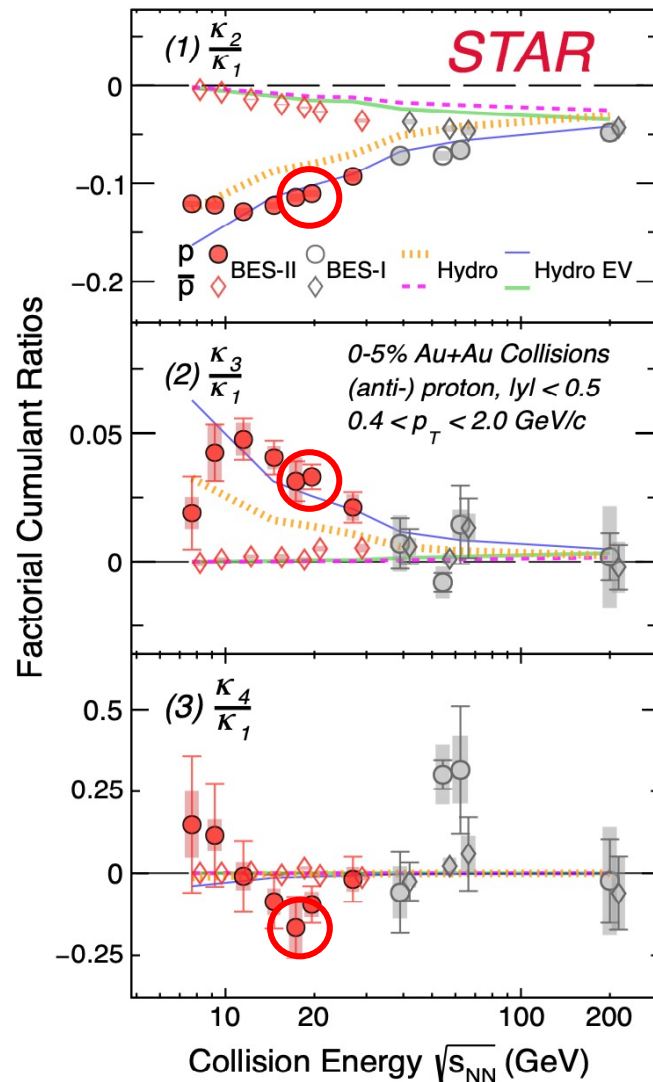
Adapted from G. Pihan, SQM2026

Our first analysis uses collider-mode BES data only.

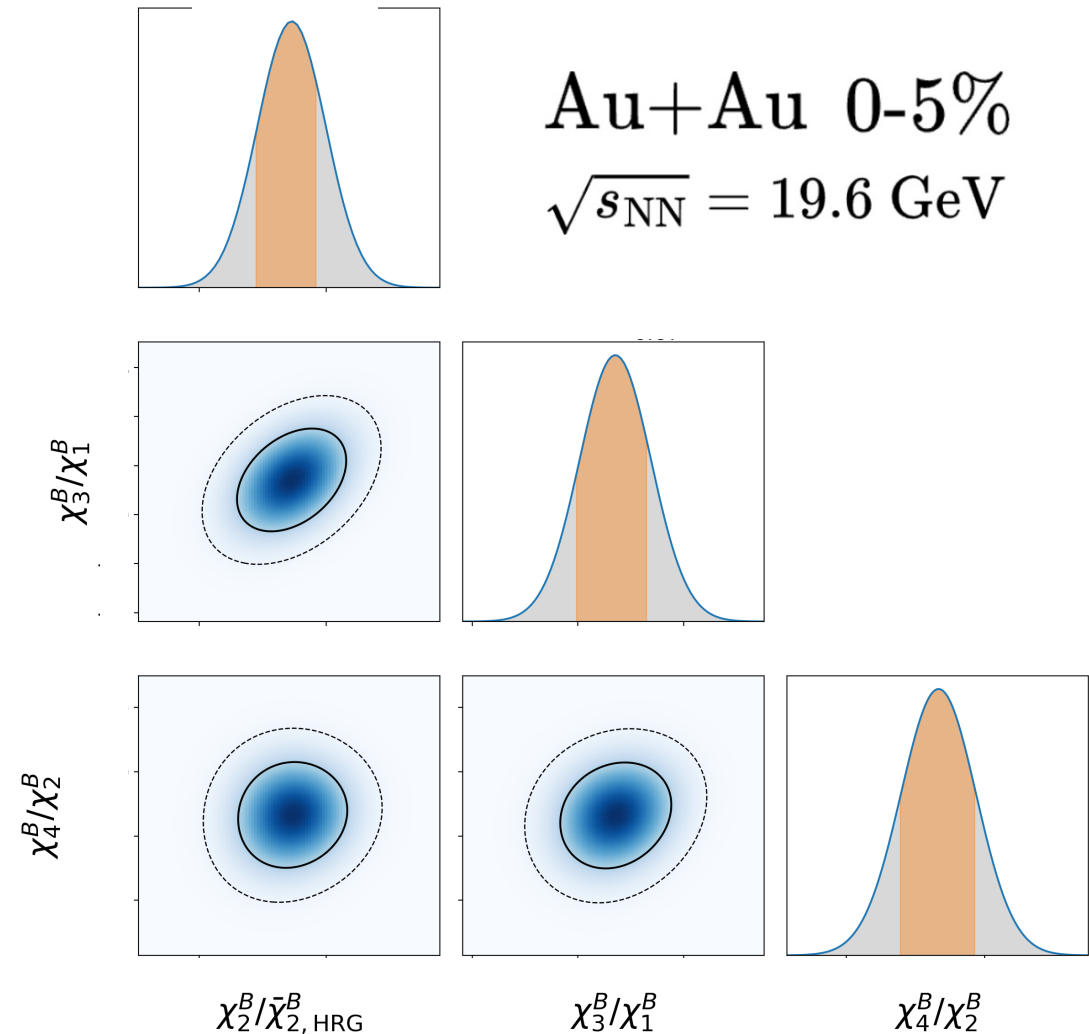
FXT inference is deferred pending improved theoretical modeling (currently no hydro below 7.7 GeV) and volume-fluctuation corrections (being refined in STAR).

Similarly, no non-equilibrium evolution of cumulants (see **talk by N. Abbasi**) or first-order phase transition dynamics (see **talk by N. Sogabe**)

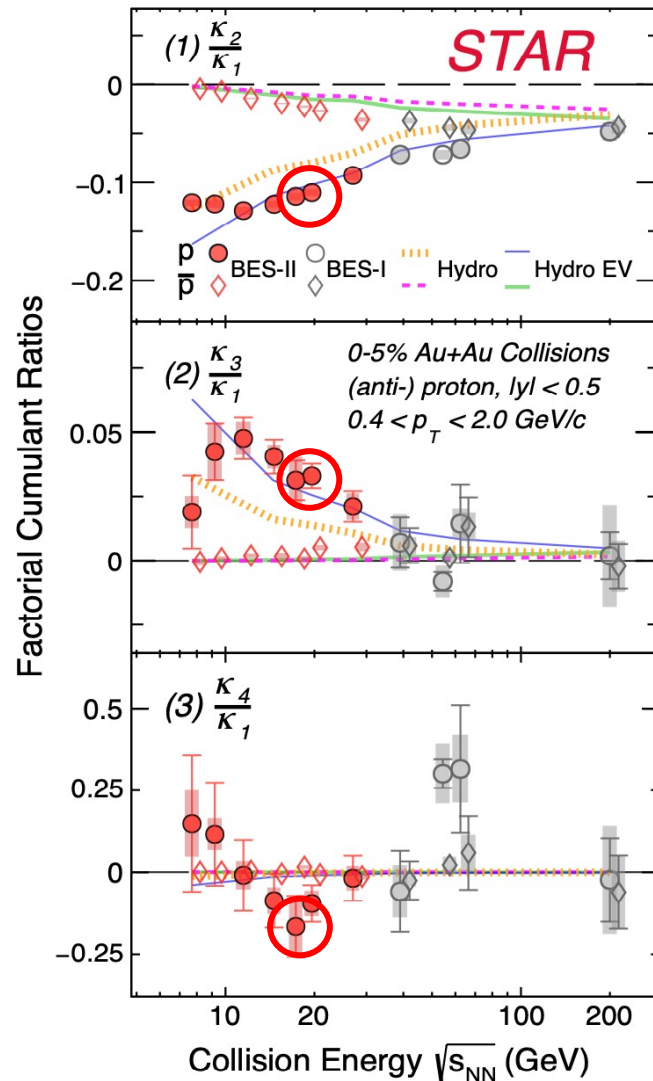
Bayesian inference: 19.6 GeV



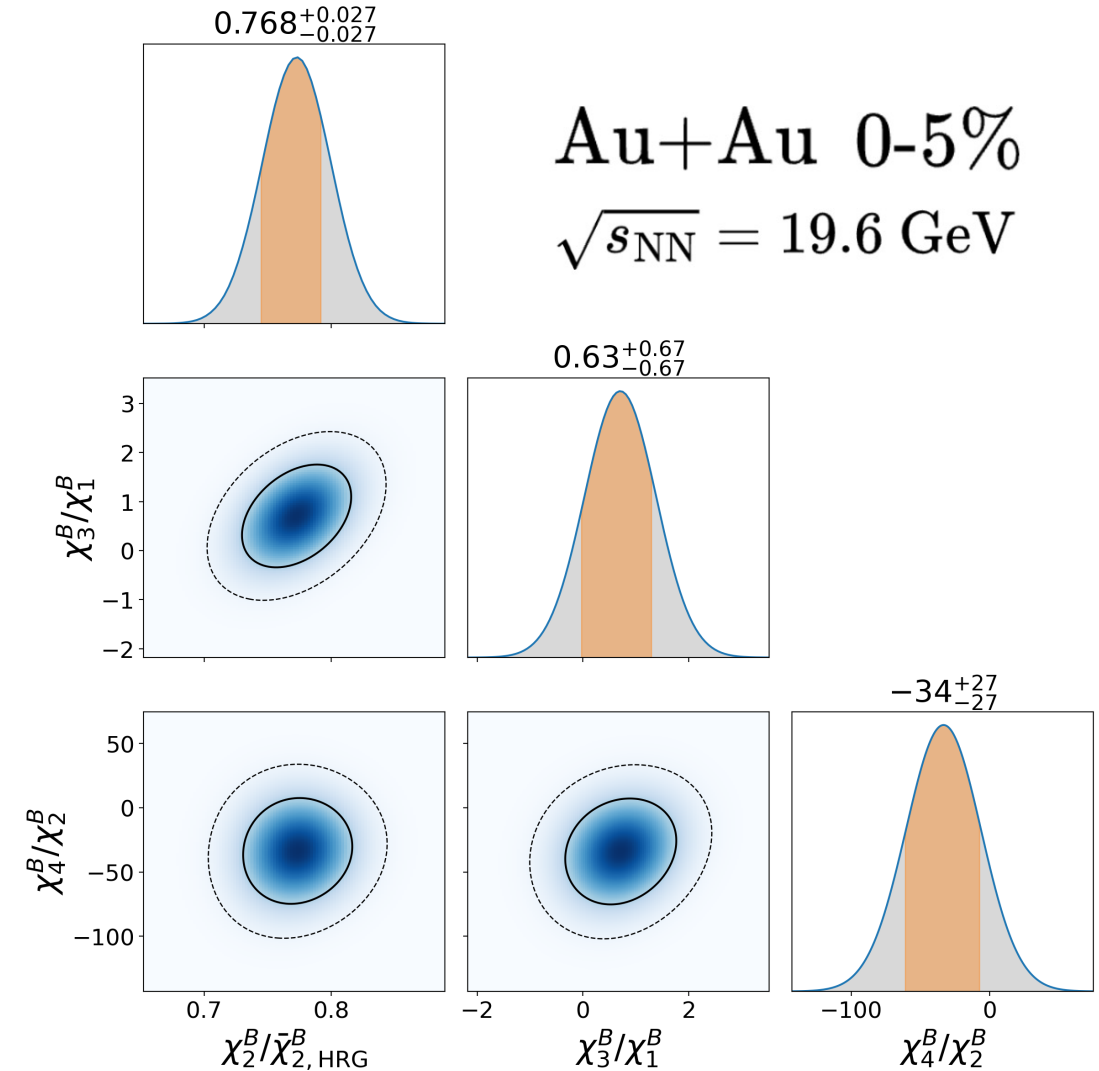
Unfolding



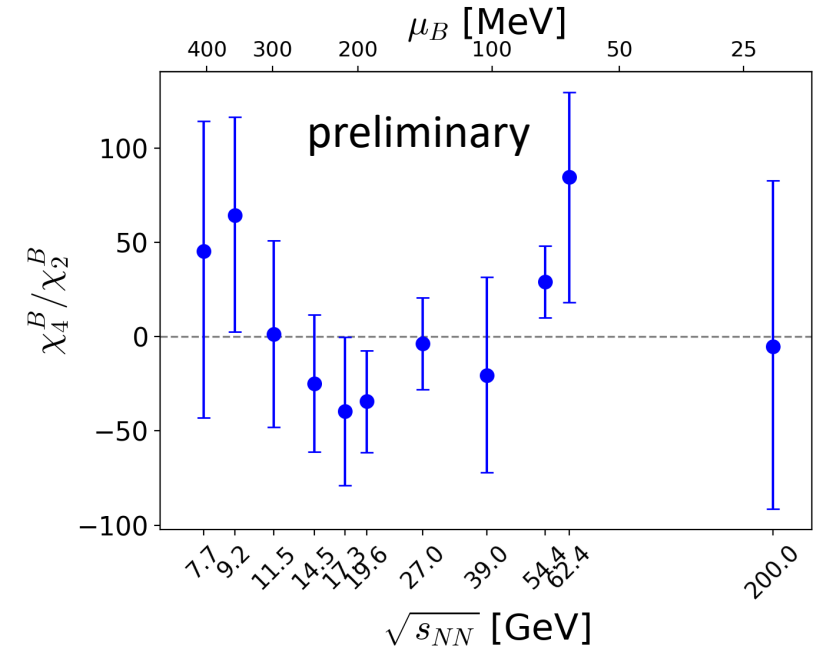
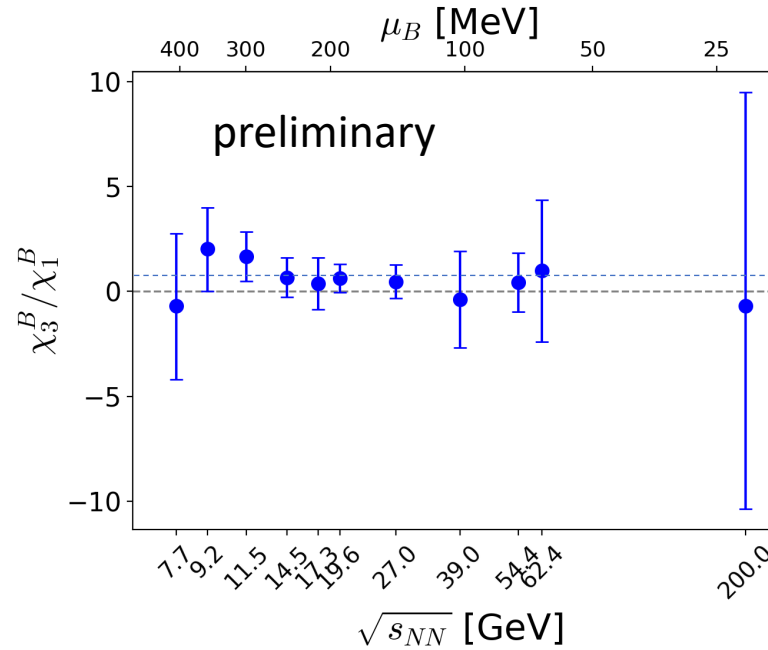
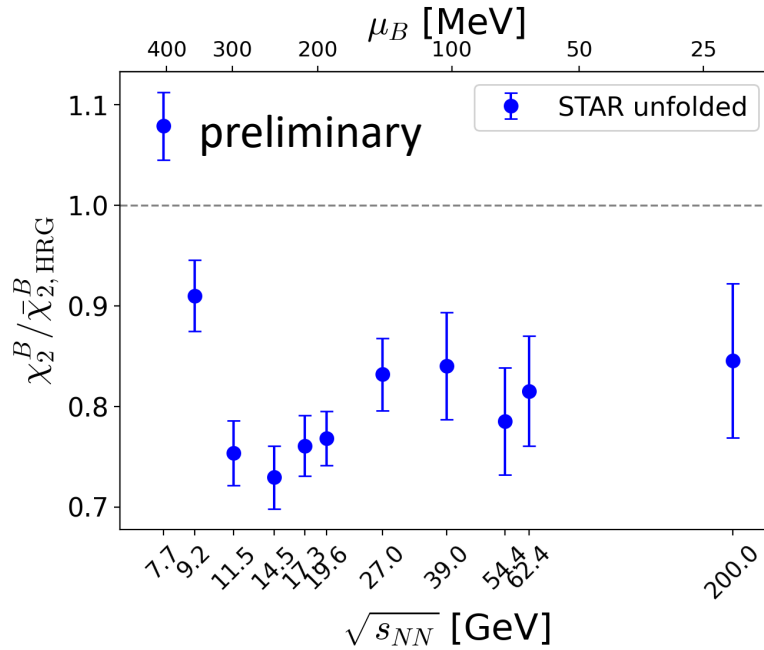
Bayesian inference: 19.6 GeV



Unfolding



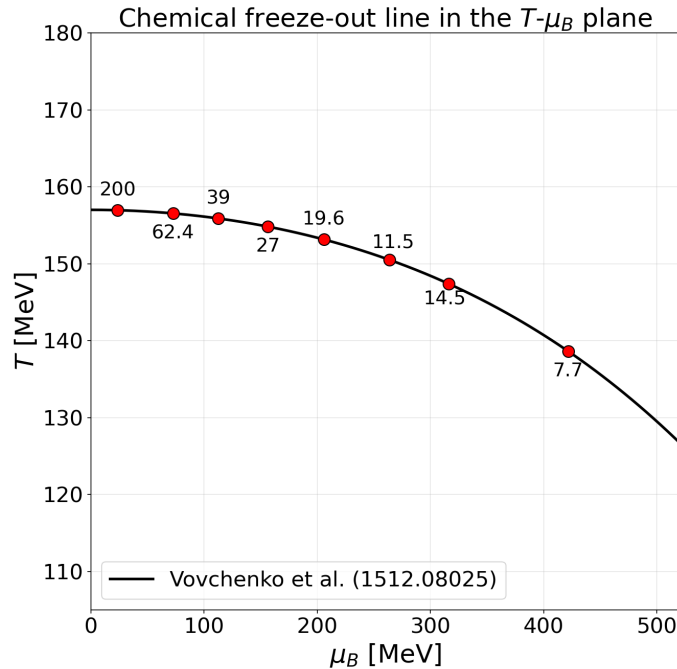
Extracted susceptibilities



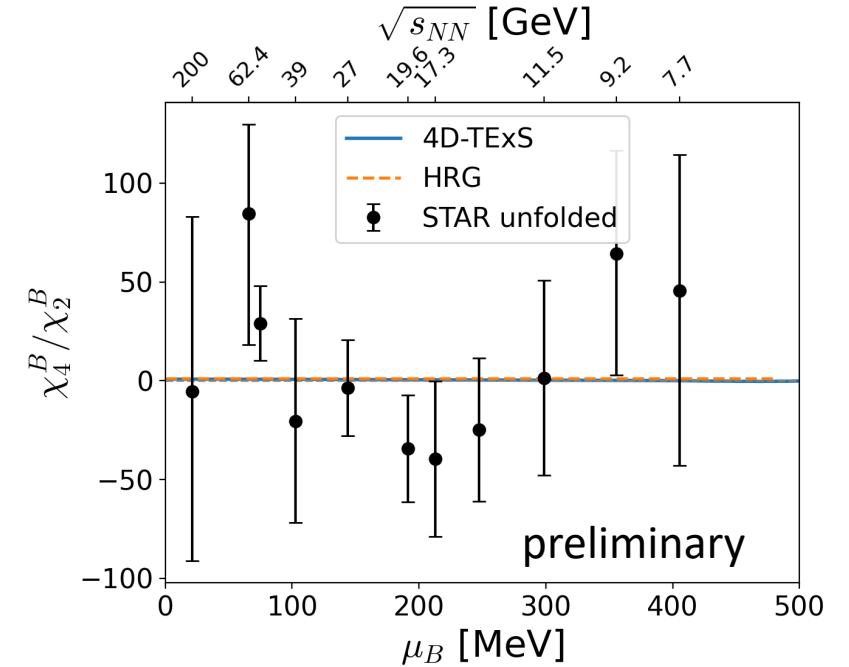
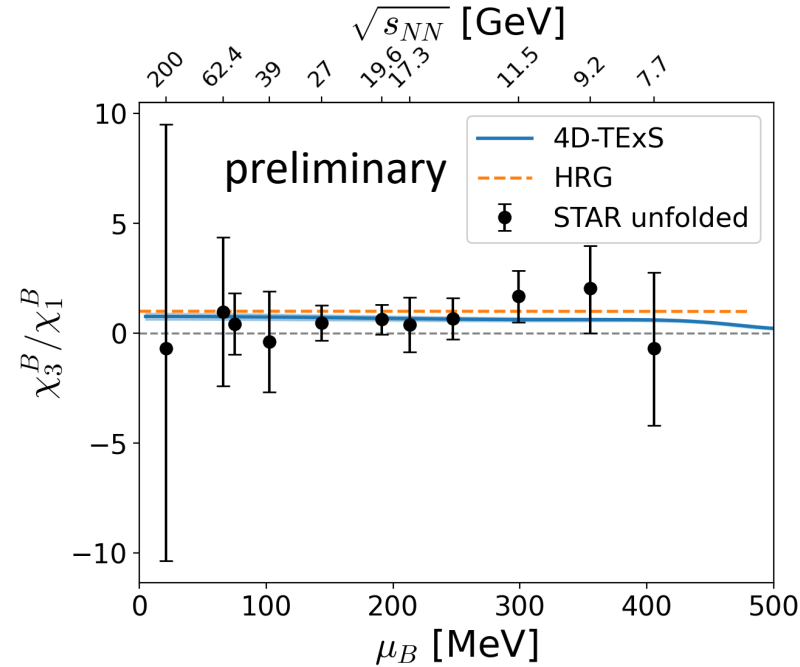
- Tight constraints on χ_2^B value relative to HRG
- Uncertainties in 3rd and 4th order susceptibilities are large
 - Intermediate steps (net-B to B⁺, kinematic cuts, B⁺→p) act akin to efficiency correction
 - Reducing uncertainties in the data would lead to reduced extraction errors
- Results appear weakly sensitive to e_{sw} , sensitivity to other hydro parameters to be studied

Comparing to lattice QCD

To map the results to QCD phase diagram use chemical freeze-out parametrization [VV et al., PRC 93, 064906 (2016)]

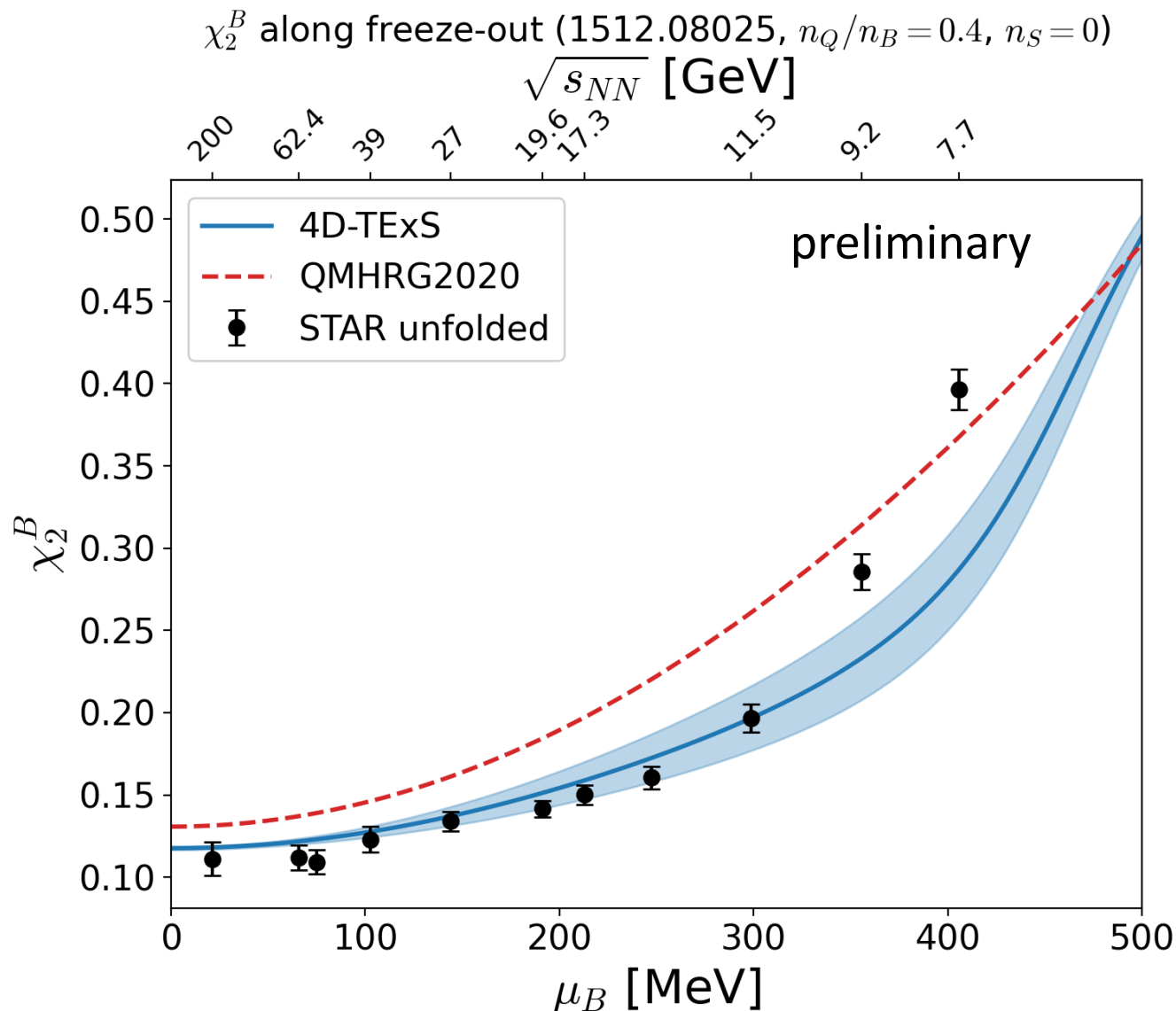


$$\varepsilon_{SW} \approx 0.35 - 0.40 \text{ GeV/fm}^3$$



4D-TExS: [A. Abuali et al., Phys. Rev. D 112, 054502 (2025)]

- Lattice-based EoS at finite baryon density
- Computed under HIC conditions ($n_Q/n_B = 0.4, n_S = 0$)



4D-TEoS: [A. Abuali et al., Phys. Rev. D 112, 054502 (2025)]

- To obtain absolute χ_2^B multiply by HRG value
 - QMHRG motivated by lattice QCD studies at $\mu_B = 0$ [HotQCD, PRD 104, 074512 (2021)]
- Agreement with lattice QCD at $\mu_B < 300$ MeV
- Enhancement relative to lattice QCD at $\mu_B > 300$ MeV

⚠ Caveats:

- Sensitive to the hadron list
- Sensitive to assumed freeze-out line
- Lattice EoS (4D-TEoS) is an extrapolation (no truncation error included)

Summary

- **Expansion from $\mu = 0$**

The entropy contour construction suggests a candidate critical region near $T \sim 110 \text{ MeV}, \mu_B \sim 600 \text{ MeV}$ conditional on the low-order extrapolation.

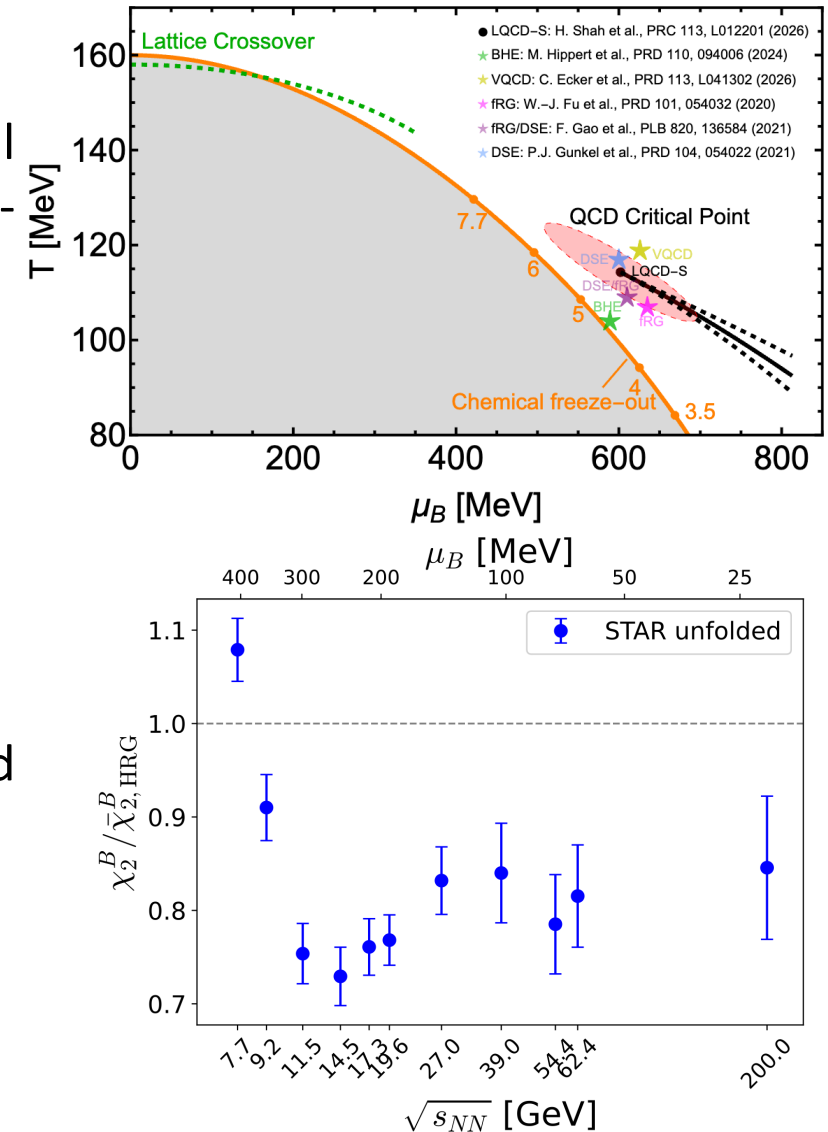
- **Extension to physical directions**

- Strangeness neutrality (HICs) moves the candidate CP to larger μ_B
- Beta equilibrium (neutron stars) changes it only mildly
- Sufficiently large isospin asymmetry (early Universe) removes it

- **Inference from heavy-ion data**

- The inferred χ_2^B agrees with lattice-based EoS at lower μ_B and shows enhancement at higher μ_B
- Large uncertainties keep higher orders weakly constrained.

Outlook: Bring these two directions together in a common 4D EoS and quantitative fluctuation analysis extended to fixed-target energies.



Thanks for your attention