Proton cumulants from hydrodynamics: From baseline to EoS inference

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Workshop on High Baryon Density Physics in High-Energy Nuclear Collisions @ LBNL

Thanks to:

V. Koch, V.A. Kuznietsov, G. Pihan, R. Poberezhniuk, H. Shah, C. Ratti

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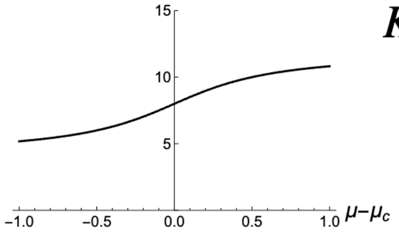
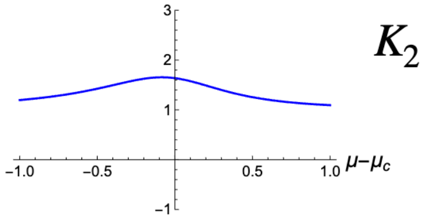
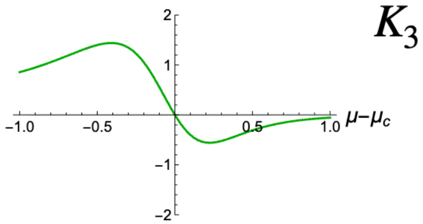
Why cumulants



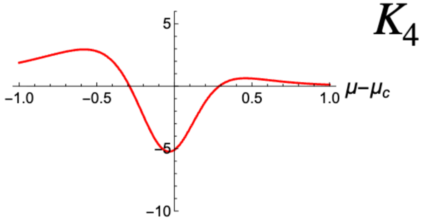
$$K_1 = \langle N \rangle$$



$$K_2 = \frac{\partial}{\partial(\mu/T)} K_1$$



$$K_3 = \frac{\partial}{\partial(\mu/T)} K_2$$



$$K_4 = \frac{\partial}{\partial(\mu/T)} K_3$$

Statistical mechanics:

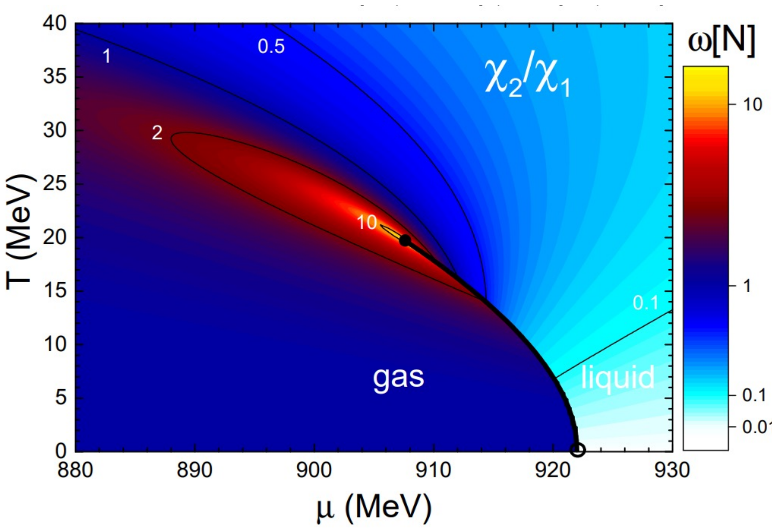
GCE: $$\ln Z^{\text{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N} Z^{\text{ce}}(T,V,N)\right],$$

$$\kappa_n \propto \frac{\partial^n (\ln Z^{\text{gce}})}{\partial(\mu_N)^n}$$

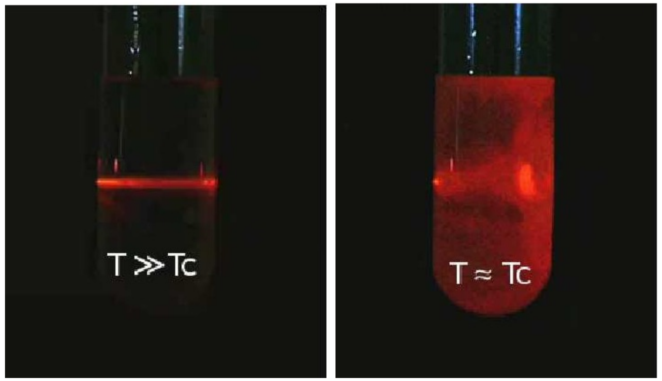
Cumulants measure μ derivatives of the (QCD) EoS

Critical point: large correlation length, equilibrium fluctuations diverge

van der Waals model



Critical opalescence



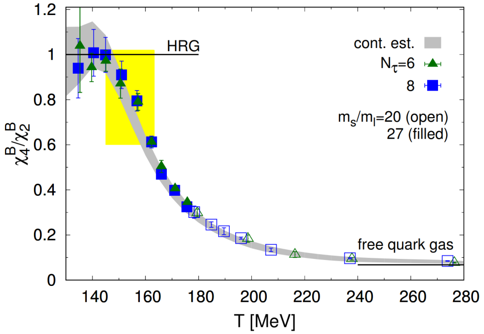
$$\langle N^2 \rangle - \langle N \rangle^2 \sim \langle N \rangle \sim 10^{23}$$

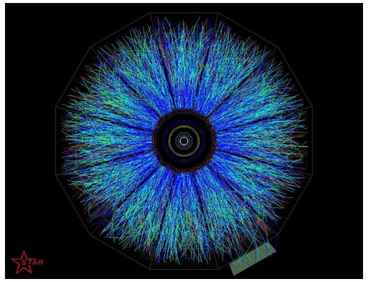
in equilibrium

Theory vs experiment

guidance from theory (e.g. lattice)

experiment (the real thing)





hydro simulation

hadronic phase

canonical effects

kinematic cuts

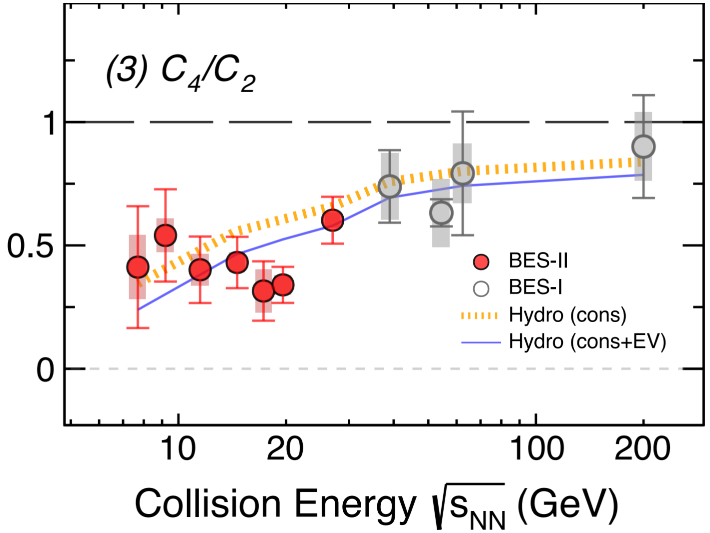
event selection

track reconstruction

efficiency correction

CBWC

purity



Theory vs experiment

guidance from theory (e.g. lattice)

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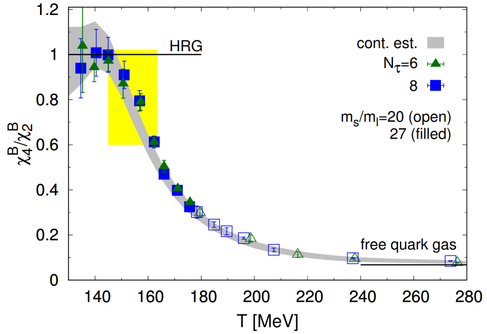
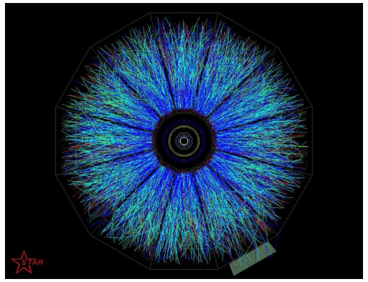


This was done in [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)]

- Full hydro simulation
- Lattice QCD-like baryon susceptibilities (interacting HRG)
- Experimental kinematic cuts
- Global baryon conservation: SAM correction

Non-critical baseline (hydro EV) prediction

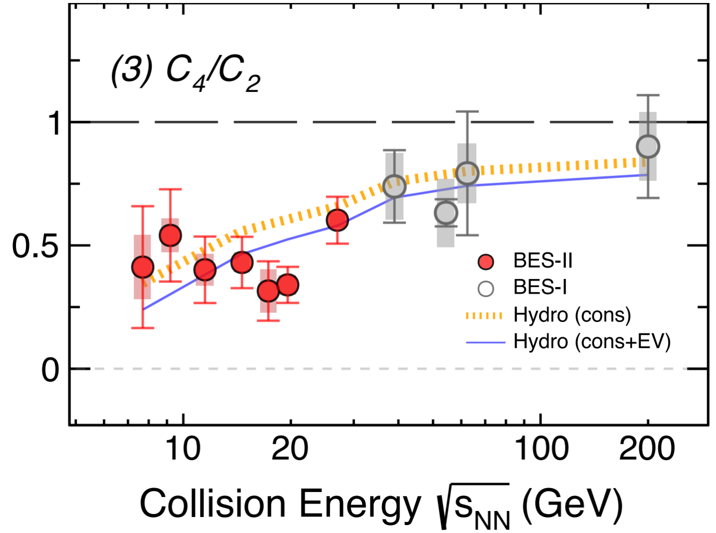


hydro simulation

hadronic phase

canonical effects

kinematic cuts



event selection

track reconstruction

efficiency correction

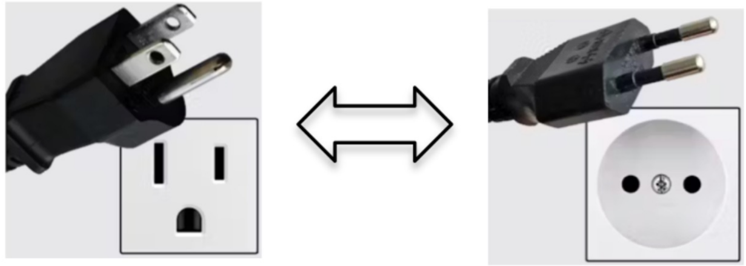
CBWC

purity

Standard disclaimer

STAR
Cumulants (C)
Factorial cumulants (κ)

Others
Cumulants (κ)
Factorial cumulants (C)



M. Arslandok, QM 2025

Cumulants (κ)
Factorial cumulants (FC)

Cumulants (κ)
Factorial cumulants ($F^{(n,0)}$)

Cumulants (κ)
Factorial cumulants ($\hat{C}$)

Measurements vs baselines

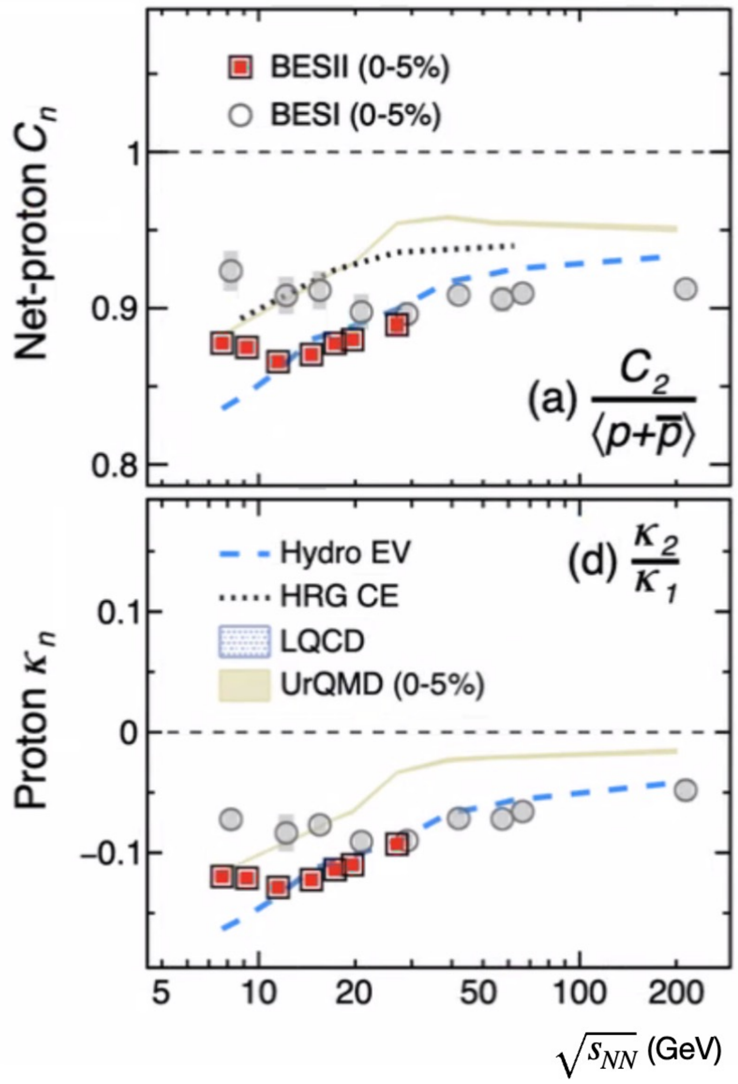
- The suppression relative to Poisson at collider energies appears to be driven by baryon conservation

$$\frac{\kappa_2[p-\bar{p}]}{\langle p+\bar{p}\rangle} \approx 1-\alpha, \qquad \alpha = \frac{\langle N_p^{\rm acc} + N_{\bar{p}}^{\rm acc}\rangle}{\langle N_B^{4\pi} + N_{\bar{B}}^{4\pi}\rangle} \qquad \sqrt{s_{NN}} \searrow \;\Rightarrow\; \alpha \nearrow$$

- Additional repulsion (excluded volume) improves the agreement
  - Same result in an alternative implementation of repulsion

Friman, Redlich, Rustamov, arXiv:2508.18879

- Change of trend emerges at $\sqrt{s_{NN}}$~10 GeV



RHIC-BES-II data and CP

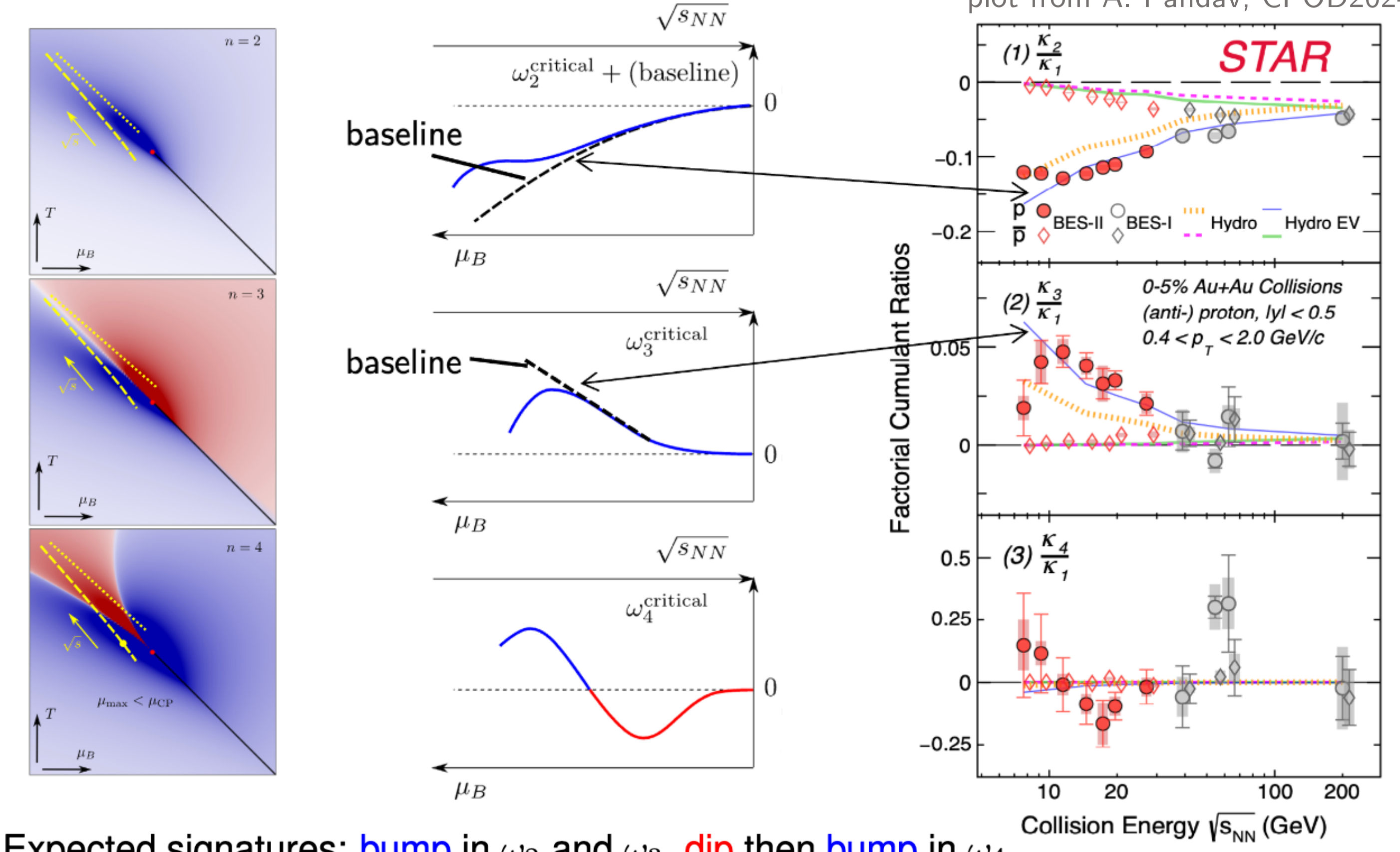
VV, Koch, arXiv:2504.01368, plot adapted from M. Stephanov, arXiv:2410.02861

$\omega_n = \hat{C}_n / \hat{C}_1$

(universal EOS) critical χ_n:

BES-II data:

plot from A. Pandav, CPOD2024



Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

Non-critical baseline (hydro EV):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

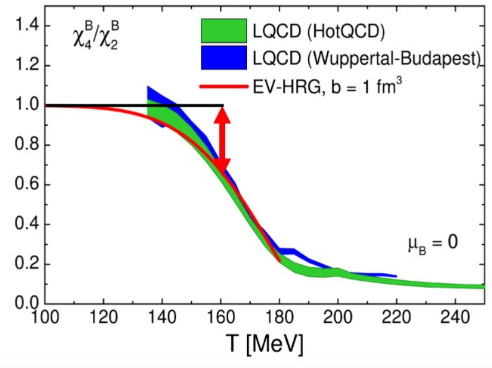
- describes right side of the peak in $\hat{C}_3$
- **signal relative to baseline:**
  - *positive* $\hat{C}_2 - \hat{C}_2^{baseline} > 0$
  - *negative* $\hat{C}_3 - \hat{C}_3^{baseline} < 0$

Controlling the non-critical baseline is essential

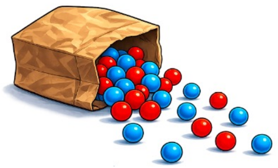
Or can we go beyond the baseline

Generalizing Hydro EV

Hydro EV [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)]



1. Smooth (3+1)-D hydro evolution with MUSIC [Shen, Alzhrani, PRC 102, 014909 (2020)]
2. Grand-canonical particlization of excluded volume HRG fitted to lattice data at $\mu_B = 0$ [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
   - model-dependent and allows only repulsion
3. Kinematic/efficiency cuts using Cooper-Frye formula and isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 86, 044904 (2012)]



4. Correction for baryon conservation using SAM-2.0 [VV, Phys. Rev. C 105, 014903 (2022)]
   - Minimum effect of baryon conservation

New developments:
- Replace EV-HRG parametrization by MaxEnt freeze-out → probe arbitrary χ_n^B
- SAM-3.0 correction: technical simplification and minor correction to SAM-2.0 results

Generalizing Hydro EV: new elements

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Maximum entropy freeze-out [M. Pradeep et al., PRL 130, 162301 (2023); arXiv:2508.19237]

- Incorporate single hydrodynamic mode – baryon density fluctuations
- Maximum entropy method defines local baryon/antibaryon joint susceptibilities G. Pihan et al., to appear

$$\chi^{+-}_{nm}(x) = \delta_{m0}\,\bar{\chi}^{+}_{1}(x) + \delta_{n0}\,\bar{\chi}^{-}_{1}(x) + (-1)^m \underbrace{\left[\chi^{B}_{n+m}(x) - \bar{\chi}^{B}_{n+m}(x)\right]}_{\text{deviations from HRG}} \frac{\bar{\chi}^{+}_{1}(x)^n\,\bar{\chi}^{-}_{1}(x)^m}{\left[\bar{\chi}^{+}_{1}(x) + \bar{\chi}^{-}_{1}(x)\right]^{n+m}}$$

$$\bar{\chi}^{B}_{n}(x) = \bar{\chi}^{+}_{1}(x) + (-1)^n\,\bar{\chi}^{-}_{1}(x)$$

HRG baseline

+ baryon, − antibaryon

- Parametrize the EoS by $\chi_2^B/\overline{\chi_2^B}$, χ_3^B/χ_1^B, χ_4^B/χ_2^B ratios at each energy

Subensemble acceptance method 3.0 R. Poberezhniuk, V.A. Kuznietsov, G. Pihan, VV, to appear

- Correction for baryon number conservation
    - Compute grand-canonically and perform correction at final step

$$P_{\rm ce}(X) \equiv P_{\rm gce}(X\,|\,B_{\rm tot}=B_0) = \frac{P_{\rm gce}(X, B_{\rm tot}=B_0)}{P_{\rm gce}(B_{\rm tot}=B_0)}$$

- Requires joint accepted proton (X) and 4π baryon (B) cumulants, summed over all CF cells

$$\kappa_2^{\rm ce}(X) = \kappa_2^X - \frac{(\kappa_{11}^{XB})^2}{\kappa_2^B}$$

$$\kappa_3^{\rm ce}(X) = \kappa_3^X - 3\,\frac{\kappa_{11}^{XB}\kappa_{21}^{XB}}{\kappa_2^B} + 3\,\frac{(\kappa_{11}^{XB})^2\kappa_{12}^{XB}}{(\kappa_2^B)^2} - \frac{(\kappa_{11}^{XB})^3\kappa_3^B}{(\kappa_2^B)^3}$$

Roadmap

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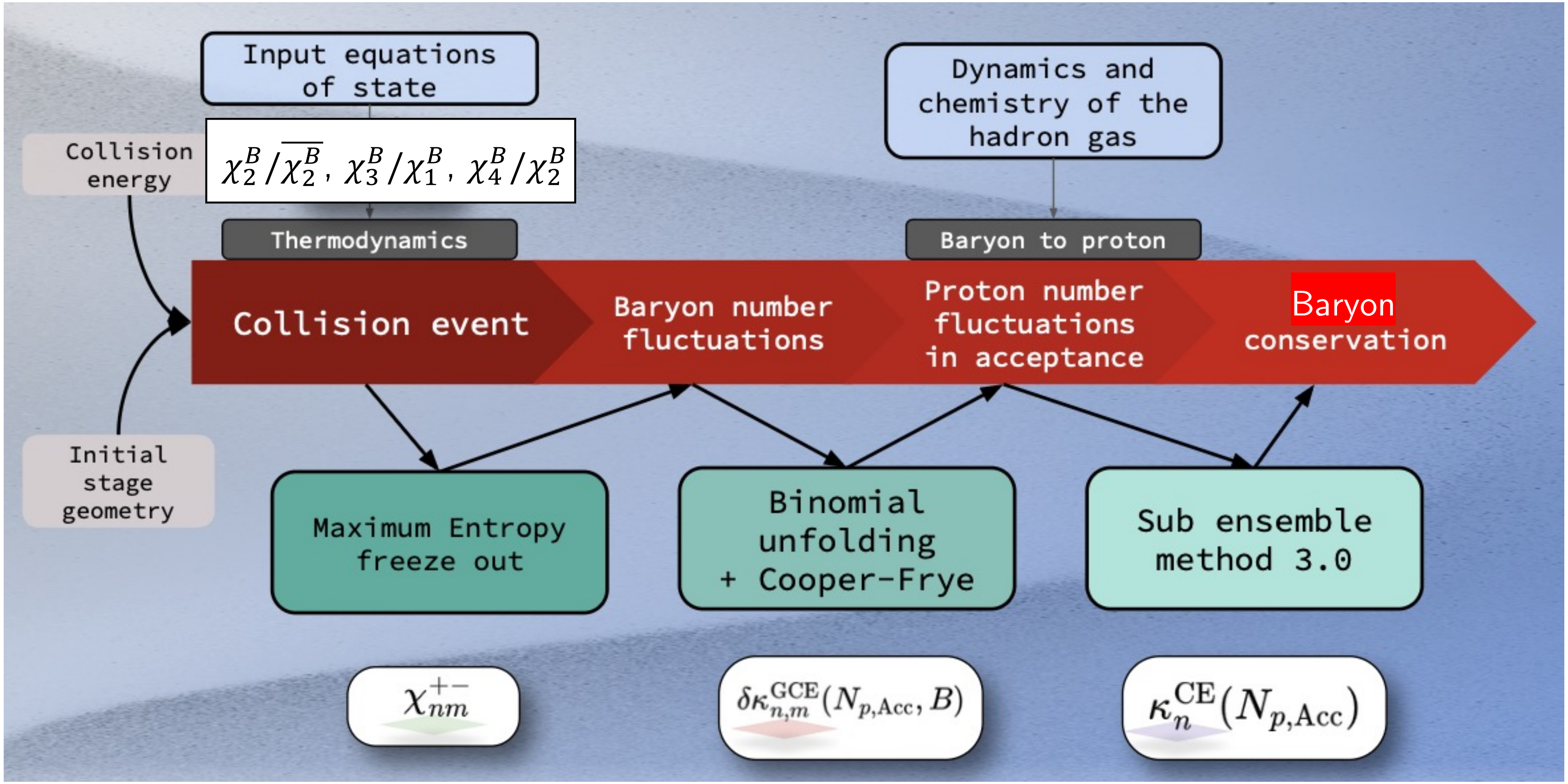
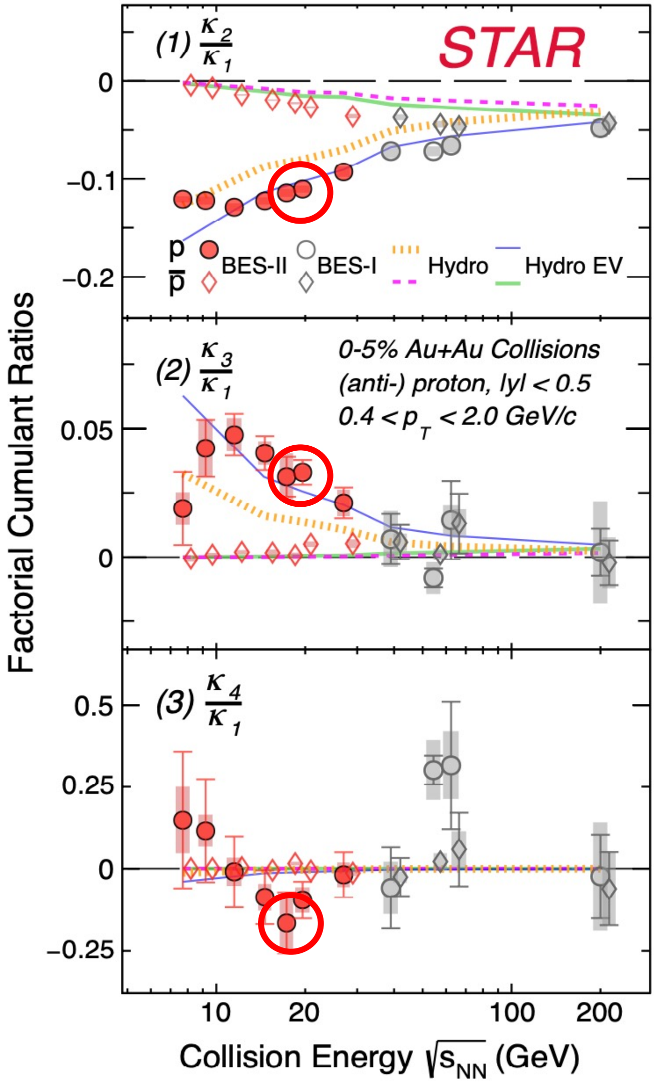
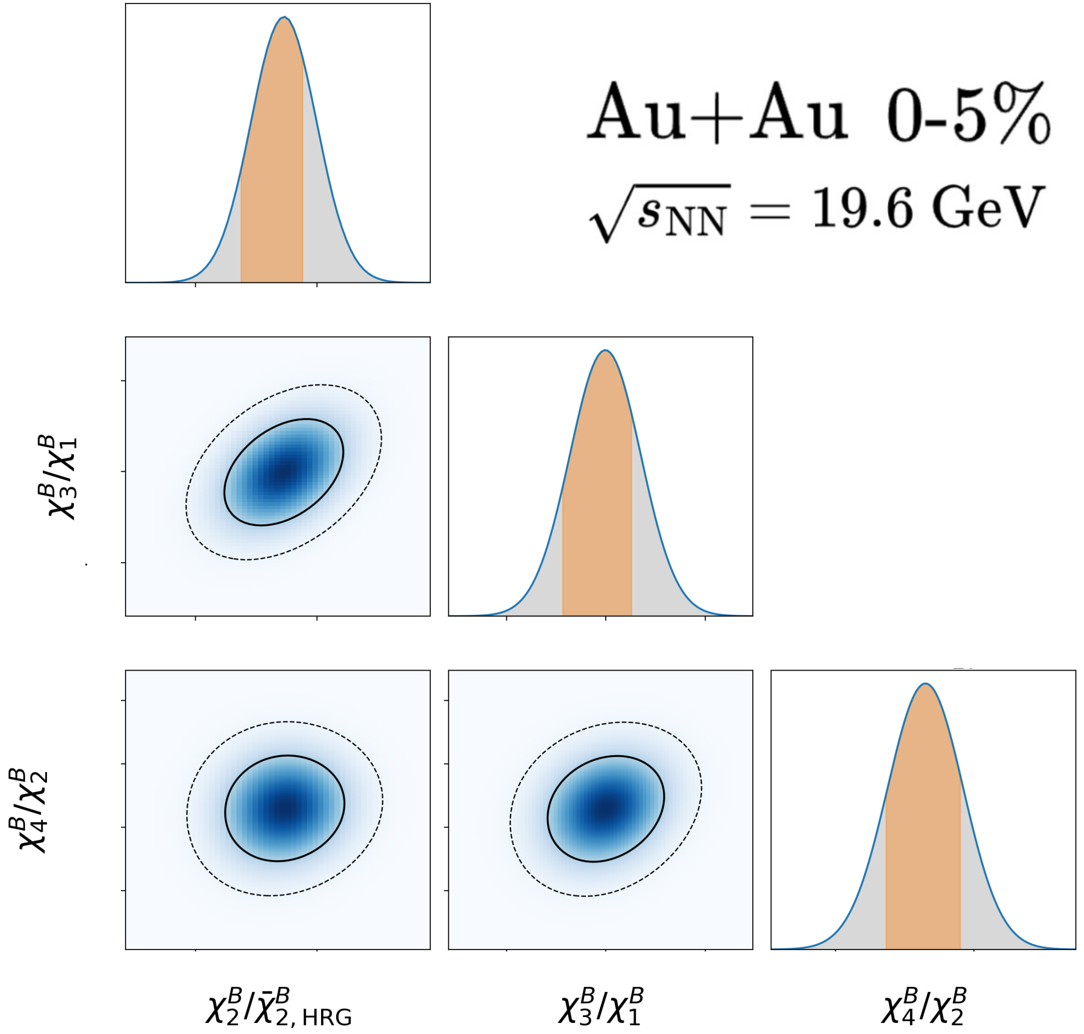


Adapted from G. Pihan

Bayesian inference

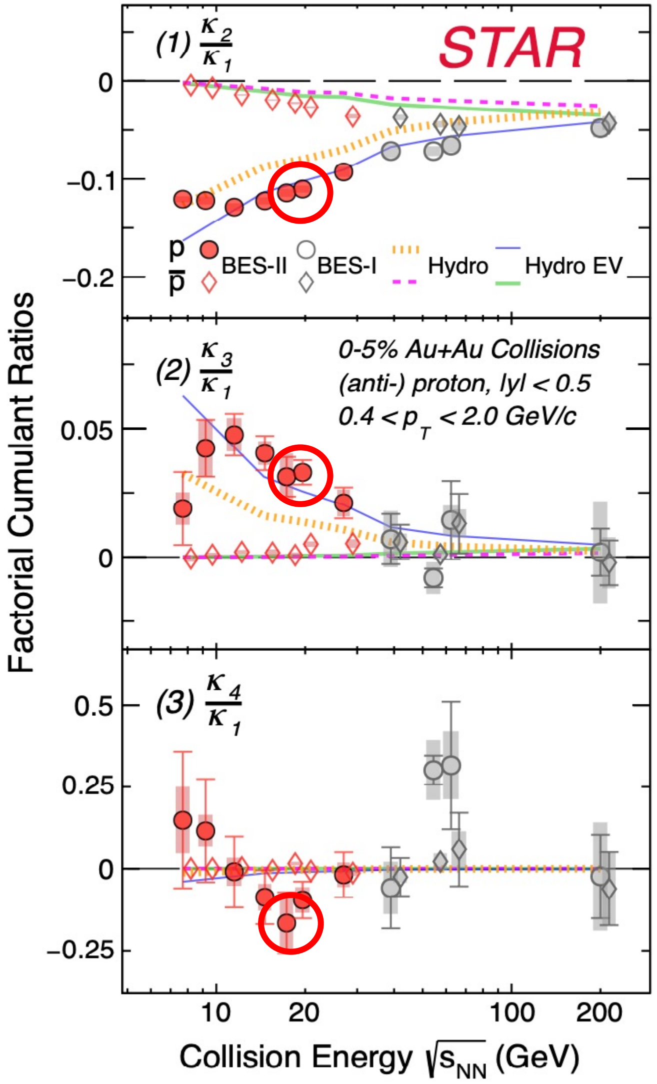


Unfolding

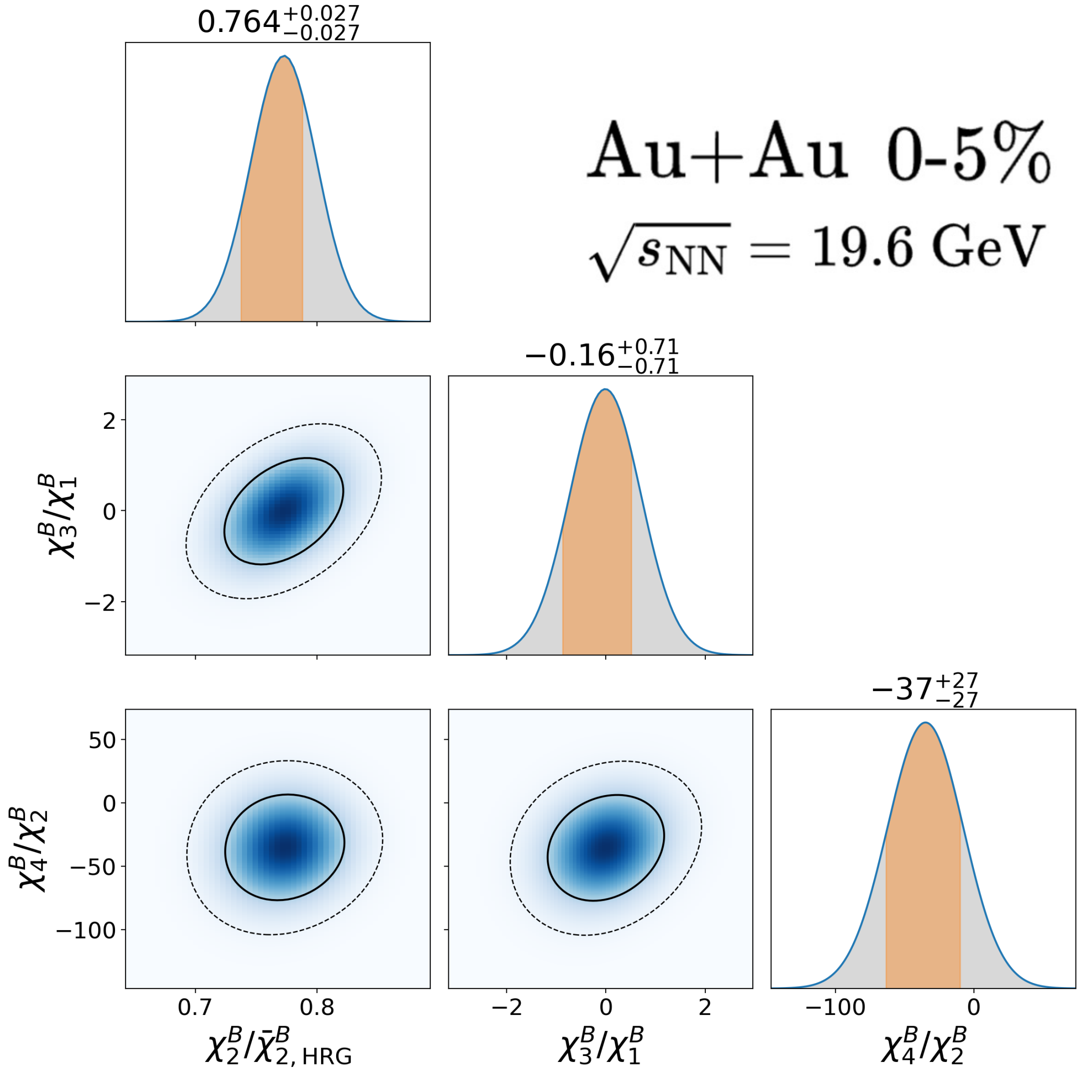


G. Pihan, R. Poberehzniuk, VV, to appear

Bayesian inference



Unfolding

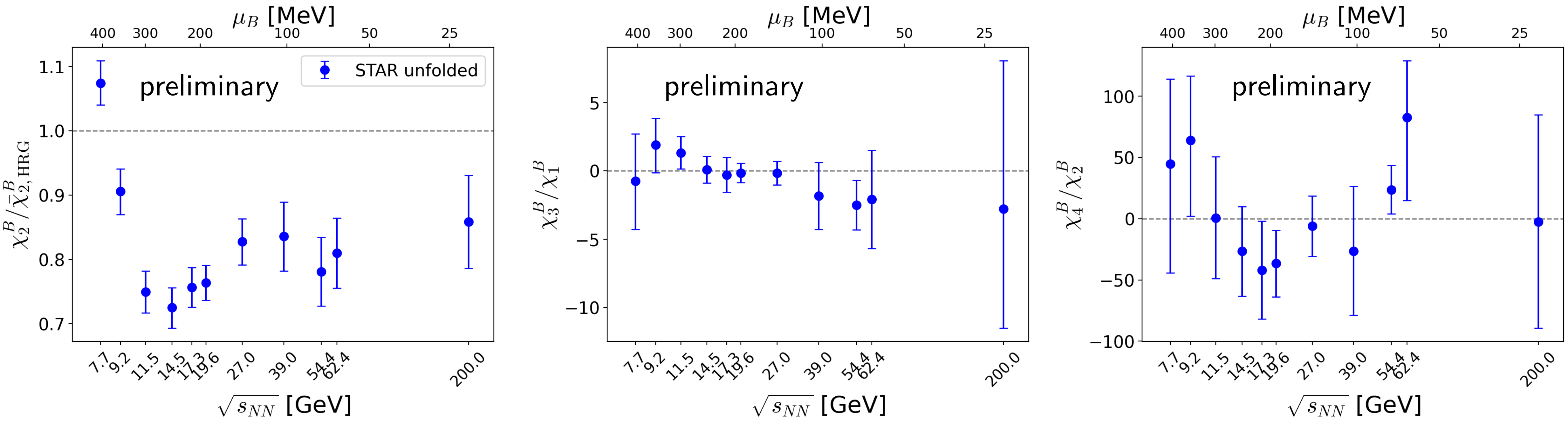


Au+Au 0-5%

$\sqrt{s_{\mathrm{NN}}} = 19.6$ GeV

G. Pihan, R. Poberehzniuk, VV, to appear

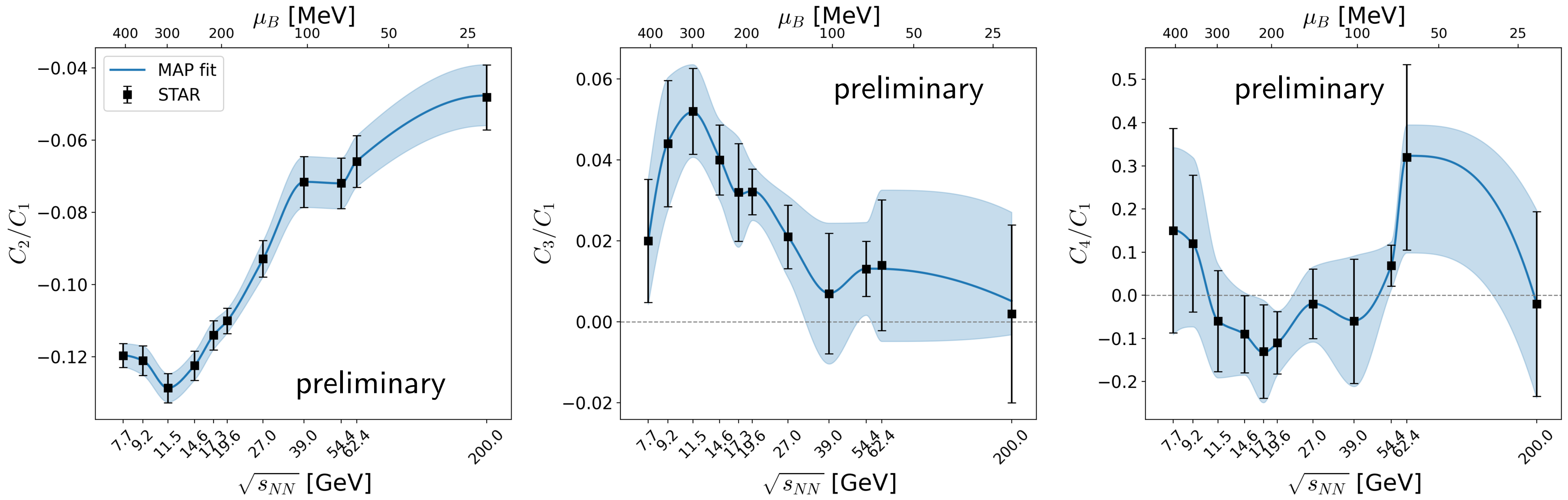
Extracted susceptibilities



- Tight constraints on χ_2^B relative to HRG
  - Switch from repulsion to attraction at lower energies?
- Uncertainties in 3rd and 4th order susceptibilities are huge
  - Intermediate steps (net-B to B$^+$, kinematic cuts, B$^+$→p) acts akin to efficiency correction
  - But reducing uncertainties in data would lead to reduced extraction errors
- Results appear weakly sensitive to e_{sw}, sensitivity to other hydro parameters to be studied

G. Pihan, R. Poberehzniuk, VV, to appear

Fit vs data



μ_B [MeV]
400 300 200 100 50 25
MAP fit
STAR
preliminary
C_2/C_1
C_3/C_1
C_4/C_1
7.7 9.2 11.5 14.6 17.3 19.6 27.0 39.0 54.4 62.4 200.0
$\sqrt{s_{NN}}$ [GeV]

Comparing to lattice QCD

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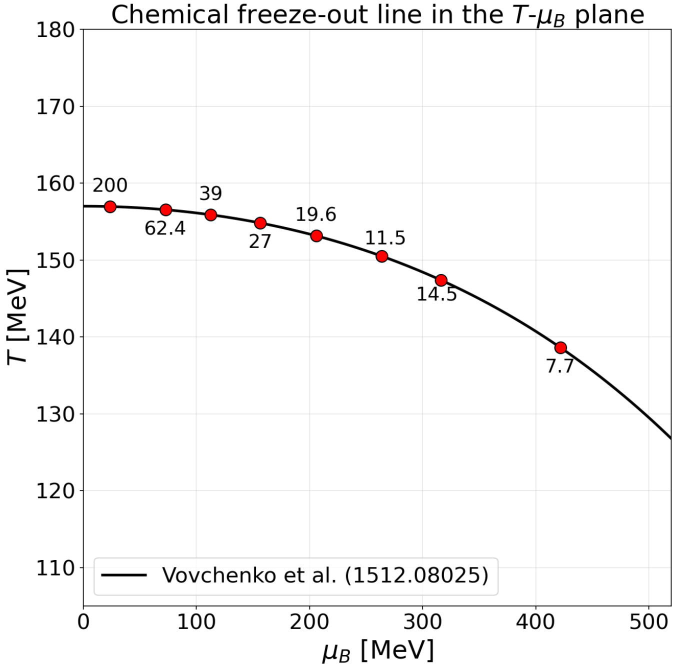
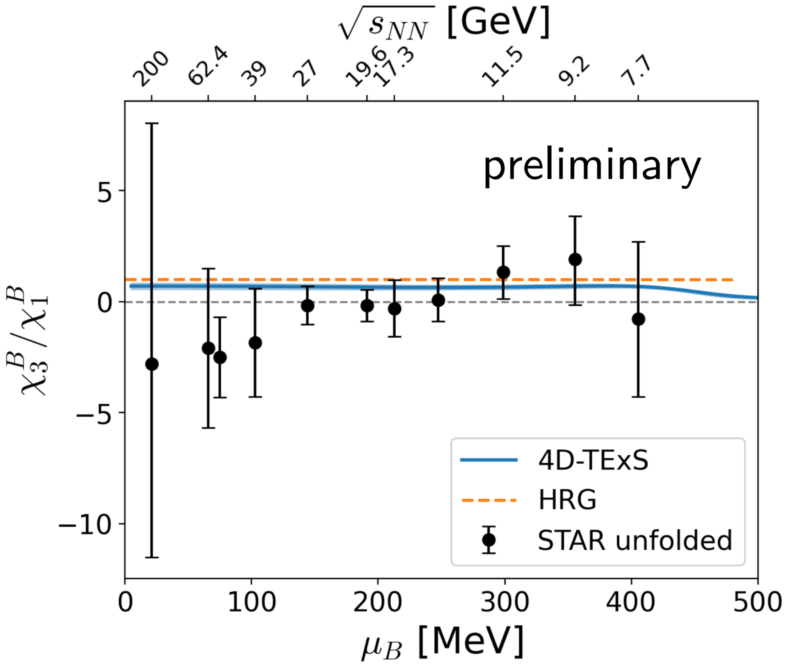
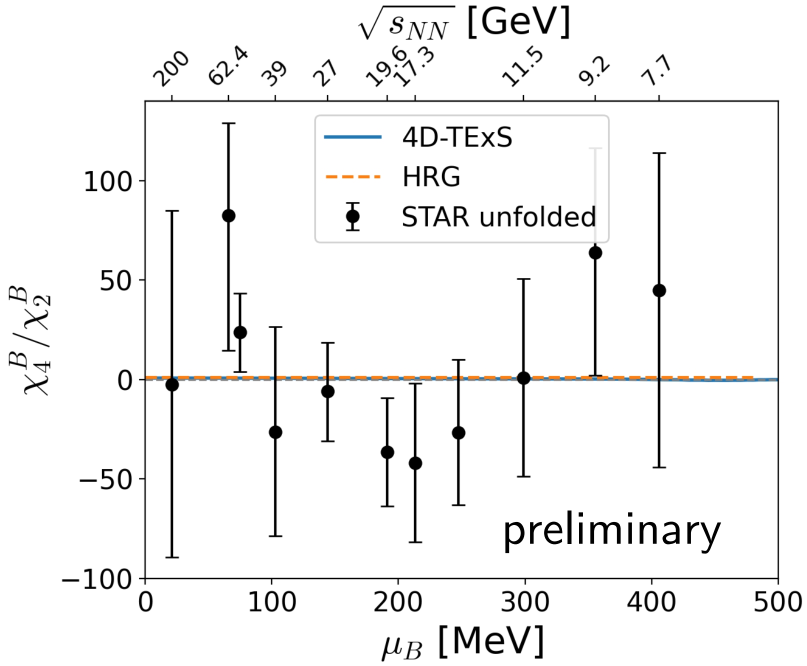
To map the results to QCD phase diagram use chemical freeze-out parametrization [VV et al., PRC 93, 064906 (2015)]







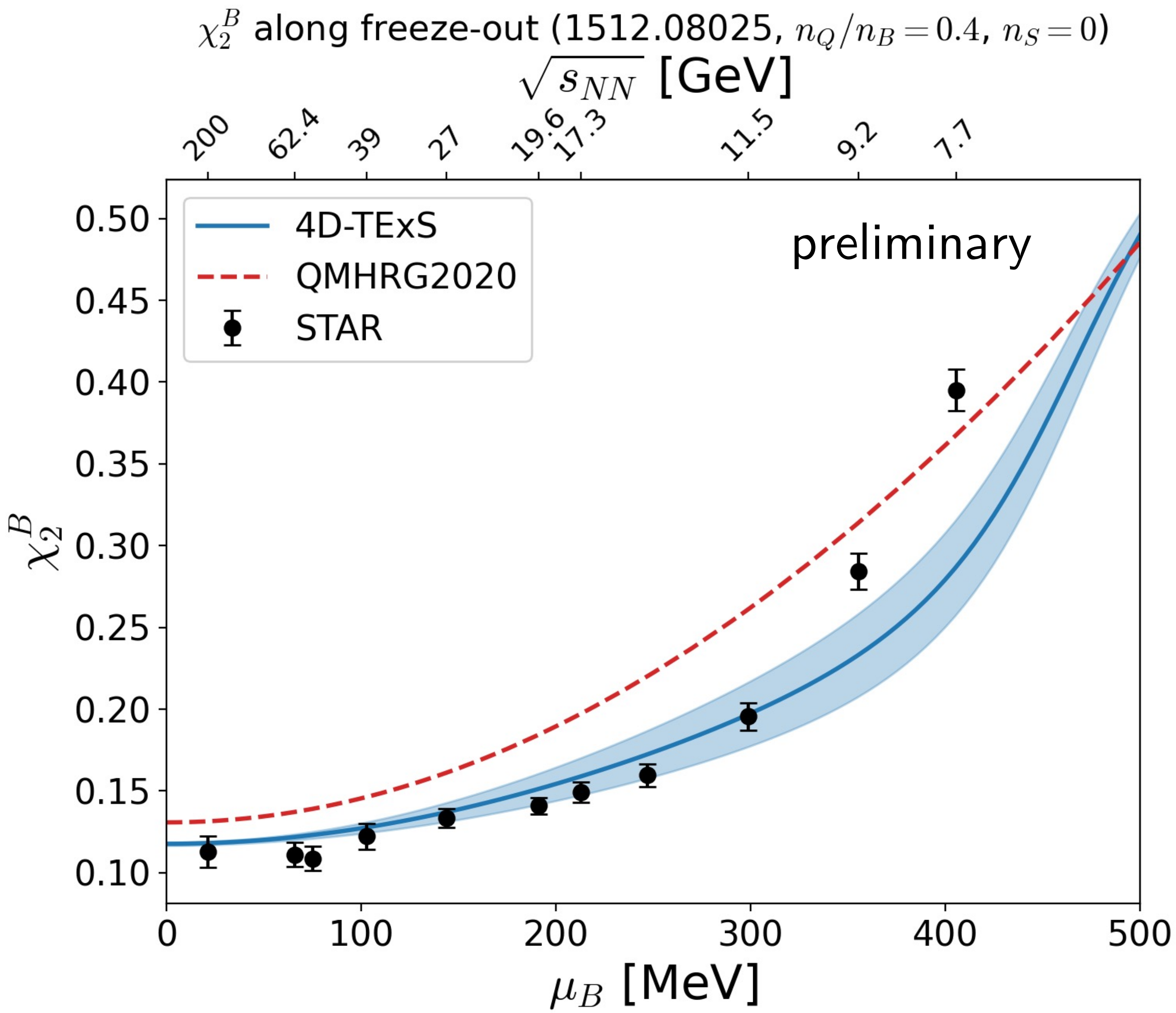
$\varepsilon_{SW} \approx 0.35 - 0.40\ \mathrm{GeV/fm^3}$

4D-TExS: [A. Abuali et al., Phys. Rev. D 112, 054502 (2025)]

- Lattice-based EoS at finite baryon density
- Computed under HIC conditions ($n_Q/n_B = 0.4, n_S = 0$)

G. Pihan, R. Poberehzniuk, VV, to appear

Baryon susceptibility χ_2^B



4D-TExS: [A. Abuali et al., Phys. Rev. D 112, 054502 (2025)]

- To obtain bare χ_2^B multiply by HRG value
  - QMHRG motivated by lattice studies at $\mu_B = 0$ [HotQCD, PRD 104, 074512 (2021)]
- Excellent agreement with lattice at $\mu_B < 300$ MeV
- Enhancement relative to lattice at $\mu_B > 300$ MeV

⚠ **Caveats:**

- Sensitive to the hadron list
- Sensitive to assumed freeze-out line
- Lattice EoS (4D-TExS) is an extrapolation (no truncation error included)

G. Pihan, R. Poberezhniuk, VV, to appear

Summary

- Recent developments allow calculations of proton cumulants that include
  - Realistic (3+1)D hydro background
  - EoS effects (χ_n^B) via maximum entropy freeze-out
  - Kinematic cuts via Cooper-Frye formula
  - Canonical baryon conservation effects

- Equilibrium baryon susceptibilities extracted from STAR data
  - Tight constraints on $\chi_2^B/\chi_{2,HRG}^B$
  - Agrees with lattice QCD at $\mu_B < 300$ MeV, enhancement at $\mu_B > 300$ MeV
  - Errors in extracted χ_3^B and χ_4^B are still too large with current data precision

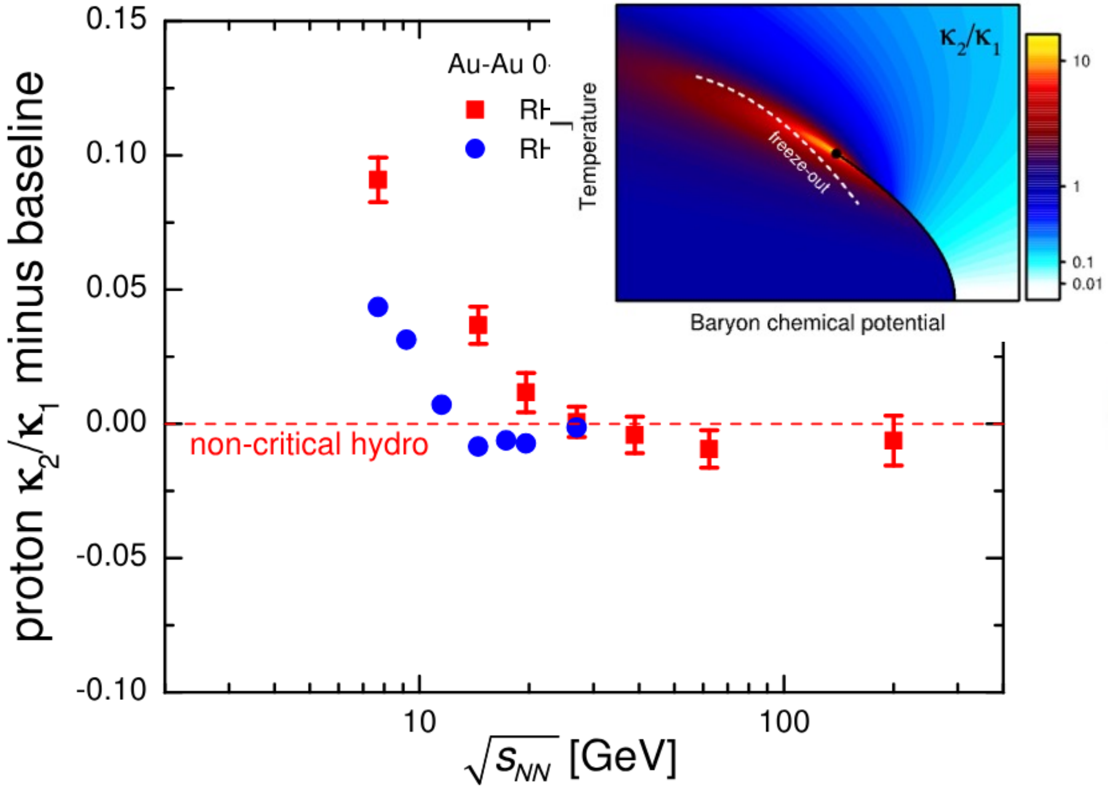
Outlook:

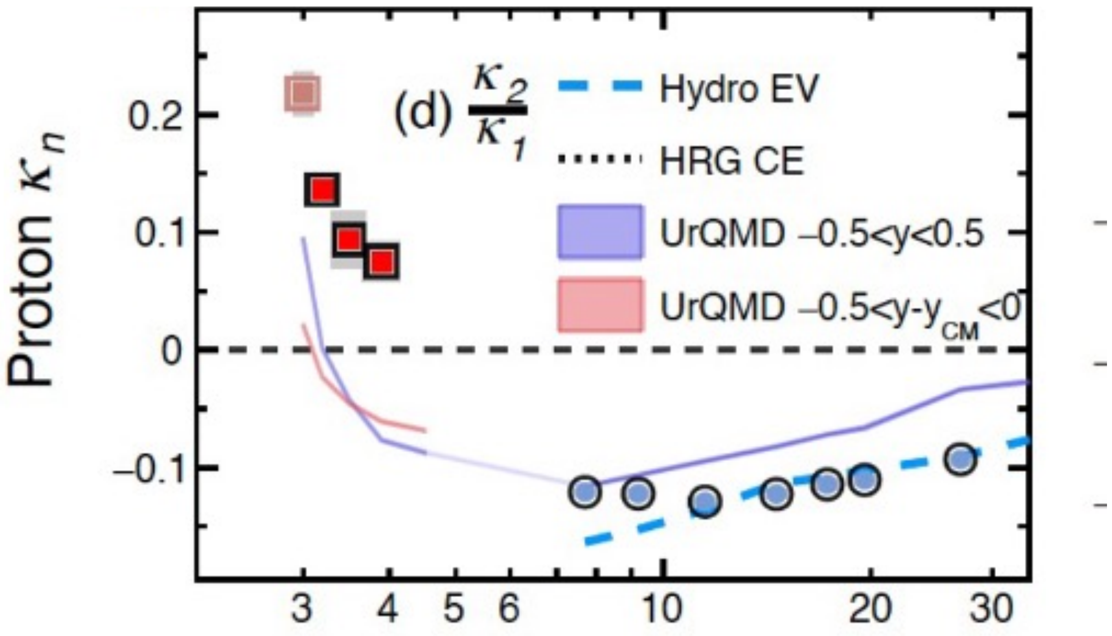
- More systematic studies of sensitivity to hydro parameters, hadron list, etc.
- Acceptance dependence, using net-proton cumulants

Additional slides

Subtracting the baseline

If $\kappa_2^{tot} \approx \kappa_2^{crit} + \kappa_2^{reg}$ try to isolate the critical part* by subtracting the baseline (here hydro EV)





Enhancement relative to the baseline at lower $\sqrt{s_{NN}}$ which continues at fixed target energies

*May be a useful quantity for finite-size scaling analysis compared to the bare κ_2/κ_1

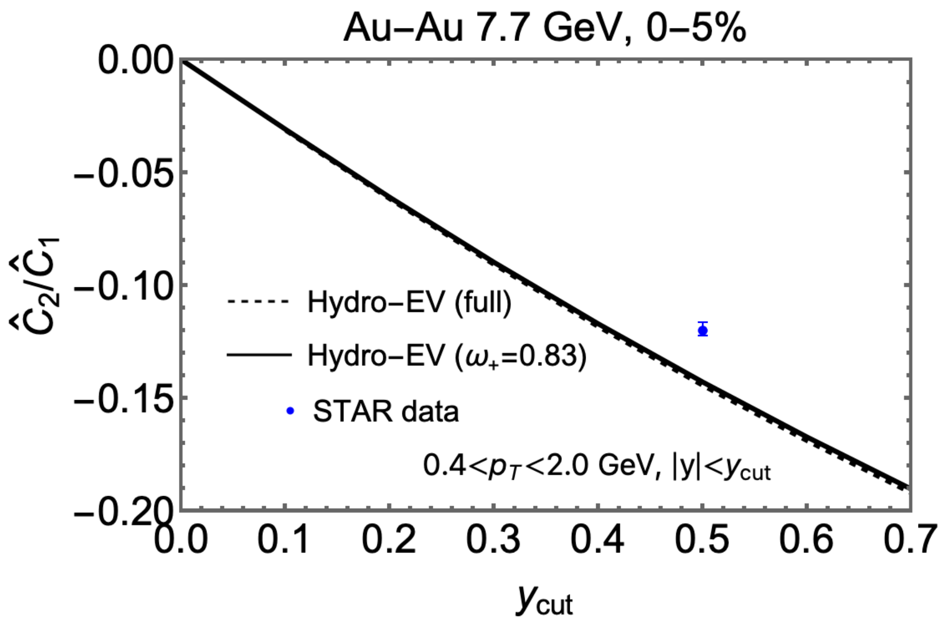
Simplified proton variance

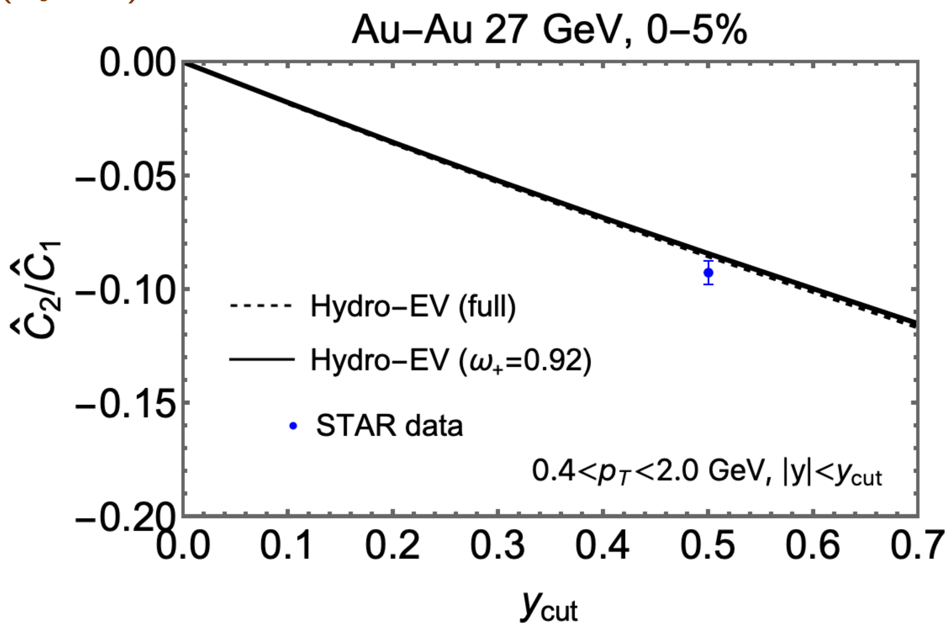
$$\frac{\kappa_2[p]}{\langle N_p \rangle} - 1 \approx (\omega_+ - 1)\frac{\langle p^2 \rangle_n}{\langle p \rangle_n} - \omega_+ \langle p \rangle_n \frac{\langle N_+ \rangle}{\langle N_+ \rangle + \langle N_- \rangle}$$

EoS

acceptance (hydro)

baryon (proton) fraction





Using average EV-HRG ω_+ shows excellent agreement with full numerical Hydro-EV calculation

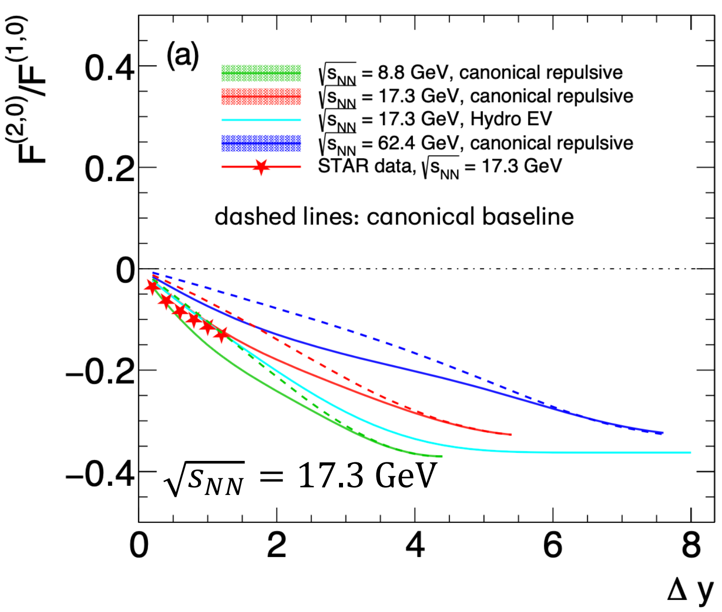
Attraction vs repulsion

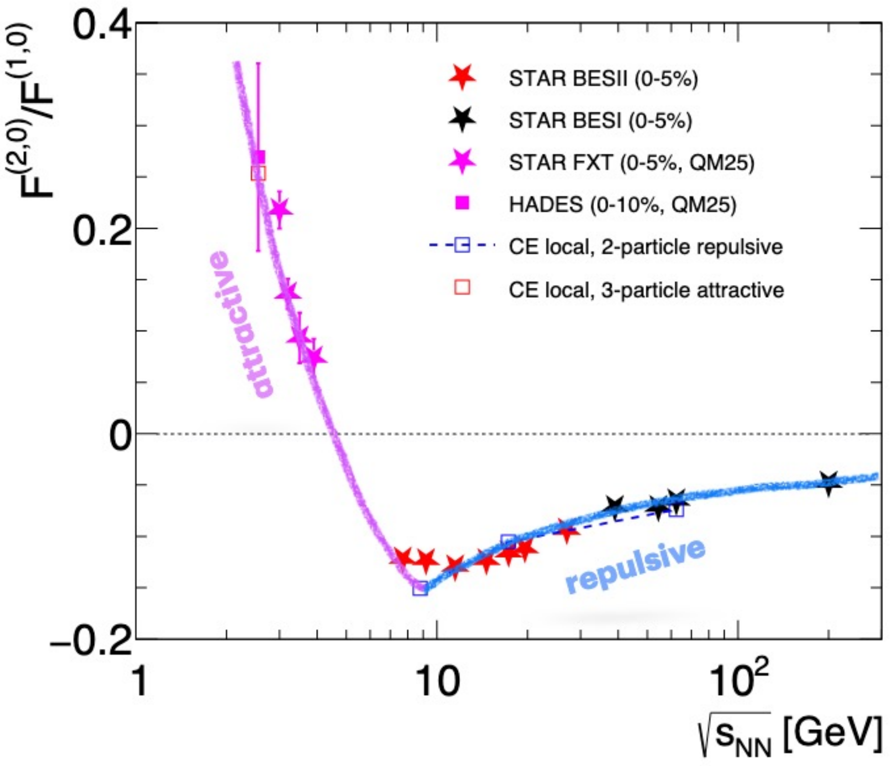
Friman, Redlich, Rustamov, arXiv:2508.18879

Recently, attractive and repulsive interactions implemented through a potential in rapidity

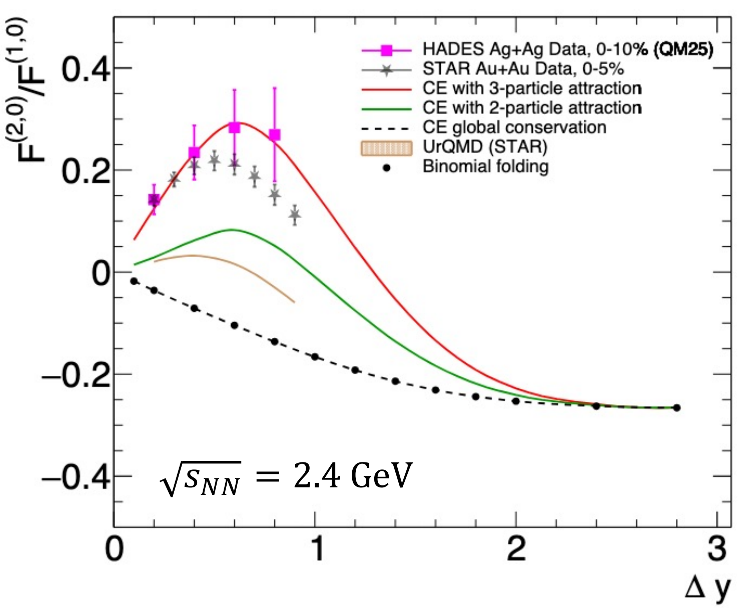
$$E_r(y_1, y_2) = \alpha_r e^{-|y_1 - y_2|/\rho_r} \qquad P(y_1, y_1) = \frac{e^{-E(y_1, y_2)}}{Z} \qquad E_a(y_1, y_2) = \alpha_a |y_1 - y_2|^{\beta_a}$$

repulsion





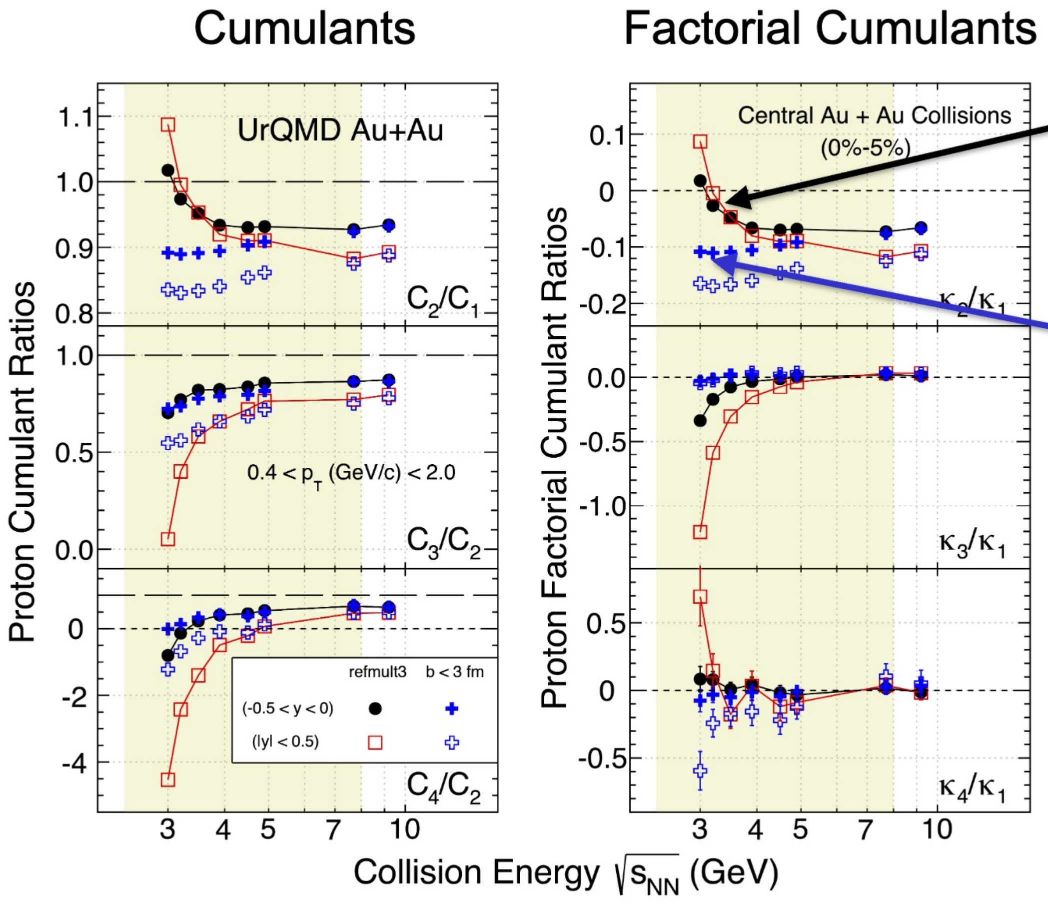
attraction



Interplay of repulsive (high $\sqrt{s_{NN}}$) and attractive (low $\sqrt{s_{NN}}$) interactions?

The (possible) culprit

X. Zhang, Y. Zhang, X. Luo, N. Xu, arXiv: 2506.18832



Fluctuating impact parameter
STAR centrality selection

Fixed impact parameter $(b < 3\ \mathrm{fm})$
minimal volume fluctuations.

N.B.: Centrality Bin Width Corrections applied to both

Possible culprit:
volume fluctuations/centrality selection

Two component model

2 sources: stopped and produced particles

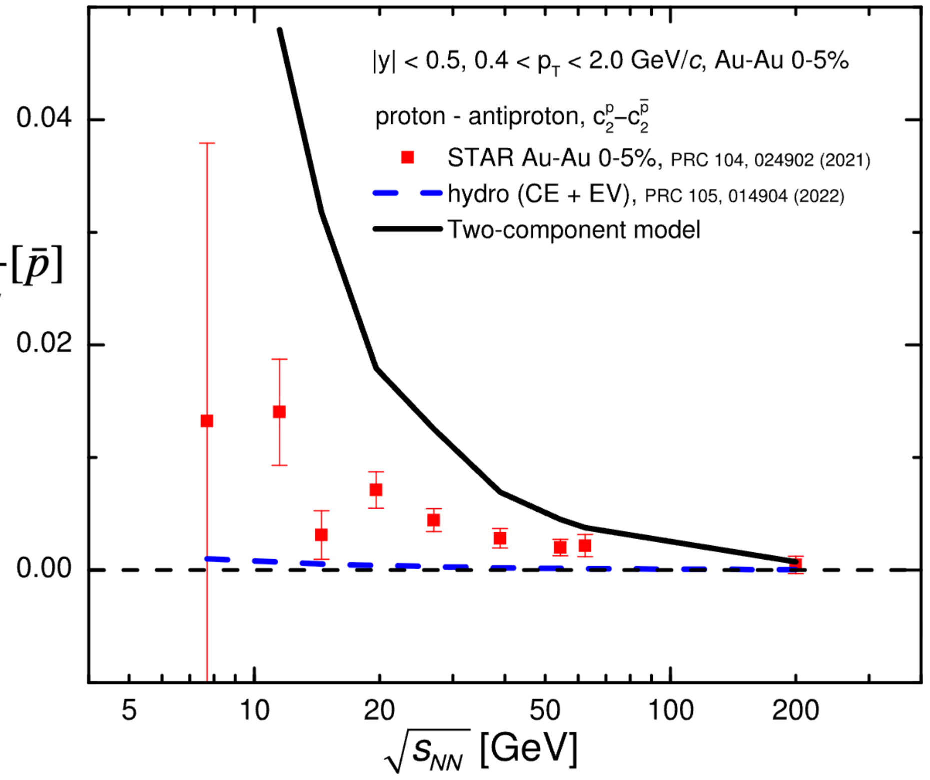
- All anti-protons are produced
- protons come from produced and stopped sources

$$N_p(\text{produced}) = \mathrm{N}_{\bar{\mathrm{p}}}$$

$$N_p = N_p(\text{stopped}) + \mathrm{N}_{\bar{\mathrm{p}}}$$

- Produced source: Thermal with zero net baryon number $\langle B - \bar{B} \rangle = 0$
- Stopped source: Follows binomial distribution

$$\frac{FC_2}{FC_1^2}[p] - \frac{FC_2}{FC_1^2}[\bar{p}]$$



Test for baseline: acceptance dependence of couplings

Bzdak et al. introduced reduced correlation functions – "couplings" [Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)]

$$\hat{c}_k = \frac{\hat{C}_k}{\langle N \rangle^k}$$

$$c_k = \frac{\int \rho_1(y_1) \cdots \rho_1(y_k) c_k(y_1, \ldots, y_k) dy_1 \cdots dy_k}{\int \rho_1(y_1) \cdots \rho_1(y_k) dy_1 \cdots dy_k}$$

integrated correlation function in rapidity

Long-range correlations in rapidity lead to acceptance-independent couplings, for example

- Global (not local) baryon conservation
  [Bzdak, Koch, Skokov, EPJC 77, 288 (2017); Bzdak, Koch, PRC 96, 054905 (2017)]
- $+$ volume fluctuations
  [Holzmann, Koch, Rustamov, Stroth, arXiv:2403.03598]

$$c_2 = -\frac{1}{B}, \qquad c_3 = \frac{2}{B^2}, \qquad c_4 = -\frac{6}{B^3}$$

$$\tilde{\hat{c}}_{i,j} = \hat{c}_{i,j} + \frac{\kappa_2[V]}{\langle V \rangle^2}, \quad \text{for} \quad i + j = 2.$$

all lead to

$$\frac{\hat{C}_k}{\langle N \rangle^k} = const.$$ at a given $\sqrt{s_{NN}}$ and $$\frac{\hat{c}_2^p}{\langle N_p \rangle^2} \approx \frac{\hat{c}_2^{\bar{p}}}{\langle N_{\bar{p}} \rangle^2} = -\frac{1}{\langle N_B + N_{\bar{B}} \rangle_{4\pi}}$$ at a given $\sqrt{s_{NN}}$

Can be tested *without* CBWC/volume fluctuations correction

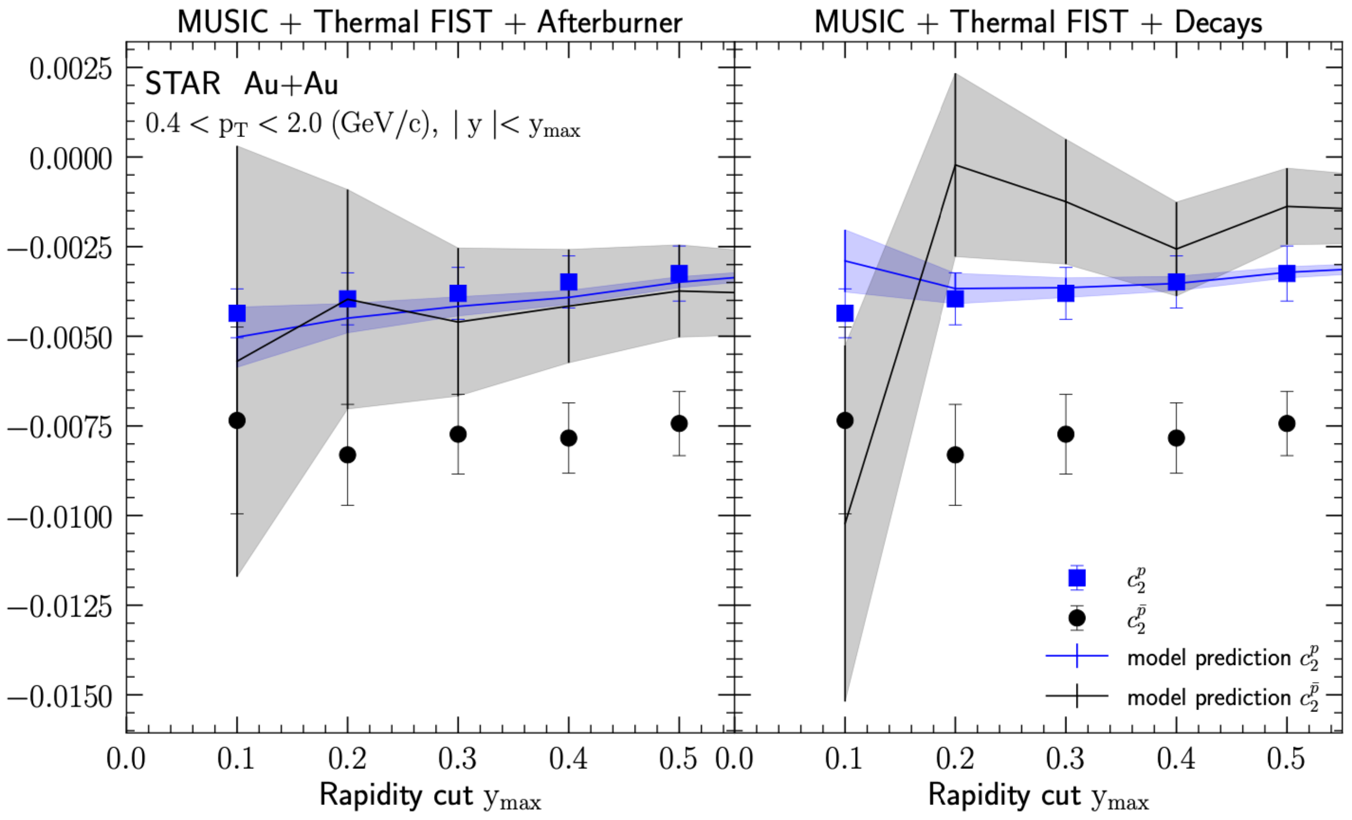
Bzdak, Koch, Vovchenko, arXiv:2503.16405

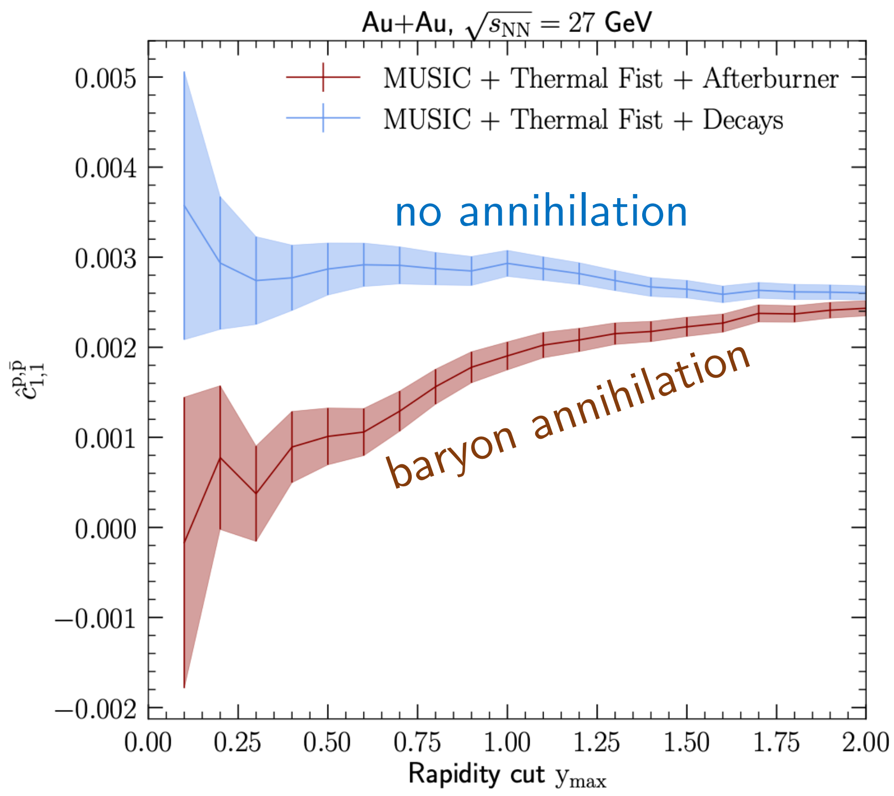
Scaled factorial cumulants and baryon annihilation

Extending Hydro EV to incorporate hadronic phase (UrQMD)

Au-Au, $\sqrt{s_{NN}} = 27$ GeV G. Pihan, VV, in progress

Covariance $c_{11}^{p\bar{p}}$



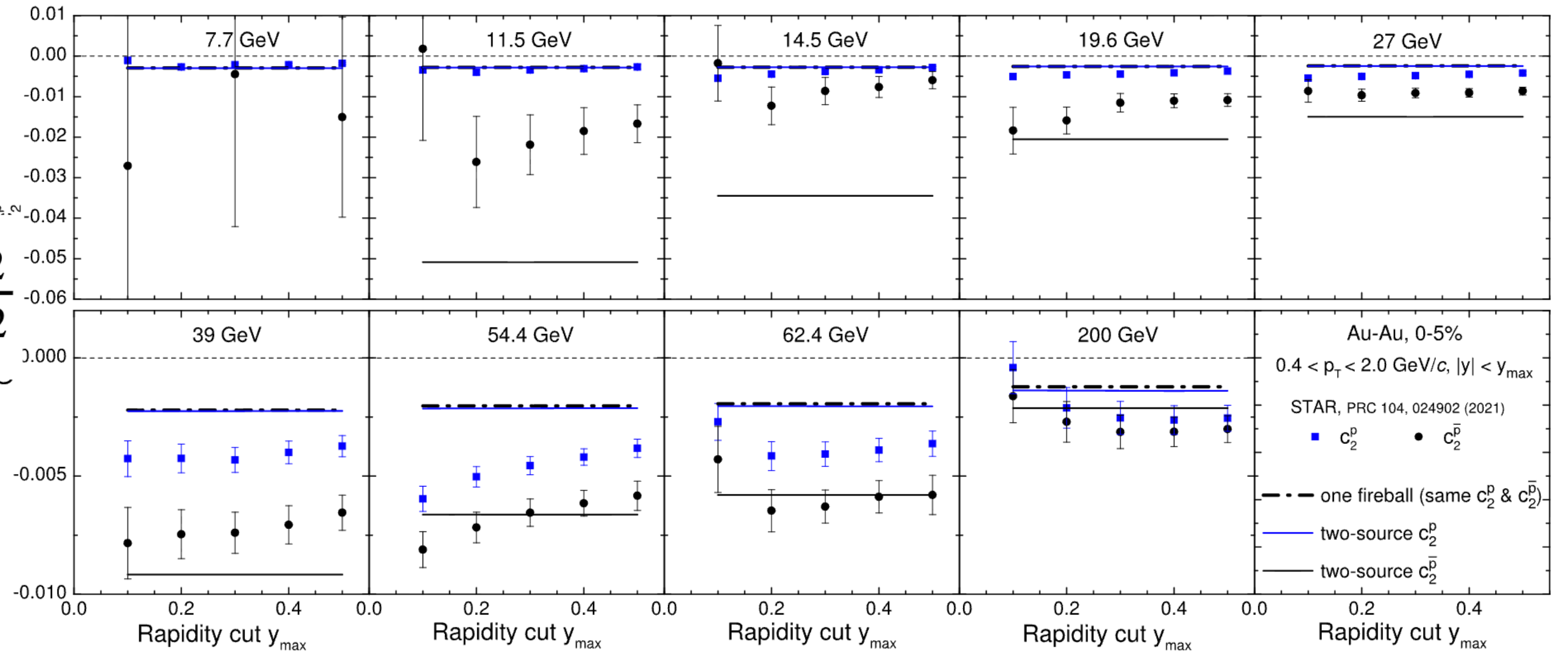


- Hadronic phase appears unlikely to resolve the antiproton puzzle (more statistics needed)
- Acceptance dependence of proton-antiproton covariance shows clear effect of hadronic phase

Test for baseline: BES-I data

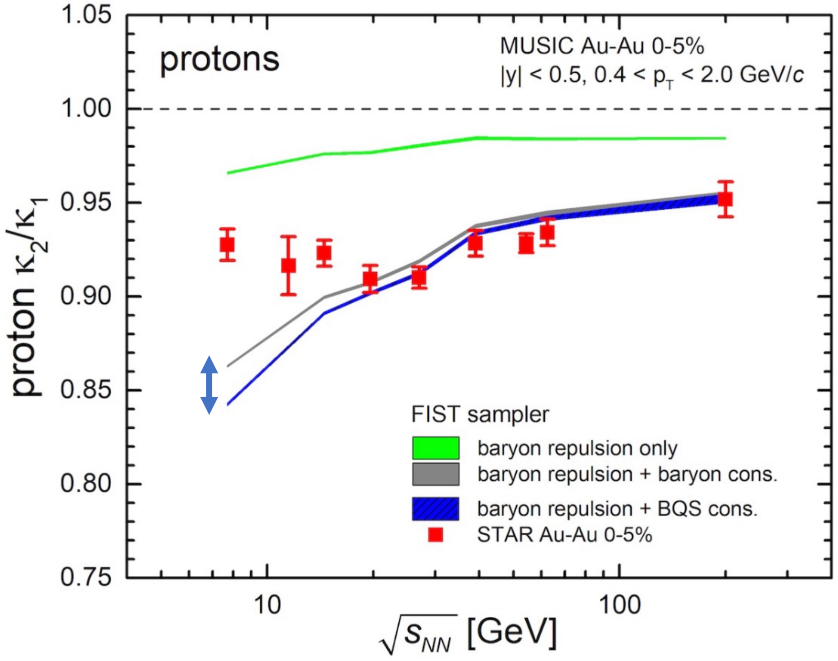
- $\dfrac{FC_2}{FC_1^2}$ more or less constant
- $\dfrac{FC_2[p]}{FC_1^2[p]} \neq \dfrac{FC_2[\bar{p}]}{FC_1^2[\bar{p}]}$
- baseline OK for protons
- No good for anti-protons ??
- How will it look with BES II data?

$\dfrac{FC_2}{FC_1^2}$



(Some of) Missing pieces at lower energies

Charge conservation



Light nuclei

particle	multiplicity	uncertainty	Ref.
p	77.6	± 2.4	[15]
$p+n \to {}^2H$	28.7	± 0.8	[15]
$p+2n \to {}^3H$	8.7	± 1.1	[15]
$p+p+n \to {}^3He$	4.6	± 0.3	[15]
p (bound)	46.5	± 1.5	[15]
π^+	9.3	± 0.6	[18]
π^-	17.1	± 1.1	[18]
K^+	$5.98\,10^{-2}$	$\pm 6.79\,10^{-3}$	[16]
K^-	$5.6\,10^{-4}$	$\pm 5.96\,10^{-5}$	[16]
Λ	$8.22\,10^{-2}$	${}^{+5.2}_{-9.2}\,10^{-3}$	[17]

TABLE I. Preliminary particle yields measured by the HADES collaboration at SIS18 accelerator, $\sqrt{s_{NN}} = 2.4$ GeV, for 10% most central Au-Au collisions. Protons bound in nuclei can be accounted all as free under the assumption that the nuclei are formed after kinetic freeze-out. The data compilation is extracted from [1].

HADES preliminary

Nuclear liquid-gas