QCD phase diagram with constant entropy contours and fluctuations in heavy-ion collisions

Volodymyr Vovchenko (University of Houston)

YITP workshop "Buenas Ideas on the QCD Phase Diagram", Kyoto, Japan

May 26, 2026

Thanks to: M. Kahangirwe, Y. Fujimoto, G. Pihan, R. Poberezhniuk, H. Shah, ...

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

UNIVERSITY OF HOUSTON

U.S. DEPARTMENT OF ENERGY | Office of Science

QCD under extreme conditions

UNIVERSITY OF HOUSTON

What we know

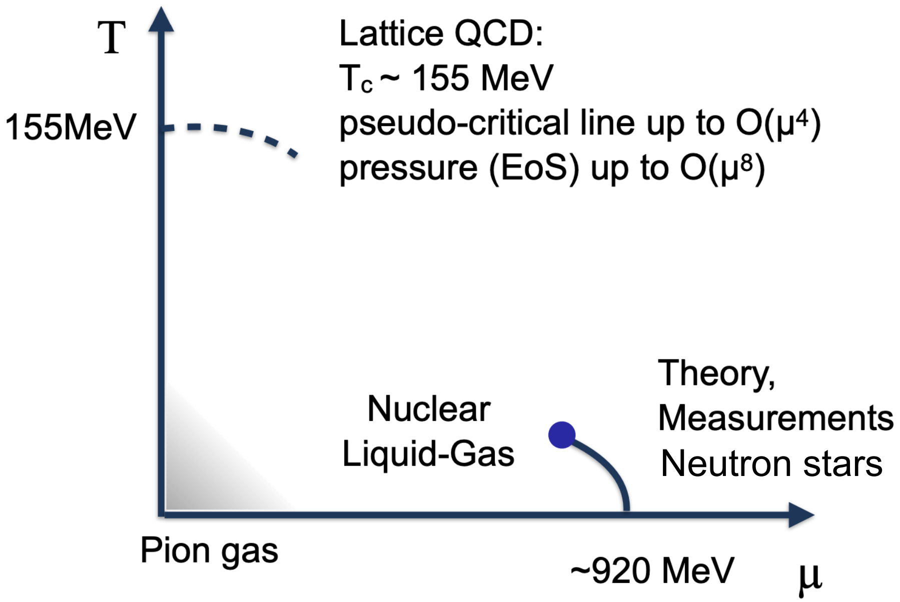


Figure courtesy of V. Koch

What we hope to know

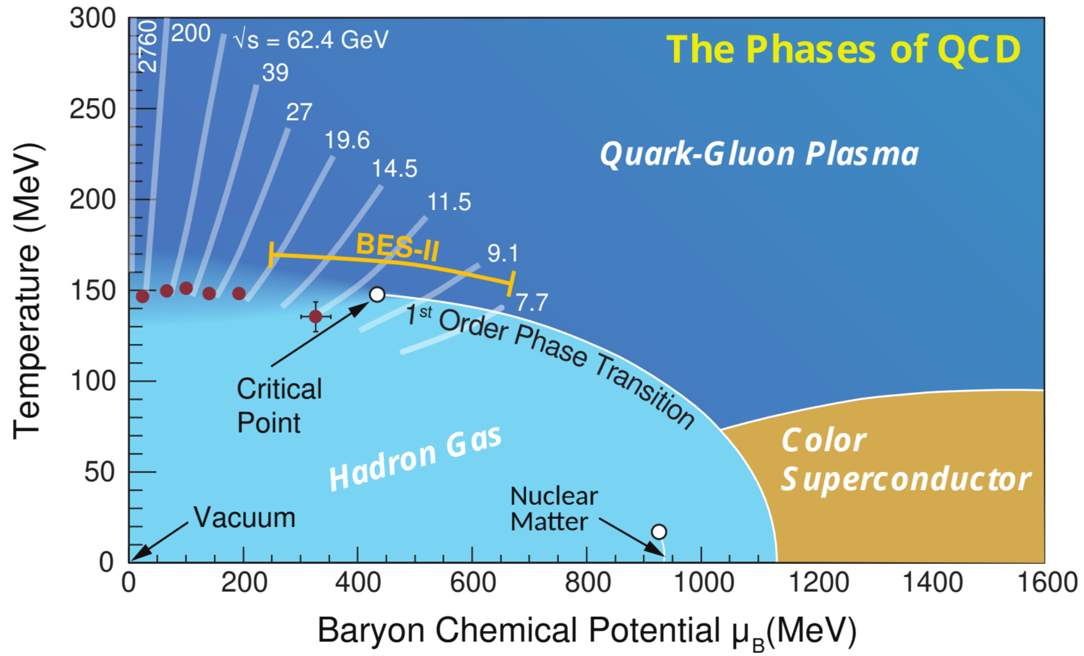


Figure from Bzdak et al., Phys. Rept. '20 & 2015 US Nuclear Long Range Plan

- Dilute hadron gas at low T & μ_B due to confinement, quark-gluon plasma high T & μ_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured
- Chiral crossover at $\mu_B = 0$ which may turn into a *first-order phase transition* at finite μ_B

QCD under extreme conditions

UNIVERSITY OF HOUSTON

What we know

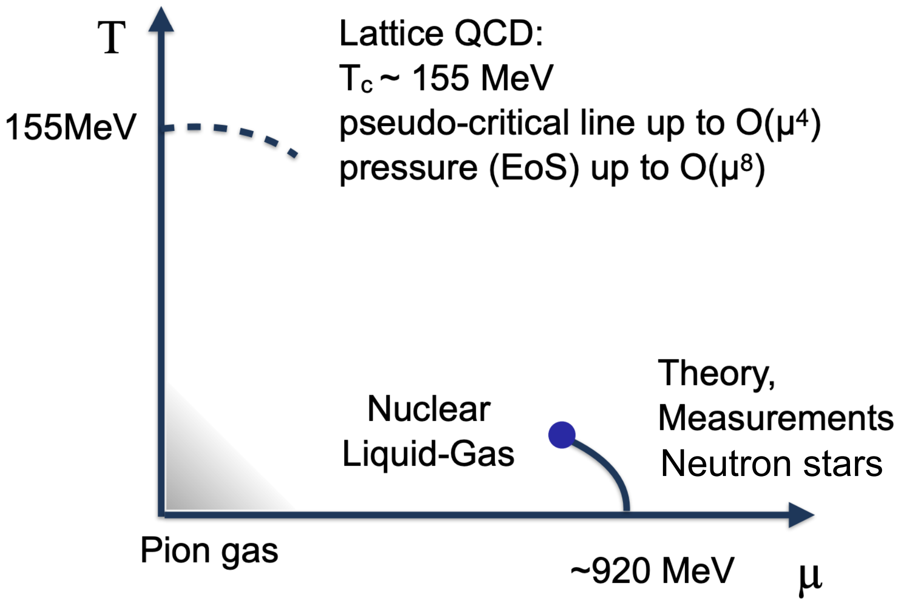
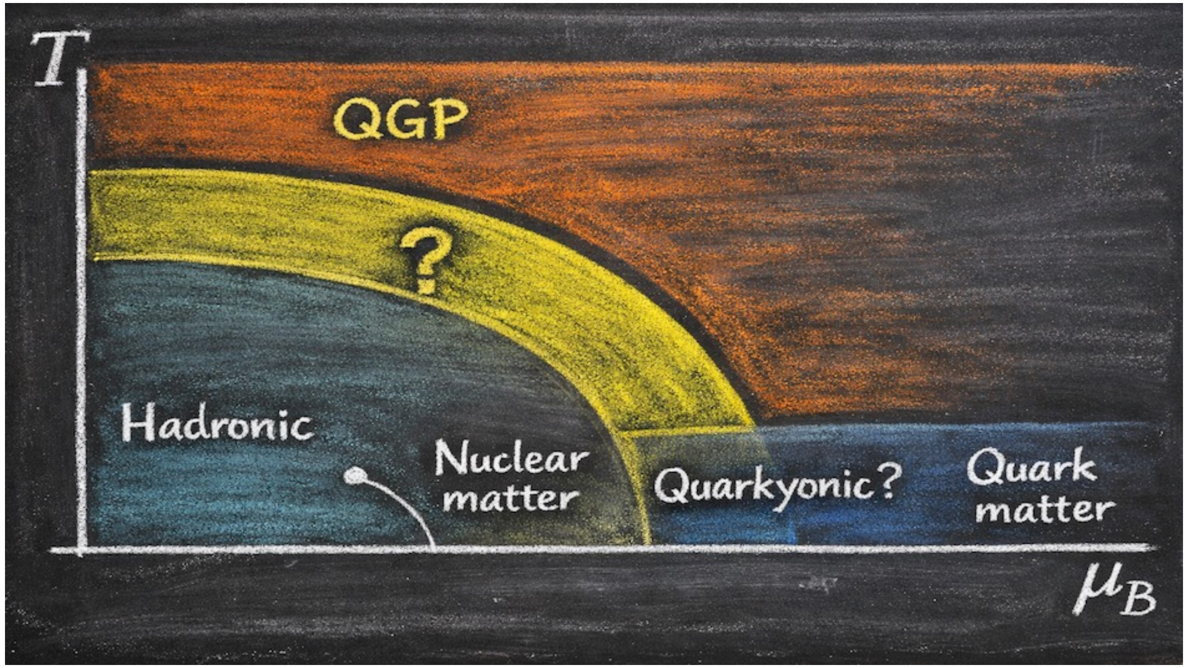


Figure courtesy of V. Koch

What we hope to know

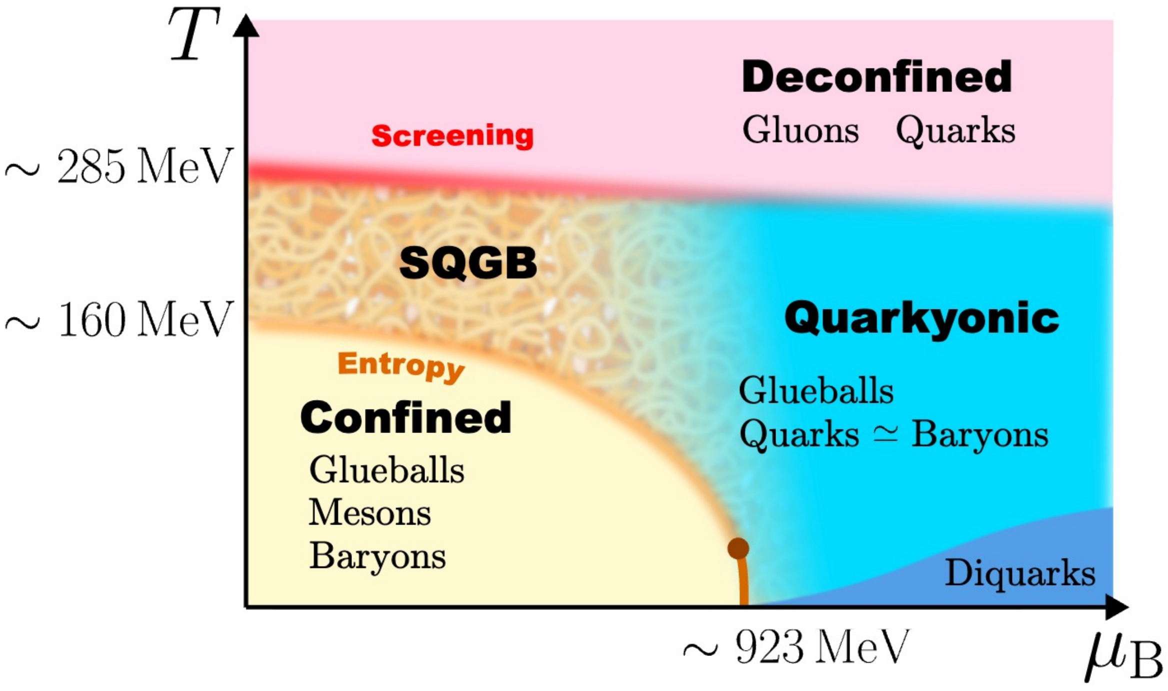


Conference poster "Buenas Ideas on the QCD Phase Diagram"

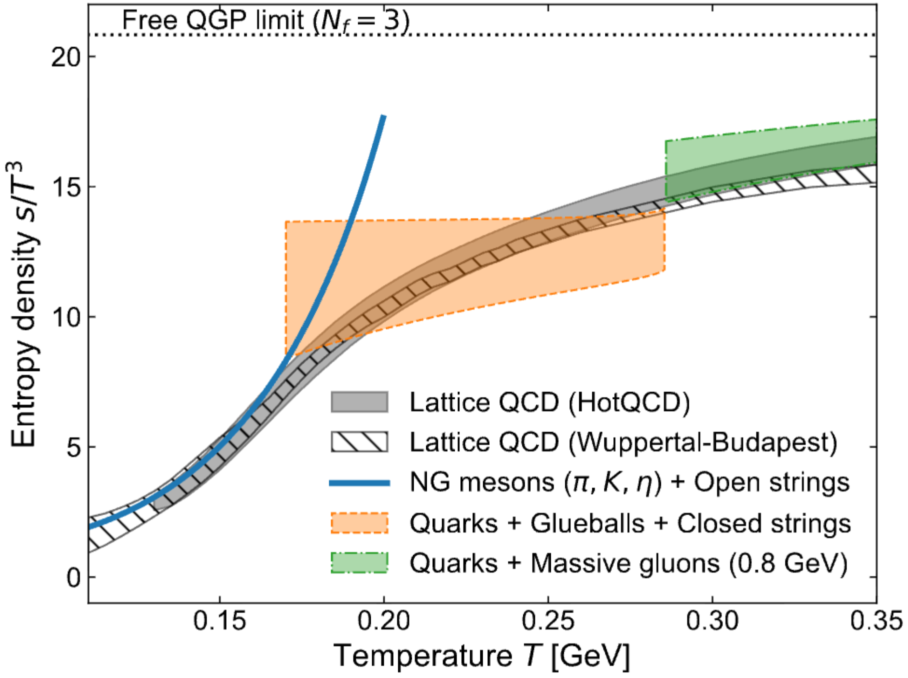
- Dilute hadron gas at low T & $\mu_{\rm B}$ due to confinement, quark-gluon plasma high T & $\mu_{\rm B}$
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured
- Chiral crossover at $\mu_B = 0$ which may turn into a *first-order phase transition* at finite μ_B
- Or perhaps a different state of matter between $T_{\rm c}$ and (2-3) $T_{\rm c}$?

Glozman et al. EPJ '24+, Fujimoto et al. PRD '25, ...

SQGB



Fujimoto, Fukushima, Hidaka, McLerran, PRD 112, 074006 (2025)



- At low T the HRG description well approximated by a gas of confining strings with a single Hagedorn-like temperature $T_H \sim 280 - 340$ MeV
- Works for glueballs [H. Meyer, PRD 80, 051502 (2009)], mesons [Marczenko et al., PRD 112, 096010, (2025)], baryons [Fujimoto, PRD 113, 054044 (2026)] and perhaps other flavors [Marczenko et al., 2603.28668].
- IM dilepton measurements point to $T_{IM} \approx T_H \approx 300$ MeV across broad range of energies

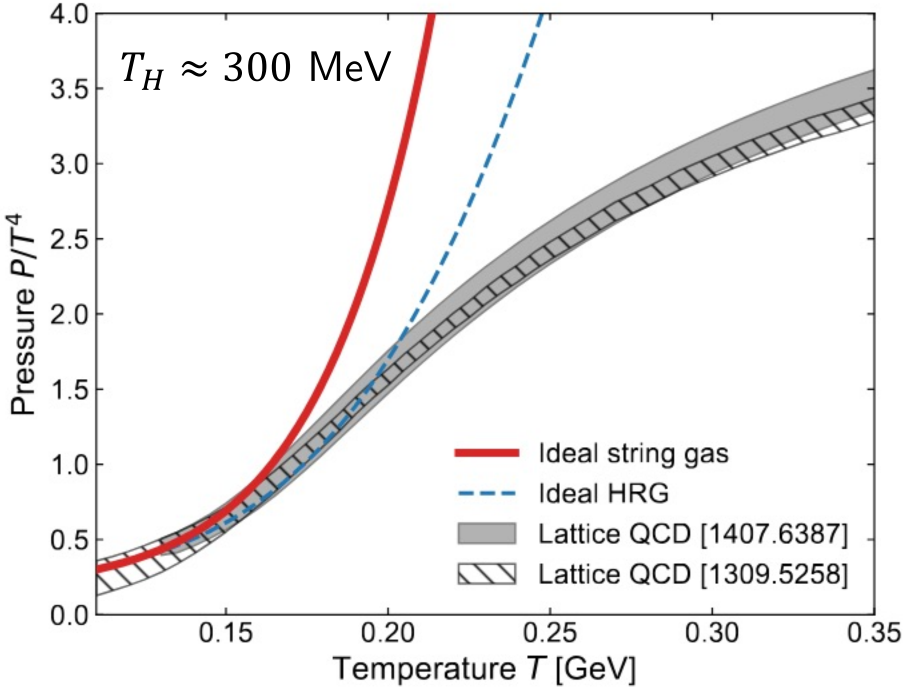
Stoecker, Satarov, VV, arXiv:2603.24219

HRG as a gas of confining strings with interaction

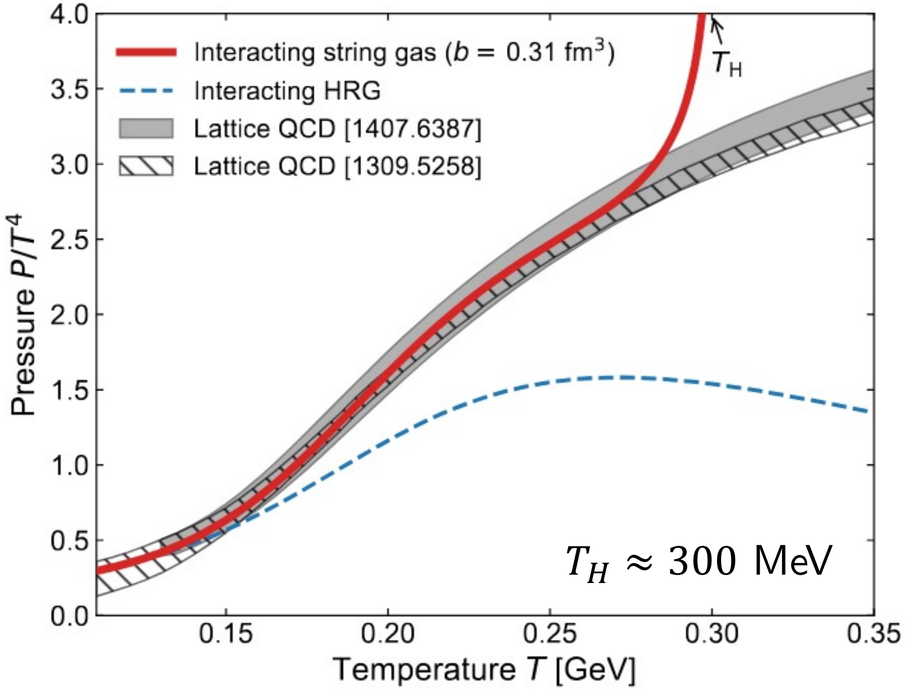
- Strings with excluded volume, a la EV/VDW-HRG model [VV, Gorenstein, Stoecker, PRL 118, 182301 (2017)]

$$P(T) = T n_{\rm id}(T) \exp\left(-\frac{bP}{T}\right) \qquad T\phi_i(T) = \int_{m_{{\rm str},i}}^{\infty} dm\, \rho_{\rm str}(m) d_{{\rm str},i} P(m) \qquad \rho_{\rm str}(m) = \frac{\sqrt{2\pi}}{6T_{\rm H}} \left(\frac{m}{T_{\rm H}}\right)^{-3/2} e^{m/T_{\rm H}}$$

Ideal string gas



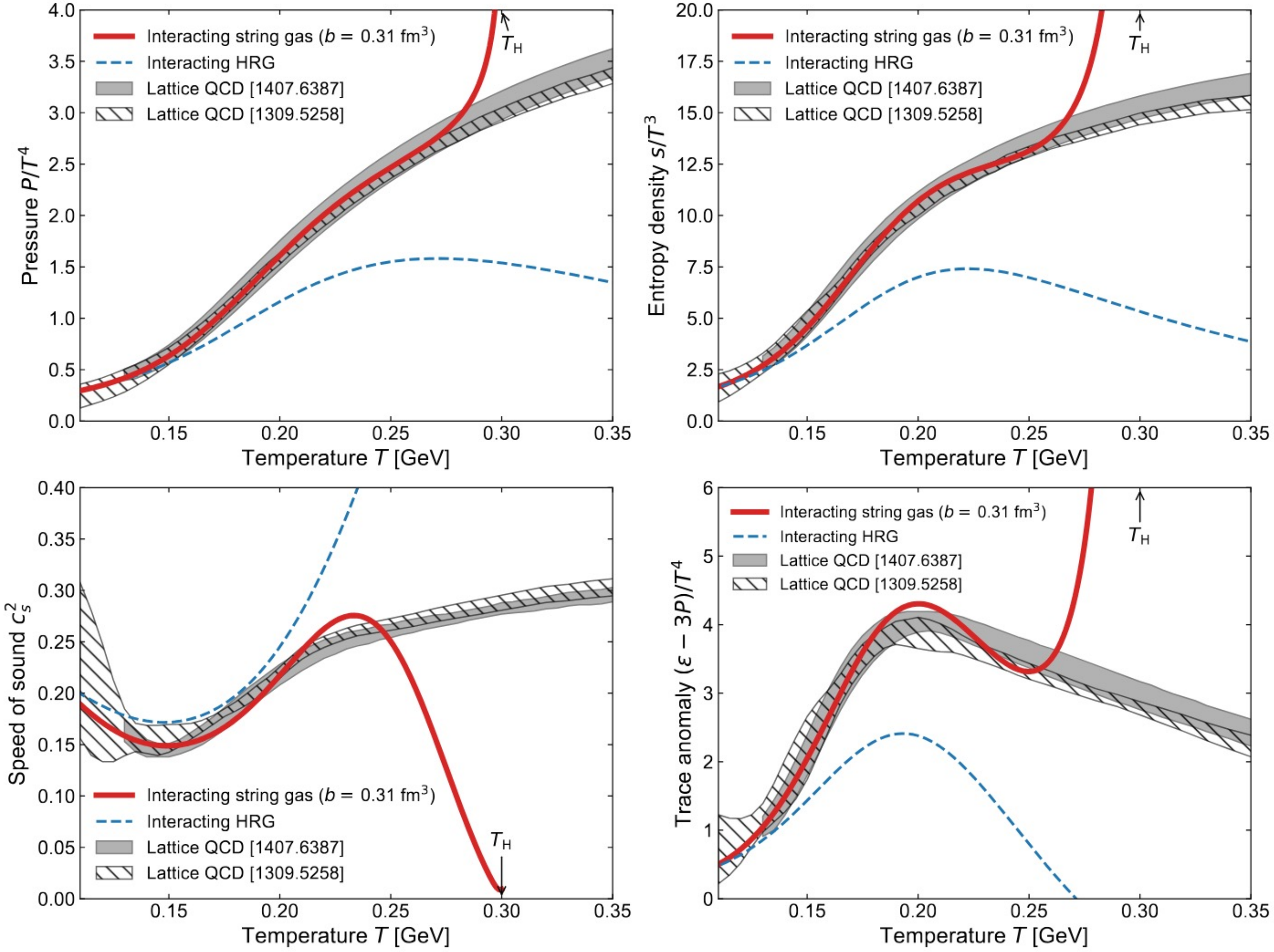
Interacting string gas



Description of thermodynamics lowers the Hagedorn temperature to $T_H \approx 300$ MeV relative to $T_H \approx 340$ MeV from PDG fits

Fujimoto, VV, to appear

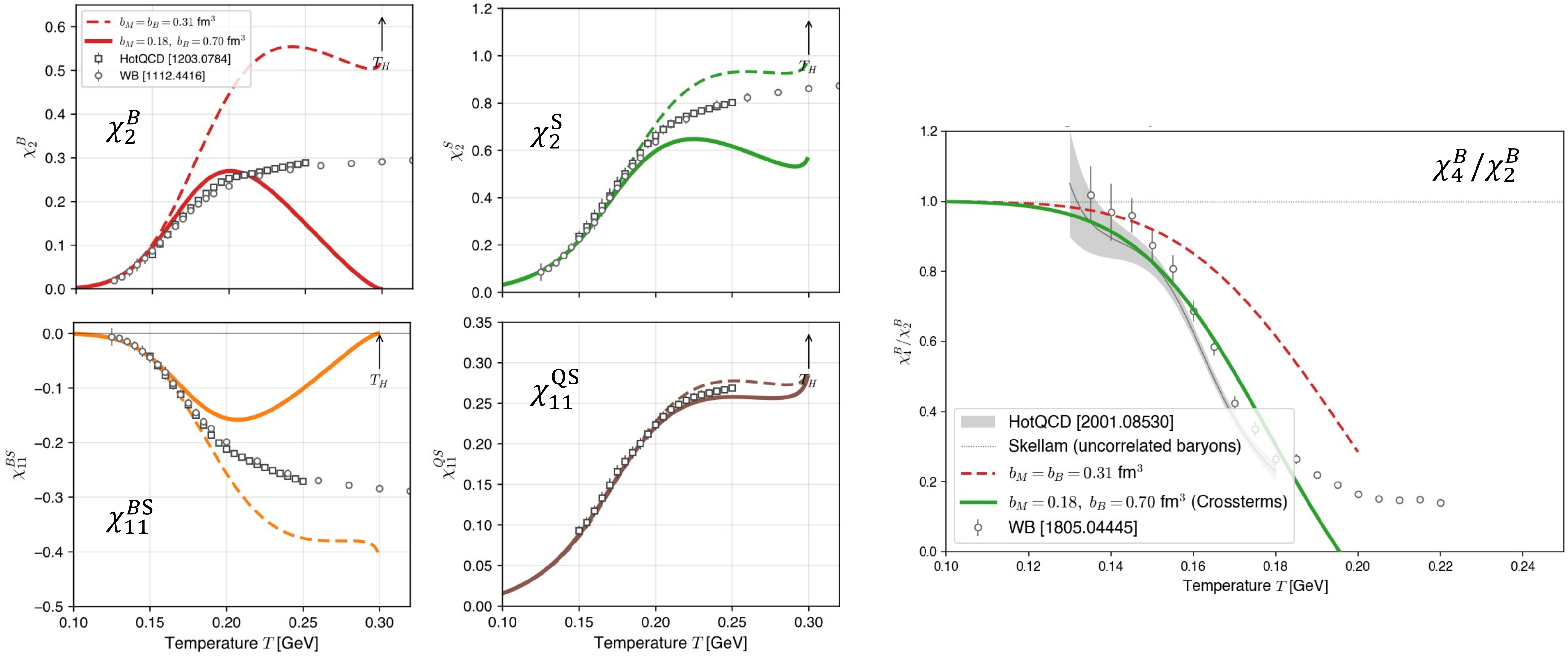
Thermodynamics of an interacting string gas



Uniform description of thermodynamics up to $T \approx 200 - 220$ MeV

Fujimoto, VV, to appear

Susceptibilities of an interacting string gas



Susceptibilities reveal a preference for a larger EV parameter for baryonic strings

Fujimoto, VV, to appear

QCD phase diagram with constant entropy contours

H. Shah, M. Hippert, J. Noronha, C. Ratti, VV, PRC 113, L012201 (2026) & 2601.08823

H. Shah, **T. Gyure**, et al., *to appear*

Critical point predictions from some years ago

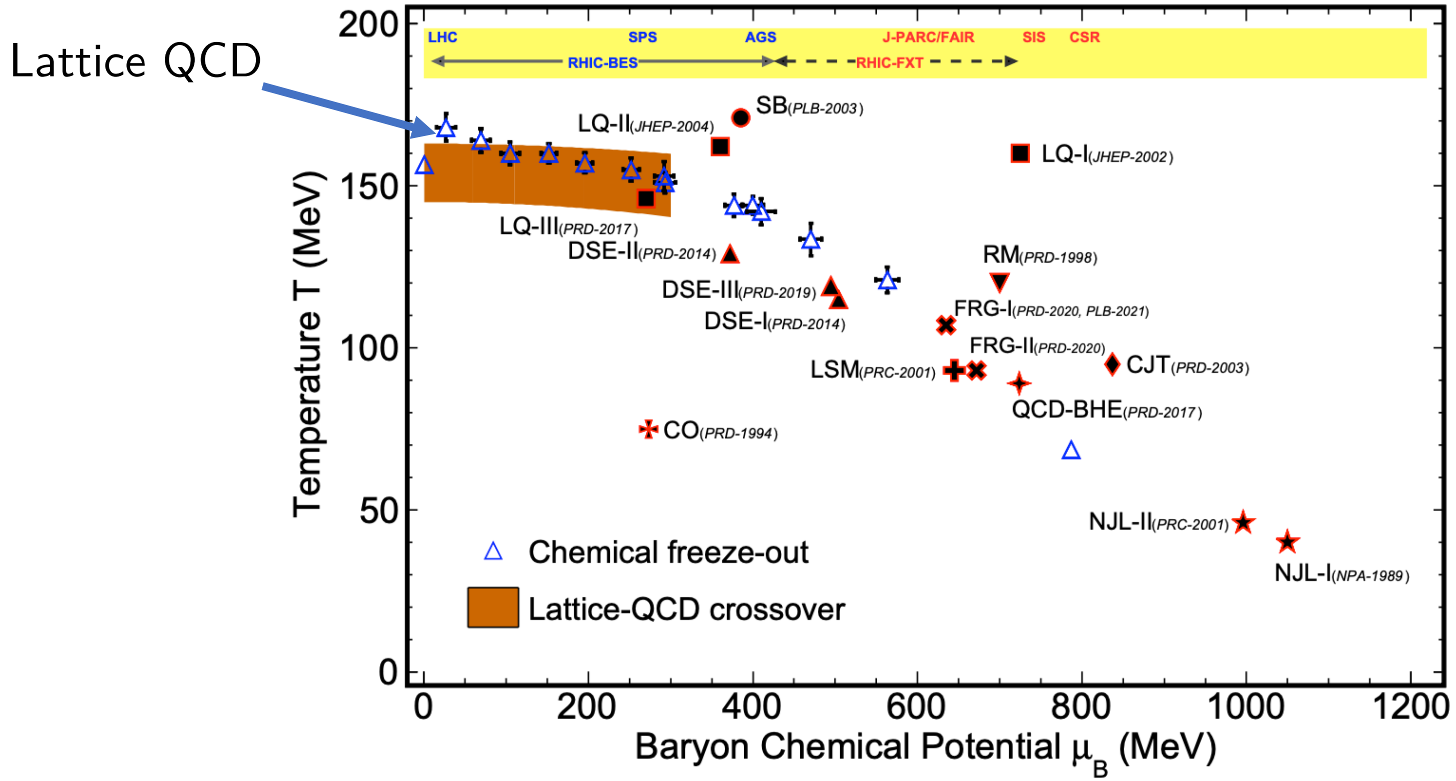


Figure adapted from A. Pandav, D. Mallick, B. Mohanty, Prog. Part. Nucl. Phys. 125 (2022)

- Including the possibility that the QCD critical point does not exist

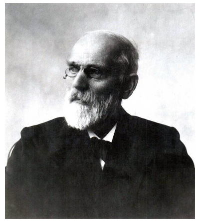
de Forcrand, Philipsen, JHEP 01, 077 (2007); VV, Steinheimer, Philipsen, Stoecker, PRD 97, 114030 (2018)

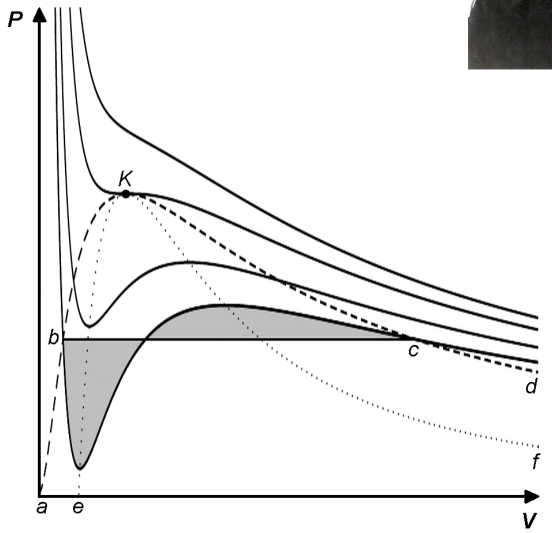
- Lattice QCD excludes the CP at $\mu_B < 450$ MeV on (one-sided) 2σ level

Borsanyi et al., PRD 112, L111505 (2025)

Extrapolating critical point from lattice QCD

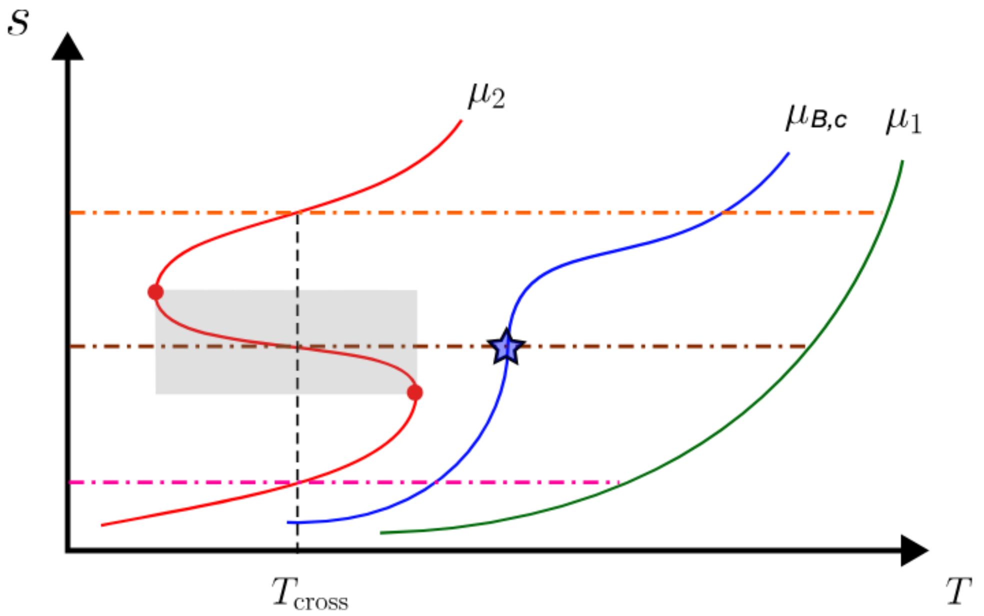
van der Waals (1873)





change of variables

Shah, Hippert, Noronha, Ratti, VV, PRC 113, L012201 (2026)



Critical Point:

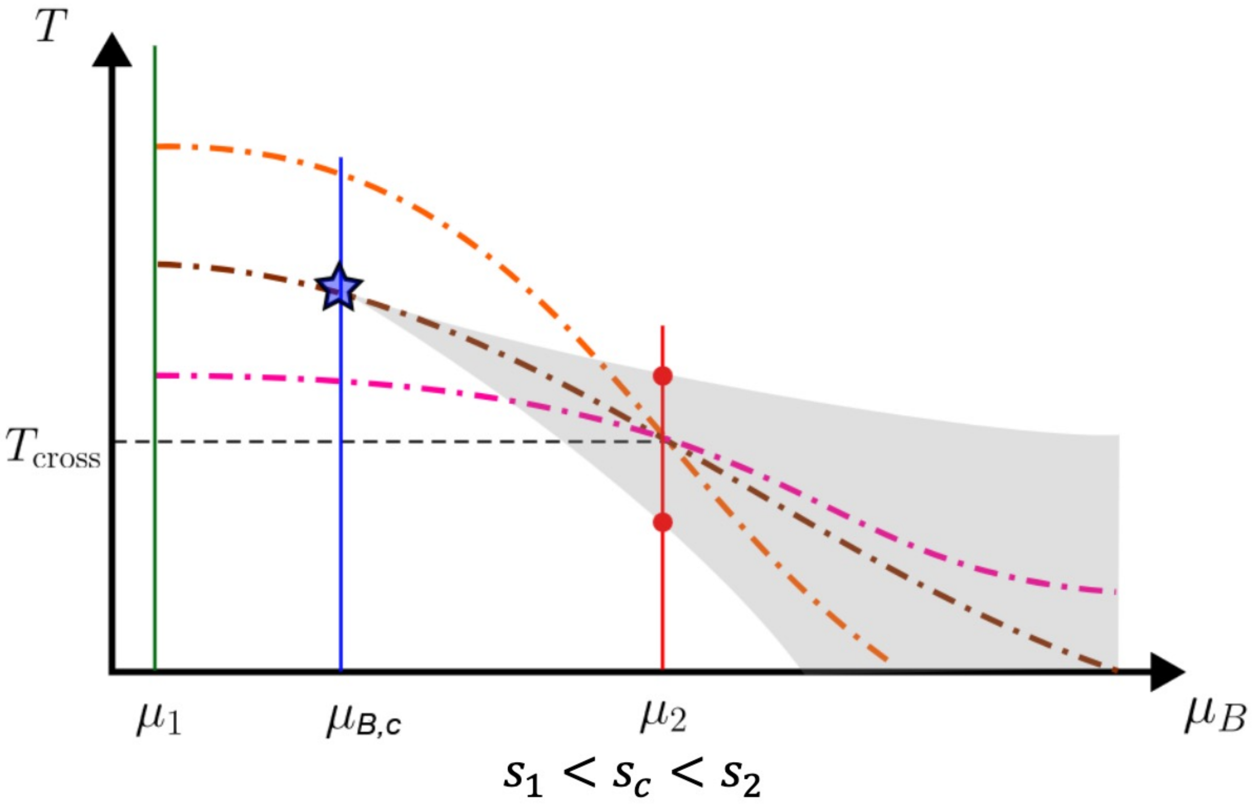
$$\left(\frac{\partial P}{\partial \rho_B}\right)_T = 0, \qquad \left(\frac{\partial^2 P}{\partial \rho_B^2}\right)_T = 0.$$

$$\left(\frac{\partial T}{\partial s}\right)_{\mu_B} = 0, \qquad \left(\frac{\partial^2 T}{\partial s^2}\right)_{\mu_B} = 0.$$

Strategy: Follow contours of constant entropy density away from $\mu_B = 0$ and look for crossings

Building the contours

Shah, Hippert, Noronha, Ratti, VV, PRC 113, L012201 (2026)



Define T-vs-μ_B line of constant entropy contour as Taylor series

$$T_s(\mu_B; T_0) = T_0 + \alpha_2(T_0)\frac{\mu_B^2}{2}$$ order $O(\mu_B^2)$

$$\alpha_2(T_0) = -\frac{2T_0\chi_2^B(T_0) + T_0^2\chi_2^{B\prime}(T_0)}{s'(T_0)}$$

- Allows multiple T_0 values to correspond to same T_s at finite μ_B
- In contrast to standard Taylor, permits the description of a first-order phase transition (mean-field-like behavior due to analyticity)

$$T_s(\mu_B; T_0) = T_0 + \alpha_2(T_0)\frac{\mu_B^2}{2}$$

expansion

$$\left(\frac{\partial T}{\partial s}\right)_{\mu_B} = 0 \;\Rightarrow\; \mu_{B,c} = \sqrt{-\frac{2}{\alpha_2'(T_{0,c})}}$$

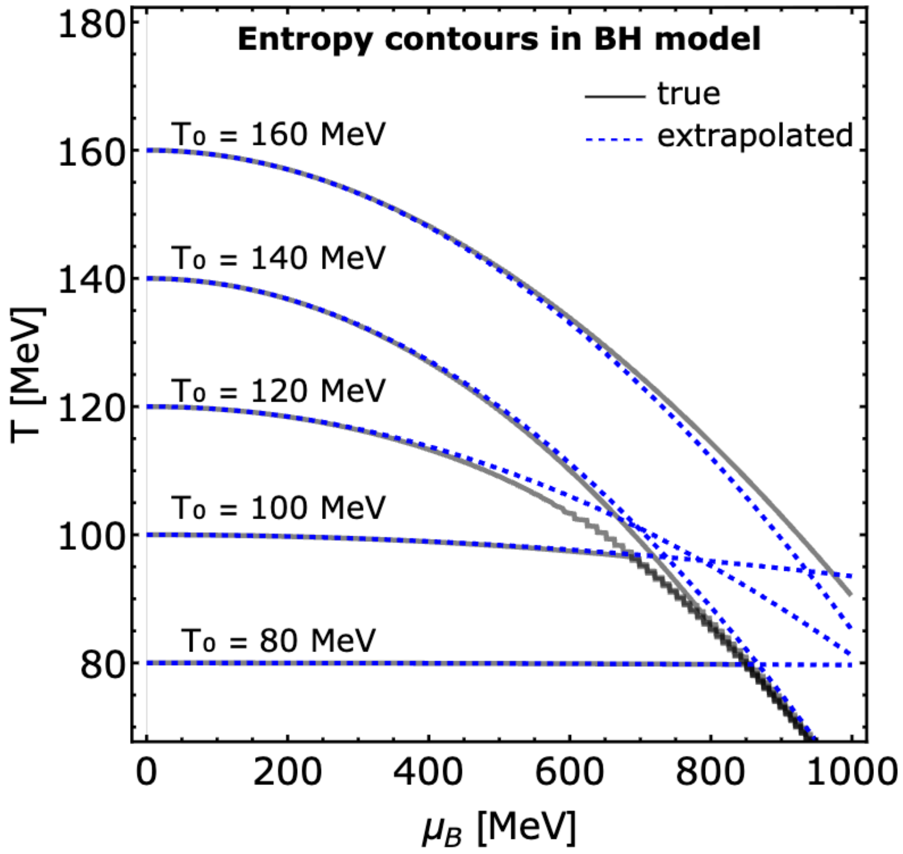
spinodals

$$\left(\frac{\partial^2 T}{\partial s^2}\right)_{\mu_B} = 0 \;\Rightarrow\; \alpha_2''(T_{0,c}) = 0$$

critical point

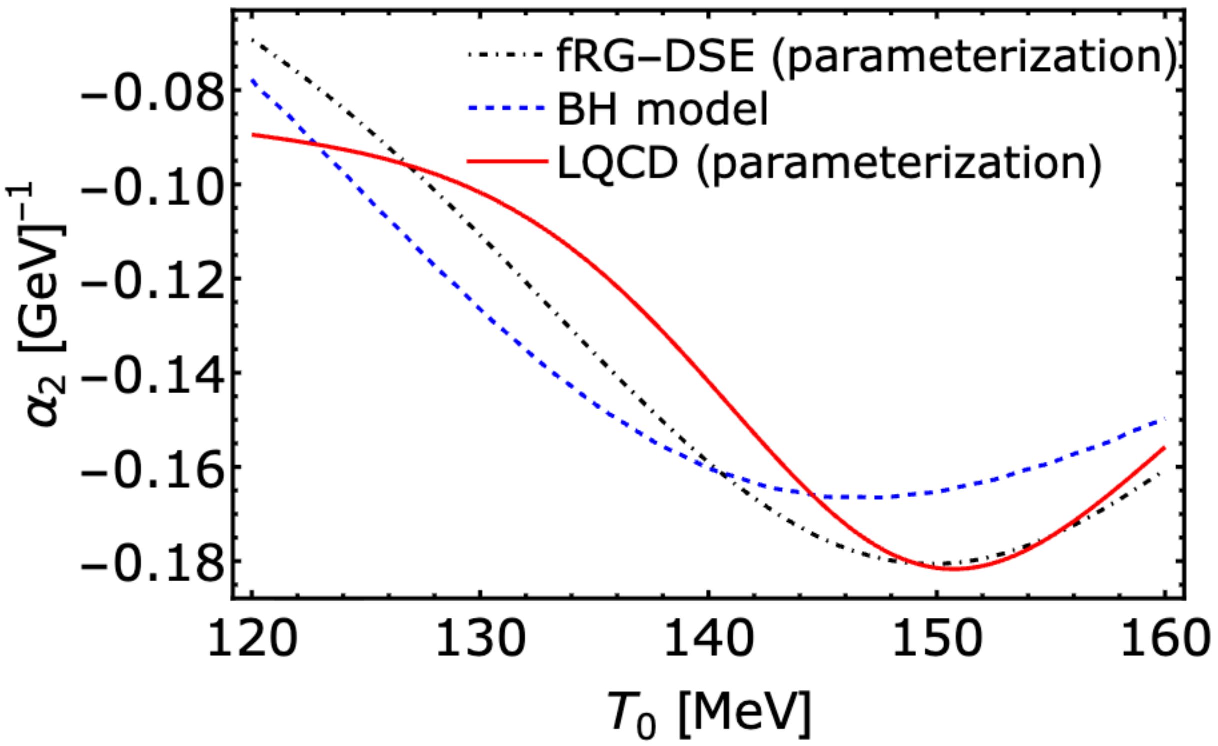
Testing with effective theories

Shah, Hippert, Noronha, Ratti, VV, arXiv:2601.08823



fRG-DSE: Lu, Gao, Liu, Pawlowski, PRD 113, 054019 (2026)

BH model: Hippert et al. PRD 110, 094006 (2024)



	true CP	**extrapolated CP**
✓ **fRG-DSE:**	(103,660)	(108,629)
✓ **BH model:**	(103,599)	(104,637)

✓ **free QGP:** no true CP and no extrapolated CP

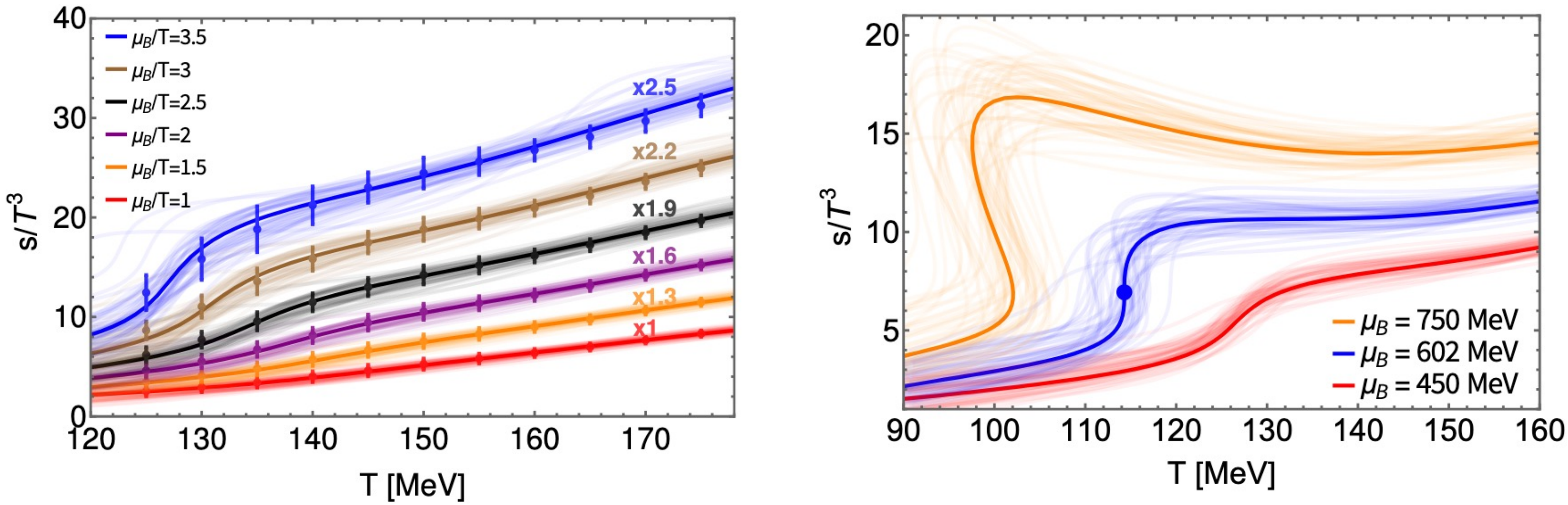
⚠ **HRG:** no true CP but fake extrapolated CP at ~(80,830) MeV

Marczenko et al., PRD 112, 034019 (2025)

Entropy density at finite μ_B at $O(\mu_B^2)$ from lattice QCD

UNIVERSITY OF HOUSTON

- Parametrized lattice QCD input from $\mu_B = 0$ [$s(T)$ and $\chi_2^B(T)$]



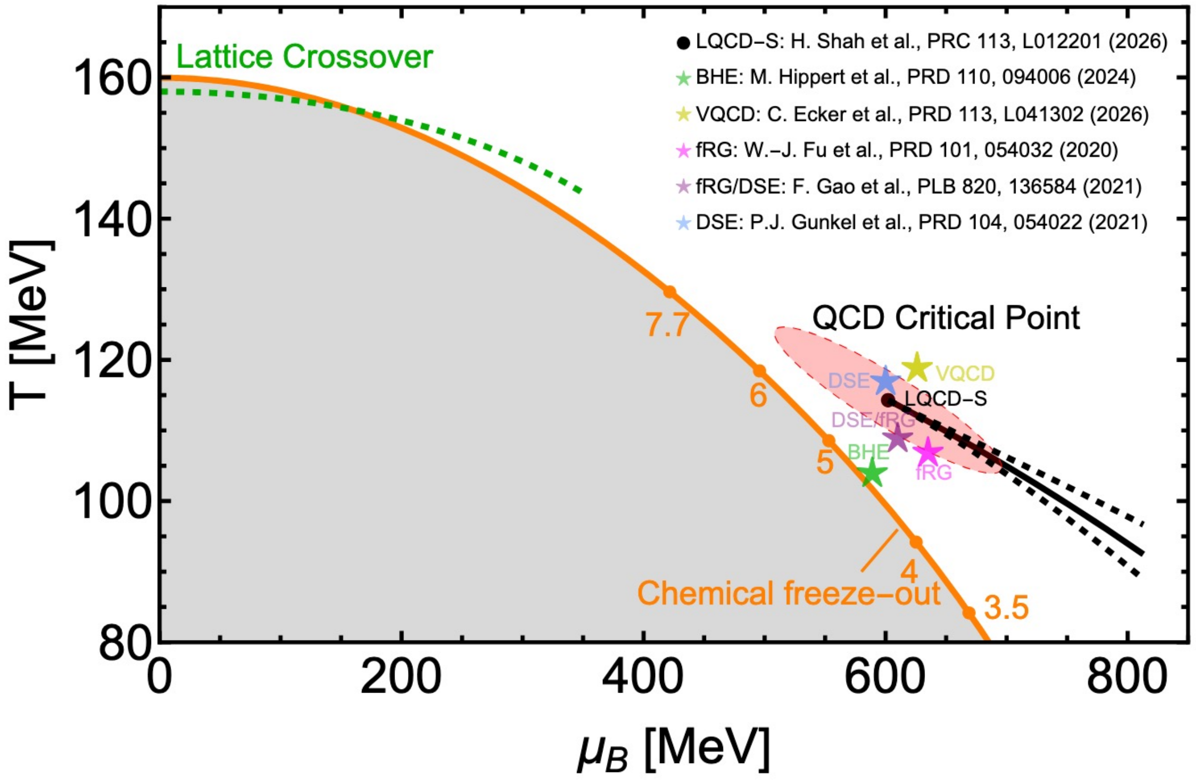
- Excellent agreement at low μ_B/T with available lattice QCD constraints from T'-expansion

Borsanyi et al., PRL 126, 232001 (2021)

- First-order phase transition emerges at $\mu_B > 600$ MeV

Shah, Hippert, Noronha, Ratti, VV, PRC 113, L012201 (2026)

Critical point estimates



Critical point estimate at $O(\mu_B^2)$:

$T_c = 114 \pm 7$ MeV, $\mu_B = 602 \pm 62$ MeV

Estimates from recent literature:

YLE-1: D.A. Clarke et al. (Bielefeld-Parma), PRD 112, L091504 (2025)

YLE-2: G. Basar, PRC 110, 015203 (2024)

BHE: M. Hippert et al., PRD 110, 094006 (2024)

fRG: W-J. Fu et al., PRD 101, 054032 (2020)

DSE/fRG: Gao, Pawlowski., PLB 820, 136584 (2021)

DSE: P.J. Gunkel et al., PRD 104, 052022 (2021)

VQCD: C. Ecker et al., PRD 113, L041302 (2026)

Optimist's view: Different estimates converge onto the same region because QCD CP is likely there

Pessimist's view: Different estimates converge onto the same region because it's the closest not yet ruled out by LQCD

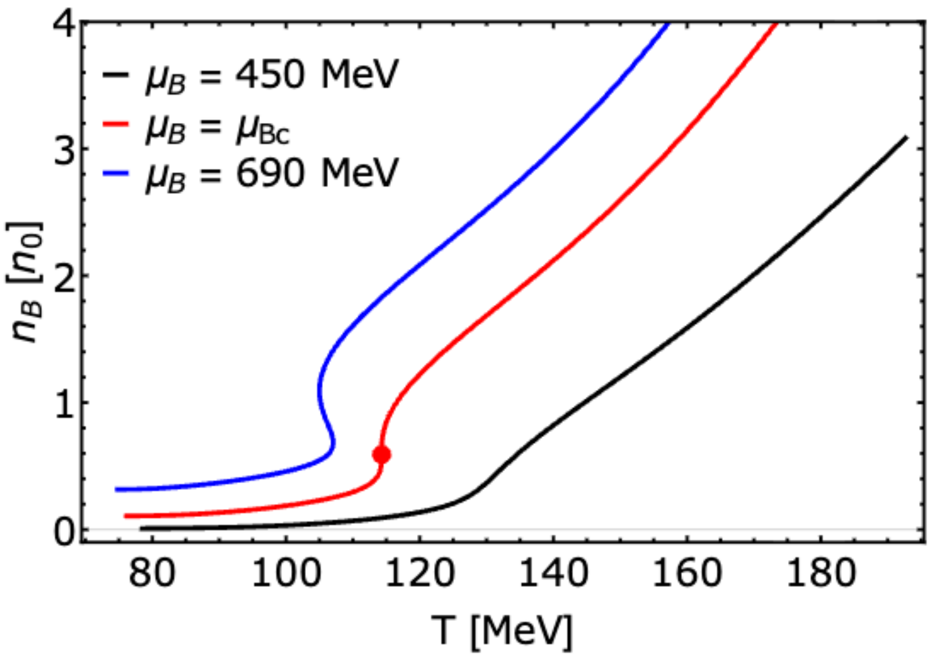
Equation of state

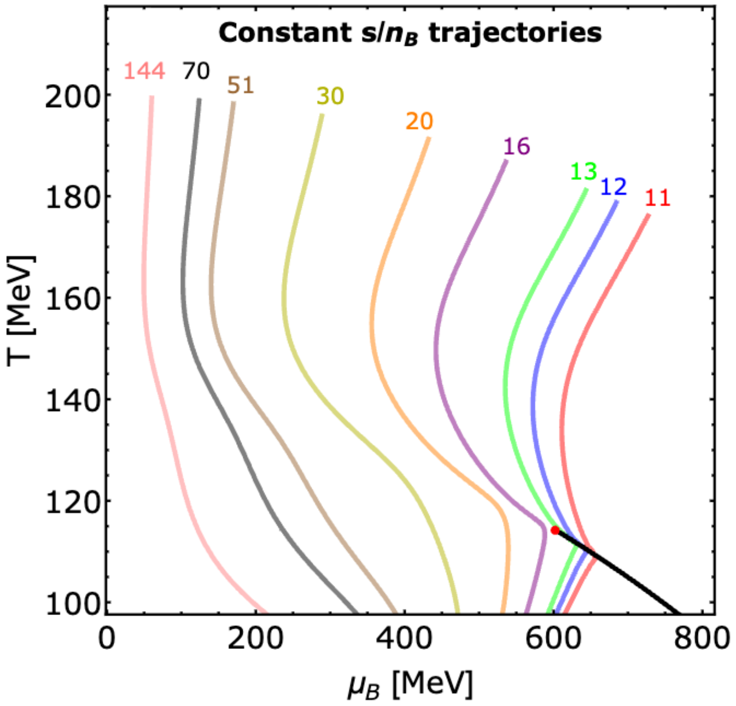
The method yields entropy density $s(T, \mu_B)$ across the (T, μ_B) phase diagram

−> Integrate over T to obtain the pressure $P(T, \mu_B)$ and the rest of the thermodynamics

$$\int_{T_{\rm low}}^{T} s(T', \mu_B) dT' = P(T, \mu_B) - P(T_{\rm low}, \mu_B)$$

Integration constant at $T_{\rm low}$ = 80 MeV matched to van der Waals HRG





Baryon density at the CP is $n_B \simeq 0.59 n_0 \simeq 0.09$ fm^{-3}

n_B in mixed phase region increases at lower T

Shah, Hippert, Noronha, Ratti, VV, arXiv:2601.08823

Critical point in 4D

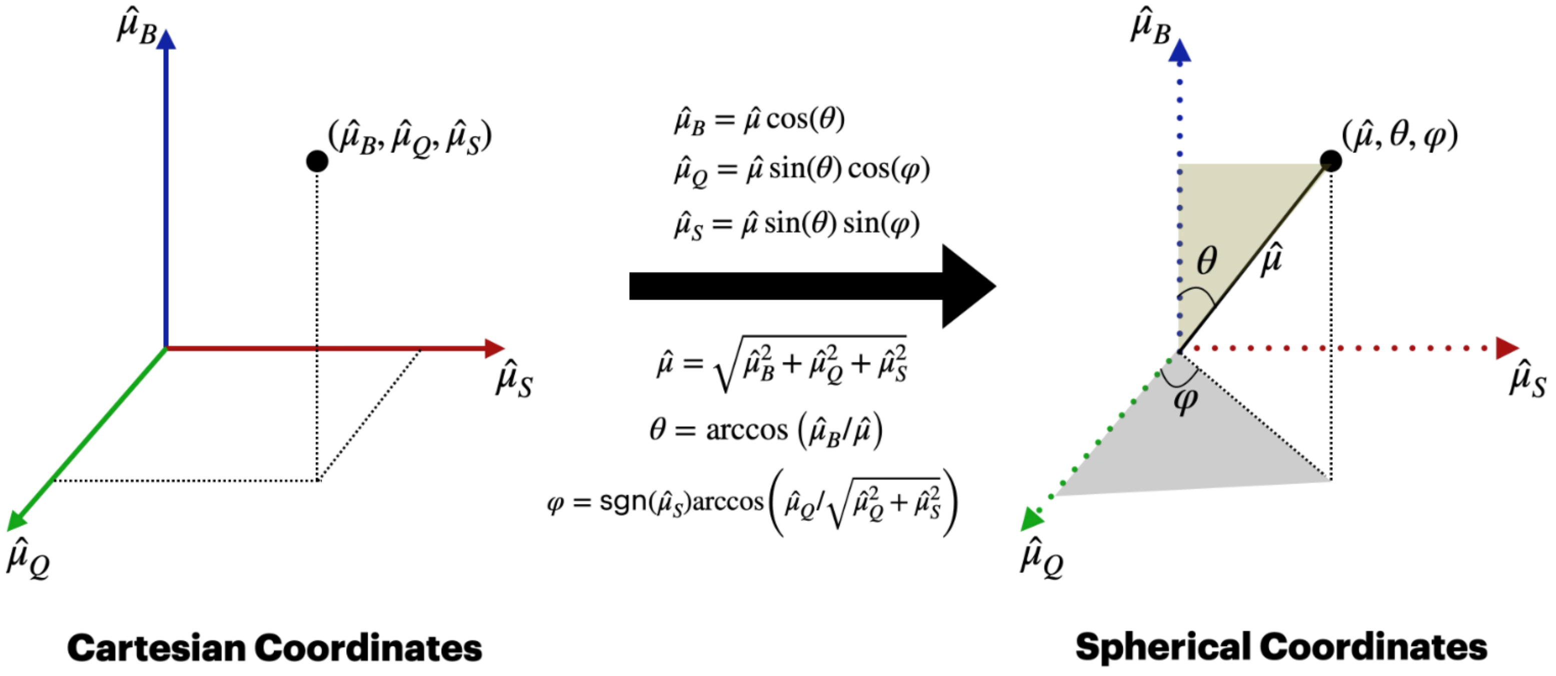
H. Shah, T. Gyure, F. Di Clemente, et al., to appear

UNIVERSITY OF HOUSTON

So far we restricted the analysis to $\mu_S = \mu_Q = 0$ conditions

General case: fix the direction in 3D space of (μ_B, μ_Q, μ_S) using spherical coordinates

A. Abuali, S. Borsanyi, et al., PRD 112, 054502 (2025)



$$\chi_2^B(T) \Rightarrow X_2^{\theta,\varphi}(T) = c_\theta^2 \chi_2^B(T) + s_\theta^2 c_\varphi^2 \chi_2^Q(T) + s_\theta^2 s_\varphi^2 \chi_2^S(T) + 2c_\theta s_\theta c_\varphi \chi_{11}^{BQ}(T) + 2c_\theta s_\theta s_\varphi \chi_{11}^{BS}(T) + 2s_\theta^2 c_\varphi s_\varphi \chi_{11}^{QS}(T)$$

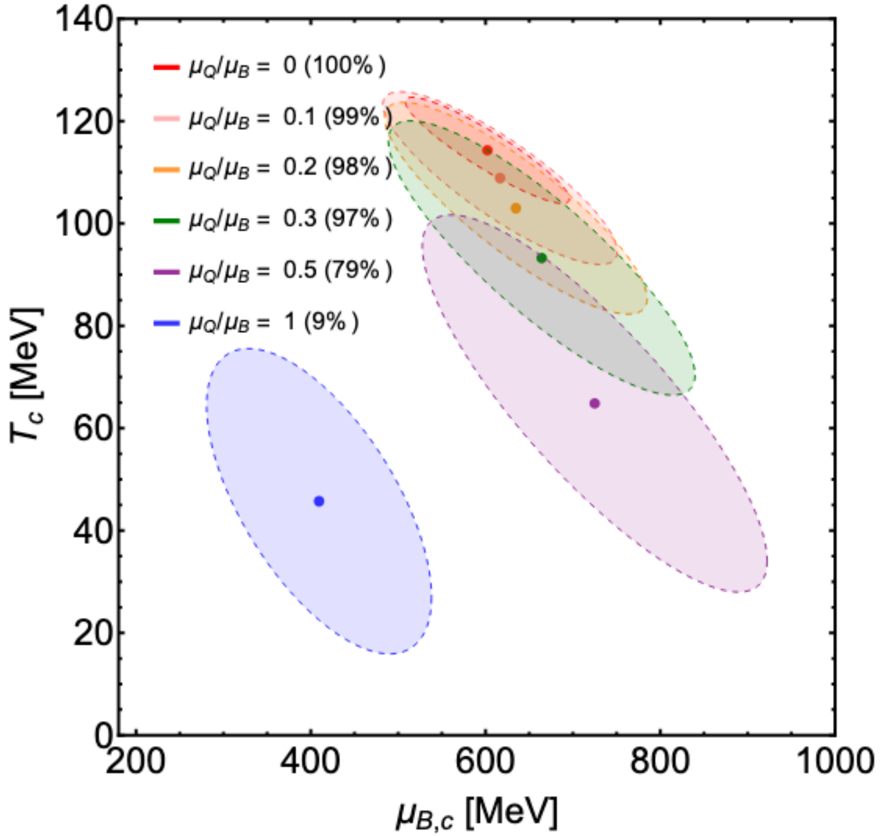
Critical point in 4D

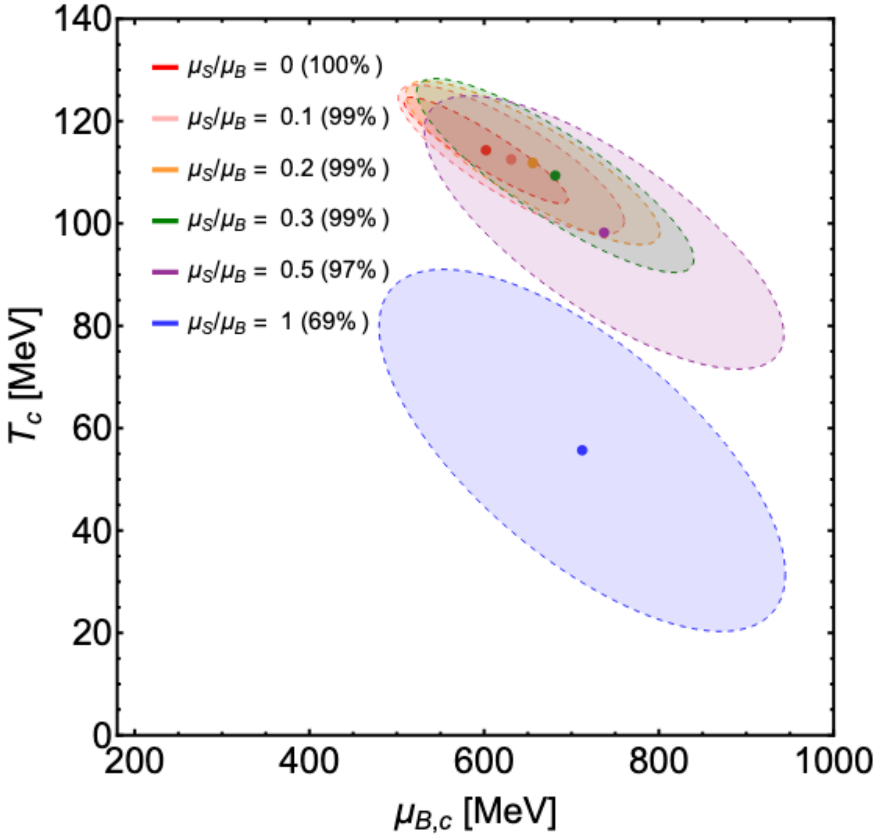
H. Shah, T. Gyure, F. Di Clemente, et al., to appear

UNIVERSITY OF HOUSTON

Using the lattice input for susceptibilities from [A. Abuali, et al., PRD 112, 054502 (2025)]

The brackets denote the percentage of MC samples giving the critical point.





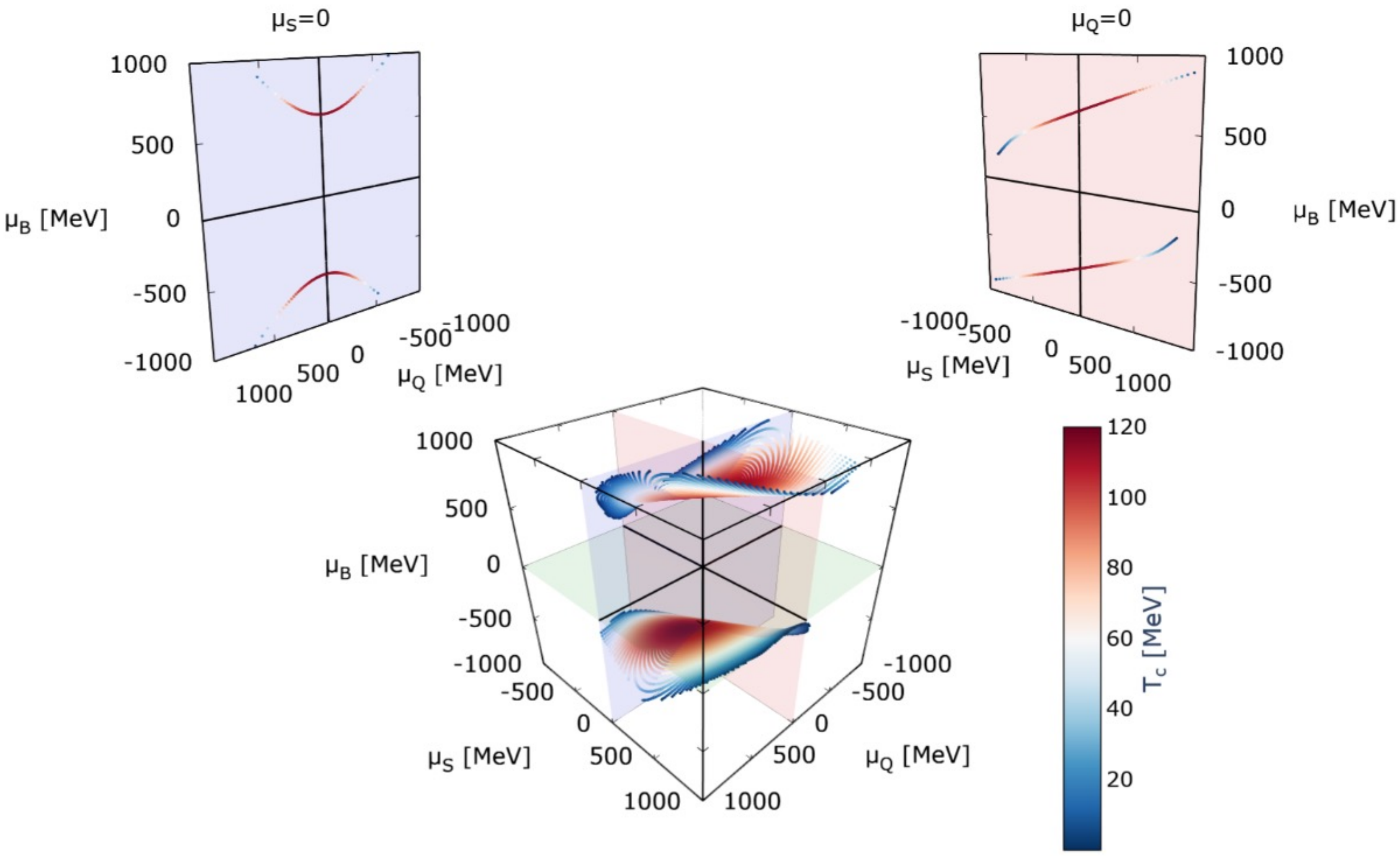
Strangeness neutrality ($\mu_S \approx \mu_B/3$): $\mu_{B,c}$ shifts to the right.

From $(T, \mu_B) = (114{,}602)$ MeV at $\mu_S = 0$ to $(T, \mu_B) = (109{,}683)$ MeV at $\mu_S = \mu_B/3$

Similar effect of an increased $\mu_{B,c}$ in recent functional QCD analysis [W.J. Fu et al., 2603.13455]

Critical point in 4D: full result

H. Shah, T. Gyure, F. Di Clemente, et al., to appear



Pure μ_Q direction: No evidence for a CP. Consistent with lattice QCD at finite isospin

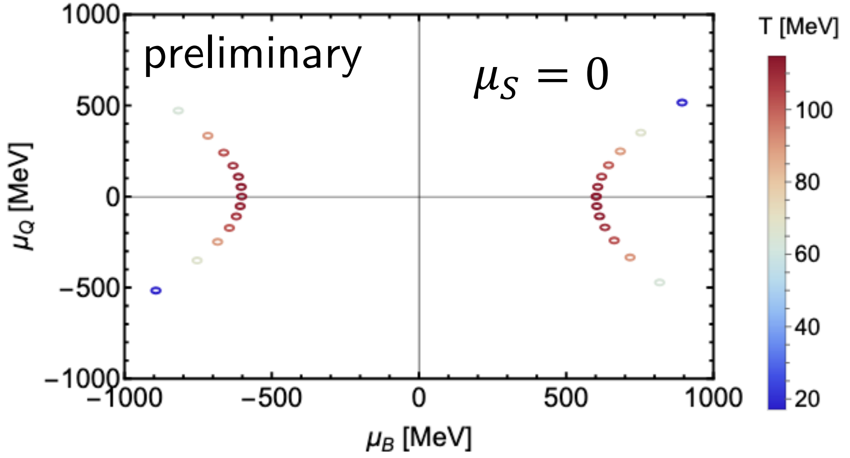
Caveats: Pion and kaon condensation

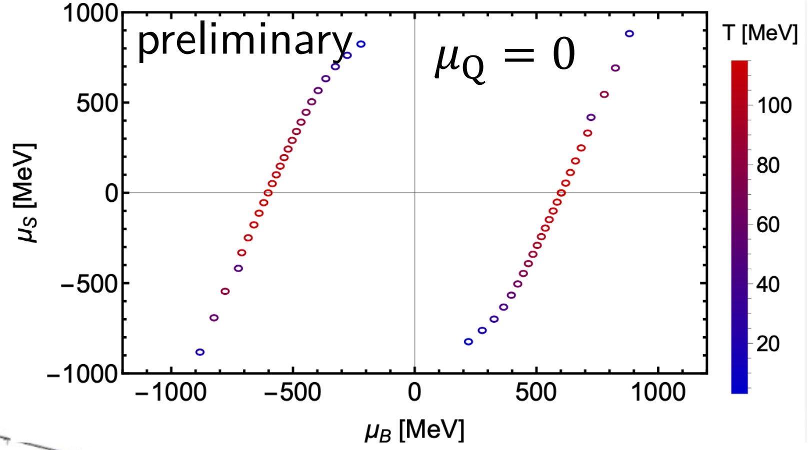
Applications: Heavy-ion collisions, cosmic trajectories with large lepton asymmetries,...

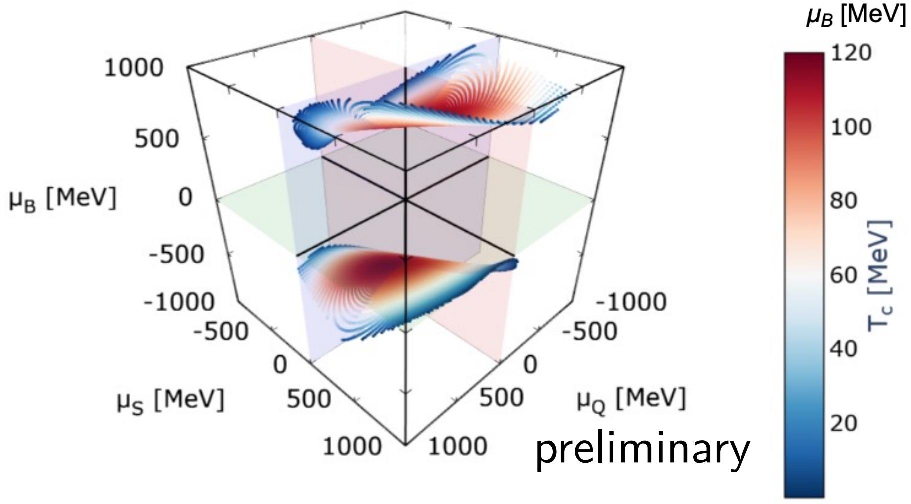
Critical point in 4D: full result

H. Shah, T. Gyure, F. Di Clemente, et al., to appear

UNIVERSITY OF HOUSTON







Pure μ_Q direction: No evidence for a CP. Consistent with lattice QCD at finite isospin

Caveats: Pion and kaon condensation

Applications: Heavy-ion collisions, cosmic trajectories with large lepton asymmetries,...

Fluctuations in heavy-ion collisions

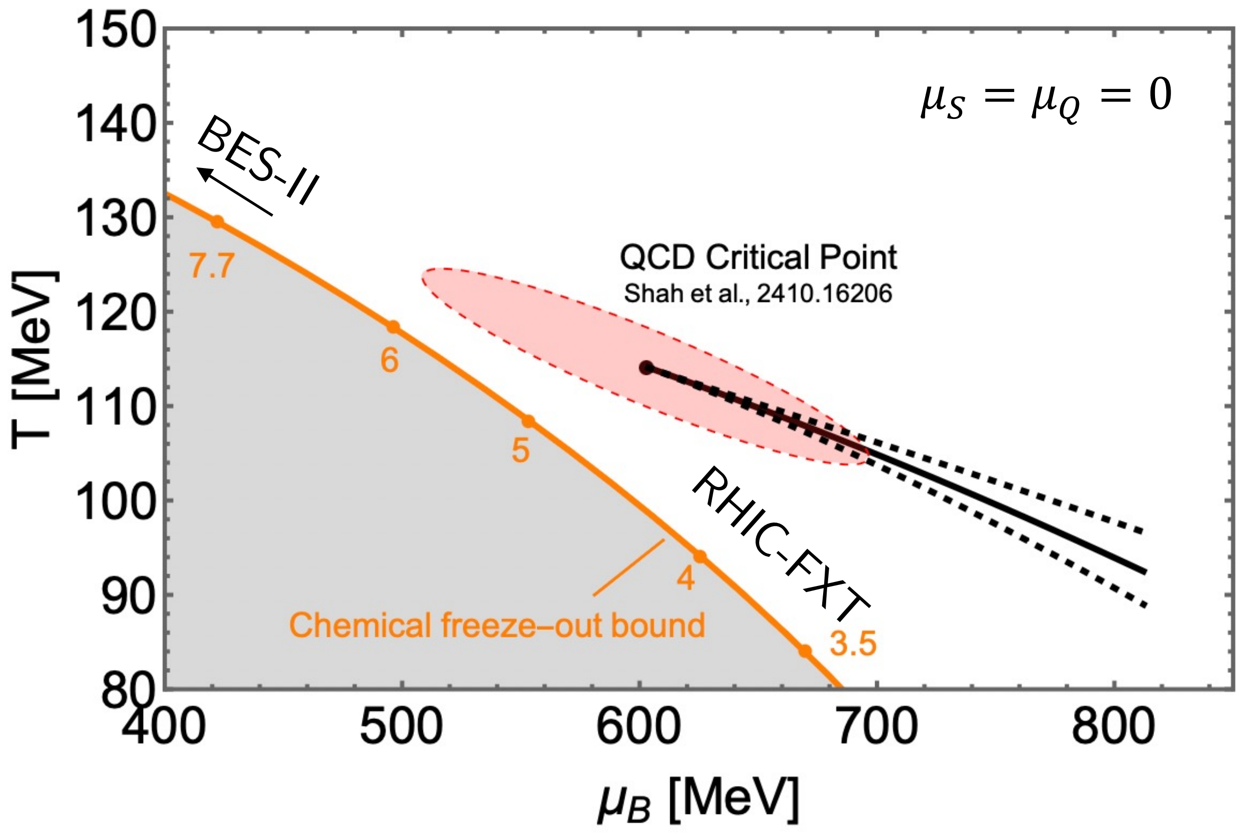
Critical point and heavy-ion collisions

Control parameters

- Collision energy $\sqrt{s_{NN}} = 2.4 - 5020$ GeV
  - Scan the QCD phase diagram
- Size of the collision region
  - Expect stronger signal in larger systems

Measurements

- Final hadron abundances and momentum distributions **event-by-event**



Chemical freeze-out curve and CP

- Sets the **lower bound** on the temperature of the CP [Lysenko, Poberezhnyuk, Gorenstein, VV, PRC 111, 054903 (2025)]
- **Caveats:** strangeness neutrality ($\mu_S \neq 0$), uncertainty in the freeze-out curve
- If the $\mu_B \simeq 600$ MeV predictions are true, the CP may be close to freeze-out line at $\sqrt{s_{NN}} \sim 3.5 - 5$ GeV

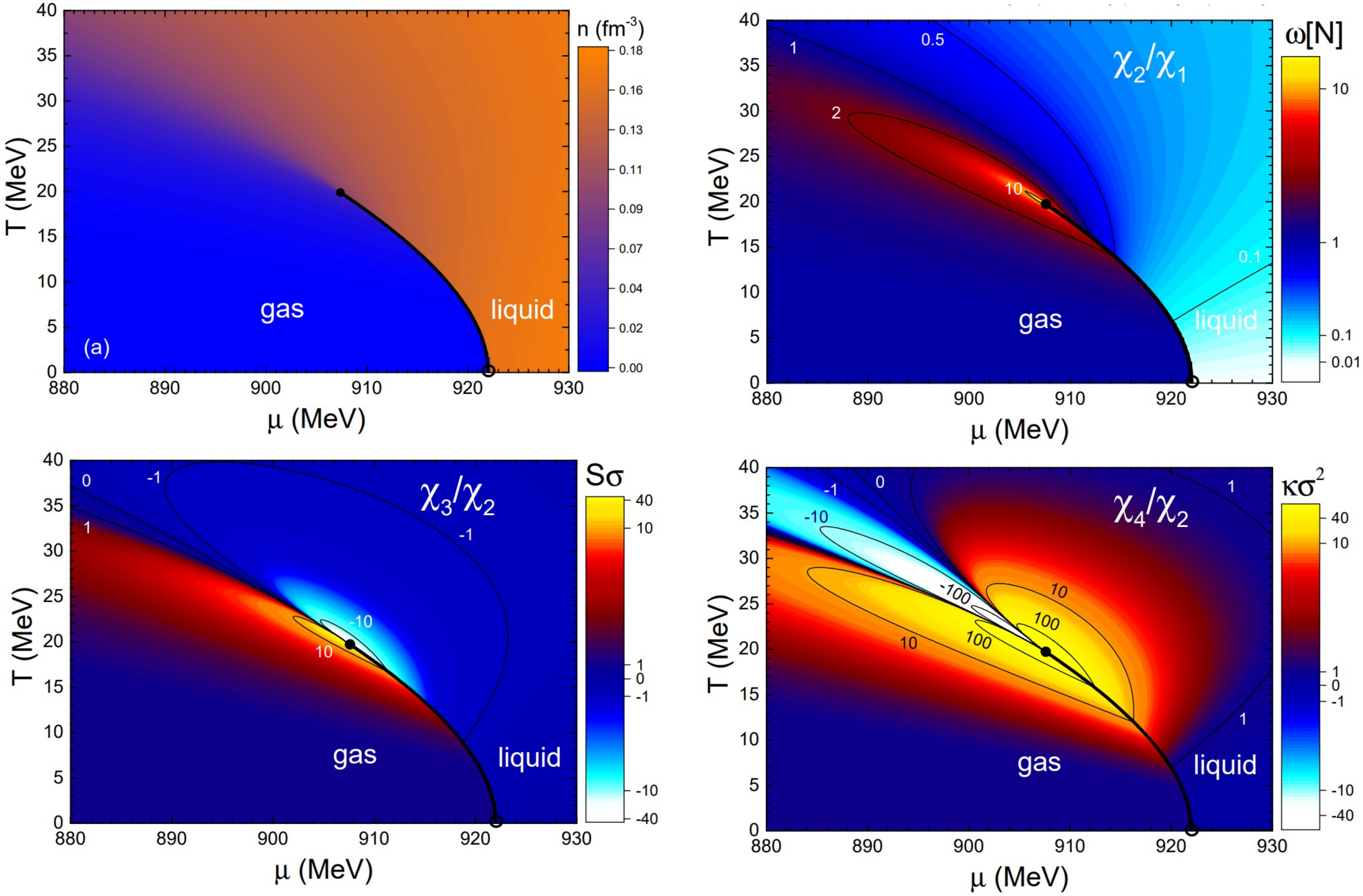
Example: (Nuclear) Liquid-gas transition

(QCD) critical point: large correlation length and fluctuations

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

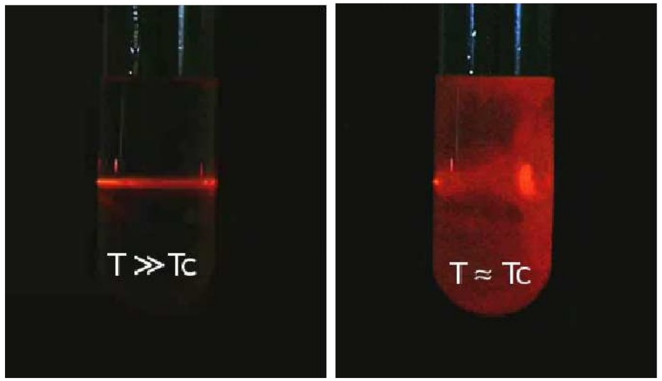
$$\xi \to \infty$$

M. Stephanov, PRL ’09, ‘11



VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Critical opalescence



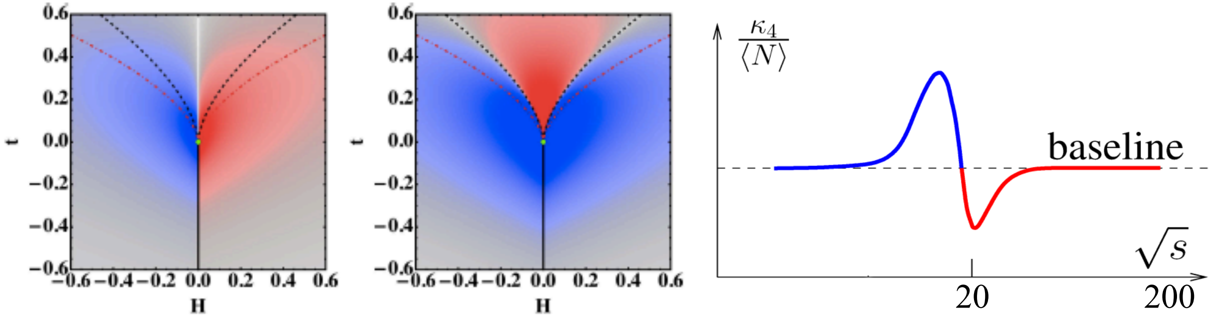
$$\langle N^2\rangle - \langle N\rangle^2 \sim \langle N\rangle \sim 10^{23}$$

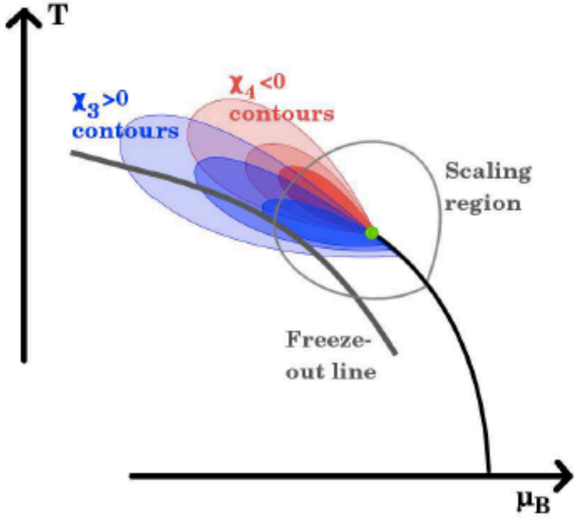
in equilibrium

Equilibrium Expectations and Beam Energy Scan

UNIVERSITY OF HOUSTON

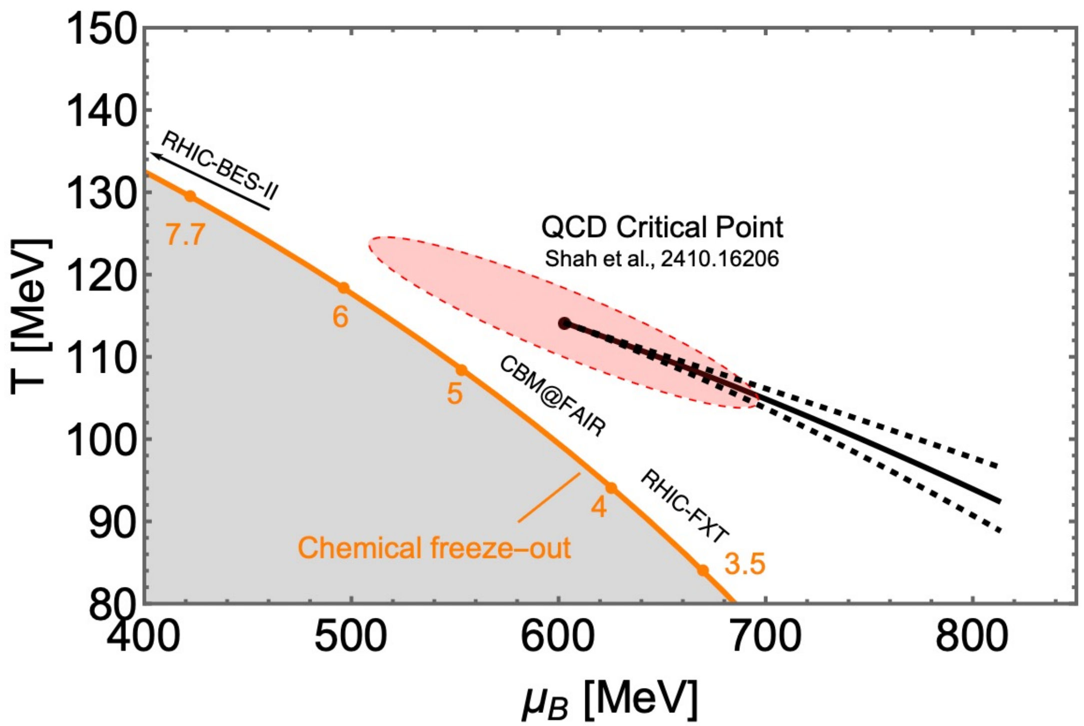
Expectation from Calculations





Characteristic "Oscillating pattern" is expected for the QCD critical point but *the exact shape depends on the location of freeze-out with respect to the location of CP*

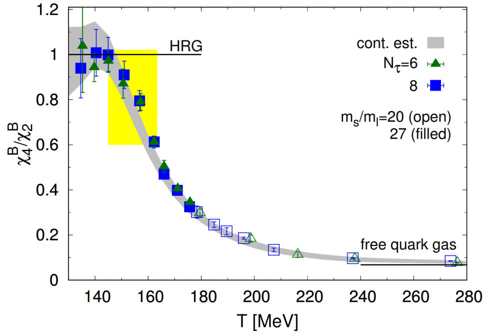
- M. Stephanov, ***PRL107***, 052301(2011)
- V. Skokov, Quark Matter 2012
- J.W. Chen, J. Deng, H. Kohyyama, arXiv: 1603.05198, Phys. Rev. **D93** (2016) 034037



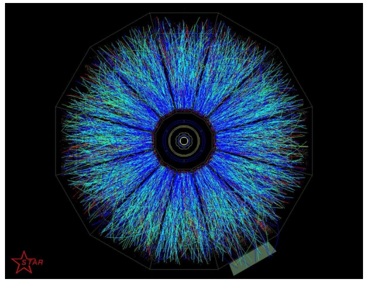
N. Xu, CPOD 2016

Theory vs experiment

guidance from theory (e.g. lattice)



experiment (the real thing)

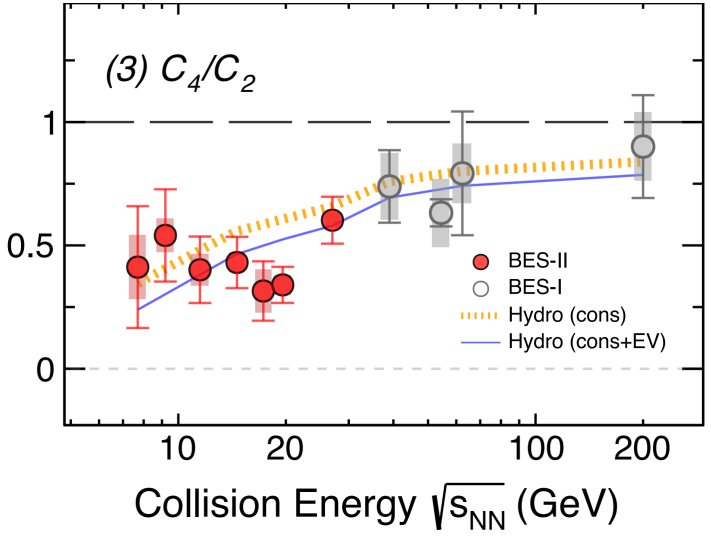


hydro simulation

hadronic phase

canonical effects

kinematic cuts



event selection

track reconstruction

efficiency correction

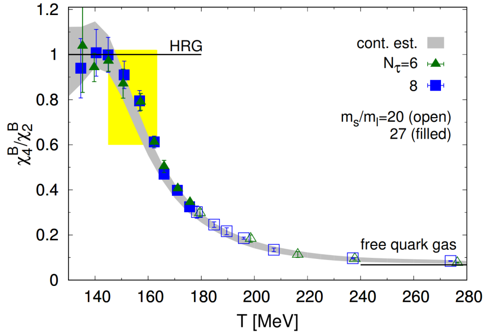
CBWC

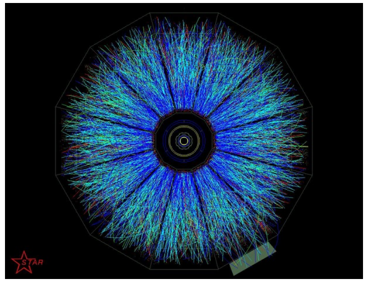
purity

Theory vs experiment

guidance from theory (e.g. lattice)

experiment (the real thing)





This was done in [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)]

- Full hydro simulation
- Lattice QCD-like baryon susceptibilities (interacting HRG)
- Experimental kinematic cuts
- Global baryon conservation: SAM correction

Non-critical baseline (hydro EV) prediction

hydro simulation

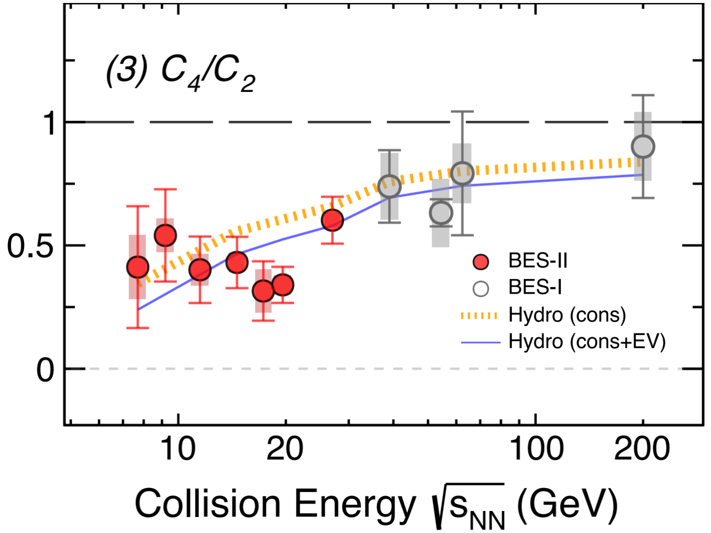
hadronic phase

canonical effects

kinematic cuts

event selection

track reconstruction



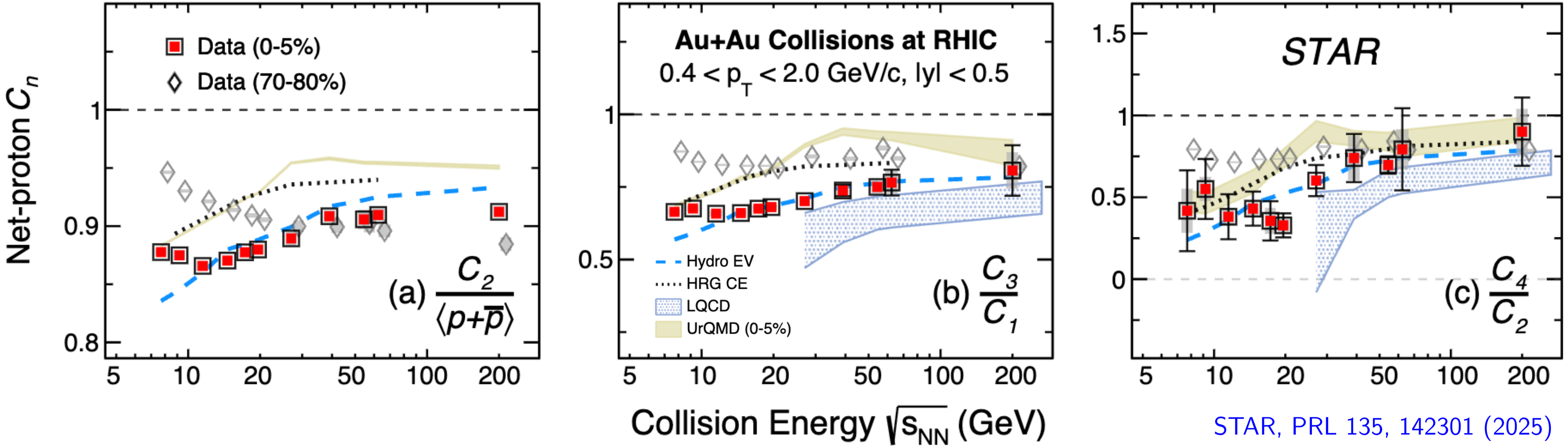
efficiency correction

CBWC

purity

RHIC-BES-II data: Ordinary cumulants

Net-proton cumulant ratios



STAR, PRL 135, 142301 (2025)

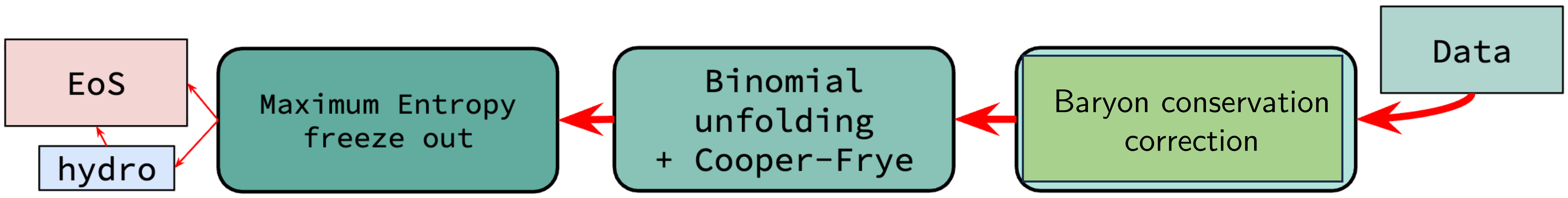
Hydro EV: VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Agreement with the baseline above $\sqrt{s_{NN}} \sim 10 - 20$ GeV

Deviations at lower energies. More pronounced in factorial cumulants.

Inference of baryon number susceptibilities from data

G. Pihan, R. Poberezhniuk, VV, to appear



Generalizing Hydro EV

Hydro EV [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)]

- Particlization at a constant energy density ~0.26 GeV/fm^3
- Canonical (baryon conservation) statistical hadronization with excluded volume repulsion
- Drawback: Cannot incorporate arbitrary EoS, in particular one with a CP

Maximum entropy freeze-out [M. Pradeep et al., PRL 130, 162301 (2023); arXiv:2508.19237]

- Incorporate single hydrodynamic mode – baryon density fluctuations
- Maximum entropy method defines local baryon/antibaryon joint susceptibilities G. Pihan et al., to appear

$$\hat{\chi}^{+-}_{nm}(x) = \delta_{m0}\delta_{n1}\bar{\chi}^{+}_{1} + \delta_{n0}\delta_{m1}\bar{\chi}^{-}_{1} + (-1)^{m}\underline{\hat{\Delta}\chi^{B}_{n+m}(x)}\frac{\bar{\chi}^{+}_{1}(x)^{n}\bar{\chi}^{-}_{1}(x)^{m}}{(\bar{\chi}^{+}_{1}(x)+\bar{\chi}^{-}_{1}(x))^{n+m}}$$

difference to HRG

$$\bar{\chi}^{B}_{n}(x) = \bar{\chi}^{+}_{1}(x) + (-1)^{n}\,\bar{\chi}^{-}_{1}(x)$$

HRG (Poisson) baseline

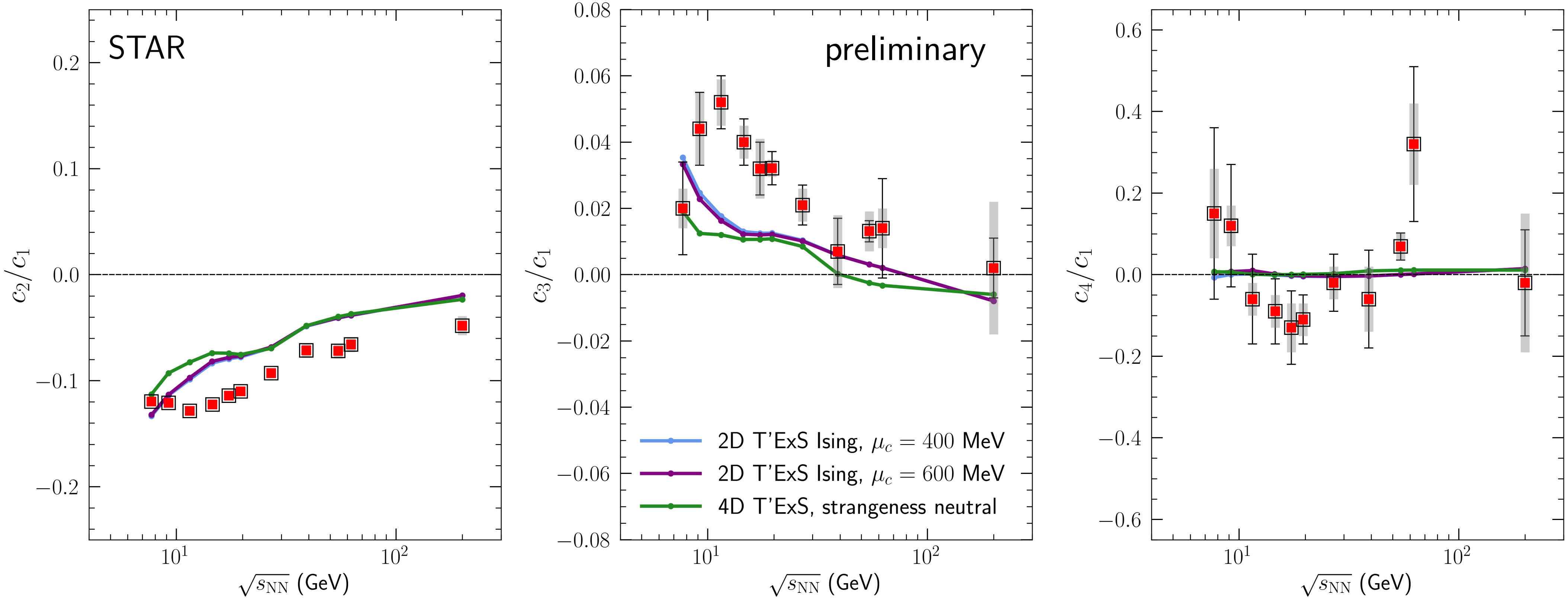
+ is baryon, – is antibaryon

- Use the χ^{B}_{n} input from *arbitrary EoS* (LQCD expansion, s-contours, functional QCD, …)

More details in talks of G. Pihan and J. Karthein next week at "QCD Critical Point and Hydrodynamic Evolution"

Lattice-based EoSs

Pihan, Kahangirwe, Shah, et al., in preparation



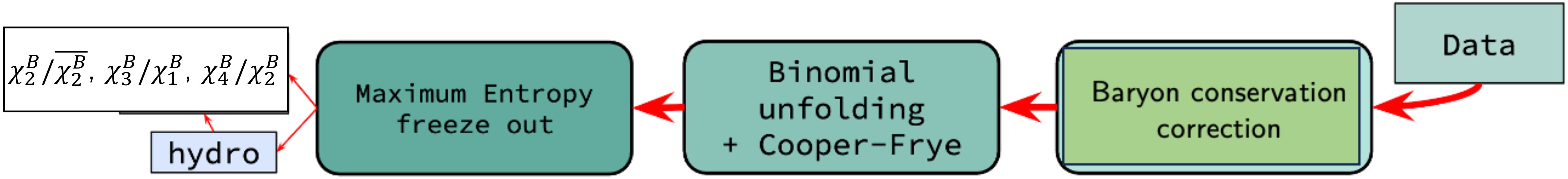
- 2D T'ExS Ising: critical point in μ_B plane $(\mu_Q = \mu_S = 0)$ [Kahangirwe et al., PRD 109, 094046 (2024)]
- 4D T'ExS: non-critical lattice based EoS [A. Abuali, et al., PRD 112, 054502 (2025)]
- Switching energy density $e_{sw} = 0.18$ GeV/fm^3 (to be varied, lower than chem. freeze-out)

Jahan, Roch, Shen, PRC 113, 024919 (2026)

Bayesian EoS inference

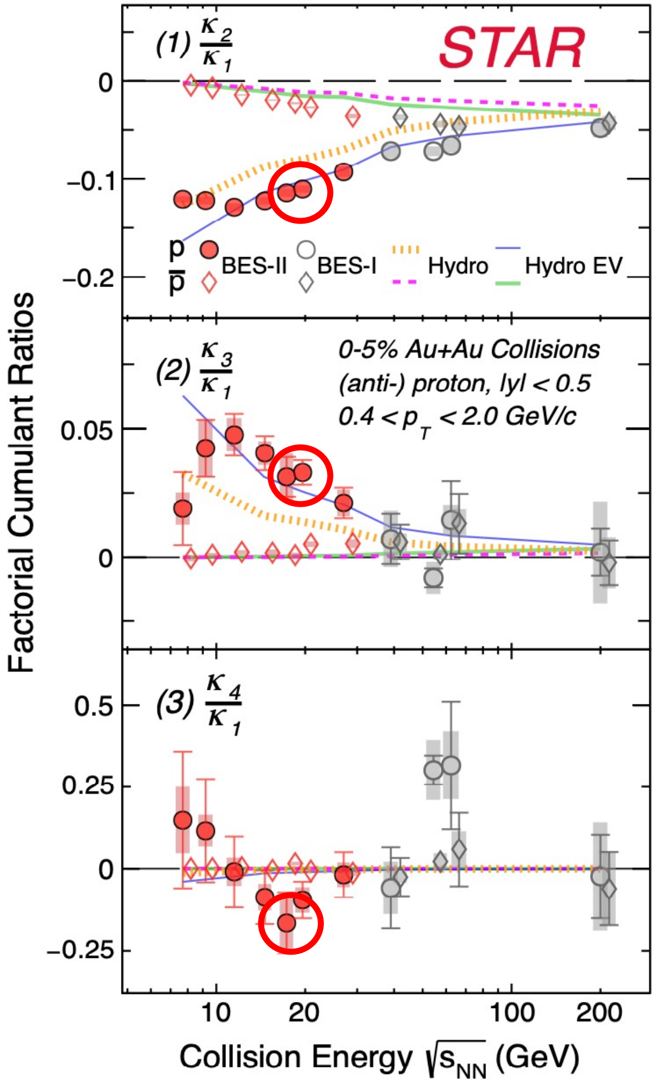
Instead of using χ_n^B as an input, extract χ_n^B's that fit data directly at each energy

Specifically, the ratios $\chi_2^B/\overline{\chi_2^B}$, χ_3^B/χ_1^B, χ_4^B/χ_2^B where $\overline{\chi_2^B}$ is the HRG baseline

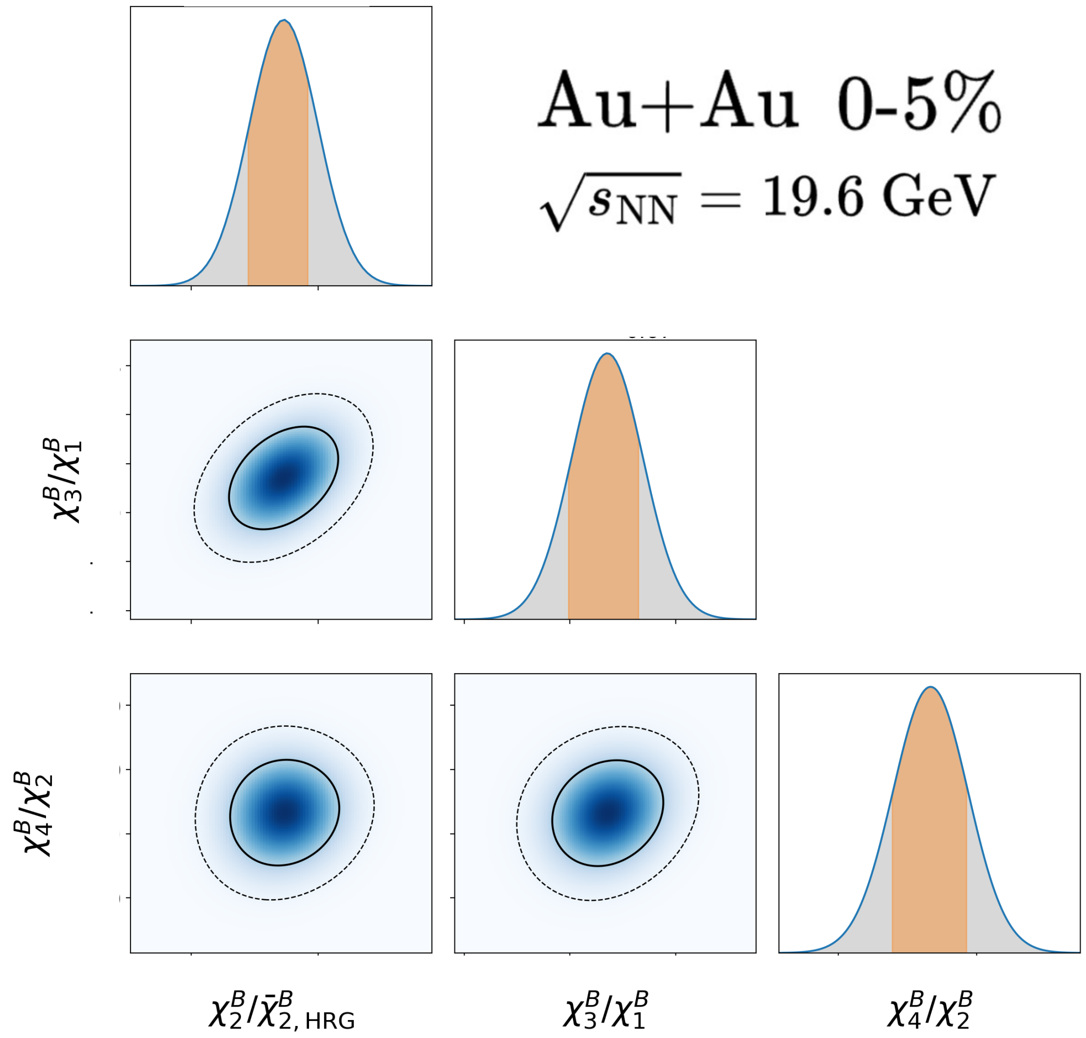


Adapted from G. Pihan, SQM2026

Bayesian inference: 19.6 GeV

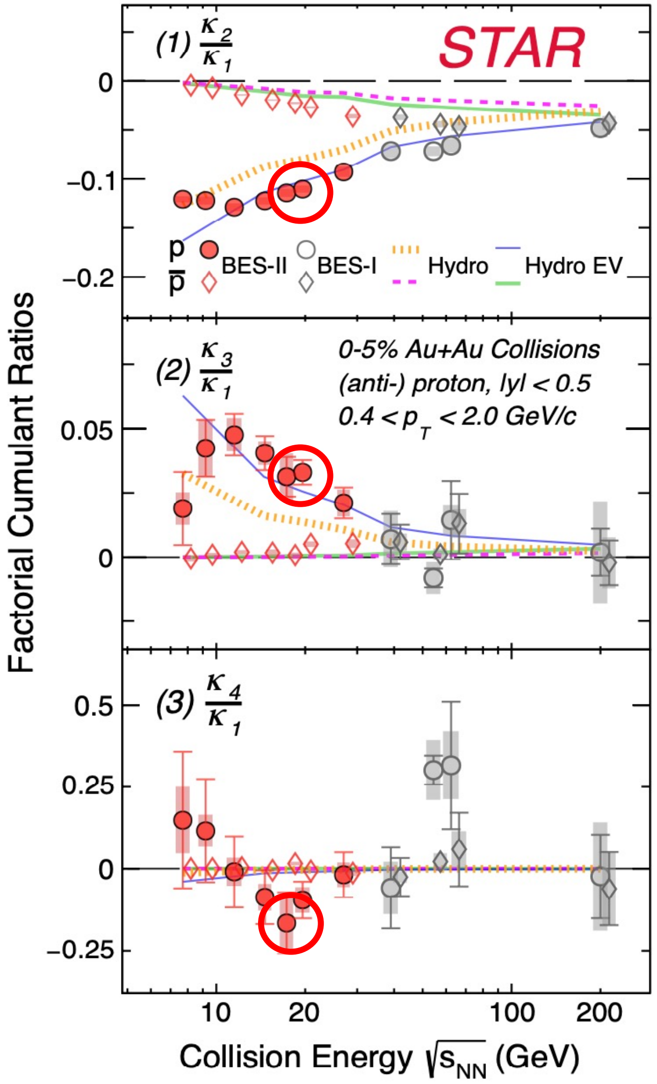


Unfolding

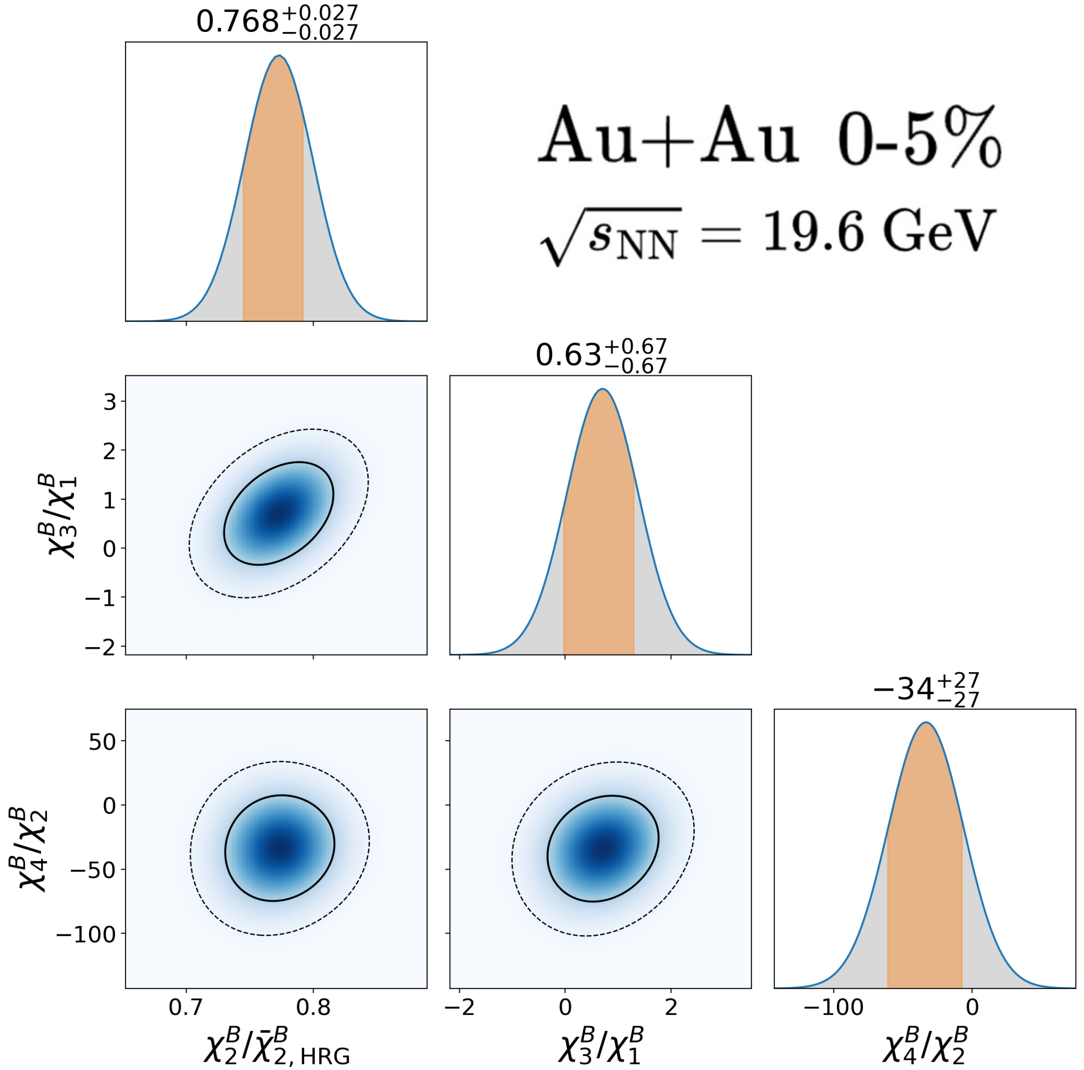


G. Pihan, R. Poberezhniuk, VV, to appear

Bayesian inference: 19.6 GeV



Unfolding

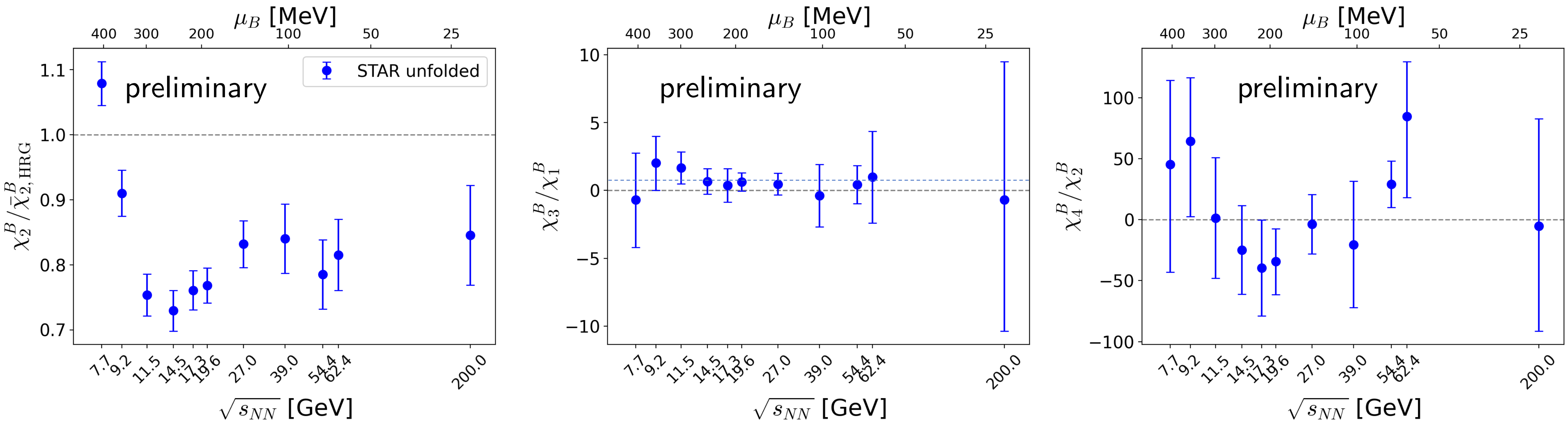


Au+Au 0-5%

$\sqrt{s_{\mathrm{NN}}} = 19.6$ GeV

G. Pihan, R. Poberezhniuk, VV, to appear

Extracted susceptibilities



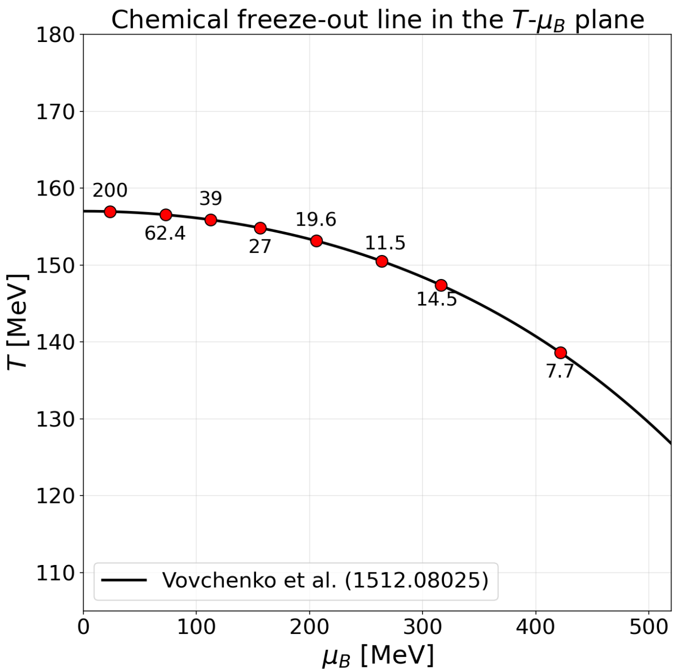
- Tight constraints on χ_2^B value relative to HRG
- Uncertainties in 3rd and 4th order susceptibilities are large---propagation of data uncertainty
  - Intermediate steps (net-B to B^+, kinematic cuts, $B^+ \to p$) acts akin to efficiency correction
  - Reducing uncertainties in the data would lead to reduced extraction errors
- Results appear weakly sensitive to e_{sw}, sensitivity to other hydro parameters to be studied

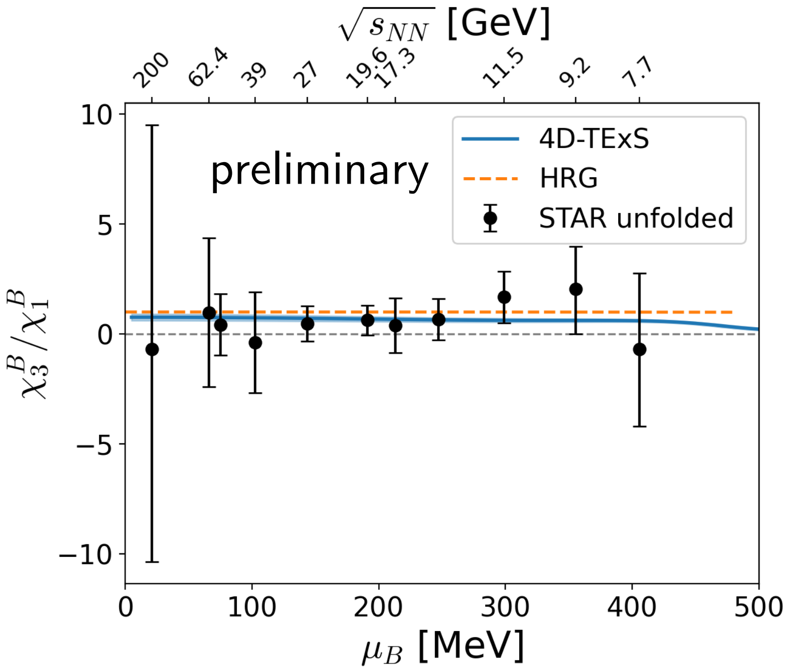
G. Pihan, R. Poberezhniuk, VV, to appear

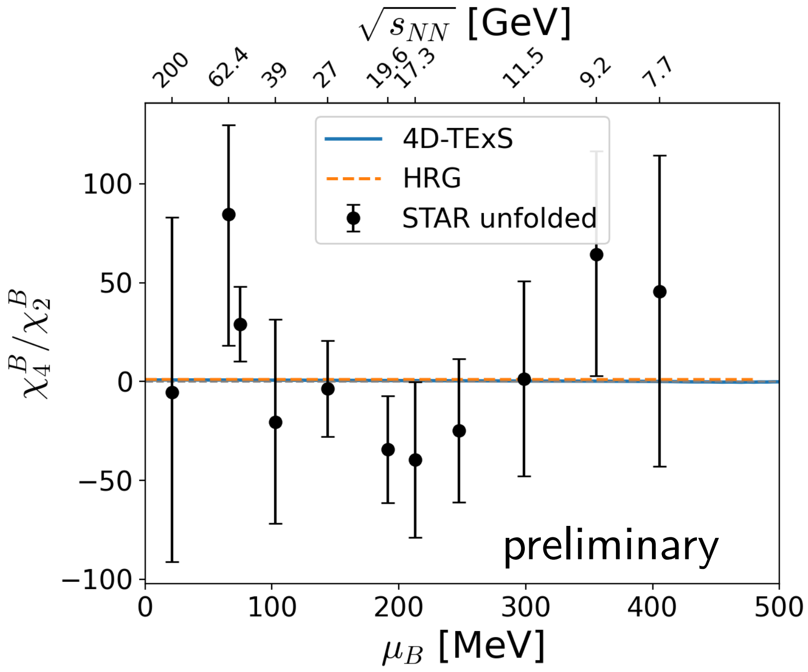
Comparing to lattice QCD

UNIVERSITY OF HOUSTON

To map the results to QCD phase diagram use chemical freeze-out parametrization [VV et al., PRC 93, 064906 (2015)]







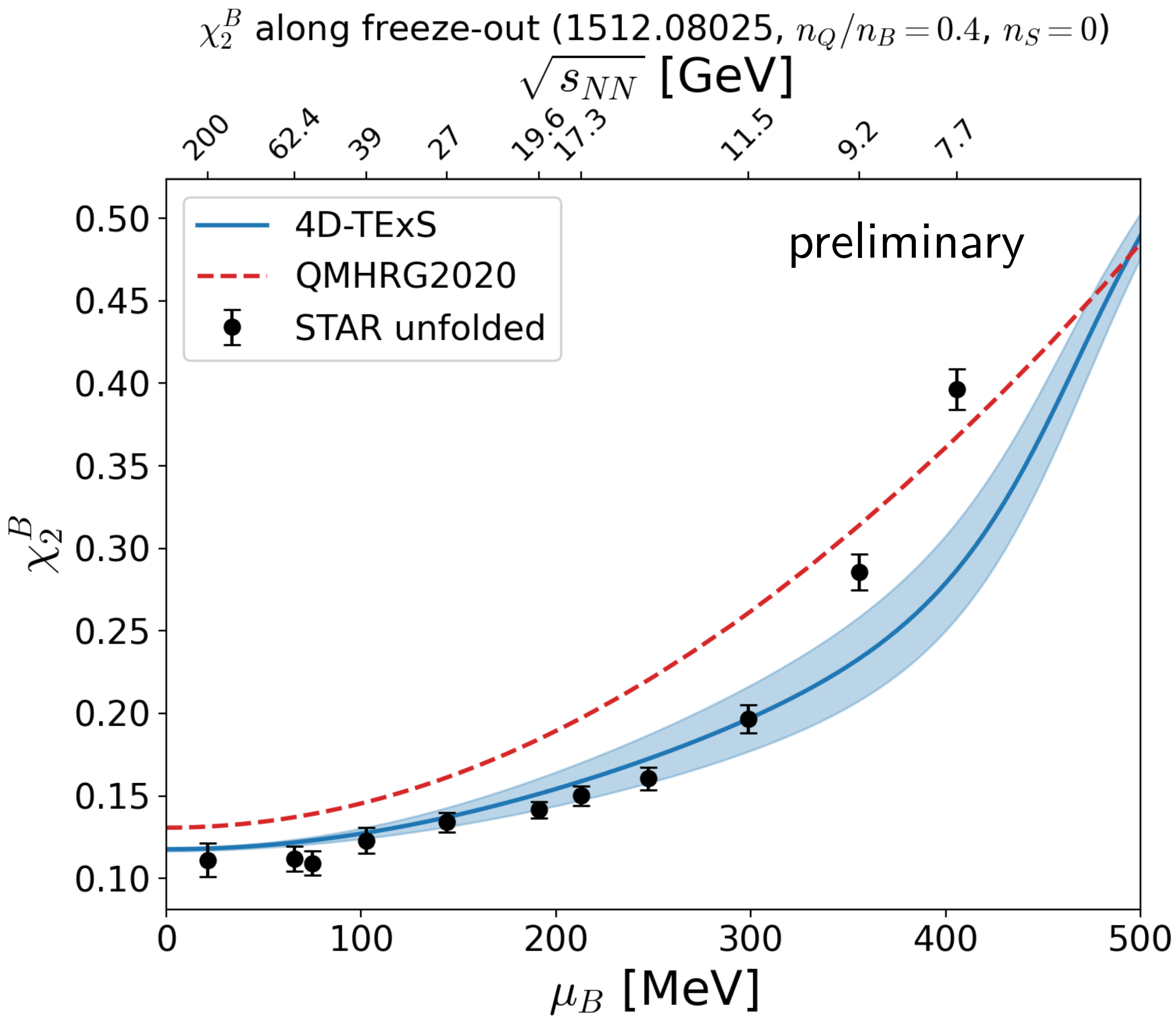
$\varepsilon_{\rm SW} \approx 0.35 - 0.40\ \mathrm{GeV/fm^3}$

4D-TExS: [A. Abuali et al., Phys. Rev. D 112, 054502 (2025)]

- Lattice-based EoS at finite baryon density
- Computed under HIC conditions $(n_Q/n_B = 0.4, n_S = 0)$

G. Pihan, R. Poberezhniuk, VV, to appear

Baryon susceptibility χ_2^B



4D-TExS: [A. Abuali et al., Phys. Rev. D 112, 054502 (2025)]

- To obtain bare χ_2^B multiply by HRG value
  - QMHRG motivated by lattice QCD studies at $\mu_B = 0$ [HotQCD, PRD 104, 074512 (2021)]
- Agreement with lattice QCD at $\mu_B < 300$ MeV
- Enhancement relative to lattice QCD at $\mu_B > 300$ MeV

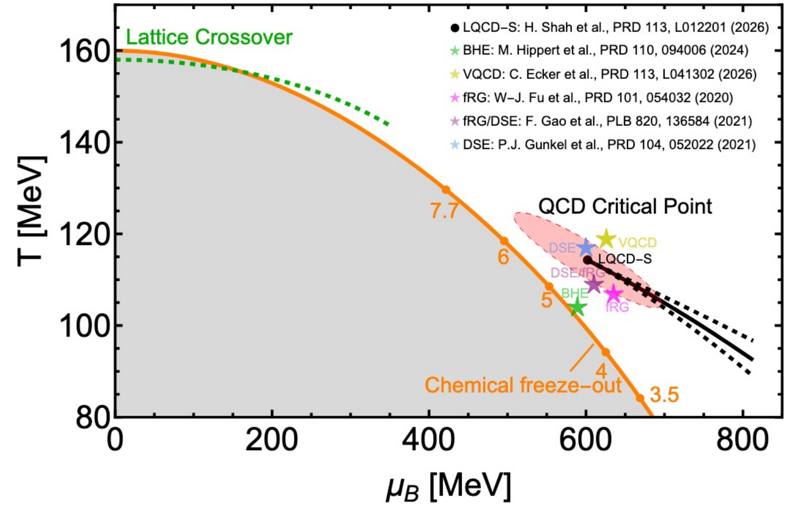
⚠ **Caveats:**

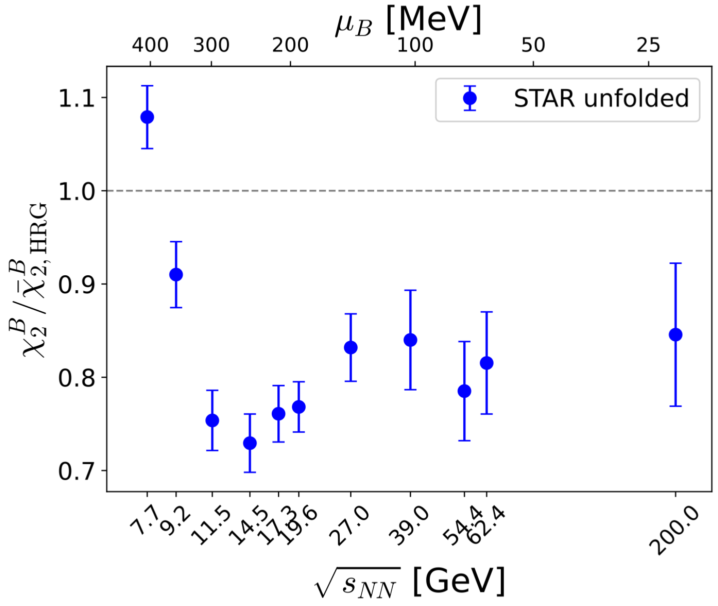
- Sensitive to the hadron list
- Sensitive to assumed freeze-out line
- Lattice EoS (4D-TExS) is an extrapolation (no truncation error included)

G. Pihan, R. Poberezhniuk, VV, to appear

Summary

- HRG is well approximated by a gas of confining strings
  - EV interactions may extend the description to T~200 MeV
- Constant entropy contours: CP estimate in the 4D phase diagram
  - Strangeness neutrality shifts the CP from ~600 MeV to ~680 MeV
- Hydrodynamics-based inference with fluctuations in heavy-ion collisio
  - Non-critical physics describe the proton data at $\sqrt{s_{NN}} \geq 20$ GeV
  - χ_2^B consistent with LQCD at $\mu_B < 300$ MeV, enhancement at larger μ_B
  - Too large errors to infer high-order susceptibilities from data





Outlook:

- 4D EoS with a critical point from constant entropy contours
- Quantitative predictions for fluctuations in heavy-ion collisions
- Bayesian inference framework for HIC fluctuations data

Thanks for your attention