The search for the QCD Critical Point

Volodymyr Vovchenko (University of Houston)

KITP conference "Probing our Understanding of Quark-Gluon matter"

March 30, 2026

Outline

UNIVERSITY OF HOUSTON

1. Lattice-based constraints and indications

2. Proton cumulants from RHIC and comparison to baselines

3. Equation of state inference from proton cumulants

QCD under extreme conditions

UNIVERSITY OF HOUSTON

What we know

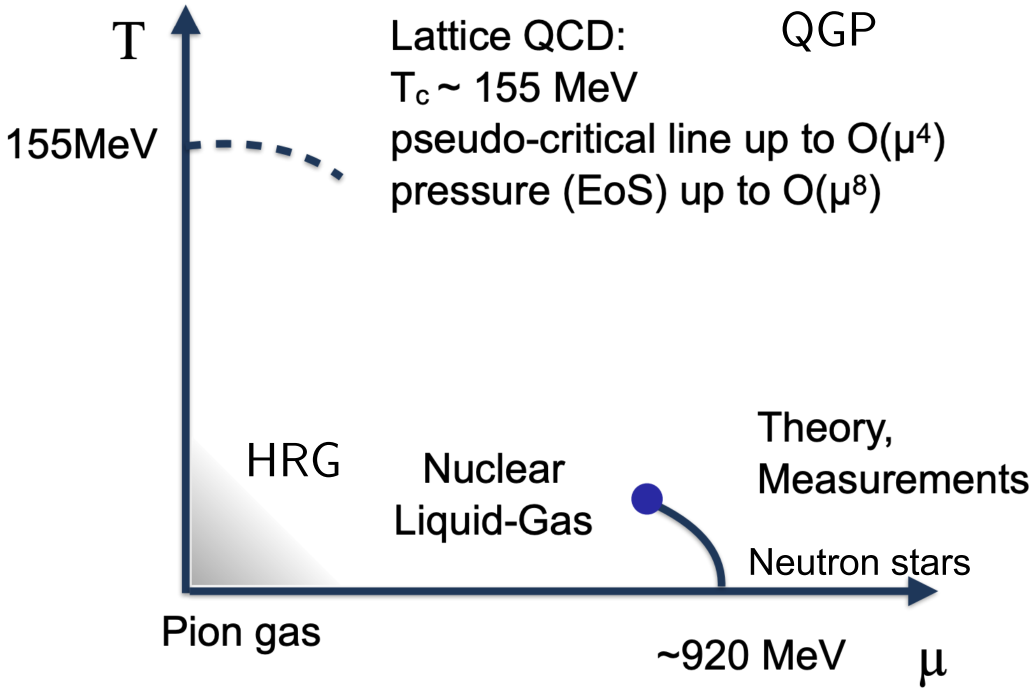


Figure courtesy of V. Koch

What we hope to know

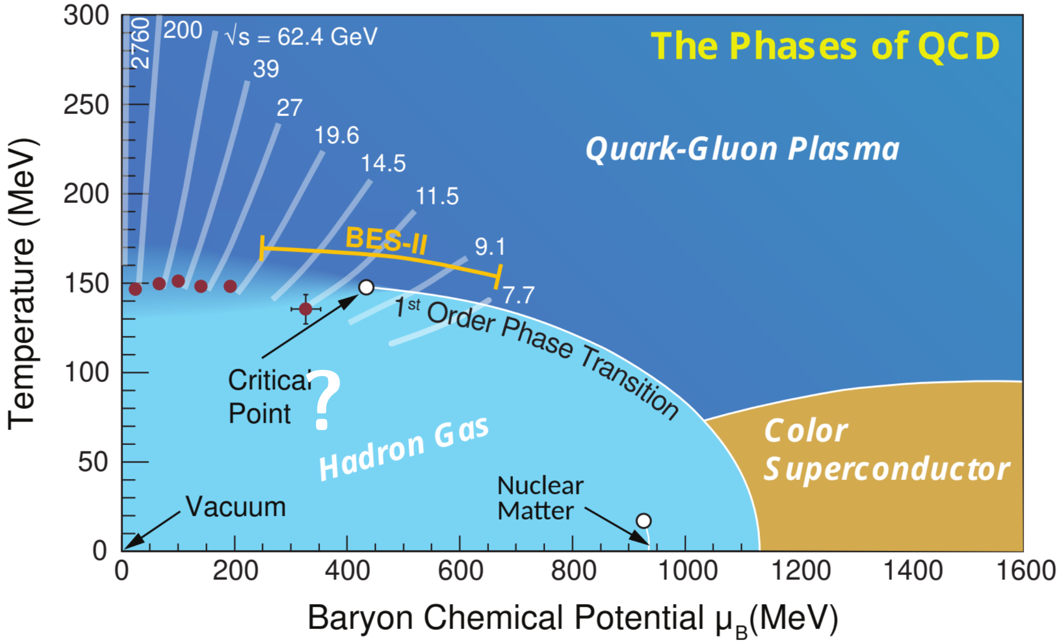


Figure from Bzdak et al., Phys. Rept. '20 & 2015 US Nuclear LRP

"The location of the transition from a gas of hadrons to QGP and the exact nature of this transition is of fundamental interest"

2023 Long Range Plan for Nuclear Science

Critical point predictions from some years ago

Lattice QCD

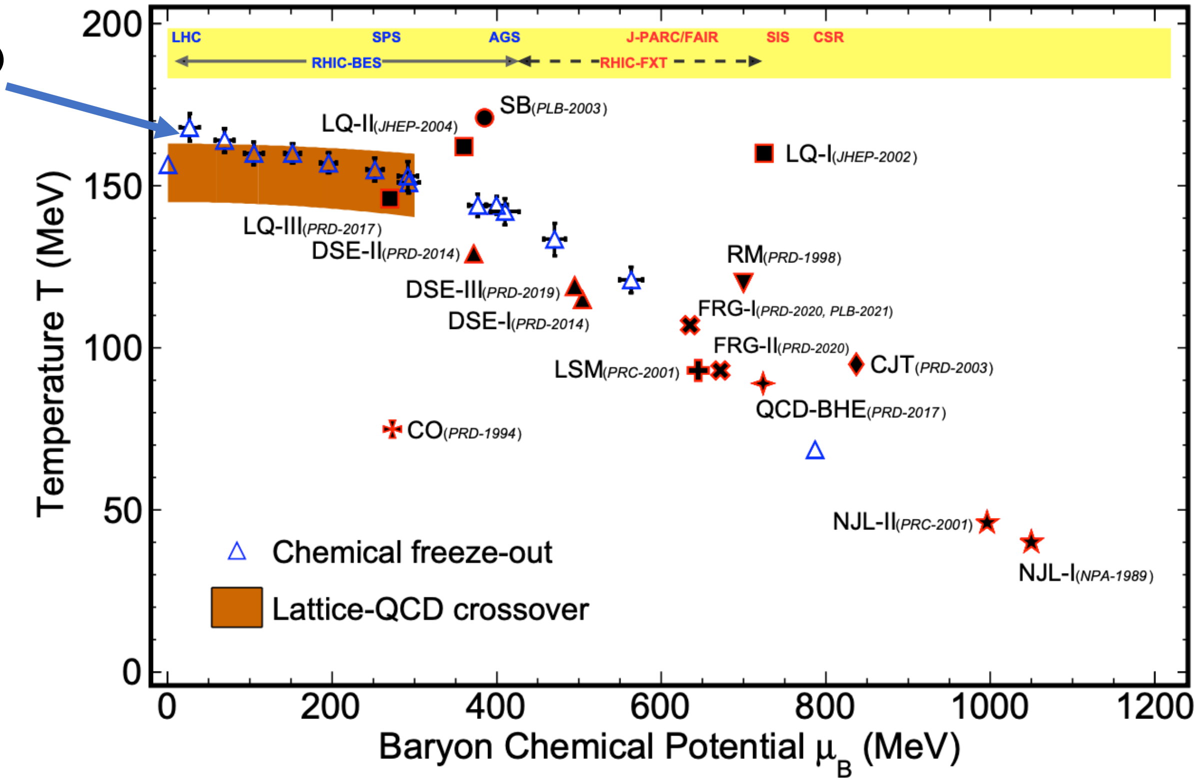


Figure adapted from A. Pandav, D. Mallick, B. Mohanty, Prog. Part. Nucl. Phys. 125 (2022)

- Including the possibility that the QCD critical point does not exist

  de Forcrand, Philipsen, JHEP 01, 077 (2007); VV, Steinheimer, Philipsen, Stoecker, PRD 97, 114030 (2018)

- Lattice QCD excludes the CP at $\mu_B < 450$ MeV on (one-sided) 2σ level

  Borsanyi et al., PRD 112, L111505 (2025)

Extrapolations from lattice QCD at $\mu_B = 0$

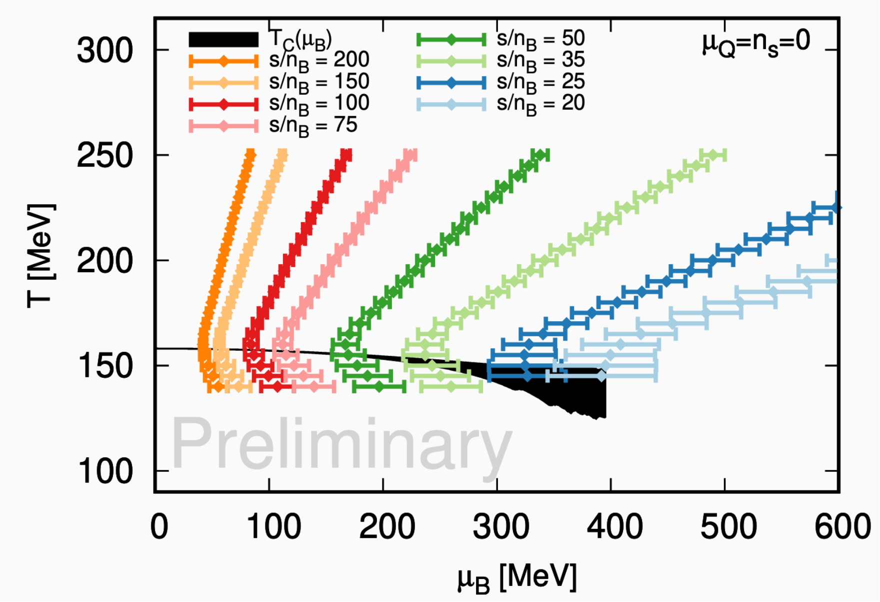
UNIVERSITY OF HOUSTON

Ideally, find the critical point through first-principle lattice QCD simulations at finite μ_B

- Challenging (sign problem), but perhaps not impossible? [Borsanyi et al., Phys. Rev. D 107, 091503L (2023)]

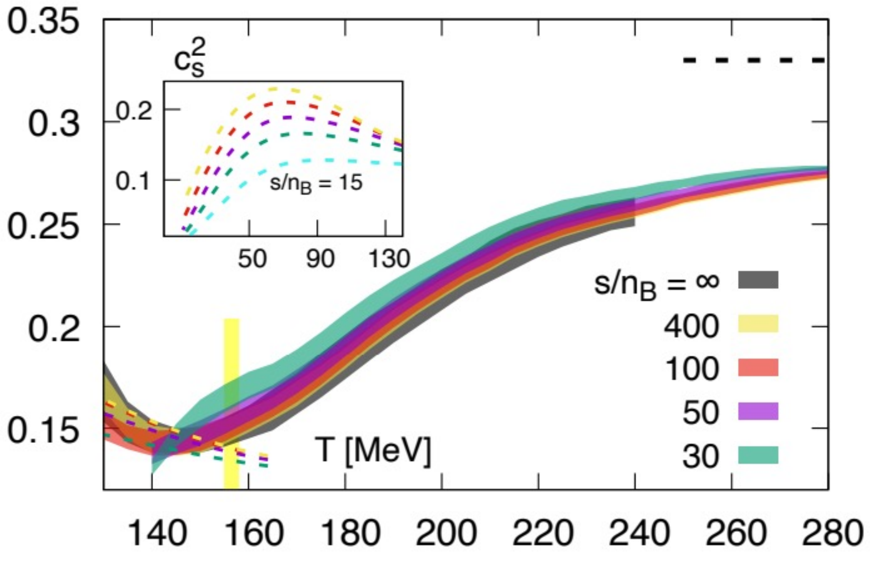
Taylor expansion + various resummations and extrapolation schemes from $\mu_B = 0$

T'-expansion



Borsanyi et al. (WB), Phys. Rev. D 105, 114504 (2022)
Abuali et al. (WB), Phys. Rev. D 112, 052502 (2025)

Padé approximants



Bollweg et al. (HotQCD), Phys. Rev. D 108, 014510 (2023)

No indications for the strengthening of the chiral crossover or critical point signals

Disfavors QCD critical point at $\frac{\mu_B}{T} < 3$

QCD critical point from chiral criticality

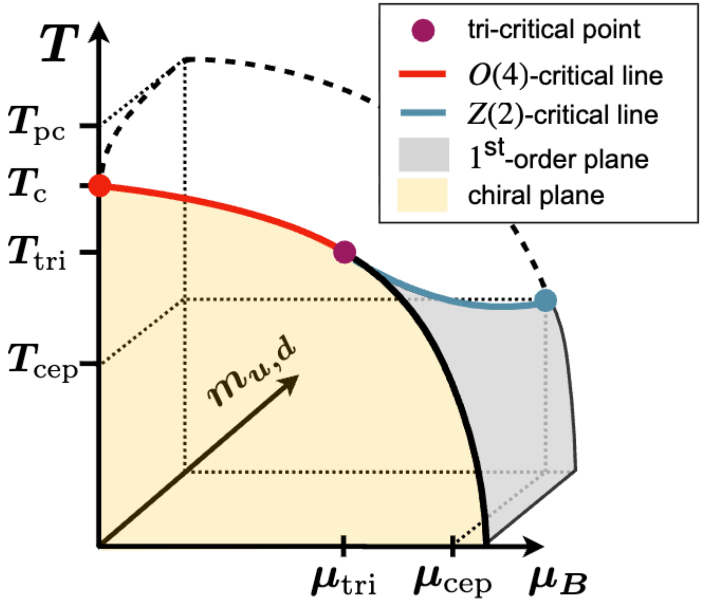
Constraints from chiral criticality

$$T_{pc}^{\text{phys}} > T_c^0 > T_c^{\text{tri}} > T_c^{\text{CP}}$$

Hatta, Ikeda, PRD 67, 014028 (2003)

$$T_{pc}^{\text{phys}} = 156.5 \pm 1.5\ \text{MeV}$$

Bazavov et al., PLB 113, 082001 (2014)

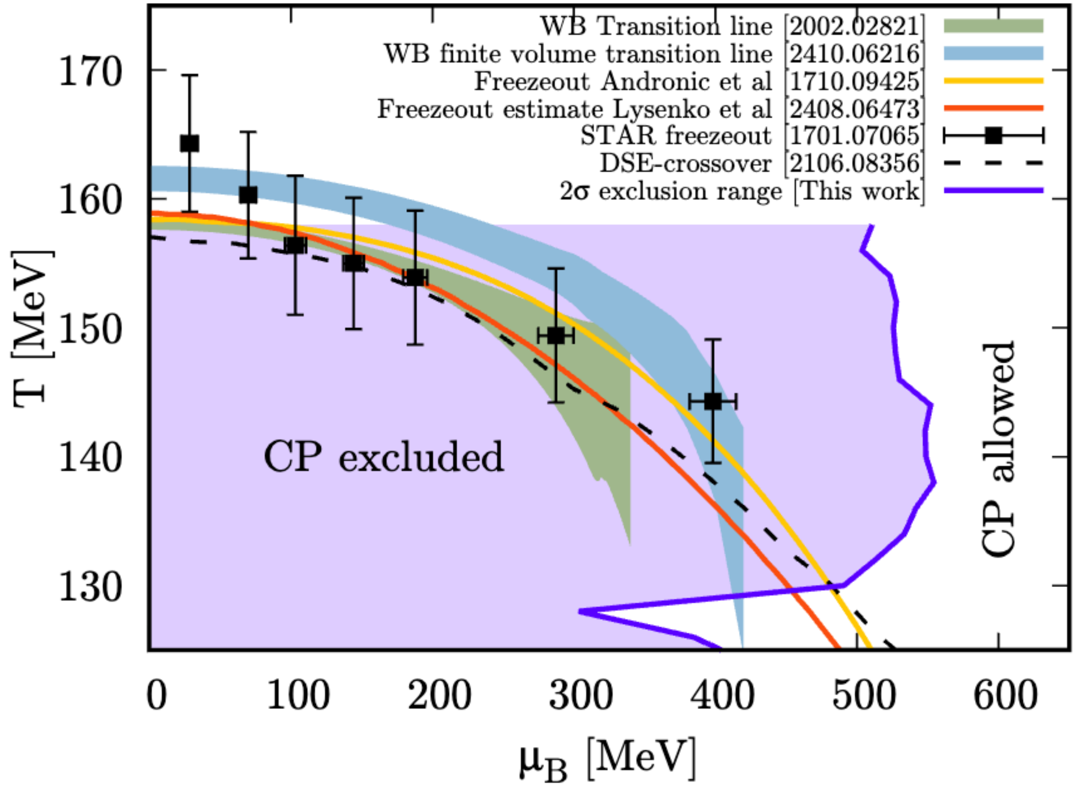


Ding et al., PRD 109, 114516 (2024)

$$T_c^{\text{CP}} < T_c^0 = 132^{+3}_{-6}\,\text{MeV}$$

Ding et al., PRL 123, 062002 (2019)

Constraints from constant entropy contours



No CP at $\mu_B < 450$ MeV (2σ level)

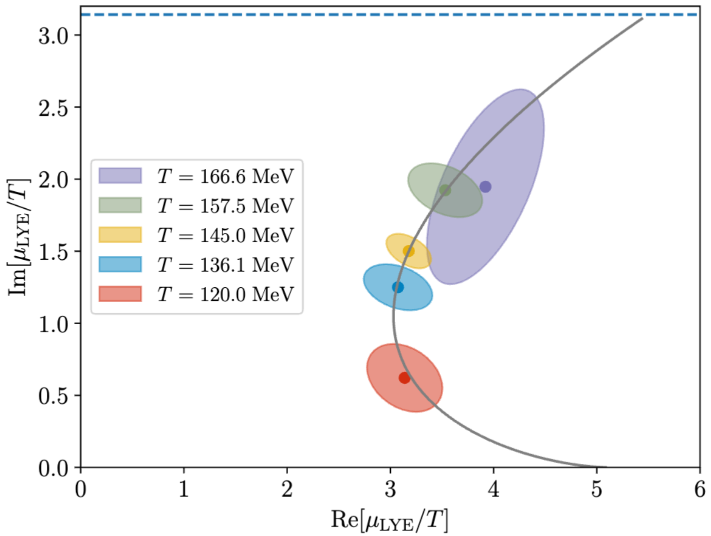
Borsanyi et al. (Budapest-Wuppertal-Houston), PRD 112, L111505 (2025)

Searching for singularities in the complex plane

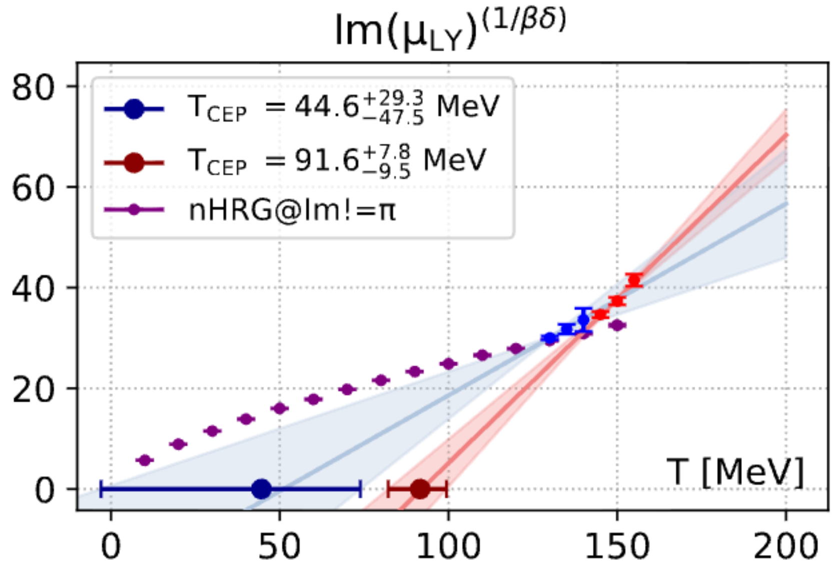
Critical point is a singularity, turns into **Yang-Lee edge singularities** above T_c **in the complex plane**

M. Stephanov, PRD 73, 094508 (2006)

Strategy: Extract YLE singularity through Pade-type fits; see if it approaches the real axis as T decreases



D.A. Clarke et al. (Bielefeld-Parma), PRD 112, L091504 (2025); also G. Basar, PRC 110, 015203 (2024)



A. Adam et al. (Wuppertal-Budapest), arXiv:2507.13254

CP Z(2) scaling inspired fit:

$$\mathrm{Im}\,\mu_{LY} = c(T - T_{CEP})^{\Delta}$$

$$\mathrm{Re}\,\mu_{LY} = \mu_{CEP} + a(T - T_{CEP}) + b(T - T_{CEP})^2$$

Extrapolated CP estimate:

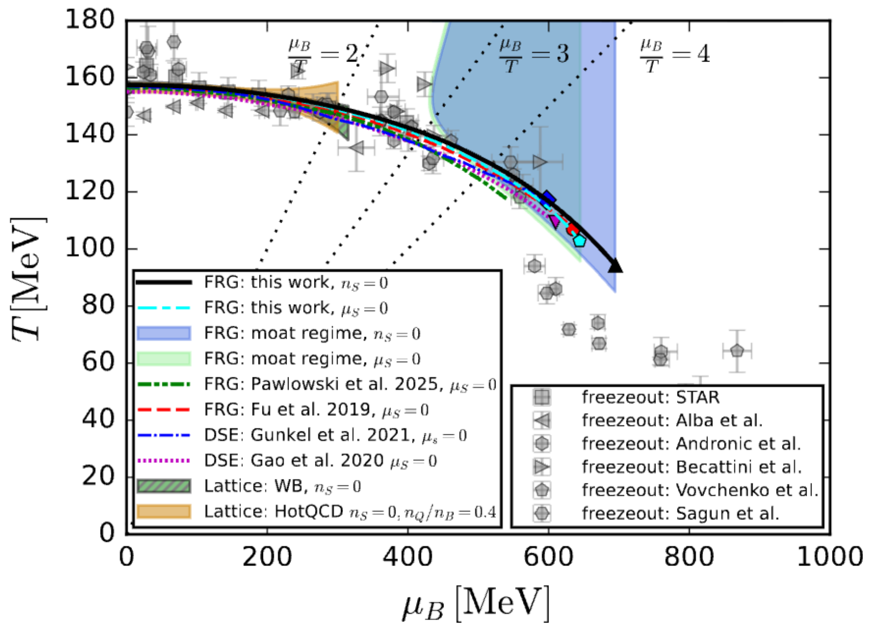
T~90-110 MeV, μ_B~400-600 MeV

Double extrapolation: Away from $\mu_B = 0$ and down in T; variations in fit range matter

Effective QCD theories predictions

UNIVERSITY OF HOUSTON

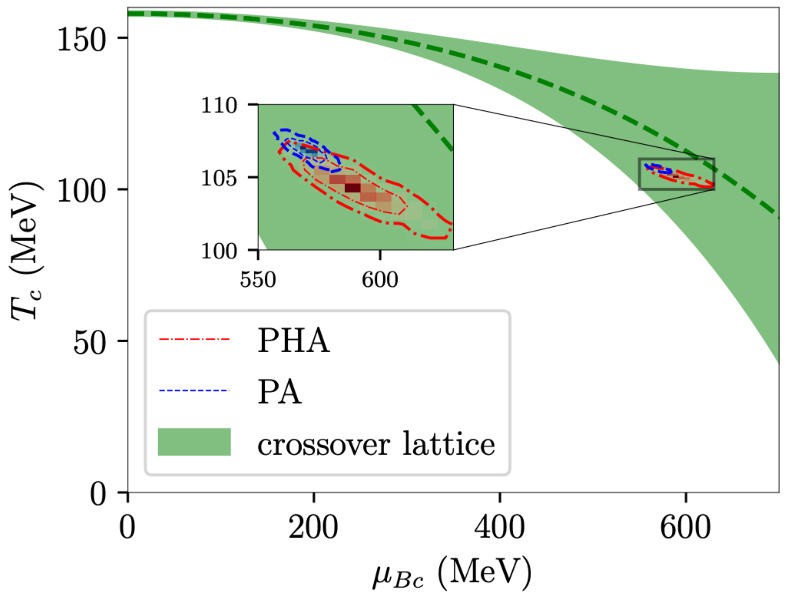
Functional methods: FRG/DSE



Recent compilation from Fu et al., arXiv:2603.13455

$T\sim 90-110$ MeV $\quad \mu_B \sim 600-700$ MeV

Black-hole engineering



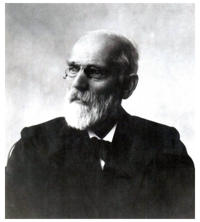
Hippert et al., PRD 110, 094006 (2024)

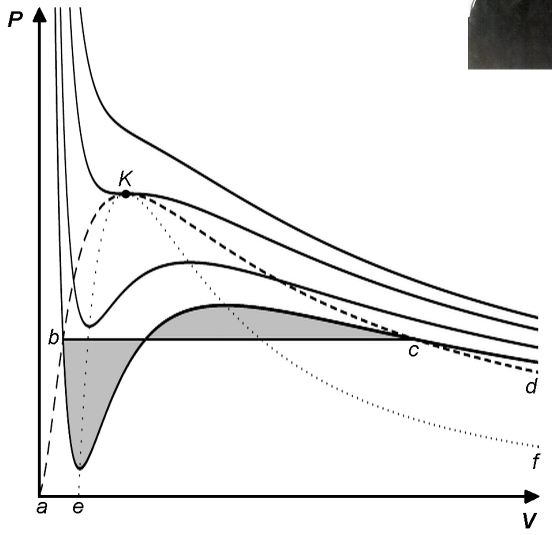
$T\sim 105$ MeV $\quad \mu_B \sim 580$ MeV

- Excellent agreement with LQCD at $\mu_B = 0$ and predict QCD critical point in a similar ballpark
- Restrictions:
  - truncations in functional methods
  - strongly-coupled regime ($\eta/s = 1/4\pi$) in holography

Extrapolating critical point from lattice QCD

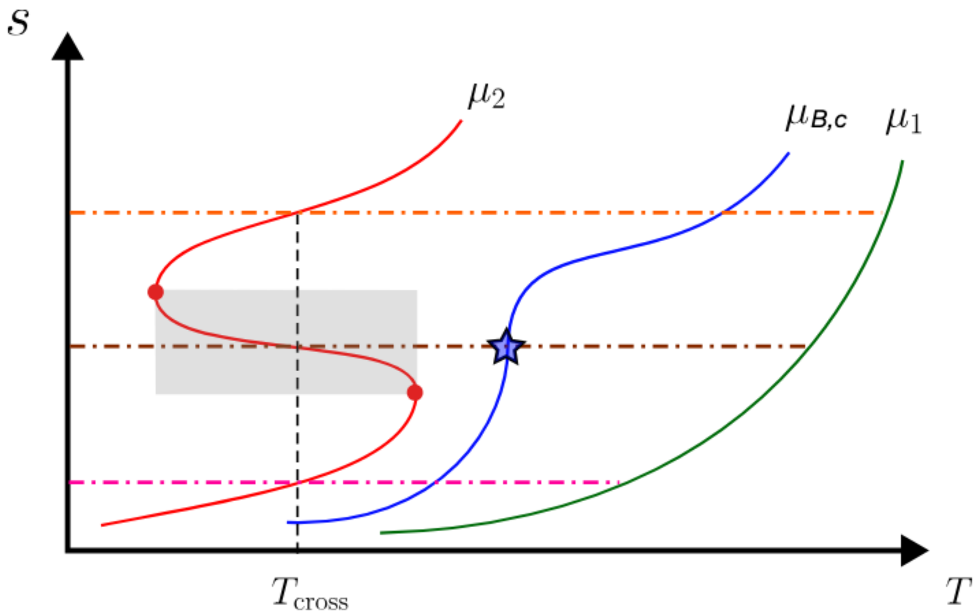
van der Waals (1873)





change of variables

Shah, Hippert, Noronha, Ratti, VV, PRC 113, L012201 (2026)



Critical Point:

$$\left(\frac{\partial P}{\partial \rho_B}\right)_T = 0, \qquad \left(\frac{\partial^2 P}{\partial \rho_B^2}\right)_T = 0.$$

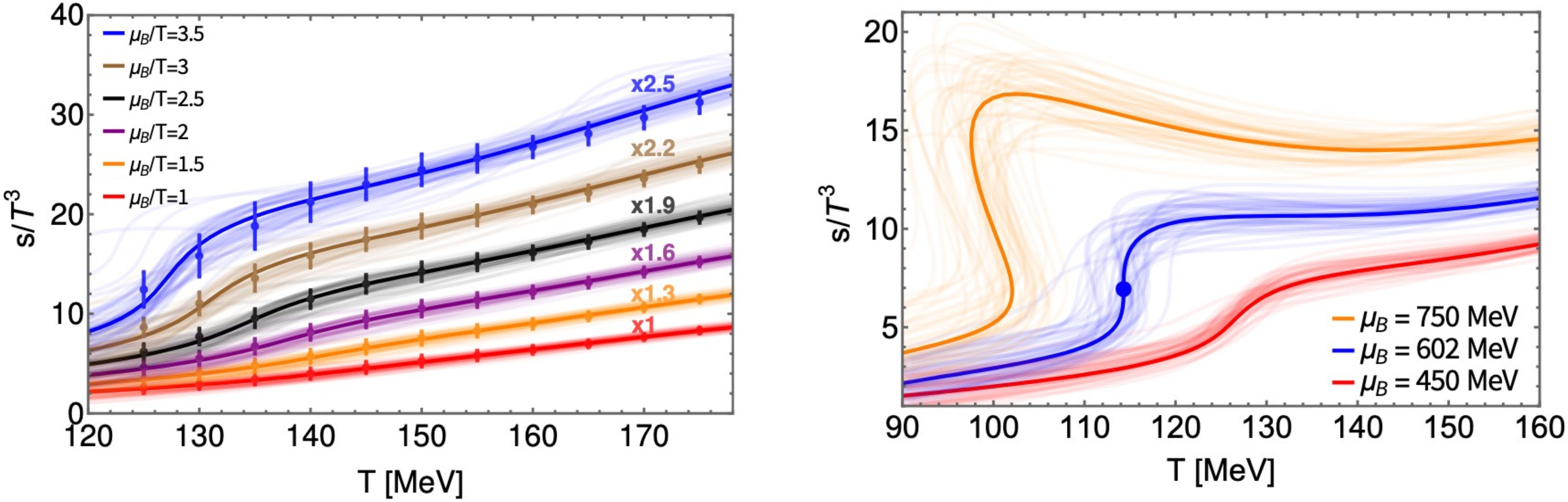
$$\left(\frac{\partial T}{\partial s}\right)_{\mu_B} = 0, \qquad \left(\frac{\partial^2 T}{\partial s^2}\right)_{\mu_B} = 0.$$

Strategy: Follow contours of constant entropy density away from $\mu_B = 0$ and look for crossings

Entropy density at finite μ_B at $O(\mu_B^2)$

UNIVERSITY OF HOUSTON

$$T_s(\mu_B; T_0) = T_0 + \alpha_2(T_0)\frac{\mu_B^2}{2} \qquad \alpha_2(T_0) = -\frac{2T_0\chi_2^B(T_0) + T_0^2\chi_2^{B'}(T_0)}{s'(T_0)}$$



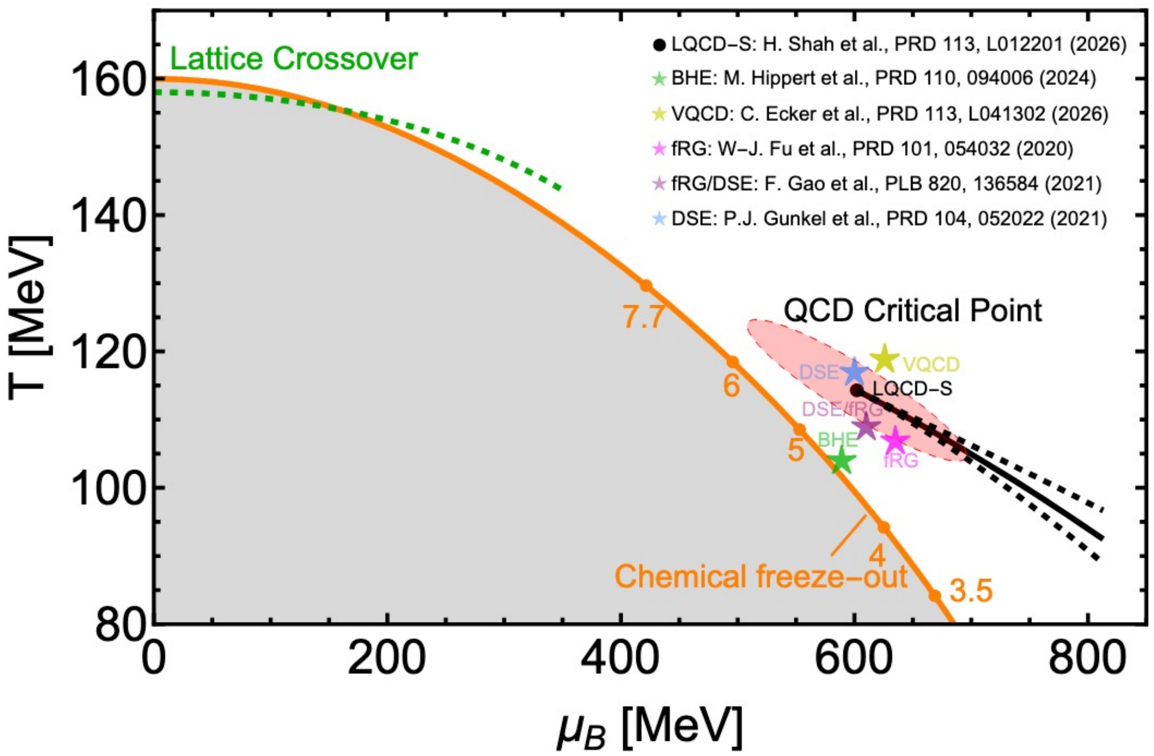
- Excellent agreement at low μ_B/T with available lattice QCD constraints from T'-expansion

Borsanyi et al., PRL 126, 232001 (2021)

- First-order phase transition emerges at $\mu_B > 600$ MeV

Shah, Hippert, Noronha, Ratti, VV, PRC 113, L012201 (2026)

Critical point estimates



Critical point estimate at $O(\mu_B^2)$:

$T_c = 114 \pm 7$ MeV, $\mu_B = 602 \pm 62$ MeV

Estimates from recent literature:

YLE-1: D.A. Clarke et al. (Bielefeld-Parma), PRD 112, L091504 (2025)

YLE-2: G. Basar, PRC 110, 015203 (2024)

BHE: M. Hippert et al., PRD 110, 094006 (2024)

fRG: W-J. Fu et al., PRD 101, 054032 (2020)

DSE/fRG: Gao, Pawlowski., PLB 820, 136584 (2021)

DSE: P.J. Gunkel et al., PRD 104, 052022 (2021)

VQCD: C. Ecker et al., PRD 113, L041302 (2026)

Optimist's view: Different estimates converge onto the same region because QCD CP is likely there

Pessimist's view: Different estimates converge onto the same region because it's the closest not yet ruled out by LQCD

"...experimental measurements are essential to determine whether a QCD critical point exists."

2023 Long Range Plan for Nuclear Science

Fluctuations in heavy-ion collisions

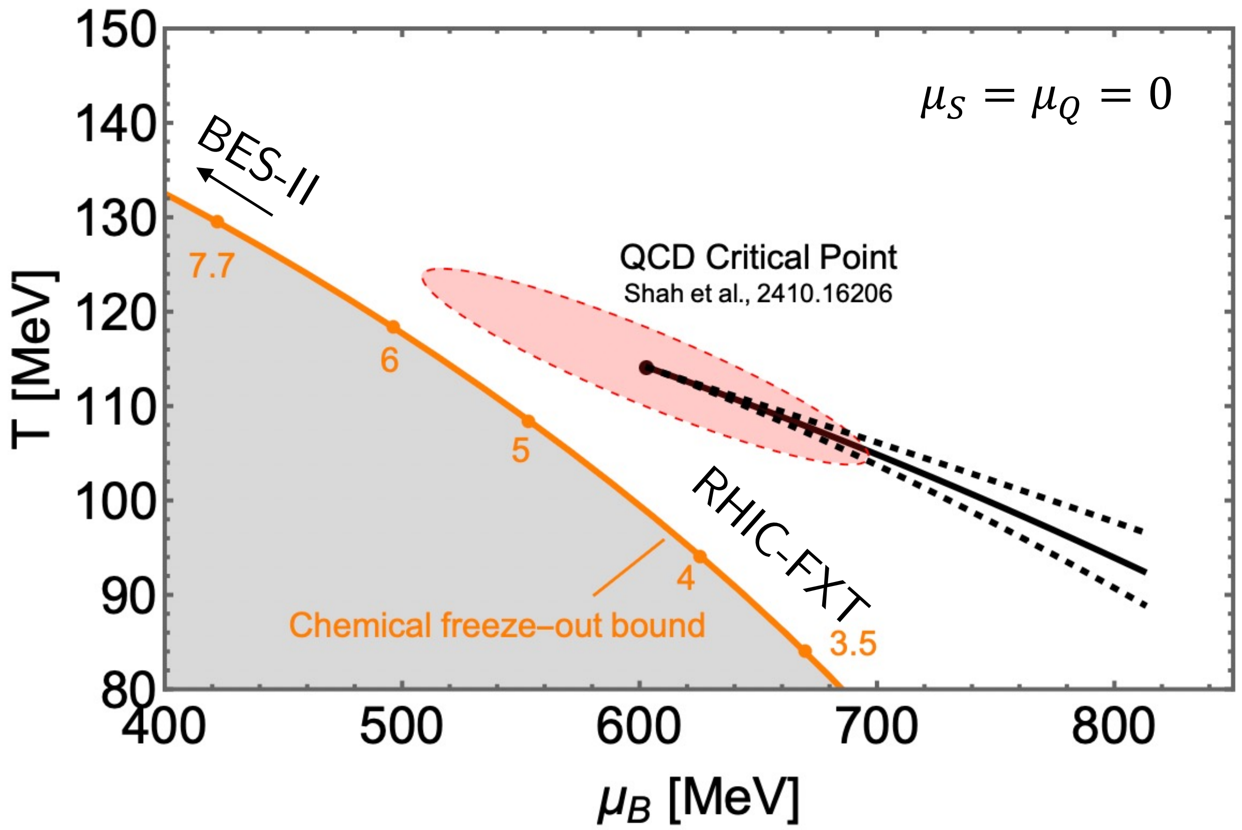
Critical point and heavy-ion collisions

Control parameters

- Collision energy $\sqrt{s_{NN}} = 2.4 - 5020$ GeV
  - Scan the QCD phase diagram
- Size of the collision region
  - Expect stronger signal in larger systems

Measurements

- Final hadron abundances and momentum distributions **event-by-event**



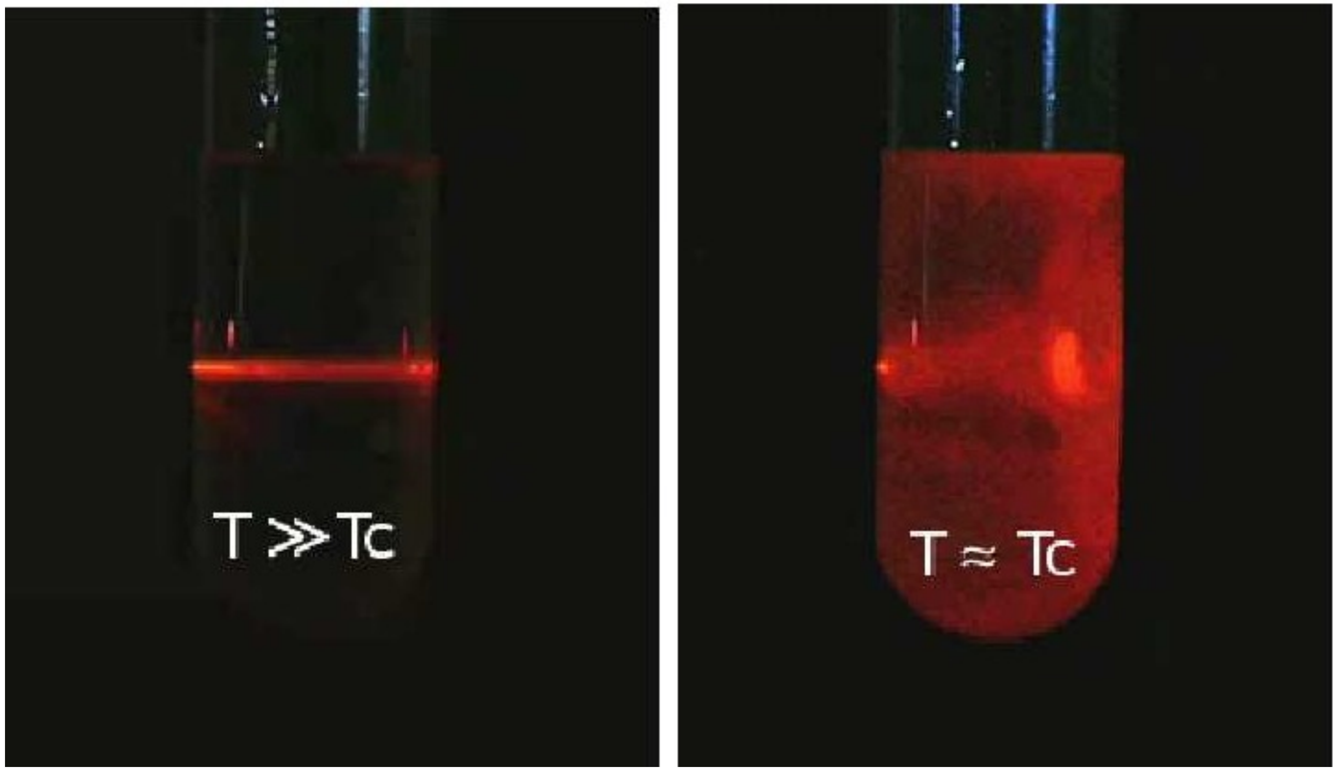
Chemical freeze-out curve and CP

- Sets the **lower bound** on the temperature of the CP [Lysenko, Poberezhnyuk, Gorenstein, VV, PRC 111, 054903 (2025)]
- **Caveats:** strangeness neutrality ($\mu_S \neq 0$), uncertainty in the freeze-out curve
- CP may be close to freeze-out line at $\sqrt{s_{NN}} \sim 3.5 - 5$ GeV

Critical point and fluctuations

Density fluctuations at macroscopic length scales

Critical opalescence



Unfortunately, we cannot do this in heavy-ion collisions. What can we do?

Event-by-event fluctuations and statistical mechanics

Consider a fluctuating number N

Cumulants: $G_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$

variance $\kappa_2 = \langle(\Delta N)^2\rangle = \sigma^2$  width

skewness $\kappa_3 = \langle(\Delta N)^3\rangle$  asymmetry

kurtosis $\kappa_4 = \langle(\Delta N)^4\rangle - 3\langle(\Delta N^2)\rangle^2$  peak shape

Experiment:

$$P(N) \sim \frac{N_{events}(N)}{N_{events}^{total}}$$

Statistical mechanics:

Grand partition function $$\ln Z^{\mathrm{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N} Z^{\mathrm{ce}}(T,V,N)\right],$$

$$\kappa_n \propto \frac{\partial^n(\ln Z^{\mathrm{gce}})}{\partial(\mu_N)^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

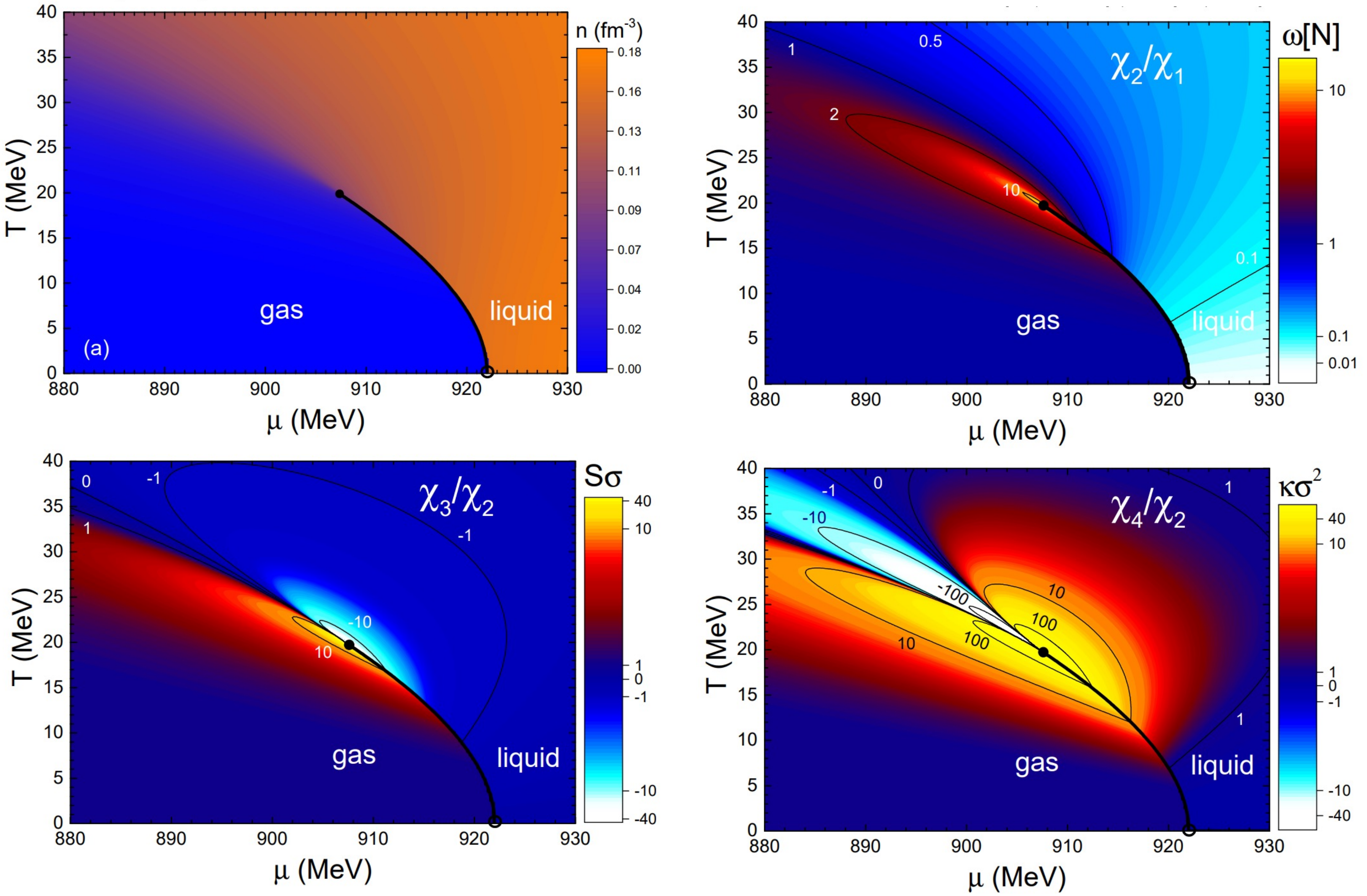
Example: (Nuclear) Liquid-gas transition

(QCD) critical point: large correlation length and fluctuations

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

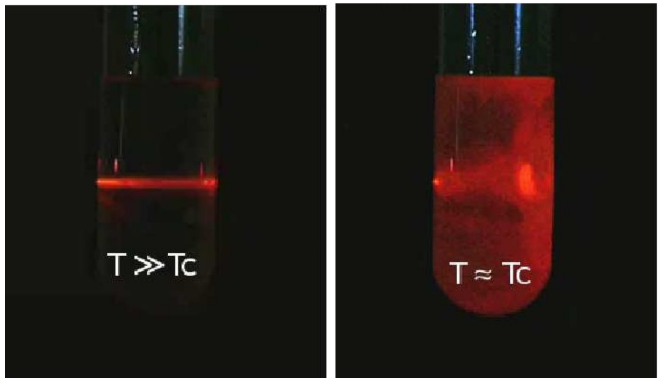
$$\xi \to \infty$$

M. Stephanov, PRL '09, '11



VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Critical opalescence

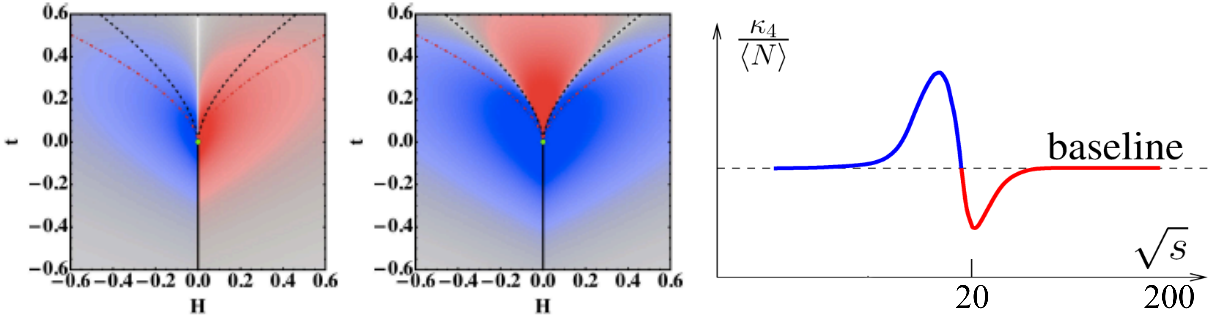


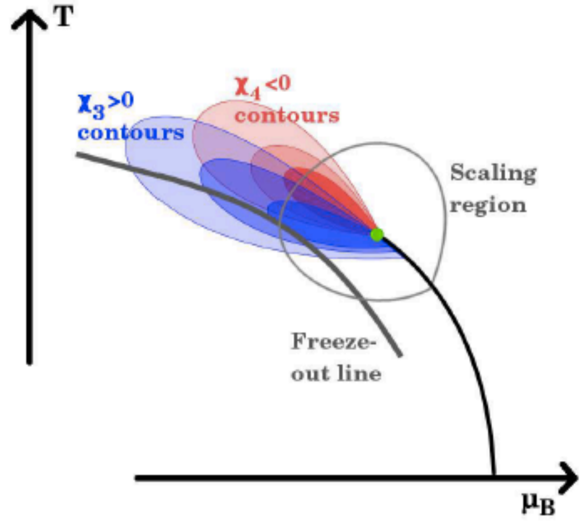
$$\langle N^2\rangle - \langle N\rangle^2 \sim \langle N\rangle \sim 10^{23}$$

in equilibrium

Equilibrium Expectations and Beam Energy Scan

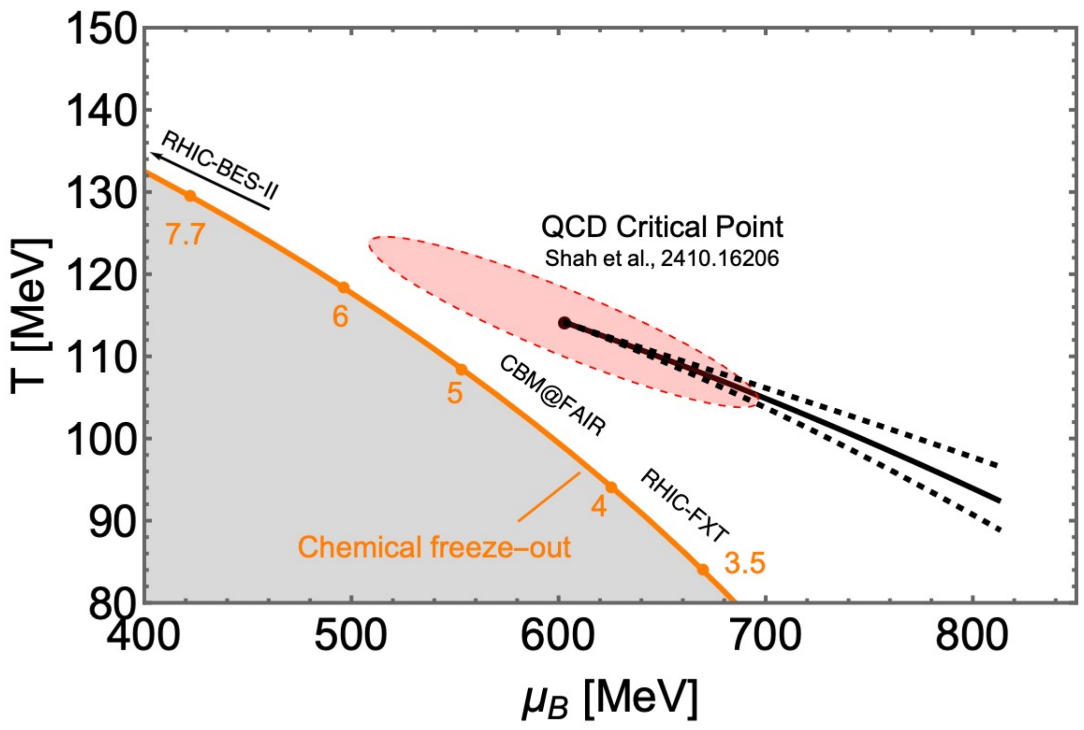
Expectation from Calculations





Characteristic "Oscillating pattern" is expected for the QCD critical point but *the exact shape depends on the location of freeze-out with respect to the location of CP*

- M. Stephanov, ***PRL107***, 052301(2011)
- V. Skokov, Quark Matter 2012
- J.W. Chen, J. Deng, H. Kohyyama, arXiv: 1603.05198, Phys. Rev. D93 (2016) 034037

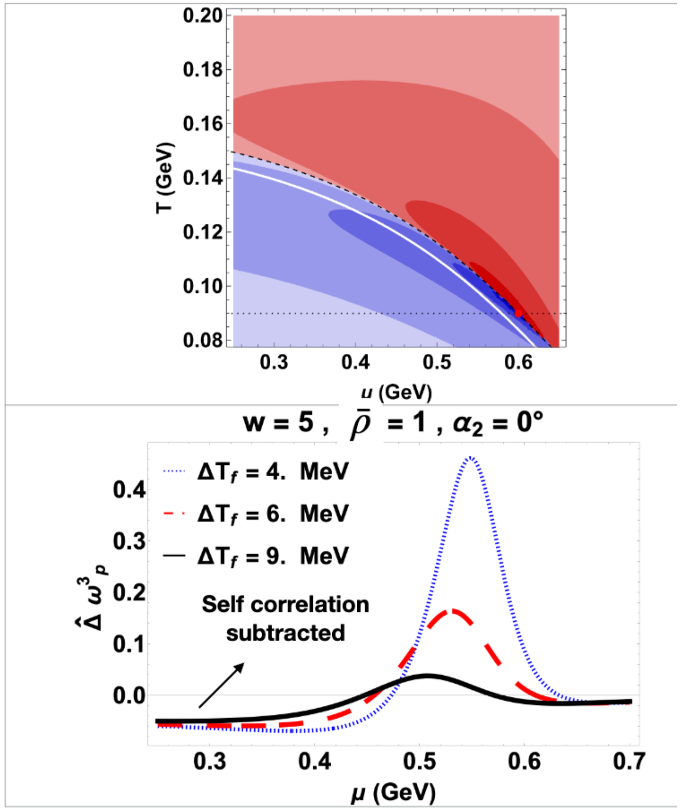


N. Xu, CPOD 2016

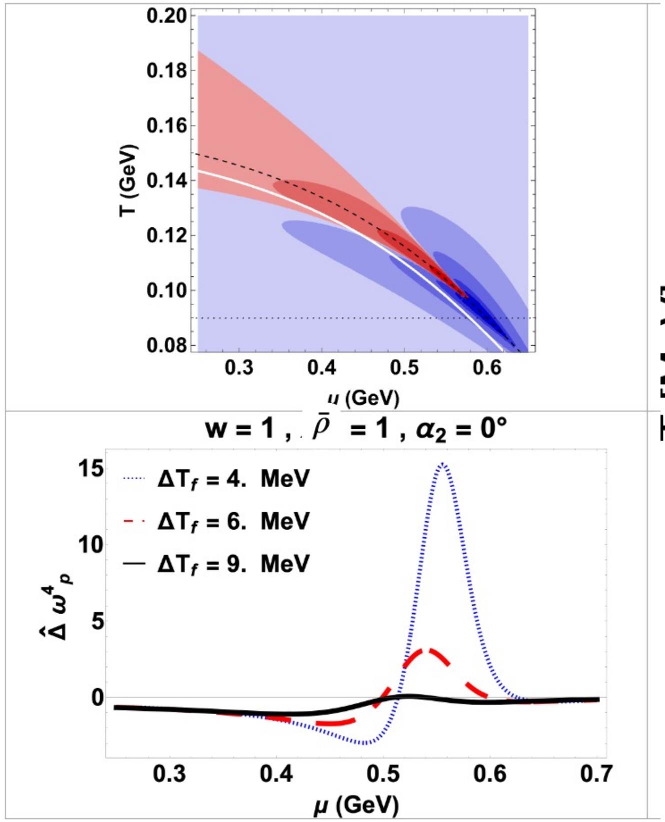
Equilibrium Expectations and Beam Energy Scan

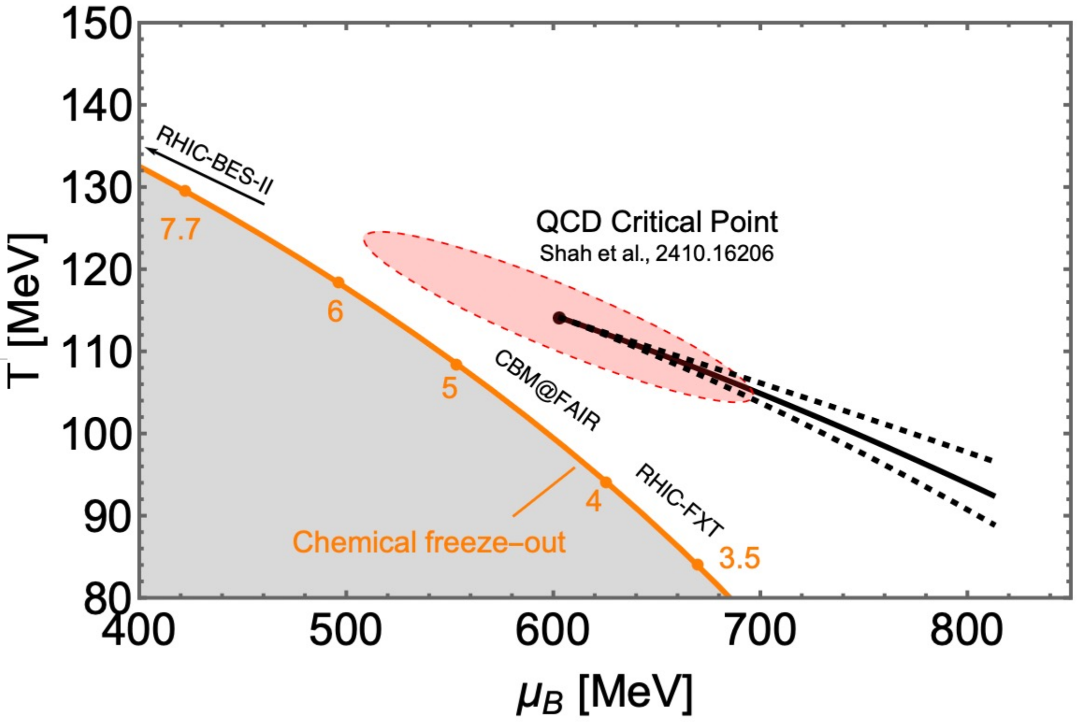
UNIVERSITY OF HOUSTON

3rd order



4th order





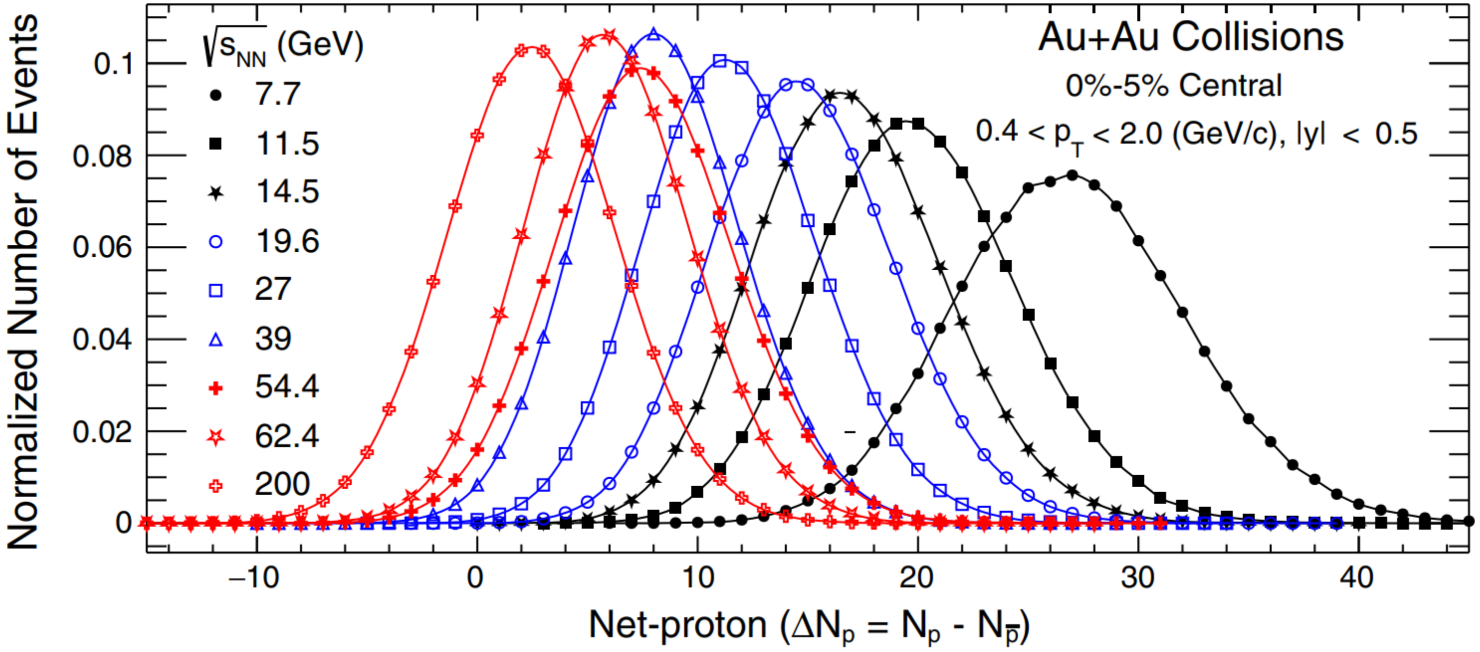
Ising-T' EoS + maximum entropy freeze-out [Karthein, et al., arXiv:2508.19237]

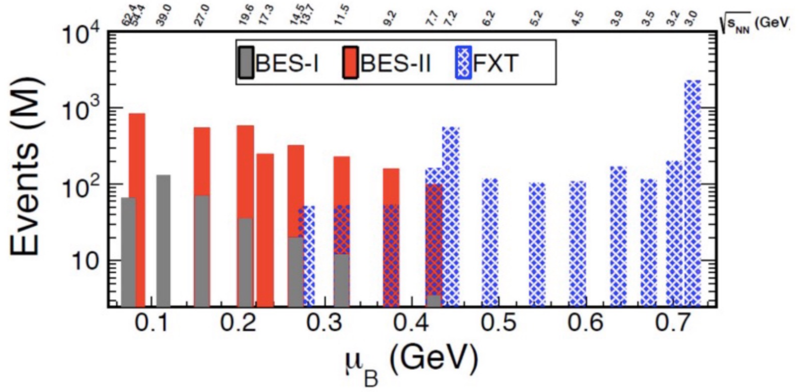
Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons

$$P(\Delta N_p) \sim \frac{N_{\text{events}}(\Delta N_p)}{N_{events}^{total}}$$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)





Statistics-hungry observables

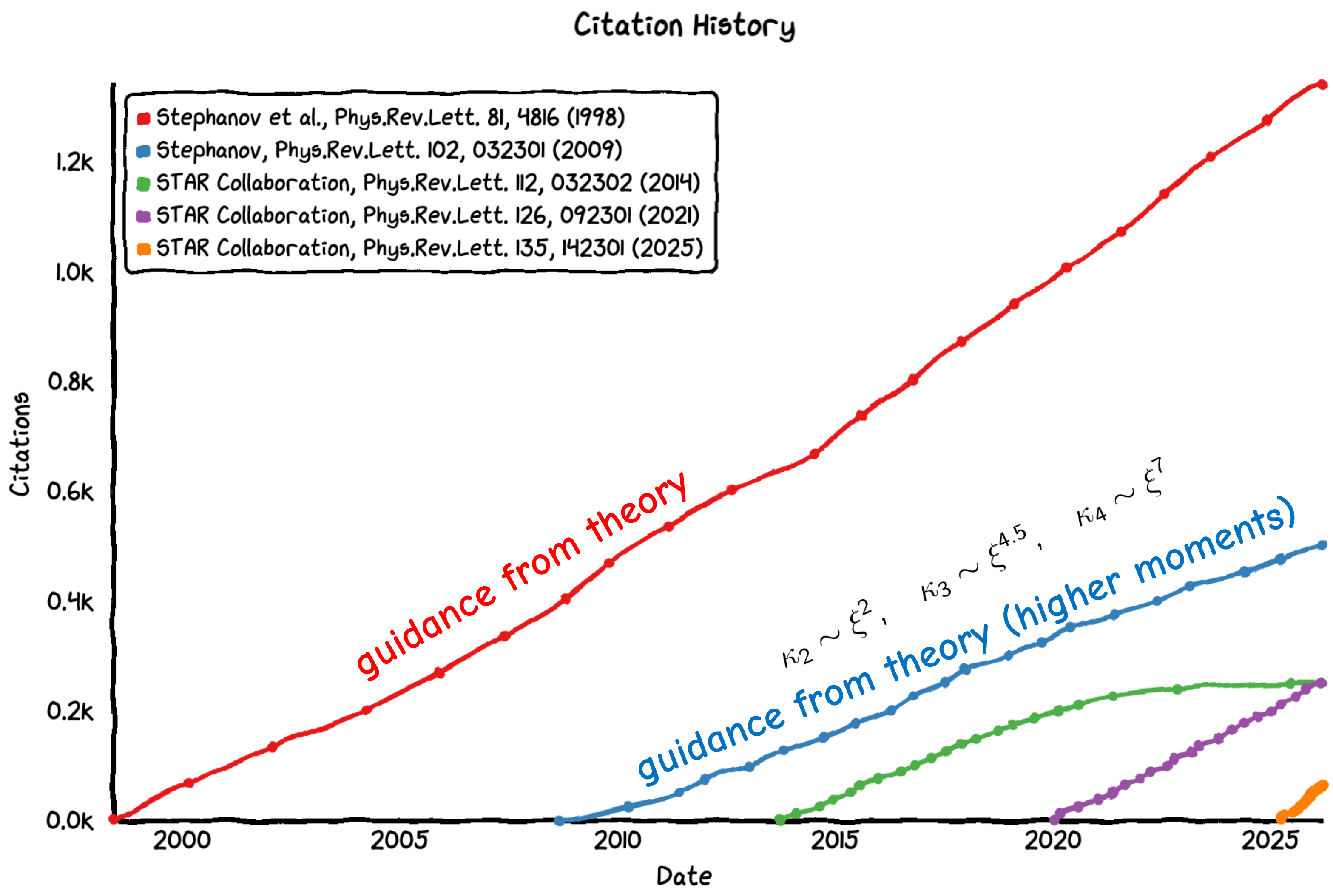
Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

$$\frac{\kappa_2}{\langle N \rangle}, \quad \frac{\kappa_3}{\kappa_2}, \quad \frac{\kappa_4}{\kappa_2}$$

Look for subtle critical point signals

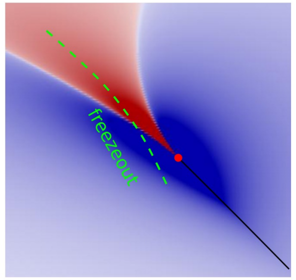
History of proton cumulants at RHIC

UNIVERSITY OF HOUSTON

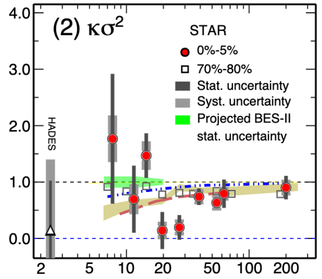


Consistently high interest over the RHIC operation lifetime

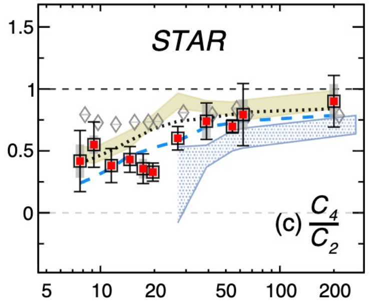
RHIC 25:
A quarter century of discovery



BES-I

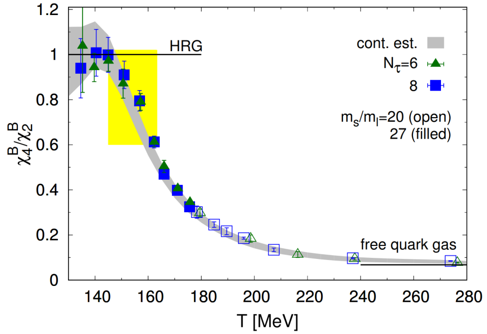


BES-II



Theory vs experiment

guidance from theory (e.g. lattice)



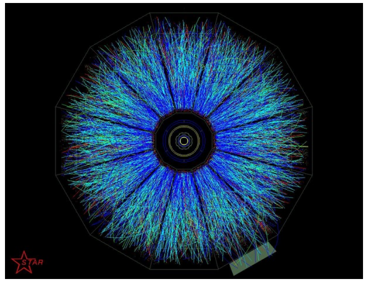
hydro simulation

hadronic phase

canonical effects

kinematic cuts

experiment (the real thing)



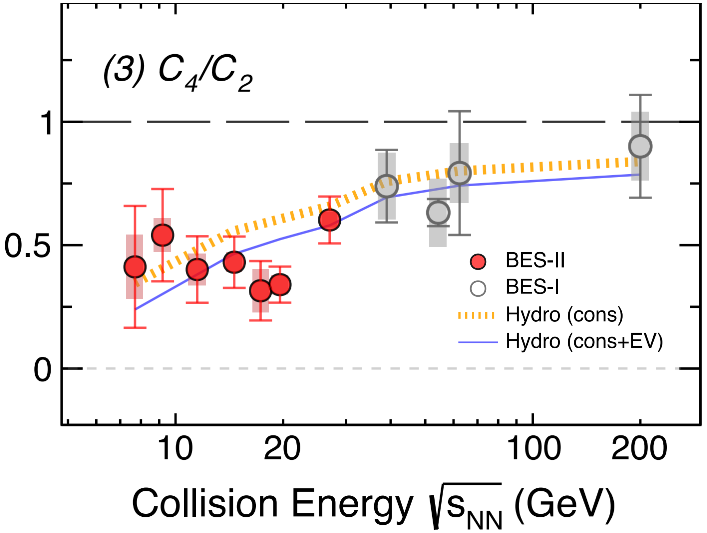
event selection

track reconstruction

efficiency correction

CBWC

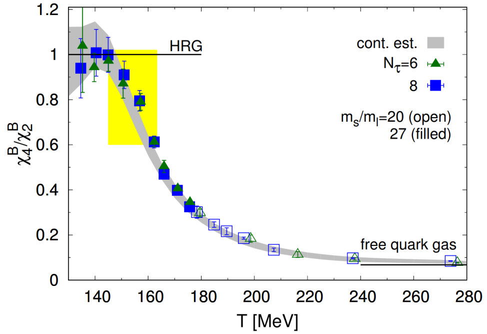
purity

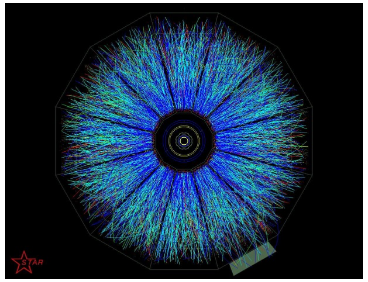


Theory vs experiment

guidance from theory (e.g. lattice)

experiment (the real thing)





This was done in [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)]

- Full hydro simulation
- Lattice QCD-like baryon susceptibilities (interacting HRG)
- Experimental kinematic cuts
- Global baryon conservation: SAM correction

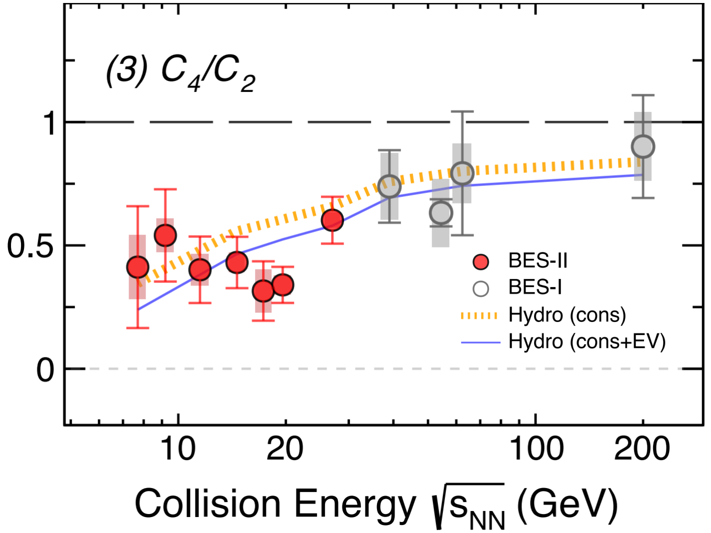
Non-critical baseline (hydro EV) prediction

hydro simulation

hadronic phase

canonical effects

kinematic cuts



event selection

track reconstruction

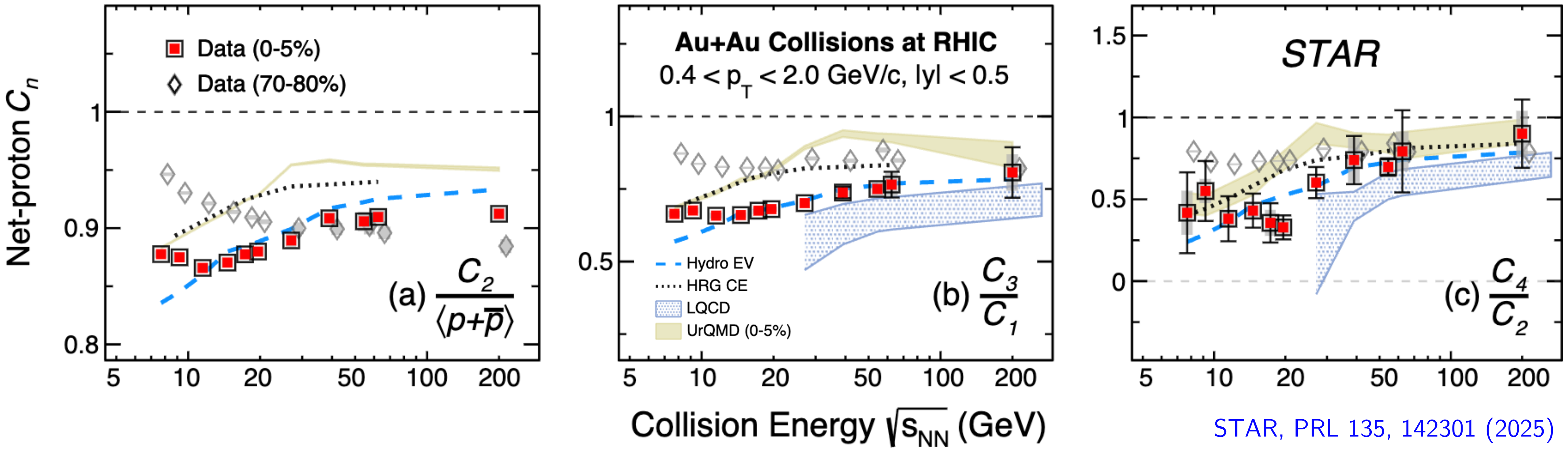
efficiency correction

CBWC

purity

RHIC-BES-II data: Ordinary cumulants

Net-proton cumulant ratios



STAR, PRL 135, 142301 (2025)

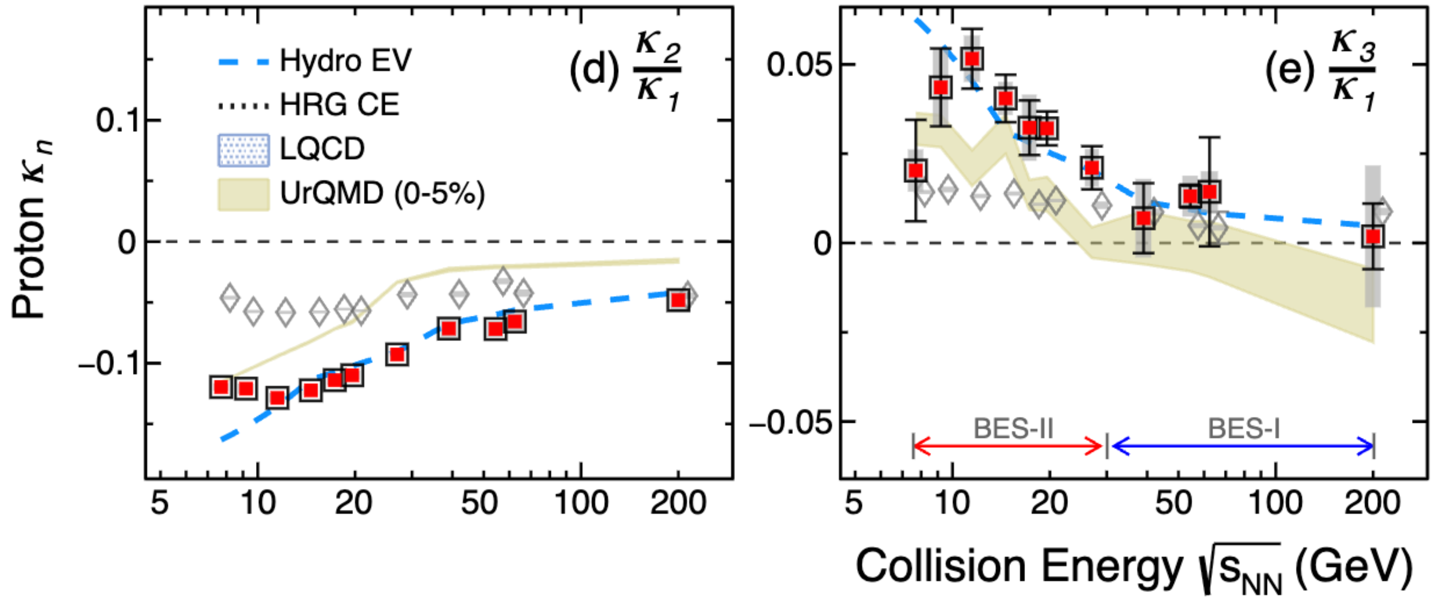
Hydro EV: VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Agreement with the baseline above $\sqrt{s_{NN}} \sim 10 - 20$ GeV

Some deviations at lower energies, but mostly uneventful. What else is there?

RHIC-BES-II data: Factorial cumulants

Proton factorial cumulant ratios

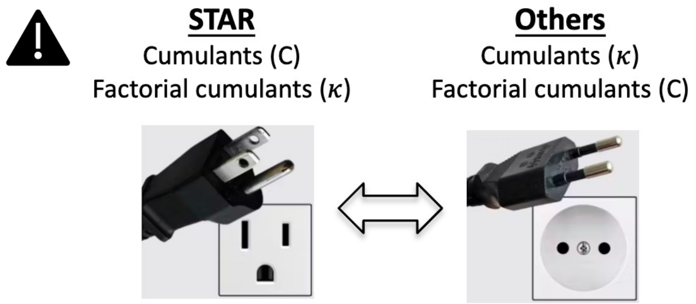


STAR, PRL 135, 142301 (2025)

Hydro EV: VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

More structure seen in factorial cumulants

- Non-monotonic κ_2/κ_1, κ_3/κ_1, and possibly κ_4/κ_1



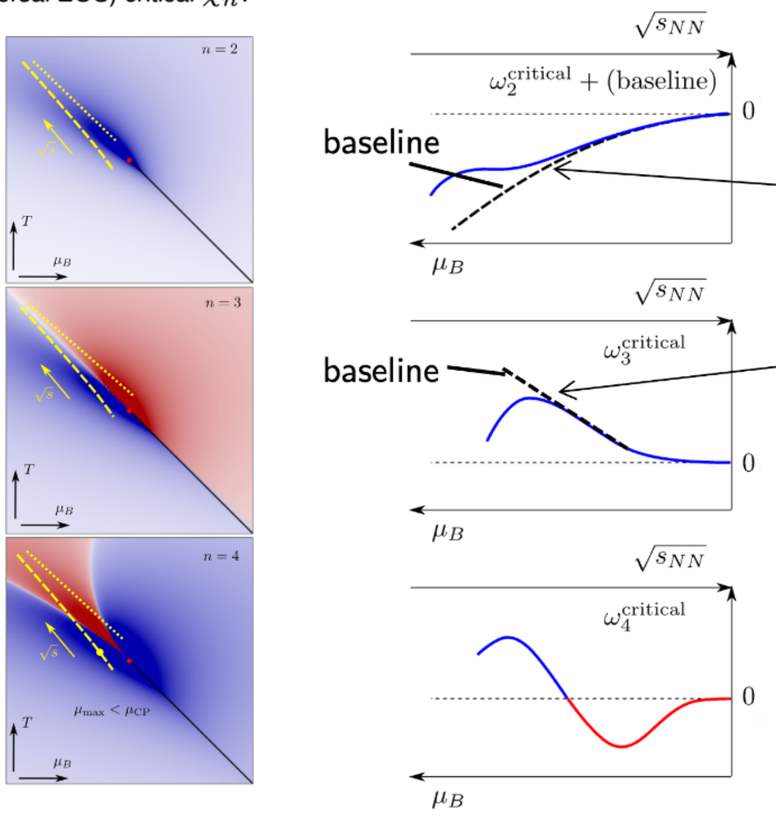
From M. Arslandok, QM2025

RHIC-BES-II data and CP

VV, Koch, arXiv:2504.01368, plot adapted from M. Stephanov, arXiv:2410.02861

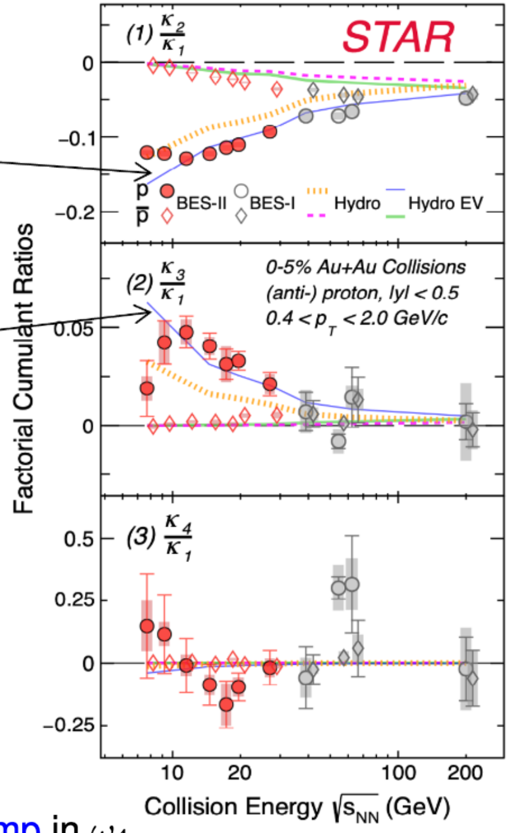
$\omega_n = \hat{C}_n / \hat{C}_1$

(universal EOS) critical χ_n:



BES-II data:

plot from A. Pandav, CPOD2024



Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

Non-critical baseline (hydro EV):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

- describes right side of the peak in $\hat{C}_3$
- **signal relative to baseline:**
  - *positive* $\hat{C}_2 - \hat{C}_2^{baseline} > 0$
  - *negative* $\hat{C}_3 - \hat{C}_3^{baseline} < 0$

New scenarios: cool critical point

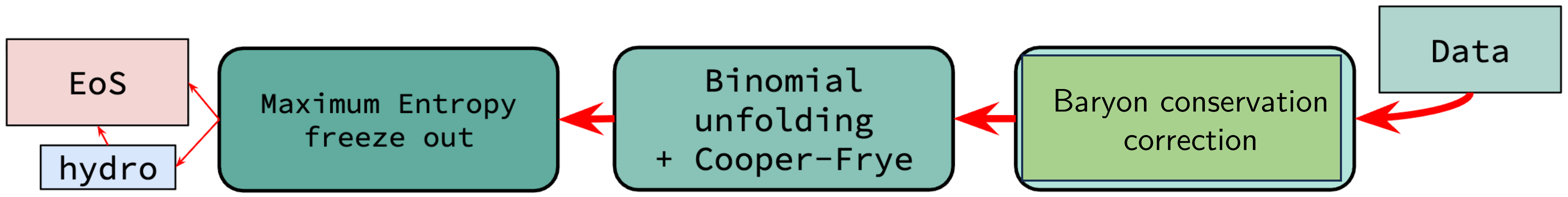
Basar, Pradeep, Stephanov, arXiv:2603.23635

Controlling the non-critical baseline is essential

Or we go beyond comparisons to baselines

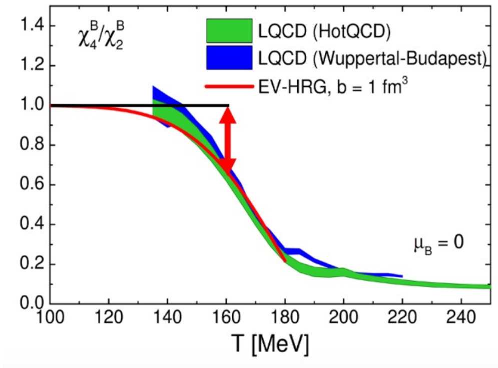
Inference of baryon number susceptibilities from data

G. Pihan, R. Poberezhniuk, VV, to appear

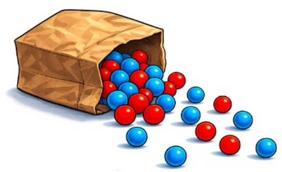


Generalizing Hydro EV

Hydro EV [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)]



1. Smooth (3+1)-D hydro evolution with MUSIC [Shen, Alzhrani, PRC 102, 014909 (2020)]
2. Grand-canonical particlization of excluded volume HRG fitted to lattice data at $\mu_B = 0$ [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
   - model-dependent and allows only repulsion
3. Kinematic/efficiency cuts using Cooper-Frye formula and isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 86, 044904 (2012)]



$0.4 < p_T < 2$ GeV, $|y| < 0.5$

4. Correction for baryon conservation using SAM-2.0 [VV, Phys. Rev. C 105, 014903 (2022)]
   - Balancing contribution from baryon conservation

New developments:

- Replace EV-HRG parametrization by MaxEnt freeze-out → probe arbitrary χ_n^B
- SAM-3.0 correction: technical simplification and minor correction to SAM-2.0 results

Generalizing Hydro EV: new elements

UNIVERSITY OF HOUSTON

Maximum entropy freeze-out [M. Pradeep et al., PRL 130, 162301 (2023); arXiv:2508.19237]

- Incorporate single hydrodynamic mode – baryon density fluctuations
- Maximum entropy method defines local baryon/antibaryon joint susceptibilities

G. Pihan et al., to appear

$$\hat{\chi}^{+-}_{nm}(x) = \delta_{m0}\delta_{n1}\bar{\chi}^{+}_{1} + \delta_{n0}\delta_{m1}\bar{\chi}^{-}_{1} + (-1)^m \underline{\hat{\Delta}\chi^{B}_{n+m}(x)} \frac{\bar{\chi}^{+}_{1}(x)^n \bar{\chi}^{-}_{1}(x)^m}{(\bar{\chi}^{+}_{1}(x) + \bar{\chi}^{-}_{1}(x))^{n+m}}$$

difference to HRG

$$\bar{\chi}^{B}_{n}(x) = \bar{\chi}^{+}_{1}(x) + (-1)^n \bar{\chi}^{-}_{1}(x)$$

HRG (Poisson) baseline

$+$ is baryon, $-$ is antibaryon

- Parametrize the EoS by $\chi_2^B/\overline{\chi_2^B}$, χ_3^B/χ_1^B, χ_4^B/χ_2^B ratios at each energy

Subensemble acceptance method 3.0 R. Poberezhniuk, V.A. Kuznietsov, G. Pihan, VV, to appear

- Correction for baryon number conservation
  - Compute grand-canonically and perform correction at the final step

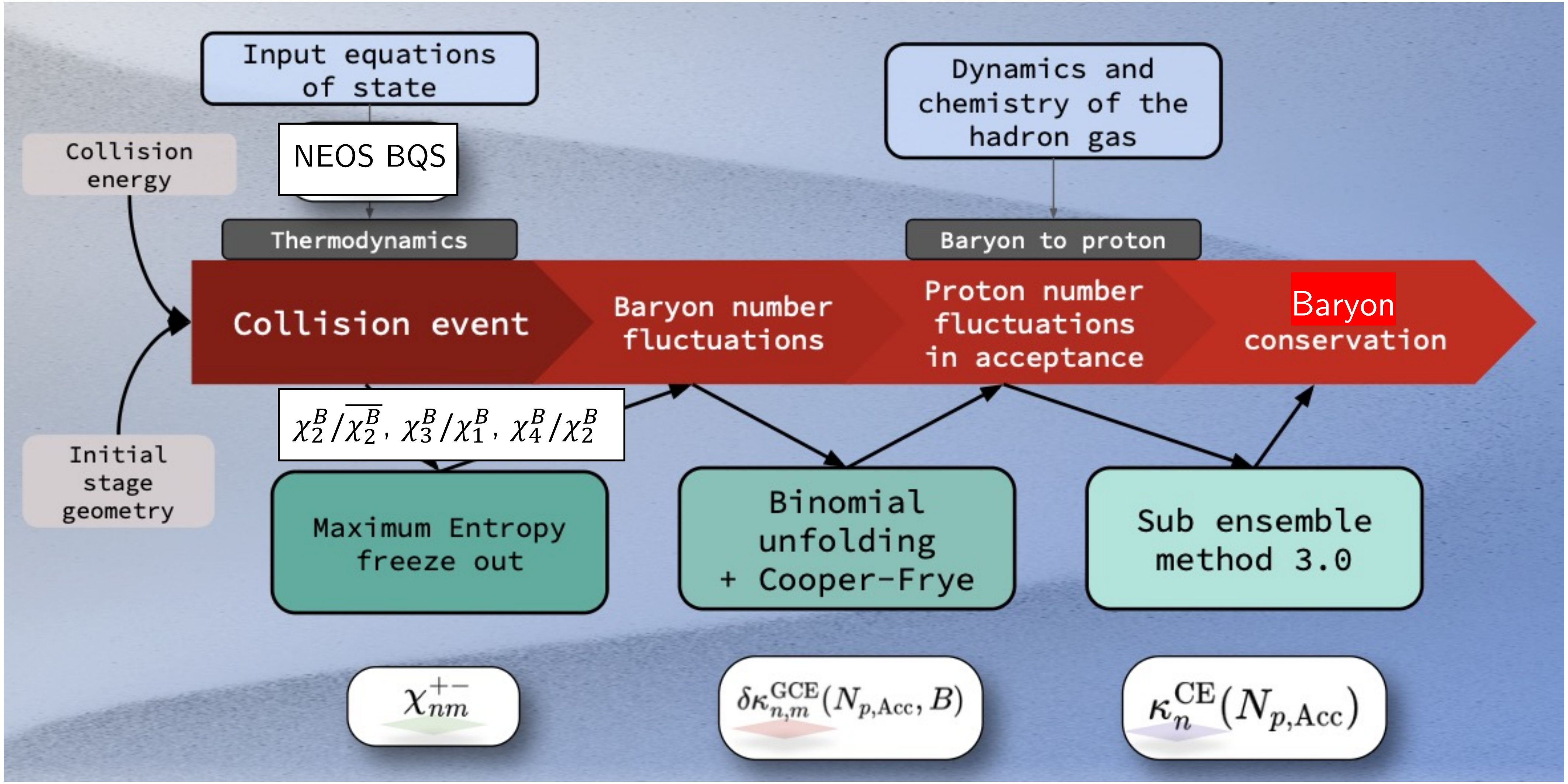
$$P_{\rm ce}(X) \equiv P_{\rm gce}(X \,|\, B_{\rm tot} = B_0) = \frac{P_{\rm gce}(X, B_{\rm tot} = B_0)}{P_{\rm gce}(B_{\rm tot} = B_0)}$$

- Requires joint accepted proton (X) and 4π baryon (B) cumulants, summed over all CF cells

$$\kappa_2^{\rm ce}(X) = \kappa_2^X - \frac{(\kappa_{11}^{XB})^2}{\kappa_2^B}$$

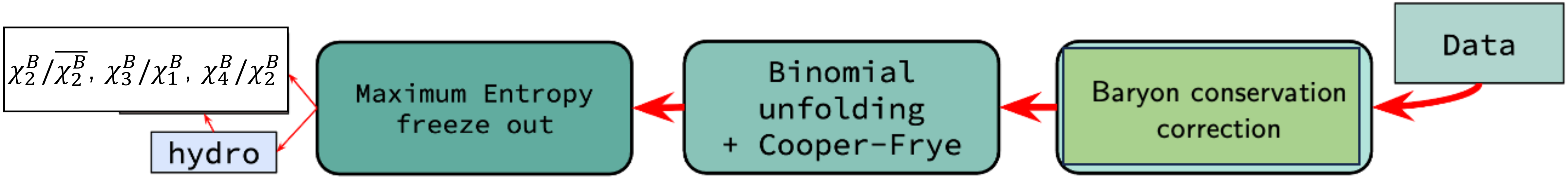
$$\kappa_3^{\rm ce}(X) = \kappa_3^X - 3\,\frac{\kappa_{11}^{XB}\kappa_{21}^{XB}}{\kappa_2^B} + 3\,\frac{(\kappa_{11}^{XB})^2\kappa_{12}^{XB}}{(\kappa_2^B)^2} - \frac{(\kappa_{11}^{XB})^3\kappa_3^B}{(\kappa_2^B)^3}$$

Roadmap to EoS inference



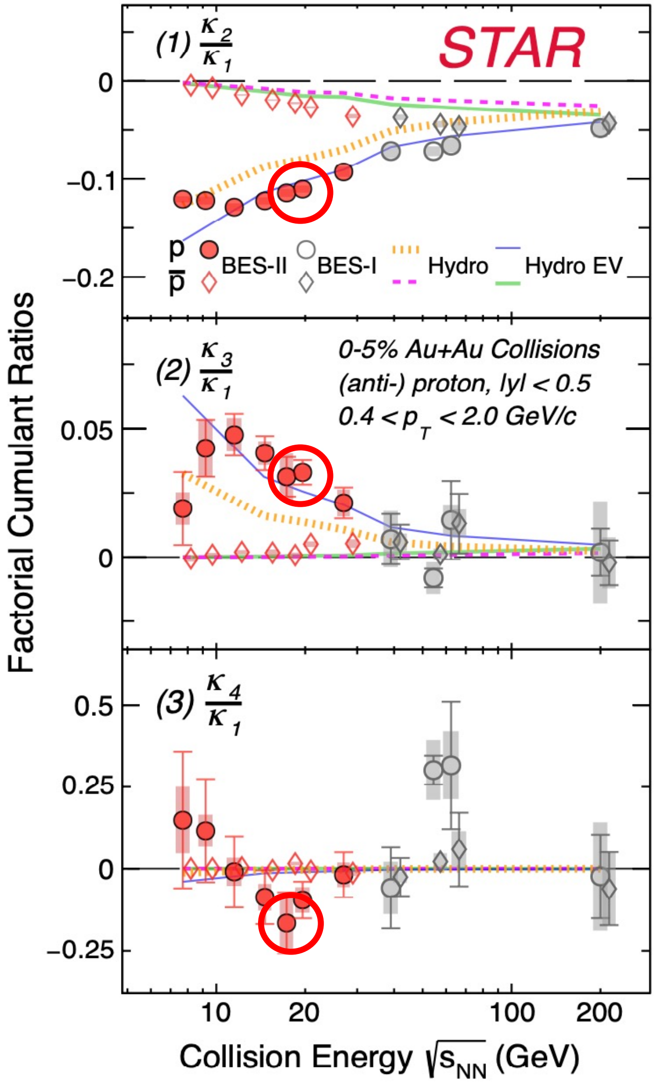
Adapted from G. Pihan, SQM2026

Roadmap to EoS inference

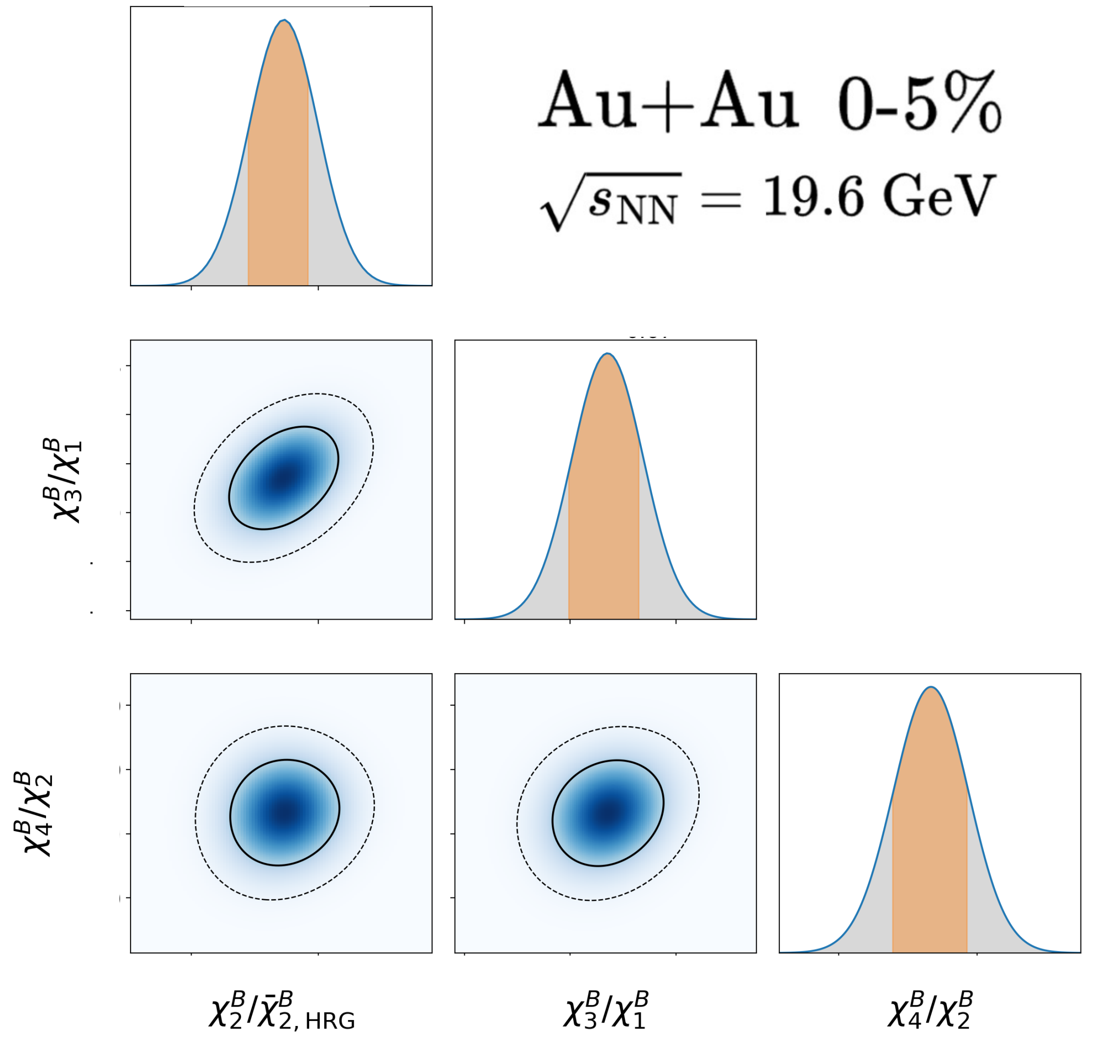


Adapted from G. Pihan, SQM2026

Bayesian inference: 19.6 GeV

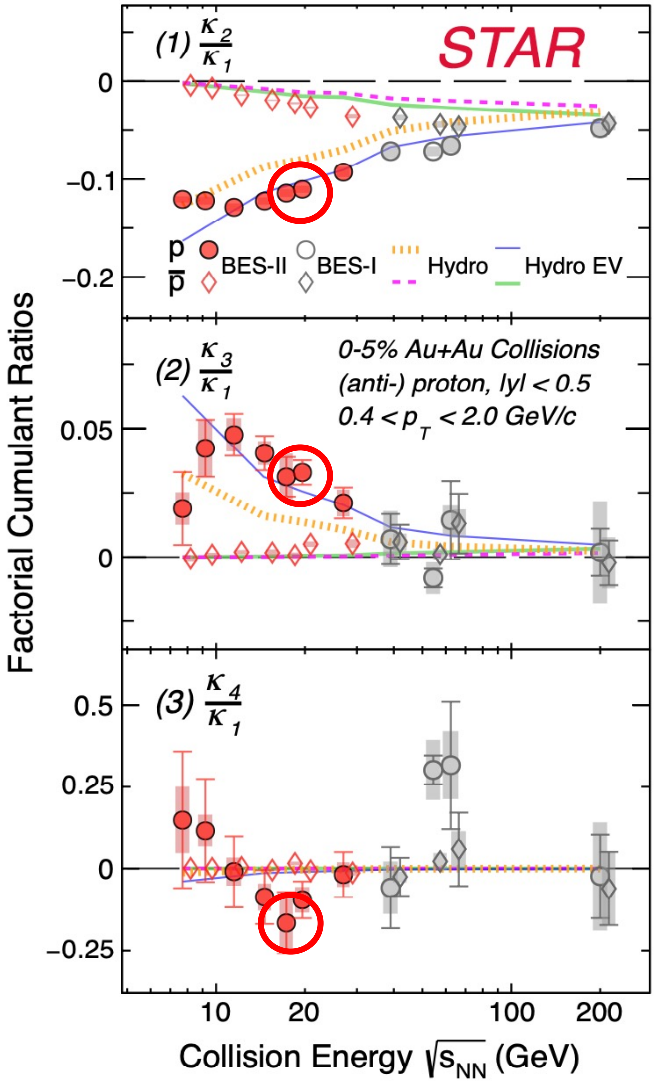


Unfolding

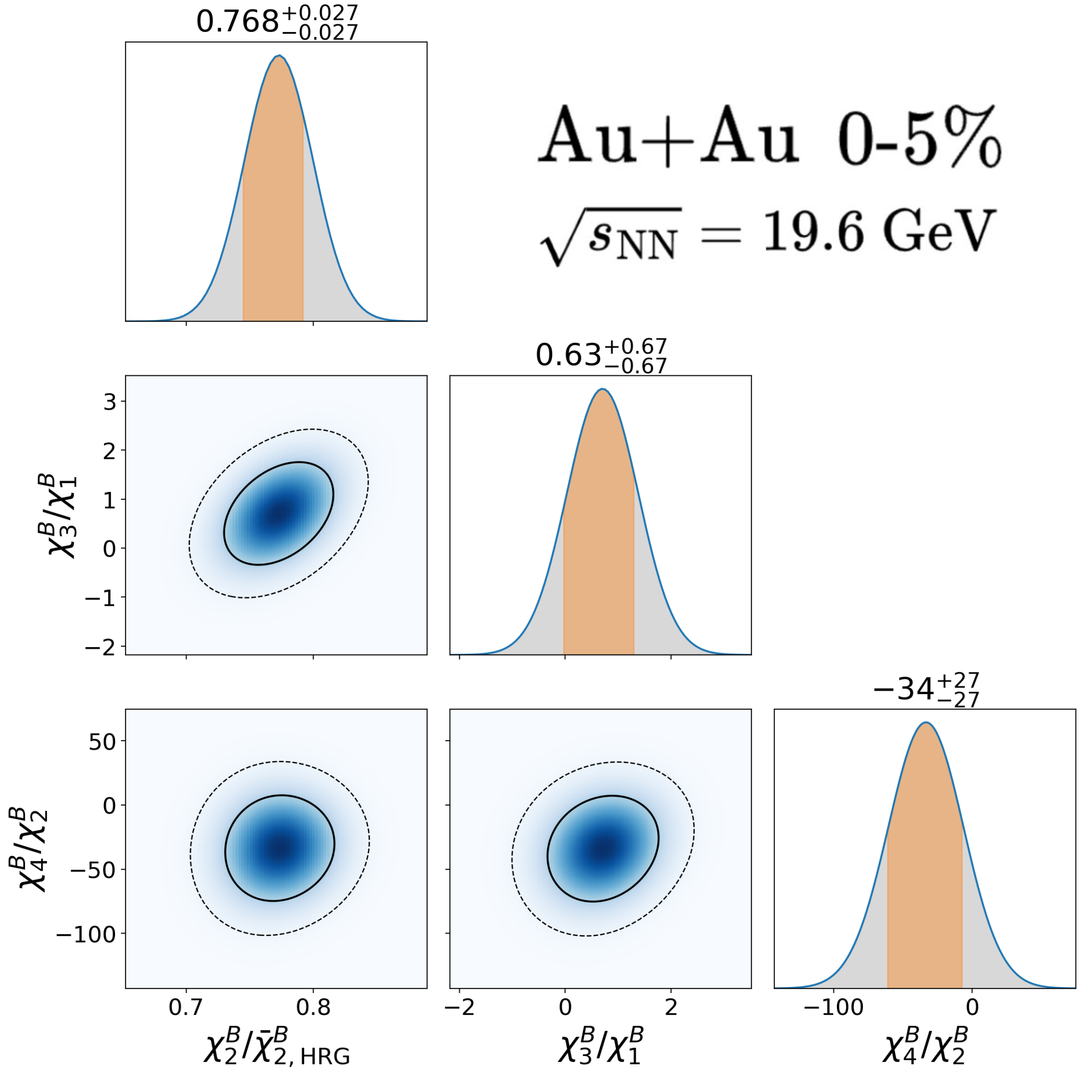


G. Pihan, R. Poberezhniuk, VV, to appear

Bayesian inference: 19.6 GeV

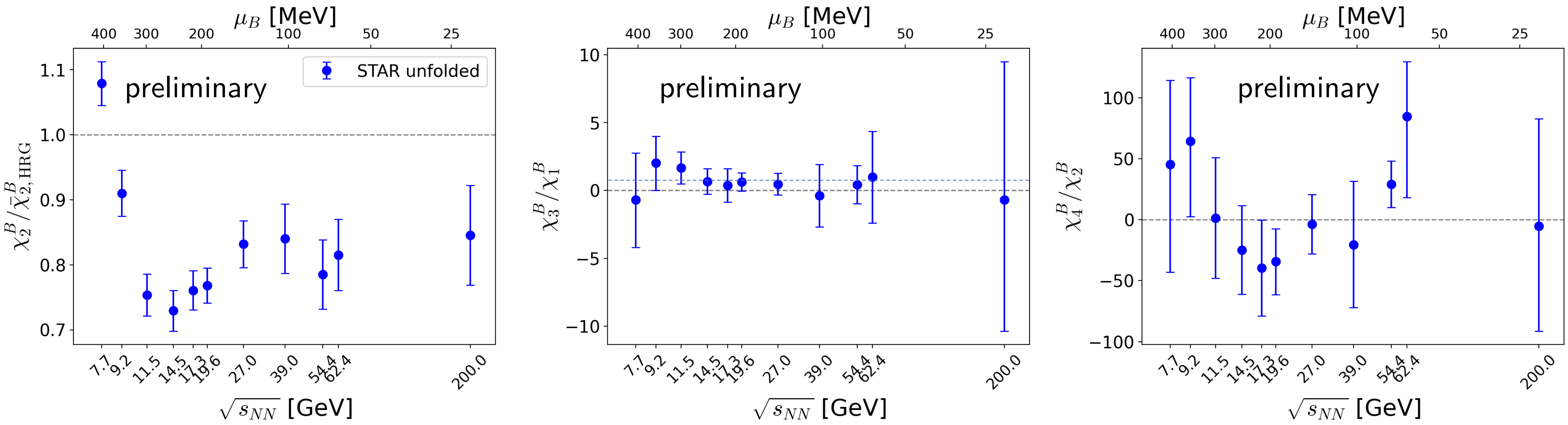


Unfolding



G. Pihan, R. Poberezhniuk, VV, to appear

Extracted susceptibilities

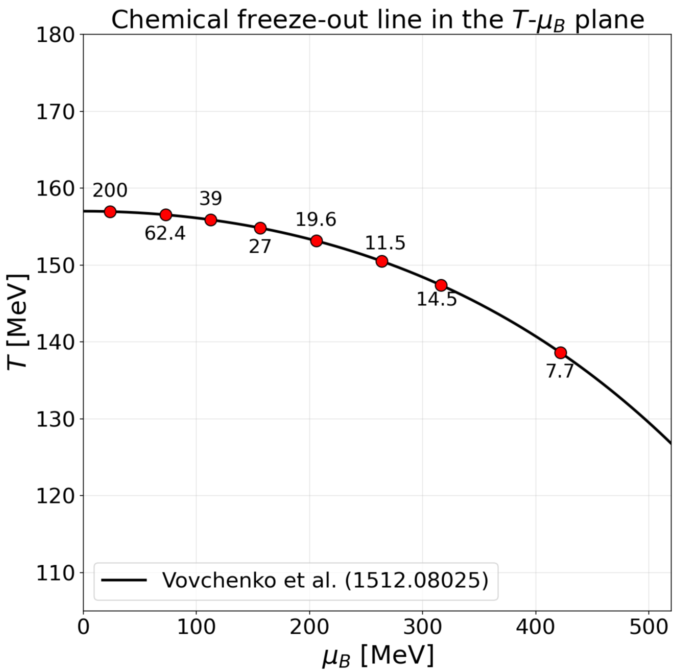


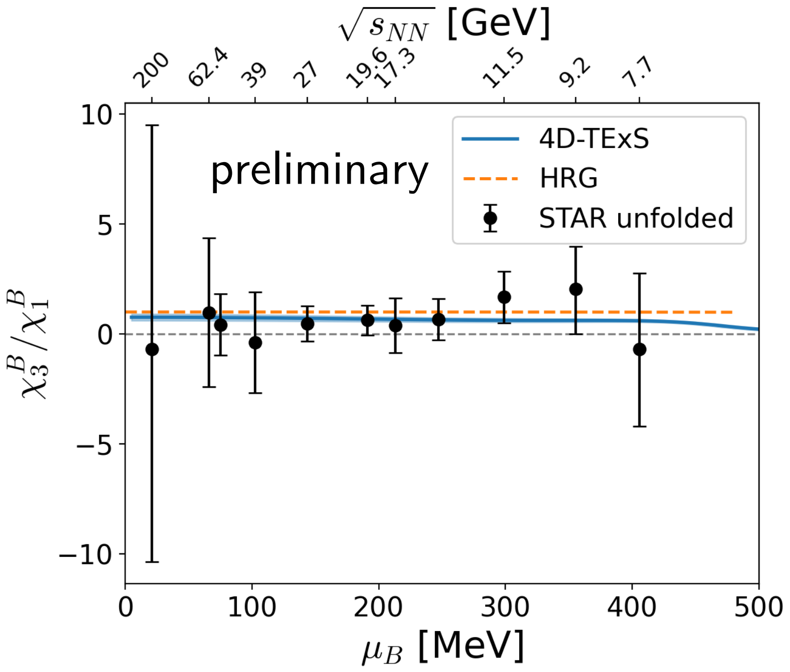
- Tight constraints on χ_2^B value relative to HRG
- Uncertainties in 3rd and 4th order susceptibilities are large---propagation of data uncertainty
  - Intermediate steps (net-B to B^+, kinematic cuts, $B^+ \rightarrow p$) acts akin to efficiency correction
  - Reducing uncertainties in the data would lead to reduced extraction errors
- Results appear weakly sensitive to e_{sw}, sensitivity to other hydro parameters to be studied

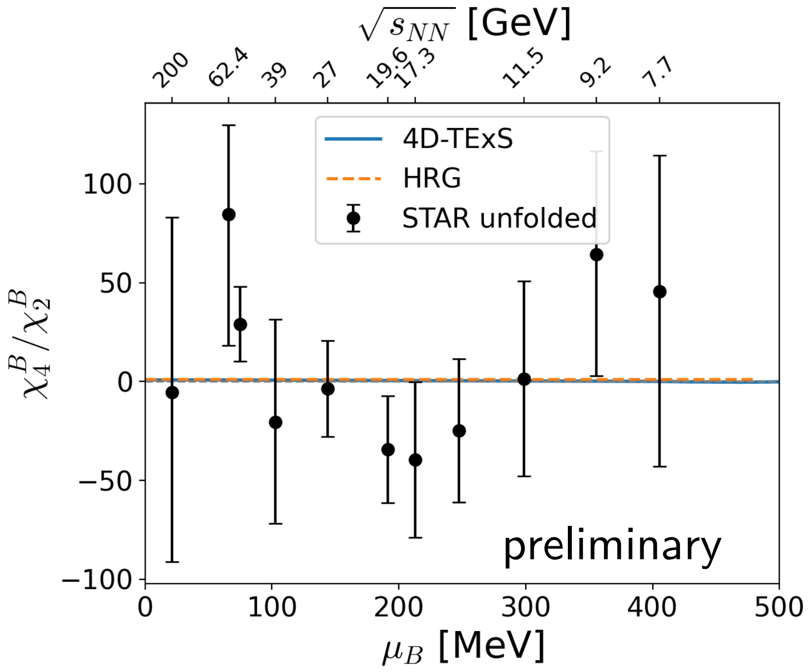
G. Pihan, R. Poberehzniuk, VV, to appear

Comparing to lattice QCD

To map the results to QCD phase diagram use chemical freeze-out parametrization [VV et al., PRC 93, 064906 (2015)]







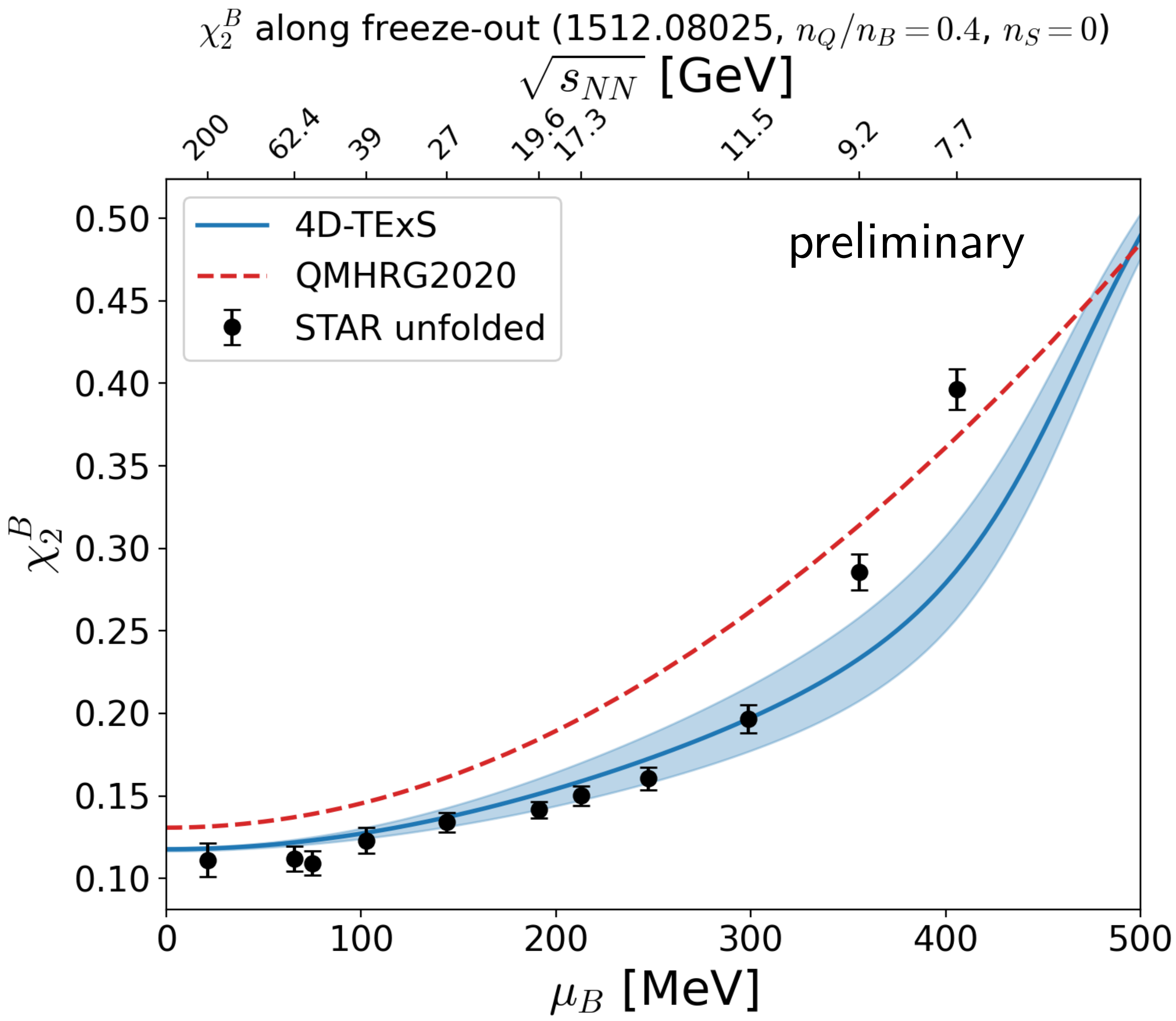
$\varepsilon_{SW} \approx 0.35 - 0.40$ GeV/fm^3

4D-TExS: [A. Abuali et al., Phys. Rev. D 112, 054502 (2025)]

- Lattice-based EoS at finite baryon density
- Computed under HIC conditions $(n_Q/n_B = 0.4, n_S = 0)$

G. Pihan, R. Poberehzniuk, VV, to appear

Baryon susceptibility χ_2^B



4D-TExS: [A. Abuali et al., Phys. Rev. D 112, 054502 (2025)]

- To obtain bare χ_2^B multiply by HRG value
  - QMHRG motivated by lattice QCD studies at $\mu_B = 0$ [HotQCD, PRD 104, 074512 (2021)]
- Agreement with lattice QCD at $\mu_B < 300$ MeV
- Enhancement relative to lattice QCD at $\mu_B > 300$ MeV

⚠ **Caveats:**

- Sensitive to the hadron list
- Sensitive to assumed freeze-out line
- Lattice EoS (4D-TExS) is an extrapolation (no truncation error included)

G. Pihan, R. Poberehzniuk, VV, to appear

Attraction vs repulsion

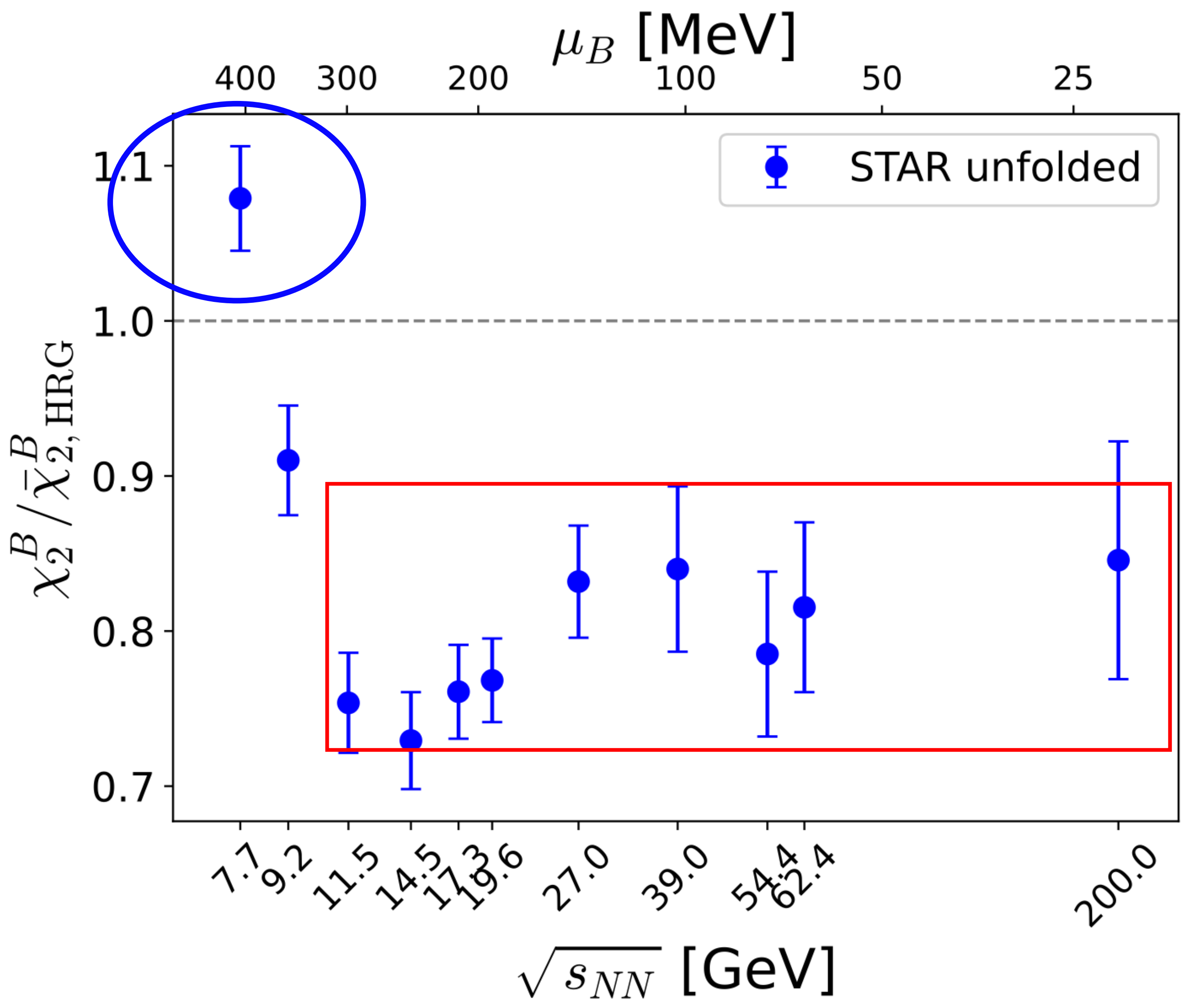
G. Pihan, R. Poberezhniuk, VV, to appear

attraction

$\chi_2^B/\overline{\chi_2^B} > 1$

VDW: $\frac{1}{1-\frac{2an}{T}}$

$a > 0$ – attraction



repulsion

$\chi_2^B/\overline{\chi_2^B} < 1$

VDW: $(1-\mathrm{bn})^2$

$\mathrm{b} > 0$ – repulsion

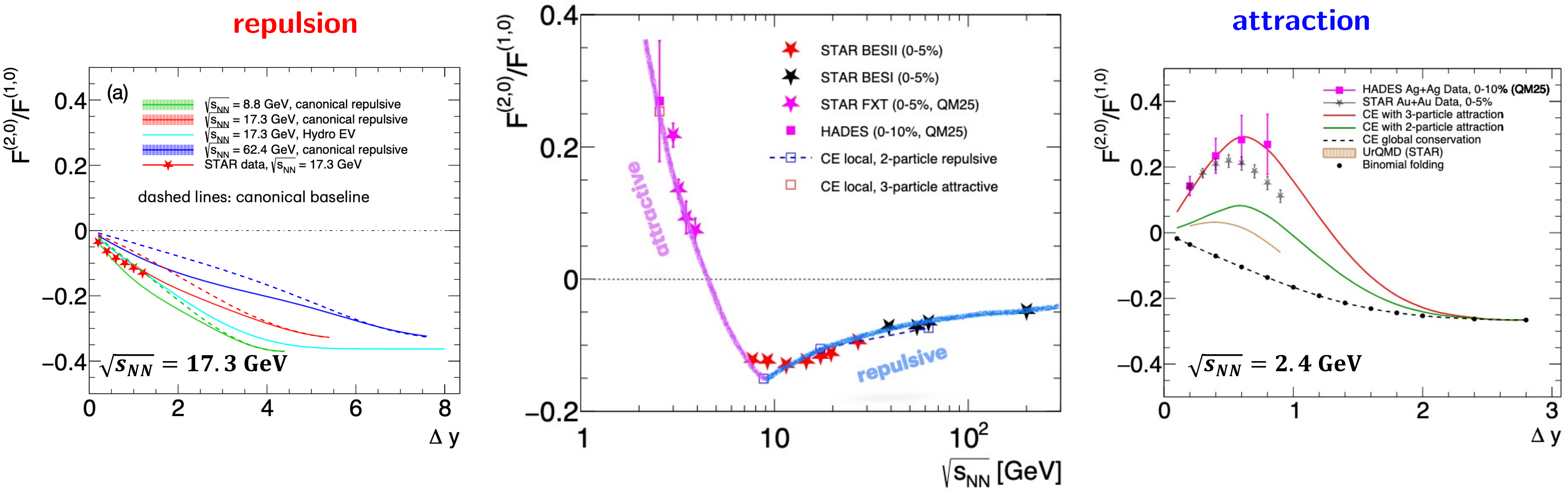
Interplay of repulsive (high $\sqrt{s_{NN}}$) and attractive (low $\sqrt{s_{NN}}$) interactions?

Attraction vs repulsion

Friman, Redlich, Rustamov, arXiv:2508.18879

Recently, attractive and repulsive interactions implemented through a potential in rapidity

$$E_r(y_1, y_2) = \alpha_r e^{-|y_1 - y_2|/\rho_r} \qquad P(y_1, y_1) = \frac{e^{-E(y_1,y_2)}}{Z} \qquad E_a(y_1, y_2) = \alpha_a |y_1 - y_2|^{\beta_a}$$



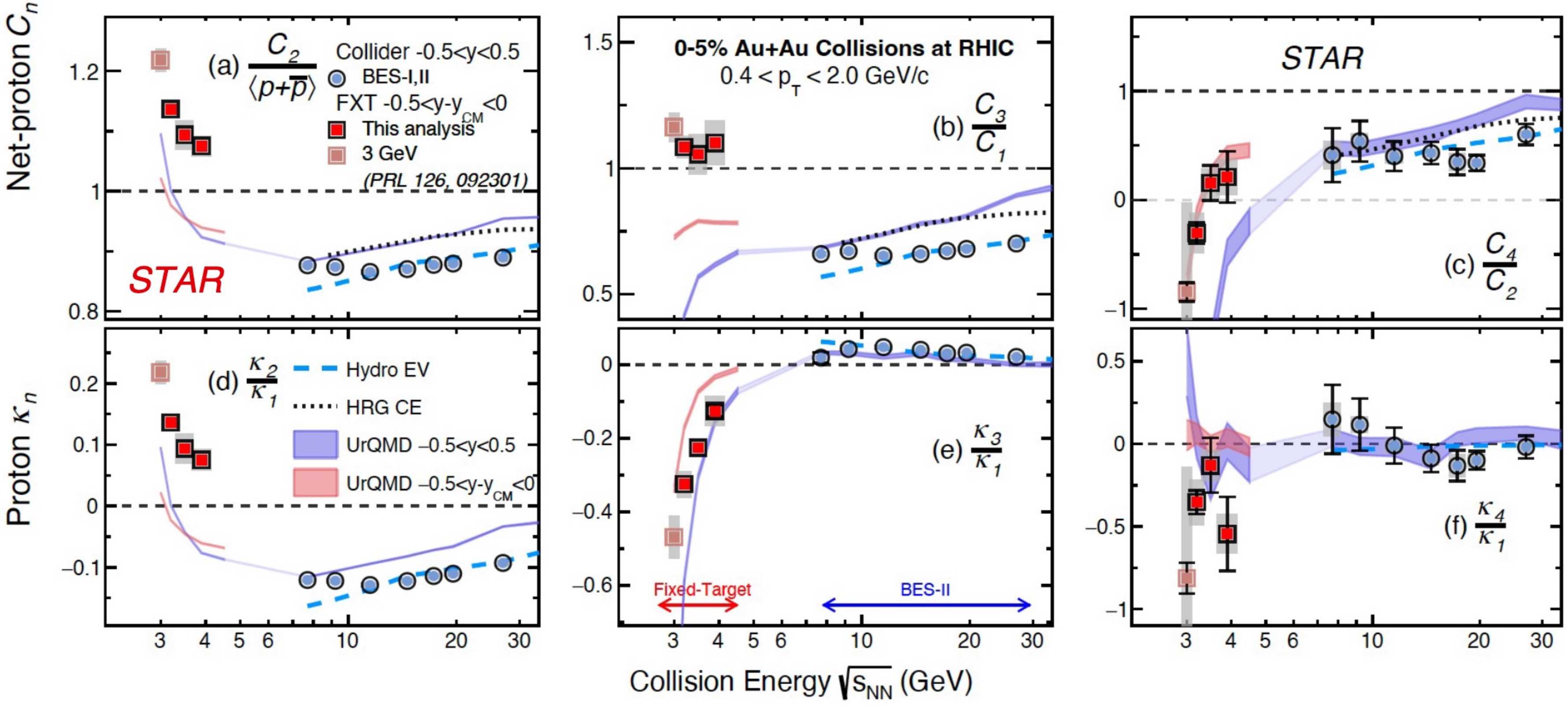
Interplay of repulsive (high $\sqrt{s_{NN}}$) and attractive (low $\sqrt{s_{NN}}$) interactions?

Time to celebrate!

Adapted from V. Koch, ERICE2025

RHIC-BES-II data including STAR-FXT

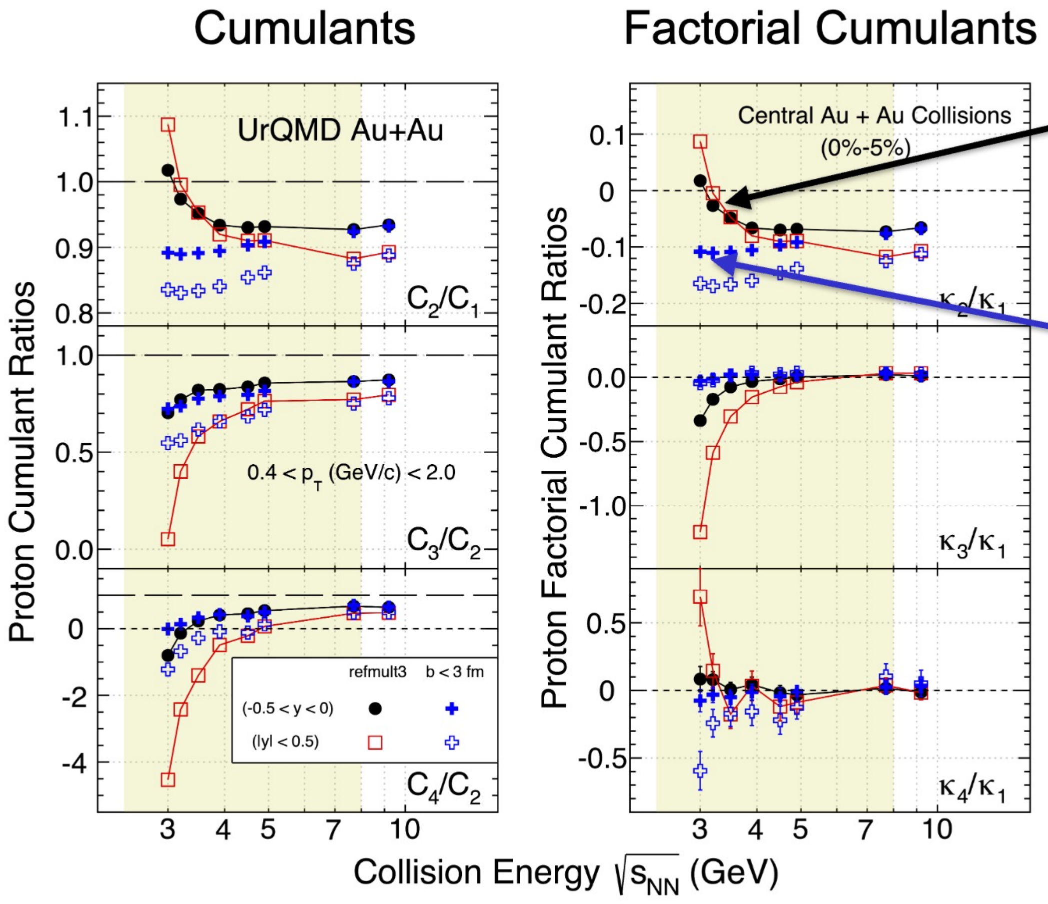
Z. Sweger (STAR), QM2025



- Continues the trends seen at lowest collider energies, in a fairly dramatic fashion
- **UrQMD (cascade) describes reasonably well the qualitative features**
  - Dominance of non-critical effects (centrality selection and spectators)?

The (possible) culprit

X. Zhang, Y. Zhang, X. Luo, N. Xu, arXiv: 2506.18832



Fluctuating impact parameter
STAR centrality selection

Fixed impact parameter $(b < 3\ \mathrm{fm})$
minimal volume fluctuations.

N.B.: Centrality Bin Width Corrections applied to both

Possible culprit:
volume fluctuations/centrality selection

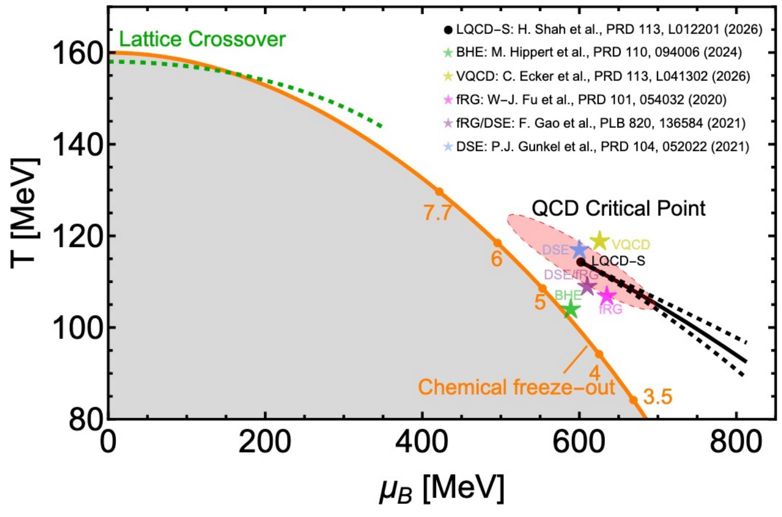
Other challenges: Antiproton puzzle (not described by hydro)

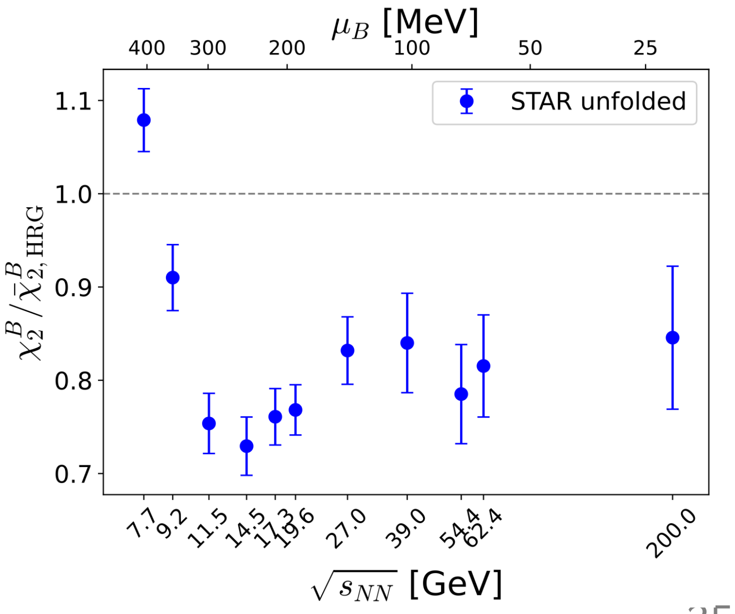
Bzdak, Koch, VV, PRC 112, 024901 (2025)

Summary

UNIVERSITY OF HOUSTON

- **Lattice-based constraints on the QCD critical point**
  - Rule out QCD CP at $\mu_B < 450$ MeV at a 2σ level
  - A few predictions around $\mu_B \sim 600 - 700$ MeV, $T \sim 90 - 110$ MeV
- **Heavy-ion collisions: RHIC-BES-II data is in**
  - Non-critical physics describe the proton data at $\sqrt{s_{NN}} \geq 20$ GeV
  - Hydrodynamics-based inference of baryon susceptibilities
    - χ_2^B agrees with LQCD at $\mu_B < 300$ MeV, shows enhancement at larger μ_B
    - Current errors are too large to infer high-order susceptibilities





Outlook:

- Continued improvement of lattice QCD constraints
- Improved quantitative description of baselines and critical fluctuations
  - Sensitivity to hydro/model parameters, other observables
  - Lower collision energies (RHIC-FXT), $\sqrt{s_{\mathrm{NN}}} \lesssim 7.7$ GeV

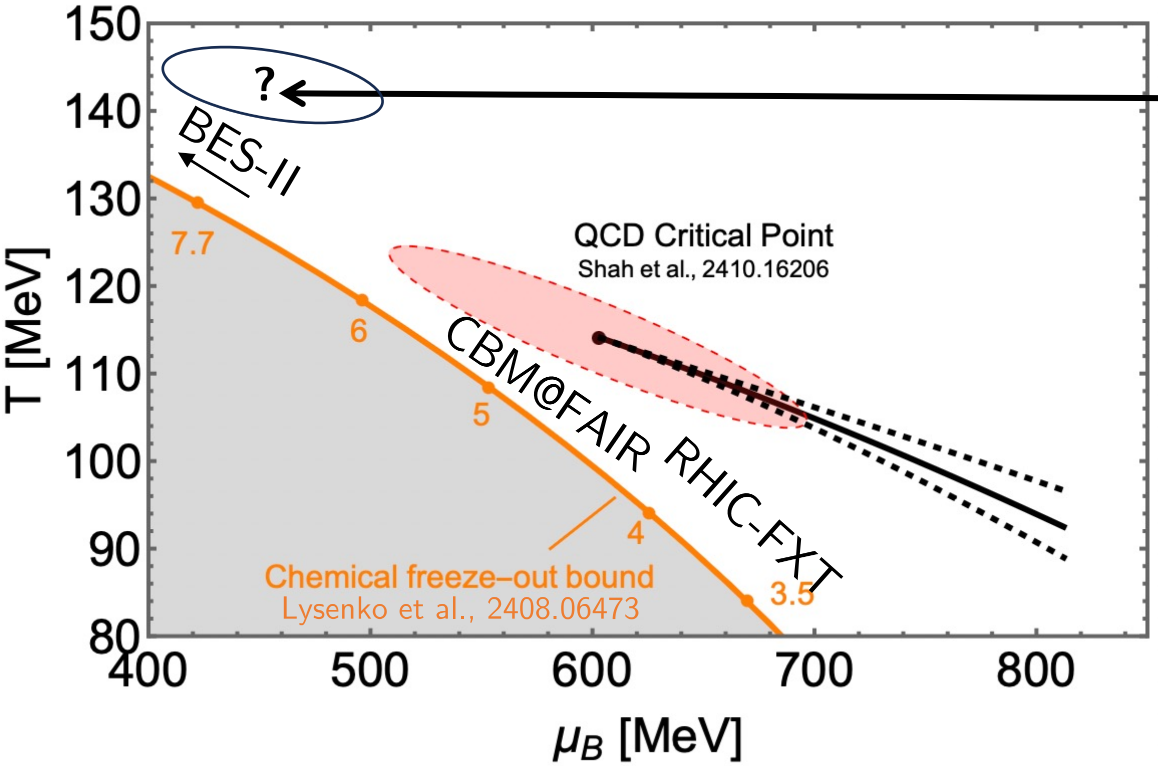
Thanks for your attention

Additional slides

If deviations from the baseline are driven by CP

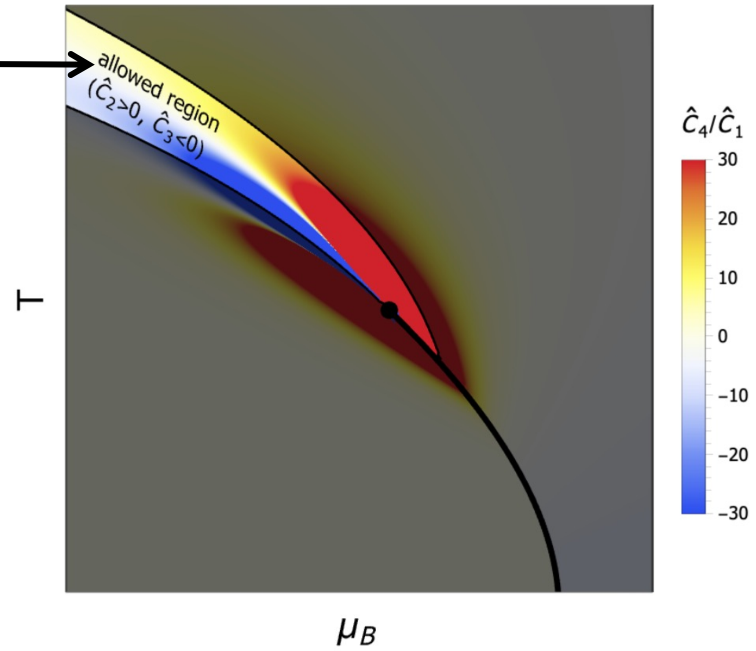
UNIVERSITY OF HOUSTON

Equilibrium expectation



Exclusion plots

Exclude $\hat{C}_2<0$ & $\hat{C}_3>0$ regions on the phase diagram near CP



Analysis adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)

- Freeze-out of fluctuations on the QGP side of the crossover?
- Due to memory effect the sign of $\hat{C}_3$ may differ from equilibrium expectation

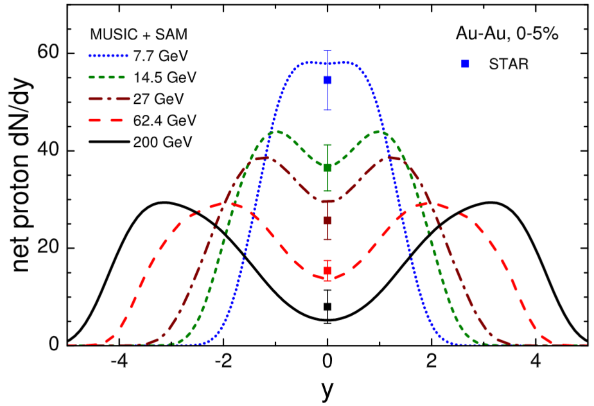
Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)

- New scenarios: cool critical point Basar, Pradeep, Stephanov, arXiv:2603.23635

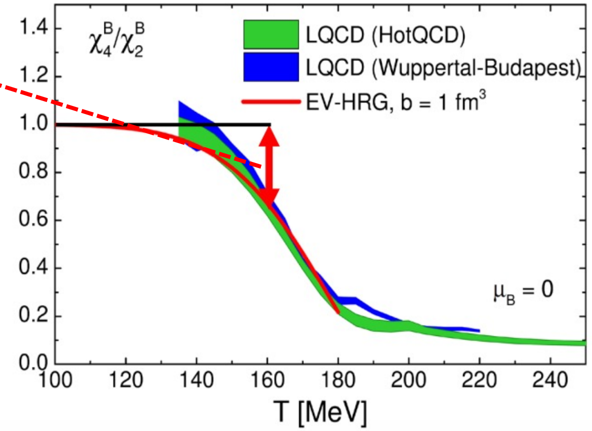
Hydro EV: Non-critical hydro baseline at RHIC-BES

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
  - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
  - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]



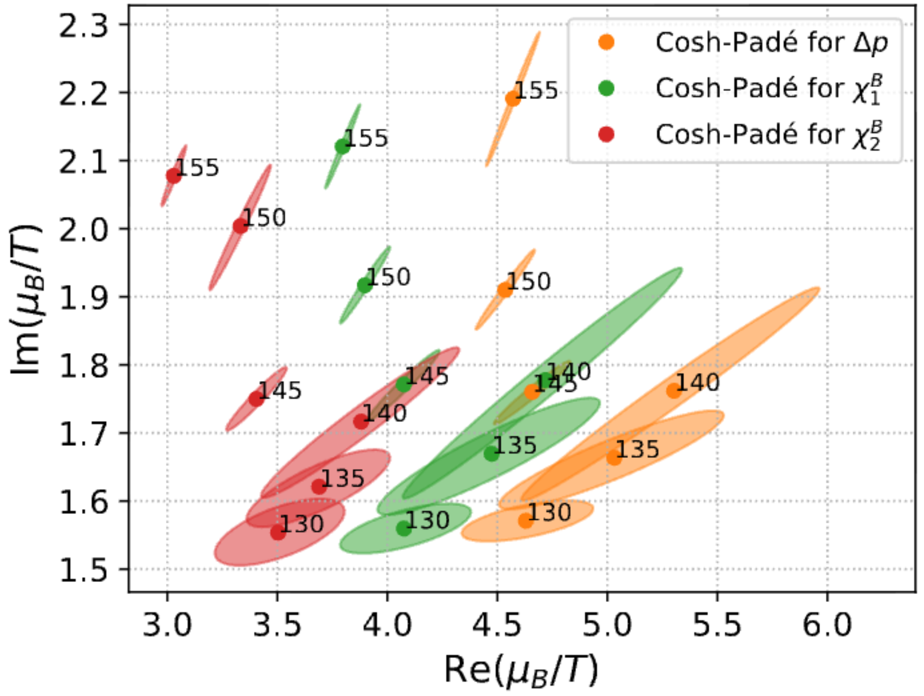
- Non-critical contributions computed at particlization ($\epsilon_{sw} = 0.26$ GeV/fm^3)
  - QCD-like baryon number distribution (χ_n^B) via **excluded volume** b = 1 fm^3 [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
  - **Exact global baryon conservation*** (and other charges)
    - Subensemble acceptance method 2.0 (analytic) [VV, Phys. Rev. C 105, 014903 (2022)]
    - or FIST sampler (Monte Carlo) [VV, Phys. Rev. C 106, 064906 (2022)] https://github.com/vlvovch/fist-sampler



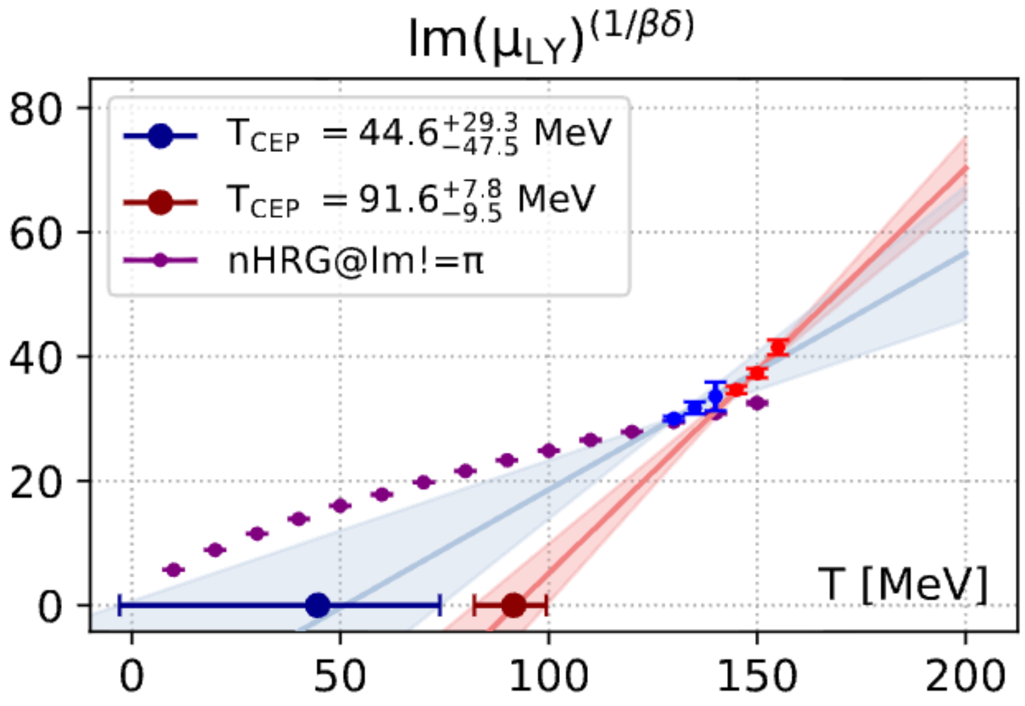
- Included: baryon conservation, repulsion, kinematical cuts
- Absent: critical point, local conservation, initial-state/volume fluctuations, hadronic phase

*If baryon conservation is the only effect (no other correlations), non-critical baseline can be computed without hydro
Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, NPA 1008, 122141 (2021)

Searching for singularities in the complex plane



Variations in fit range matter



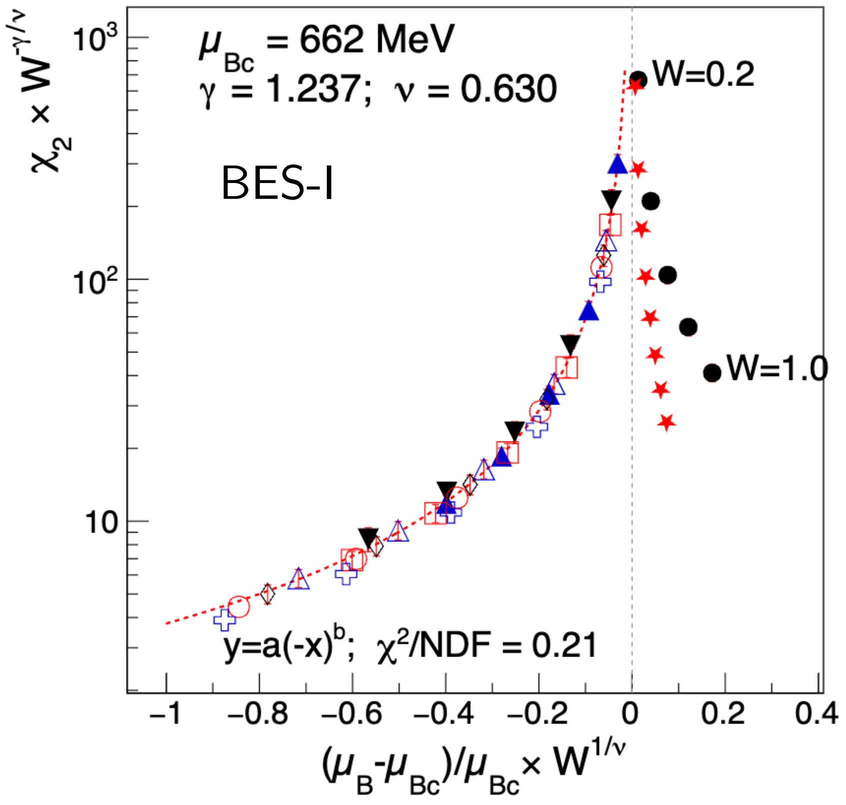
A. Adam et al. (Wuppertal-Budapest), arXiv:2507.13254

Finite Size Scaling

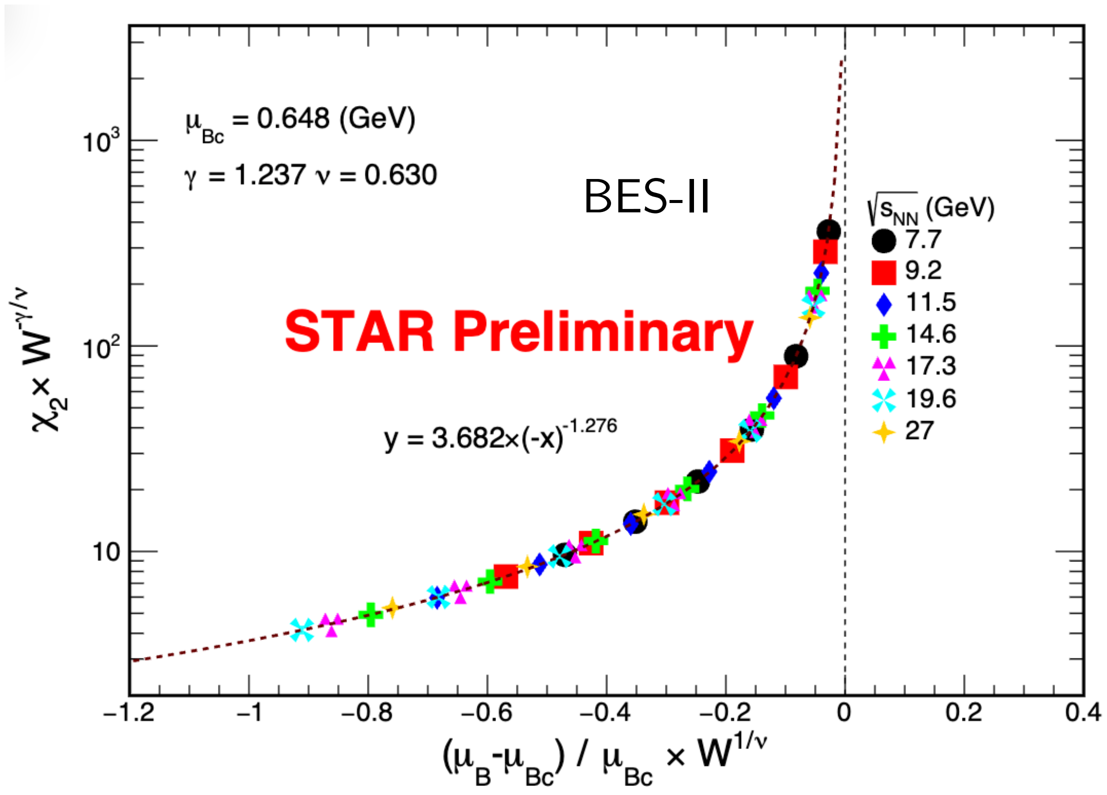
Near the CP: $\chi(L,t) = L^{\gamma/\nu}\Phi(tL^{1/\nu})$

Rapidity bin width W to vary size: $\chi_2(W,\mu_{\text{fo}}) = \frac{C_2(W,\mu_{\text{fo}})}{T_{\text{fo}}^3 W dV_{\text{fo}}/dy}$

A. Sorensen, P. Sorensen, arXiv:2405.10278



Y. Huang (STAR), QM2025



Related analysis using centralities in lieu of bin width, yields CP at $\sqrt{s_{NN}}$~ 33 GeV (μ_B~130 MeV)

R. Lacey, arXiv:2411.09139

Mean p_T fluctuations

Mean p_T fluctuations:

$$\langle \Delta p_{T,i} \Delta p_{T,i} \rangle \sim \langle \Delta \langle p_T \rangle^2 \rangle$$

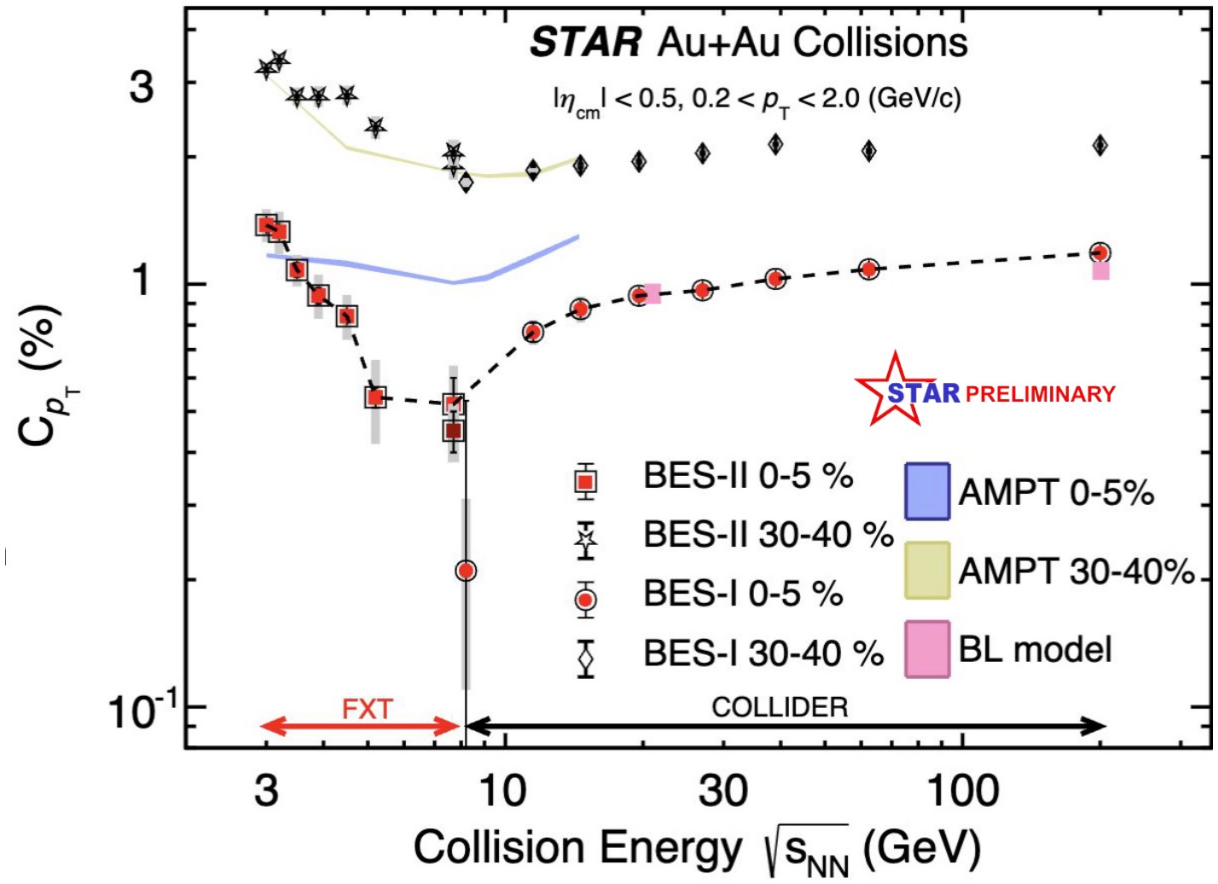
Mean p_T probes the temperature

Gardim et al, Nature Phys. (2020)

$$\langle p_T \rangle \propto Teff$$

$$\langle \Delta p_{T,i} \Delta p_{T,i} \rangle \sim \langle \Delta T^2 \rangle$$

In equilibrium: $\langle \Delta T^2 \rangle = \frac{T^2}{V c_V}$



R. Manikandhan (STAR), QM2025

At the critical point $c_V \to \infty$ ➡ Minimum in $\sqrt{s_{NN}}$ dependence?

Factorial cumulants $\hat{C}_n$ vs ordinary cumulants C_n

Factorial cumulants: ~irreducible n-particle correlations

$$\hat{C}_n \sim \langle N(N-1)(N-2)\dots\rangle_c$$

$$\hat{C}_1 = C_1$$
$$\hat{C}_2 = C_2 - C_1$$
$$\hat{C}_3 = C_3 - 3C_2 + 2C_1$$
$$\hat{C}_4 = C_4 - 6C_3 + 11C_2 - 6C_1$$

Ordinary cumulants: mix correlations of different orders

$$C_n \sim \langle \delta N^n \rangle_c$$

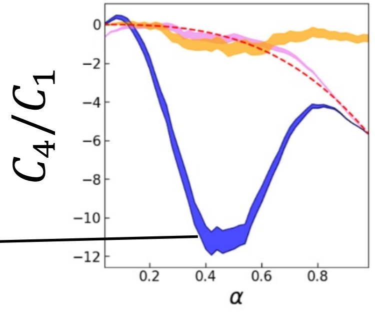
$$C_1 = \hat{C}_1$$
$$C_2 = \hat{C}_2 + \hat{C}_1$$
$$C_3 = \hat{C}_3 + 3\hat{C}_2 + \hat{C}_1$$
$$C_4 = \hat{C}_4 + 6\hat{C}_3 + 7\hat{C}_2 + \hat{C}_1$$

[Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017); Kitazawa, Luo, PRC 96, 024910 (2017); C. Pruneau, PRC 100, 034905 (2019)]

Factorial cumulants and different effects

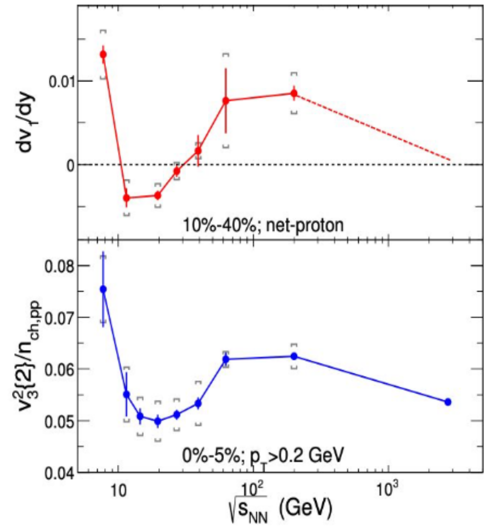
- Baryon conservation [Bzdak, Koch, Skokov, EPJC '17] — $\hat{C}_n^{\text{cons}} \propto (\hat{C}_1)^n / \langle N_{\text{tot}} \rangle^{n-1}$ *small*
- Excluded volume [VV et al, PLB '17] — $\hat{C}_n^{\text{EV}} \propto b^n$ *small*
- Volume fluctuations [Holzman et al., NPA '24] — $\hat{C}_n^{CF} \sim (\hat{C}_1)^n \kappa_n[V]$ *depends on volume cumulants*
- **Critical point** [Ling, Stephanov, PRC '16] — $\hat{C}_2^{CP} \sim \xi^2$, $\hat{C}_3^{CP} \sim \xi^{4.5}$, $\hat{C}_4^{CP} \sim \xi^7$ ***large***

- proton vs baryon [Kitazawa, Asakawa, PRC '12] — $\hat{C}_n^B \sim 2^n \times \hat{C}_n^p$ **same sign!**

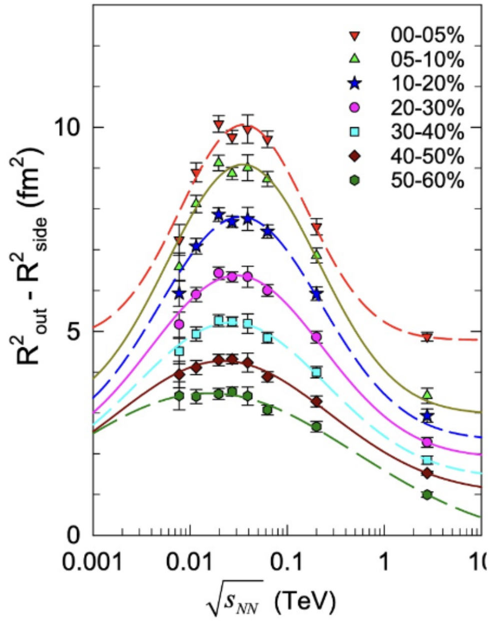


Holes and bumps at ~20 GeV

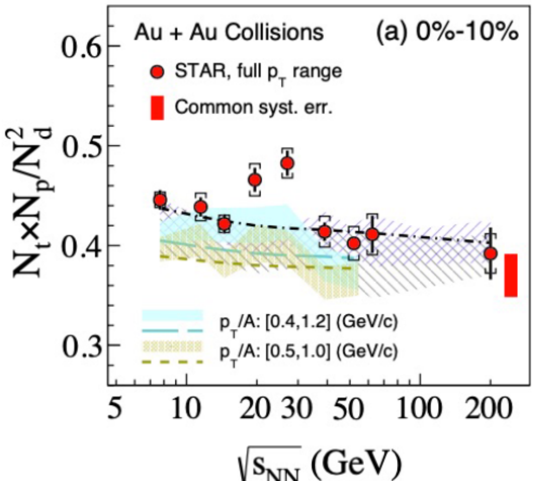
- Flow $\frac{dv_1}{dy}$, $\frac{v_3^2}{n_{ch}}$ show minimum



- HBT shows maximum



- Clusters $\frac{tp}{d^2p}$ show peak



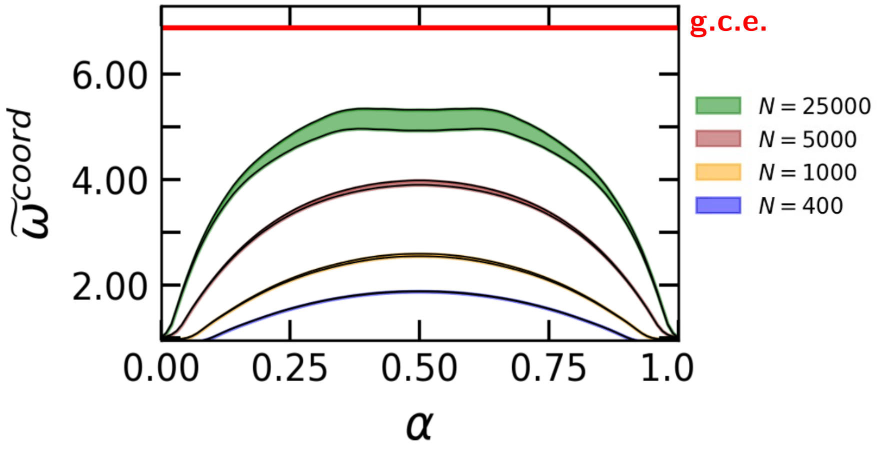
Example: Critical fluctuations in microscopic simulation

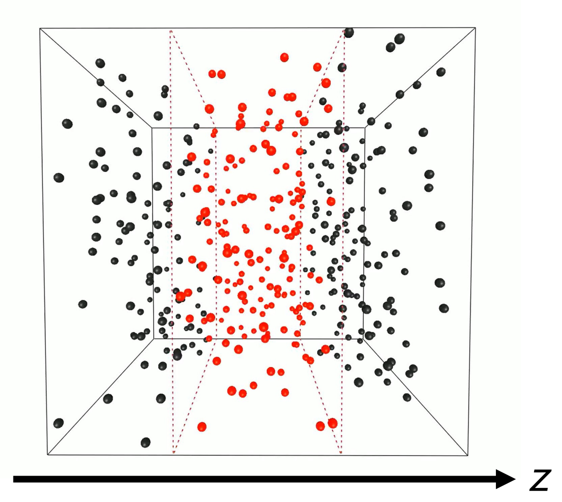
V. Kuznietsov et al., Phys. Rev. C 105, 044903 (2022)

Classical molecular dynamics simulations of the **Lennard-Jones fluid** near Z(2) critical point ($T \approx 1.06T_c$, $n \approx n_c$) of the liquid-gas transition

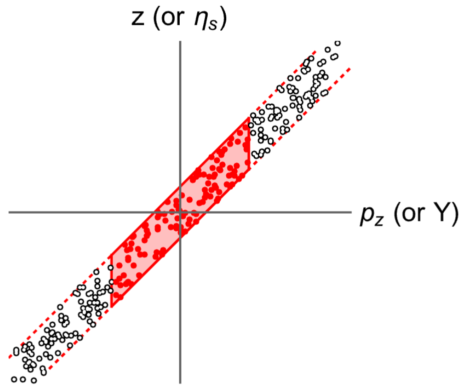
Scaled variance in coordinate space acceptance $|z| < z^{max}$

$$\tilde{\omega}^{\text{coord}} = \frac{1}{1-\alpha}\frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle}$$





Heavy-ion collisions:
flow correlates p_z and z cuts



- Large fluctuations survive despite strong finite-size effects
- Need coordinate space cuts (collective flow helps)
- Here no finite-time effects

The maximum entropy freeze-out

The Max Ent method provides the least-biased reconstruction of particle multiplicity fluctuations consistent with the hydrodynamic correlations.

$$\hat{\Delta} G_{A_1 \ldots A_k} = \hat{\Delta} \mathcal{H}_{a_1 \ldots a_k} \prod_{i=1}^{k} P_{A_i}^{a_i}$$

$\hat{\Delta}$ Irreducible relative cumulants (IRC) operator

M.S. Pradeep, M.A. Stephanov Phys. Rev. Lett. 130, 162301

$\hat{\Delta} \mathcal{H}_{a_1 \ldots a_k}$ IRC of the K-points correlator of hydrodynamic fields $a_i = \epsilon, n_B$

$\hat{\Delta} G_{A_1 \ldots A_k}$ IRC of the K-points correlator of particles $A_i \in \text{Particles}$

$P_{A_i}^{a_i}$

Hydro matching conditions projectors

The maximum entropy freeze-out

Assumptions

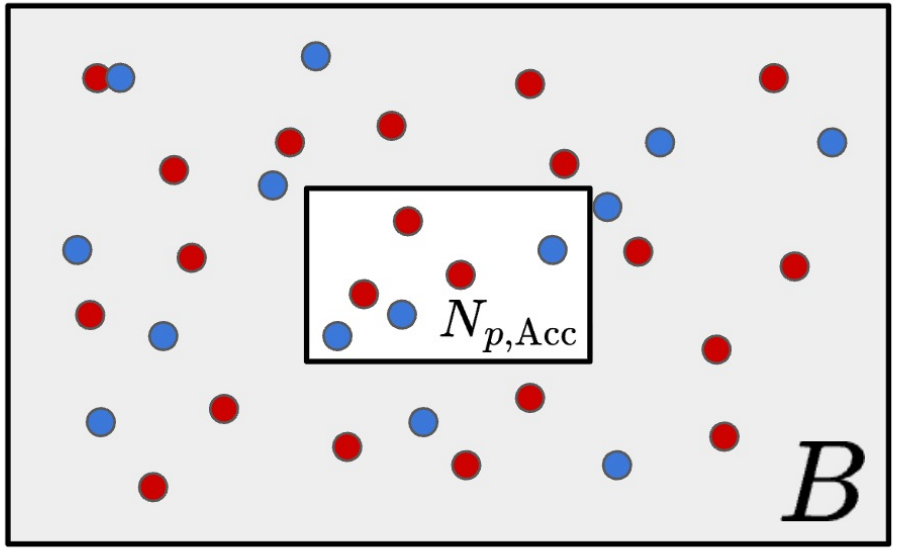
- B^+ B^- joint cumulants — $B^\pm = \left\{ \hat{A}_i \;/\; q_{A_i} = \pm 1 \right\}_{i \in \text{Particles}}$
- Boltzmann approximation — Particle IRCs = Particle factorial cumulants
- Cells are statistically independent — No correlation between cells
- Net-baryon density field only — $a_i = n_B \, \forall\, i$

Maximum Entropy B^+ B^- factorial cumulants and cumulants

$$\tilde{\kappa}^{+-}_{nm}(x) = \tilde{\tilde{\kappa}}^{+-}_{nm}(x) + \hat{\Delta}\mathcal{H}_{n+m}(x)\big(P_+\big)^n\big(P_-\big)^m$$

$$\tilde{\kappa}^{+-}_{nm}(x) \longrightarrow \kappa^{+-}_{nm}(x)$$

Towards the global B conservation



$$G(t_+, t_-) = \ln \left\langle e^{\,t_+ B_+ + t_- B_-} \right\rangle$$

$$\widetilde{G}(t, t_B) = \ln \left\langle e^{\,t\, N_{p,\mathrm{Acc}} + t_B\, B} \right\rangle$$

$$B = B^+ - B^-$$

$$\widetilde{G}(t, t_B) = G\big(t_B + \theta(t, \alpha_+),\, -t_B\big)$$

GCE proton, total Baryon number joint cumulants

$$\kappa^{\mathrm{GCE}}_{n,m}(N_{p,\mathrm{Acc}}, B) = \sum_{k=1}^{n} B_{n,k}(c_1, c_2, \ldots, c_{n-k+1}) \sum_{q=0}^{m} \binom{m}{q} (-1)^{m-q} \kappa^{+-}_{k+q,\, m-q}$$

$B_{n,k}$ Partial bell polynomials

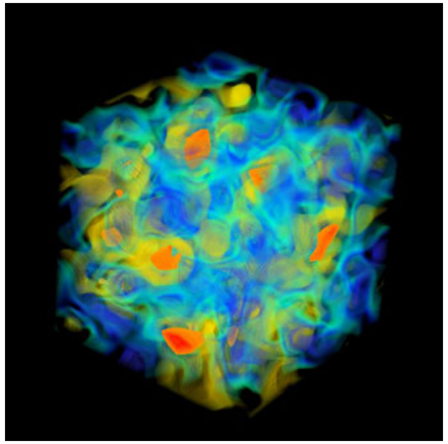
$c_k \equiv \partial_t^k \theta(t, \alpha_+)\big|_{t=0}$

$\theta(t, \alpha_+) \equiv \ln\big(1 - \alpha_+ + \alpha_+ e^t\big)$

Theory vs experiment: Challenges for fluctuations

UNIVERSITY OF HOUSTON

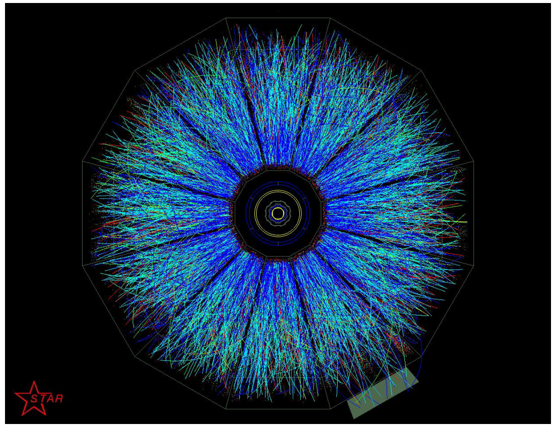
Theory



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

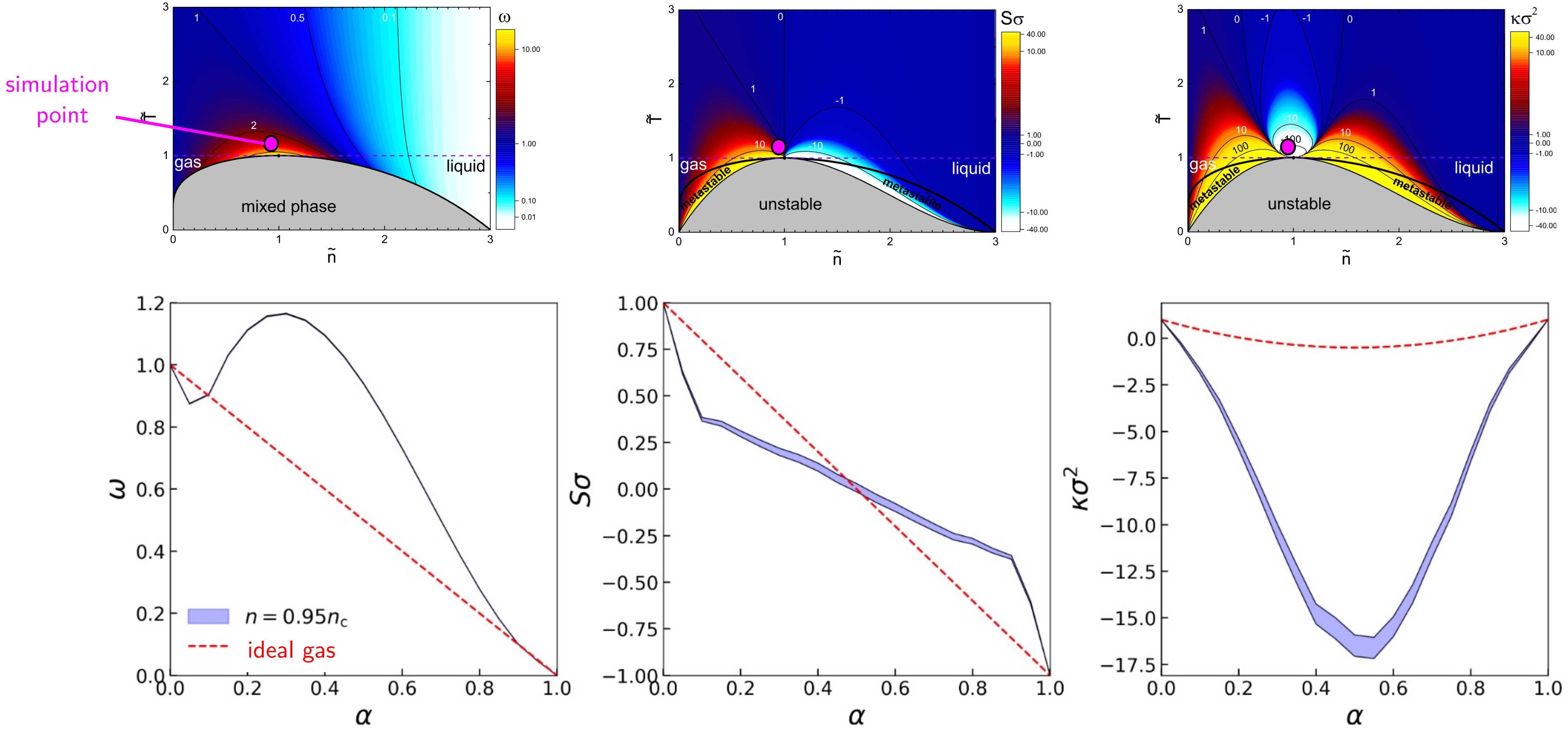
- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

Comparing theory and experiment should be done very carefully

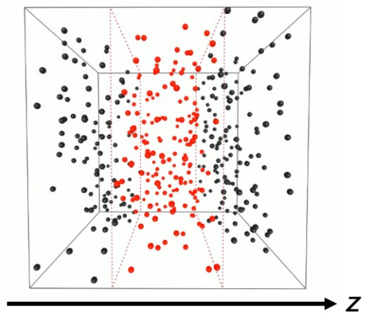
Non-Gaussian fluctuations from molecular dynamics

V. Kuznietsov, Gorenstein, Koch, VV, to appear

Scaled variance κ_2/κ_1 Skewness κ_3/κ_2 Kurtosis κ_4/κ_2



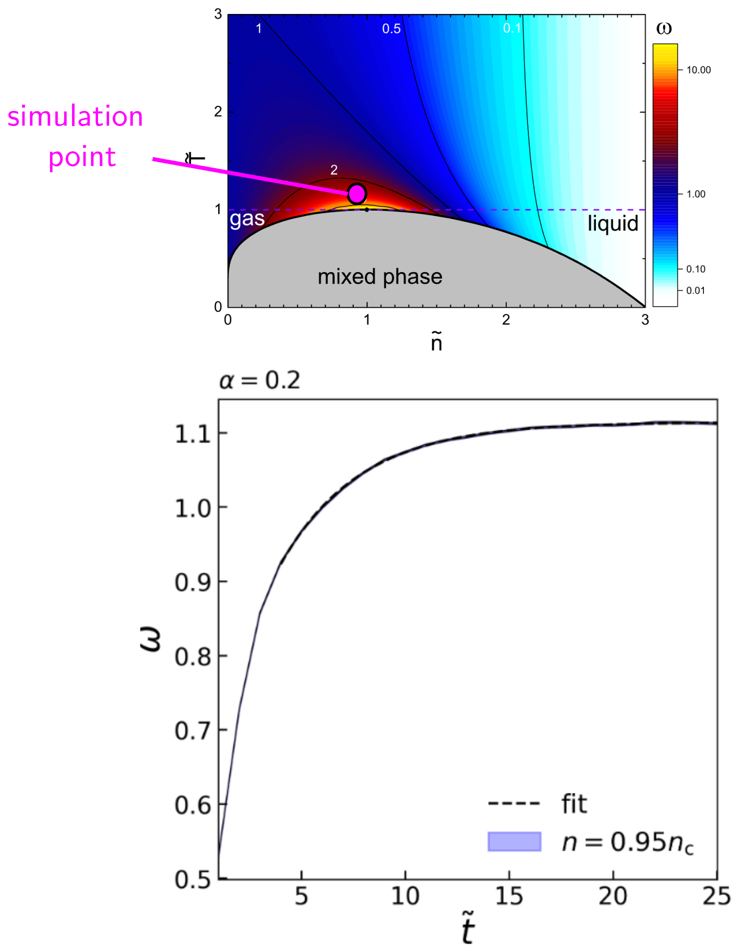
400 nucleons in a box



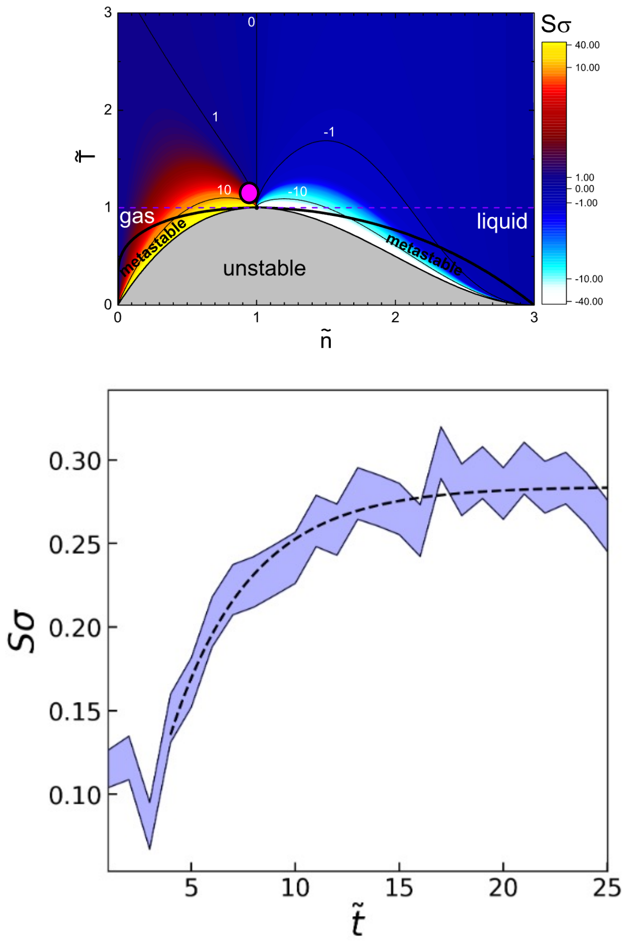
Non-Gaussian fluctuations from molecular dynamics

V. Kuznietsov, Gorenstein, Koch, VV, to appear

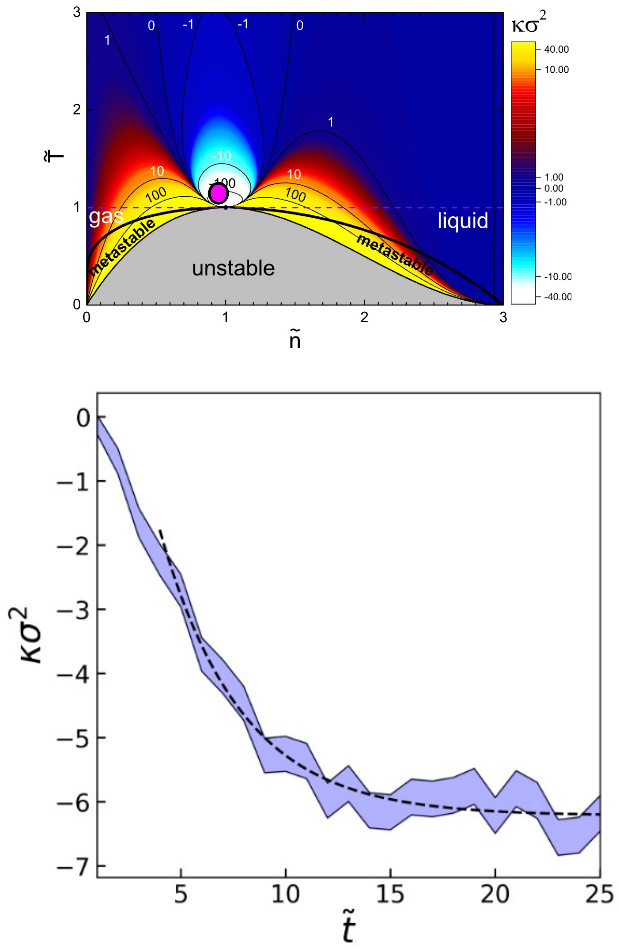
Scaled variance κ_2/κ_1



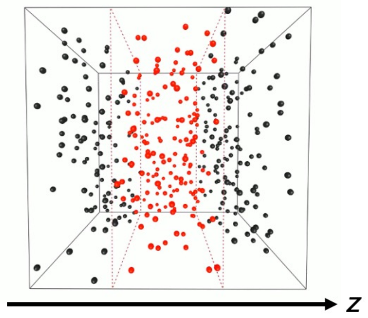
Skewness κ_3/κ_2



Kurtosis κ_4/κ_2



400 nucleons in a box

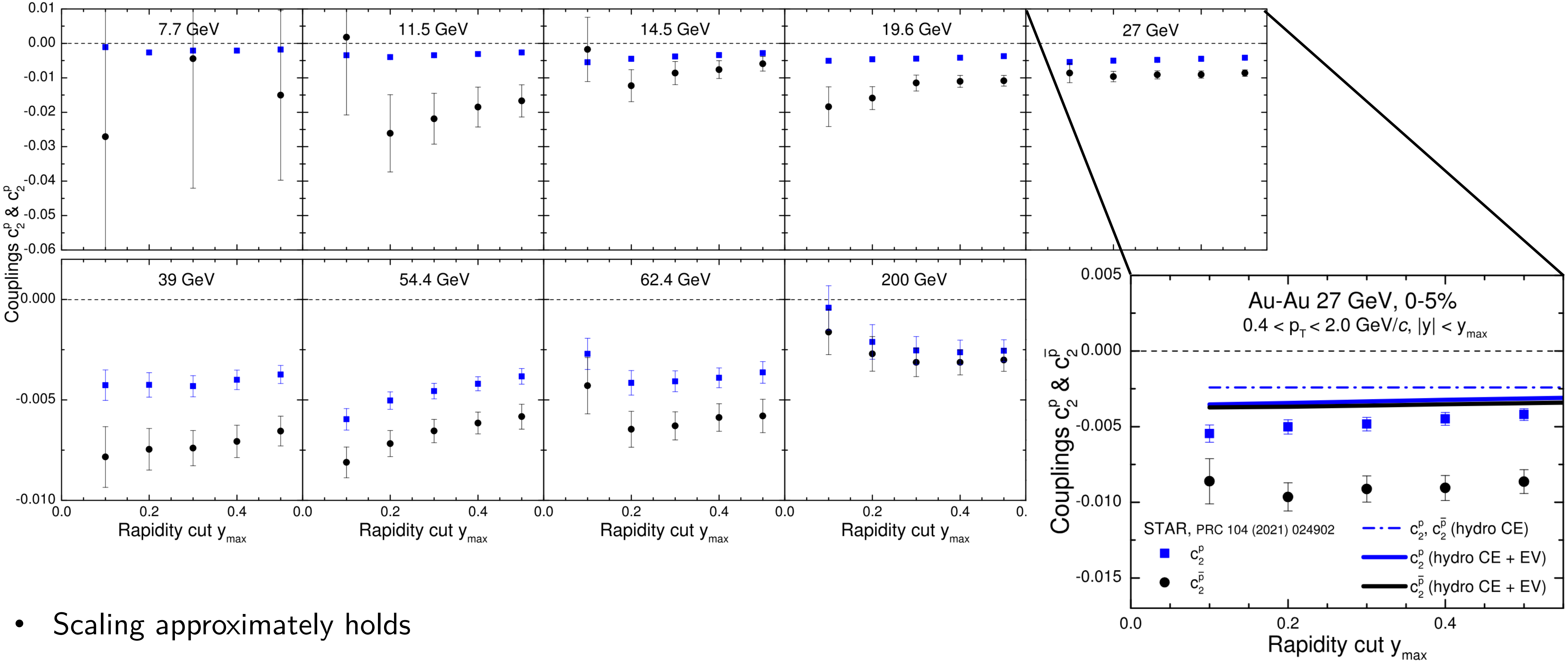


- (Non-)Gaussian cumulants equilibrate on comparable time scales

see also X. An et al., PRL 127, 072301 (2021); C. Chattopadhyay et al., PRL 133, 032301 (2024)

Scaled factorial cumulants from RHIC-BES-I

A. Bzdak, V. Koch, VV, arXiv:2503.16405



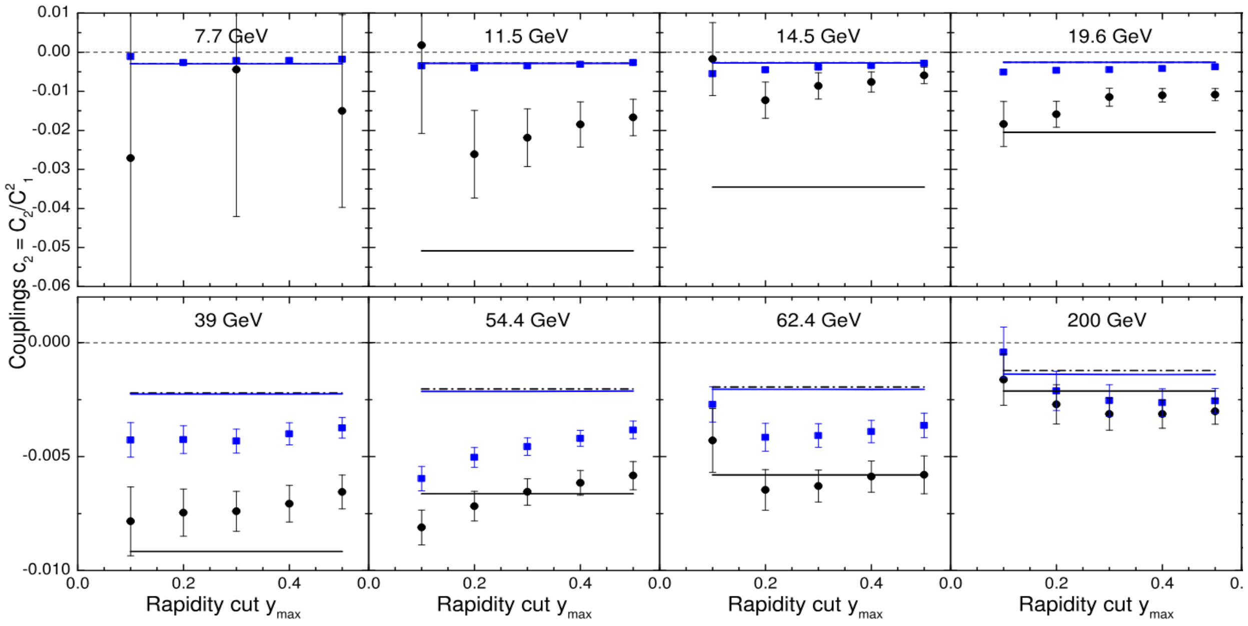
- Scaling approximately holds
- But significant difference between p and $\bar{p}$ in BES-I and hydro fails – **the antiproton puzzle**

no single thermalized fireball?

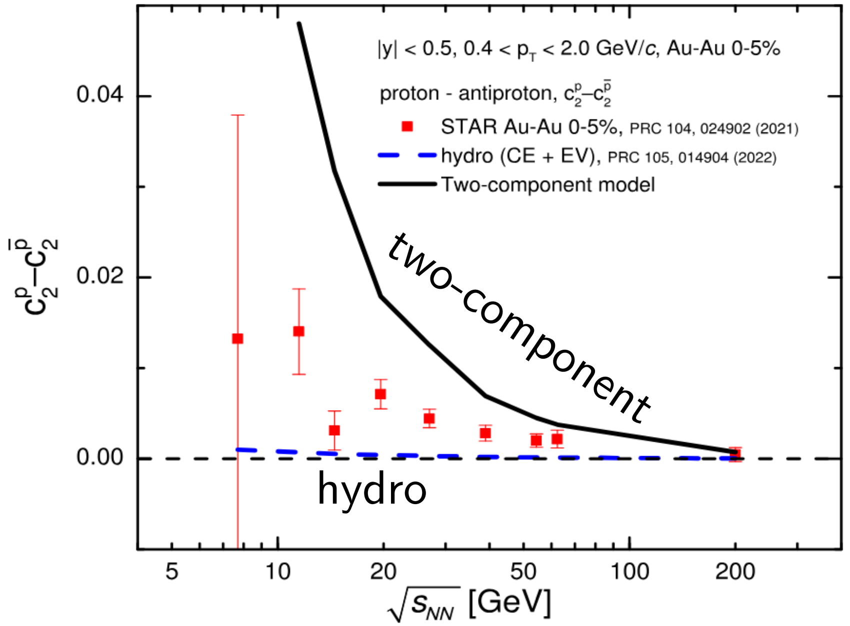
The antiproton puzzle and the two-component model

Two-component model: produced ($p\bar{p}$ pairs) and stopped protons comprise from two independent sources

The data lie in-between single and two-fireball models



Difference between p and $\bar{p}$



A. Bzdak, V. Koch, VV, arXiv:2503.16405

Opportunities for BES-II:

- Further tests of the splitting between p and $\bar{p}$ in 2nd order cumulants with extended y coverage
- Critical point signal expected to break the scaling

$$\frac{\hat{C}_n}{(\hat{C}_1)^n} = \text{const.}$$

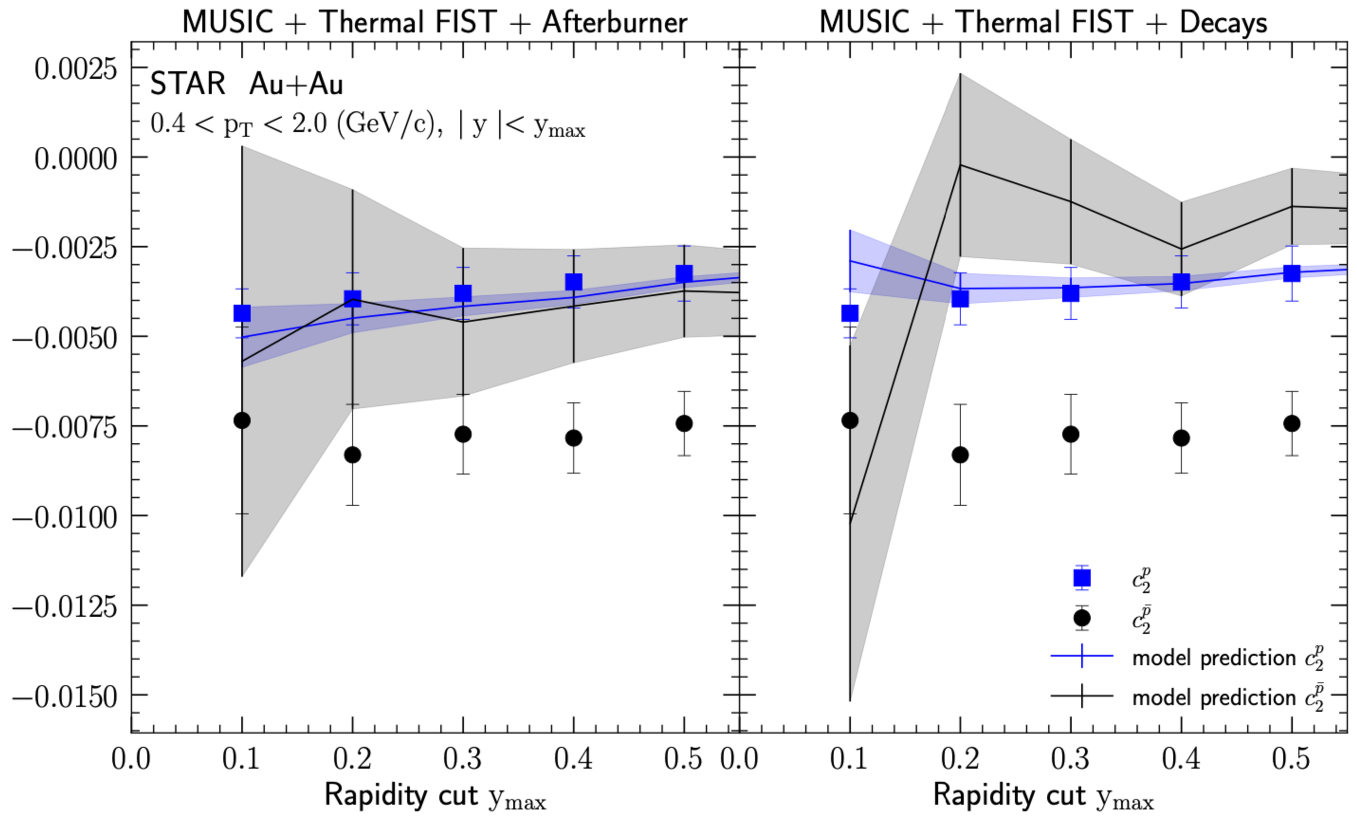
[Ling. Stephanov, PRC 93, 034915 (2016)]

Scaled factorial cumulants and baryon annihilation

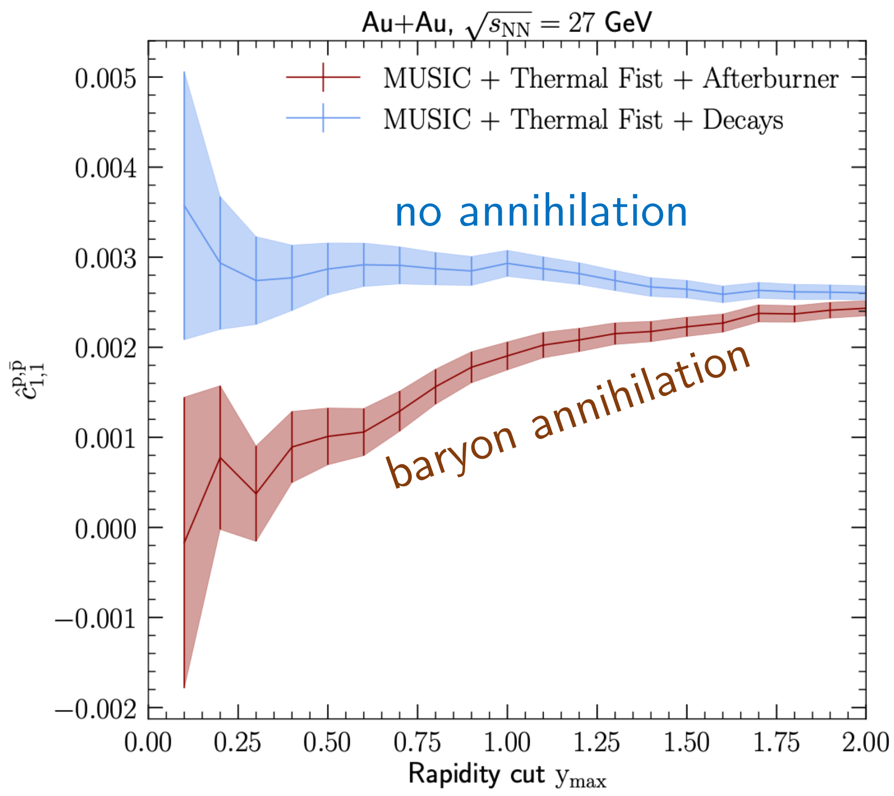
UNIVERSITY OF HOUSTON

Extending Hydro EV to incorporate hadronic phase (UrQMD)

Au-Au, $\sqrt{s_{NN}} = 27$ GeV G. Pihan, VV, in preparation



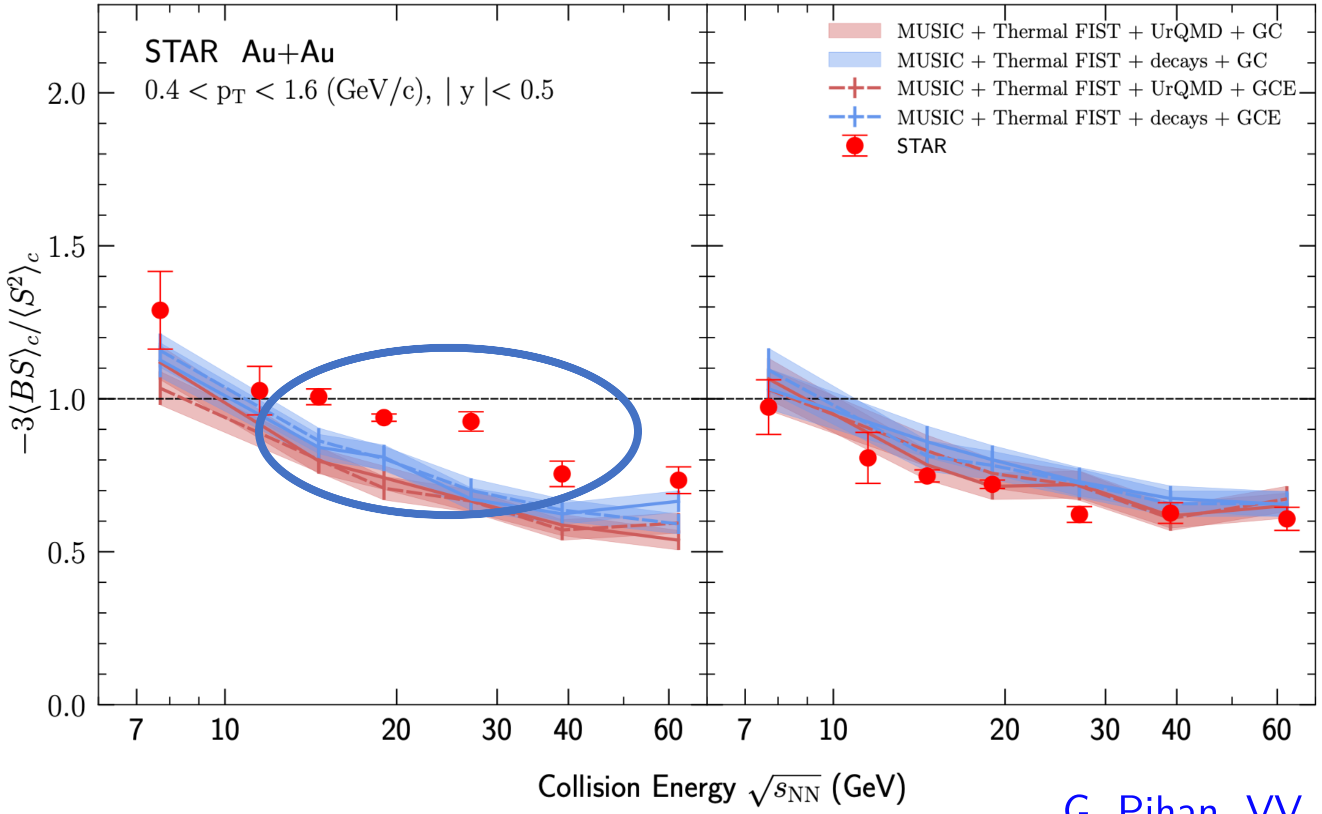
Covariance $c_{11}^{p\bar{p}}$



- Hadronic phase appears unlikely to resolve the antiproton puzzle (more statistics needed)
- Acceptance dependence of proton-antiproton covariance shows clear signature of hadronic phase

Baryon-strangeness correlator

Baryon-strangeness correlator is a diagnostic of QCD matter   Koch, Majumder, Randrup, PRL (2005)

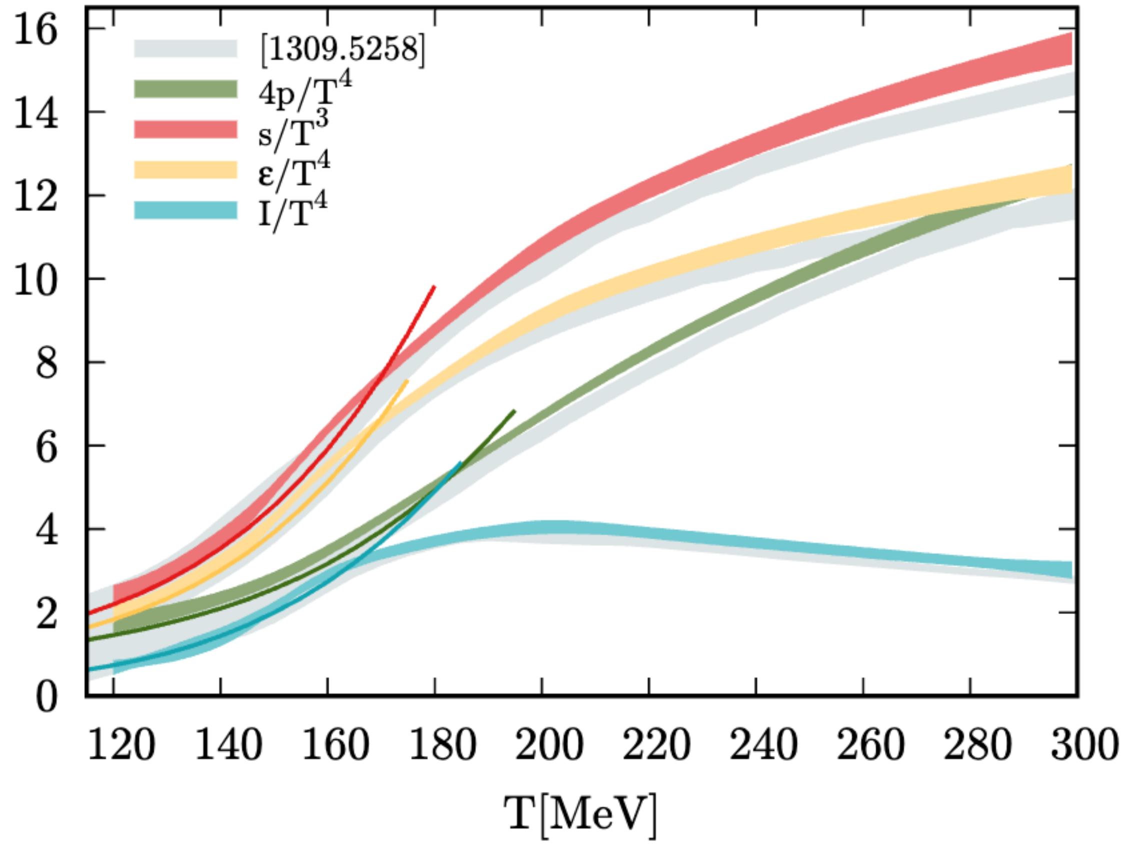


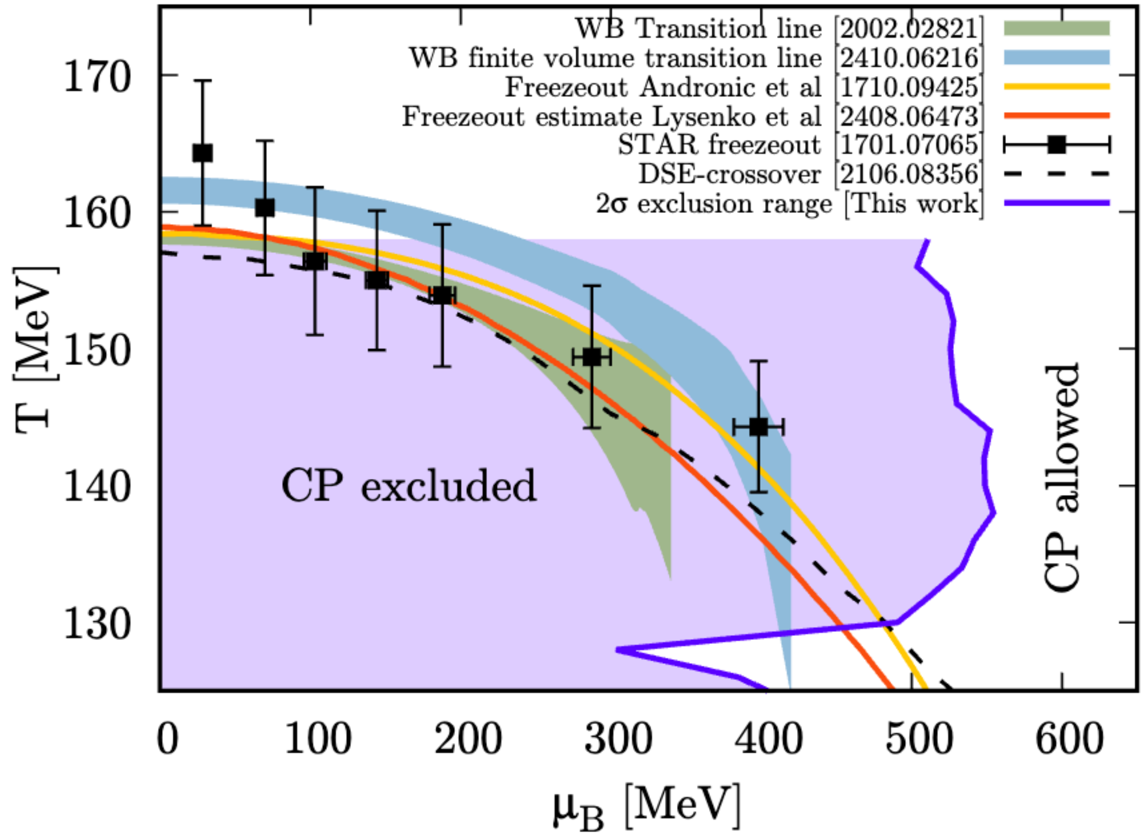
G. Pihan, VV, in preparation

- Hadronic phase appears unlikely to resolve the antiproton puzzle (more statistics needed)
- Acceptance dependence of proton-antiproton covariance shows clear signature of hadronic phase

Other observables: light nuclei production, balance functions, HBT,…

New CP constraints from lattice QCD

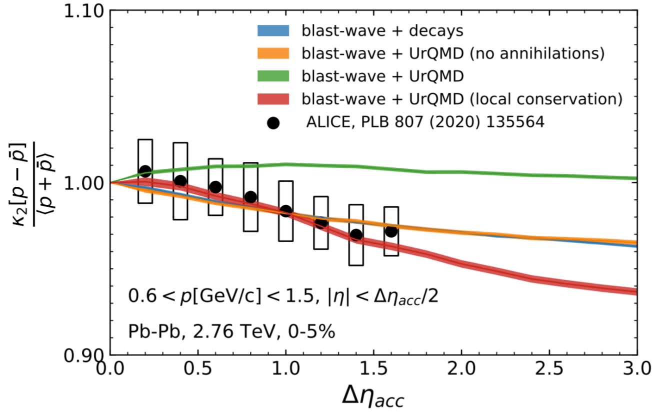




Proton cumulants at high energy

Second-order cumulants such as $\kappa_2[p-\bar{p}]/\langle p+\bar{p}\rangle$:

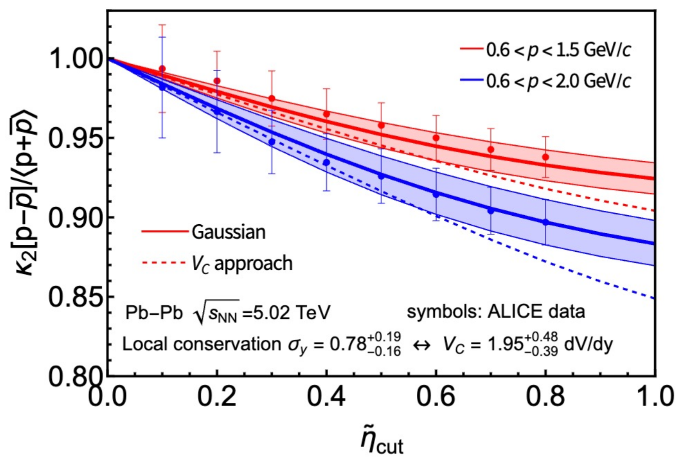
Pb-Pb 2.76 TeV



O. Savchuk et al., PLB 827, 136983 (2022)

- Largely understood as driven by baryon conservation
- baryon annihilation(↗) vs local conservation(↘)
  - Additional measurement of $\kappa_2[p+\bar{p}]$ can resolve it
- For some quantities like net-charge (or net-pion/net-kaon) fluctuations, resonance decays are improtant

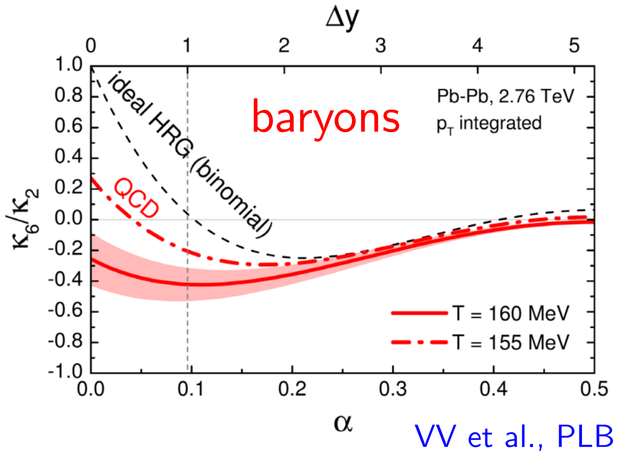
Pb-Pb 5.02 TeV



VV, arXiv:2409.01397

High-order cumulants: probe remnants of chiral criticality

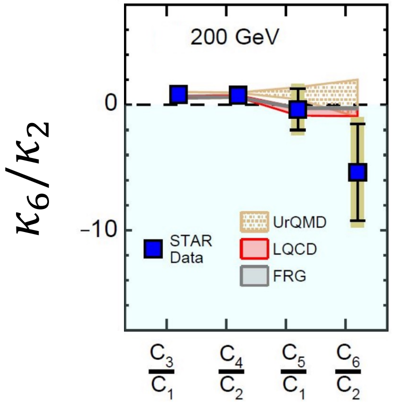
Friman et al., EPJC 71, 1694 (2011)



- negative κ_6 of baryons

VV et al., PLB 811, 135868 (2020)

RHIC 200 GeV: hints of negative $\kappa_6 < 0$ (protons)



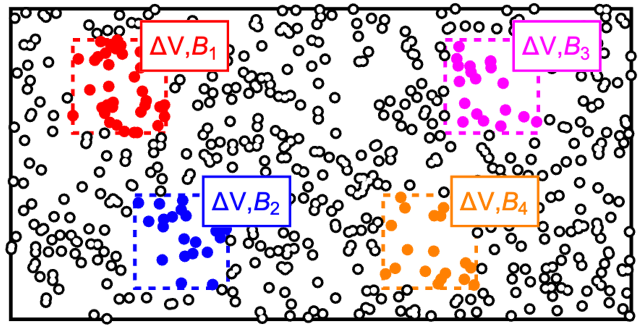
- are baryons even more negative?

STAR Collaboration, PRL 130, 082301 (2023)

Exact charge conservation

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020); VV, arXiv:2409.01397

Utilizing the canonical partition function in thermodynamic limit
compute **n-point density correlators**



$$\mathcal{C}_1(\mathbf{r}_1) \;=\; \rho(\mathbf{r}_1)$$

$$\mathcal{C}_2(\mathbf{r}_1,\mathbf{r}_2) = \chi_2\delta(\mathbf{r}_1-\mathbf{r}_2) - \frac{\chi_2}{V}$$

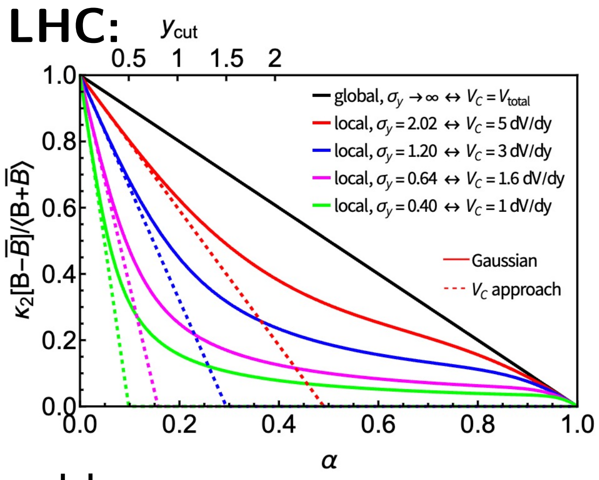
local correlation — balancing contribution (e.g. baryon conservation)

$$\mathcal{C}_3(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = \chi_3\delta_{1,2,3} - \frac{\chi_3}{V}[\delta_{1,2}+\delta_{1,3}+\delta_{2,3}] + 2\frac{\chi_3}{V^2} \qquad \delta_{1,\dots,n} = \prod_{i=2}^{n}\delta(\mathbf{r}_1-\mathbf{r}_i)$$

local correlation — balancing contributions

$$\mathcal{C}_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = \chi_4\delta_{1,2,3,4} - \frac{\chi_4}{V}[\delta_{1,2,3}+\delta_{1,2,4}+\delta_{1,3,4}+\delta_{2,3,4}] - \frac{(\chi_3)^2}{\chi_2 V}[\delta_{1,2}\delta_{3,4}+\delta_{1,3}\delta_{2,4}+\delta_{1,4}\delta_{2,3}]$$
$$+\frac{1}{V^2}\left[\chi_4+\frac{(\chi_3)^2}{\chi_2}\right][\delta_{1,2}+\delta_{1,3}+\delta_{1,4}+\delta_{2,3}+\delta_{2,4}+\delta_{3,4}] - \frac{3}{V^3}\left[\chi_4+\frac{(\chi_3)^2}{\chi_2}\right].$$

local correlation — balancing contributions



Integrating the correlator yields cumulant inside a subsystem of the canonical ensemble

$$\kappa_n[B_{V_s}] = \int_{\mathbf{r}_1\in V_s} d\mathbf{r}_1 \dots \int_{\mathbf{r}_n\in V_s} d\mathbf{r}_n\, \mathcal{C}_n(\{\mathbf{r}_i\})$$

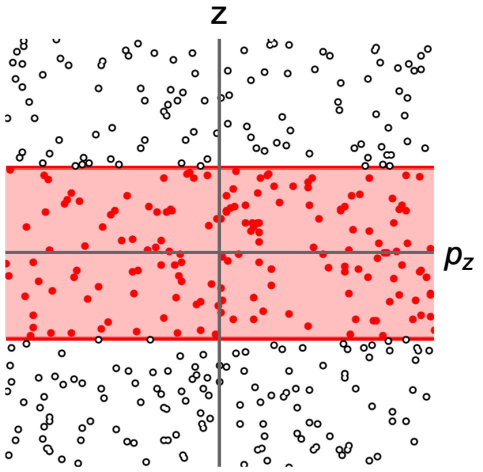
Momentum space: Fold with Maxwell-Boltzmann in LR frame and integrate out the coordinates

Coordinate vs Momentum space

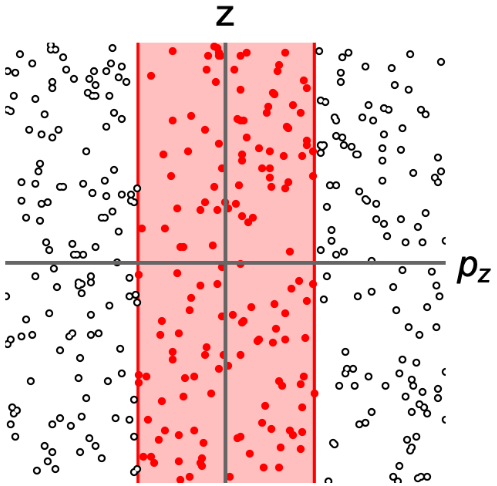
Box setup: Coordinates and momenta are uncorrelated

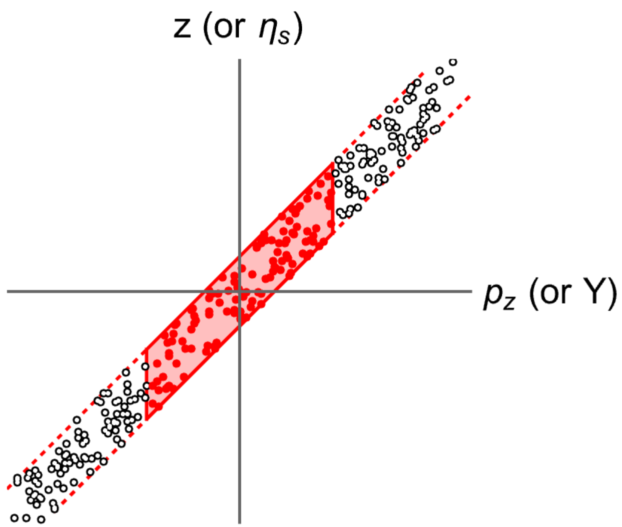
HICs: Flow (e.g. Bjorken)

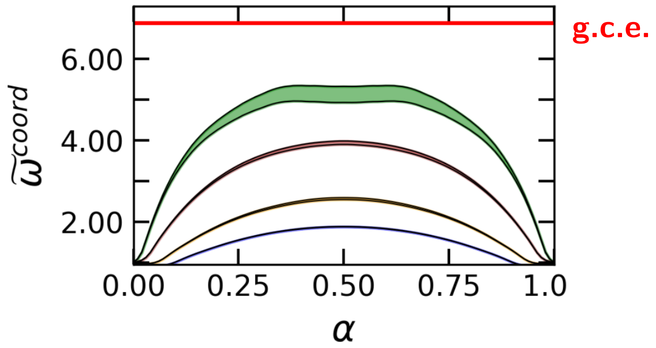
Coordinate space cut



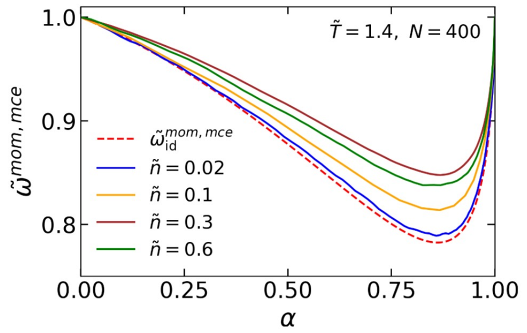
Momentum space cut



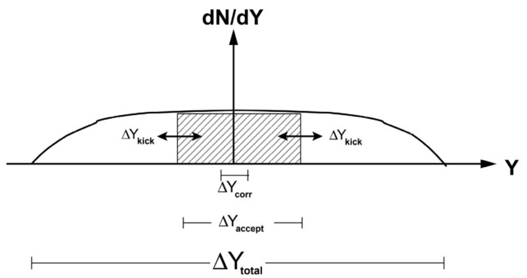




Large correlations



Nothing left

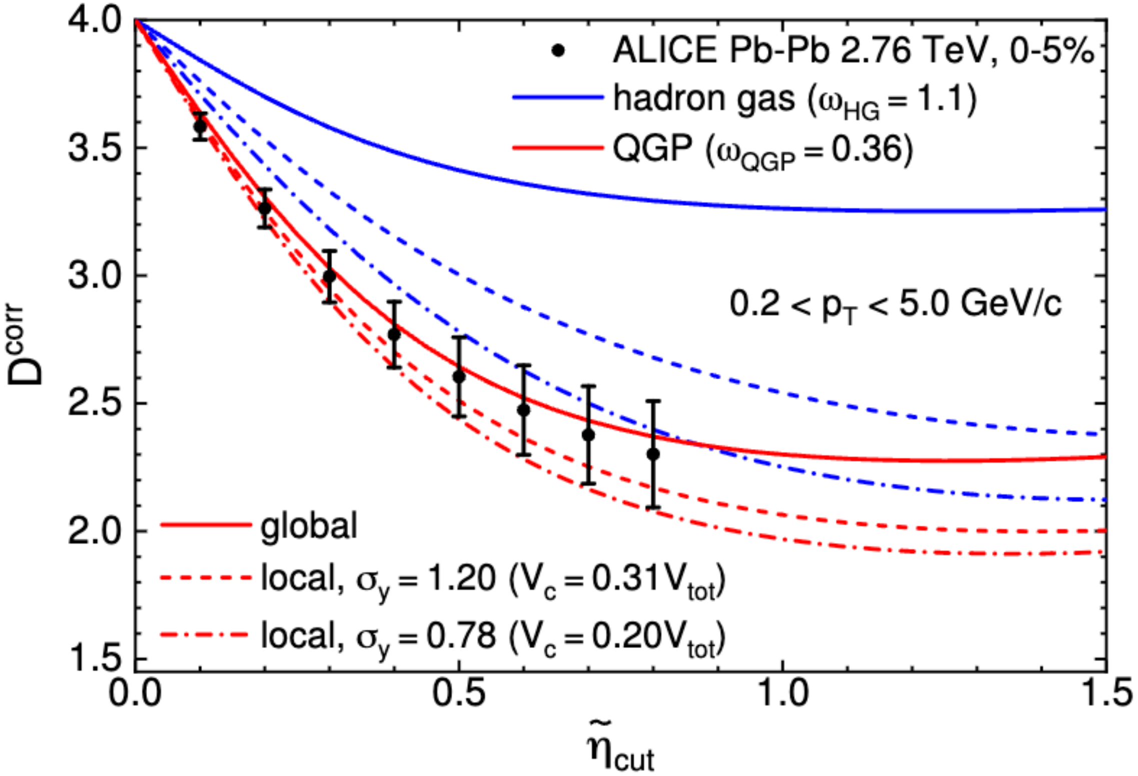


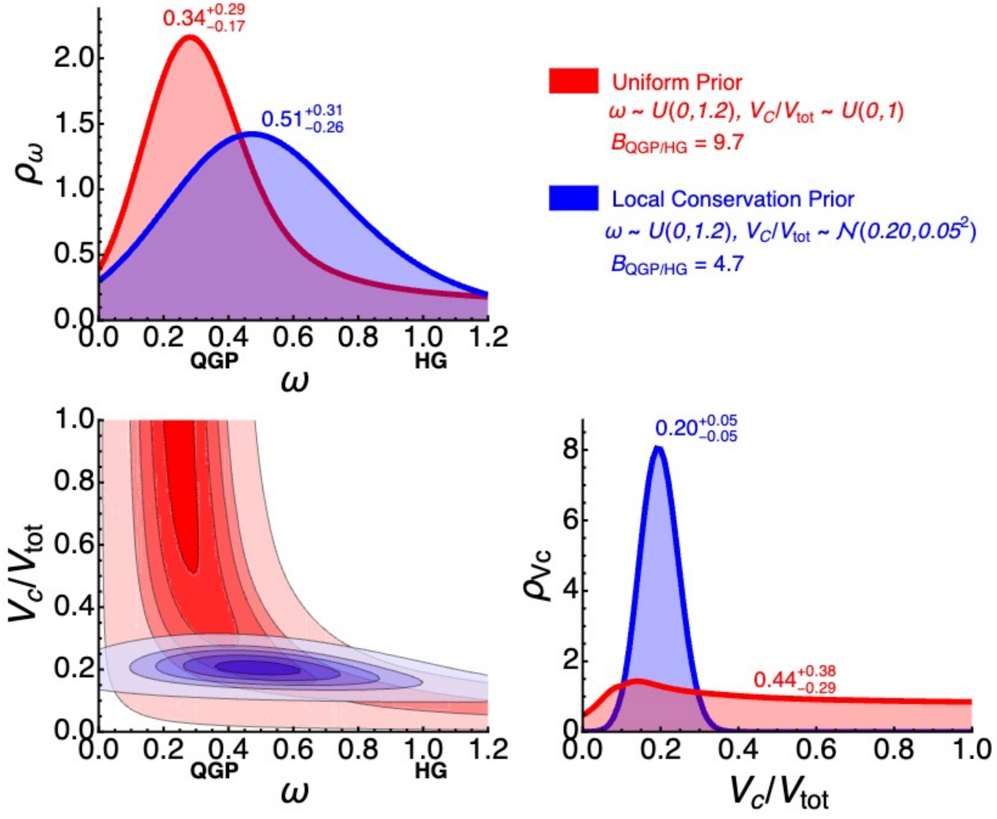
V. Koch, arXiv:0810.2520

momentum cut $\sim$ coordinate cut $+$ smearing

D-measure of charge fluctuations

$$D = 4\frac{\kappa_2[N_+ - N_-]}{\langle N_{\rm ch}\rangle} = 4\frac{\kappa_2[Q]}{\langle Q^+ + Q^-\rangle} = 4\left\{1 - \left(1 - \frac{\omega}{\gamma_Q}\right)\frac{\langle p^2(\eta)\rangle}{\langle p(\eta)\rangle} - \frac{\omega}{\gamma_Q}\frac{\langle p(\eta_1)p(\eta_2)\rangle_{\varkappa}}{\langle p(\eta)\rangle}\right\}$$





J. Parra, R. Poberezhniuk, V. Koch, C. Ratti, arXiv:2504.02085

Dynamical approaches to the QCD critical point search

UNIVERSITY OF HOUSTON

1. Dynamical model calculations of critical fluctuations — BEST COLLABORATION [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]
    - Fluctuating hydrodynamics (hydro+) and (non-equilibrium) evolution of fluctuations
    - Equation of state with a tunable critical point [P. Parotto et al, PRC 101, 034901 (2020); J. Karthein et al., EPJ Plus 136, 621 (2021)]
    - Generalized Cooper-Frye particlization [M. Pradeep, et al., PRD 106, 036017 (2022); PRL 130, 162301 (2023)]

Alternatives at high μ_B: hadronic transport/molecular dynamics with a critical point

[A. Sorensen, V. Koch, PRC 104, 034904 (2021); V. Kuznietsov et al., PRC 105, 044903 (2022)]

2. **Deviations from precision calculations of non-critical fluctuations**
    - Non-critical baseline is not flat [Braun-Munzinger et al., NPA 1008, 122141 (2021)]
    - Include essential non-critical contributions to (net-)proton number cumulants
    - Exact baryon conservation + hadronic interactions (hard core repulsion)
    - Based on realistic hydrodynamic simulations tuned to bulk data

[VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

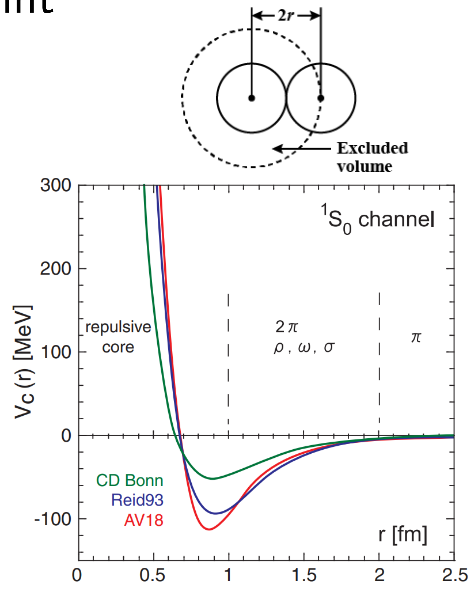
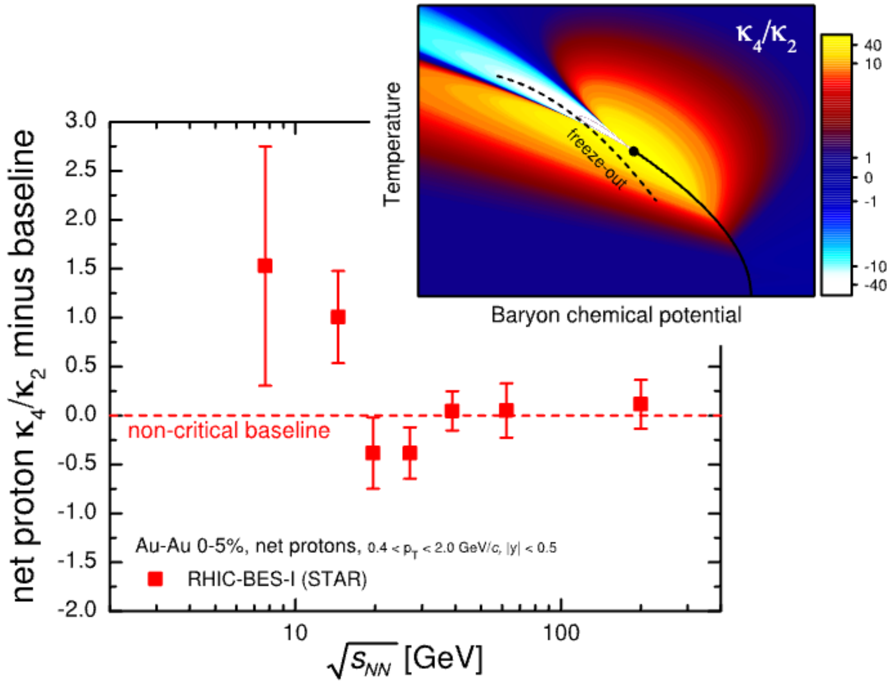


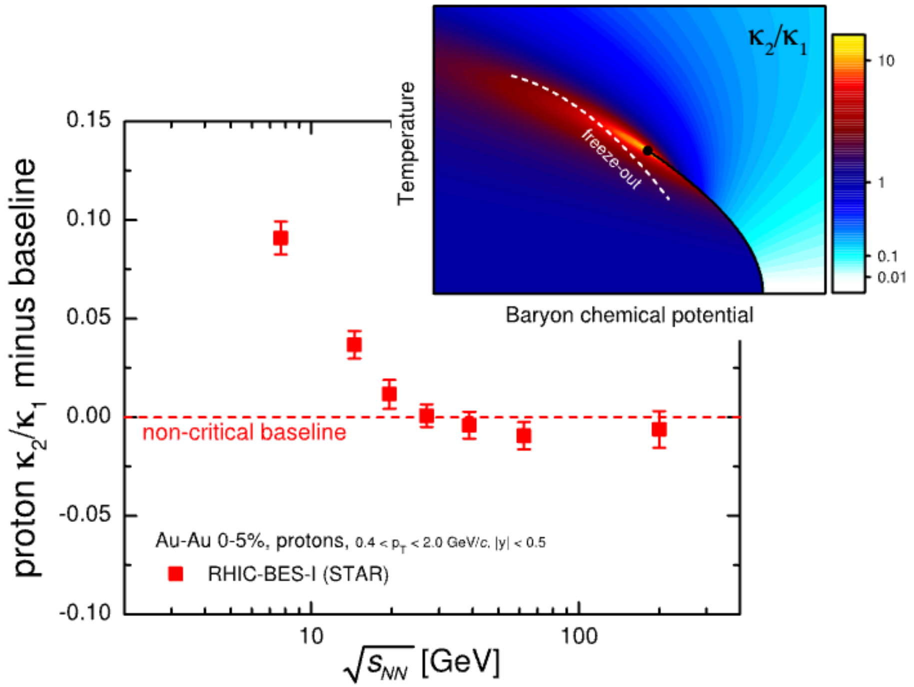
Figure from Ishii et al., PRL '07

Hints from RHIC-BES-I

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Subtracting the hydrodynamic non-critical baseline

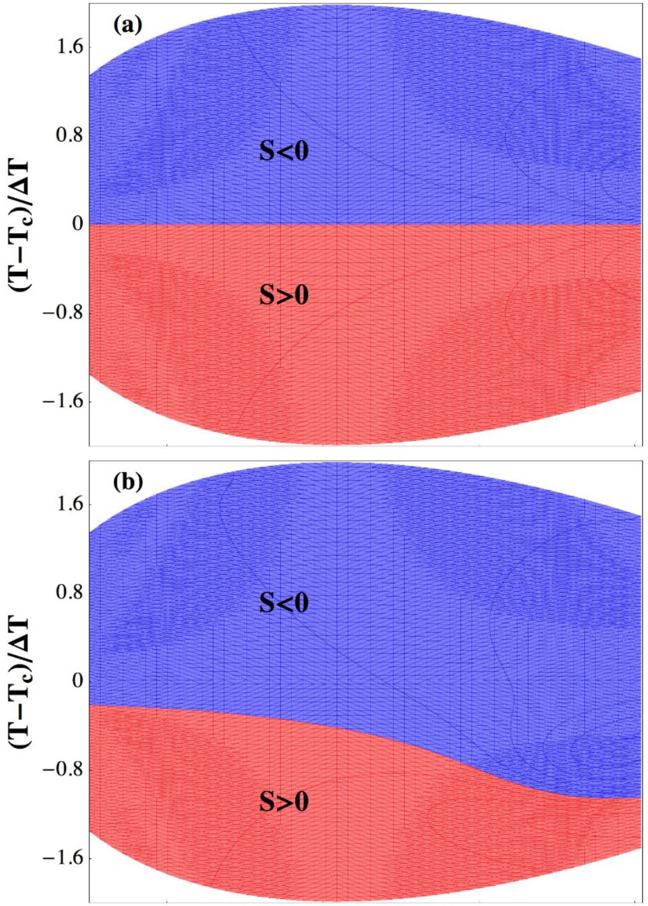




⚠️ **Notation:** Here we use κ_n for cumulants and $\hat{C}_n$ for factorial cumulants, STAR Collaboration uses the opposite

Factorial cumulants from RHIC-BES-II and CP

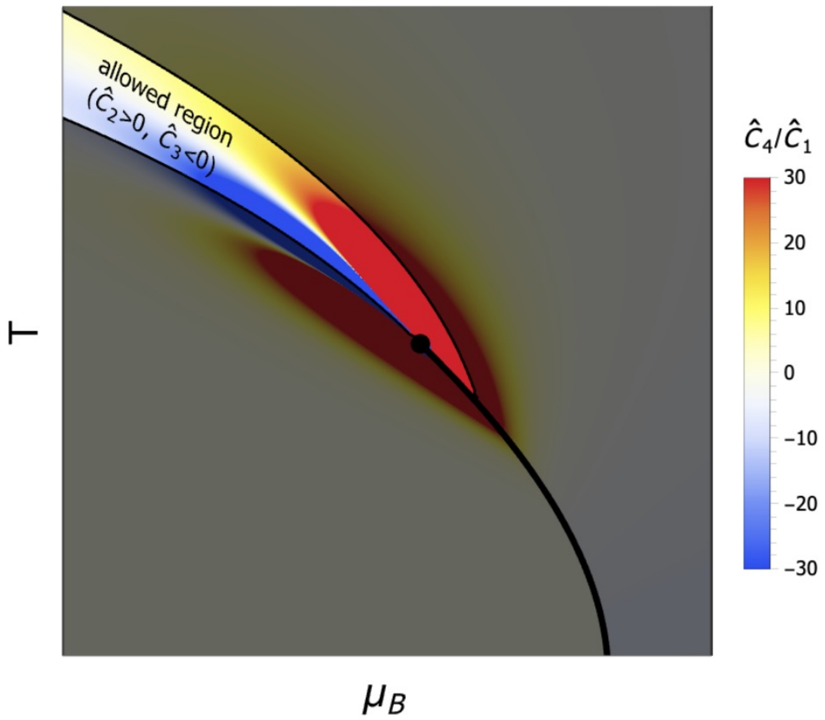
Memory effect



Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)

Exclusion plots

Exclude $\hat{C}_2<0$ & $\hat{C}_3>0$ regions on the phase diagram near CP

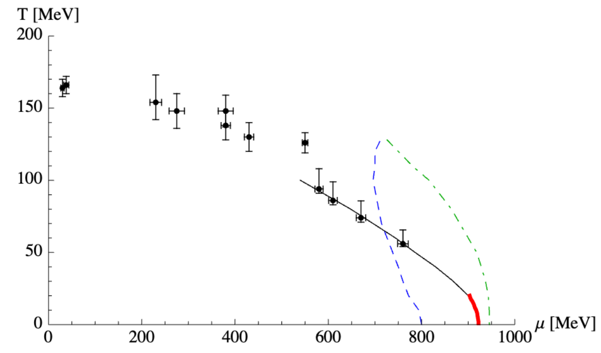


Adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)
and based on the model from
VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

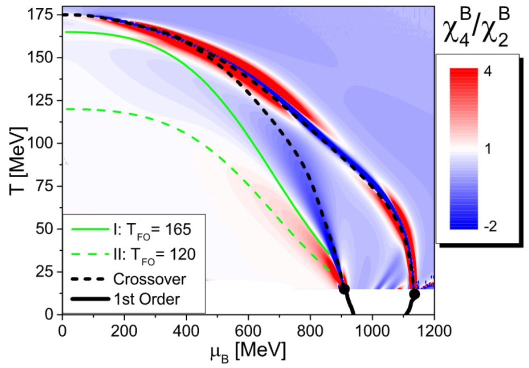
Freeze-out of fluctuations on the QGP side of the crossover?

Interplay with nuclear liquid-gas transition

UNIVERSITY OF HOUSTON

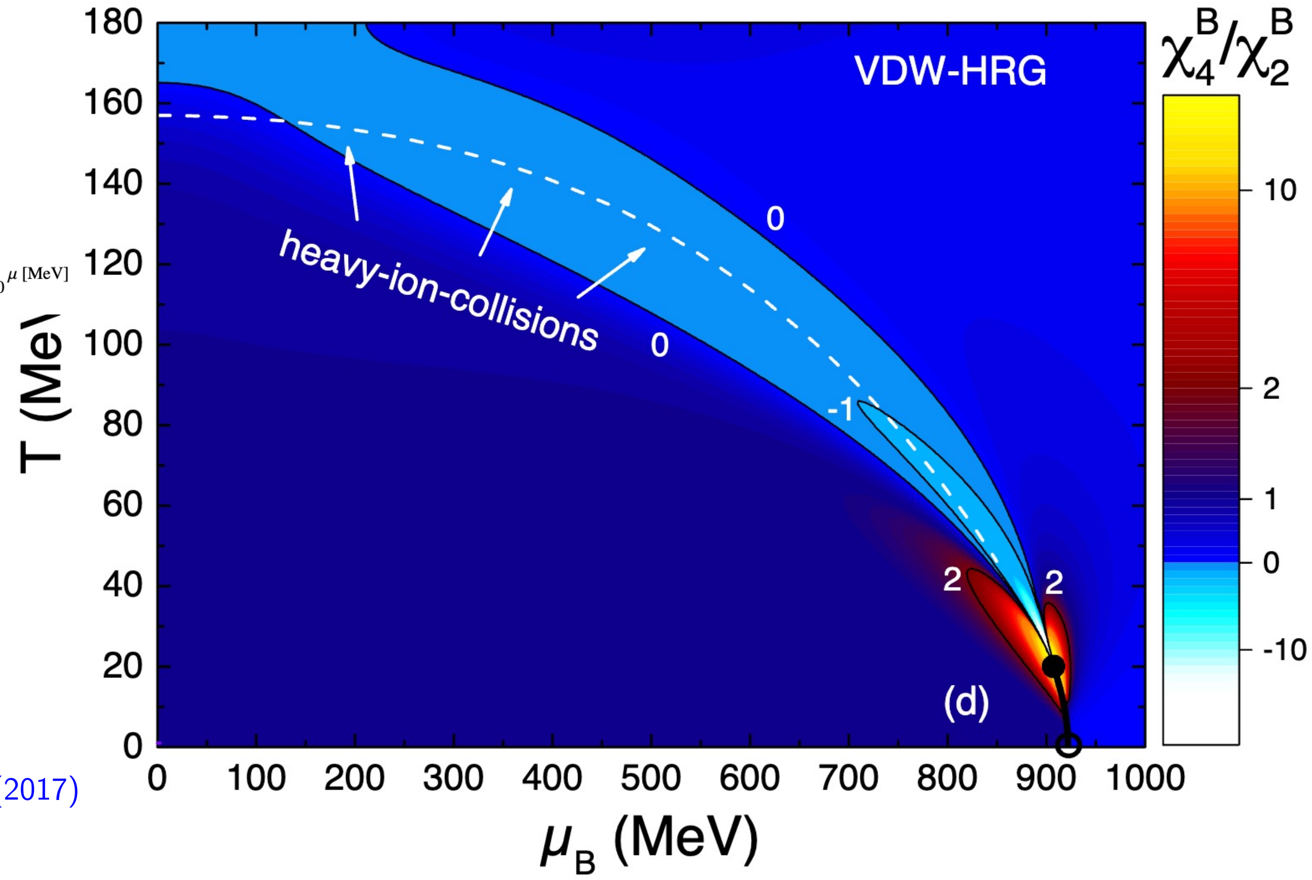


Floerchinger, Wetterich, NPA (2012)

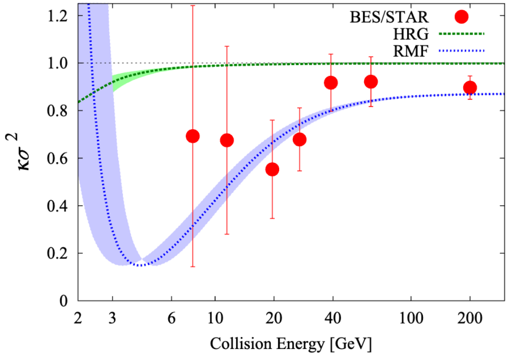


Mukherjee, Steinheimer, Schramm, PRC (2017)

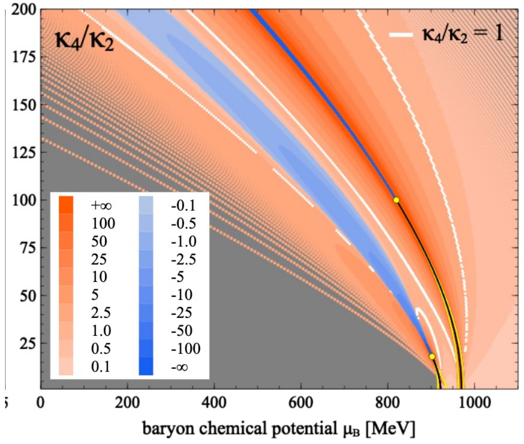
HRG with attractive and repulsive interactions among baryons



VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)



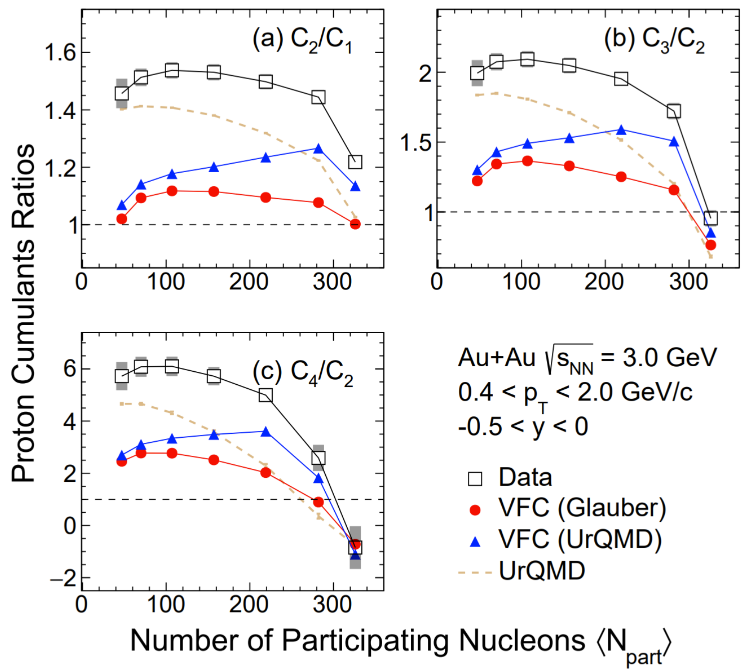
Fukushima, PRC (2014)



Sorensen, Koch, PRC (2020)

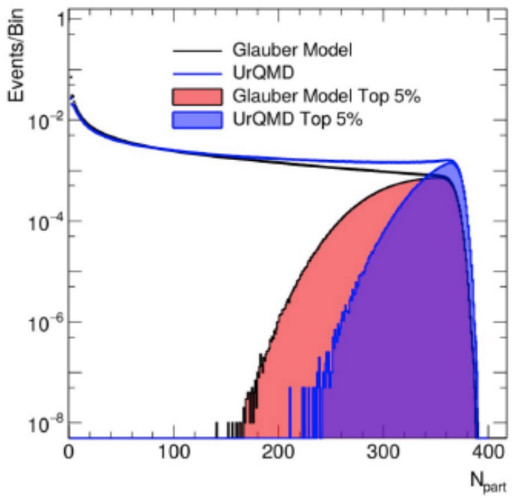
Increasingly relevant at lower energies probed through RHIC-FXT

Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV



STAR Collaboration, Phys. Rev. Lett. 128 (2022) 202303

STAR-FXT



HADES

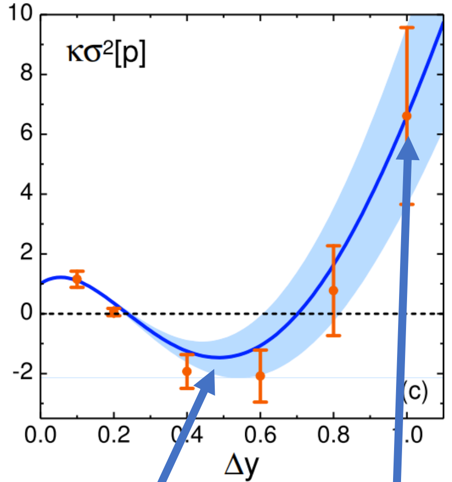


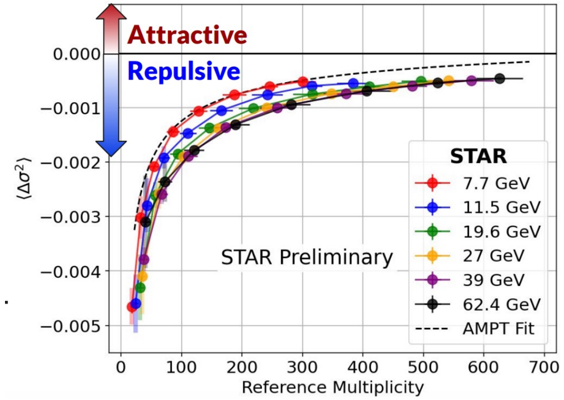
Figure from O. Savchuk et al., PLB 835, 137540 (2022)

- Volume fluctuations/centrality selection appear to play an important role
  - UrQMD is useful for understanding basic systematics associated with it
- Indications for enhanced scaled variance, $\kappa_2/\kappa_1 > 1$
- κ_4/κ_2 negative and described by UrQMD (purely hadronic?), note -0.5<y<0 instead of |y|<0.5

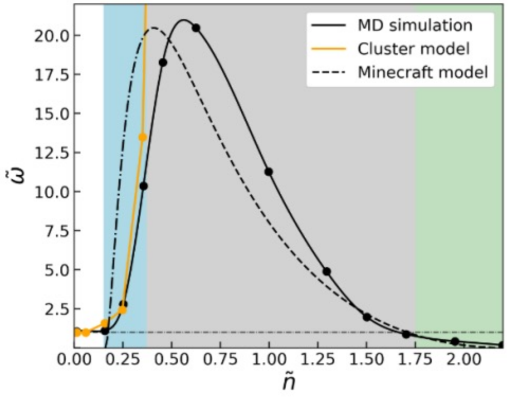
Proper understanding of $\kappa_2/\kappa_1 > 1$ in both HADES and STAR-FXT is missing

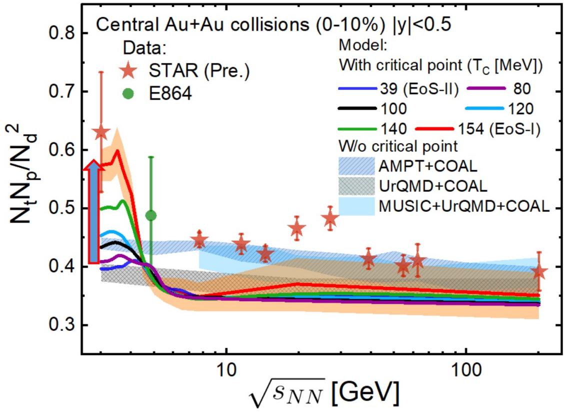
Other observables

- Azimuthal correlations of protons
  - points to repulsion at RHIC-BES



- Light nuclei
  - Spinodal/critical point enhancement of density fluctuations and light nuclei production





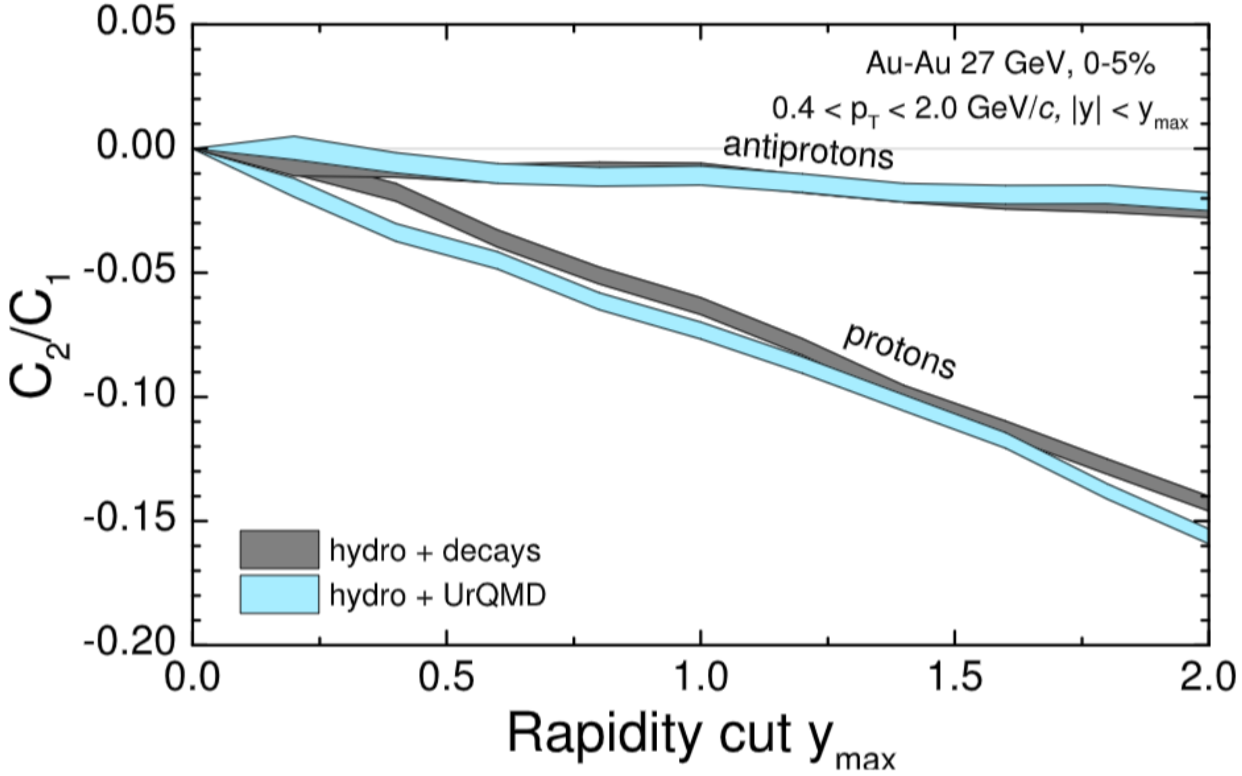
- Proton intermittency
  - No structure indicating power-law seen by NA61/SHINE

- Directed flow, speed of sound

Consistency in understanding all the observables is required

Effect of the hadronic phase

Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD



Dependence on the switching energy density