Canonical statistical hadronization with local charge conservation for fluctuations and correlations

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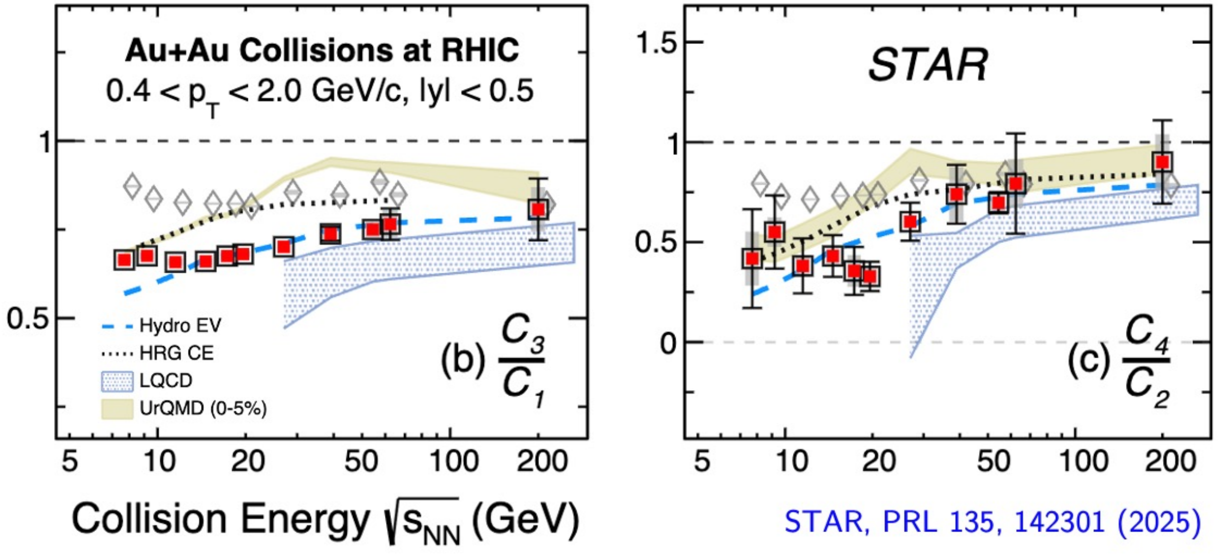
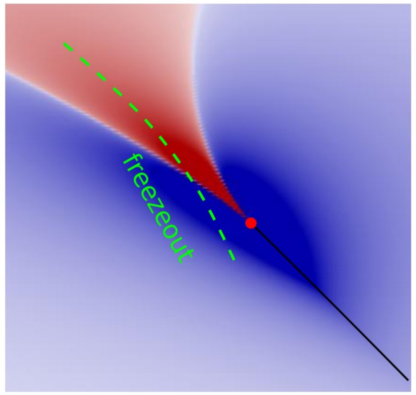
QCD phase structure with fluctuations

In equilibrium, the cumulants probe the derivatives of the QCD partition function



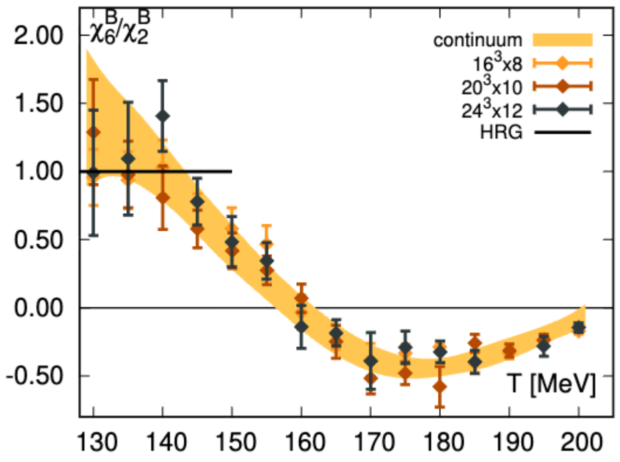
STAR, PRL 135, 142301 (2025)

Exact baryon conservation is a primary ingredient of non-critical baselines



QCD critical point
with beam energy scan

M. Stephanov, PRL '09, '11
Motivation for RHIC-BES



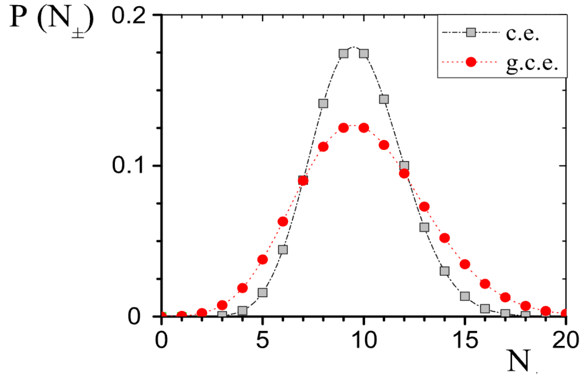
Probe chiral crossover and remnants of chiral criticality with high-order cumulants at $\mu_B = 0$

Wuppertal-Budapest, PRD 110, L011501 (2024)

Hadron resonance gas in the canonical ensemble

Begun, Gazdzicki, Gorenstein, Zozulya, PRC 70, 034901 (2004)

Canonical partition function of an ideal gas of **particles and antiparticles:**



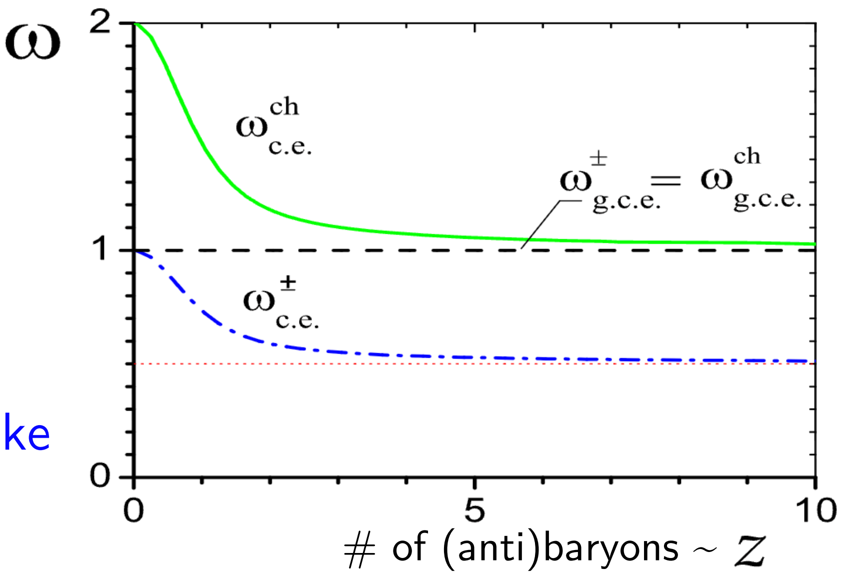
$$Z_{c.e.}(V,T) = \sum_{N_+=0}^{\infty} \sum_{N_-=0}^{\infty} \frac{(\lambda_+ z)^{N_+}}{N_+!} \frac{(\lambda_- z)^{N_-}}{N_-!} \delta(N_+ - N_-) =$$

$$= \frac{1}{2\pi} \int_0^{2\pi} d\phi \ \exp\left[z\left(\lambda_+ e^{i\phi} + \lambda_- e^{-i\phi}\right)\right] = I_0(2z)$$

Skellam distribution

$$P_{c.e.}(N_+) = \frac{1}{I_0(2z)} \cdot \left(\frac{z^{N_+}}{N_+!}\right)^2$$

Fluctuations:



$$\omega^{\pm}_{c.e.} = \frac{\langle N_\pm^2 \rangle_{c.e.} - \langle N_\pm \rangle^2_{c.e.}}{\langle N_\pm \rangle_{c.e.}} = 1 - z\left[\frac{I_1(2z)}{I_0(2z)} - \frac{I_2(2z)}{I_1(2z)}\right] \cong \frac{1}{2}$$

Exact (baryon) charge conservation introduces correlation among unlike charges ($B\bar{B}$) and anticorrelation among like charges (BB and $\bar{B}\bar{B}$)

Further developments evaluate high-order cumulants in acceptance

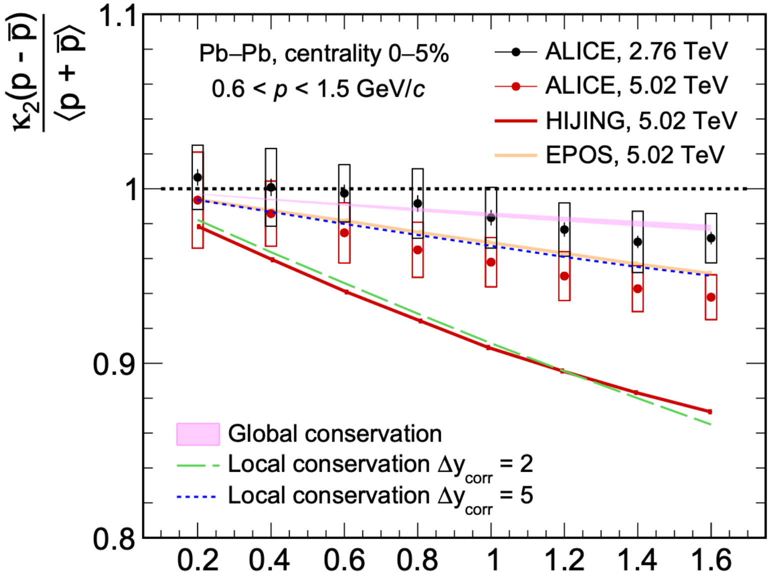
Bzdak, Koch, Skokov, PRC 87, 014901 (2013); Braun-Munzinger et al., NPA 1008, 122141 (2021)

LHC and local conservation

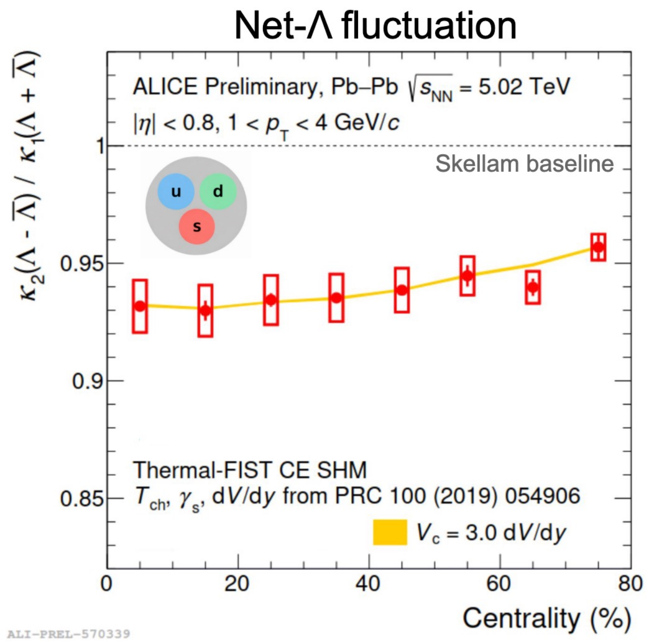
Global conservation correlates all baryons everywhere in the fireball

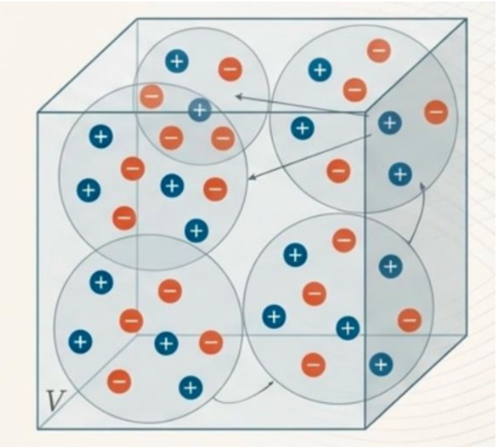
This requires very early production of the baryon charge...

or introducing **local baryon conservation**

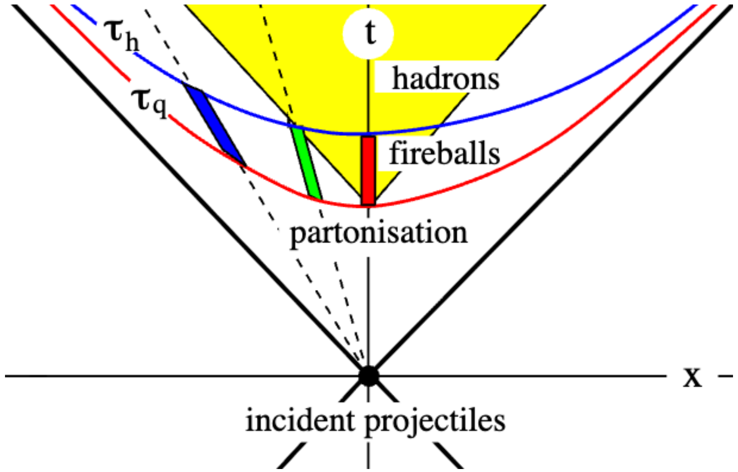


ALICE, PLB 844, 137545 (2023)





Castorina, Satz, IJMPE 23, 1450019 (2014)

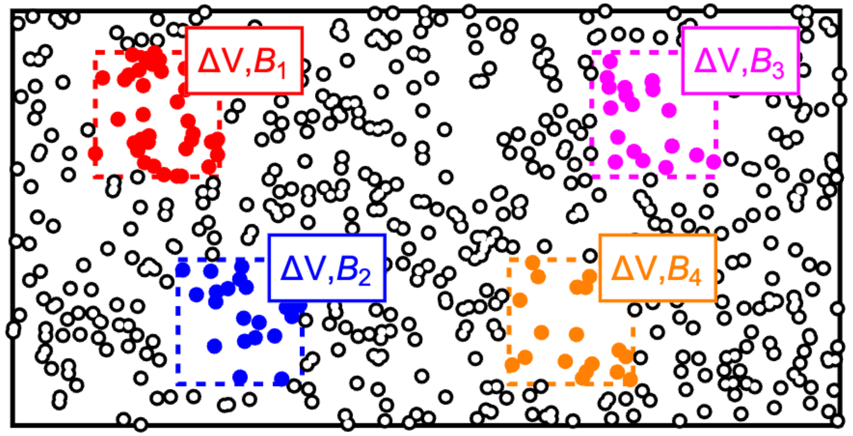


- balance functions [Schlichting, Pratt, PRC 83, 014913 (2011)]
- diffusion master equation [Sakaida et al., PRC 90, 064911 (2014)]
- V_c approach [VV, Donigus, Stoecker, PRC 100, 054906 (2019)]
- Correlated sampling [Braun-Munzinger, Redlich, Rustamov, Stachel, JHEP 08, 113 (2024)]

Density correlations framework

VV, PRC 110, L061902 (2024)

Split a thermal system into multiple subvolumes ΔV and consider joint distribution of baryon number inside the subvolumes



$$G_{\boldsymbol{B}}(\boldsymbol{t}) = \ln \left\langle e^{\sum_{i=1}^{n} t_i B_i} \right\rangle = \ln \left[\sum_{\boldsymbol{B}} \exp\left(\sum_{i=1}^{n} t_i B_i \right) P(\boldsymbol{B}) \right]$$

$$\boldsymbol{B} = (B_1, \dots, B_N), \qquad B_1 + \dots + B_N = B_{tot}$$

In large system (thermodynamic limit), the joint probability factorizes into a product of partition functions

$$P(\boldsymbol{B}) \propto \left[\prod_{j=1}^{n} Z(\Delta V, B_j) \right] Z(V - n\Delta V, B - \sum_{j=1}^{n} B_j) \propto \left[\prod_{j=1}^{n} e^{-\Delta V f(\rho_j)} \right] e^{-(V-n\Delta V) f(\rho_{n+1})}$$

$f(\rho)$ – free energy density

$(\partial \mu_B / \partial \rho_B)_T = [T^3 \chi_2^B]^{-1}$

$$\left\langle \delta B_1^{k_1} \dots \delta B_n^{k_n} \right\rangle_c = \left. \frac{\partial^{k_1+\dots+k_n} G_{\boldsymbol{B}}(\boldsymbol{t})}{\partial t_1^{k_1} \dots \partial t_n^{k_n}} \right|_{t_1=\dots=t_n=0} \quad \xrightarrow{\Delta V \to 0} \quad \mathcal{C}_n(\eta_1, \dots, \eta_n) \equiv \left\langle \prod_{i=1}^{n} \delta\rho_i \right\rangle_c, \qquad n \geq 2,$$

Density correlations framework

VV, PRC 110, L061902 (2024)

$$\mathcal{C}_2(\eta_1,\eta_2) = \boldsymbol{\chi_2^B \delta_{1,2}} - \frac{\chi_2^B}{V}$$

GCE 2-point

$$\mathcal{C}_3(\eta_1,\eta_2,\eta_3) = \boldsymbol{\chi_3^B \delta_{1,2,3}} - \frac{\chi_3^B}{V}[\delta_{1,2}+\delta_{1,3}+\delta_{2,3}] + \frac{2\chi_3^B}{V^2}$$

GCE 2-point 3-point

$$\mathcal{C}_4(\eta_1,\eta_2,\eta_3,\eta_4) = \boldsymbol{\chi_4^B \delta_{1,2,3,4}} - \frac{\chi_4^B}{V}[\delta_{1,2,3}+\delta_{1,2,4}+\delta_{1,3,4}+\delta_{2,3,4}] - \frac{(\chi_3^B)^2}{\chi_2^B V}[\delta_{1,2}\delta_{3,4}+\delta_{1,3}\delta_{2,4}+\delta_{1,4}\delta_{2,3}]$$

GCE 2-point 2-point

$$+\frac{1}{V^2}\left[\chi_4^B + \frac{(\chi_3^B)^2}{\chi_2^B}\right][\delta_{1,2}+\delta_{1,3}+\delta_{1,4}+\delta_{2,3}+\delta_{2,4}+\delta_{3,4}] - \frac{3}{V^3}\left[\chi_4^B + \frac{(\chi_3^B)^2}{\chi_2^B}\right]$$

3-point 4-point

All terms apart from the local one are *balancing contributions*

Integrals yield canonical ensemble cumulants in subvolume $\prod_{i=1}^{n}\int d\eta_i\, \mathcal{C}_n(\eta_1,\ldots,\eta_n) = \kappa_n[B].$

Introducing local charge conservation

VV, PRC 110, L061902 (2024)

Introduce Gaussian (spatial) rapidity correlation into baryon-conservation balancing term

global conservation

$$C_2^B(\eta_1,\eta_2) = \langle n_B + n_{\bar{B}} \rangle \left[\delta(\eta_1 - \eta_2) - \frac{1}{2\eta_{\max}} \right]$$

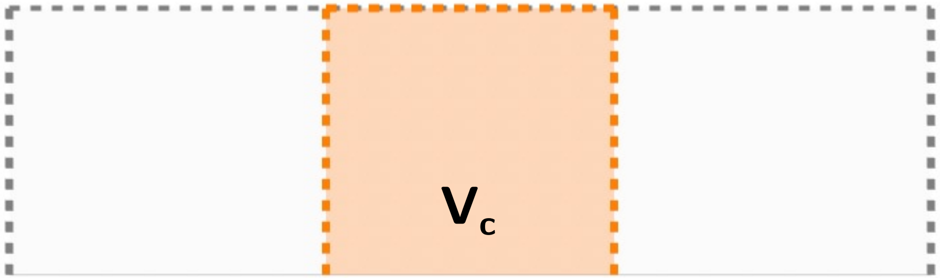
local correlation — balancing contribution (e.g. baryon conservation)

+ local conservation

$$C_2^B(\eta_1,\eta_2) = \langle n_B + n_{\bar{B}} \rangle \left[\delta(\eta_1 - \eta_2) - \frac{\tilde{A}\, e^{-\frac{(\eta_1-\eta_2)^2}{2\sigma_\eta^2}}}{2\eta_{\max}} \right]$$

local correlation — local balancing contribution

truncated fireball



vs

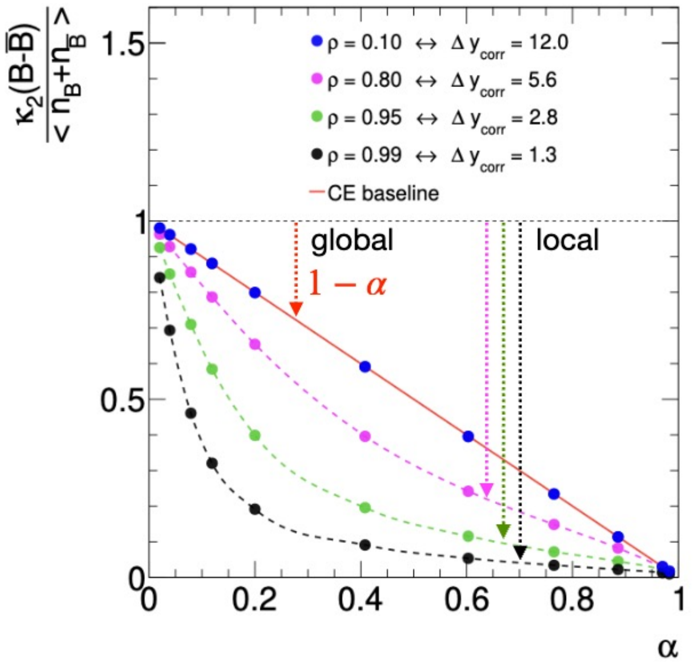
Gaussian correlation

Gaussian correlation captures the diminishing contributions of hadrons at forward/backward rapidities

Applicable also to model local conservation of other conserved charges

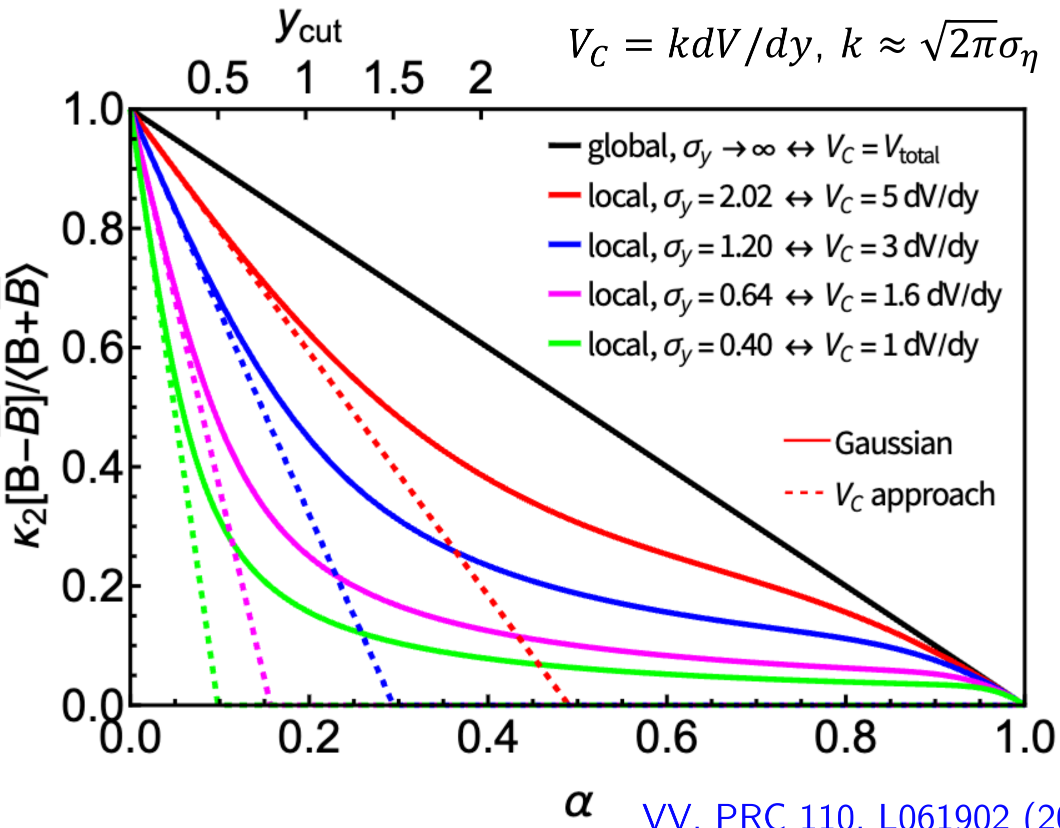
Local charge conservation in coordinate space

correlated sampling



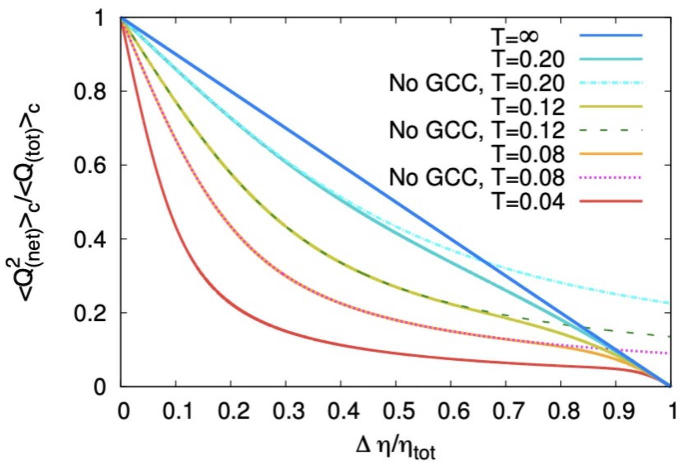
Braun-Munzinger et al., JHEP 08, 113 (2024)

$V_C = k dV/dy,\ k \approx \sqrt{2\pi}\sigma_\eta$



VV, PRC 110, L061902 (2024)

diffusion equation



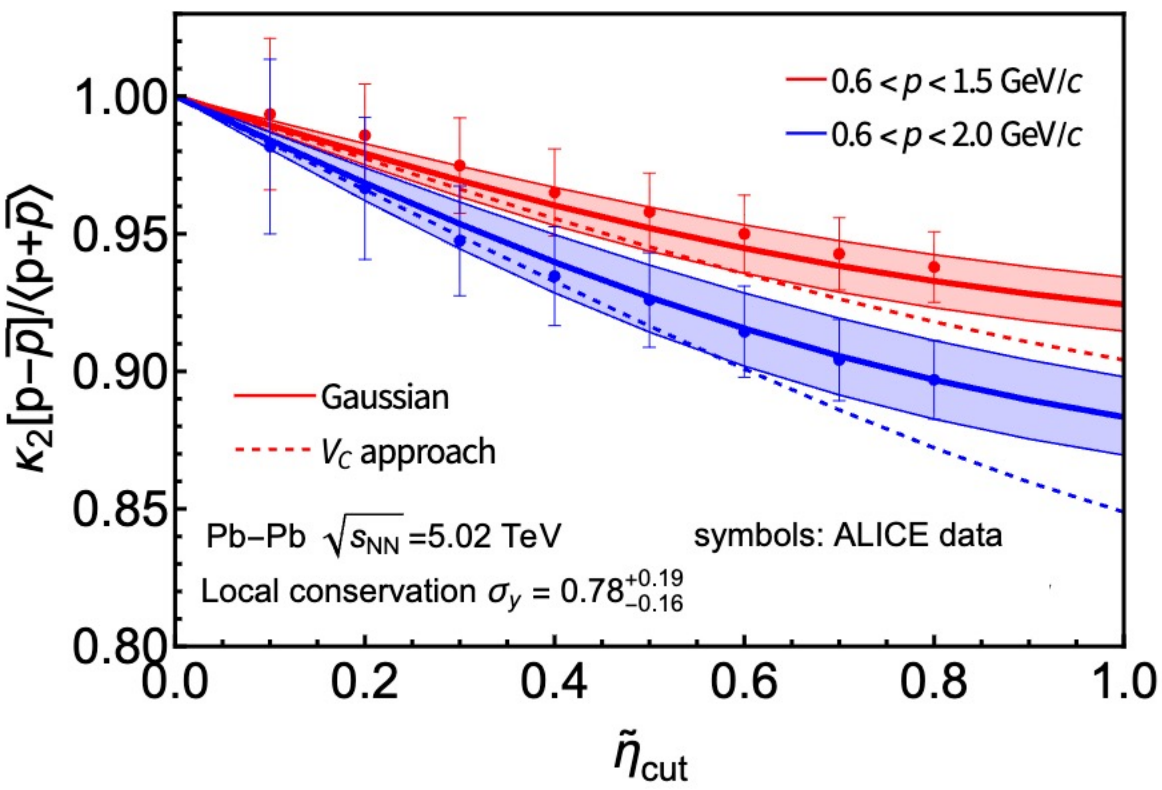
Sakaida et al., PRC 90, 064911 (2014)

- Good agreement between different implementations of local charge conservation at 2nd order
- V_c approach works at small α, smaller for a more local range of conservation
  - Experiment (LHC): $|y| < 0.5$ corresponds to $\alpha < 0.1$
  - Measuring protons (not baryons) + p_T cuts decrease effective α to 0.025

Braun-Munzinger et al., NPA 1008, 122141 (2021)

Local charge conservation: 2^{nd} order

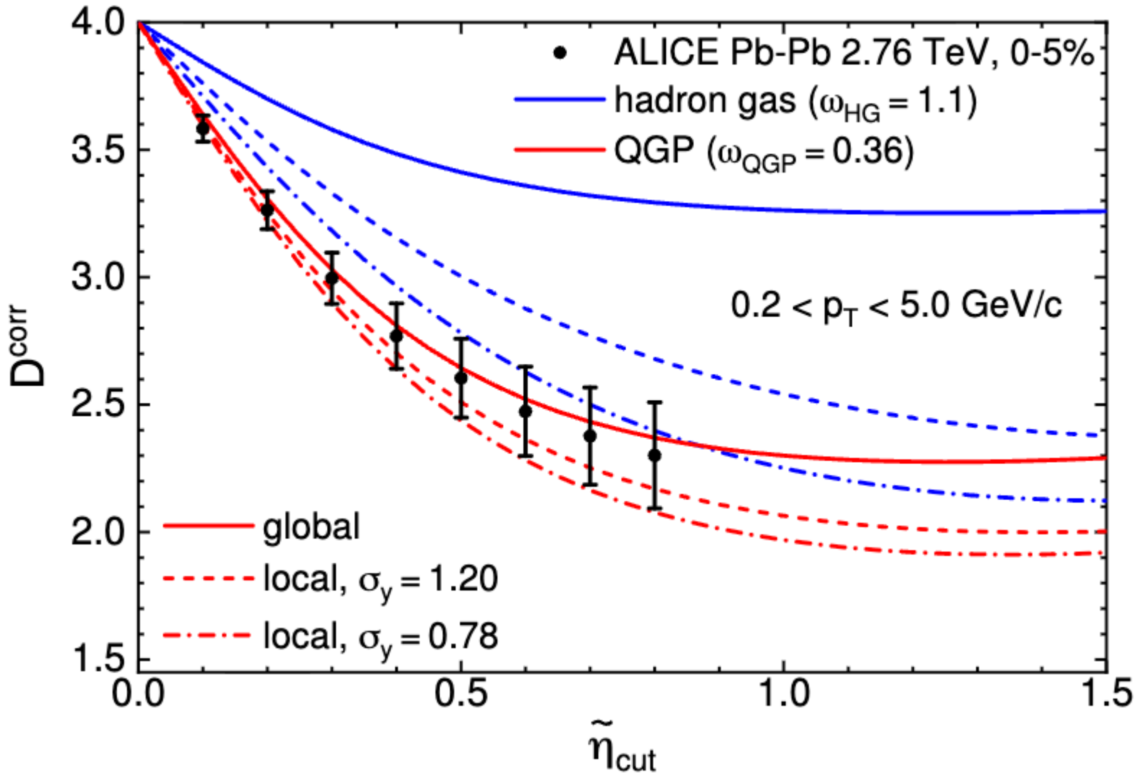
net-proton fluctuations



VV, PRC 110, L061902 (2024)

Hadronic scenario describe data with $\sigma_y \sim 0.78$

net-charge fluctuations



J. Parra et al., PRL 135, 242302 (2025)

Moderate evidence for freeze-out in QGP, hadronic scenario can only work for local charge conservation

Local charge conservation: 2nd order generalizations

- Non-conserved quantities correlated to a conserved charge

$$\mathrm{C}_{11}^{ij}(x_1,x_2) = \chi_{ij}\left[\delta(x_1-x_2) - \varkappa(x_1,x_2)\frac{\chi_{11}^{iB}\chi_{11}^{jB}}{\chi_2^B\,\mathrm{V}}\right]$$

$$\mathrm{C}_2^{\mathrm{B}-\bar{\mathrm{B}}}(x_1,x_2) = \langle \mathrm{B}+\bar{\mathrm{B}}\rangle\left[\delta(x_1-x_2) - \varkappa(x_1,x_2)\frac{1}{\mathrm{V}}\right]$$

self-correlation **balancing term**

$$\mathrm{C}_2^{\mathrm{B}+\bar{\mathrm{B}}}(x_1,x_2) = \langle \mathrm{B}+\bar{\mathrm{B}}\rangle\,\delta(x_1-x_2)$$

Local baryon conservation does not affect $\boldsymbol{B}+\bar{\boldsymbol{B}}$

$$\mathrm{C}_2^{\bar{\mathrm{B}}}(x_1,x_2) = \langle \bar{\mathrm{B}}\rangle\left[\delta(x_1-x_2) - \varkappa(x_1,x_2)\frac{1}{2\mathrm{V}}\right]$$

Anticorrelation among like baryons

$$\mathrm{C}_{11}^{\mathrm{B}\bar{\mathrm{B}}}(x_1,x_2) = \langle \bar{\mathrm{B}}\rangle\,\varkappa(x_1,x_2)\frac{1}{2\mathrm{V}}$$

Correlation among unlike baryons

- Multiple conserved charges, $\boldsymbol{Q} = (B, Q, S, \dots)$

$$\mathrm{C}_{11}^{ij}(x_1,x_2) = \chi_{ij}\left[\delta(x_1-x_2) - \varkappa(x_1,x_2)\frac{\chi_{11}^{i\boldsymbol{Q}_\mathrm{k}}\left(\chi_{kl}^{\boldsymbol{Q}}\right)^{-1}\chi_{11}^{j\boldsymbol{Q}_\mathrm{l}}}{\mathrm{V}}\right]$$

- Balance functions (ideal gas, LHC)

$$B(x_1,x_2) = \frac{n_B + n_{\bar{B}}}{\langle B+\bar{B}\rangle}\varkappa(x_1,x_2)$$

Local charge conservation: high-order cumulants

Ciacco, Kuznietsov, et al., to appear

Introduce **n-point local conservation kernel**

$$\mathcal{C}_2(\eta_1,\eta_2) = \chi_2^B \left[\delta_{1,2} - \frac{\varkappa_2(\eta_1,\eta_2)}{V}\right]$$ second-order

$$\mathcal{C}_3(\eta_1,\eta_2,\eta_3) = \chi_3^B\,\delta_{1,2,3} - \frac{\chi_3^B}{V}\left[\delta_{1,2}\,\varkappa_2(\eta_1,\eta_3) + \delta_{1,3}\,\varkappa_2(\eta_1,\eta_2) + \delta_{2,3}\,\varkappa_2(\eta_2,\eta_1)\right] + \frac{2\chi_3^B}{V^2}\,\varkappa_3(\eta_1,\eta_2,\eta_3)]$$ third-order

$$\mathcal{C}_4(\eta_1,\ldots,\eta_4) = \chi_4^B\delta_{1,2,3,4} - \frac{\chi_4^B}{3!V}\sum_{\sigma\in S_4}\delta_{\sigma_1,\sigma_2,\sigma_3}\varkappa_2(\eta_{\sigma_1},\eta_{\sigma_4}) - \frac{(\chi_3^B)^2}{(2!)^2\chi_2^B V}\sum_{\sigma\in S_4}\delta_{\sigma_1,\sigma_2}\delta_{\sigma_3,\sigma_4}\varkappa_2(\eta_{\sigma_1},\eta_{\sigma_3})$$
$$+ \frac{1}{2!V^2}\left[\chi_4^B + \frac{(\chi_3^B)^2}{\chi_2^B}\right]\sum_{\sigma\in S_4}\delta_{\sigma_1,\sigma_2}\varkappa_3(\eta_{\sigma_1},\eta_{\sigma_3},\eta_{\sigma_4}) - \frac{3}{V^3}\left[\chi_4^B + \frac{(\chi_3^B)^2}{\chi_2^B}\right]\varkappa_4(\eta_1,\eta_2,\eta_3,\eta_4).$$ fourth-order

Sum rule:

$$\int d\eta_n\,\mathcal{C}_n = 0$$ for any *n* ⟺ $$\int d\eta_n\,\varkappa_n(\eta_1,\ldots,\eta_n) = V\,\varkappa_{n-1}(\eta_1,\ldots,\eta_{n-1})$$

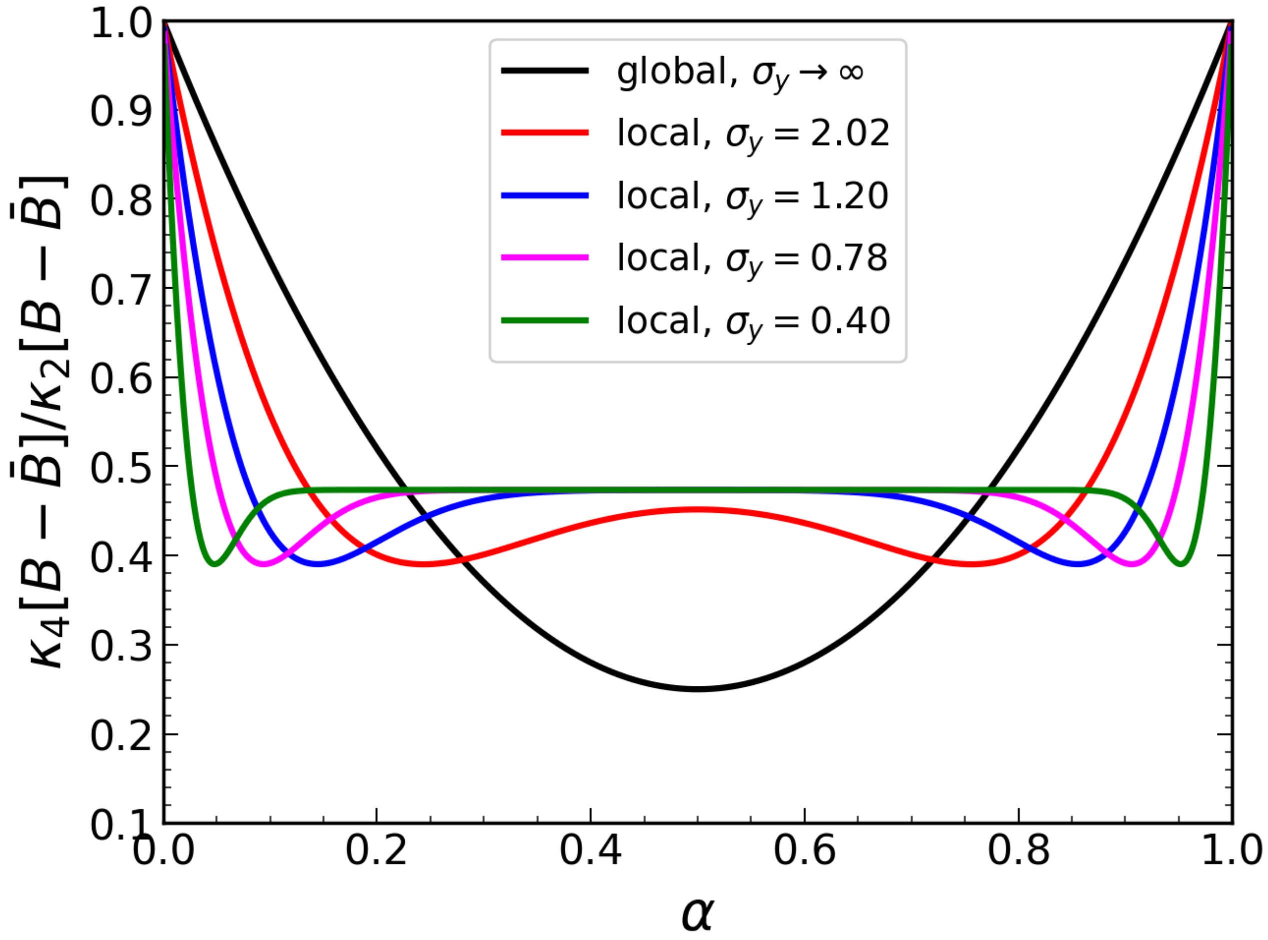
Symmetric n-point Gaussian kernel:

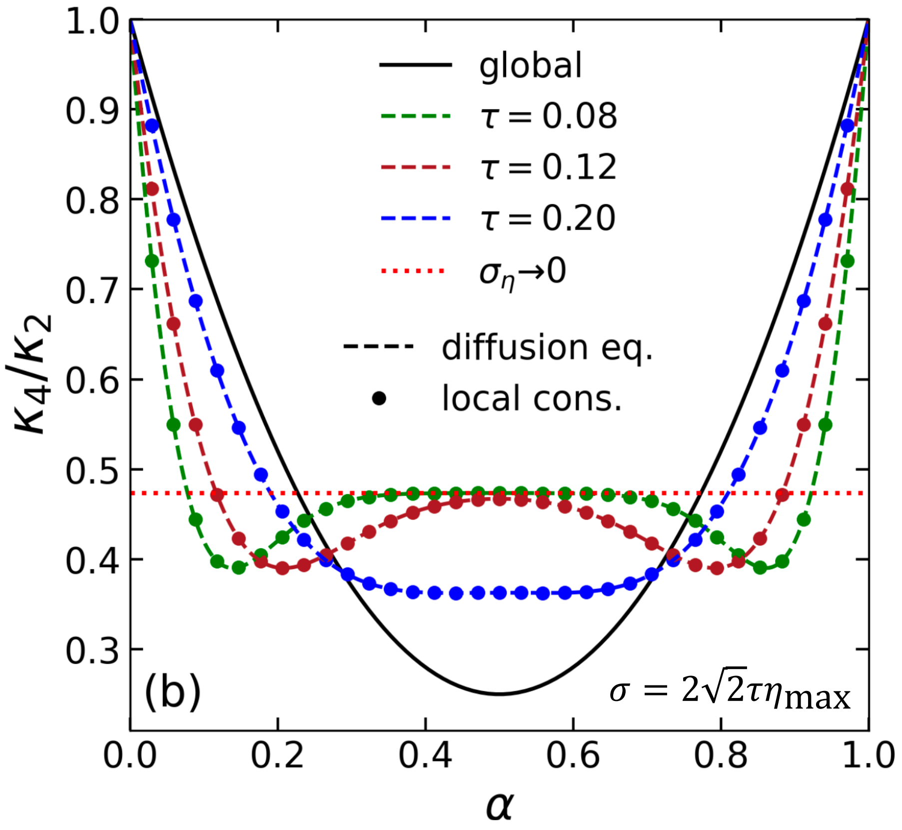
$$\varkappa_2(\eta_1,\eta_2) \propto \exp\left[-\frac{(\eta_1-\eta_2)^2}{2\sigma_\eta^2}\right]$$ ⟹ $$\varkappa_n(\eta_1,\ldots,\eta_n) \propto A_n\exp\left[-\frac{1}{n\sigma_\eta^2}\sum_{1\le i<j\le n}(\eta_i-\eta_j)^2\right]$$

Fourth-order net-baryon fluctuations

Ciacco, Kuznietsov, et al., to appear

Coordinate space: cut in spatial rapidity $|\eta| < \eta_{cut}$





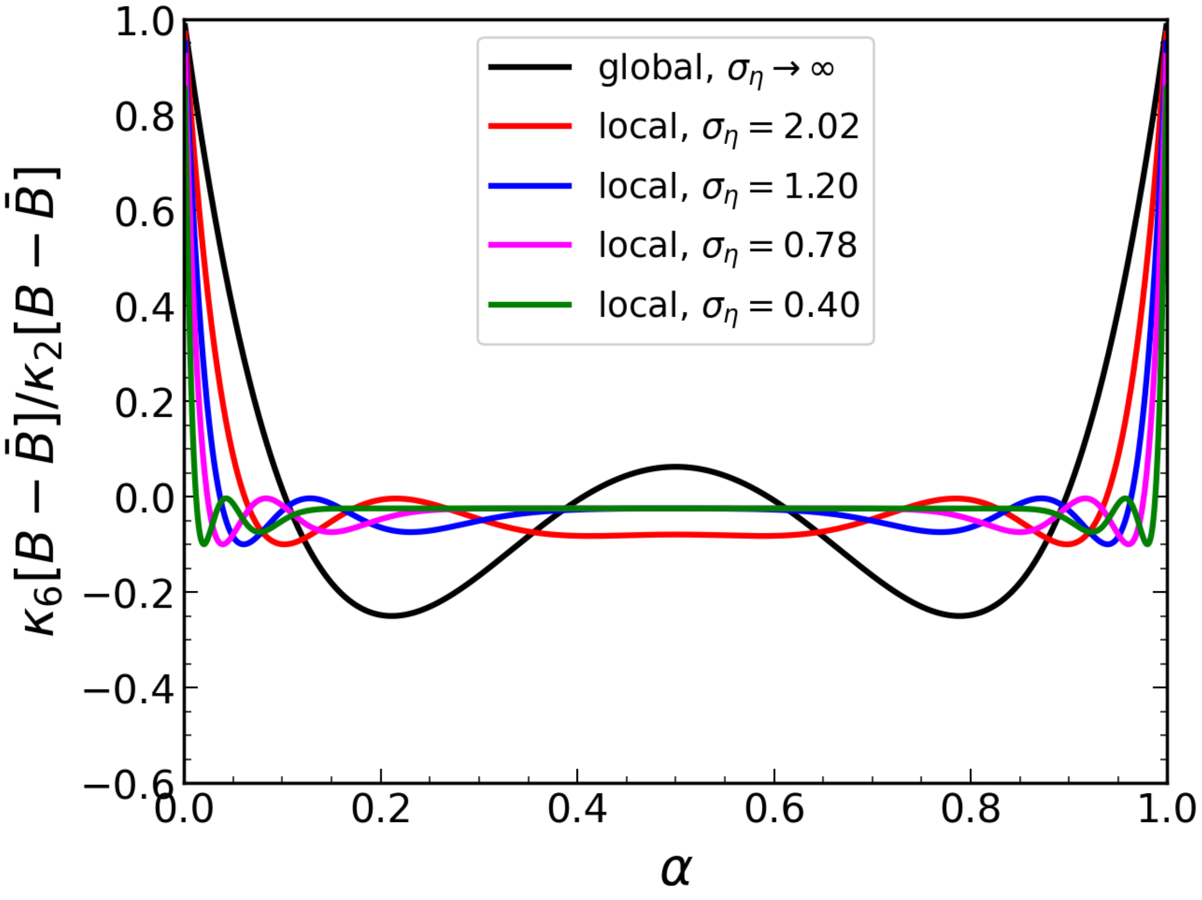
Small σ limit: plateau at $-\frac{1}{2} + \frac{9}{\pi}\arcsin(1/3) \approx 0.474$

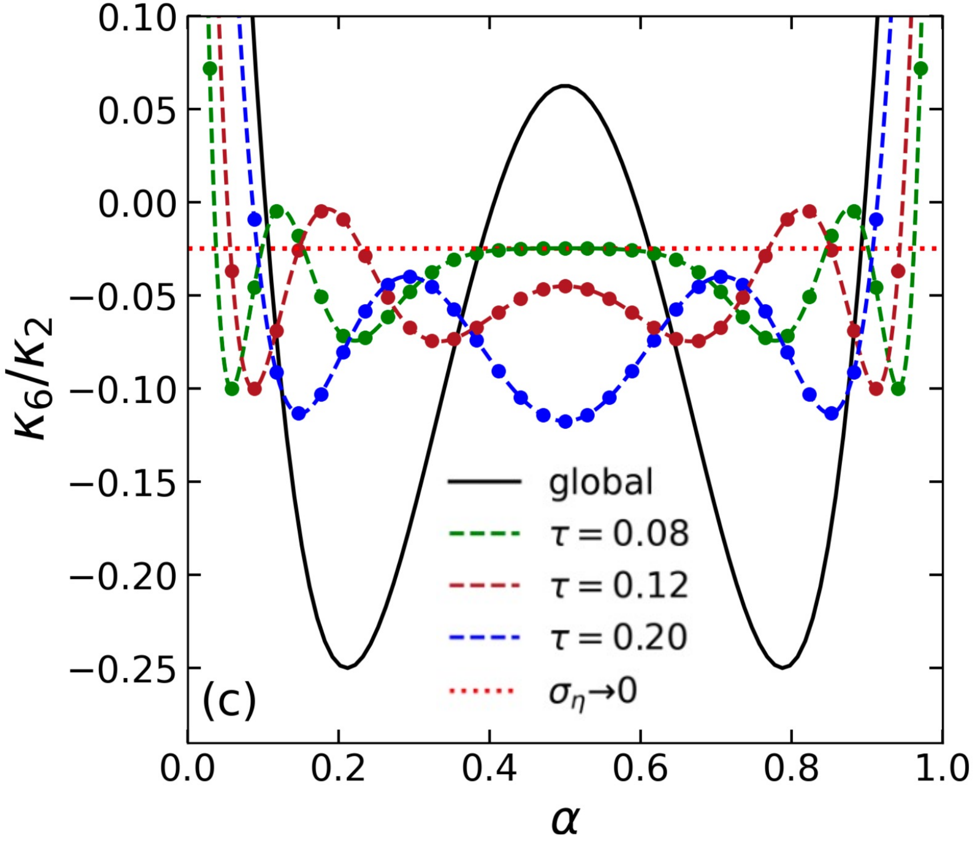
Excellent (exact?) agreement with hadronic diffusion model of Sakaida et al., PRC 90, 064911 (2014)

Sixth-order net-baryon fluctuations

Ciacco, Kuznietsov, et al., to appear

Coordinate space: cut in spatial rapidity $|\eta| < \eta_{cut}$





Small σ limit: plateau at $\frac{31}{16} - \frac{225}{4\pi}\arcsin\left(\frac{1}{3}\right) + \frac{675}{16\sqrt{\pi}} I_4 \simeq -0.02465$

Excellent (exact?) agreement with hadronic diffusion model of Sakaida et al., PRC 90, 064911 (2014)

Predictions for O-O collisions

Ciacco, Kuznietsov, et al., to appear

Momentum-space measurements → Acceptance factors $p(\eta)$ at each spatial rapidity

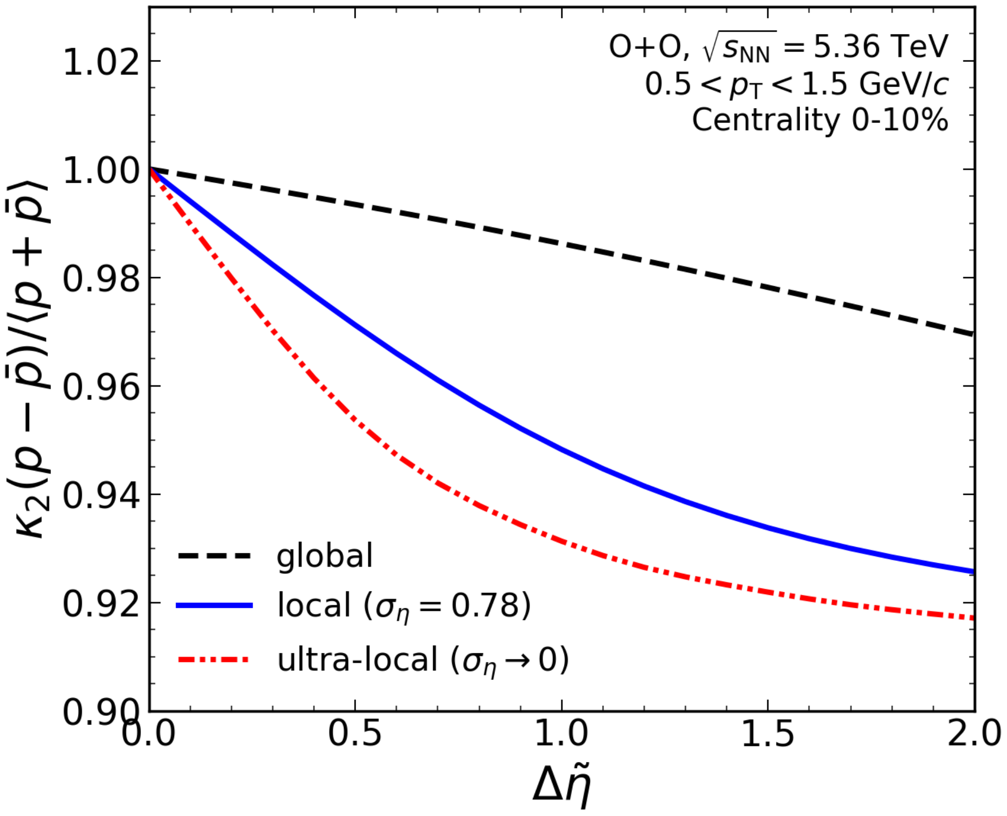
Input from the blast-wave model

$$\kappa_2 = \frac{\langle B+\bar{B}\rangle}{V}\left[\mathcal{J}_1 - \mathcal{J}_2\right],$$

$$\kappa_4 = \frac{\langle B+\bar{B}\rangle}{V}\left[\mathcal{J}_1 - 4\,\mathcal{J}_2 + 6\,\mathcal{J}_3 - 3\,\mathcal{J}_4\right],$$

$$\kappa_6 = \frac{\langle B+\bar{B}\rangle}{V}\left[\mathcal{J}_1 - 16\,\mathcal{J}_2 + 75\,\mathcal{J}_3 - 150\,\mathcal{J}_4 + 135\,\mathcal{J}_5 - 45\,\mathcal{J}_6\right].$$

$$\mathcal{J}_n \equiv \frac{1}{V^{n-1}} \int d\eta_1 \ldots d\eta_n\, p(\eta_1) \ldots p(\eta_n)\, \varkappa_n(\eta_1, \ldots, \eta_n)$$



Preferred scenario: $\sigma_\eta = 0.78$

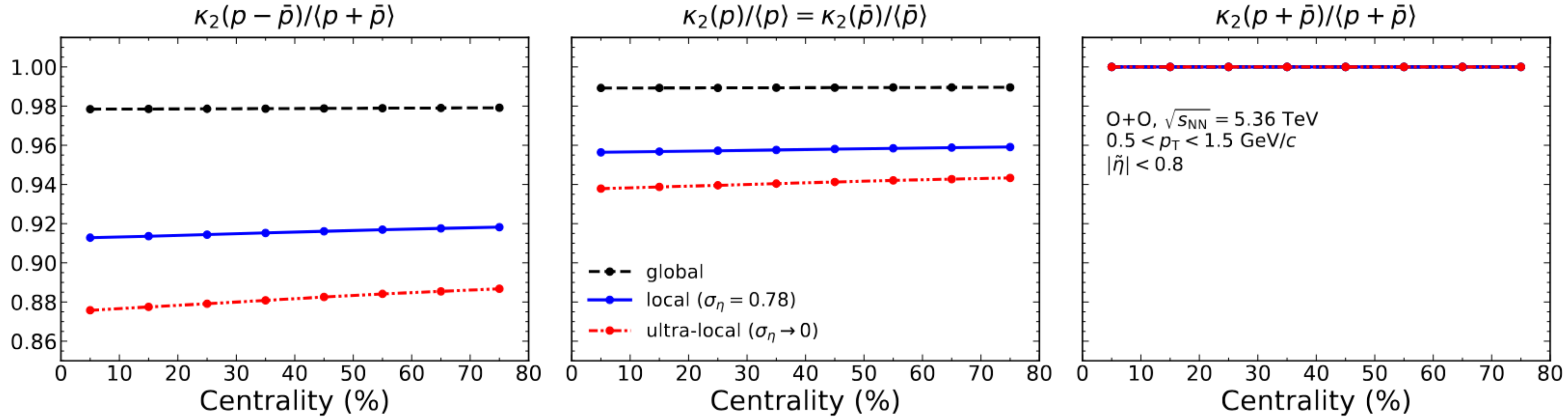
- Constraint from 5.02 TeV Pb-Pb data

Ultra-local limit ($\sigma_\eta \to 0$):

- maximum effect of baryon conservation
- non-zero cumulants in this limit due to momentum cut and absence of neutrons

Predictions for O-O collisions

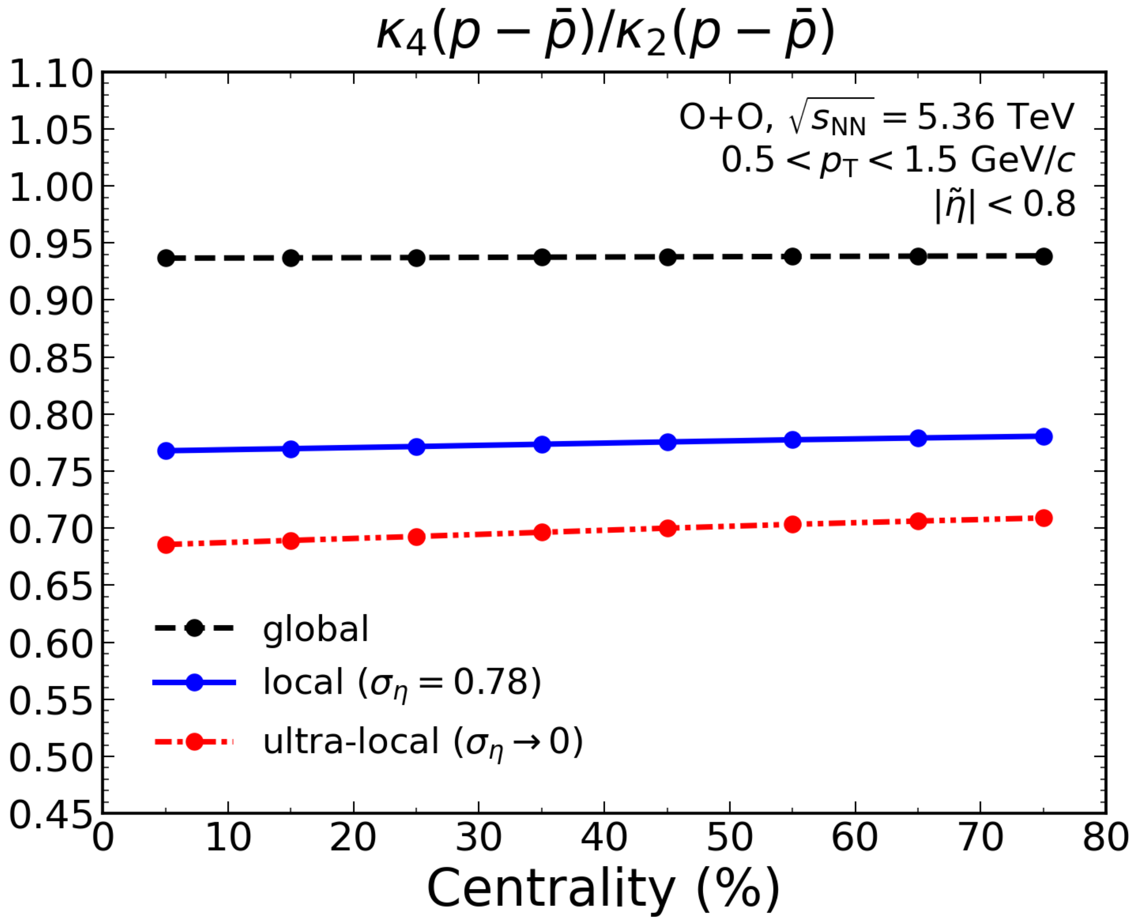
Ciacco, Kuznietsov, et al., to appear

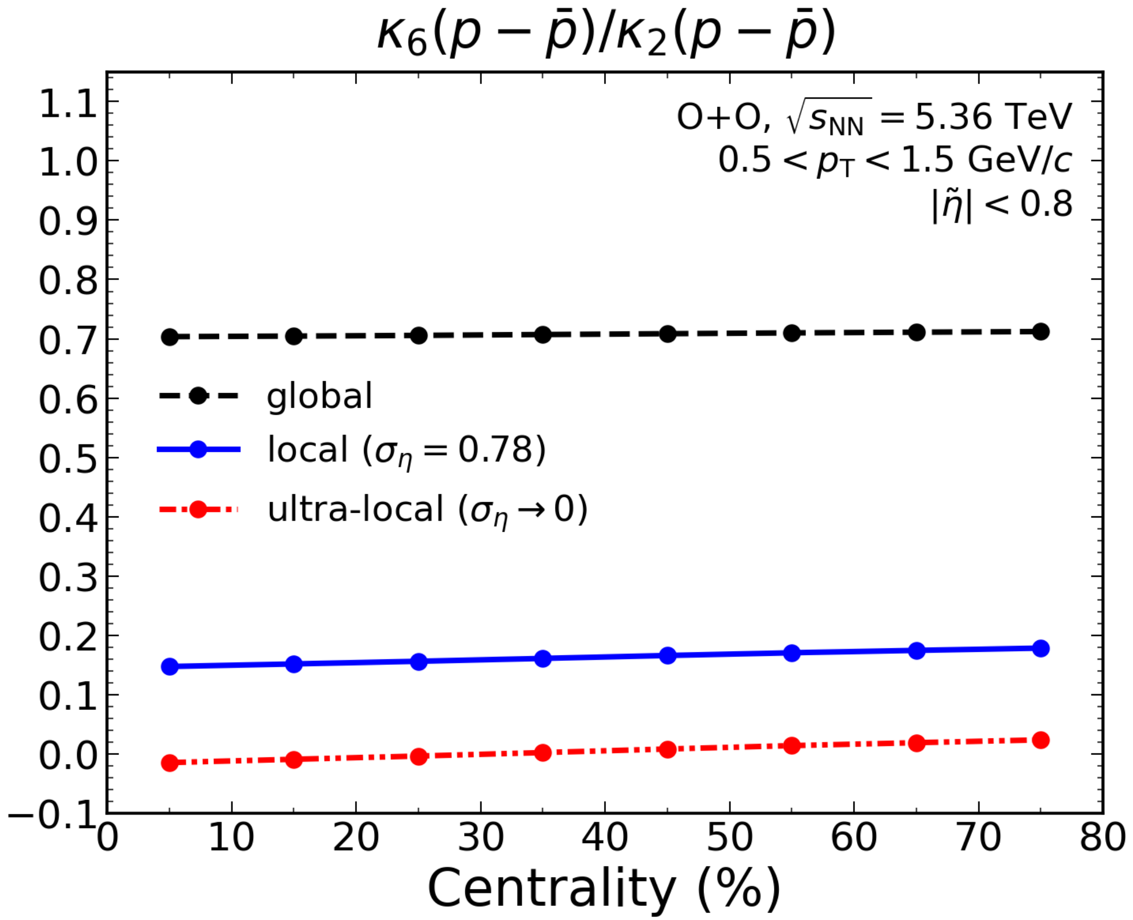


- Measurements of individual variances carry additional information
- $\kappa_2[p]/\langle p\rangle$ closer to Poisson than $\kappa_2[p-\bar{p}]/\langle p+\bar{p}\rangle$
- $\boldsymbol{\kappa_2[p+\bar{p}]/\langle p+\bar{p}\rangle = 1}$
  - consequence of symmetry at LHC: $\mathrm{cov}(p+\bar{p}, B_{\mathrm{tot}} - \bar{B}_{\mathrm{tot}}) = 0$
  - clear test of physics beyond baryon conservation

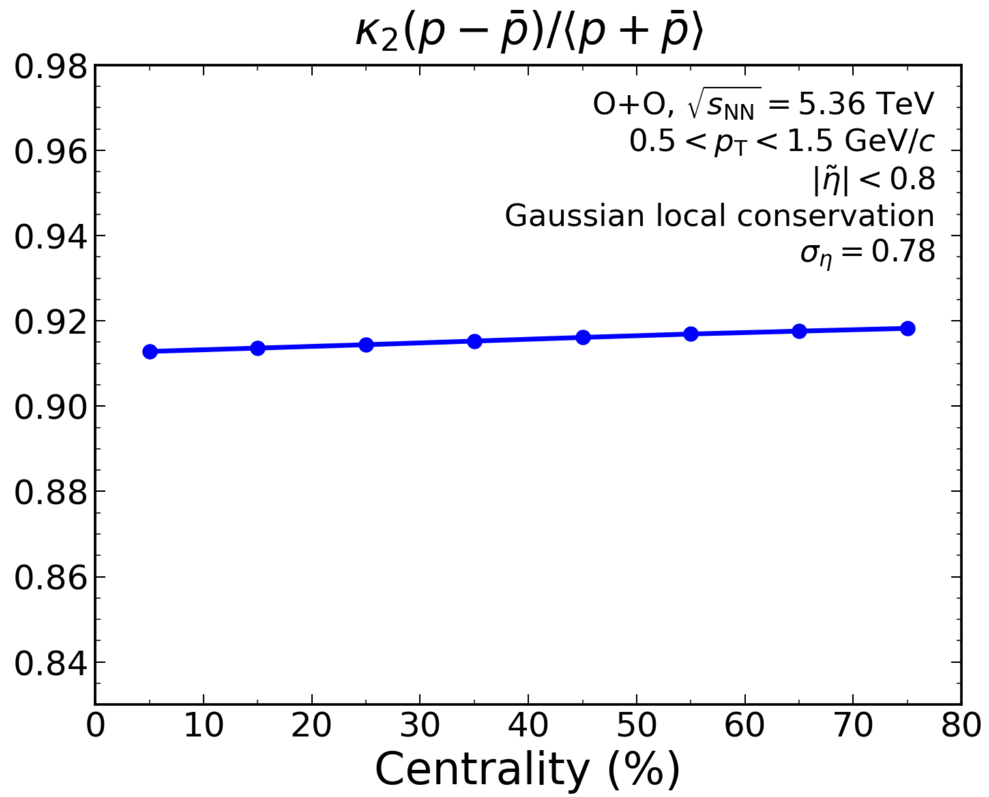
Predictions for O-O collisions: high-order cumulants

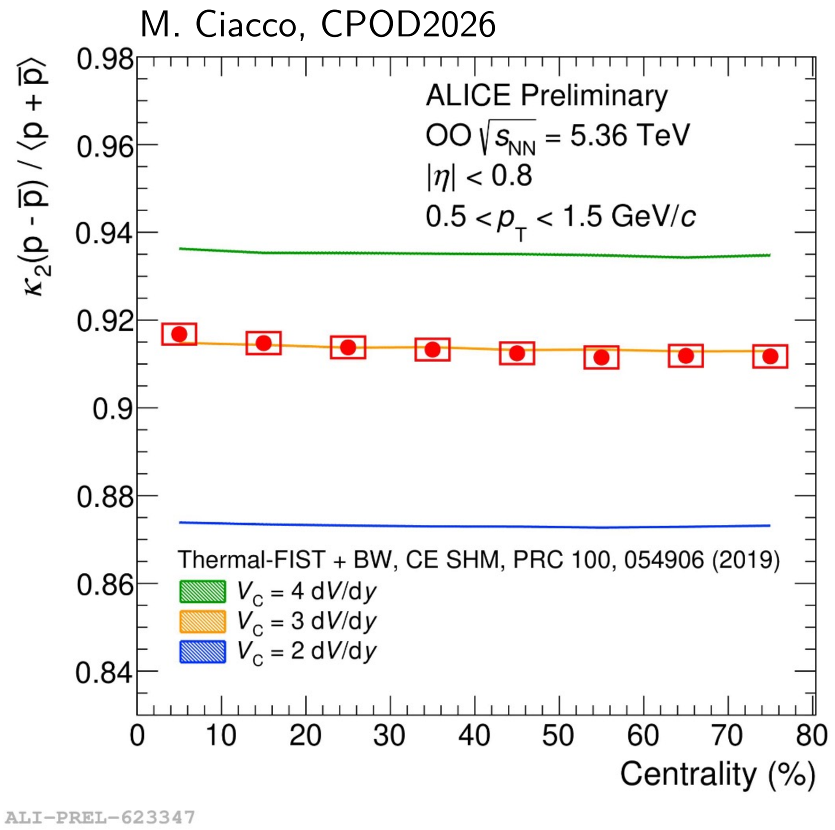
Ciacco, Kuznietsov, et al., to appear





Comparison to data and other implementations: κ_2





- ✓ Good agreement with preliminary O-O data (no need to retune σ_η from Pb-Pb)
- ✓ Excellent (exact?) agreement with the diffusion model of PRC 90, 064911 (2014)
- ✓ Good agreement with V_c approach (Thermal-FIST SHM $V_c = 3dV/dy$) PRC 100, 054906 (2019)

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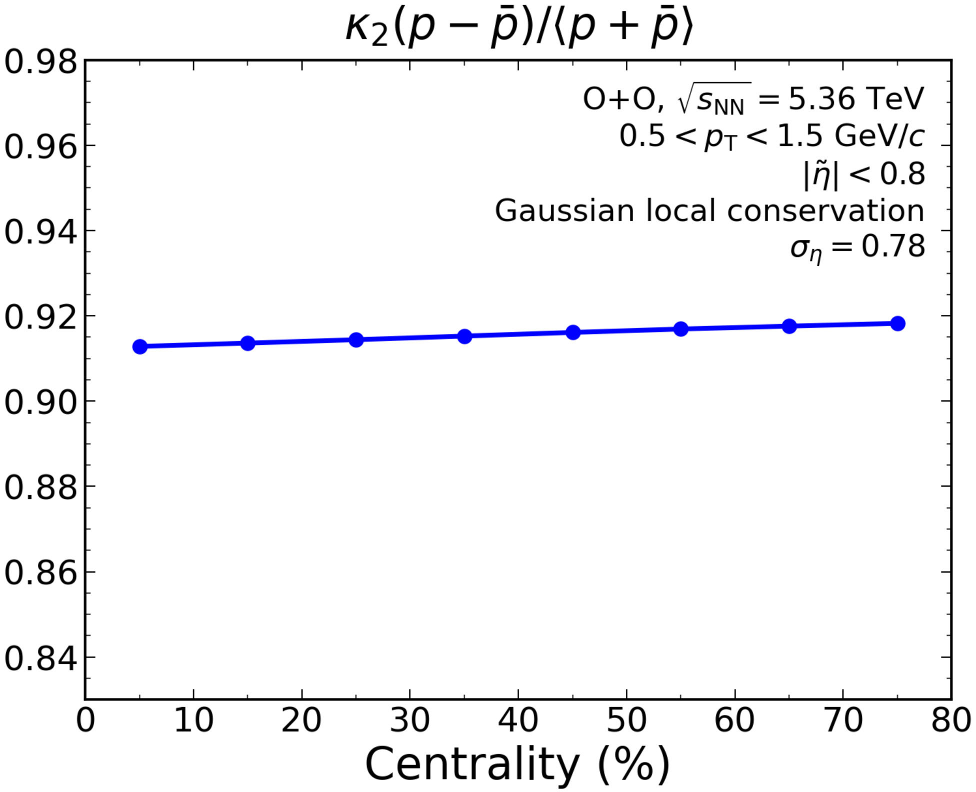
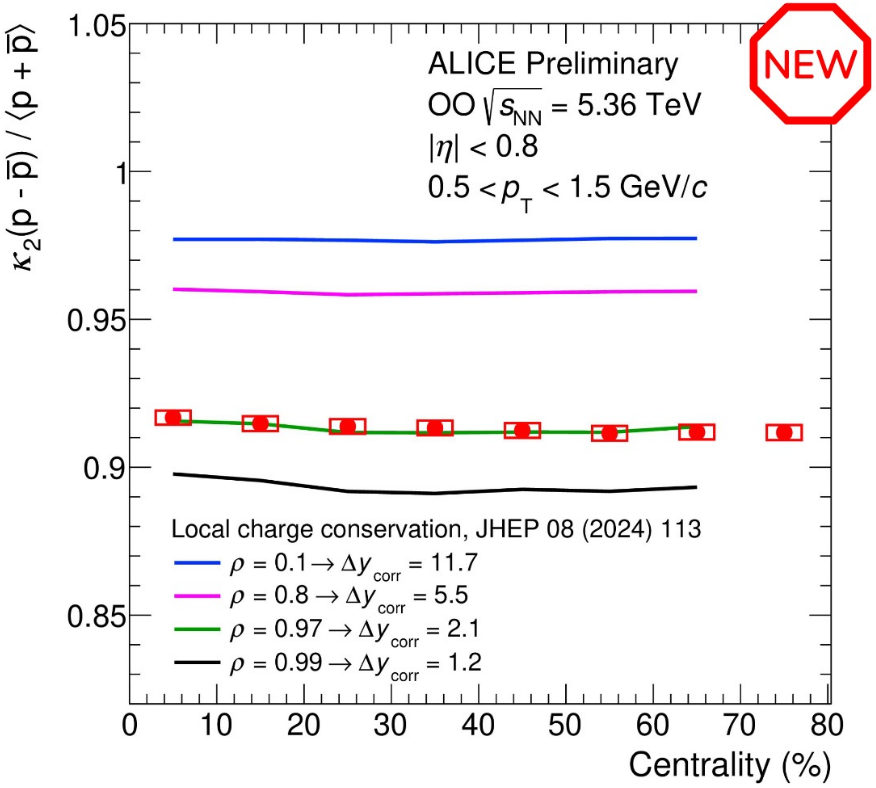


M. Ciacco, CPOD2026



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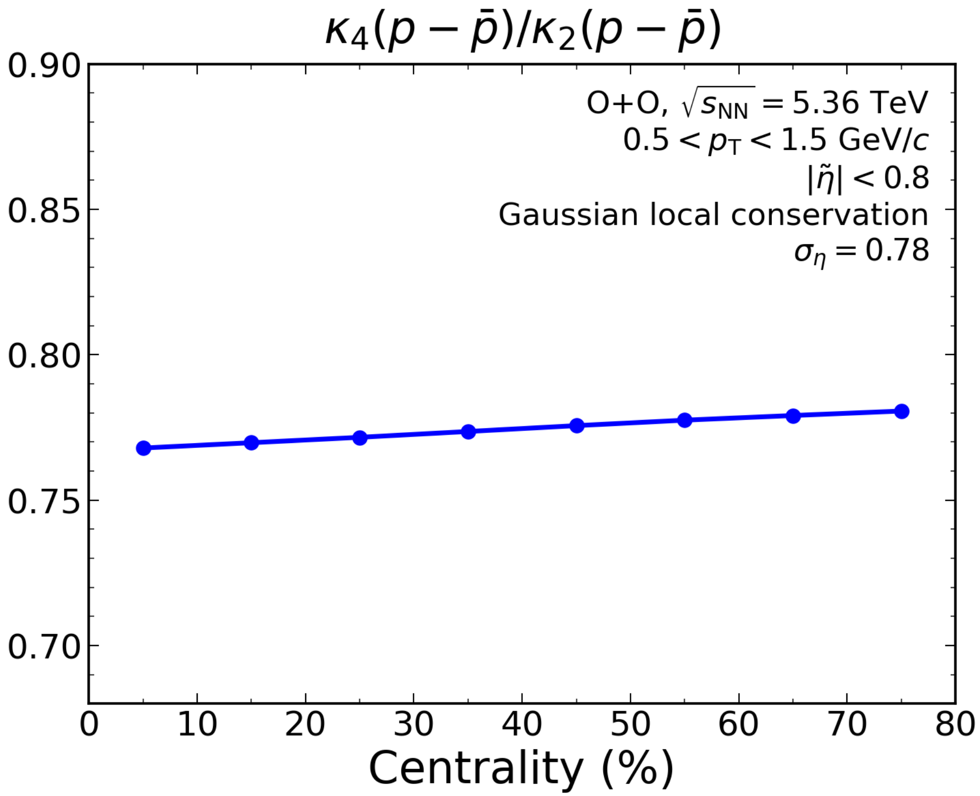
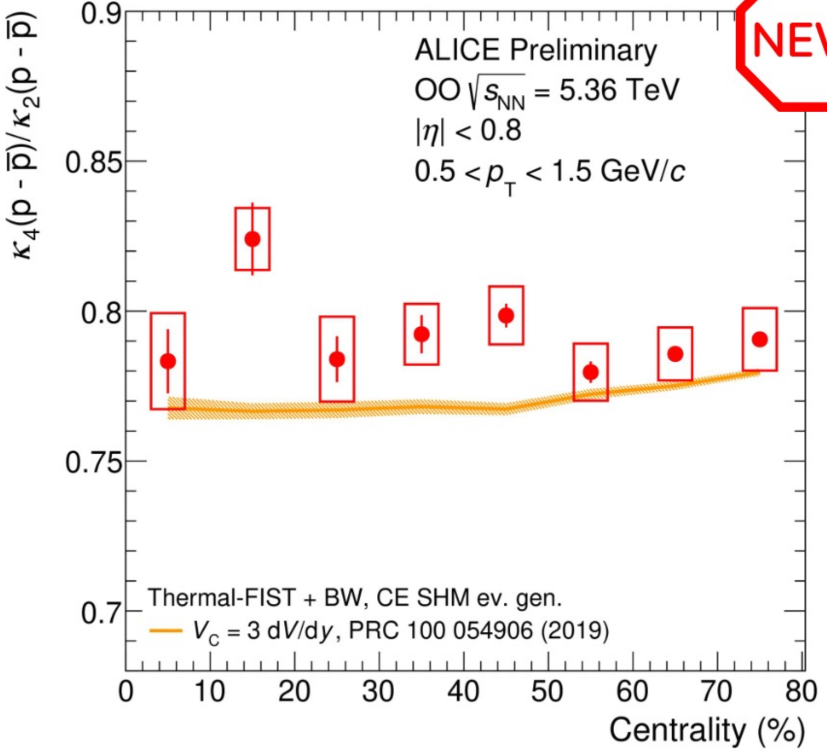
- ✓ Good agreement with the correlated sampling model of JHEP 08, 113 (2024)

Comparison to data and other implementations: κ_4/κ_2



M. Ciacco, CPOD2026



- ◐ Fair agreement with preliminary O-O data
- ✓ Excellent (exact?) agreement with the diffusion model of PRC 90, 064911 (2014)
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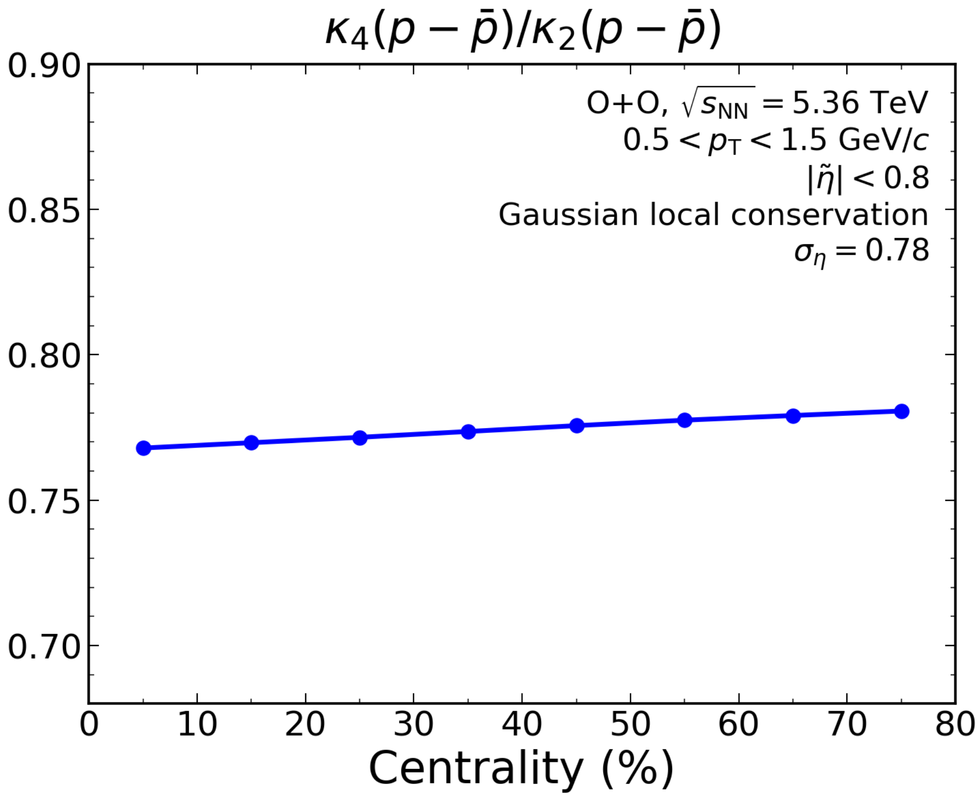
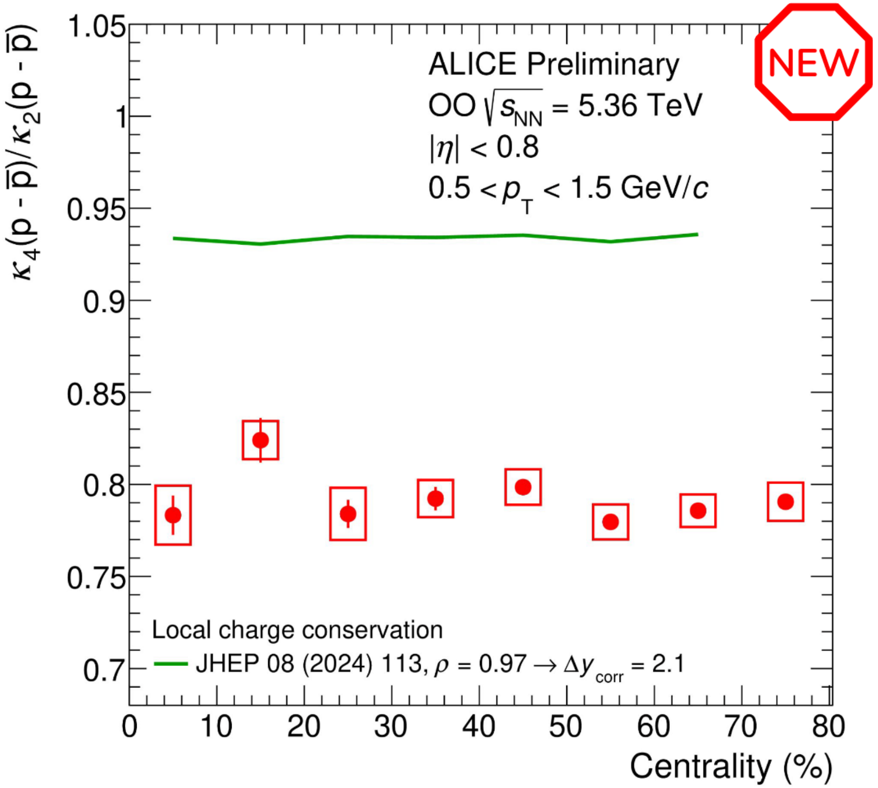
Comparison to data and other implementations: κ_4/κ_2



M. Ciacco, CPOD2026; A. Rustamov, CPOD2026



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- ✓ Excellent (exact?) agreement with the diffusion model of PRC 90, 064911 (2014)
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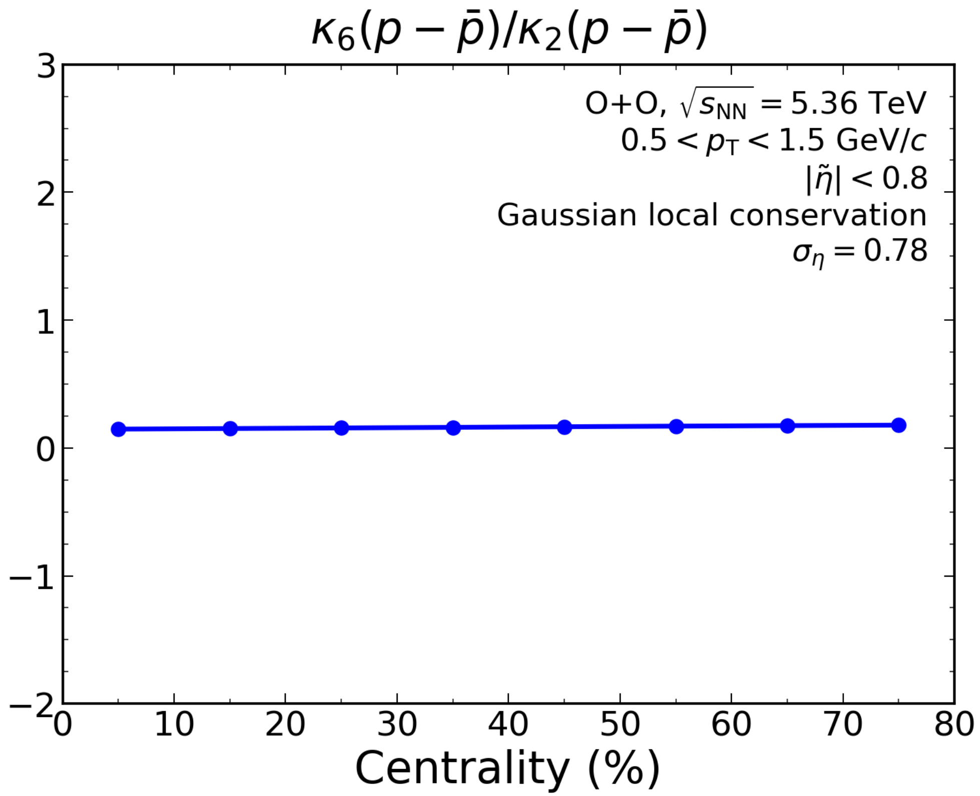
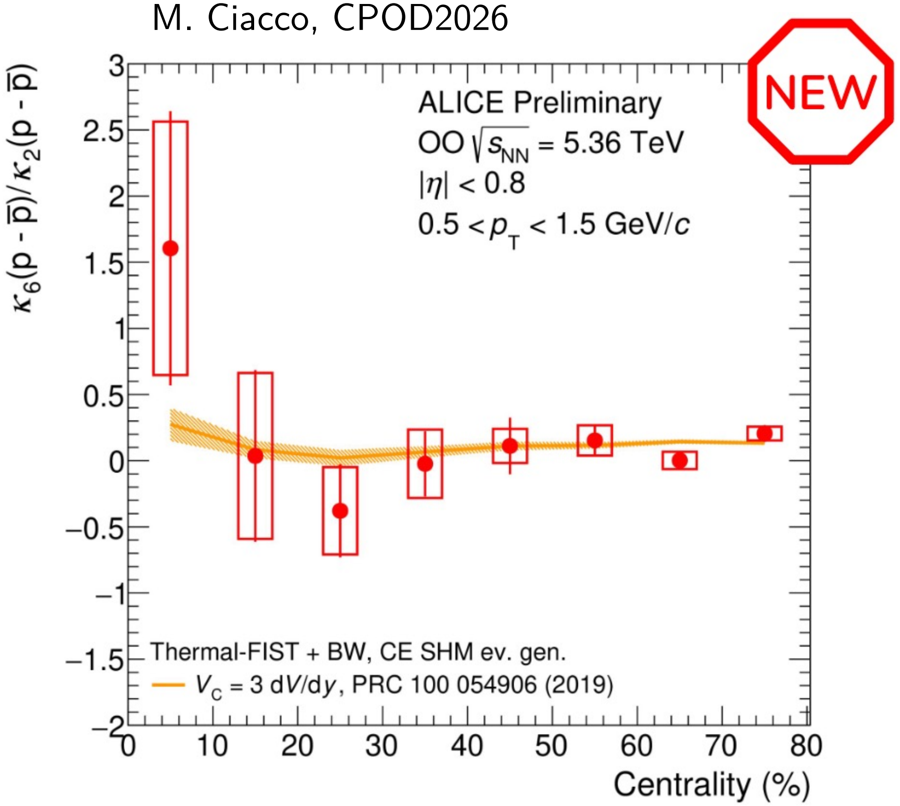
Comparison to data and other implementations: κ_6/κ_2



M. Ciacco, CPOD2026



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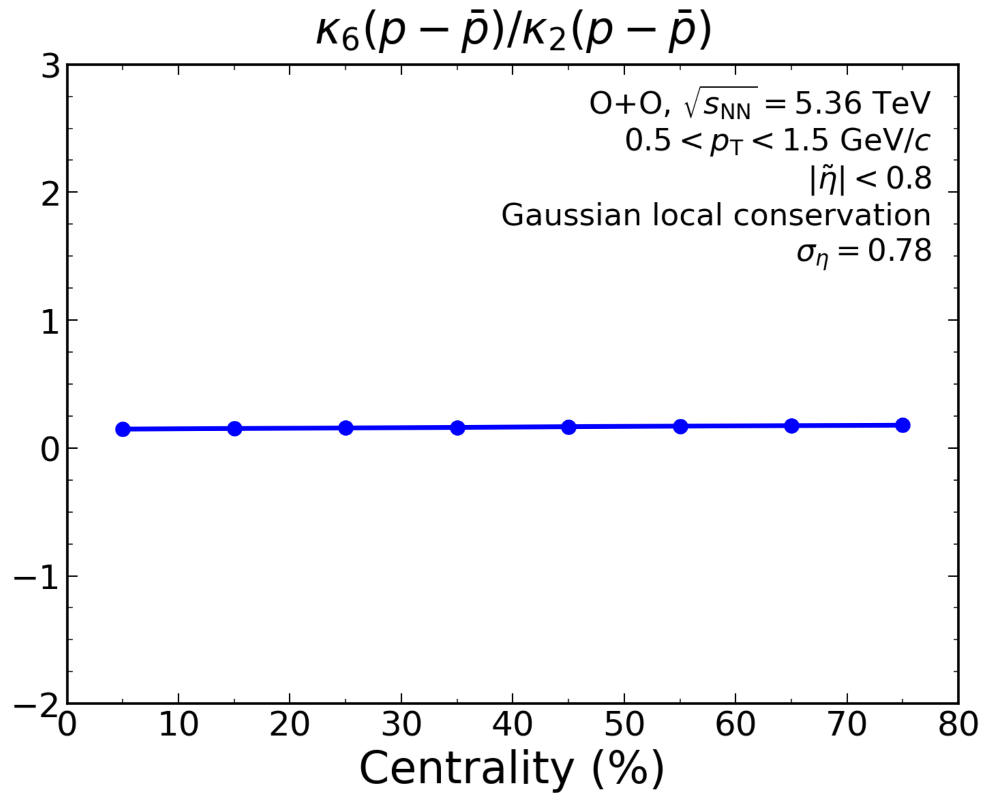
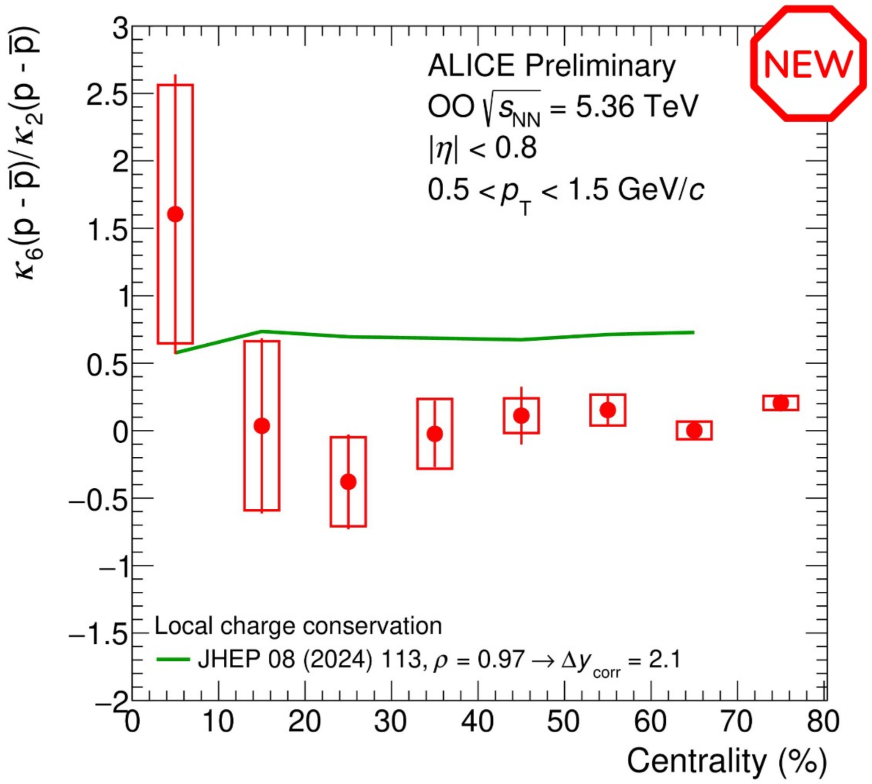
Comparison to data and other implementations: κ_6/κ_2



M. Ciacco, CPOD2026; A. Rustamov, CPOD2026



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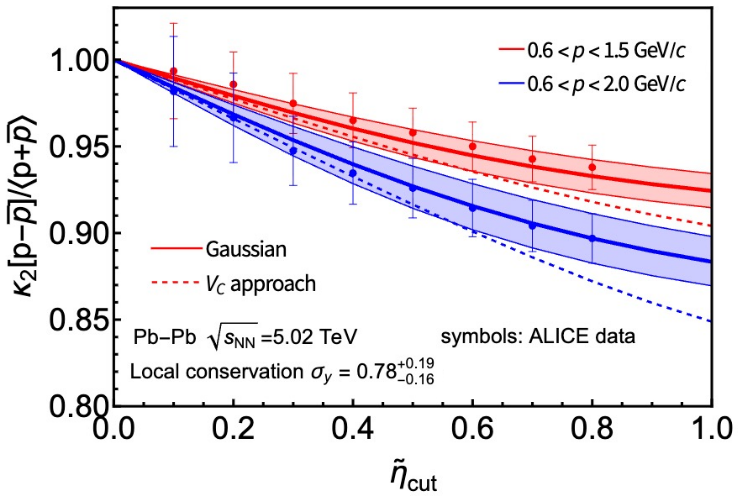
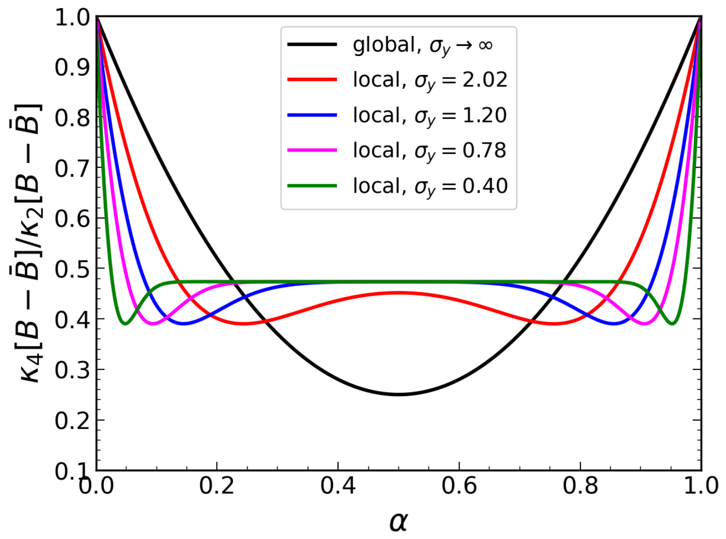
Summary

- Density correlations: statistical hadronization with local charge conservation for correlations/fluctuations of hadron numbers
  - Gaussian local conservation in rapidity space
  - Net protons in Pb-Pb collisions: $\sigma_\eta \approx 0.78$
- Baseline for high-order net proton cumulants at LHC
  - Good agreement with diffusion model and the V_c approach
  - Fair description of cumulants in O-O collisions up to κ_6
  - No clear signal of physics beyond hadron gas



Outlook:

- Going beyond baselines and incorporating lattice QCD susceptibilities
  - E.g. through maximum entropy freeze-out prescription
- Additional observables and conserved charges (BQS)
  - Balance functions, non-conserved cumulants
- Extension to RHIC



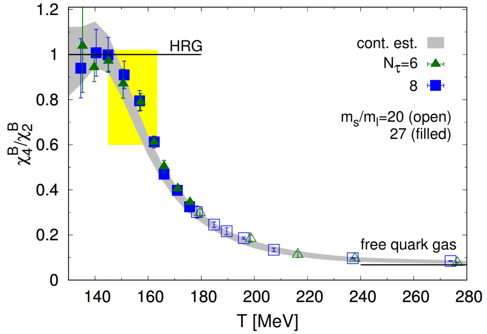
Thanks for your attention

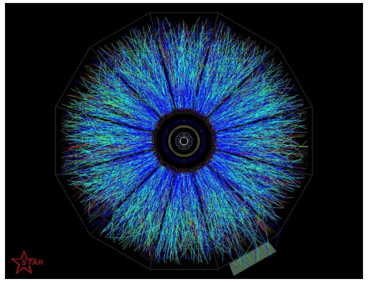
Additional slides

Theory vs experiment

guidance from theory (e.g. lattice)

experiment (the real thing)





hydro simulation

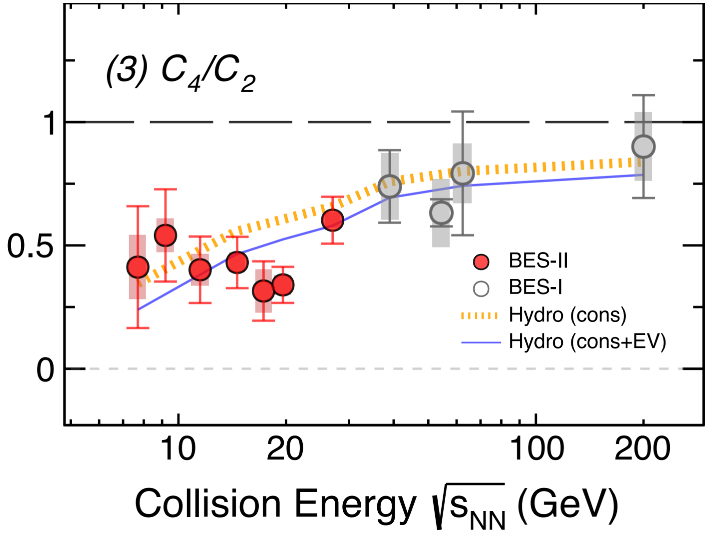
hadronic phase

canonical effects

kinematic cuts

event selection

track reconstruction



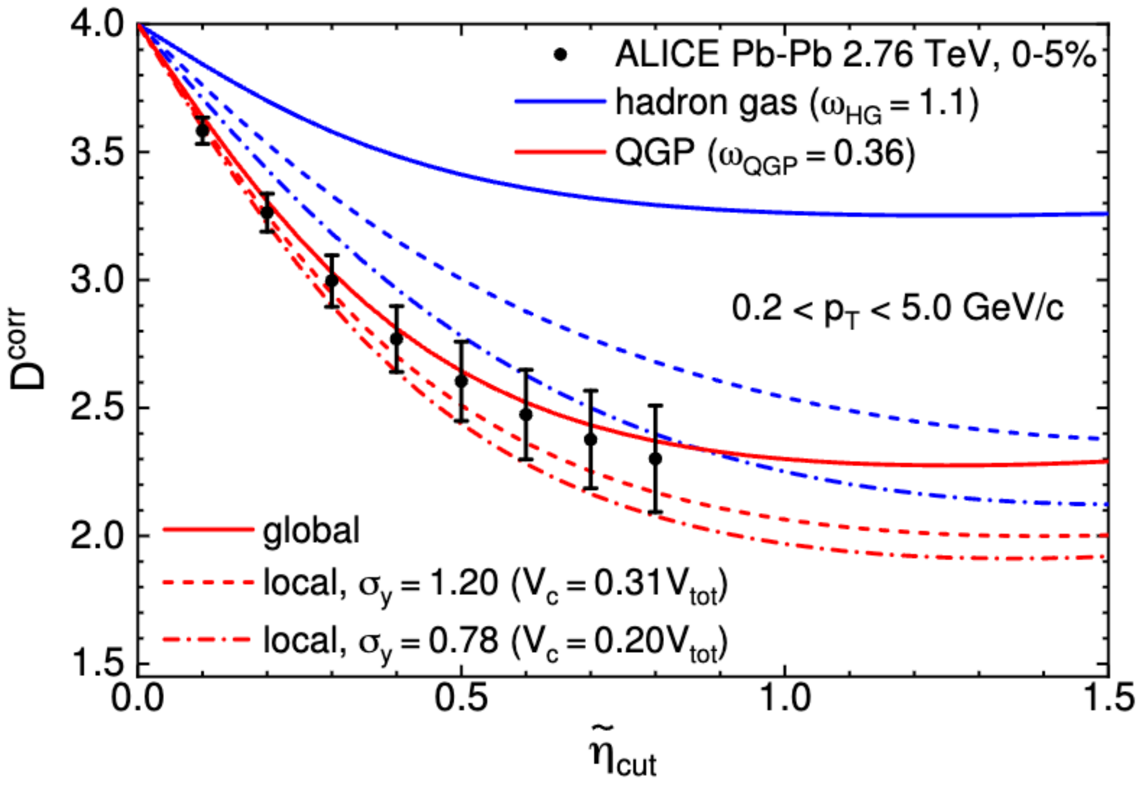
efficiency correction

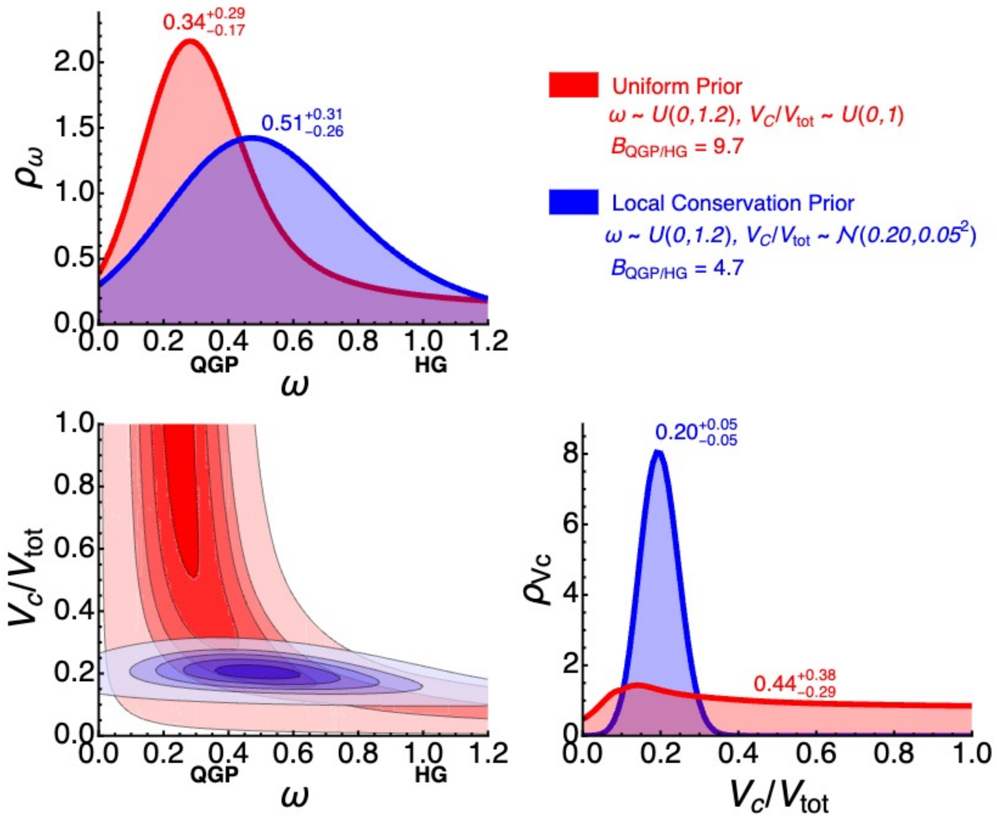
CBWC

purity

D-measure of charge fluctuations

$$D = 4\frac{\kappa_2[N_+ - N_-]}{\langle N_{ch}\rangle} = 4\frac{\kappa_2[Q]}{\langle Q^+ + Q^-\rangle} = 4\left\{1 - \left(1 - \frac{\omega}{\gamma_Q}\right)\frac{\langle p^2(\eta)\rangle}{\langle p(\eta)\rangle} - \frac{\omega}{\gamma_Q}\frac{\langle p(\eta_1)p(\eta_2)\rangle_\varkappa}{\langle p(\eta)\rangle}\right\}$$





J. Parra, R. Poberezhniuk, V. Koch, C. Ratti, arXiv:2504.02085

Cumulants in the canonical ensemble

Net-baryon cumulants in the acceptance, non-zero total baryon number B

$$g(t) = \ln\left(\sum_n P_B(n)e^{nt}\right) = \ln\left[\left(\frac{q_+}{q_-}\right)^{B/2} \frac{I_B(2z\sqrt{q_+q_-})}{I_B(2z)}\right]$$

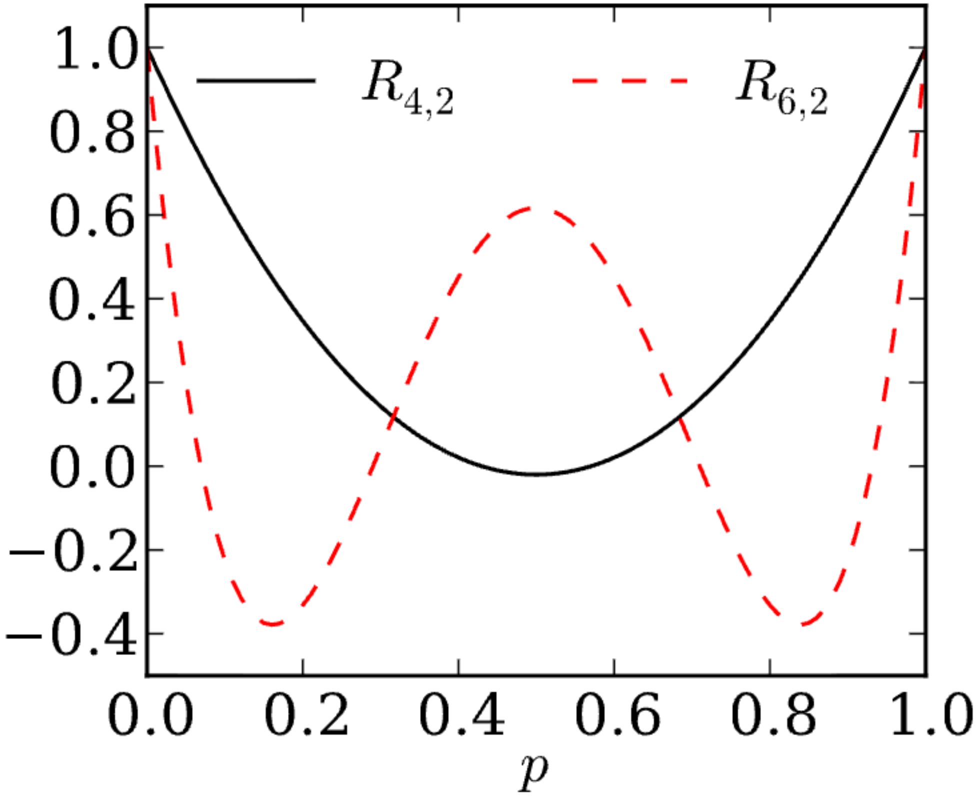
$$q_+ = 1 - p_B + p_B e^t$$
$$q_- = 1 - p_{\bar{B}} + p_{\bar{B}} e^{-t}$$

Bzdak, Koch, Skokov, PRC 87, 014901 (2013)

$$\frac{\kappa_2[B-\bar{B}]}{\langle \mathrm{B}+\bar{\mathrm{B}}\rangle} \approx 1 - p$$

$$\frac{\kappa_4[B-\bar{B}]}{\kappa_2[B-\bar{B}]} \approx 1 - 3p[1\text{-p}(1+r_B^2)]$$

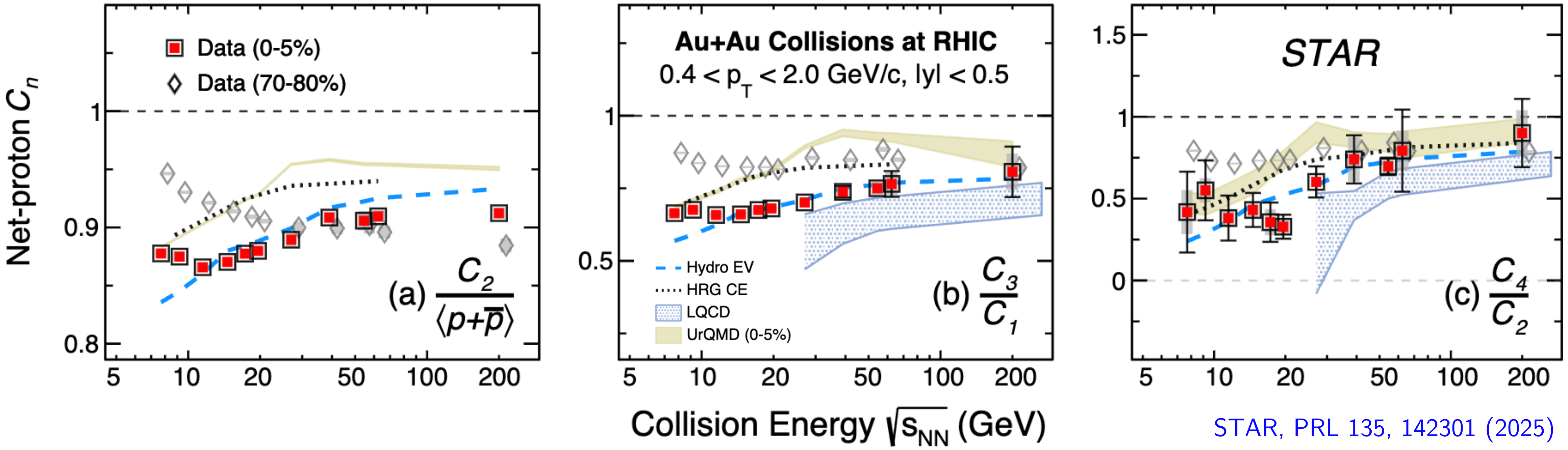
$$\frac{\kappa_6[B-\bar{B}]}{\kappa_2[B-\bar{B}]} \approx 1 - 15p(1-p)[1+r_B^2 - p(1-p)](3+6r_B^2 - r_B^4)$$



see also Braun-Munzinger et al., NPA 1008, 122141 (2021)

Conservation, baselines, and data

Net-proton cumulant ratios



STAR, PRL 135, 142301 (2025)

HRG CE: Braun-Munzinger et al., NPA 1008, 122141 (2021)

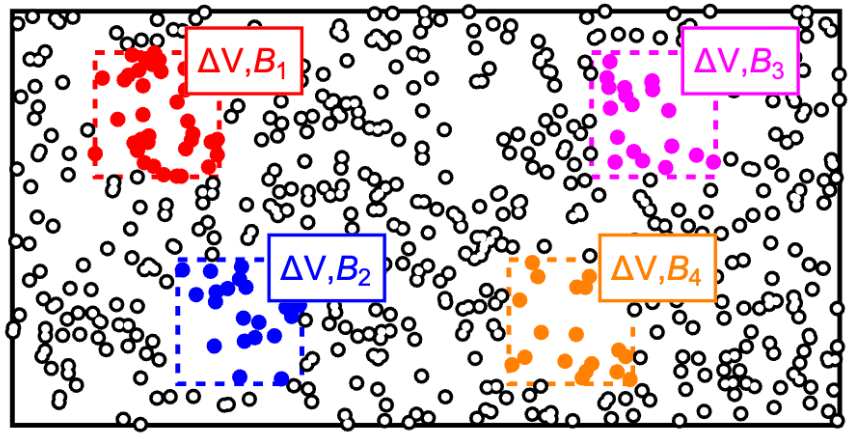
Hydro EV: VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Exact baryon conservation is a primary ingredient of non-critical baselines

Density correlations framework

VV, PRC 110, L061902 (2024)

Evaluate the cumulants in thermodynamic limit using maximum term (saddle-point) method



$$\langle \delta B_i \delta B_j \rangle = \Delta V \chi_2 \left[\delta_{ij} - \frac{\Delta V}{V} \right]$$

$$\langle \delta B_i \delta B_j \delta B_k \rangle = \delta_{ijk} \chi_3 \Delta V - (\delta_{ij} + \delta_{ik} + \delta_{kj}) \chi_3 (\Delta V)^2 + 2 \chi_3 (\Delta V)^3$$

$$\langle \delta B_i \delta B_j \delta B_k \delta B_l \rangle_c = \Delta V \chi_4 \delta_{ijkl} - \chi_4 \frac{(\Delta V)^2}{V} [\delta_{ijk} + \delta_{ijl} + \delta_{ikl} + \delta_{jkl}] - \frac{(\chi_3)^2}{\chi_2} \frac{(\Delta V)^2}{V} [\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}] + \frac{(\Delta V)^3}{V^2} \left[\chi_4 + \frac{(\chi_3)^2}{\chi_2} \right] [\delta_{ij} + \delta_{ik} + \delta_{il} + \delta_{jk} + \delta_{jl} + \delta_{kl}] - \frac{3(\Delta V)^4}{V^3} \left[\chi_4 + \frac{(\chi_3)^2}{\chi_2} \right] .$$

Taking "continuum" limit ($\Delta V \to 0$) yields *n*-point density correlation functions

$$\mathcal{C}_n(\eta_1, \ldots, \eta_n) \equiv \left\langle \prod_{i=1}^{n} \delta\rho_i \right\rangle_c , \qquad n \geq 2, \qquad \prod_{i=1}^{n} \int d\eta_i \, \mathcal{C}_n(\eta_1, \ldots, \eta_n) = \kappa_n[B].$$

N-point local conservation kernel

$$\varkappa_2(\eta_1,\eta_2) \propto \exp\left[-\frac{(\eta_1-\eta_2)^2}{2\sigma_\eta^2}\right]$$

2-point Gaussian kernel

$$\varkappa_n(\eta_1,\ldots,\eta_n) \propto A_n \exp\left[-\frac{1}{n\sigma_\eta^2}\sum_{1\le i<j\le n}(\eta_i-\eta_j)^2\right]$$

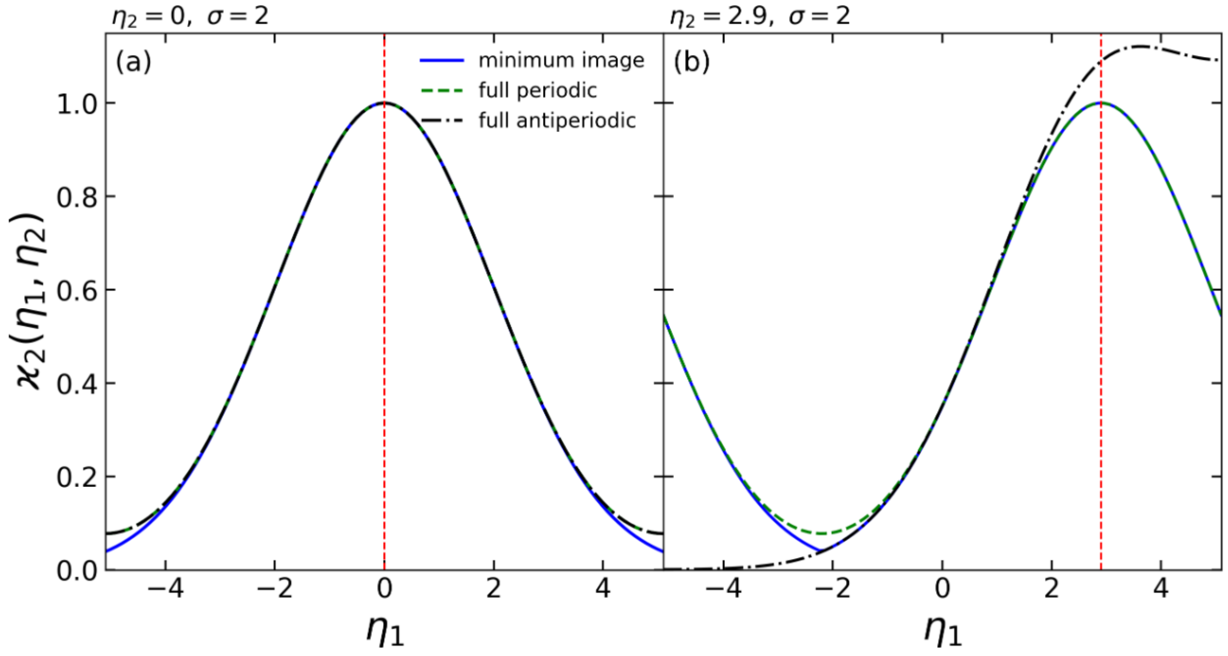
Symmetric n-point Gaussian kernel

Reflecting boundary conditions in a finite system → Sum over antiperiodic Gaussian images

$$\varkappa_n^{\mathrm{BC}}(\eta_1,\ldots,\eta_n) = A_n \sum_{k_2,\ldots,k_n=-\infty}^{\infty} \exp\left[-\frac{1}{n\sigma_\eta^2}\sum_{1\le i<j\le n}(\tilde{\eta}_i-\tilde{\eta}_j)^2\right],$$

$\tilde{\eta}_j = \eta_j + 4k_j\eta_{\max}$ and $\tilde{\eta}_j = 2\eta_{\max} - \eta_j + 4k_j\eta_{\max}$

In practice, these boundary conditions are largely irrelevant for midrapidity



Fifth and sixth order

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$$\mathcal{C}_5(\eta_1,\ldots,\eta_5) = \chi_5^B\delta_{1,2,3,4,5} - \frac{\chi_5^B}{4!V}\sum_{\sigma\in S_5}\delta_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}\varkappa_2(\eta_{\sigma_1},\eta_{\sigma_5}) - \frac{\chi_3^B\chi_4^B}{3!2!\chi_2^B V}\sum_{\sigma\in S_5}\delta_{\sigma_1,\sigma_2,\sigma_3}\delta_{\sigma_4,\sigma_5}\varkappa_2(\eta_{\sigma_1},\eta_{\sigma_5})$$
$$+ \frac{1}{2!3!V^2}\left[\chi_5^B + \frac{\chi_3^B\chi_4^B}{\chi_2^B}\right]\sum_{\sigma\in S_5}\delta_{\sigma_1,\sigma_2,\sigma_3}\varkappa_3(\eta_{\sigma_1},\eta_{\sigma_4},\eta_{\sigma_5}) + \frac{2\chi_3^B\chi_4^B}{(2!)^3V^2}\sum_{\sigma\in S_5}\delta_{\sigma_1,\sigma_2}\delta_{\sigma_3,\sigma_4}\varkappa_3(\eta_{\sigma_1},\eta_{\sigma_4},\eta_{\sigma_5})$$
$$- \frac{1}{3!2!V^3}\left[\chi_5^B + 5\frac{\chi_3^B\chi_4^B}{\chi_2^B}\right]\sum_{\sigma\in S_5}\delta_{\sigma_1,\sigma_2}\varkappa_4(\eta_{\sigma_1},\eta_{\sigma_3},\eta_{\sigma_4},\eta_{\sigma_5}) + \frac{4}{V^4}\left[\chi_5^B + 5\frac{\chi_3^B\chi_4^B}{\chi_2^B}\right]\varkappa_5(\eta_{\sigma_1},\ldots,\eta_{\sigma_5})$$

$$\mathcal{C}_6(\eta_1,\ldots,\eta_6) = \chi_6^B\delta_{1,2,3,4,5,6} - \frac{\chi_6^B}{5!V}\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2,\sigma_3,\sigma_4\sigma_5}\varkappa_2(\eta_{\sigma_1},\eta_{\sigma_6}) - \frac{\chi_3^B\chi_5^B}{4!2!\chi_2^B V}\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}\delta_{\sigma_1,\sigma_5}\varkappa_2(\eta_{\sigma_1},\eta_{\sigma_6})$$
$$- \frac{(\chi_4^B)^2}{2!(3!)^2V\chi_2^B}\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2,\sigma_3}\delta_{\sigma_4,\sigma_5,\sigma_6}\varkappa_2(\eta_{\sigma_1},\eta_{\sigma_6}) + \frac{1}{4!2!V^2}\left[\chi_6^B + \frac{\chi_3^B\chi_5^B}{\chi_2^B}\right]\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}\varkappa_3(\eta_{\sigma_1},\eta_{\sigma_5},\eta_{\sigma_6})$$
$$+ \frac{1}{(2!)^2 3!V^2}\left[\frac{(\chi_4^B)^2 + \chi_3^B\chi_5^B}{\chi_2^B}\right]\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2,\sigma_3}\delta_{\sigma_4,\sigma_5}\varkappa_3(\eta_{\sigma_1},\eta_{\sigma_5},\eta_{\sigma_6}) + \frac{1}{3!(2!)^3V^2}\left[\frac{3\chi_2^B(\chi_3^B)^2\chi_4^B - (\chi_3^B)^4}{\chi_2^B}\right]$$
$$\times \sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2}\delta_{\sigma_3,\sigma_4}\delta_{\sigma_5,\sigma_6}\varkappa_3(\eta_{\sigma_1},\eta_{\sigma_5},\eta_{\sigma_6}) - \frac{1}{(3!)^2V^3}\left[\frac{2(\chi_4^B)^2 + 3\chi_3^B\chi_5^B + \chi_2^B\chi_6^B}{\chi_2^B}\right]\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2,\sigma_3}\varkappa_4(\{\eta_{\sigma_i}\}_{i=1,4,5,6})$$
$$+ \frac{1}{(2!)^5V^3}\left[\frac{(\chi_3^B)^4 - 3\chi_2^B(\chi_3^B)^2\chi_4^B - 2(\chi_2^B)^2(\chi_4^B)^2 - 2(\chi_2^B)^2\chi_3^B\chi_5^B}{(\chi_2^B)^3}\right]\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2}\delta_{\sigma_3,\sigma_4}\varkappa_4(\{\eta_{\sigma_i}\}_{i=1,4,5,6})$$
$$+ \frac{1}{4!2!V^4}\left[\frac{-3(\chi_3^B)^4 + 9\chi_2^B\chi_4^B(\chi_3^B)^2 + 9(\chi_2^B)^2\chi_5^B\chi_3^B + (\chi_2^B)^2(8(\chi_4^B)^2 + \chi_2^B\chi_6^B)}{(\chi_2^B)^3}\right]\sum_{\sigma\in S_6}\delta_{\sigma_1,\sigma_2}\varkappa_5(\{\eta_{\sigma_i}\}_{i=1,3,4,5,6})$$
$$- \frac{5}{V^5}\left[\frac{-3(\chi_3^B)^4 + 9\chi_2^B\chi_4^B(\chi_3^B)^2 + 9(\chi_2^B)^2\chi_5^B\chi_3^B + (\chi_2^B)^2(8(\chi_4^B)^2 + \chi_2^B\chi_6^B)}{(\chi_2^B)^3}\right]\varkappa_6(\{\eta_i\}_{i=1,2,3,4,5,6})$$