

Proton cumulants at RHIC: The antiproton puzzle, factorial cumulants, and baryon susceptibilities

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16th Workshop on Critical Point and Onset of Deconfinement

April 17, 2026

*This talk (part I) is given on behalf of **Adam Bzdak***

Thanks to: V. Koch, G. Pihan, R. Poberezhniuk



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1. Acceptance dependence of factorial cumulants and the antiproton puzzle

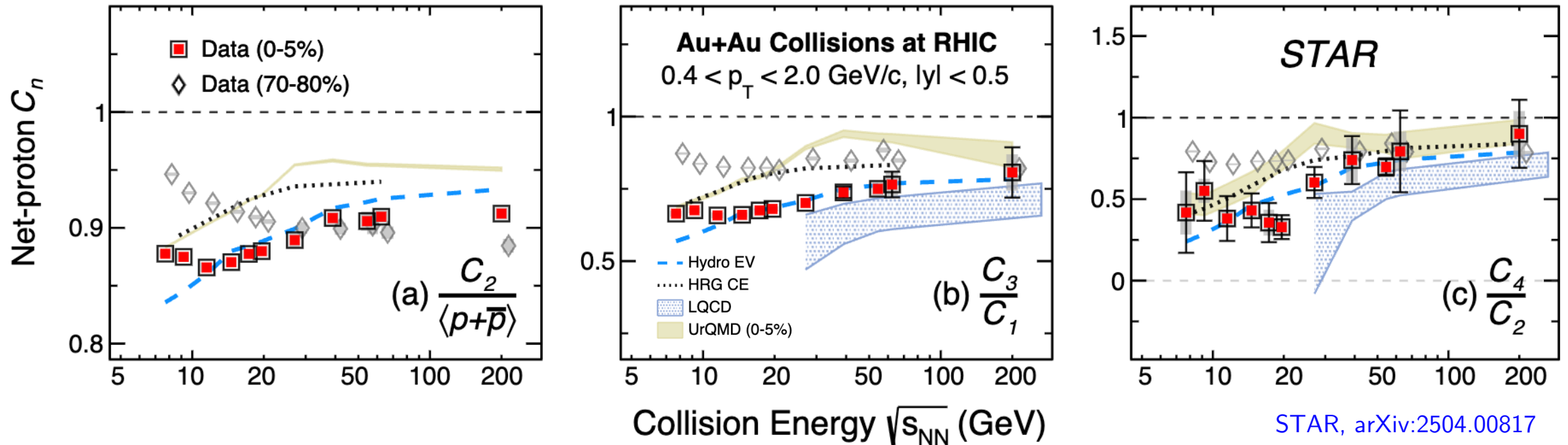
[A. Bzdak, V. Koch, VV, PRC 112, 024901 \(2025\)](#)

2. Inference of baryon number susceptibilities from data

[G. Pihan, R. Poberezhniuk, VV, to appear](#)

Acceptance dependence of factorial
cumulants and the antiproton puzzle

Net-proton cumulant ratios

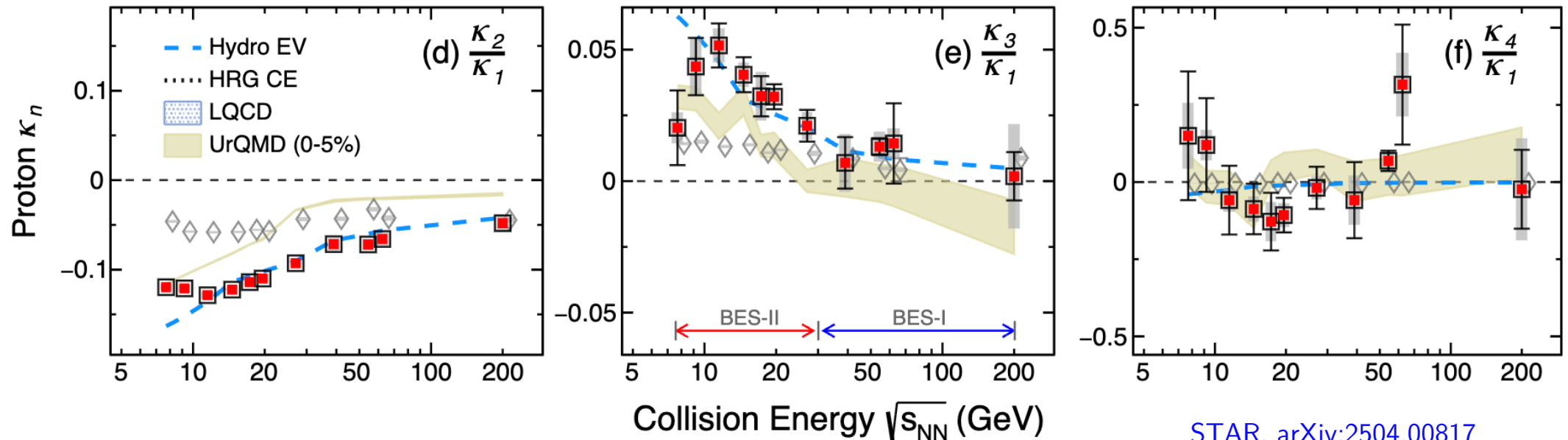


Hydro EV: [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 \(2022\)](#)

Agreement with the baseline above $\sqrt{s_{NN}} \sim 10 - 20$ GeV, maybe something in C_4/C_2 at 20 GeV

But otherwise mostly uneventful. What else is there?

Proton factorial cumulant ratios



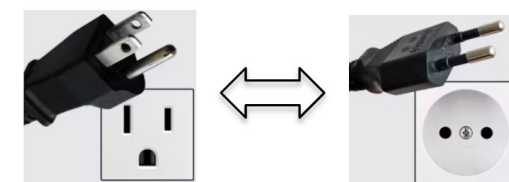
More structure seen in factorial cumulants

- Non-monotonic κ_2/κ_1 , κ_3/κ_1 , and possibly κ_4/κ_1



STAR
 Cumulants (C)
 Factorial cumulants (κ)

Others
 Cumulants (κ)
 Factorial cumulants (C)



Factorial cumulants \hat{C}_n vs ordinary cumulants C_n

Factorial cumulants

$$\hat{C}_n \sim \langle N(N-1)(N-2)\dots \rangle_c$$

$$\hat{C}_1 = C_1$$

$$\hat{C}_2 = C_2 - C_1$$

$$\hat{C}_3 = C_3 - 3C_2 + 2C_1$$

$$\hat{C}_4 = C_4 - 6C_3 + 11C_2 - 6C_1$$

Ordinary cumulants

$$C_n \sim \langle \delta N^n \rangle_c$$

$$C_1 = \hat{C}_1$$

$$C_2 = \hat{C}_2 + \hat{C}_1$$

$$C_3 = \hat{C}_3 + 3\hat{C}_2 + \hat{C}_1$$

$$C_4 = \hat{C}_4 + 6\hat{C}_3 + 7\hat{C}_2 + \hat{C}_1$$

[Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017); Kitazawa, Luo, PRC 96, 024910 (2017); C. Pruneau, PRC 100, 034905 (2019)]

Ordinary cumulants measure deviations from the **Gaussian distribution**

- Natural variables for net-charge density fluctuations
- Connection to lattice QCD susceptibilities
- But they mix particle correlations of different orders

Factorial cumulants measure deviations from the **Poisson distribution**

- Natural variables for particle number fluctuations
- Irreducible n-particle correlations (lower orders are subtracted)
- Critical fluctuations are as singular as in ord. cumulants, non-critical are suppressed [Ling, Stephanov, PRC '16]

Test for baseline: acceptance dependence of couplings

Bzdak ratios: reduced correlation functions – “couplings”

[Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)]

$$\hat{c}_k = \frac{\hat{C}_k}{\langle N \rangle^k}$$

$$c_k = \frac{\int \rho_1(y_1) \cdots \rho_1(y_k) c_k(y_1, \dots, y_k) dy_1 \cdots dy_k}{\int \rho_1(y_1) \cdots \rho_1(y_k) dy_1 \cdots dy_k}$$

integrated correlation function in rapidity

Long-range correlations in rapidity lead to acceptance-independent couplings, for example

- Global (not local) baryon conservation

[Bzdak, Koch, Skokov, EPJC 77, 288 (2017); Bzdak, Koch, PRC 96, 054905 (2017)]

- + volume fluctuations

[Holzmann, Koch, Rustamov, Stroth, arXiv:2403.03598]

$$c_2 = -\frac{1}{B}, \quad c_3 = \frac{2}{B^2}, \quad c_4 = -\frac{6}{B^3}$$

$$\hat{c}_{i,j} = \hat{c}_{i,j} + \frac{\kappa_2[V]}{\langle V \rangle^2}, \quad \text{for } i + j = 2.$$

all lead to

$$\frac{\hat{C}_k}{\langle N \rangle^k} = \text{const.} \quad \text{at a given } \sqrt{s_{NN}}$$

and

$$\frac{\hat{C}_2^p}{\langle N_p \rangle^2} \approx \frac{\hat{C}_2^{\bar{p}}}{\langle N_{\bar{p}} \rangle^2} = -\frac{1}{\langle N_B + N_{\bar{B}} \rangle_{4\pi}} \quad \text{at a given } \sqrt{s_{NN}}$$

Can be tested *without* CBWC/volume fluctuations correction [Bzdak, Koch, VV, PRC 112, 024901 (2025)]

Test for baseline: BES-I data

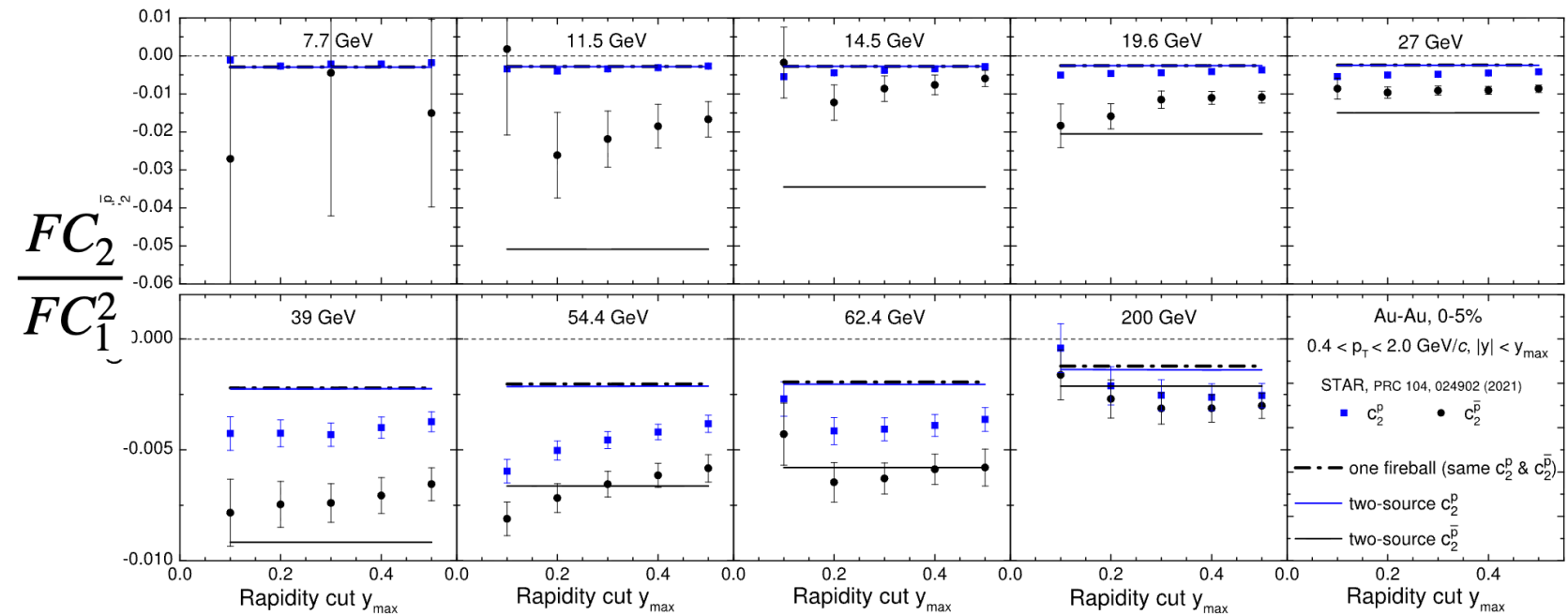
- $\frac{FC_2}{FC_1^2}$ more or less constant

- $\frac{FC_2[p]}{FC_1^2[p]} \neq \frac{FC_2[\bar{p}]}{FC_1^2[\bar{p}]}$

- baseline OK for protons

- No good for anti-protons ??

- How will it look with BES II data?



Test for baseline: BES-I data

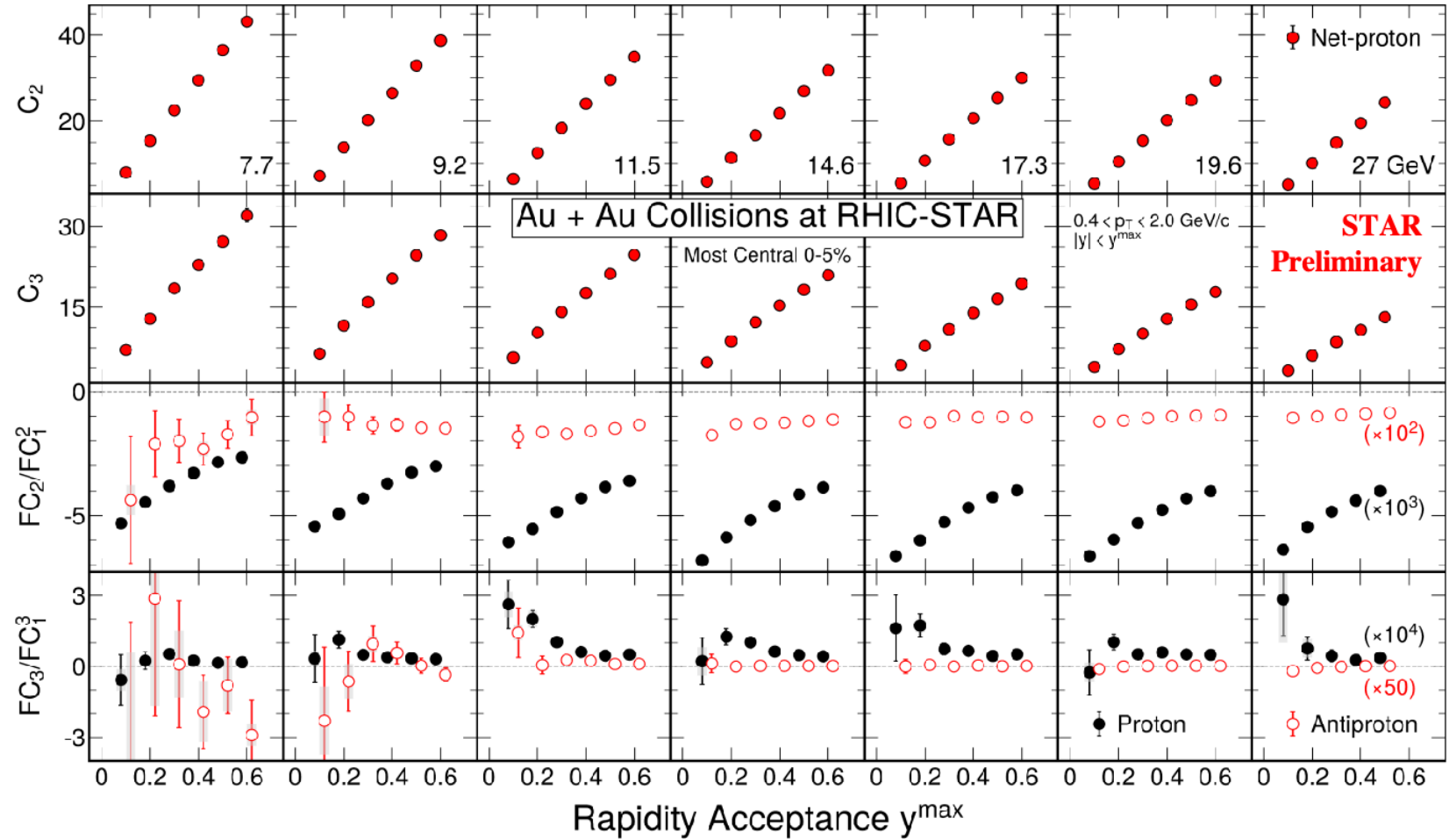
- $\frac{FC_2}{FC_1^2}$ more or less constant

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- **not** baseline OK for protons ?
- No good for anti-protons ??
- How will it look with BES II data?

Antiproton puzzle seems preserved in BES-II

Preliminary BES-II data, Yige Huang (CPOD2026)



Two component model

2 sources: stopped and produced particles

- All anti-protons are produced
- protons come from produced and stopped sources

$$N_p(\text{produced}) = N_{\bar{p}}$$

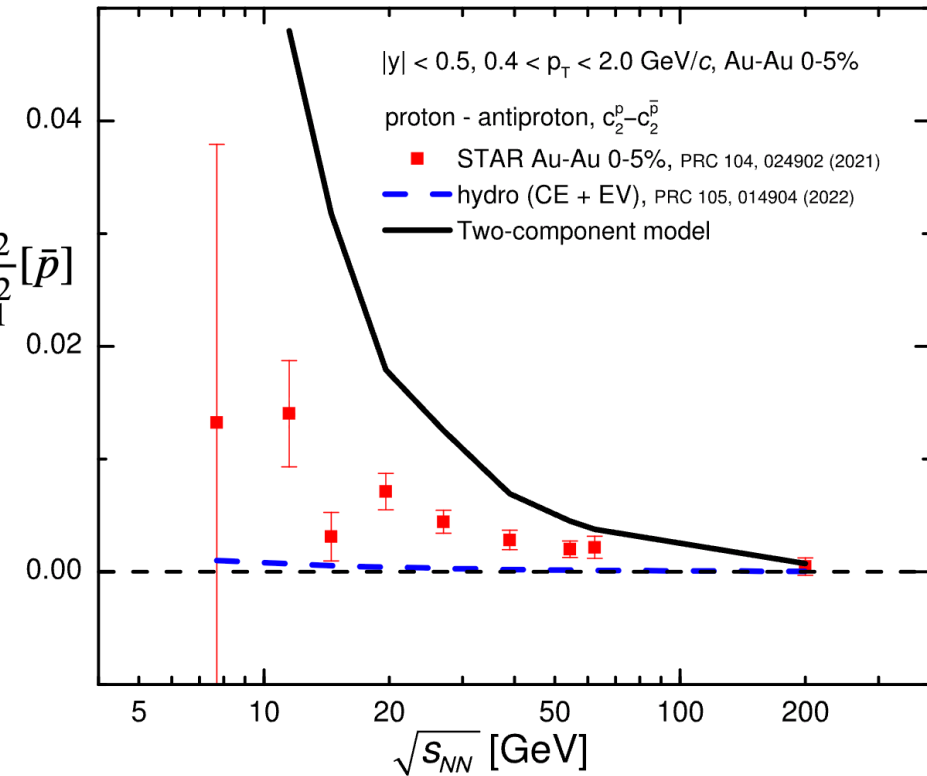
$$N_p = N_p(\text{stopped}) + N_{\bar{p}}$$

$$\frac{FC_2}{FC_1^2}[p] - \frac{FC_2}{FC_1^2}[\bar{p}]$$

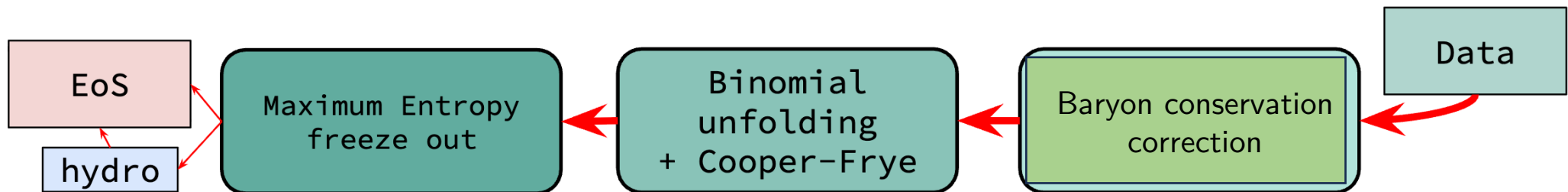
- Produced source: Thermal with zero net baryon number $\langle B - \bar{B} \rangle = 0$

- Stopped source: Follows binomial distribution

- If accurate, may indicate incomplete equilibration of the fireball
- Baryon annihilation needs to be ruled out first



Inference of baryon number susceptibilities from data



Generalizing Hydro EV: new elements

Hydro EV: particlization of excluded volume HRG in the canonical ensemble following hydro

[VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)]

Hydro EV: particlization of ~~excluded volume~~ HRG in the canonical ensemble following hydro

[VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)]

Maximum entropy freeze-out [M. Pradeep et al., PRL 130, 162301 (2023); J.M. Karthein et al., PRD 113, 074010 (2026)]

- Incorporate single hydrodynamic mode – **baryon density fluctuations**
- Maximum entropy method defines local **baryon/antibaryon joint susceptibilities** G. Pihan et al., to appear

$$\hat{\chi}_{nm}^{+-}(x) = \delta_{m0}\delta_{n1}\bar{\chi}_1^+ + \delta_{n0}\delta_{m1}\bar{\chi}_1^- + (-1)^m \frac{\hat{\Delta}\chi_{n+m}^B(x)}{(\bar{\chi}_1^+(x) + \bar{\chi}_1^-(x))^{n+m}} \frac{\bar{\chi}_1^+(x)^n \bar{\chi}_1^-(x)^m}{(\bar{\chi}_1^+(x) + \bar{\chi}_1^-(x))^{n+m}}$$

$$\bar{\chi}_n^B(x) = \bar{\chi}_1^+(x) + (-1)^n \bar{\chi}_1^-(x)$$

+ is baryon, – is antibaryon

difference to HRG

HRG (Poisson) baseline

- Parametrize the EoS by $\chi_2^B/\bar{\chi}_2^B$, χ_3^B/χ_1^B , χ_4^B/χ_2^B ratios at each energy

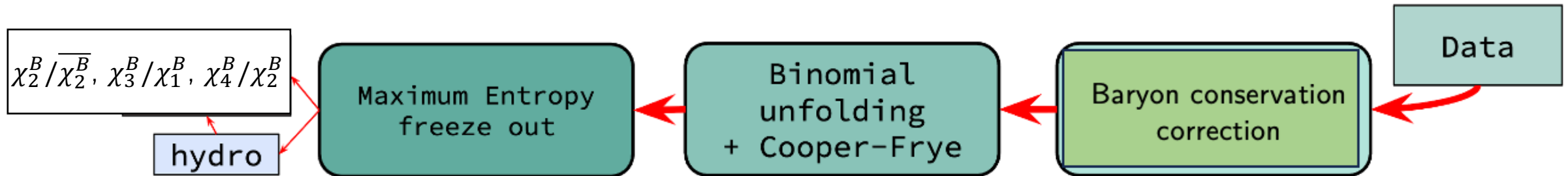
Subensemble acceptance method 3.0 R. Poberezhniuk, V.A. Kuznietsov, G. Pihan, VV, to appear

- Correction for **baryon number conservation**
- Requires joint accepted proton (X) and 4π baryon (B) cumulants, summed over all CF cells

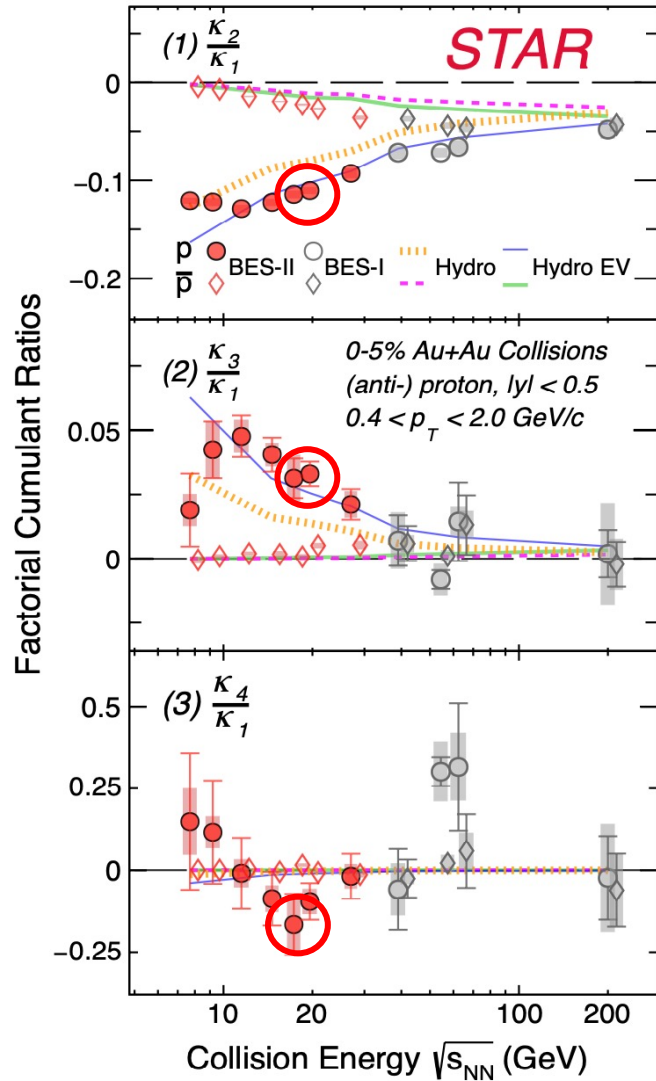
$$P_{ce}(X) \equiv P_{gce}(X | B_{tot} = B_0) = \frac{P_{gce}(X, B_{tot} = B_0)}{P_{gce}(B_{tot} = B_0)}$$

$$\kappa_2^{ce}(X) = \kappa_2^X - \frac{(\kappa_{11}^{XB})^2}{\kappa_2^B}$$

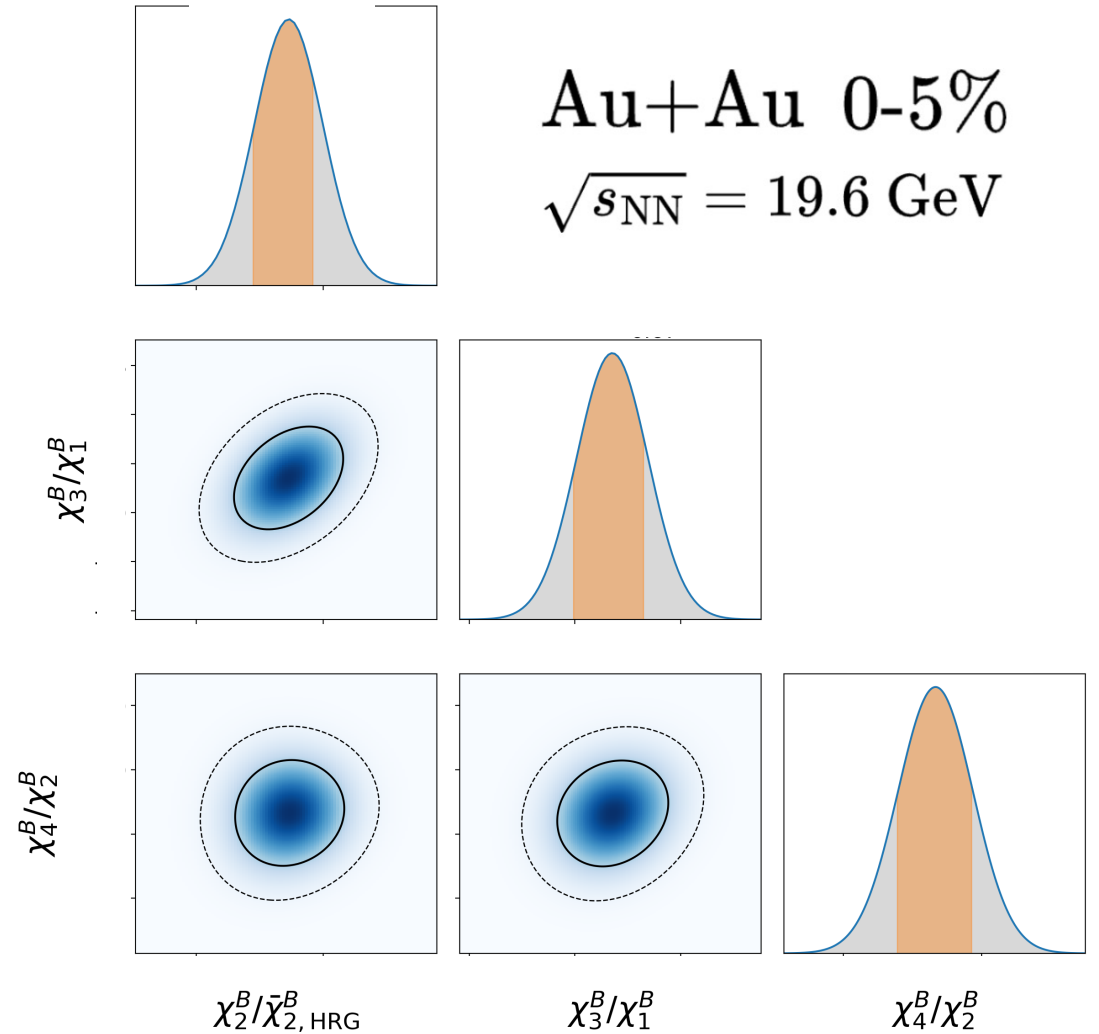
Roadmap to EoS inference



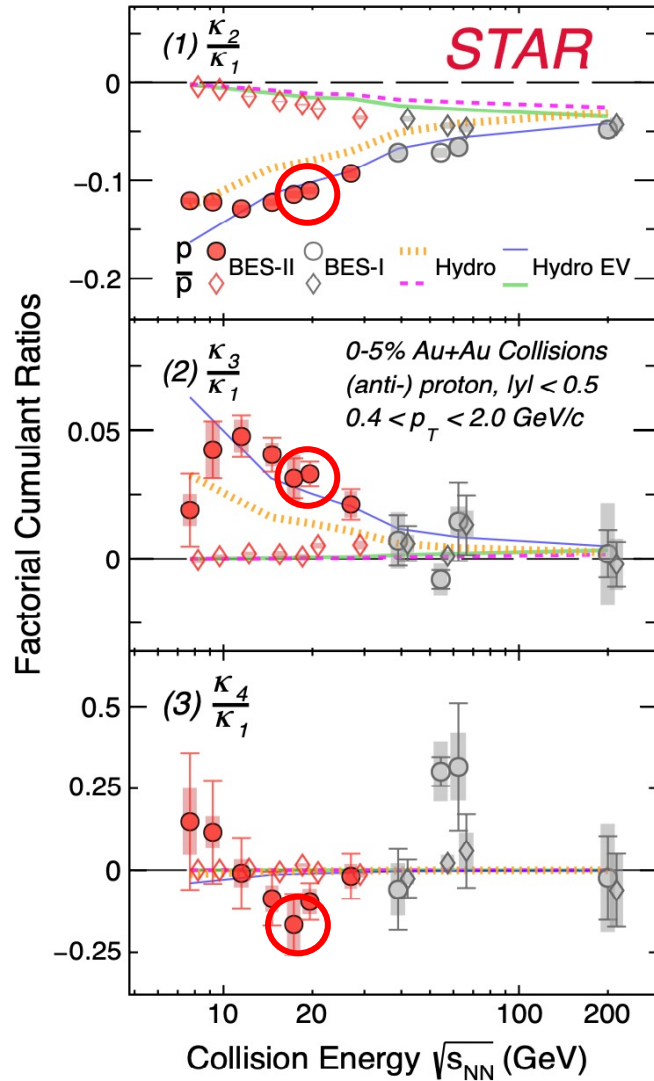
Bayesian inference: 19.6 GeV



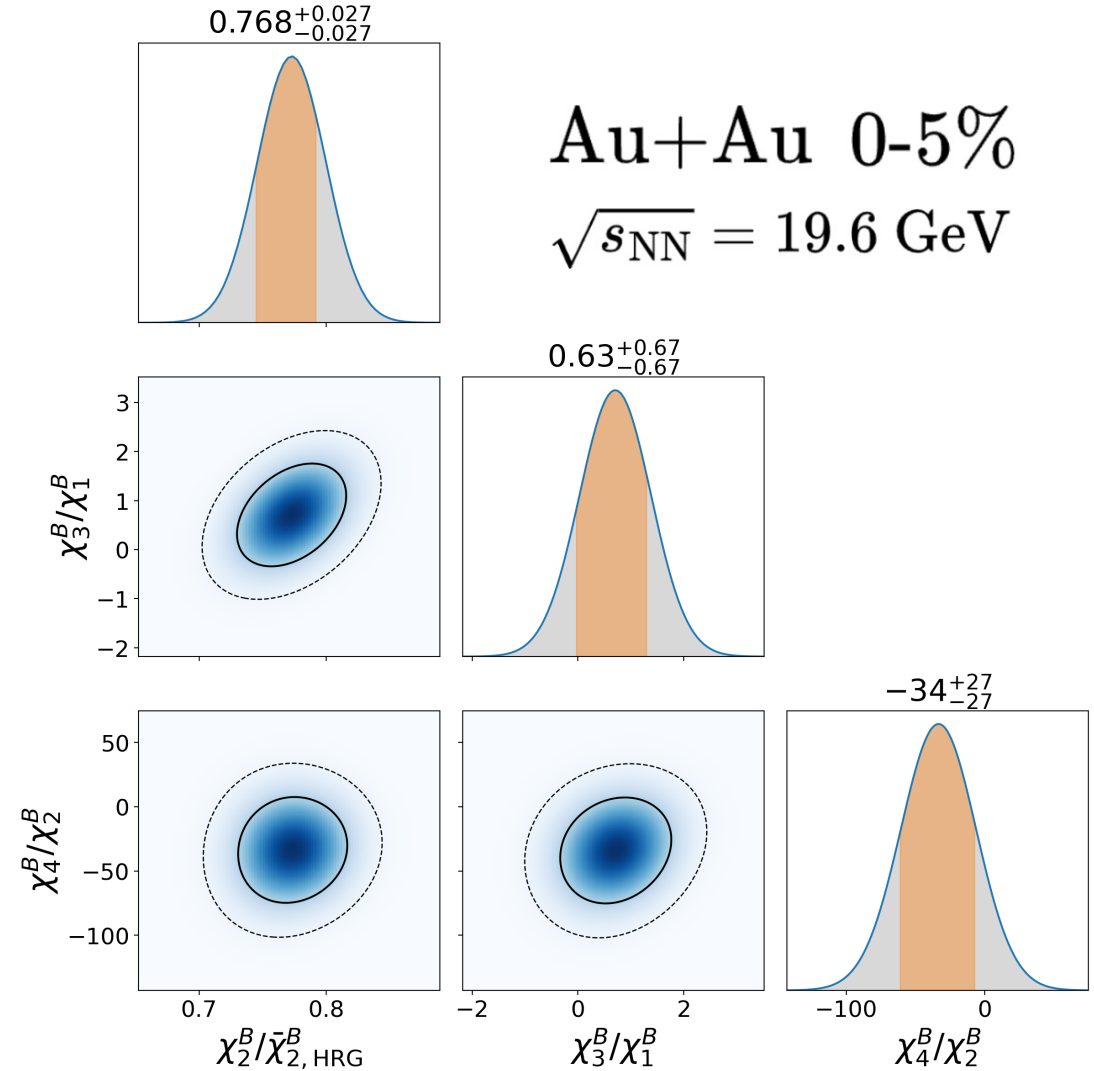
Unfolding



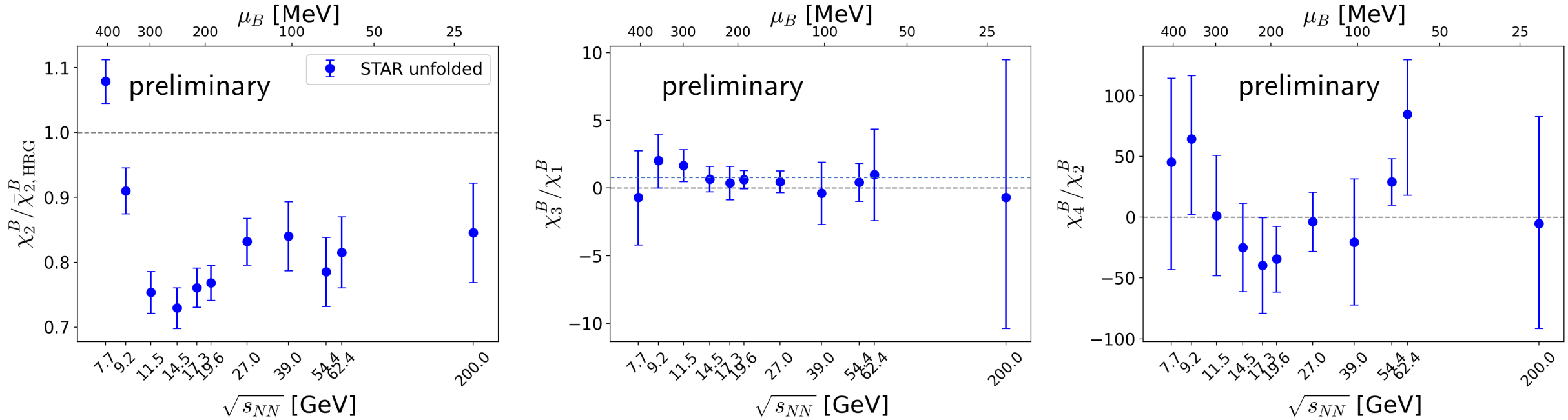
Bayesian inference: 19.6 GeV



Unfolding



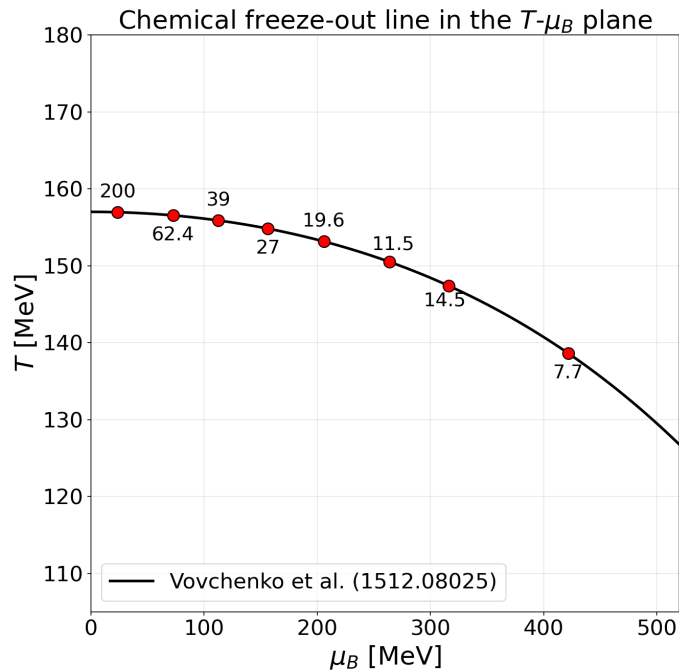
Extracted susceptibilities



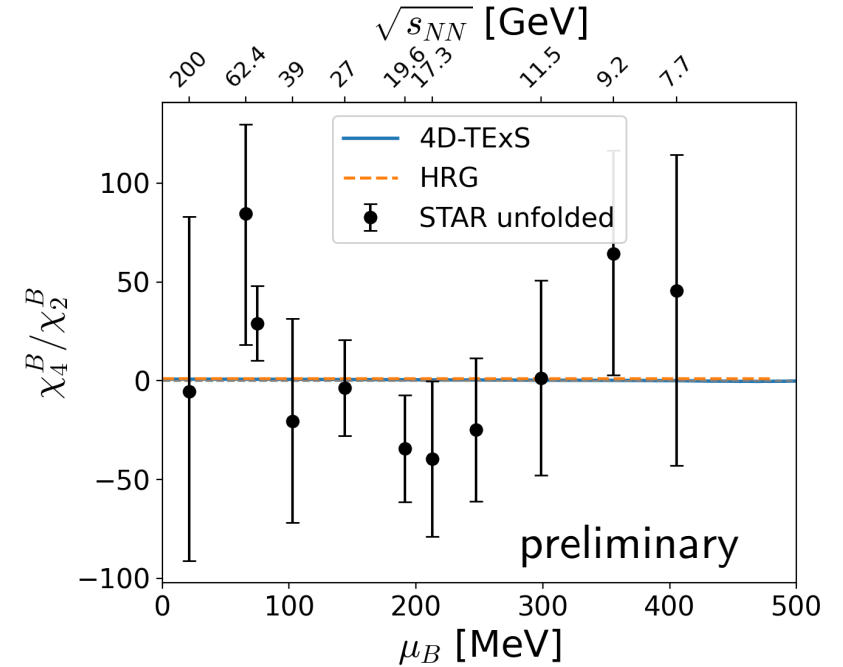
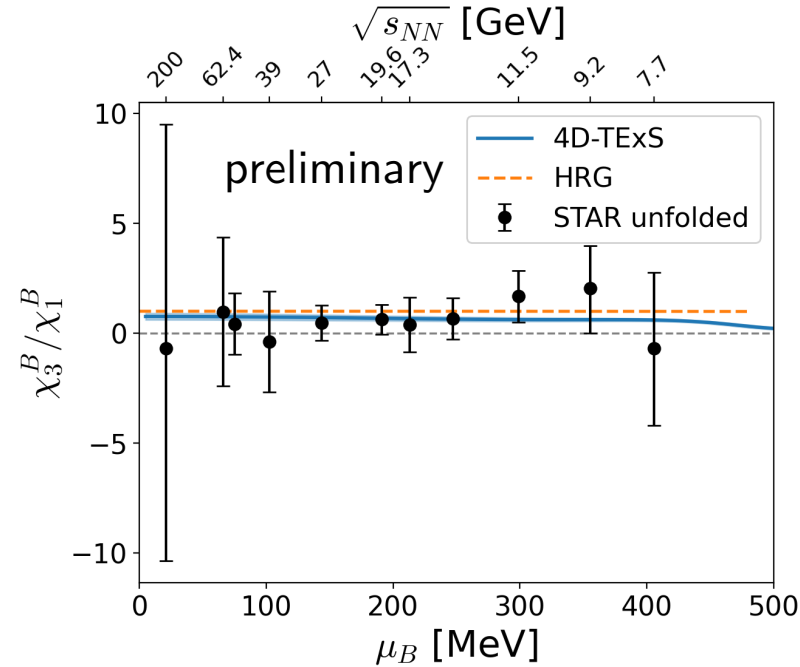
- Tight constraints on χ_2^B value relative to HRG
- Uncertainties in 3rd and 4th order susceptibilities are large---propagation of data uncertainty
 - Intermediate steps (net-B to B^+ , kinematic cuts, $B^+ \rightarrow p$) acts akin to efficiency correction
 - Reducing uncertainties in the data would lead to reduced extraction errors
- Results appear weakly sensitive to e_{sw} , sensitivity to other hydro parameters to be studied

Comparing to lattice QCD

To map the results to QCD phase diagram use chemical freeze-out parametrization [VV et al., PRC 93, 064906 (2015)]



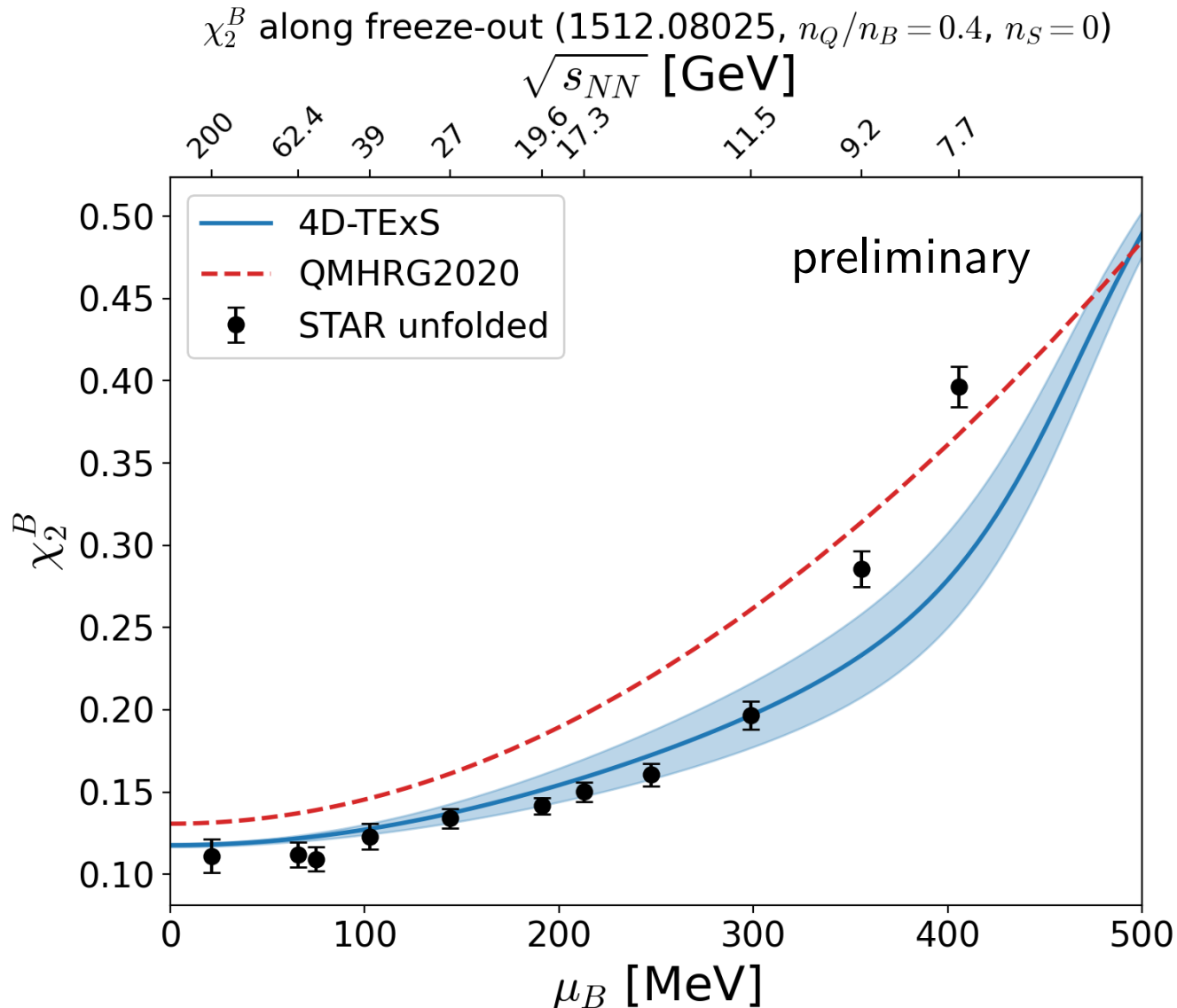
$$\varepsilon_{SW} \approx 0.35 - 0.40 \text{ GeV}/\text{fm}^3$$



4D-TExS: [A. Abuali et al., Phys. Rev. D 112, 054502 (2025)]

- Lattice-based EoS at finite baryon density
- Computed under HIC conditions ($n_Q/n_B = 0.4, n_S = 0$)

Baryon susceptibility χ_2^B



- Bare χ_2^B : multiply by HRG value
 - QMHRG motivated by lattice QCD studies at $\mu_B = 0$ [HotQCD, PRD 104, 074512 (2021)]
- Agreement with lattice QCD at $\mu_B < 300$ MeV
- Enhancement relative to lattice QCD at $\mu_B > 300$ MeV
- Attraction vs repulsion? see also A. Rustamov's talk

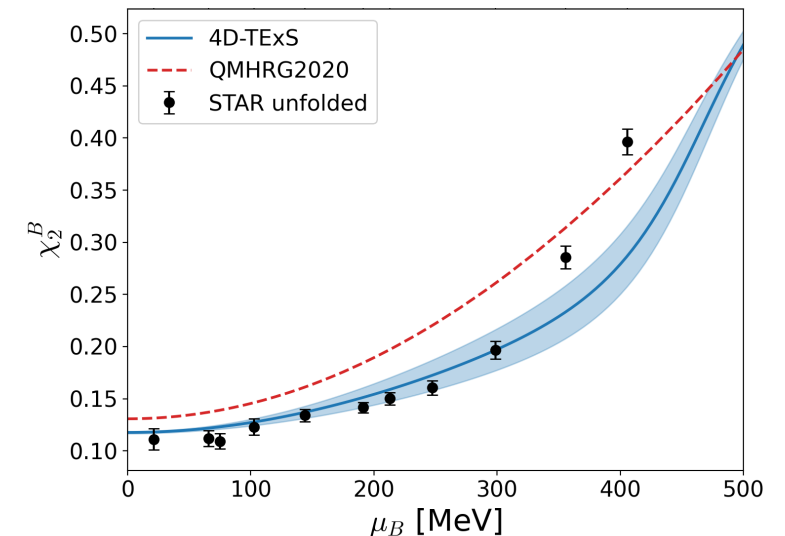
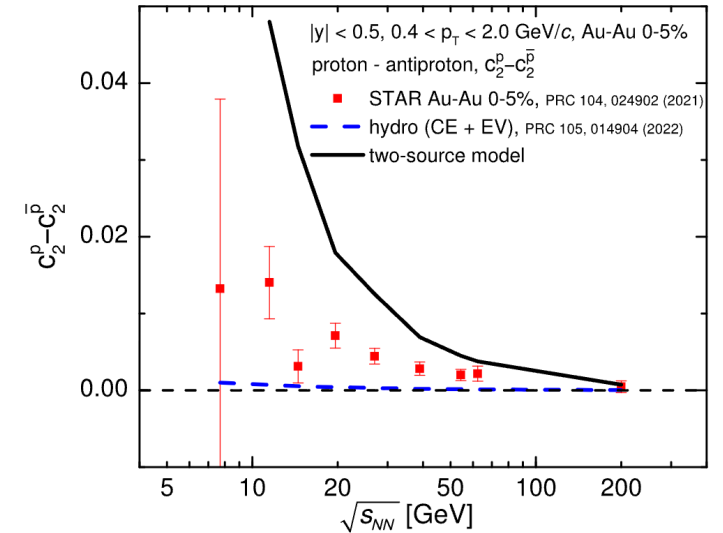
⚠ Caveats:

- Sensitive to the hadron list
- Sensitive to assumed freeze-out line
- Lattice EoS (4D-TEoS) is an extrapolation (no truncation error included)

- **Antiproton puzzle**
 - Large splitting between normalized factorial cumulants of p and \bar{p}
 - Existing baselines fail to describe it, BES-II data to be analyzed
- **Hydrodynamics-based inference of baryon susceptibilities**
 - χ_2^B agrees with LQCD at $\mu_B < 300$ MeV, shows enhancement at larger μ_B
 - Current errors are too large to infer high-order susceptibilities

Outlook:

- Detailed analysis of acceptance dependence, baryon annihilation
- Improved quantitative description of baselines and critical fluctuations
 - Sensitivity to hydro/model parameters, other observables
 - Lower collision energies (RHIC-FXT), $\sqrt{s_{NN}} \lesssim 7.7$ GeV



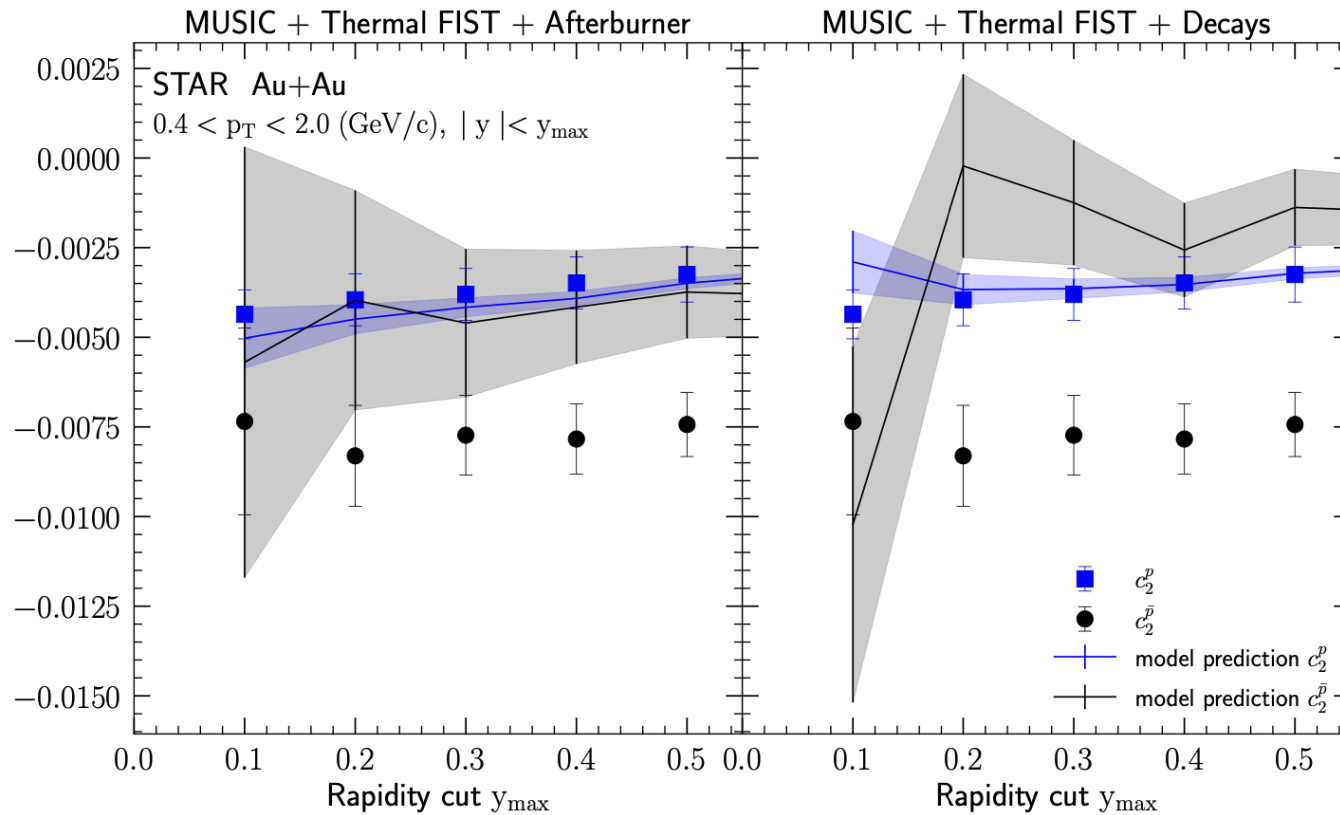
Thanks for your attention

Additional slides

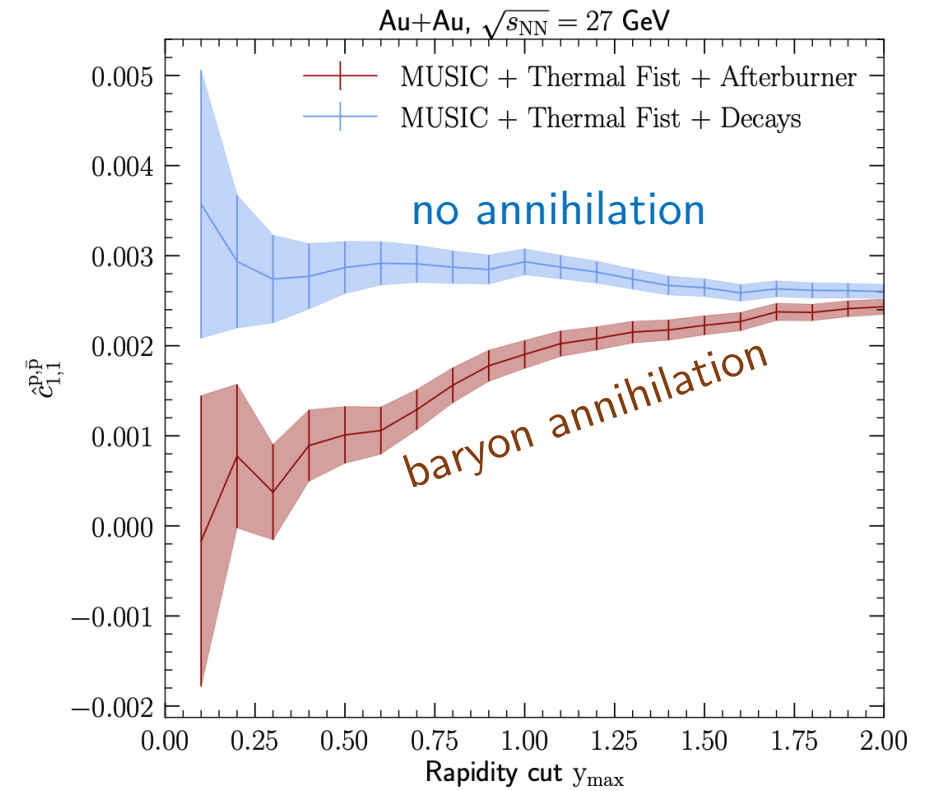
Scaled factorial cumulants and baryon annihilation

Extending Hydro EV to incorporate hadronic phase (UrQMD)

Au-Au, $\sqrt{s_{NN}} = 27$ GeV G. Pihan, VV, in progress



Covariance $c_{11}^{p\bar{p}}$



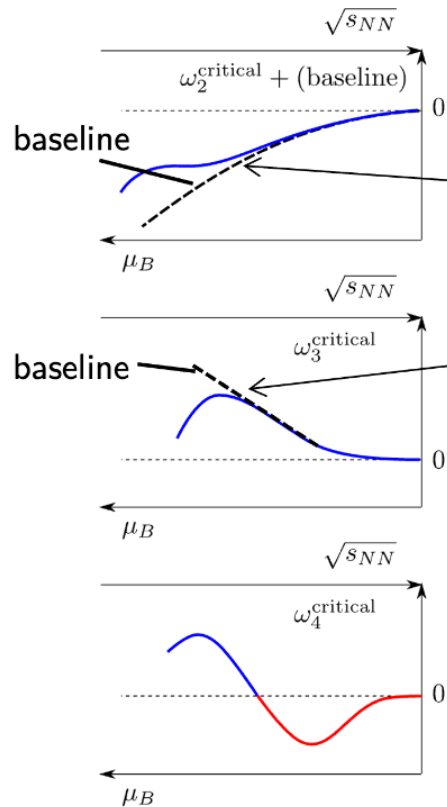
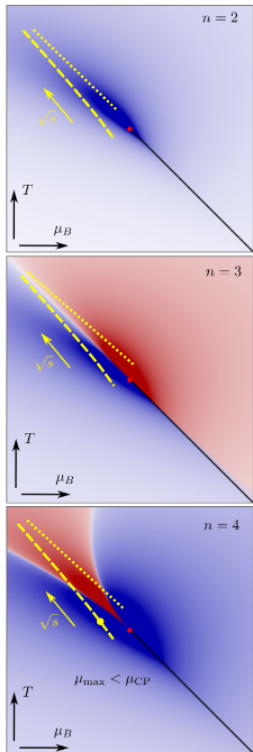
- Hadronic phase appears unlikely to resolve the antiproton puzzle (more statistics needed)
- Acceptance dependence of proton-antiproton covariance shows clear effect of hadronic phase

RHIC-BES-II data and CP

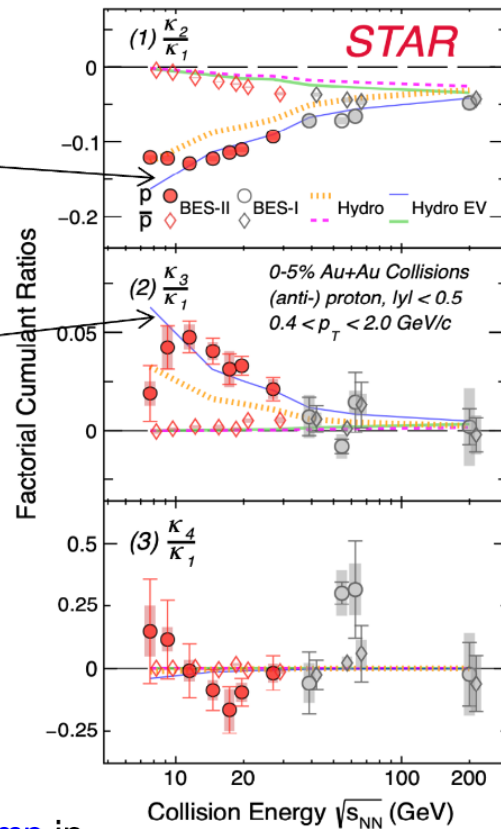
VV, Koch, arXiv:2504.01368, plot adapted from M. Stephanov, arXiv:2410.02861

$$\omega_n = \hat{C}_n / \hat{C}_1$$

(universal EOS) critical χ_n :



BES-II data:
plot from A. Pandav, CPOD2024



Non-critical baseline (hydro EV):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

- describes right side of the peak in \hat{C}_3
- **signal relative to baseline:**
 - *positive* $\hat{C}_2 - \hat{C}_2^{baseline} > 0$
 - *negative* $\hat{C}_3 - \hat{C}_3^{baseline} < 0$

New scenarios: **cool critical point**

Basar, Pradeep, Stephanov, arXiv:2603.23635

Controlling the non-critical baseline is essential

Or one can go beyond comparisons to baselines

Expected signatures: **bump** in ω_2 and ω_3 , **dip then bump** in ω_4
for CP at $\mu_B > 420$ MeV

Hydro EV [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)]

1. Smooth (3+1)-D hydro evolution with MUSIC [Shen, Alzhrani, PRC 102, 014909 (2020)]
2. Grand-canonical particlization of excluded volume HRG fitted to lattice data at $\mu_B = 0$ [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]

- model-dependent and allows only repulsion

3. Kinematic/efficiency cuts using Cooper-Frye formula and isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 86, 044904 (2012)]

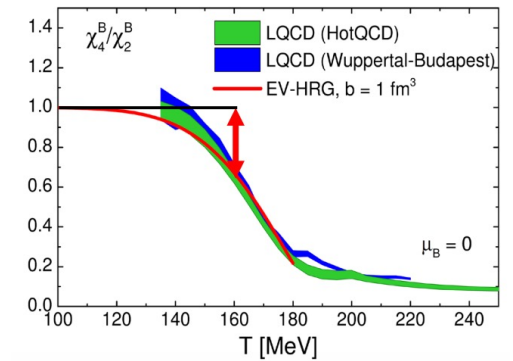


$$0.4 < p_T < 2 \text{ GeV}, |y| < 0.5$$

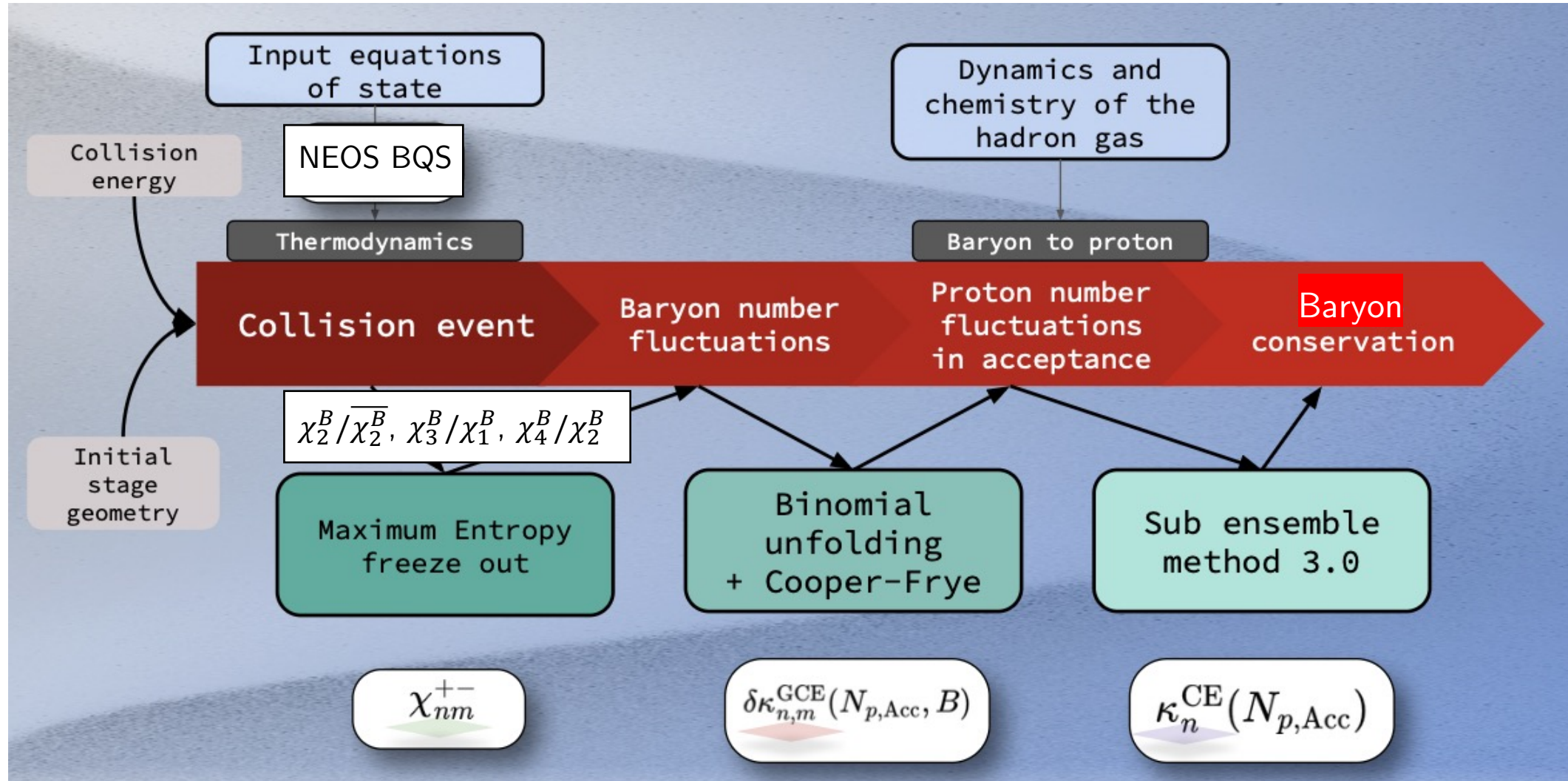
4. Correction for baryon conservation using SAM-2.0 [VV, Phys. Rev. C 105, 014903 (2022)]

- Balancing contribution from baryon conservation

- **Included:** baryon conservation, repulsion, kinematical cuts
- **Absent:** critical point, local conservation, initial-state/volume fluctuations, hadronic phase



Roadmap to EoS inference



Attraction vs repulsion

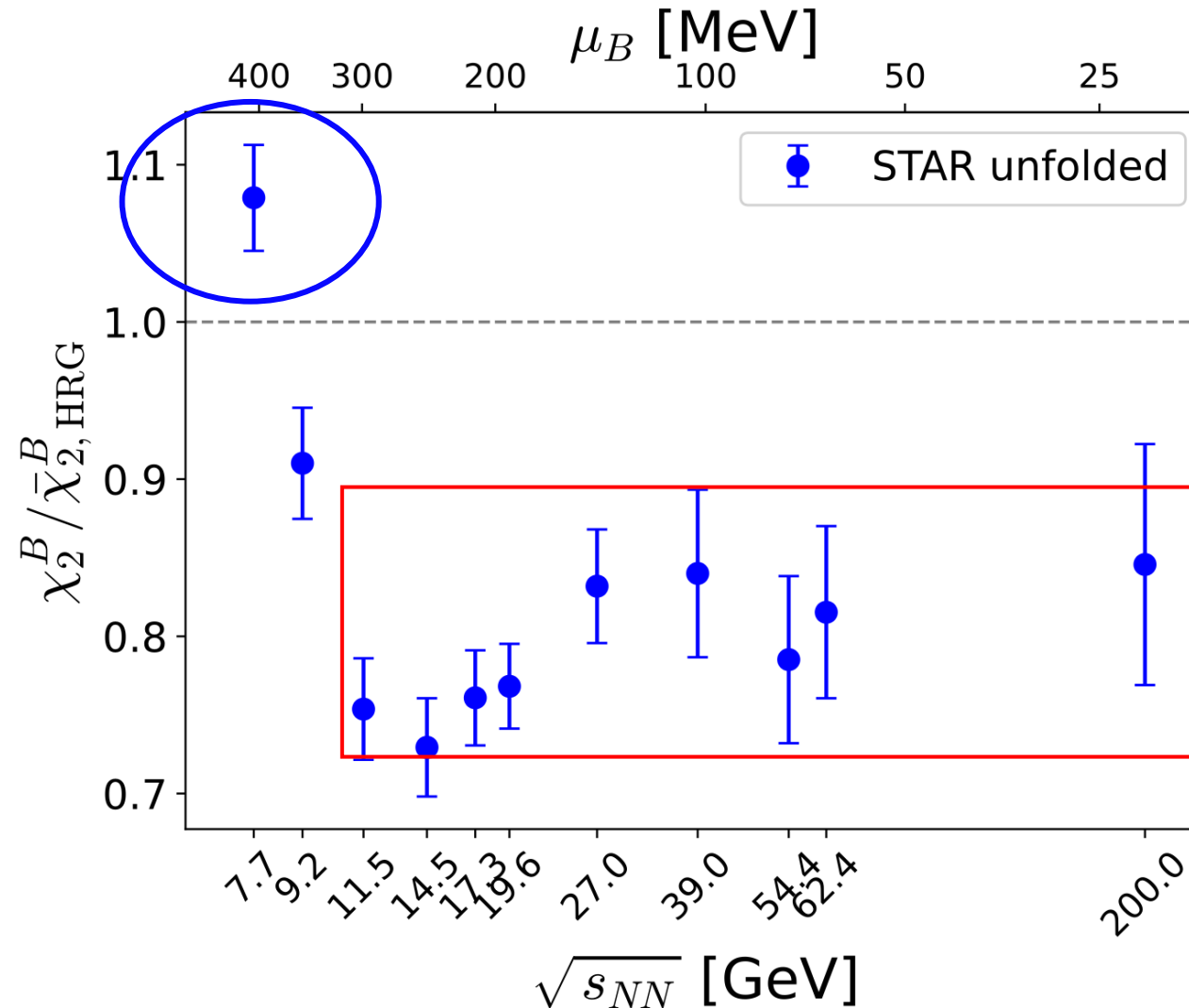
G. Pihan, R. Poberezhniuk, VV, to appear

attraction

$$\chi_2^B / \bar{\chi}_2^B > 1$$

$$\text{VDW: } \frac{1}{1 - \frac{2an}{T}}$$

$a > 0$ – attraction



repulsion

$$\chi_2^B / \bar{\chi}_2^B < 1$$

$$\text{VDW: } (1 - bn)^2$$

$b > 0$ – repulsion

Interplay of repulsive (high $\sqrt{s_{NN}}$) and attractive (low $\sqrt{s_{NN}}$) interactions?

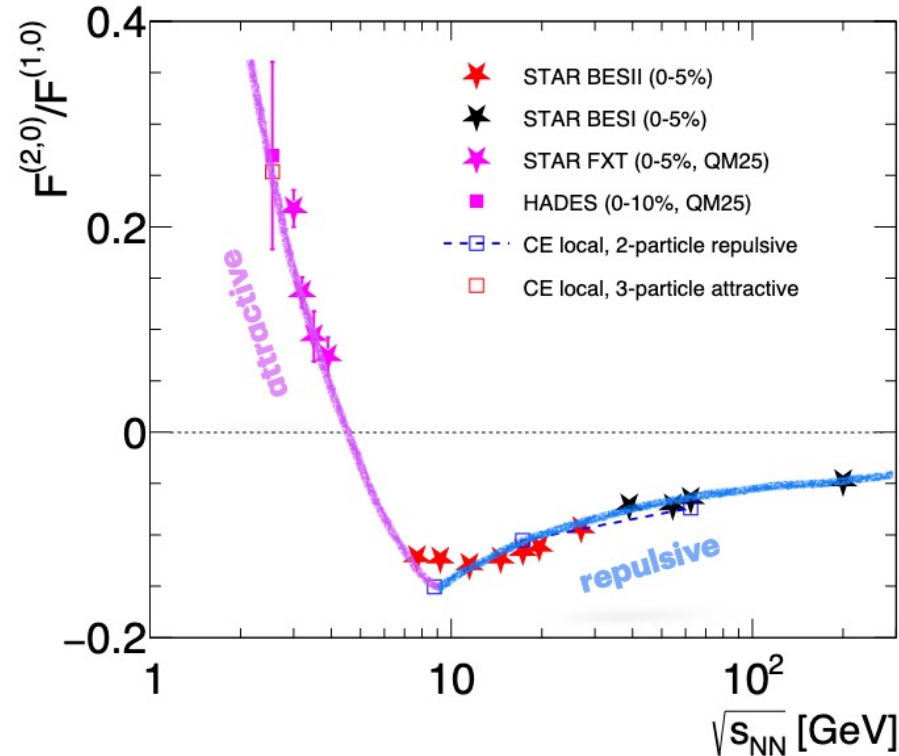
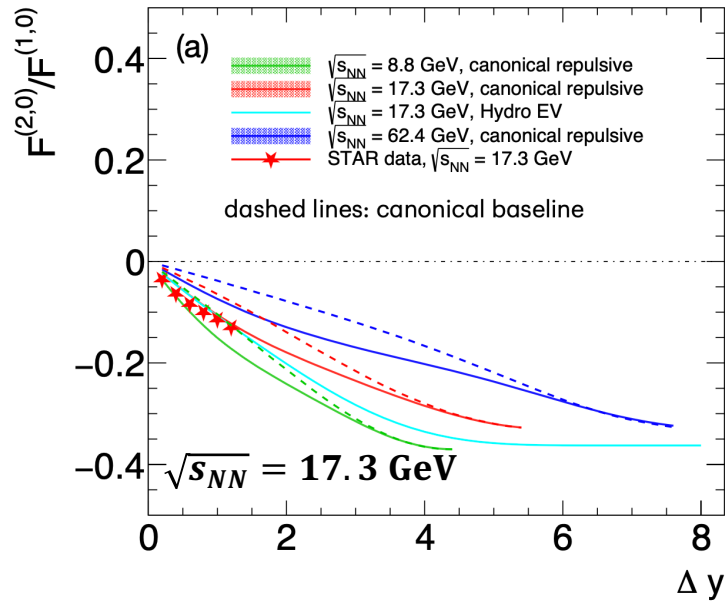
Attraction vs repulsion

Friman, Redlich, Rustamov, arXiv:2508.18879

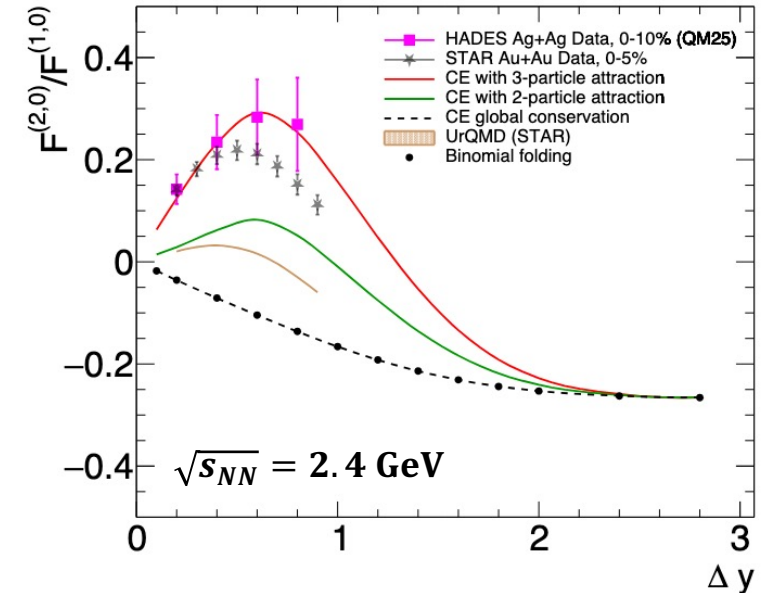
Recently, attractive and repulsive interactions implemented through a potential in rapidity

$$E_r(y_1, y_2) = \alpha_r e^{-|y_1 - y_2|/\rho_r} \quad P(y_1, y_2) = \frac{e^{-E(y_1, y_2)}}{Z} \quad E_a(y_1, y_2) = \alpha_a |y_1 - y_2|^{\beta_a}$$

repulsion



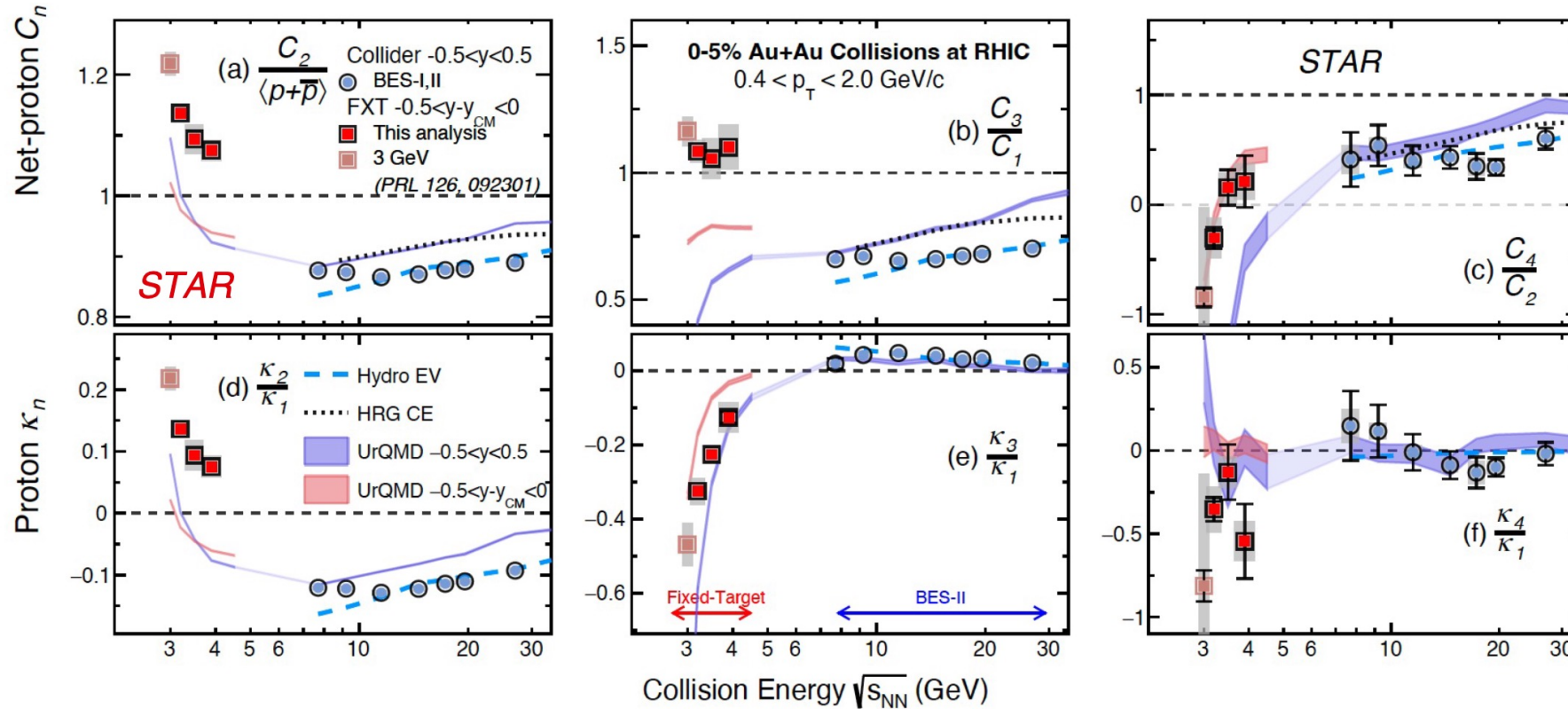
attraction



Interplay of repulsive (high $\sqrt{s_{NN}}$) and attractive (low $\sqrt{s_{NN}}$) interactions?

RHIC-BES-II data including STAR-FXT

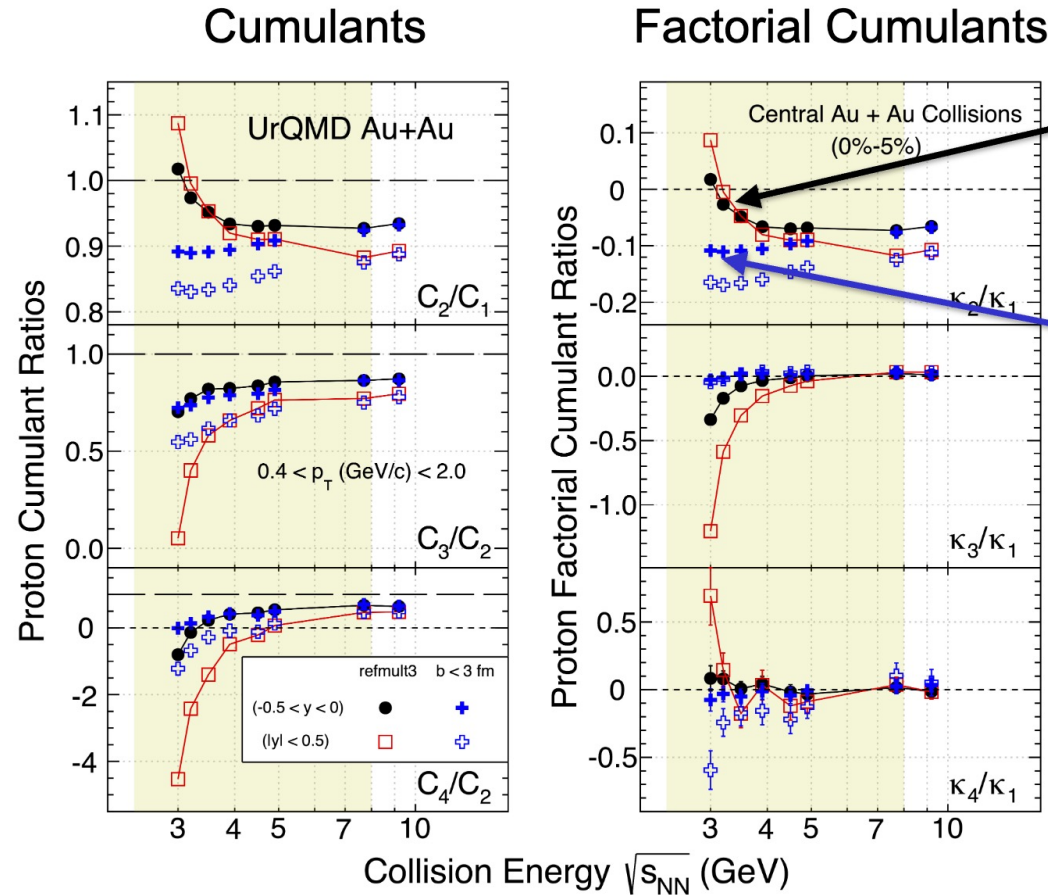
Z. Sweger (STAR), QM2025



- Continues the trends seen at lowest collider energies, in a fairly dramatic fashion
- **UrQMD (cascade) describes reasonably well the qualitative features**
 - Dominance of non-critical effects (centrality selection and spectators)?

The (possible) culprit

X. Zhang, Y. Zhang, X. Luo, N. Xu, arXiv: 2506.18832



Fluctuating impact parameter
STAR centrality selection

Fixed impact parameter ($b < 3$ fm)
minimal volume fluctuations.

N.B.: Centrality Bin Width Corrections
applied to both

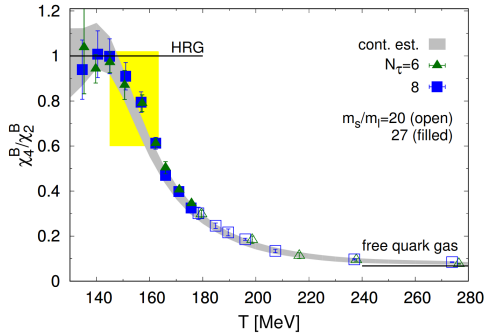
Possible culprit:
volume fluctuations/centrality selection

Other challenges: Antiproton puzzle (not described by hydro)

Theory vs experiment

guidance from theory (e.g. lattice)

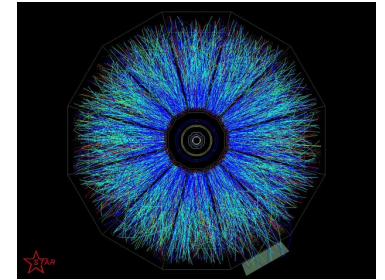
experiment (the real thing)



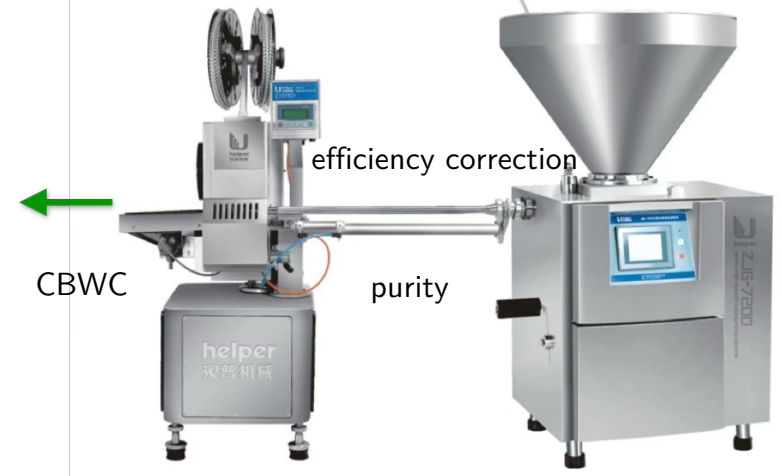
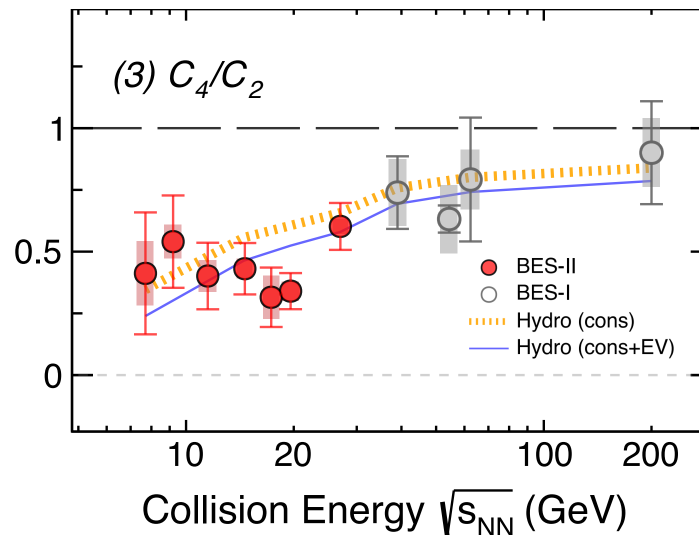
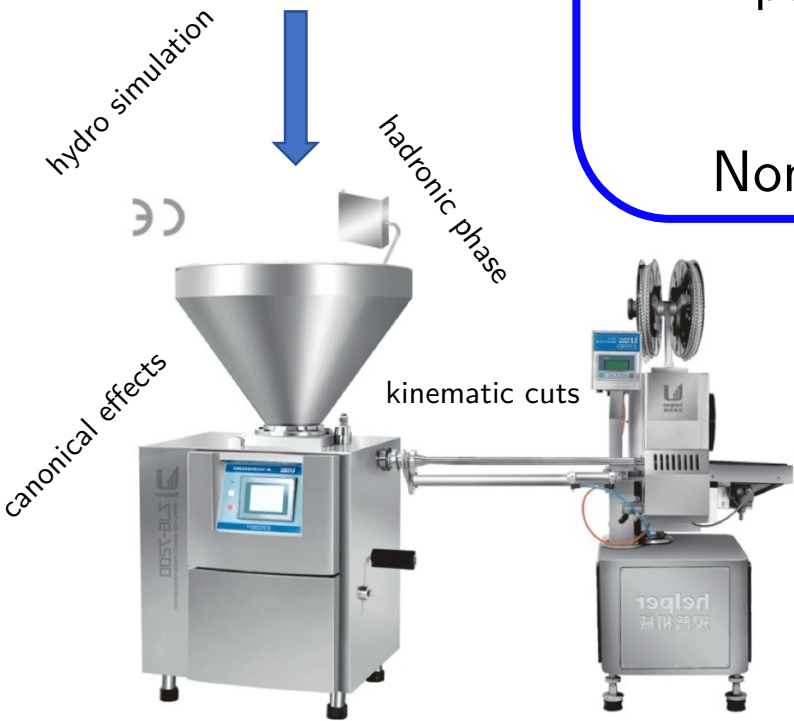
This was done in [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)]

- Full hydro simulation
- Lattice QCD-like baryon susceptibilities (interacting HRG)
- Global baryon conservation (SAM)
- Experimental kinematic cuts

Non-critical baseline (hydro EV) prediction



event selection
track reconstruction



CBWC