# Disentangling equation of state and canonical ensemble effects on proton fluctuations via a density correlations formalism

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*Thanks to:*

V. Koch, V.A. Kuznietsov, G. Pihan, R. Poberezhniuk

UNIVERSITY OF HOUSTON

APS Global Physics Summit

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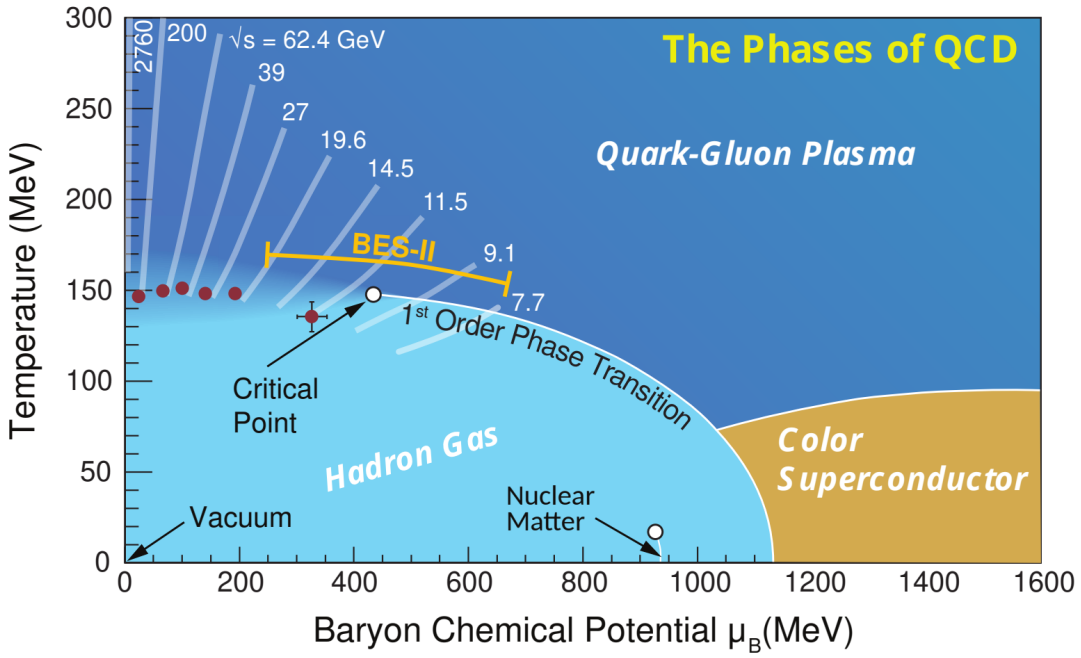
# QCD under extreme conditions




From 2015 US Nuclear Long Range Plan

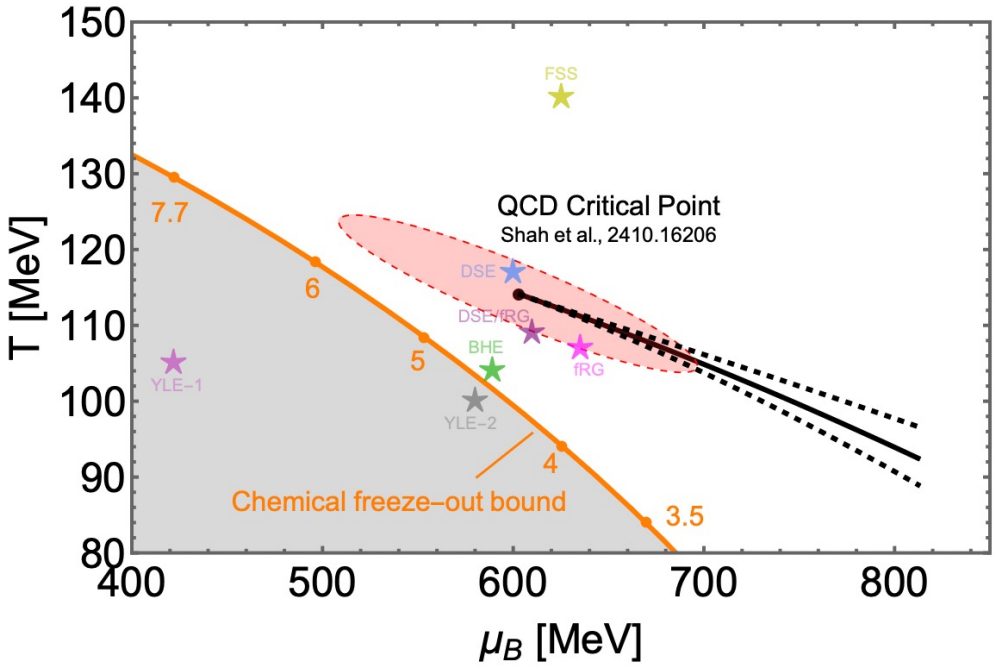
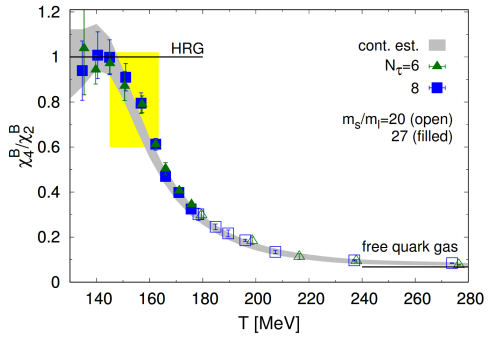



Figure from H. Shah et al., PRC 113, L012201 (2026)

- One of key questions: *Is there a QCD critical point and how to find it?*
- Several predictions place it in the vicinity of the freeze-out curve in heavy-ion collisions
- Probe with measurements of proton cumulants at RHIC

# Theory vs experiment

guidance from theory (e.g. lattice)

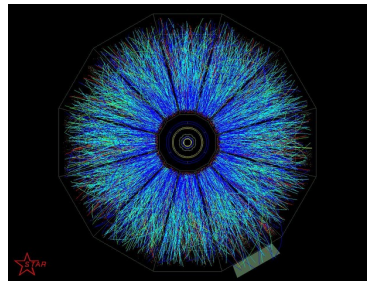



This was done in [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)]

- Full hydro simulation
- Lattice QCD-like baryon susceptibilities (interacting HRG)
- Experimental kinematic cuts
- Global baryon conservation (SAM) correction

Non-critical baseline (Hydro EV) **prediction**

experiment (the real thing)

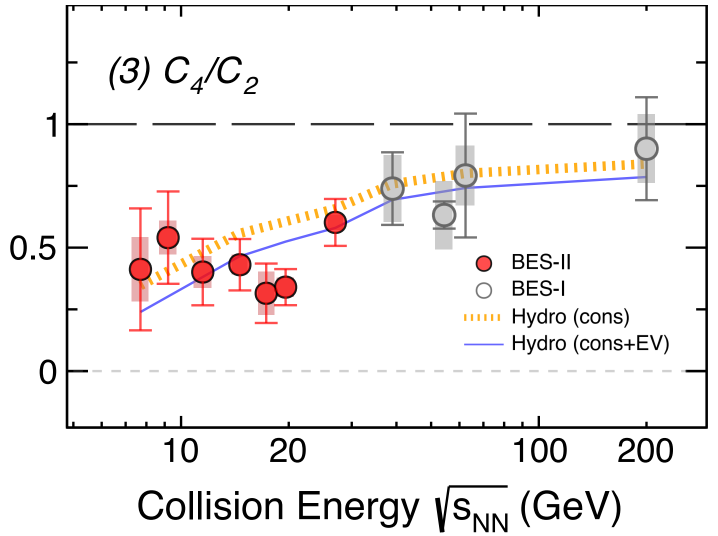


hydro simulation

hadronic phase

canonical effects

kinematic cuts



efficiency correction

CBWC

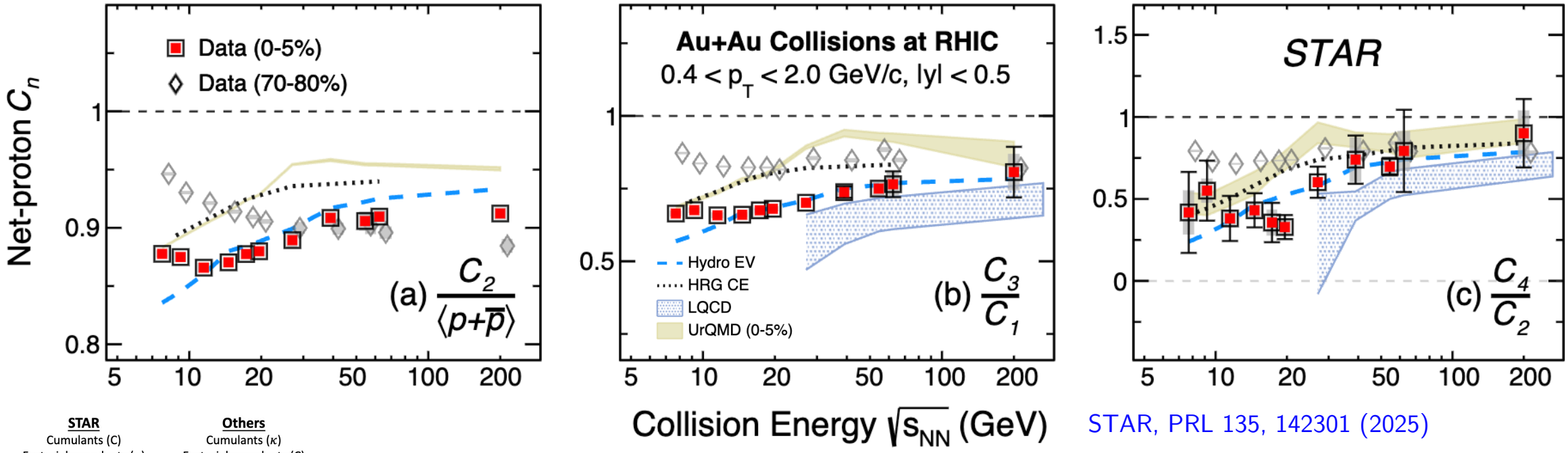
purity

event selection

track reconstruction

# RHIC-BES-II data: Ordinary cumulants

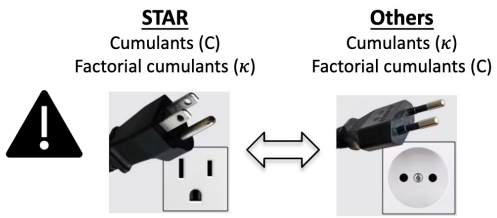
## Net-proton cumulant ratios




STAR, PRL 135, 142301 (2025)

Hydro EV: VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

HRG CE: Braun-Munzinger et al., Nucl. Phys. A 1008, 122141 (2021)




From M. Arslandok, QM2025

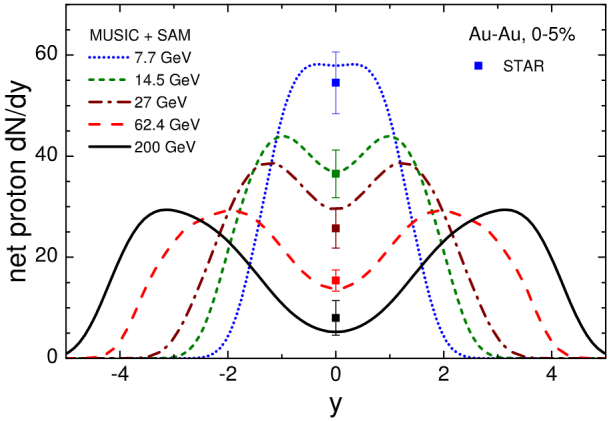
Fair agreement with the baseline above $\sqrt{s_{NN}} \sim 10 - 20$ GeV

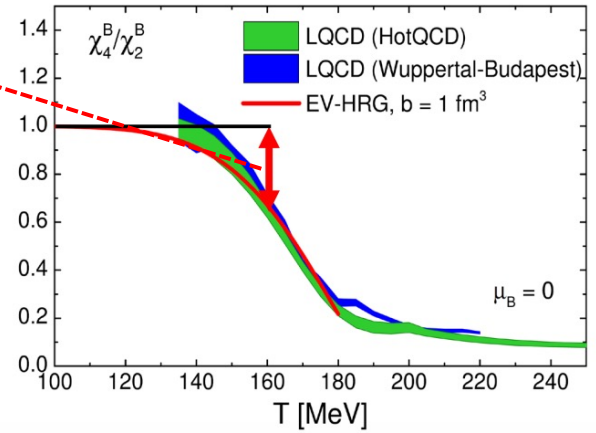
Can we learn more about the EoS?

# Hydro EV description

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
  - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
  - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]
- Non-critical contributions computed at particlization ($\epsilon_{sw} = 0.26$ GeV/fm$^3$)
  - Lattice QCD-like baryon number distribution ($\chi_n^B$) via **excluded volume** b = 1 fm$^3$ [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
  - Kinematic cuts via Cooper-Frye formula




- Exact global baryon conservation via correction step at the end
  - Subensemble acceptance method (SAM-2.0) (analytic) [VV, Phys. Rev. C 105, 014903 (2022)]
- Included: baryon conservation, repulsion, kinematical cuts
- However, effects mix together, are difficult to disentangle, and require complex numerical calculation each time

# Reevaluate using density-density correlator

Density-density correlator of net baryon number (uniform system) [VV, Phys. Rev. C 110, L061902 (2024)]

$$\mathcal{C}_2(\mathbf{r}_1, \mathbf{r}_2) = \chi_2 \delta(\mathbf{r}_1 - \mathbf{r}_2) - \frac{\chi_2}{V}$$

local fluctuations (EoS)

balancing contribution (e.g. baryon conservation)

Generalized: Non-conserved quantity $X$ (e.g. protons) correlated to B on Cooper-Frye surface

$$\mathcal{C}_2^X(x_1, x_2) = \chi_2^X(x_1)\delta(x_1 - x_2) - \frac{\chi_{11}^{XB}(x_1)\chi_{11}^{XB}(x_2)}{\kappa_2^{B,\mathrm{gce}}}$$

local fluctuations (self-correlations + two-particle)

balancing contribution (e.g. baryon conservation)

$$\kappa_2^{B,\mathrm{gce}} = \int_{\sigma(x)} dV(x)\chi_2^B(x)$$

Cooper-Frye (CF) integral

$$dV(x) = d\sigma_\mu(x)u^\mu(x)$$

CF volume element

## Ingredients and differences to Hydro-EV

- Baryon conservation (total net baryon number does not fluctuate) enforced straight away
- EoS effects encoded in $\chi_2$ (not restricted to excluded volume repulsion)
- Both effects clearly disentangled so far

# Proton variance

Consider fluctuations of positive baryons ($X = B^+$)

- Parametrize GCE baryon correlations by scaled variance $\omega_+$
- No correlations among baryons and antibaryons
- Neglect variations in $\omega_+$ along freeze-out hypersurface

$\omega_+ < 1$ **repulsion** $\quad$ $\omega_+ = 1$ **Poisson** $\quad$ $\omega_+ > 1$ **attraction**

$$\chi_2^X(\mathrm{x}) = \omega_+ \langle n_+ \rangle(\mathrm{x}) \qquad \chi_{11}^{X\mathrm{B}}(\mathrm{x}) = \omega_+ \langle n_+ \rangle(\mathrm{x}) \qquad \kappa_2^{B,\mathrm{gce}} = \omega_+ \langle N_+ \rangle + \omega_- \langle N_- \rangle$$

- Kinematic acceptance + efficiency factor $p(x)$: probability to detect baryon emitted from x

Density-density correlator becomes

$$\mathcal{C}_2^X(x_1, x_2) = p(x_1)\langle n_+ \rangle(x_1)\delta(x_1 - x_2) + [p(x_1)]^2(\omega_+ - 1)\langle n_+ \rangle(x_1)\delta(x_1 - x_2) - \frac{(\omega_+)^2 p(x_1)\langle n_+ \rangle(x_1) p(x_2)\langle n_+ \rangle(x_2)}{\omega_+ \langle N_+ \rangle + \omega_- \langle N_- \rangle}$$

**self-correlation (Poisson)** $\quad$ **local two-particle correlations** $\quad$ **balancing contribution from baryon conservation**

Integrating the correlator yields measurable proton variance

$$\frac{\kappa_2[p]}{\langle N_p \rangle} = 1 + (\omega_+ - 1)\frac{\langle p^2 \rangle_n}{\langle p \rangle_n} - (\omega_+)^2 \langle p \rangle_n \frac{\langle N_+ \rangle}{\omega_+ \langle N_+ \rangle + \omega_- \langle N_- \rangle}$$

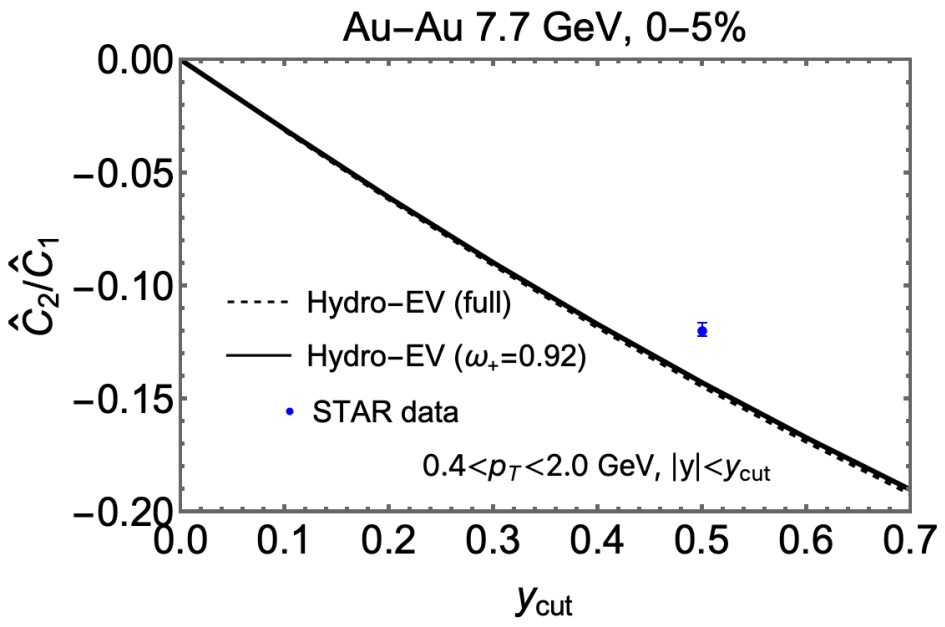
# Simplified proton variance

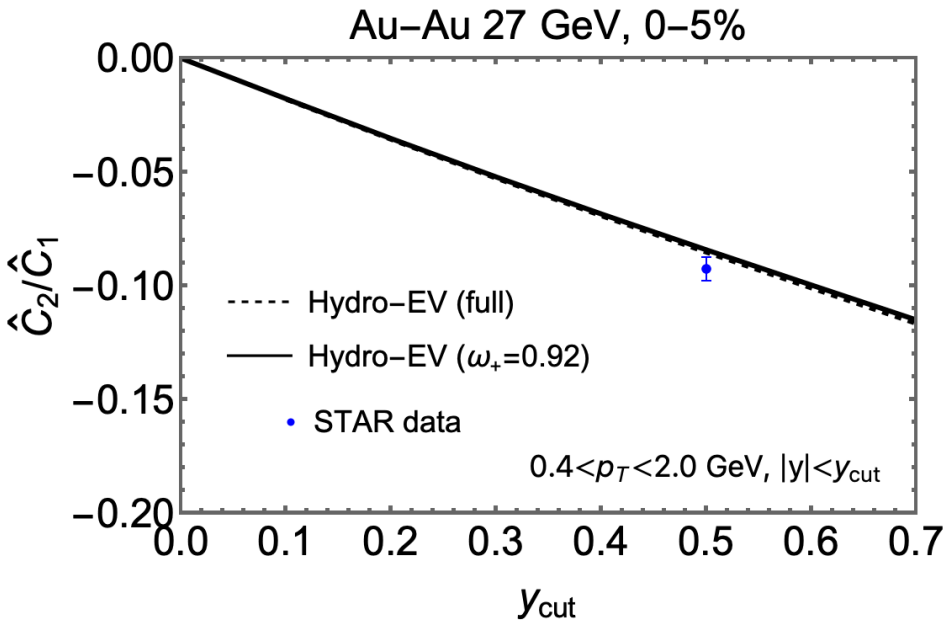
$$\frac{\kappa_2[p]}{\langle N_p \rangle} - 1 \approx (\omega_+ - 1)\frac{\langle p^2 \rangle_n}{\langle p \rangle_n} - \omega_+ \langle p \rangle_n \frac{\langle N_+ \rangle}{\langle N_+ \rangle + \langle N_- \rangle}$$

EoS

acceptance (hydro)

baryon (proton) fraction



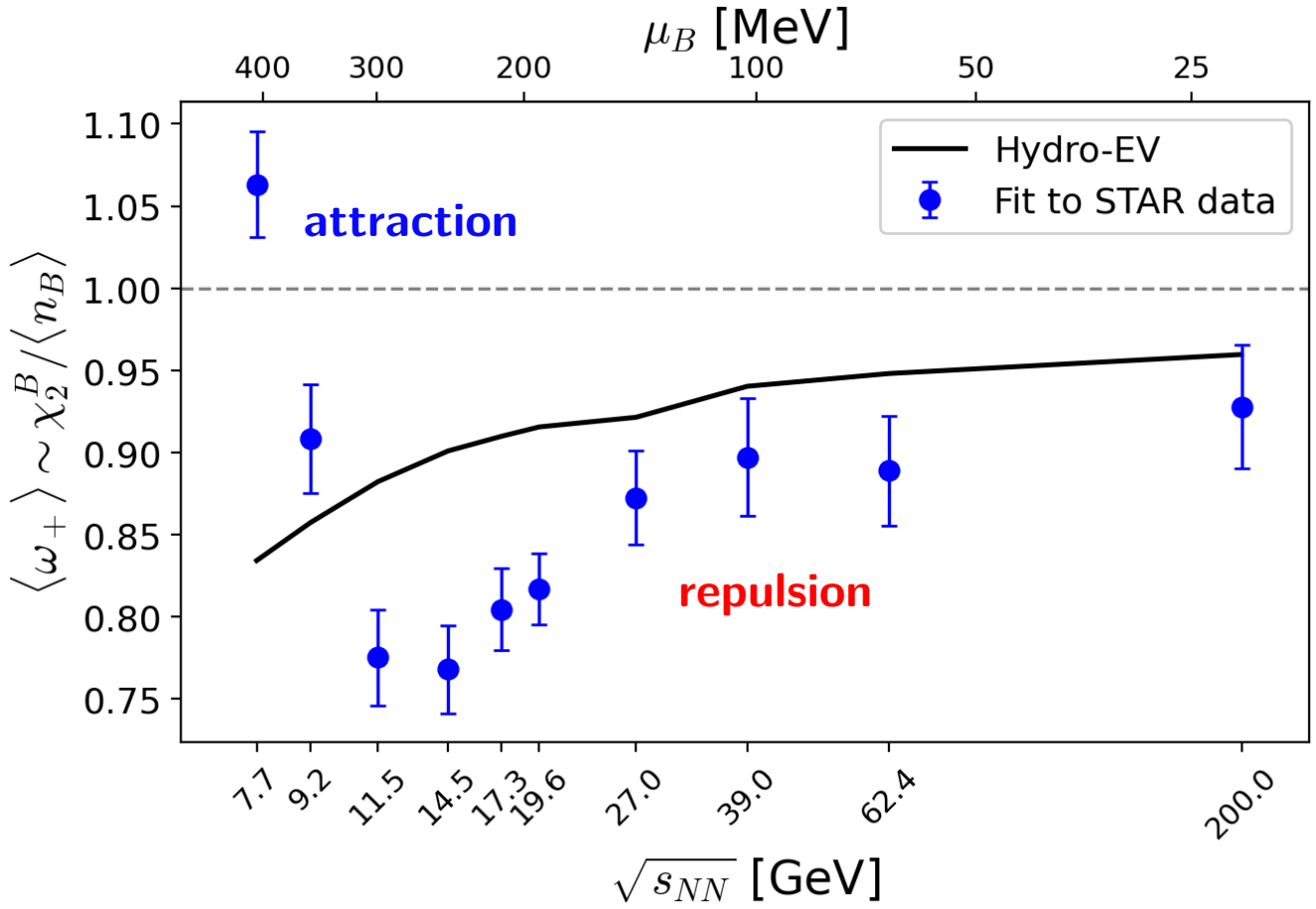




Simplified expression is in excellent agreement with full numerical Hydro-EV calculation

Now have freedom to vary $\omega_+$ and constrain using data

# Extracting scaled variance from data

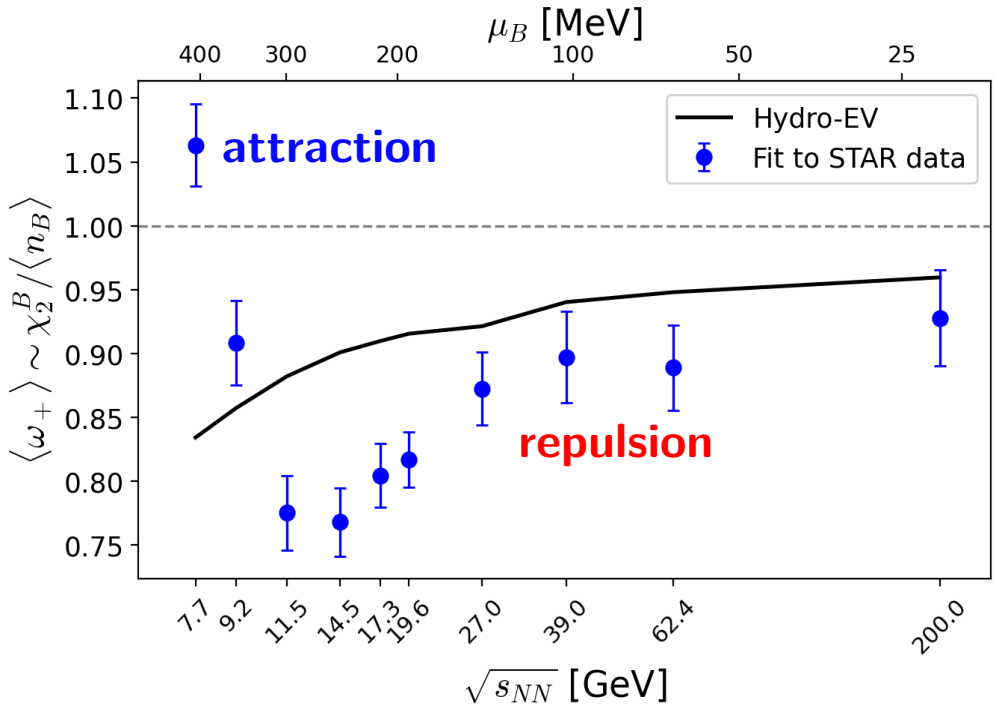
$\omega_+ > 1$
**attraction**

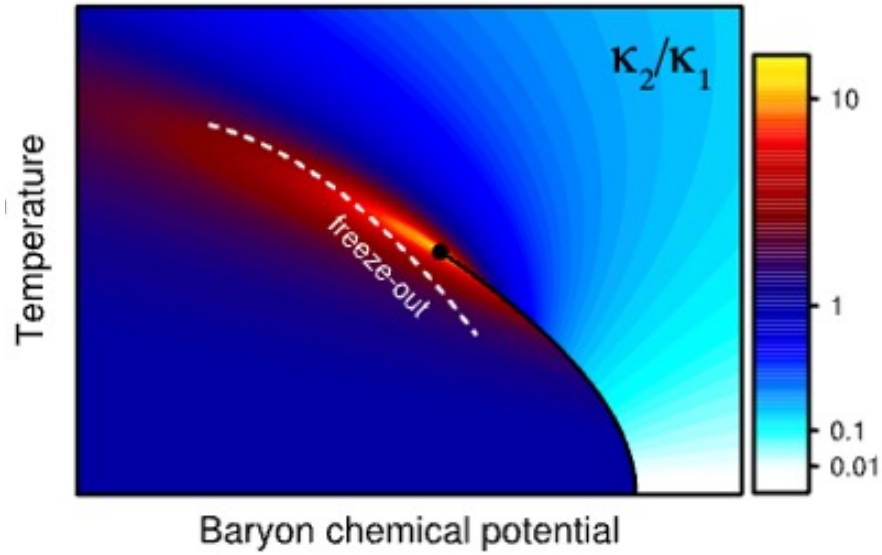



$\omega_+ < 1$
**repulsion**

Change of trend around 14 GeV from repulsion to attraction as $\sqrt{s_{\mathrm{NN}}}$ is decreased

# Attraction vs repulsion and the CP




VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Interplay of attraction and repulsion is typical for systems with a **critical point**

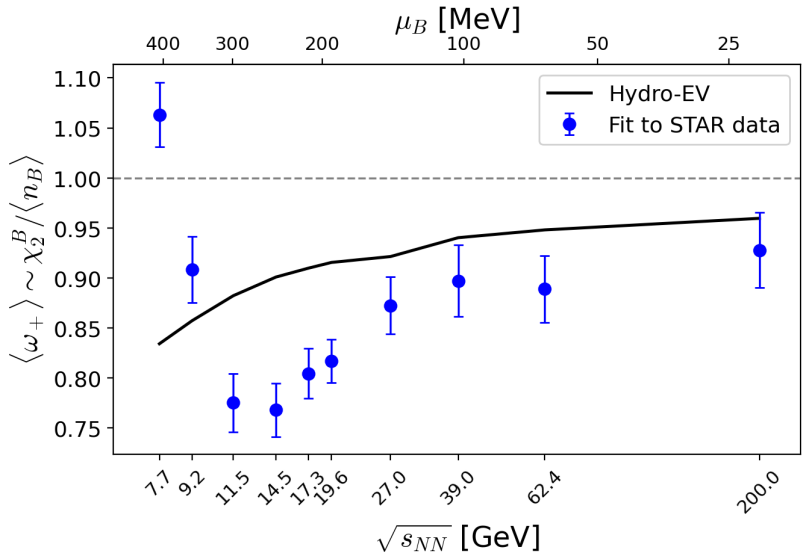
⚠ **Caveats:**

- Local conservation* not included → would require more attraction/less repulsion
- Neglects volume fluctuations (STAR data are corrected with CBWC**)
- No hadronic phase; antiproton puzzle*** not resolved
- Assumes hydro validity; additional tests with acceptance dependence of cumulants***

*Parra et al., PRL 135, 242302 (2025); **Friman, Koch, arXiv:2511.11869; ***Bzdak, Koch, VV, PRC 112, 024901 (2025)

# Summary

- Proton cumulants in heavy-ion collisions are complex observables sensitive to underlying QCD dynamics
- Density correlations approach simplifies hydrodynamical description of fluctuations and disentangles EoS from kinematical and conservation effects
- Analysis of RHIC-BES-II data indicate a switch from baryon repulsion to attraction around $\sqrt{s_{\mathrm{NN}}}$~15 GeV




## Outlook:

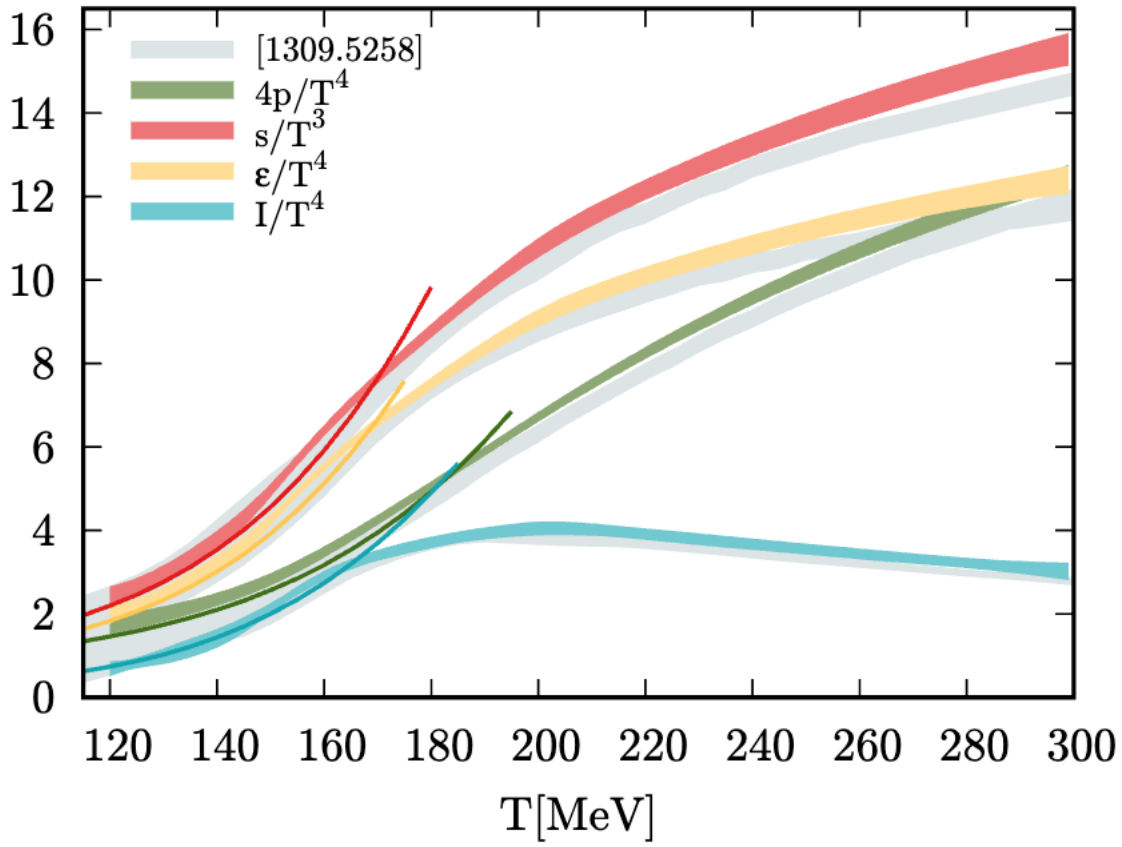
- High-order (factorial) cumulants
- Acceptance dependence and other observables
- Antiprotons

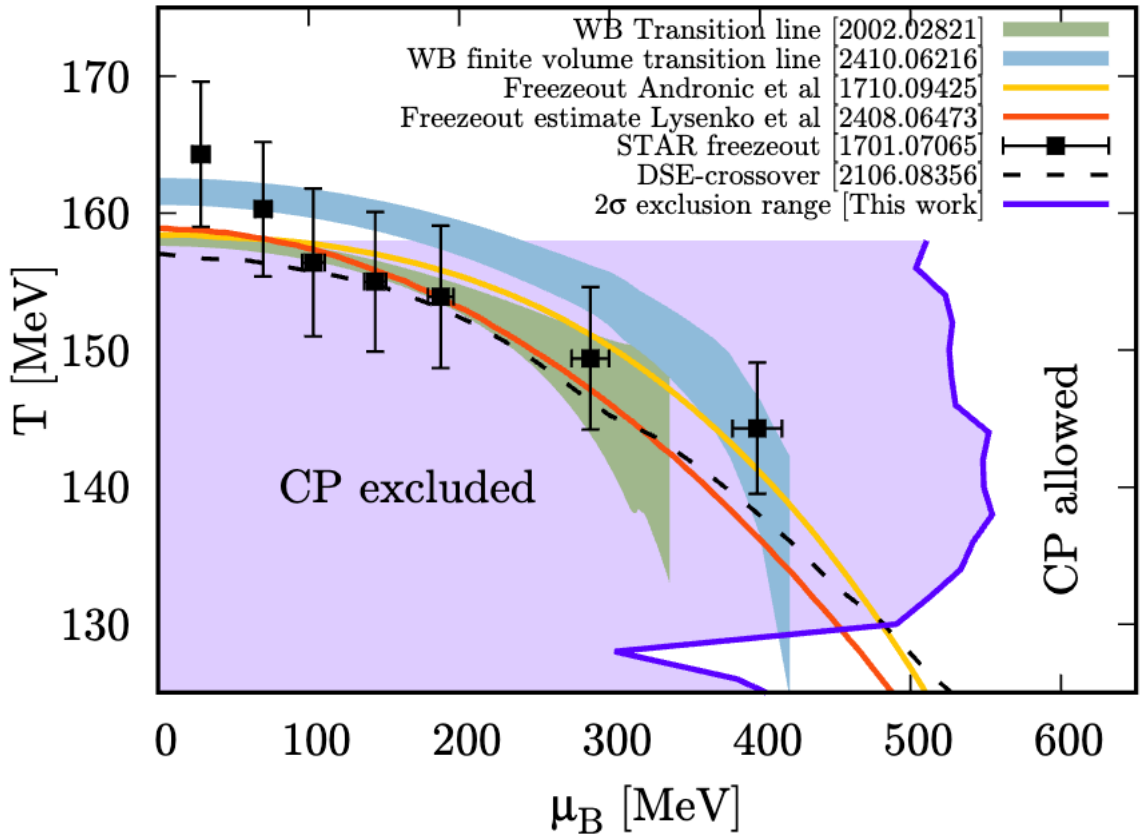
$$\frac{\kappa_2[p]}{\langle N_p \rangle} - 1 \approx (\omega_+ - 1)\frac{\langle p^2 \rangle_n}{\langle p \rangle_n} - \omega_+ \langle p \rangle_n \frac{\langle N_+ \rangle}{\langle N_+ \rangle + \langle N_- \rangle}$$

**Thanks for your attention!**

# Backup slides

# New constraints from lattice QCD



# Event-by-event fluctuations and statistical mechanics

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*Cumulant generating function*

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

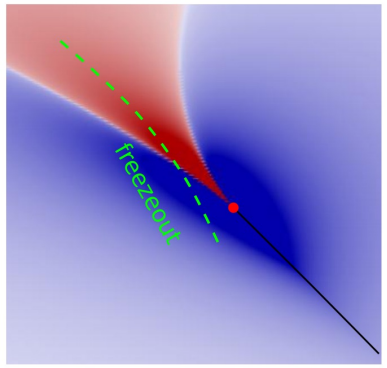
$$\kappa_n \propto \frac{\partial^n(\ln Z^{\mathrm{gce}})}{\partial \mu^n}$$

*Grand partition function*

$$\ln Z^{\mathrm{gce}}(T, V, \mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\mathrm{ce}}(T, V, N)\right]$$

*Cumulants measure chemical potential derivatives of the (QCD) equation of state*

- **(QCD) critical point:** large correlation length and fluctuations




M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR) and CERN-SPS (NA61/SHINE)

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

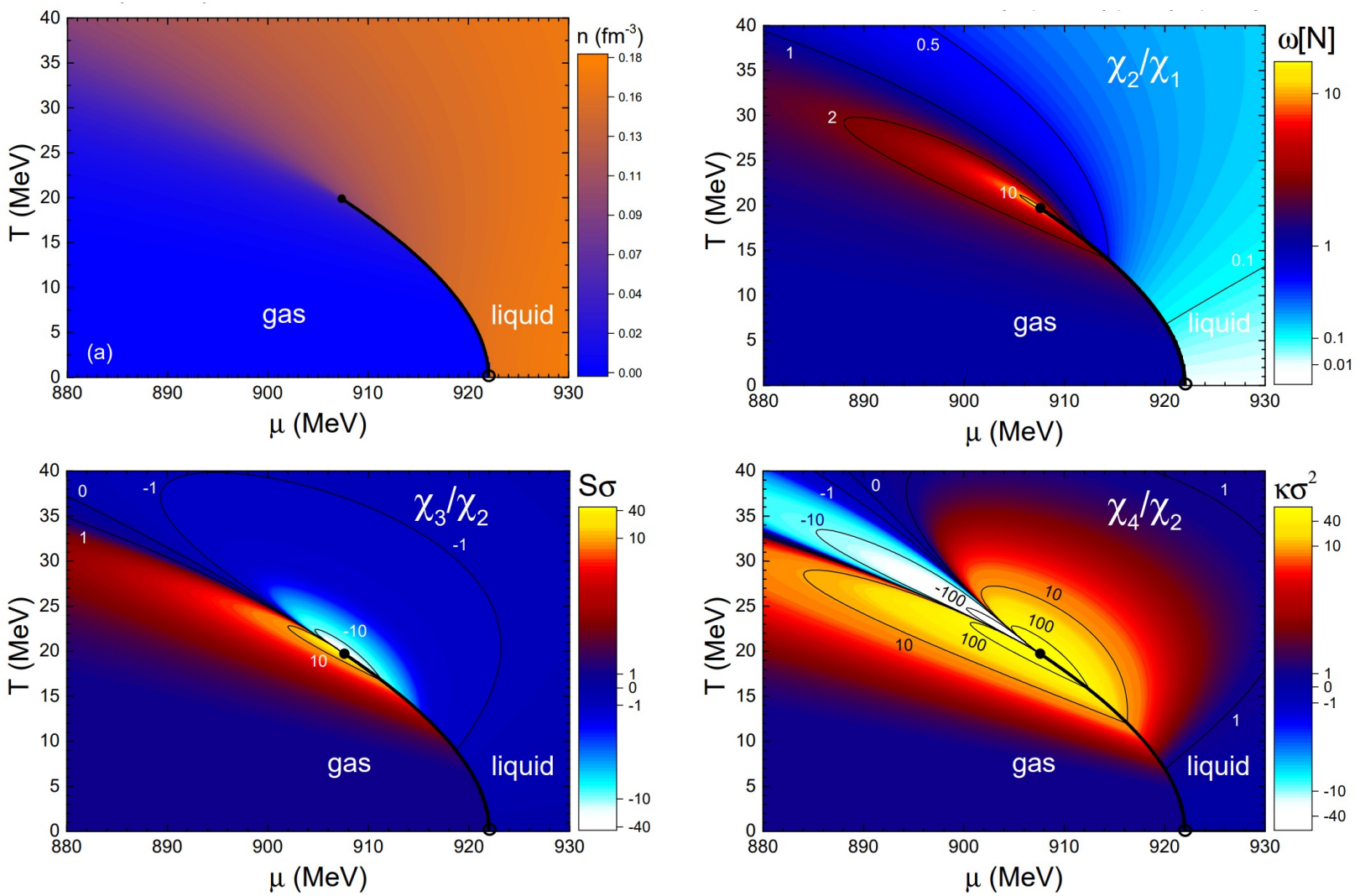
$$\xi \to \infty$$

Looking for enhanced fluctuations and non-monotonicities

**Other uses of cumulants:**

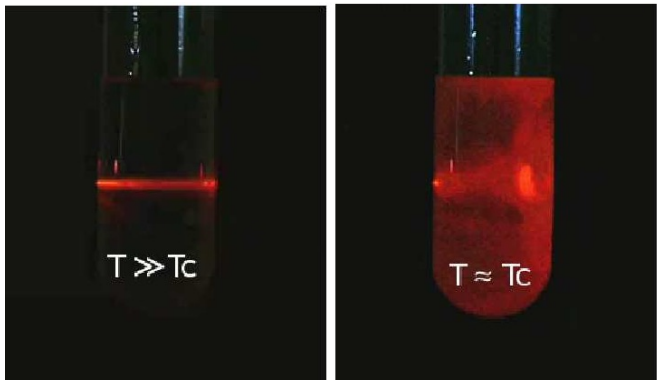
- **QCD degrees of freedom**
  Jeon, Koch, PRL 85, 2076 (2000)
  Asakawa, Heinz, Muller, PRL 85, 2072 (2000)
  J. Parra et al., PRL 135, 242302 (2025)
- Extracting the speed of sound
  A. Sorensen et al., PRL 127, 042303 (2021)
- Conservation volume $V_C$
  VV, Donigus, Stoecker, PRC 100, 054906 (2019)

# Example: (Nuclear) Liquid-gas transition




VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

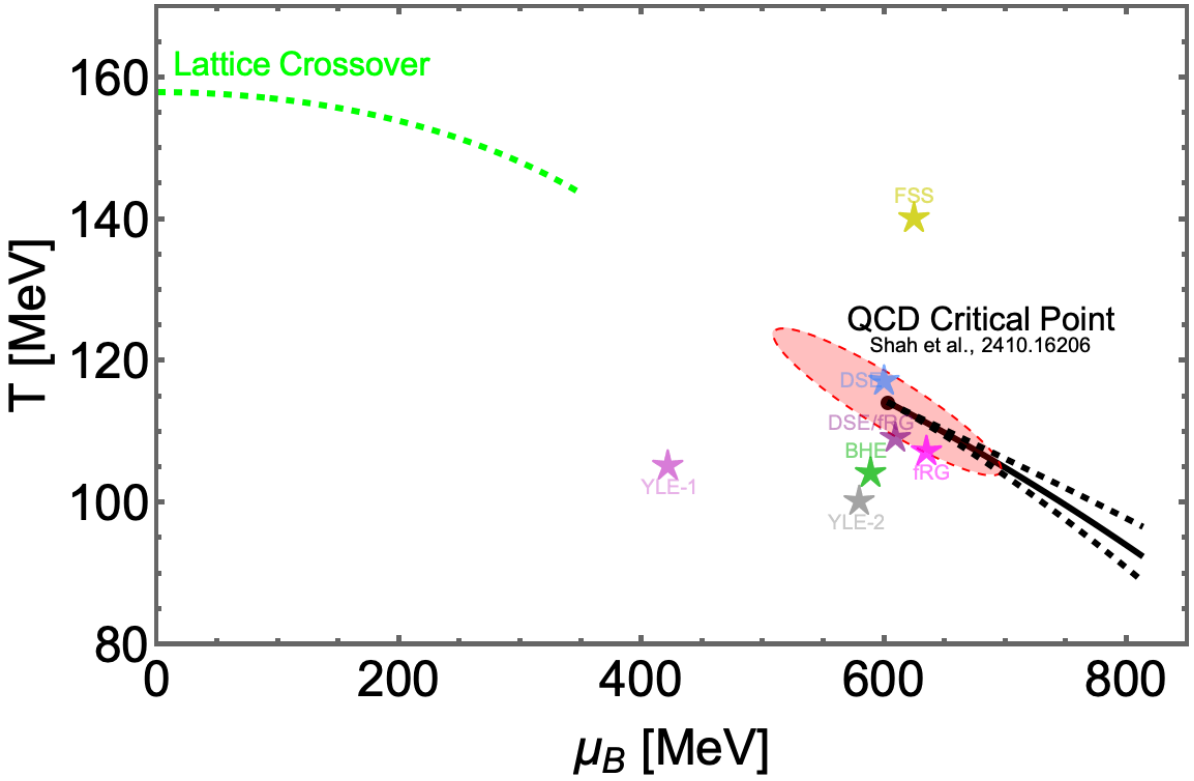
**Critical opalescence**




$\langle N^2 \rangle - \langle N \rangle^2 \sim \langle N \rangle \sim 10^{23}$

in equilibrium

# Critical point estimates




Critical point estimate at $O(\mu_B^2)$:

$T_c = 114 \pm 7$ MeV, $\mu_B = 602 \pm 62$ MeV

**Estimates from recent literature:**

YLE-1: D.A. Clarke et al. (Bielefeld-Parma), arXiv:2405.10196

YLE-2: G. Basar, PRC 110, 015203 (2024)

BHE: M. Hippert et al., arXiv:2309.00579

fRG: W-J. Fu et al., PRD 101, 054032 (2020)

DSE/fRG: Gao, Pawlowski., PLB 820, 136584 (2021)

DSE: P.J. Gunkel et al., PRD 104, 052022 (2021)

FSS: A. Sorensen et al., arXiv:2405.10278

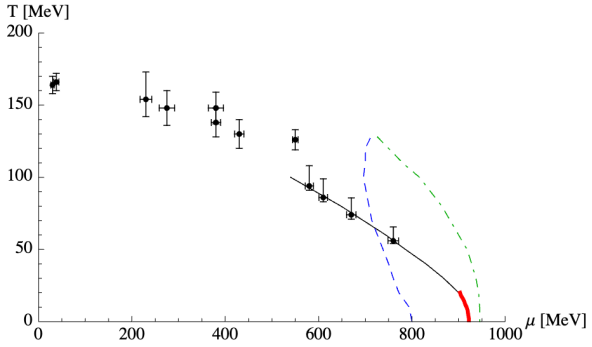
**Optimist's view:** Different estimates converge onto the same region because QCD CP is likely there

**Pessimist's view:** Different estimates converge onto the same region because it's the closest not yet ruled out by LQCD

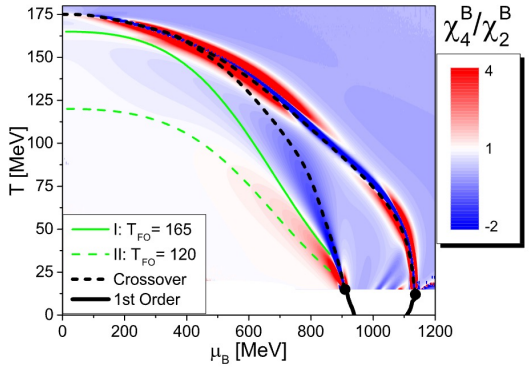
"...experimental measurements are essential to determine whether a QCD critical point exists."

**2023 Long Range Plan for Nuclear Science**

# Interplay with nuclear liquid-gas transition

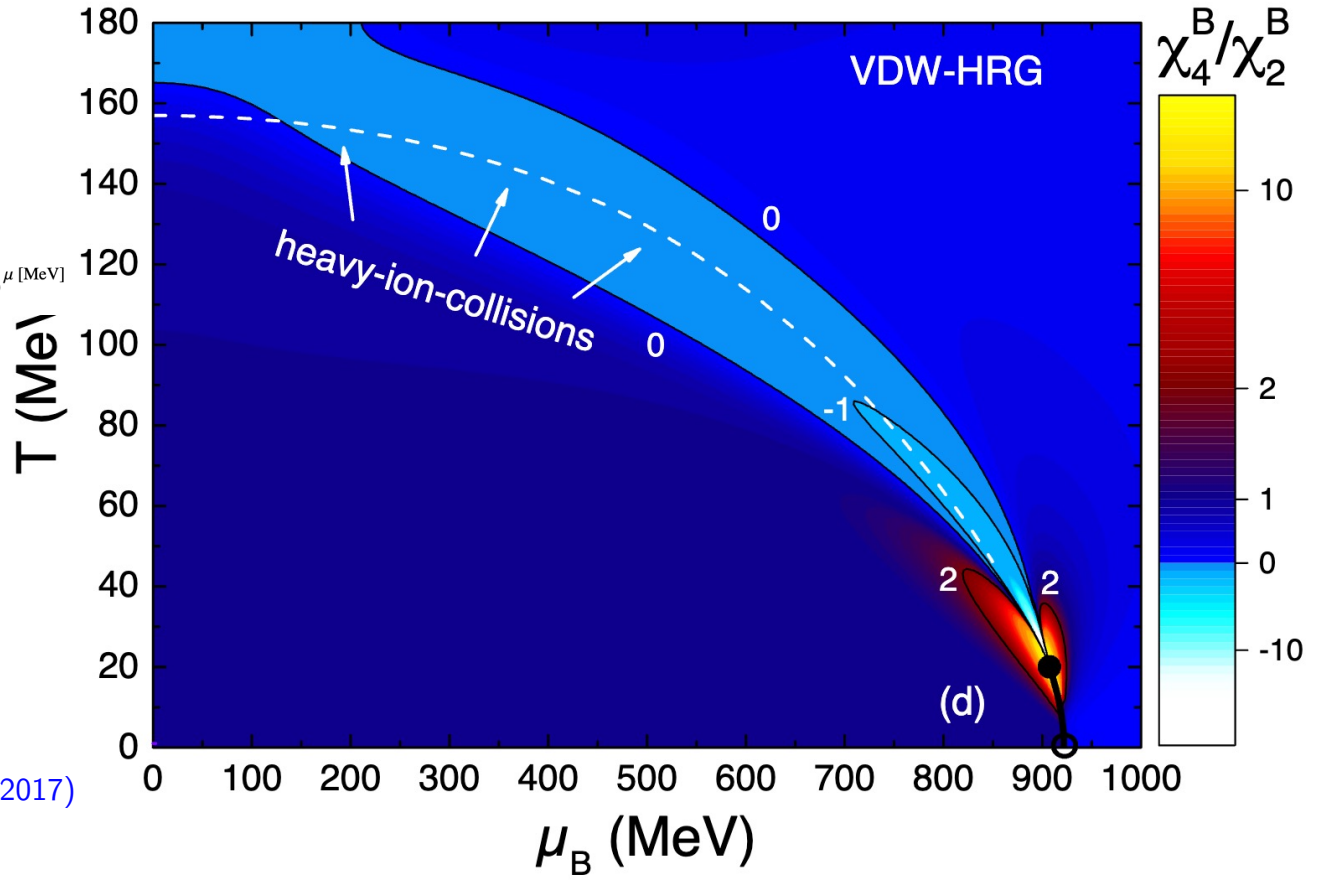



Floerchinger, Wetterich, NPA (2012)

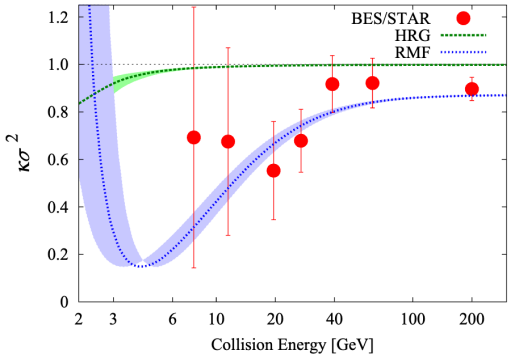



Mukherjee, Steinheimer, Schramm, PRC (2017)

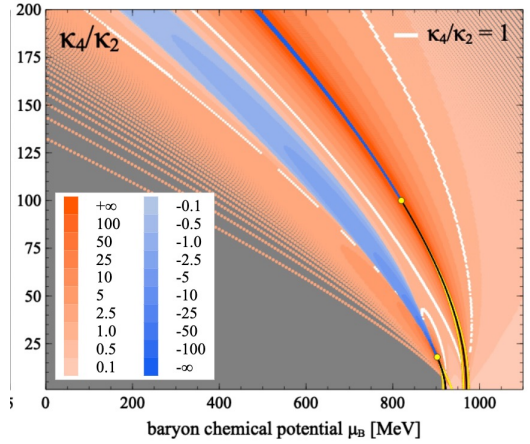
HRG with attractive and repulsive interactions among baryons




VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)




Fukushima, PRC (2014)




Sorensen, Koch, PRC (2020)

Increasingly relevant at lower energies probed through RHIC-FXT

# Hints from RHIC-BES-I

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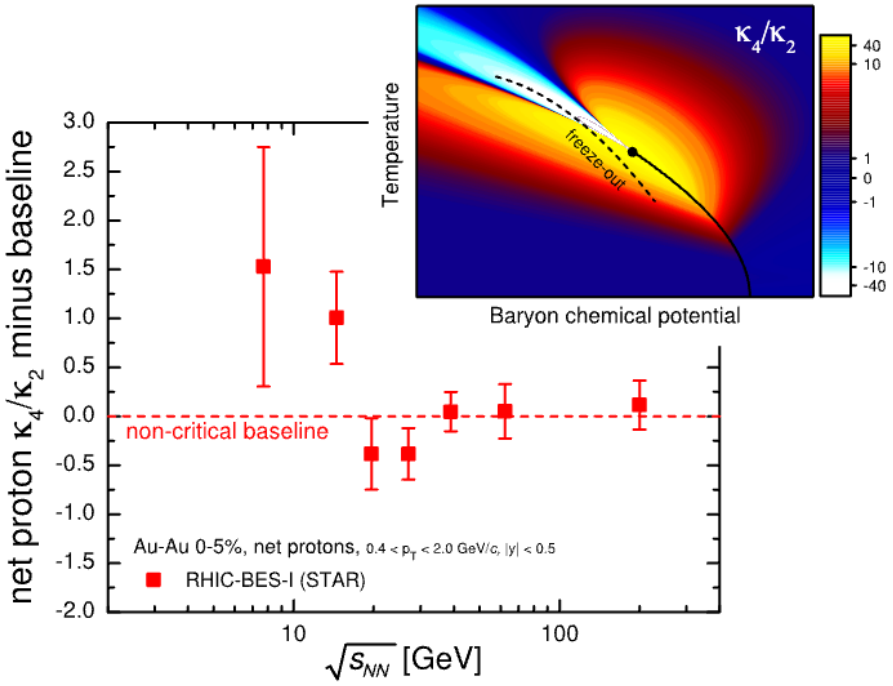
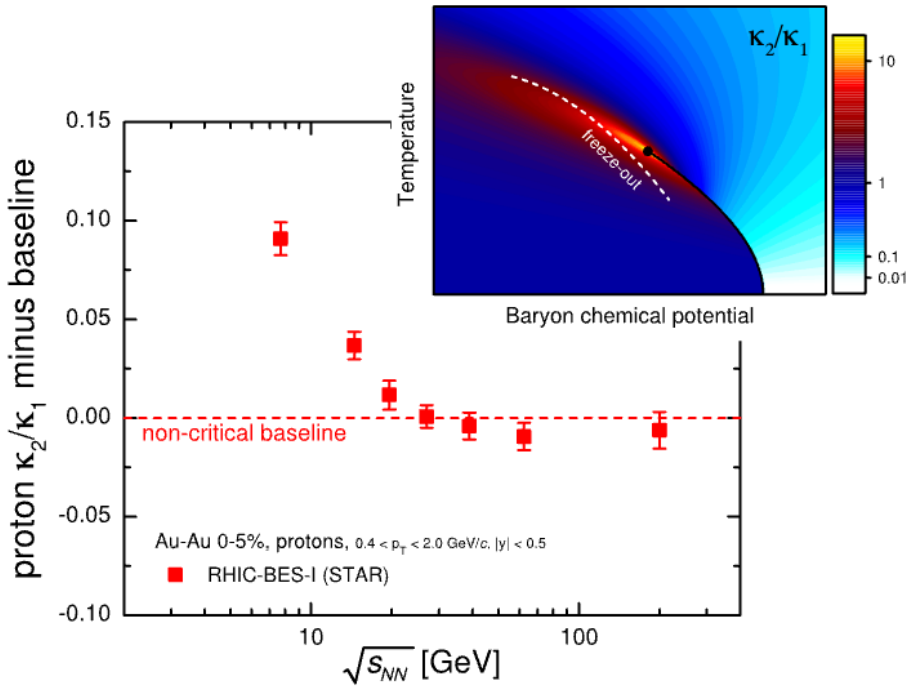
VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Subtracting the hydrodynamic non-critical baseline

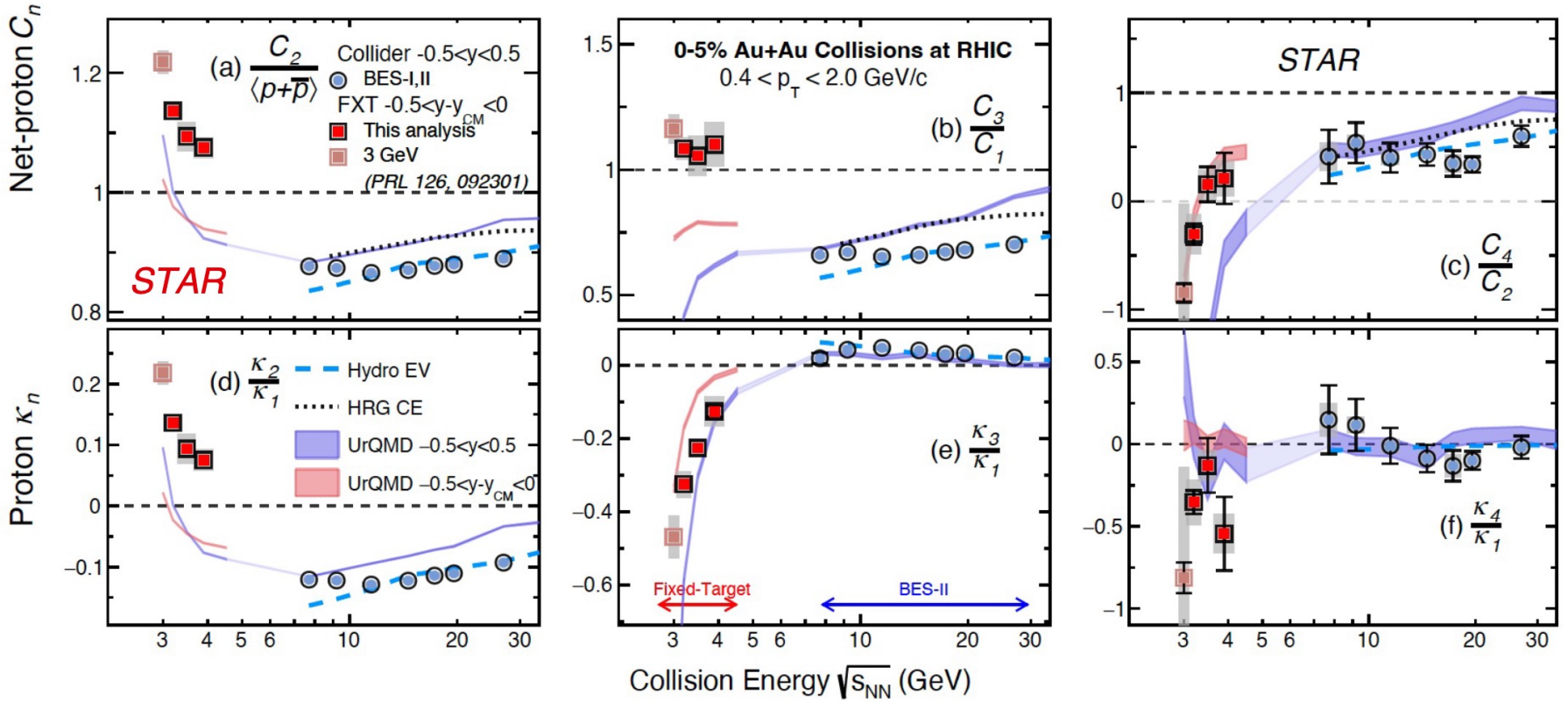



⚠ **Notation:** Here we use $\kappa_n$ for cumulants and $\hat{C}_n$ for factorial cumulants, STAR Collaboration uses the opposite

# RHIC-BES-II data including STAR-FXT

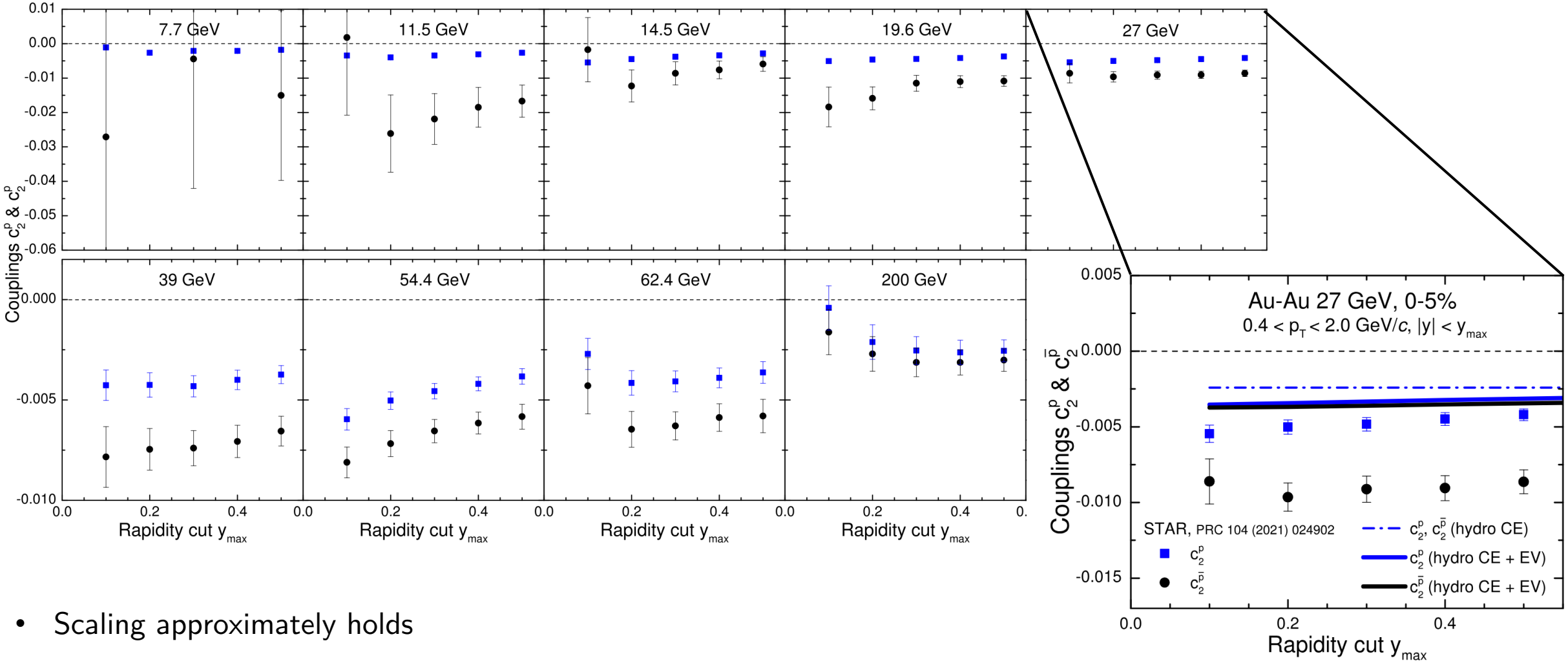
Z. Sweger (STAR), QM2025




- Continues the trends seen at lowest collider energies, in a fairly dramatic fashion
- **UrQMD (cascade) describes reasonably well the qualitative features**
  - Dominance of non-critical effects (centrality selection and spectators)?

# Scaled factorial cumulants from RHIC-BES-I

A. Bzdak, V. Koch, VV, arXiv:2503.16405



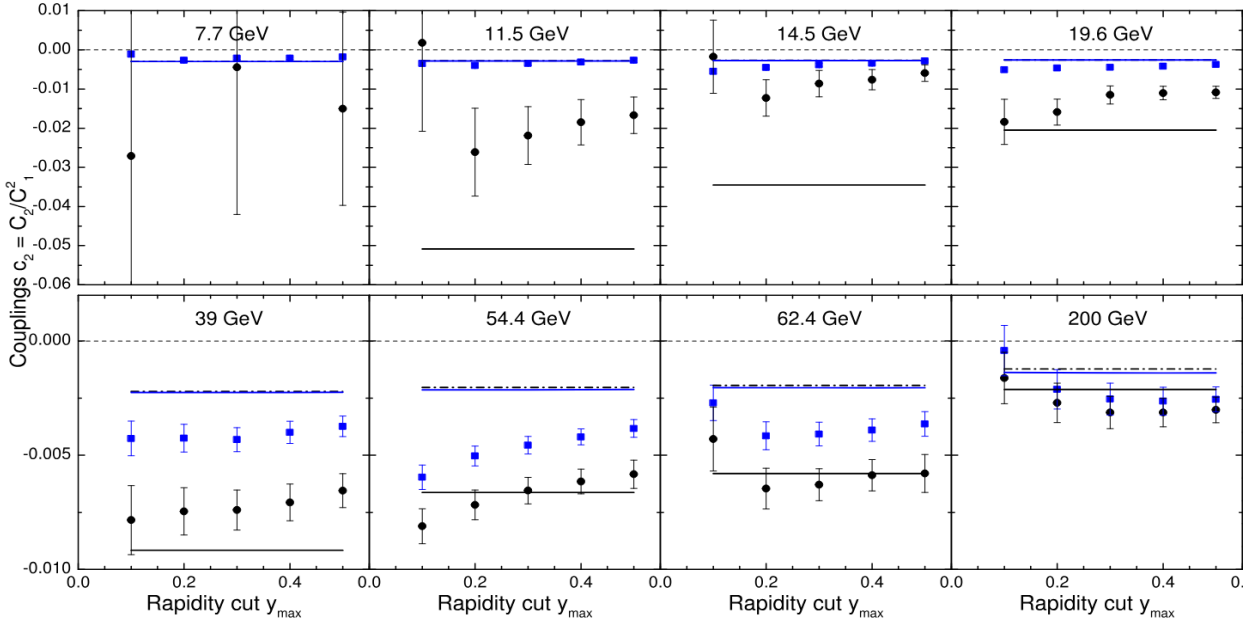

- Scaling approximately holds
- But significant difference between $p$ and $\bar{p}$ in BES-I and hydro fails – **the antiproton puzzle**

no single thermalized fireball?

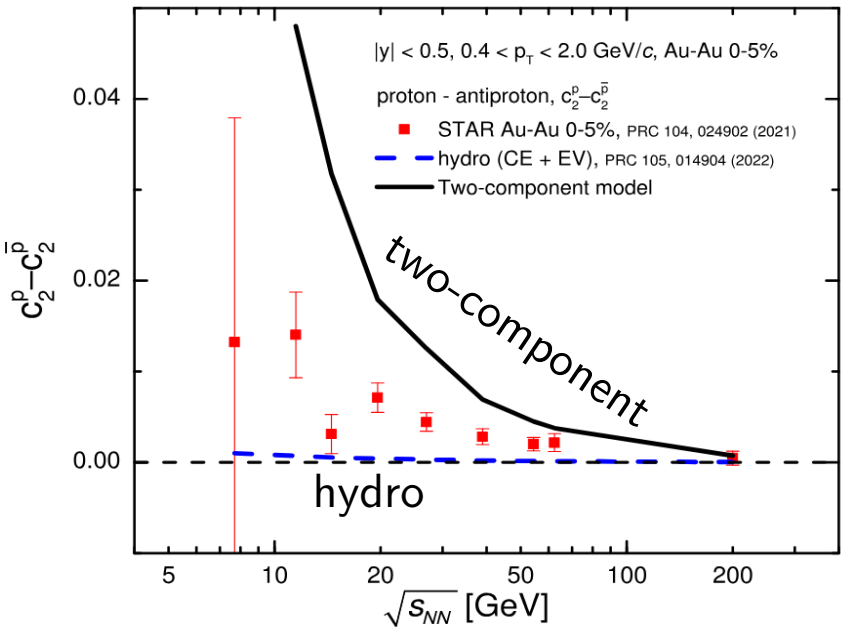
# The antiproton puzzle and the two-component model

**Two-component model:** produced ($p\bar{p}$ pairs) and stopped protons comprise from two independent sources

*The data lie in-between single and two-fireball models*




*Difference between $p$ and $\bar{p}$*




A. Bzdak, V. Koch, VV, arXiv:2503.16405

**Opportunities for BES-II:**

- Further tests of the splitting between $p$ and $\bar{p}$ in 2nd order cumulants with extended $y$ coverage
- Critical point signal expected to break the scaling

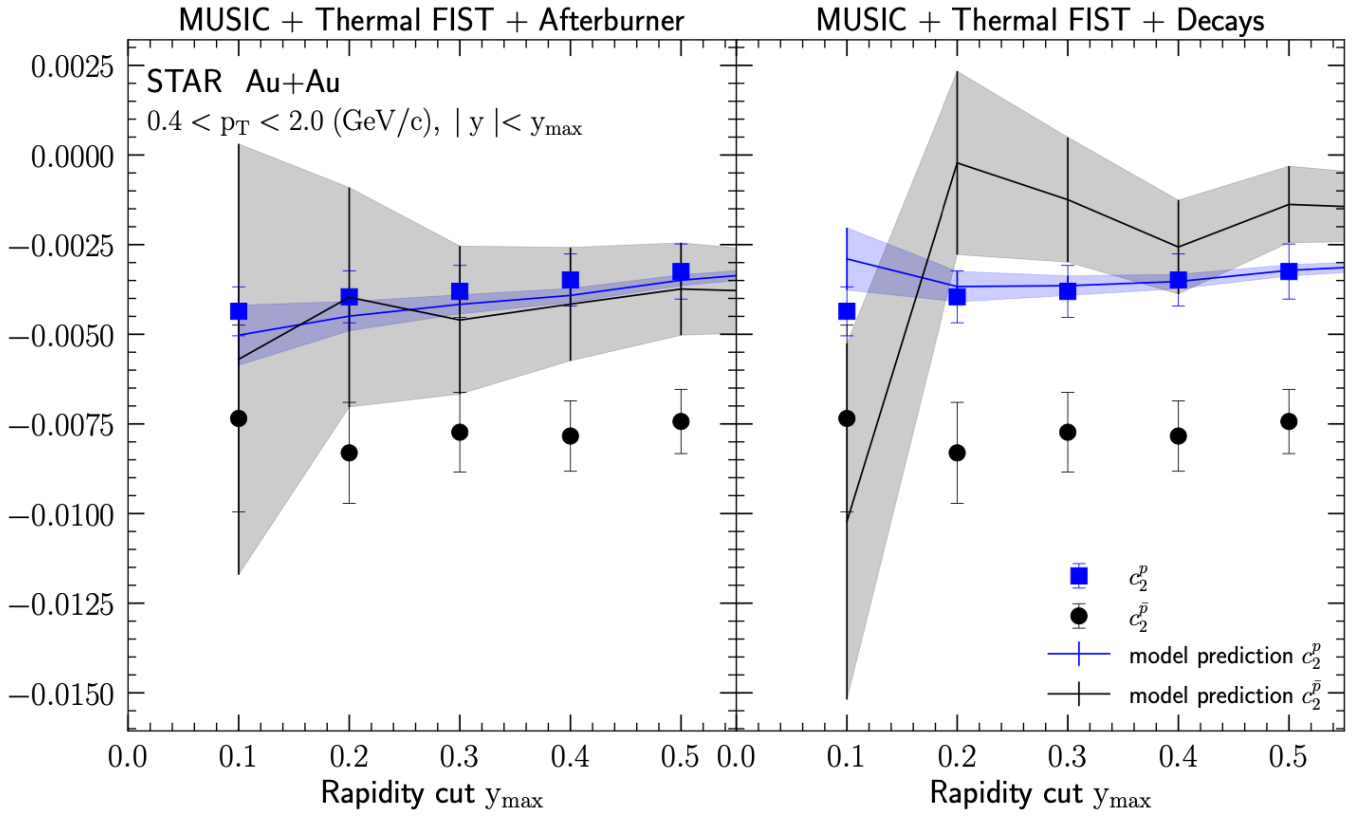
$$\frac{\hat{C}_n}{(\hat{C}_1)^n} = \text{const.}$$

[Ling. Stephanov, PRC 93, 034915 (2016)]

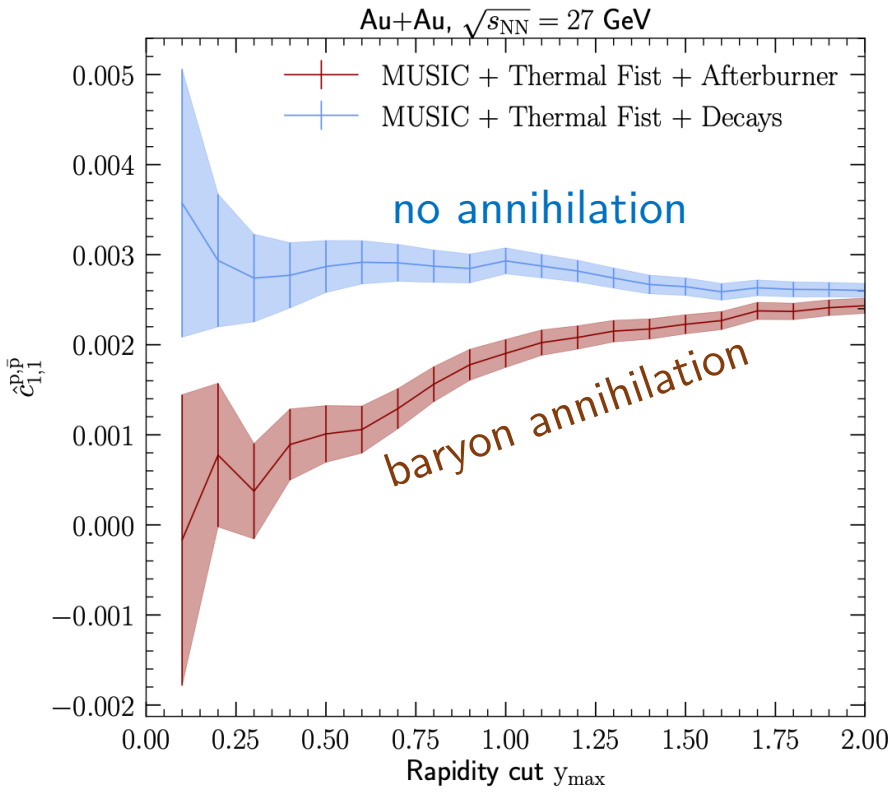
# Scaled factorial cumulants and baryon annihilation

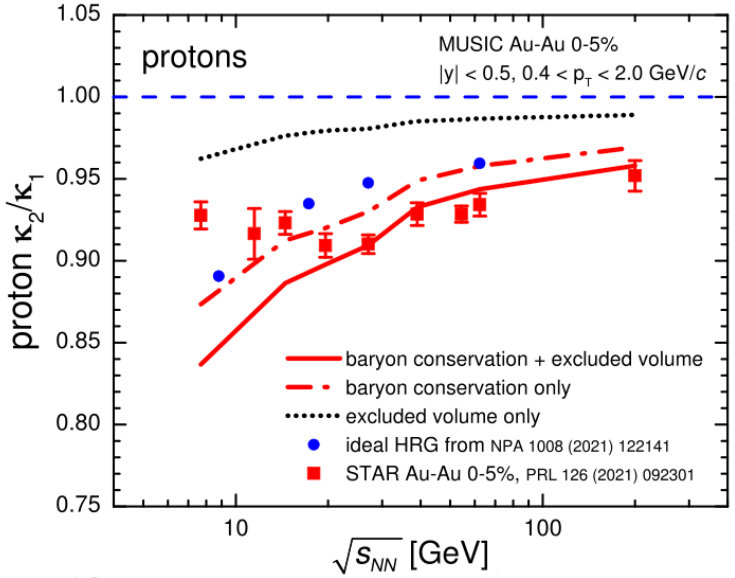
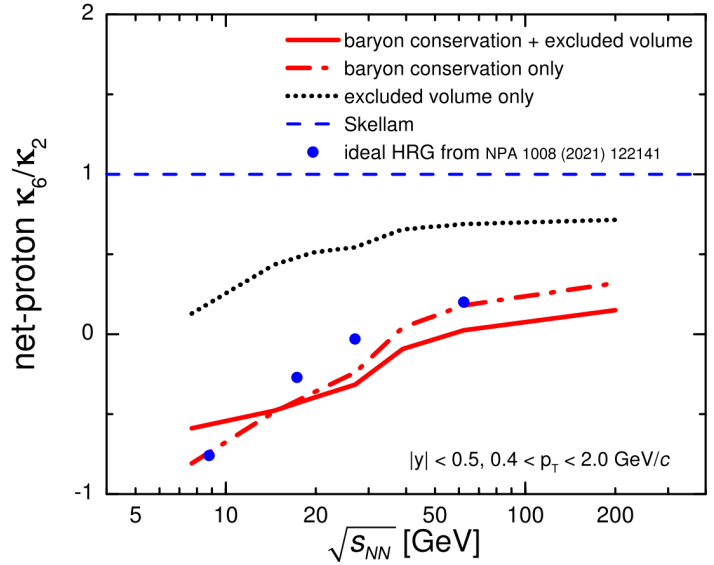
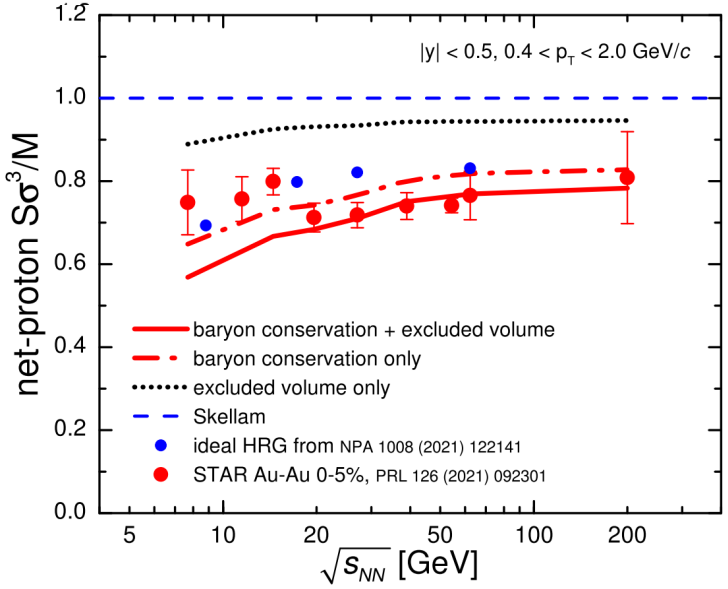
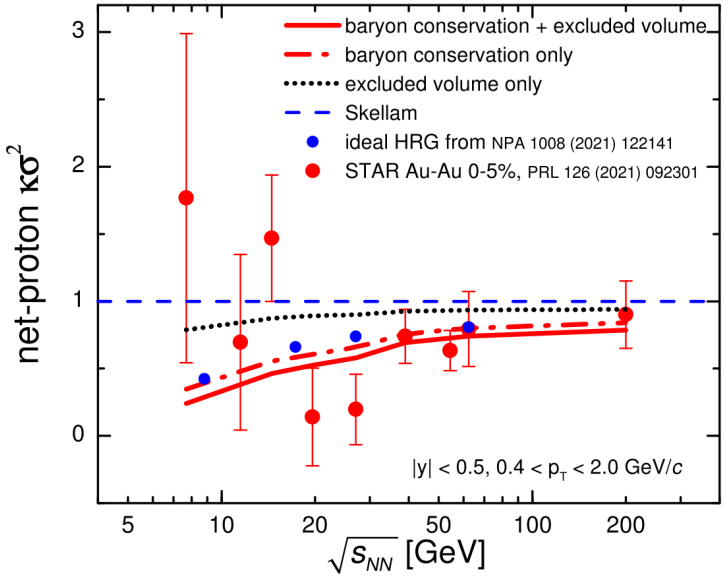
Extending Hydro EV to incorporate hadronic phase (UrQMD)

**Au-Au, $\sqrt{s_{NN}} = 27$ GeV** G. Pihan, VV, in preparation




Covariance $c_{11}^{p\bar{p}}$




- Hadronic phase appears unlikely to resolve the antiproton puzzle (more statistics needed)
- Acceptance dependence of proton-antiproton covariance shows clear signature of hadronic phase

# Non-critical cumulants