

Hadron resonance gas with density-dependent interactions for neutron stars and heavy-ion collisions

Volodymyr Vovchenko

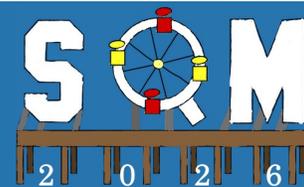
SQM 2026 - The 22nd International Conference on Strangeness in Quark Matter

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HRG model on the phase diagram

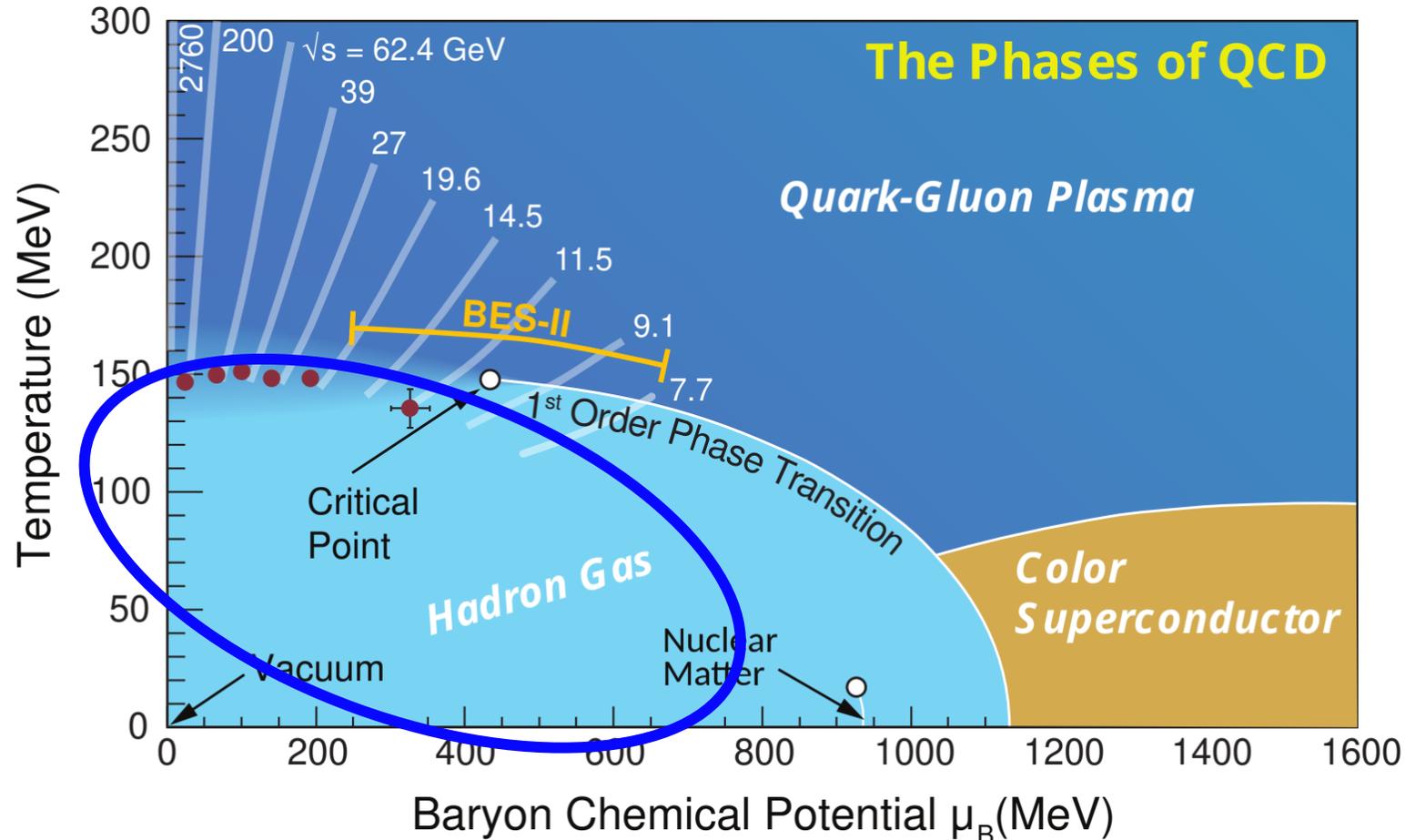


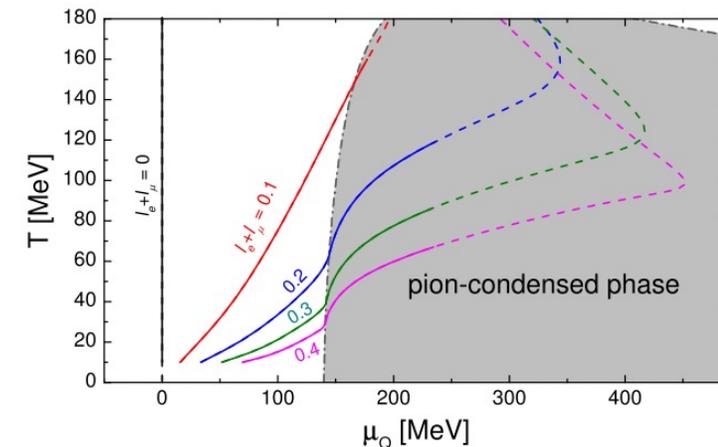
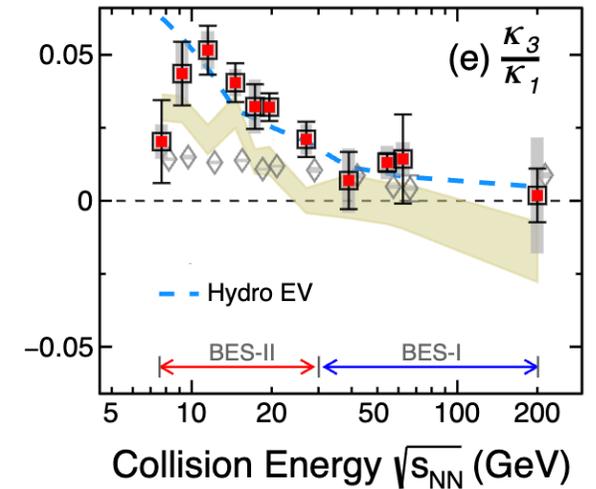
Figure from Bzdak et al., Phys. Rept. '20 & 2015 US Nuclear Long Range Plan

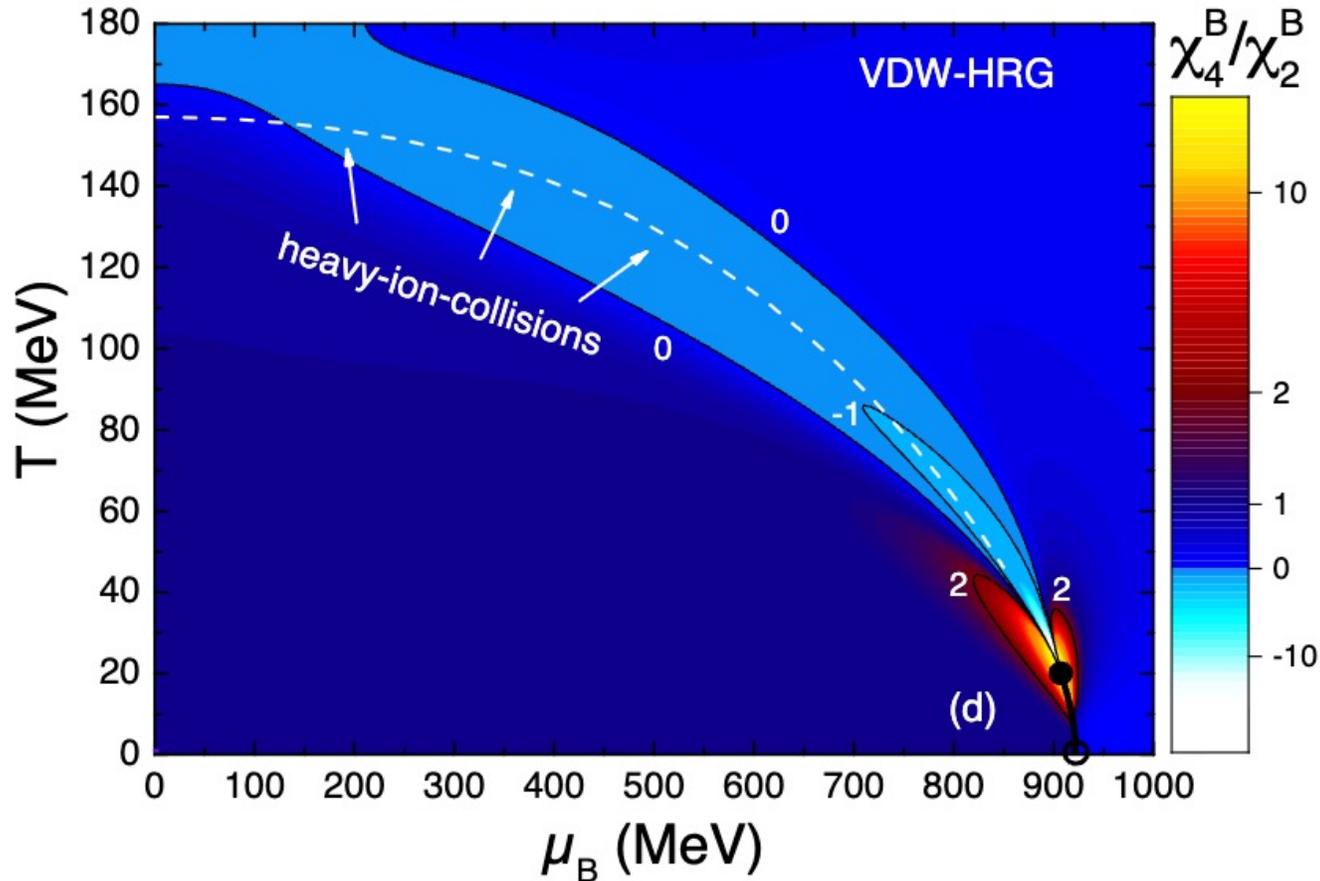
Traditional HRG territory: finite-temperature hadronic matter, chemical freeze-out in heavy-ion collisions

This talk: interacting HRG for both hot and cold/dense matter, and neutron stars

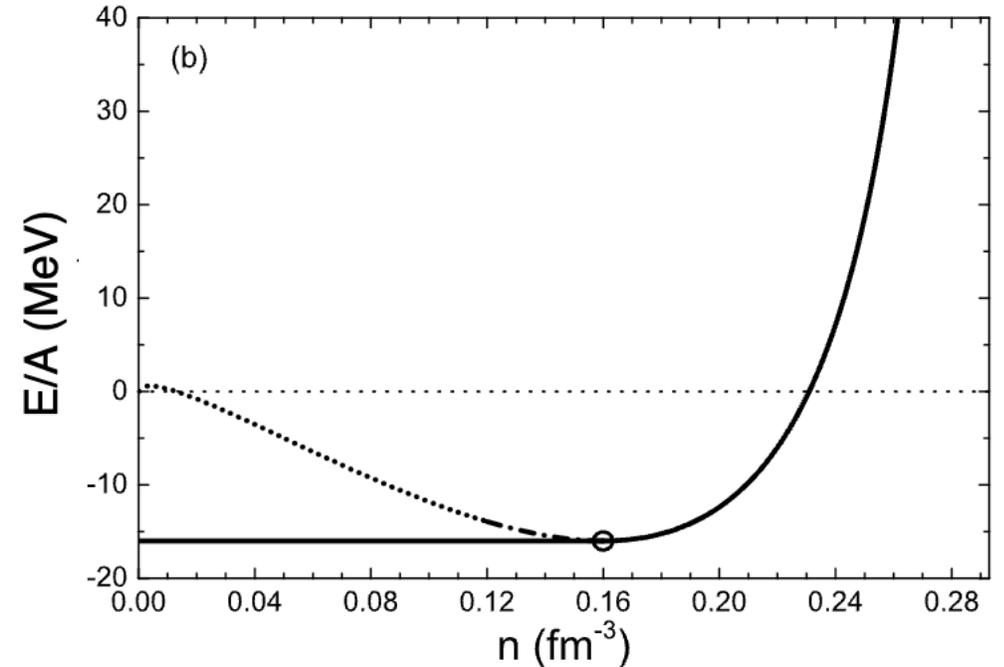
Non-resonant interactions in HRG

- Excluded volume effect [Yen, Gorenstein, Greiner, Yang, PRC 56, 2210 (1997)]
 - Lattice susceptibilities [Karthein et al., PRD 104, 094009 (2021)]
 - Proton cumulants [VV, Koch, Shen, PRC 105, 014904 (2022)]
 - Merging with high-T EoS [Albright, Kapusta, Young, PRC 90, 024915 (2014)]
- (Repulsive) mean field [Huovinen, Petreczky, PLB 77, 125 (2018)]
- S-Matrix approach [Andronic et al., PLB 792, 304 (2019)]
 - Rigorous low-density calculation of non-resonant interactions
 - Challenging to apply past inelastic threshold
- Effective mass model [Savchuk et al., PRC 102, 035202 (2020)]
 - Pion condensation e.g. in Early Universe [VV et al., PRL 126, 012701 (2021)]
- van der Waals interactions [VV et al., PRL 126, 012701 (2021)]
 - Repulsive excluded volume and attractive mean-field
 - Yield nuclear liquid-gas transition
 - Exploratory study suggests possible NS applications [Fujimoto et al., PLB 835, 137524 (2022)]





$$p(T, n_1, \dots, n_h) = \sum_i \frac{T n_i}{1 - \sum_j \tilde{b}_{ji} n_j} - \sum_{i,j} a_{ij} n_i n_j$$



- Liquid-gas transition and associated criticality captured, has effect on $\mu_B = 0$ observables
- Useful for merging with other EoSs, [talk by P. Garella, Tue 5:45 PM](#)
- The $T = 0$ EoS is stiff ($K_0 \sim 760$ MeV), hard packing limit at $n_B \sim 1.8n_0$

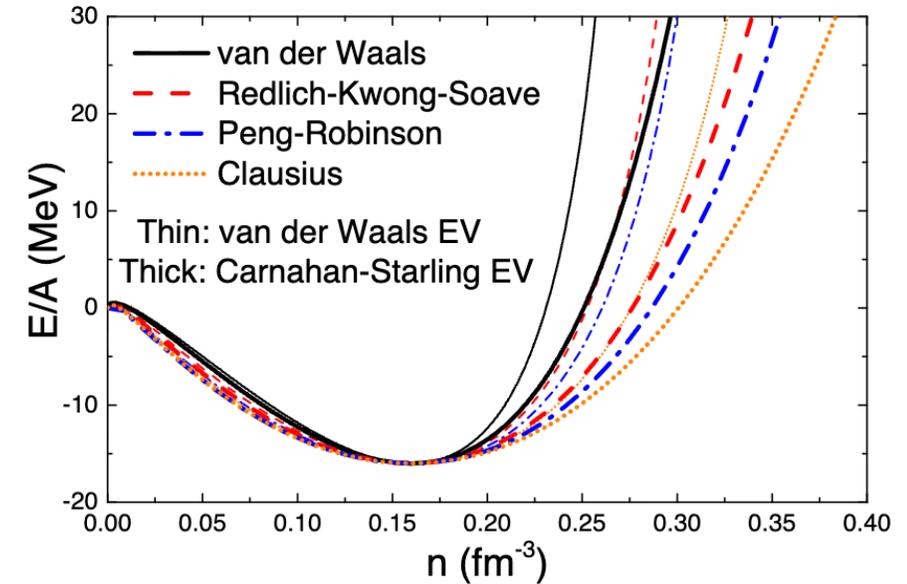
Density-dependent van der Waals: single-component

$$F(T, V, N) = F^{\text{id}}(T, Vf(\eta), N) + Vv(n)$$

- Generalized excluded volume prescription

- $f(\eta)$ – available volume fraction
- van der Waals: $f(\eta) = 1 - 4\eta$ where $\eta = bn/4$ and $V \rightarrow V - bN$
- Carnahan-Starling: $f(\eta) = \exp\left(-\frac{(4-3\eta)\eta}{(1-\eta)^2}\right)$, 4x the VDW limit
- tri-virial model: $f(\eta) = \exp(-4\eta - 8\eta^2)$, no packing limit

Density-dependent effective EV parameter $b^{\text{eff}}(n) = \frac{1-f(\eta)}{n}$



VV, PRC 96, 015206 (2017)

EV suppresses occupation number, $\rho(p) \rightarrow f(\eta)\rho(p)$, Fermi sea no longer saturated

Effectively models quark Pauli exclusion principle, in similarity to quarkyonic models and in contrast to typical RMF models or ChPT

Poberezhniuk et al, PRC 108, 045202 (2023)

Fujimoto et al, PRL 132, 112701 (2024)

- Density-dependent mean-field

- E.g. van der Waals: $v(n) = -an^2$ or Skyrme $v(n) = -an + \beta n^\gamma$
- Can be attractive or repulsive

$$a^{\text{eff}}(n) = -\frac{v(n)}{n^2}$$

Multi-component generalization

$$F(T, V, \{N_i\}) = \sum_i F_i^{\text{id}}(T, V f_i(\{n\}), N_i) + V v(\{n\})$$

$$f_{i,j} \equiv \partial f_i / \partial n_j$$

$$v_i \equiv \partial v / \partial n_i$$

- $f_i(\{n\})$ – available volume fraction for species i
- $v(\{n\})$ is the mean-field energy density describing the attractive interactions
- Both can in principle be functions of all hadron densities

System of transcendental equations:

$$\mu_i = \mu_i^* - \sum_k f_{k,i} p_k^{\text{id}} + v_i$$

$$n_i = f_i n_i^{\text{id}}, \quad n_i^{\text{id}} \equiv n_i^{\text{id}}(T, \mu_i^*)$$

Thermodynamics:

$$P = \sum_i \left(f_i - \sum_j n_j f_{i,j} \right) p_i^{\text{id}} - v + \sum_i v_i n_i$$

$$s = \sum_i f_i s_i^{\text{id}}$$

$$\varepsilon = \sum_i f_i \varepsilon_i^{\text{id}} + v$$

Partial cases:

- **EV-HRG** and **VDW-HRG**
- Mean field HRG ($f_i = 1$)
- VDF model
 - $f_i = 1, v(\{n\}) = v(\rho_B)$

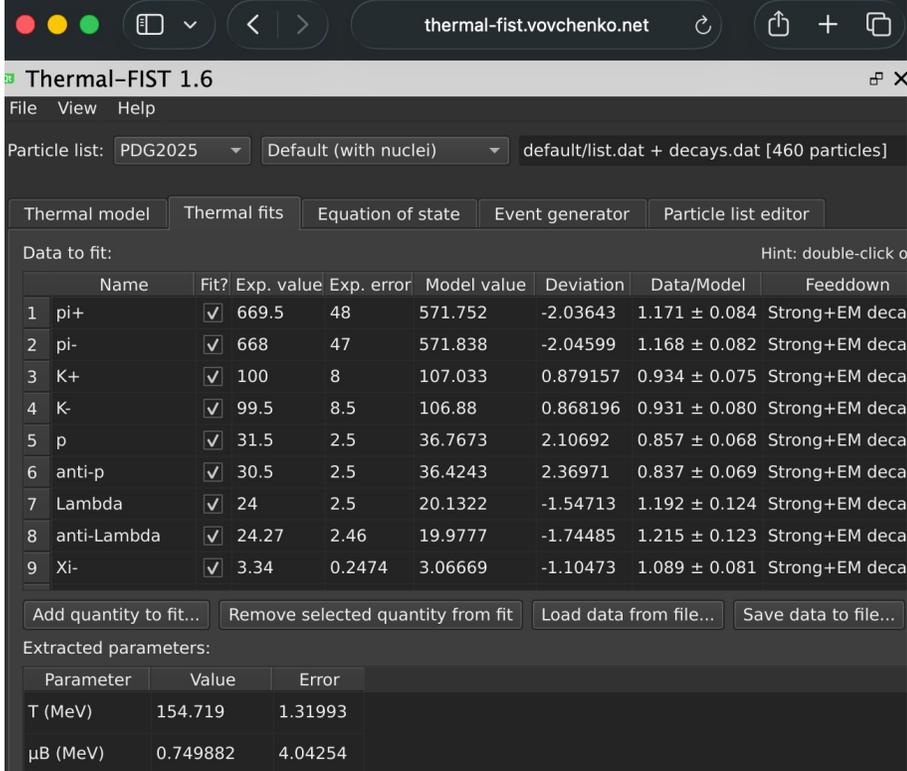
NEW: Version 1.5 (Mar 22, 2025) and 1.6 (Feb 16, 2026)

- Full support for real gas HRG
 - Specify $v(\{n\})$ and $f_i(\{n\})$ via C++ classes
- Lepton degrees of freedom and beta-equilibrium
 - Cosmic trajectories, neutron-star matter EoS
- Magnetic field, Hessian matrix of the pressure
- PDG2025 list, charm, MUSES interface 

• Browser-based calculations (WASM) – thermal-fist.vovchenko.net

- **AI integration (since v1.6):**
 - Documentation, unit tests, agentic workflow

<https://github.com/vlvovch/Thermal-FIST>



The screenshot shows the Thermal-FIST 1.6 web interface in a browser. The page title is "Thermal-FIST 1.6". The interface includes a menu (File, View, Help) and a "Particle list" dropdown set to "PDG2025". Below the menu are tabs for "Thermal model", "Thermal fits", "Equation of state", "Event generator", and "Particle list editor". The "Thermal fits" tab is active, displaying a table of fit data. The table has columns for Name, Fit?, Exp. value, Exp. error, Model value, Deviation, Data/Model, and FeedException. The data rows are:

	Name	Fit?	Exp. value	Exp. error	Model value	Deviation	Data/Model	FeedException
1	pi+	<input checked="" type="checkbox"/>	669.5	48	571.752	-2.03643	1.171 ± 0.084	Strong+EM deca
2	pi-	<input checked="" type="checkbox"/>	668	47	571.838	-2.04599	1.168 ± 0.082	Strong+EM deca
3	K+	<input checked="" type="checkbox"/>	100	8	107.033	0.879157	0.934 ± 0.075	Strong+EM deca
4	K-	<input checked="" type="checkbox"/>	99.5	8.5	106.88	0.868196	0.931 ± 0.080	Strong+EM deca
5	p	<input checked="" type="checkbox"/>	31.5	2.5	36.7673	2.10692	0.857 ± 0.068	Strong+EM deca
6	anti-p	<input checked="" type="checkbox"/>	30.5	2.5	36.4243	2.36971	0.837 ± 0.069	Strong+EM deca
7	Lambda	<input checked="" type="checkbox"/>	24	2.5	20.1322	-1.54713	1.192 ± 0.124	Strong+EM deca
8	anti-Lambda	<input checked="" type="checkbox"/>	24.27	2.46	19.9777	-1.74485	1.215 ± 0.123	Strong+EM deca
9	Xi-	<input checked="" type="checkbox"/>	3.34	0.2474	3.06669	-1.10473	1.089 ± 0.081	Strong+EM deca

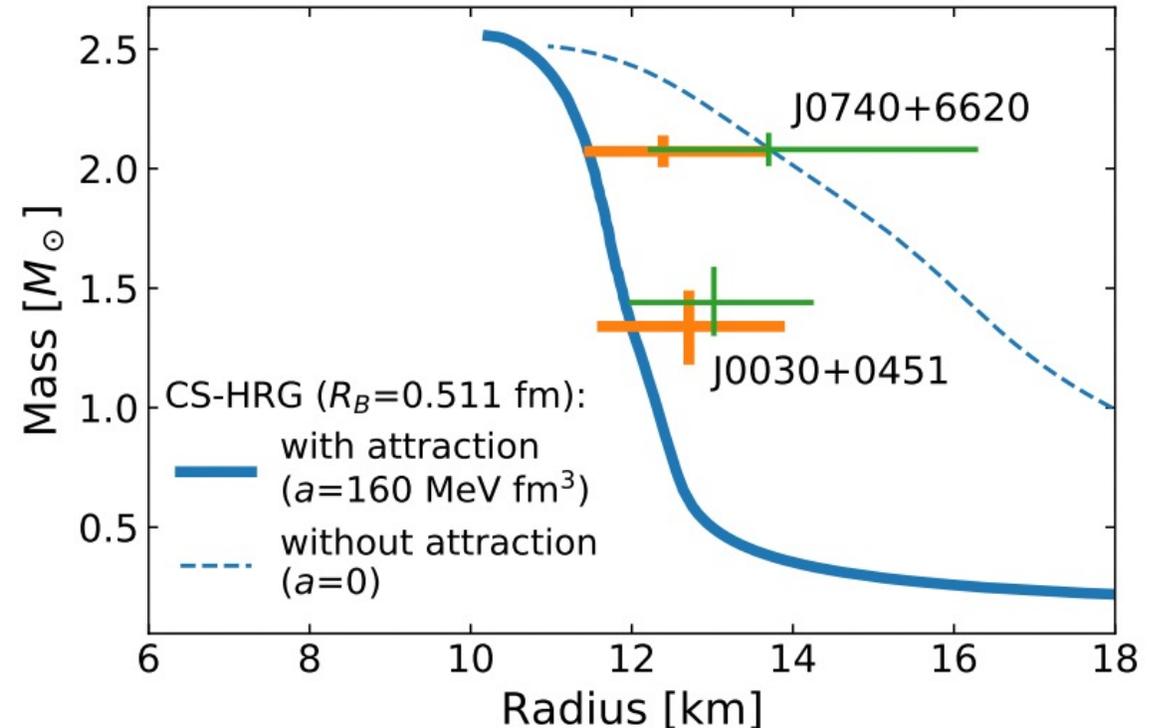
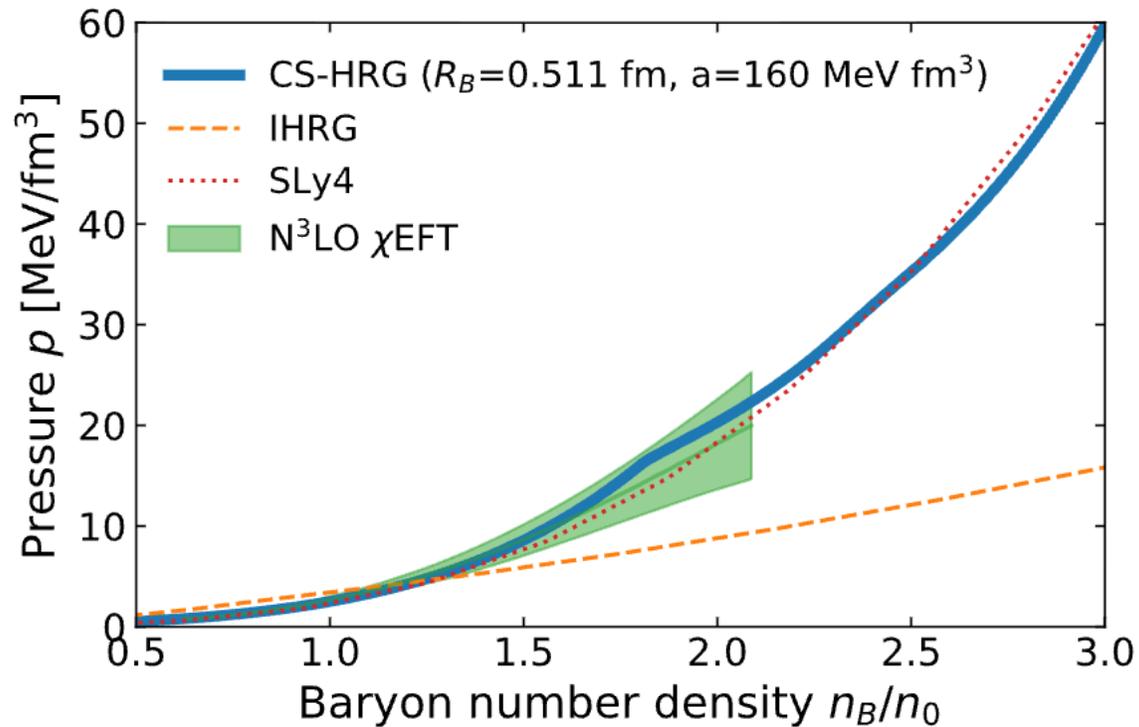
Below the table are buttons for "Add quantity to fit...", "Remove selected quantity from fit", "Load data from file...", and "Save data to file...". An "Extracted parameters" section shows:

Parameter	Value	Error
T (MeV)	154.719	1.31993
μB (MeV)	0.749882	4.04254



HRG for neutron-star matter

Earlier proof-of-principle: Carnahan-Starling HRG with constant a and b yields neutron stars



[Fujimoto, Fukushima, Hidaka, Hiraguchi, Iida, PLB 835, 137524 (2022)]

Areas for improvement:

- Restricted to single-component interaction (isospin-blind parameters)
- Part of neutron-star curve is acausal

DD-HRG: Fixing the parameters

Asymmetric nuclear matter: proton (charge) fraction $0 < y < 1$ $y = \frac{\rho_Q}{\rho_B}$

Mixture of protons, neutrons, and hyperons/resonances

- Isospin-like (a_p, b_p) and isospin-unlike (a_{pn}, b_{pn}) int. parameters
- $y = 1/2$: symmetric matter for $a = (a_n + a_{pn})/2$ and $b = (b_n + b_{pn})/2$

VDW+Clausius:

$$v(n_j) = - \sum_{i,j \in B} \frac{a_{ij} n_i n_j}{1 + c n_B}$$

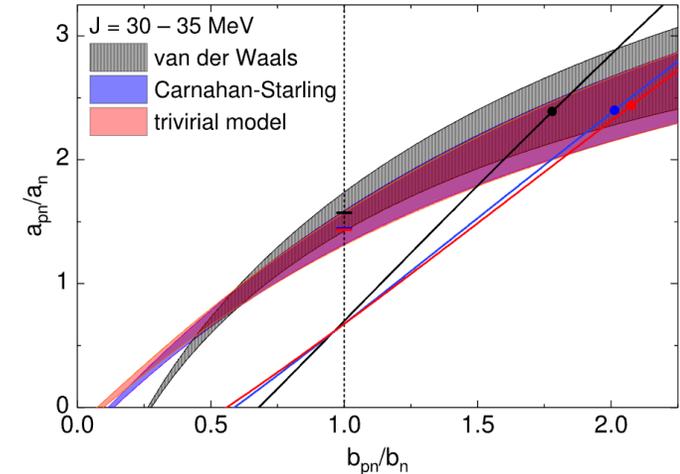
TVM:

$$f_i(\{n_j\}) = \exp(-4\eta_i - 8\eta_i^2)$$

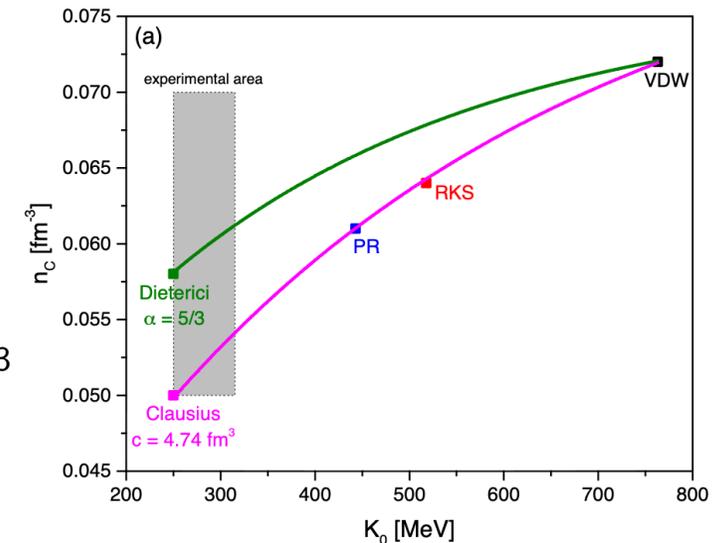
$$\eta_i = \frac{1}{4} \sum_{j \in B} b_{ij} n_j$$

Constraints:

- Binding energy 16 MeV at saturation, $a \approx 350 \text{ MeV fm}^3$, $b \approx 4.3 \text{ fm}^3$
- Symmetry energy and its slope, $a_n \approx 200 \text{ MeV fm}^3$, $b_n \approx 3.1 \text{ fm}^3$
- Incompressibility $K_0 \sim 240\text{-}320 \text{ MeV}$, $c \sim 3 - 4.5 \text{ fm}^3$



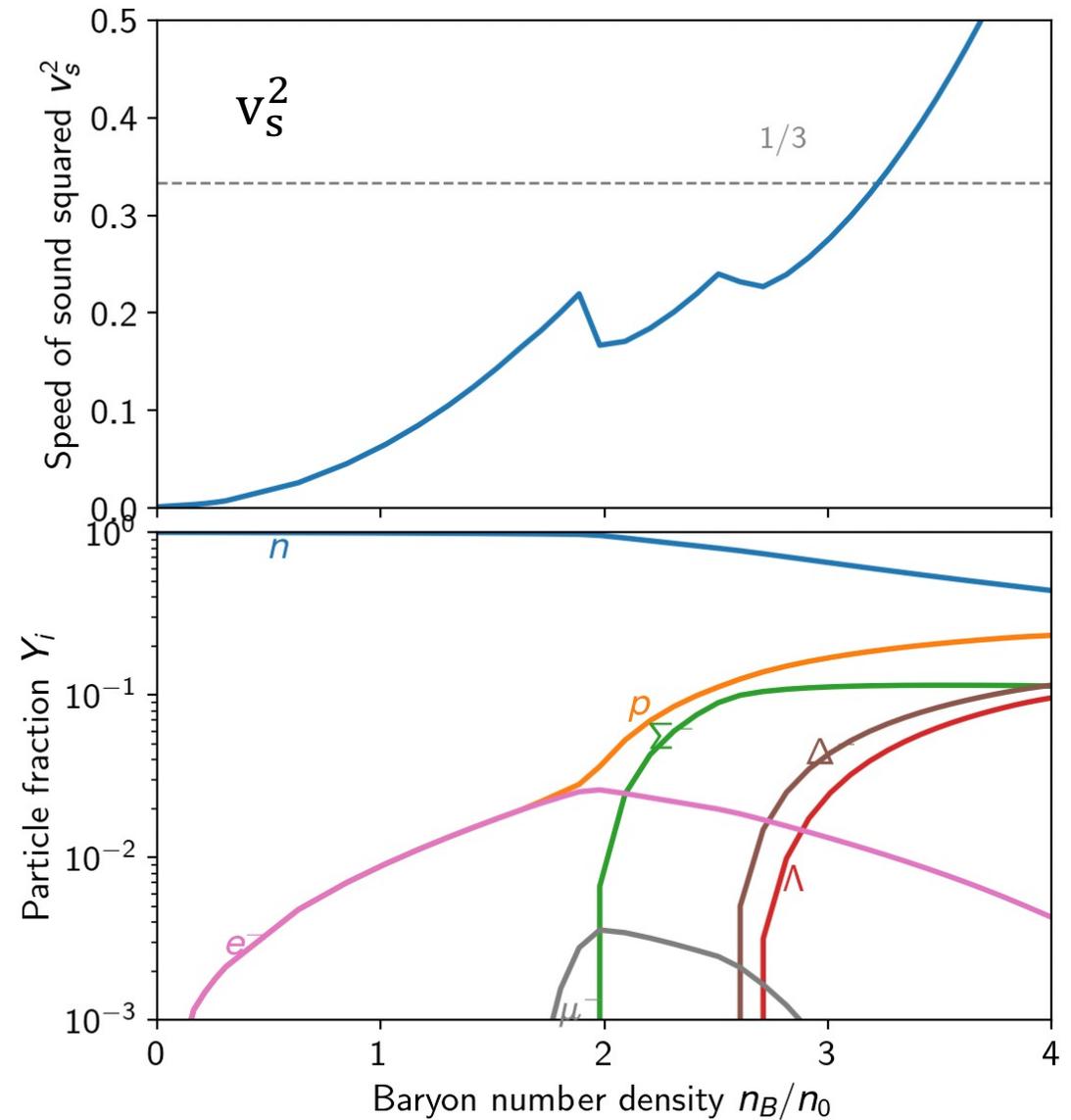
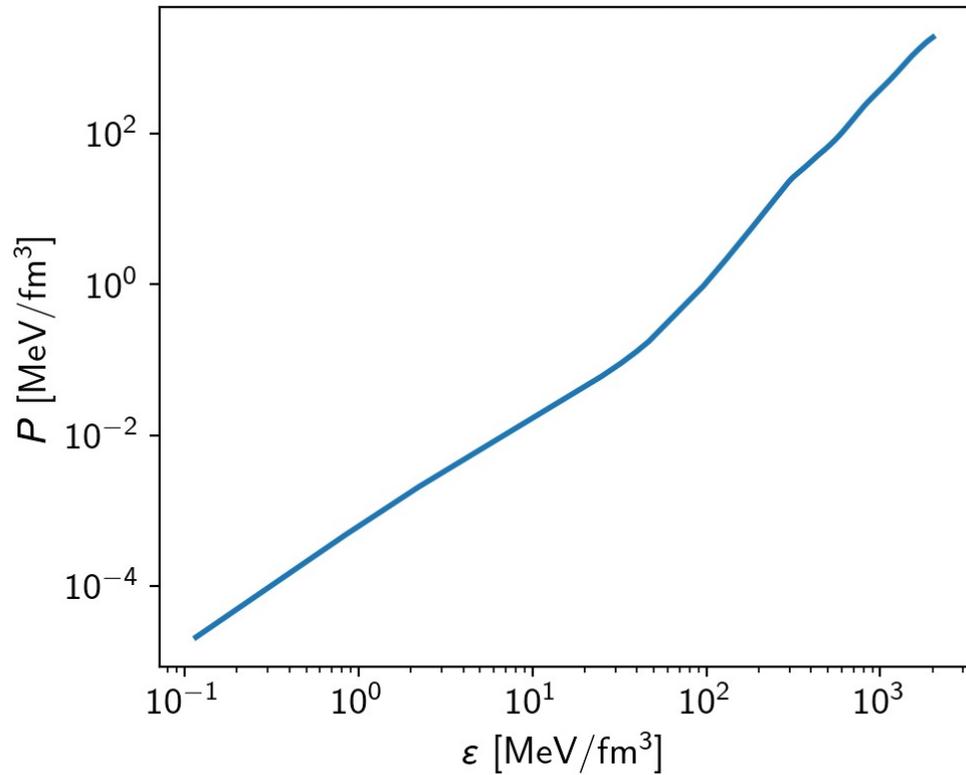
Moss, Poberezhniuk, VV, PRC 111, 025803 (2025)



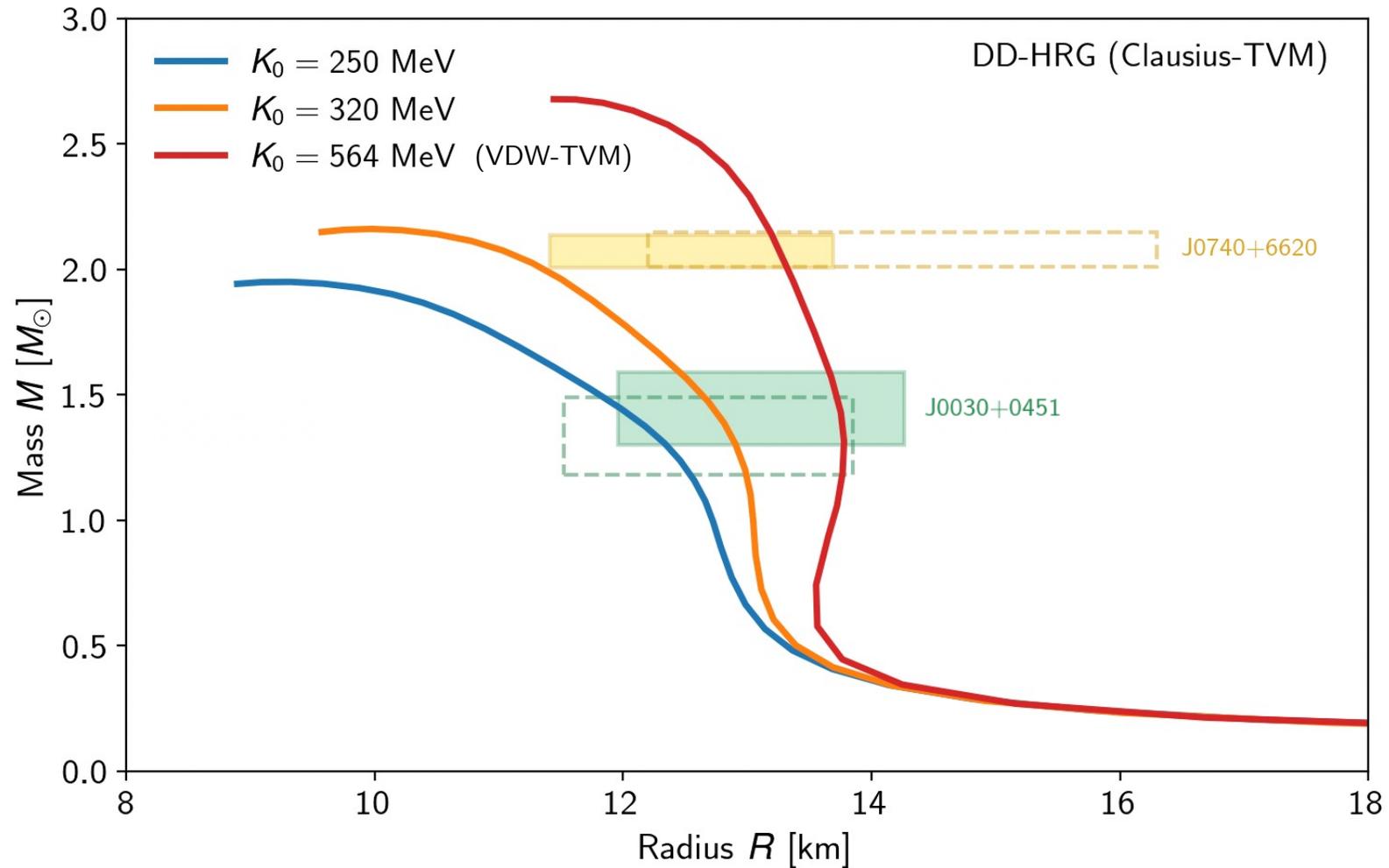
Lysenko et al., PRC 111, 035204 (2025)

DD-HRG: neutron-star matter

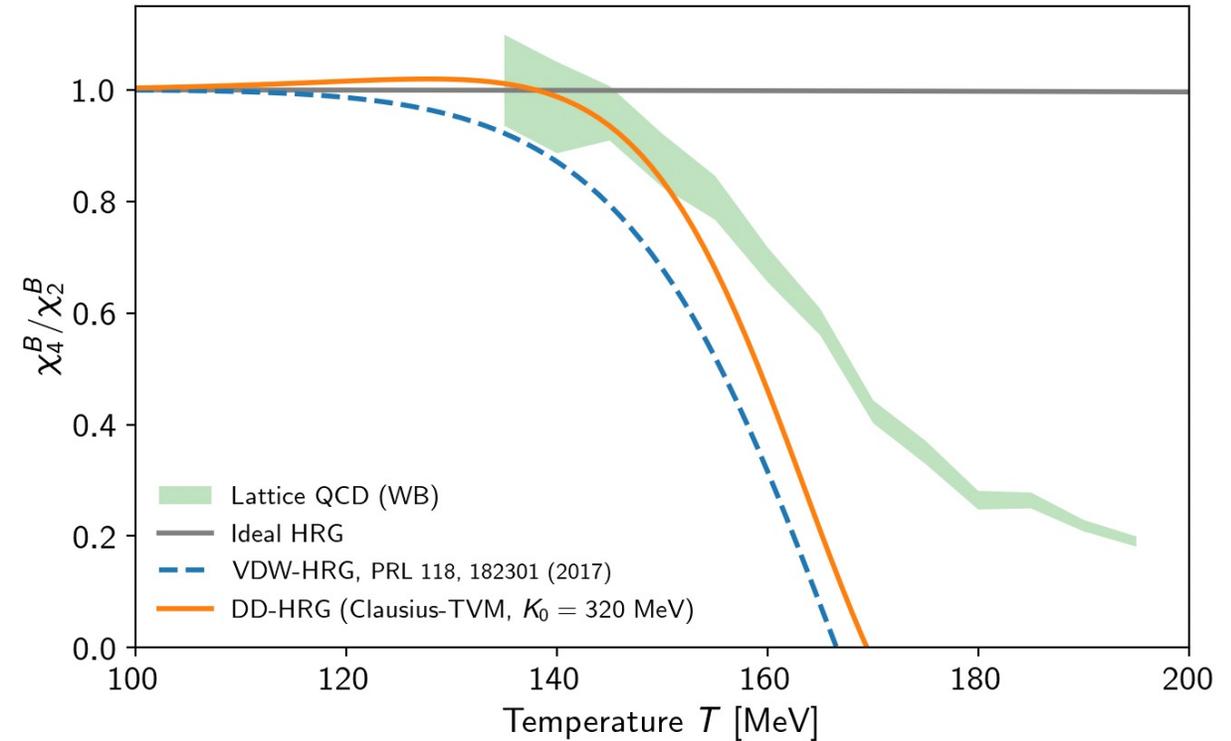
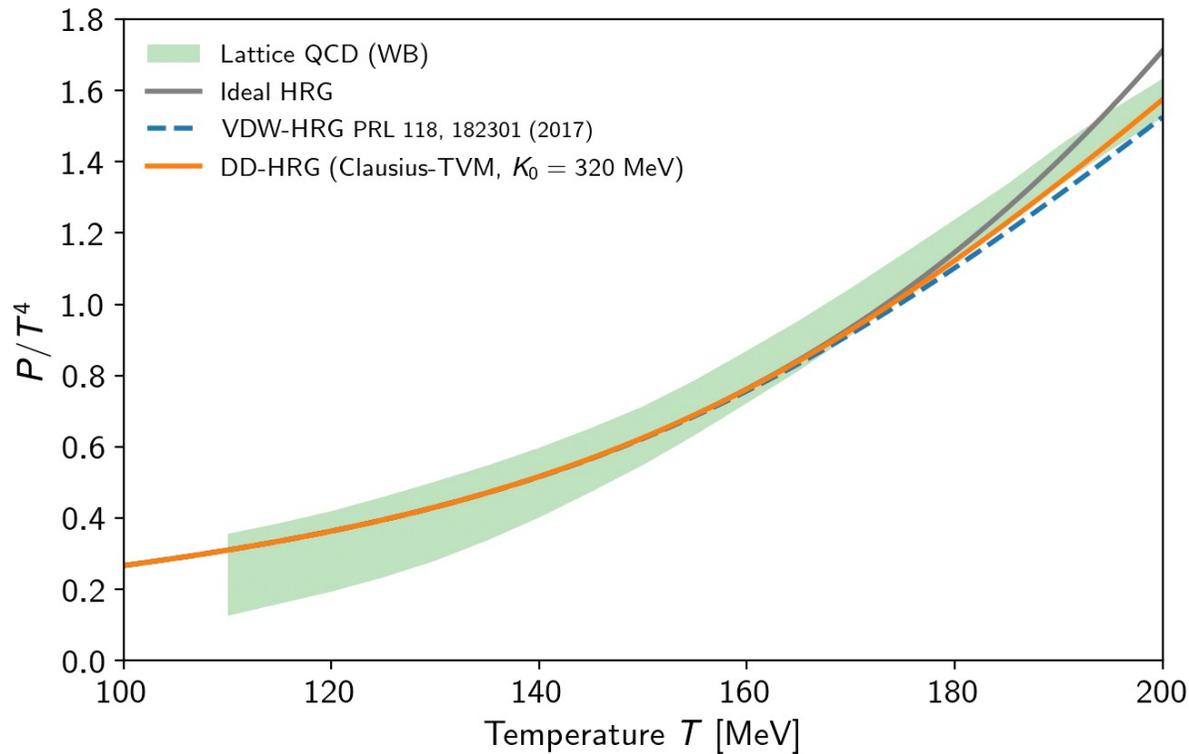
Beta-equilibrium: $n_Q = n_{l_e} + n_{l_\mu}$, $\mu_S = 0$



DD-HRG: mass-radius relation



DD-HRG: EoS at $\mu_B = 0$



Lattice data from Wuppertal-Budapest collaboration, PLB 730, 99 (2014); JHEP 10, 205 (2018)

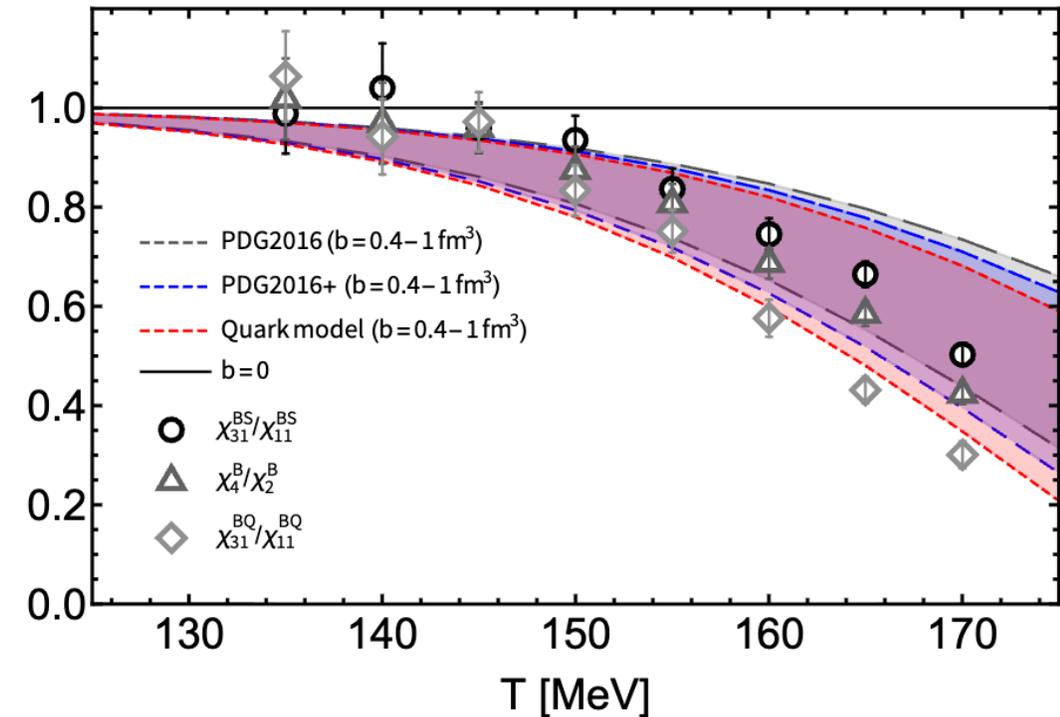
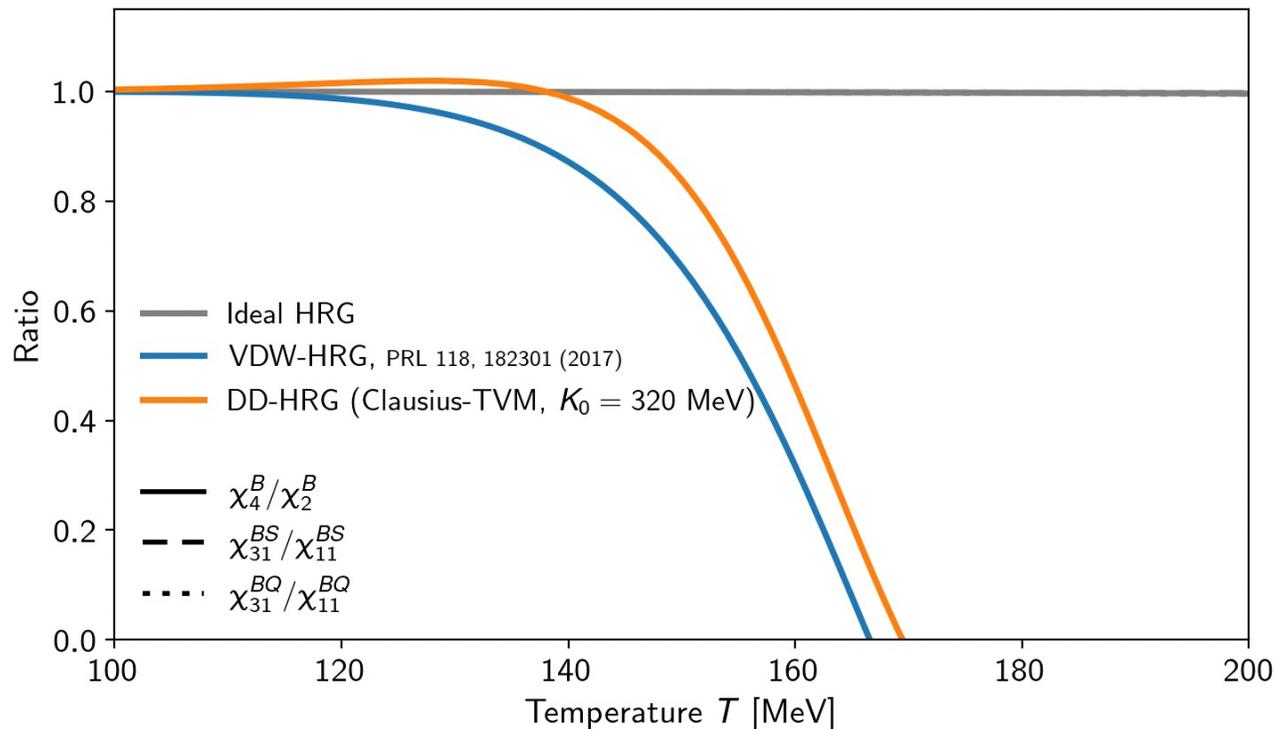
DD-HRG improves both the description of nuclear matter and agreement with lattice QCD at $\mu_B = 0$

DD-HRG: Probing strangeness interaction

Consider 4th-to-2nd order ratios $\chi_4^B/\chi_2^B, \chi_{31}^{BQ}/\chi_{11}^{BQ}, \chi_{31}^{BS}/\chi_{11}^{BS}$

For strangeness-independent interactions $\chi_4^B/\chi_2^B = \chi_{31}^{BQ}/\chi_{11}^{BQ} = \chi_{31}^{BS}/\chi_{11}^{BS}$

Lattice suggests ordering $\chi_{31}^{BS}/\chi_{11}^{BS} > \chi_4^B/\chi_2^B > \chi_{31}^{BQ}/\chi_{11}^{BQ}$



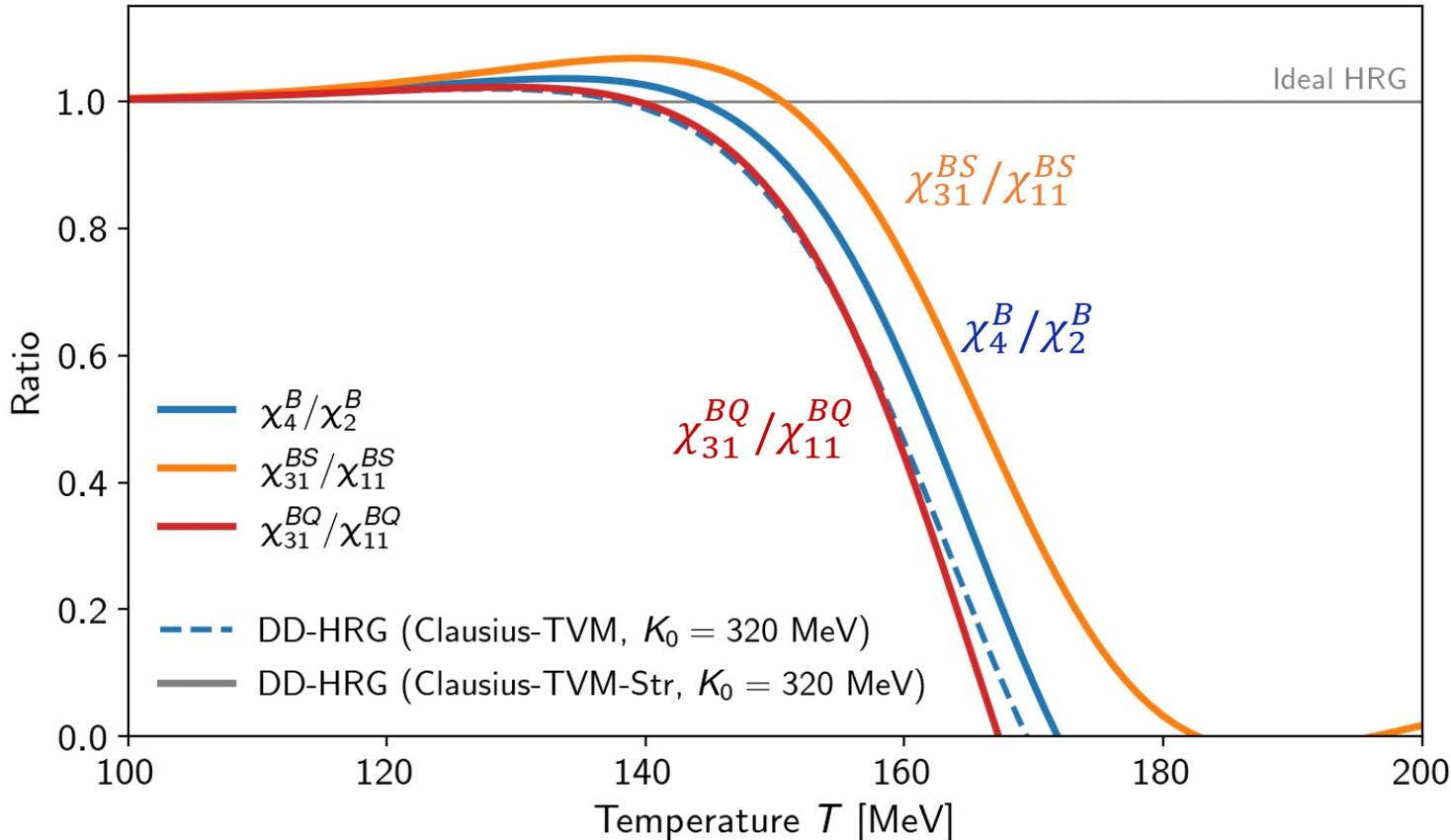
DD-HRG: Strangeness-dependent EV

Introduce strangeness-dependent excluded volume among baryons

$$b_{ij} = \bar{b} + \delta b_I I_{Z,i} I_{Z,j} + b_{BS} \frac{1}{2} (B_i S_j + S_i B_j)$$

average
isospin
strangeness

$b_{BS} = 0.3 \text{ fm}^3$
yields less repulsion for hyperons



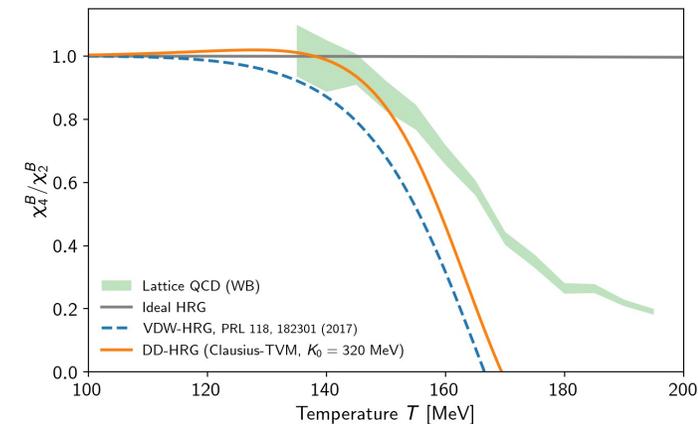
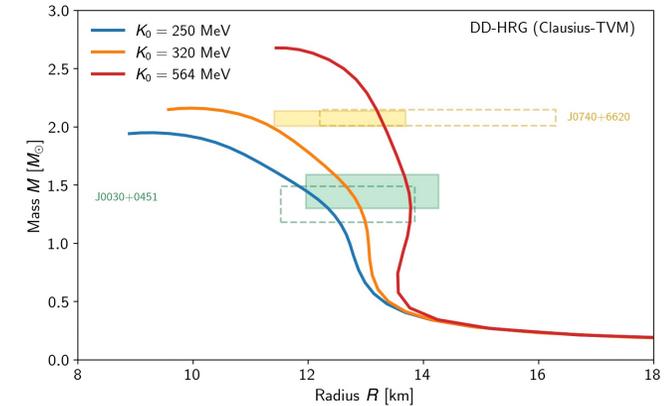
$$\chi_{31}^{BS} / \chi_{11}^{BS} > \chi_4^B / \chi_2^B > \chi_{31}^{BQ} / \chi_{11}^{BQ}$$

Summary

- Density-dependent van der Waals HRG formulated
 - Arbitrary mean field and generalized excluded volume mechanism
 - DD-HRG thermodynamics implemented and available in Thermal-FIST-1.6
- DD-HRG provides a reasonable description of neutron-star matter
 - Density and isospin dependent parameters constrained with nuclear matter data
 - Extended causality region, supports $\geq 2M_{\odot}$ stars
- Same DD-HRG EoS provides reasonable thermodynamics at $\mu_B = 0$
 - Lattice susceptibility ratios suggest weaker repulsion for strange baryons

Outlook:

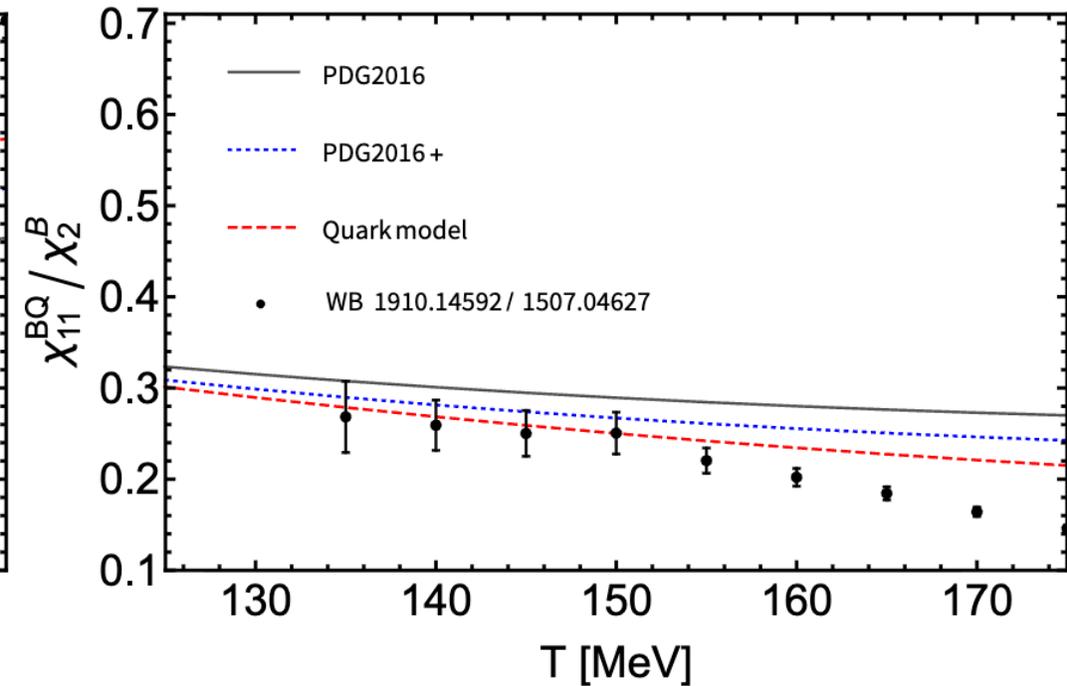
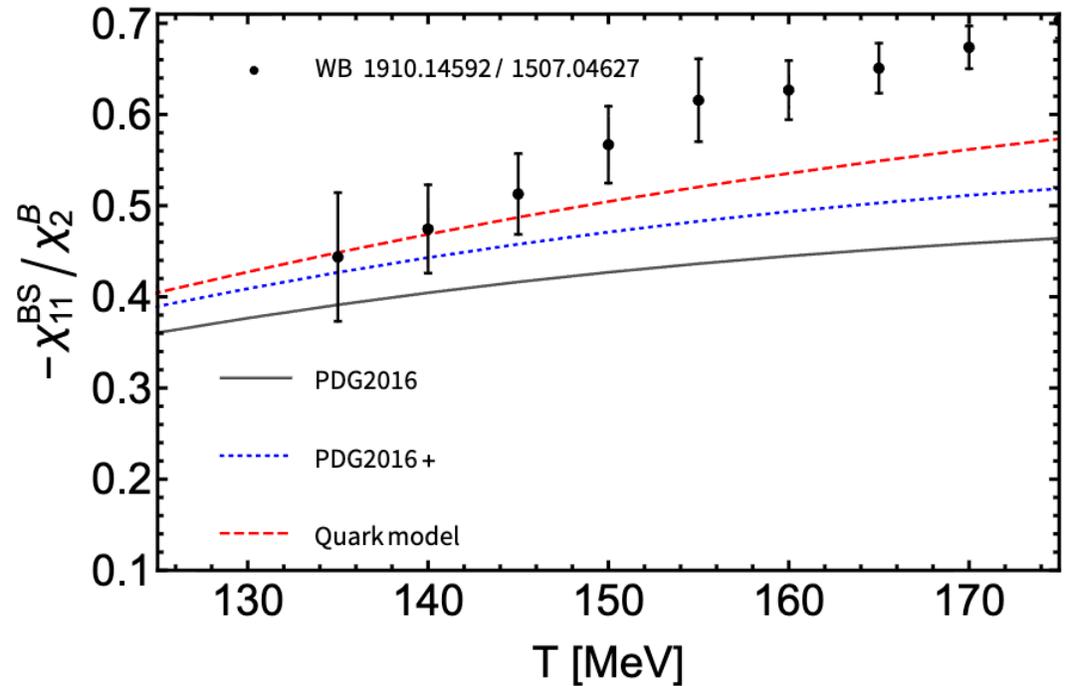
- Constraining strangeness/isospin dependence further, including mean fields
- Bayesian analysis analysis combined constraints
- Applications to heavy-ion collisions and NS mergers, merging with other EoSs



Thanks for your attention!

Backup slides

Susceptibilities in EV-HRG



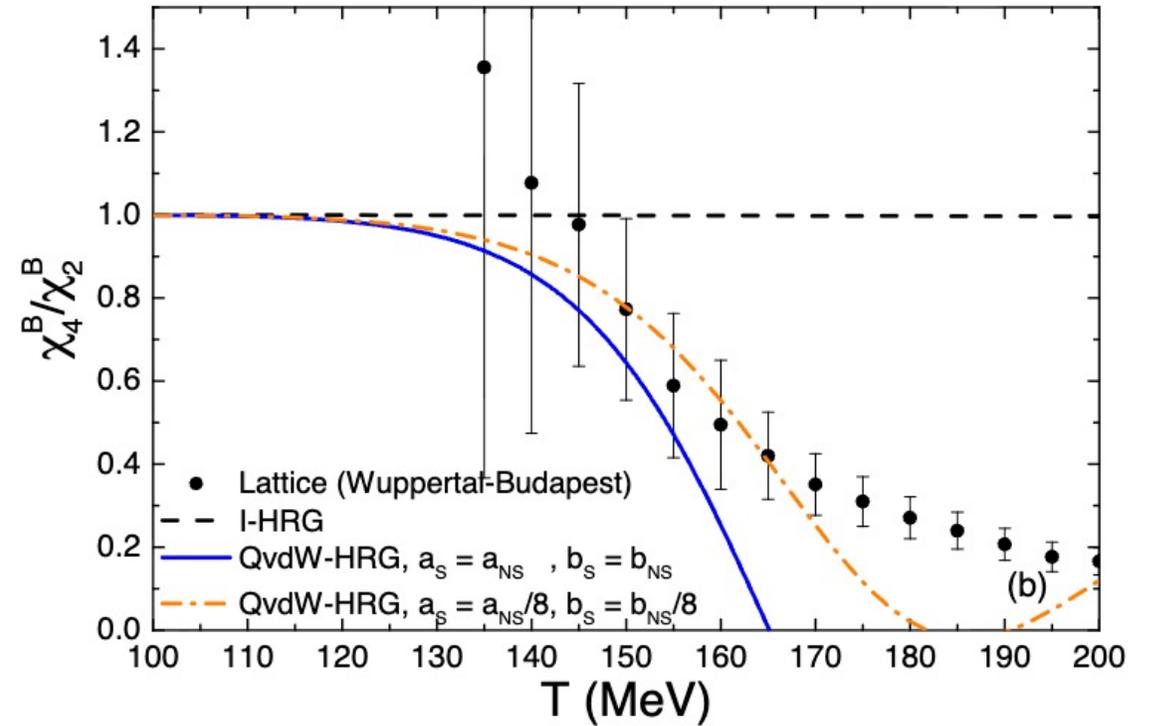
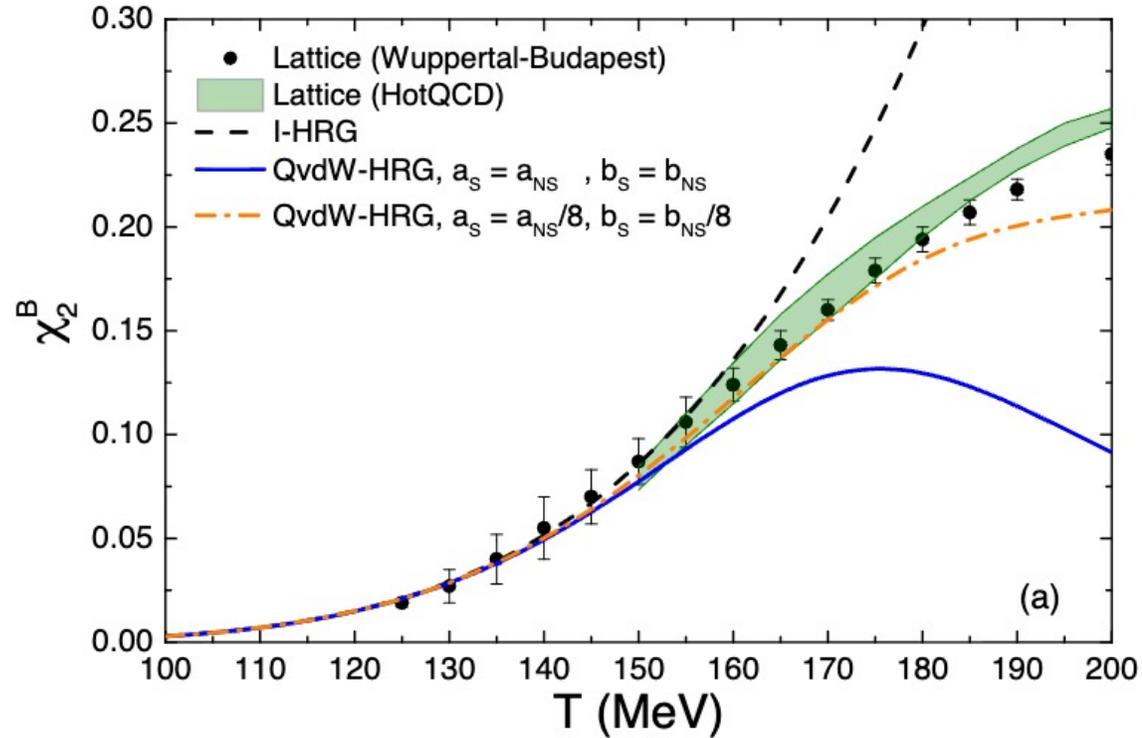
$$\frac{\chi_{11}^{BQ}}{\chi_2^B} = \frac{\sum_{j \in \text{sectors}} B_j Q_j \tilde{\phi}_j(T)}{\sum_{j \in \text{sectors}} B_j^2 \tilde{\phi}_j(T)},$$

$$\frac{\chi_{11}^{BS}}{\chi_2^B} = \frac{\sum_{j \in \text{sectors}} B_j S_j \tilde{\phi}_j(T)}{\sum_{j \in \text{sectors}} B_j^2 \tilde{\phi}_j(T)}.$$

$$\frac{\chi_4^B}{\chi_2^B} = \frac{\chi_{31}^{BS}}{\chi_{11}^{BS}} = \frac{\chi_{31}^{BQ}}{\chi_{11}^{BQ}} = \frac{1 - 8W(\kappa_B) + 6[W(\kappa_B)]^2}{[1 + W(\kappa_B)]^4}$$

$$= 1 - 12\kappa_B + O(\kappa_B^2).$$

Strangeness in VDW-HRG



VV et al., PRC 96, 045202 (2017)