

Contribution of critical fluctuations and baryon annihilation to proton number cumulants at $\sqrt{s_{NN}} = 7.7 - 200$ GeV from hydrodynamics

Gregoire Pihan

Collaborators:

V. Vovchenko, J. Parra
R. Poberezhniuk, V.A. Kuznietsov,
M. Kahangirwe, H. Shah, C. Ratti



U.S. DEPARTMENT OF
ENERGY

Office of
Science

The QCD phase diagram

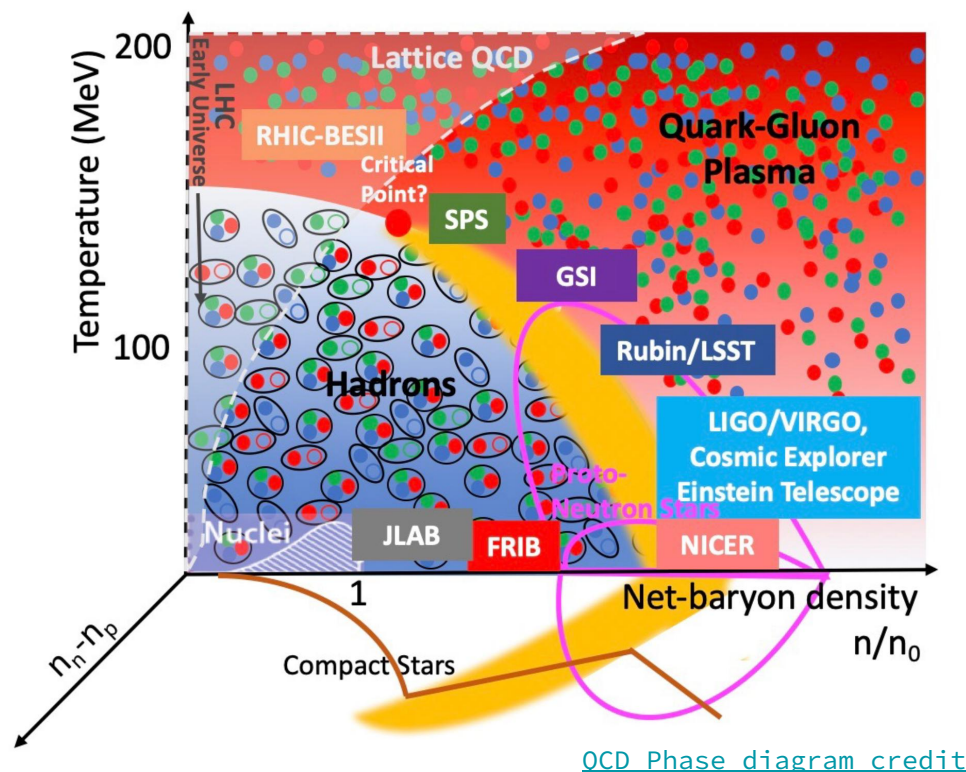
Different phase of nuclear matter

- Dilute hadron gas low T , μ_B
- Quark-gluon plasma high T , μ_B

Different type of transition?

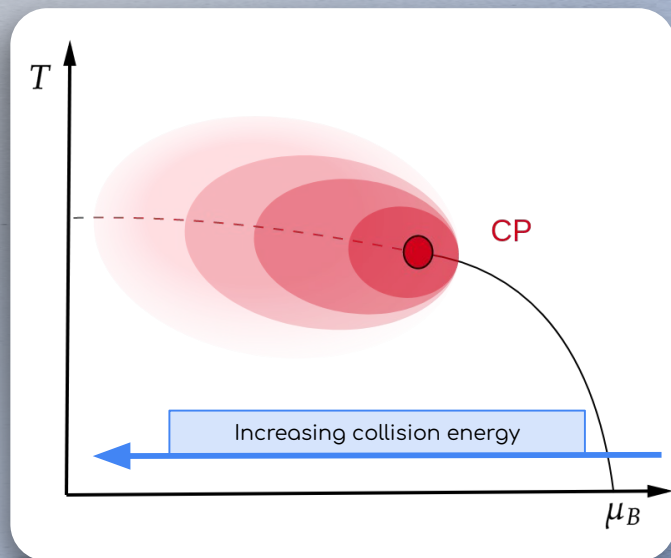
- Lattice: Crossover at $\mu_B = 0$ GeV
[Y. Aoki et al., Nature 443, 675](#)
- Effective QCD: 1st order?

Is there a critical point in the QCD phase diagram?



The phenomenology of the QCD critical point

In the critical point region the fluctuations are largely enhanced

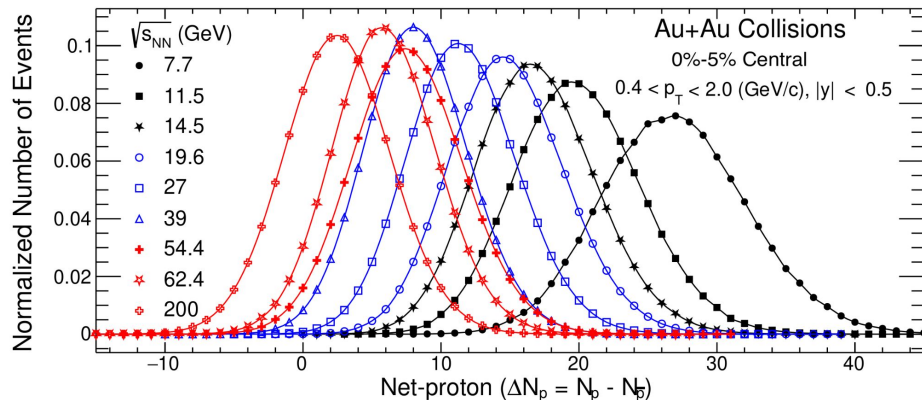


Higher order moments of the net-proton number distribution

$$\langle (\delta N_p)^2 \rangle \propto \xi^2 \quad \langle (\delta N_p)^3 \rangle \propto \xi^{4.5} \quad \langle (\delta N_p)^4 \rangle \propto \xi^7$$

[M.A. Stephanov, Phys. Rev. Lett. 102, 032301](#)

Net-proton distribution as a function of the collision energy in Au-Au collisions at STAR



[STAR Collaboration Phys. Rev. Lett. 126, 092301](#)

Multi stage challenges for fluctuations observables

Highly dynamical environment
Short lived

Finite size effects

[V. Vovchenko, R. Poberezhnyuk et al Phys. Lett. B 811, 135868](#)

Proxies observables

[VV, Jiang, Gorenstein, Stoecker, PRC 98, 024910](#)

Finite acceptance

[O. Savchuk, R. V. Poberezhnyuk, V. Vovchenko and M. I. Gorenstein Phys. Rev. C 101, 024917](#)

Coordinate / momentum space

[Ling, Stephanov, PRC 93, 034915](#)

Late stage interactions

[Steinheimer, Vovchenko, Aichelin, Bleicher, Stoecker, PLB 776, 32](#)

Equilibrium?

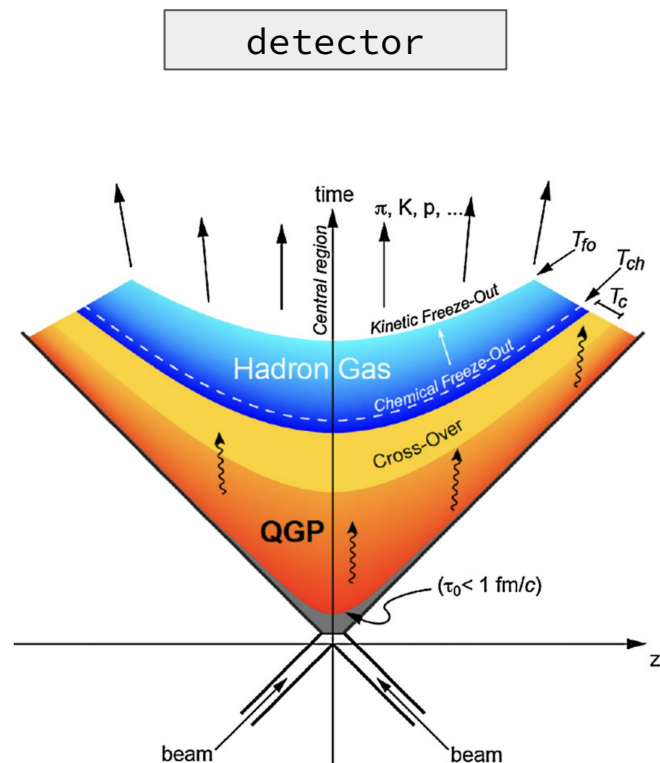
Retardation effects?

Grand Canonical
Ensemble?

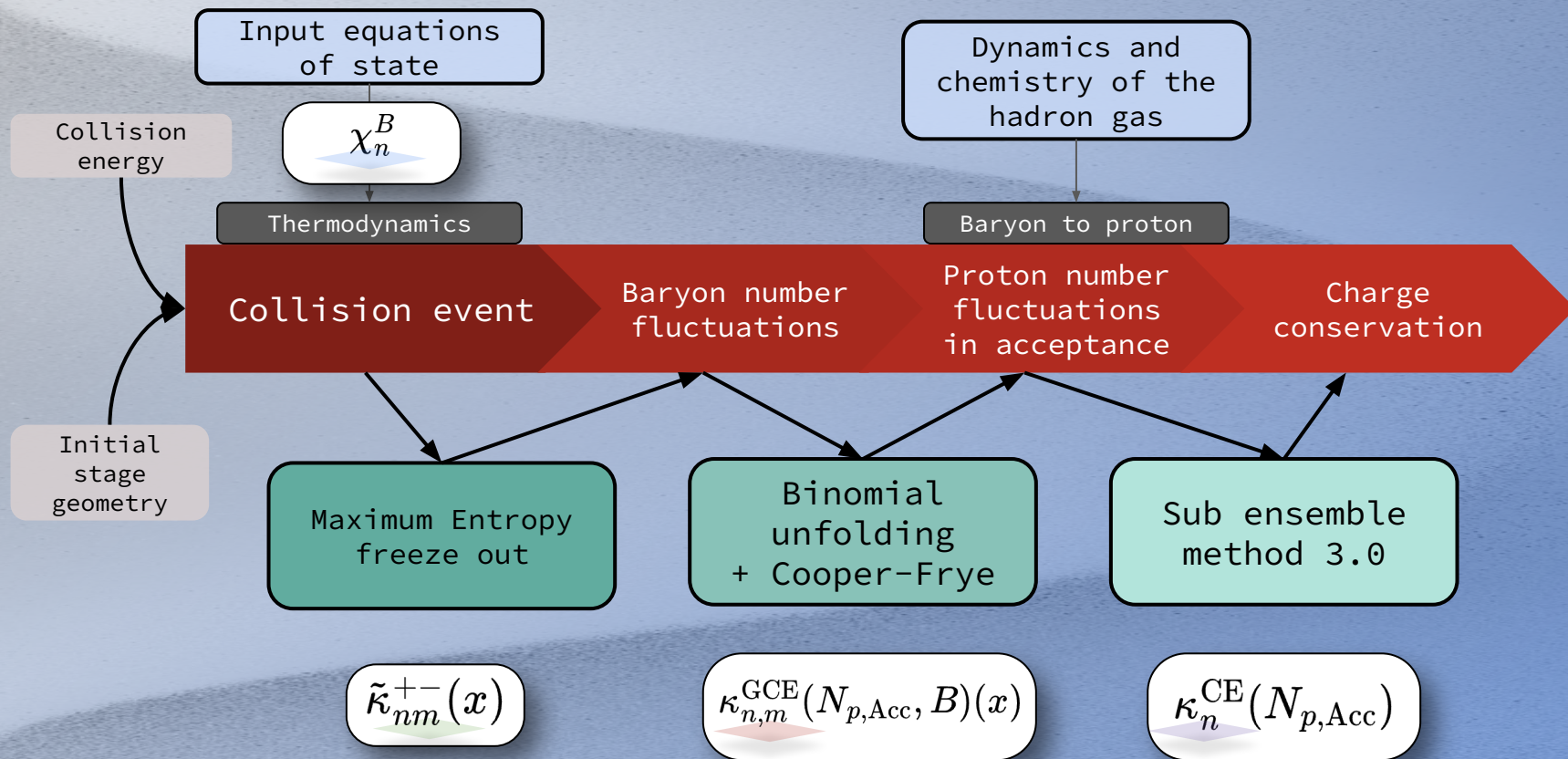
Baryon number
~
Proton number?

Thermal smearing?

Impact of hadron gas
dynamics?



Roadmap from theory to experiment



Simulation of the collision event

Smooth collision event: [iEBE-MUSIC](#)

Collision event

Baryon number
fluctuations

Proton number
fluctuations
in acceptance

Charge
conservation

Transverse: Glauber model

$$\epsilon(x, y, \eta_s) \propto e(\eta_s) T_{\text{Proj}}(x, y) T_{\text{Targ}}(x, y)$$

$$n_B(x, y, \eta_s) \propto f_{\text{Proj}}^{n_B}(\eta_s) T_{\text{Proj}}(x, y) + f_{\text{Targ}}^{n_B}(\eta_s) T_{\text{Targ}}(x, y)$$

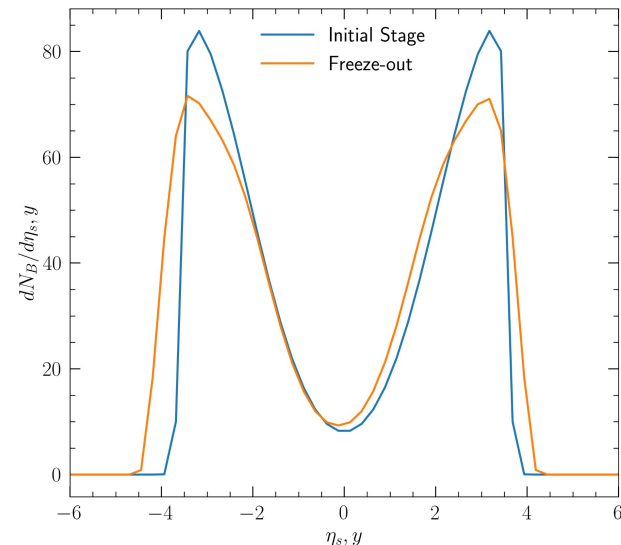
Longitudinal: Density envelopes

$$e(\eta_s, y_{CM}) \propto \exp\left[-\frac{(\eta_s - y_{CM} - \eta_0)^2}{2\sigma_\eta^2}\right] \theta(|\eta_s - y_{CM}| - \eta_0)$$

$$f_{\text{Proj}}^{n_B} \propto \theta(\eta_s - \eta_{B,0}) \exp\left[-\frac{(\eta_s - \eta_{B,0})^2}{2\sigma_{B,\text{out}}^2}\right] + \theta(\eta_s + \eta_{B,0}) \exp\left[-\frac{(\eta_s + \eta_{B,0})^2}{2\sigma_{B,\text{in}}^2}\right]$$

$$f_{\text{Targ}}^{n_B} \propto \theta(\eta_s + \eta_{B,0}) \exp\left[-\frac{(\eta_s + \eta_{B,0})^2}{2\sigma_{B,\text{in}}^2}\right] + \theta(-\eta_s - \eta_{B,0}) \exp\left[-\frac{(\eta_s + \eta_{B,0})^2}{2\sigma_{B,\text{out}}^2}\right]$$

0-5% Au+Au $\sqrt{s_{NN}} = 200$ GeV



Simulation of the collision event

Smooth collision event: [iEBE-MUSIC](#)

Collision event

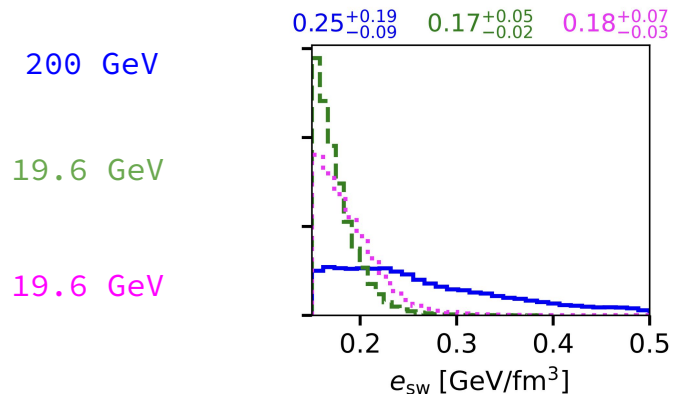
Baryon number
fluctuations

Proton number
fluctuations
in acceptance

Charge
conservation

Freeze-out: constant energy density

Posteriors energy density for the BES energies

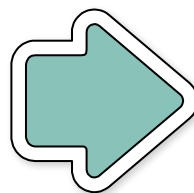


[Jahan et al. Phys. Rev. C 113. 024919](#)

Working Values

$$e_{\text{sw}} = 0.18 \text{ GeV}/\text{fm}^3$$

$$e_{\text{sw}} = 0.25 \text{ GeV}/\text{fm}^3$$



Net Baryon number fluctuations

NetB fluctuations on the hypersurface

Each hydro cell is in local equilibrium (GCE)

$$\kappa_n^{B,\text{GCE}}(x) = \delta V(x) T(x)^3 \chi_n^B(x)$$

$$\delta V = d\Sigma_\mu u^\mu (> 0) \quad x_i \equiv (\tau_i, x_i, y_i, \eta_i) \quad \chi_1^B(x) = n_B(x)$$

Cells are statistically independent

$$\kappa_n^B = \sum_{i \in \text{cells}} \delta \kappa_n^B$$

Collision event

Baryon number
fluctuations

Proton number
fluctuations
in acceptance

Charge
conservation

Ideal HRG

- Ideal Poisson/Skellam baseline
- Deviations?

4D TExS

- Lattice QCD expansion
- Strangeness neutrality (HIC)

[Abuali, Jahan et al, Phys. Rev. D 112, 054502](#)

2D TExS Ising

- Lattice QCD expansion
- Custom position of the CEP

[Kahangirwe et al, Phys. Rev. D 109, 094046](#)

Constant Entropy Contour (CEC)

- Lattice QCD expansion
- Prediction of CP from CEC

[Shah et al, arXiv:2601.08823](#)

Net Baryon number fluctuations

NetB fluctuations on the hypersurface

Each hydro cell is in local equilibrium (GCE)

$$\kappa_n^{B,\text{GCE}}(x) = \delta V(x) T(x)^3 \chi_n^B(x)$$

$$\delta V = d\Sigma_\mu u^\mu (> 0) \quad x_i \equiv (\tau_i, x_i, y_i, \eta_i) \quad \chi_1^B(x) = n_B(x)$$

Cells are statistically independent

$$\kappa_n^B = \sum_{i \in \text{cells}} \delta \kappa_n^B$$

Collision event

Baryon number
fluctuations

Proton number
fluctuations
in acceptance

Charge
conservation

Ideal HRG

- Ideal Poisson/Skellam baseline
- Deviations?

Any other equation of
state adapted for
heavy-ion collisions
conditions
(strangeness neutral)
can be used too!

Constant Entropy Contour (CEC)

- Lattice QCD expansion
- Prediction of CP from CEC
[Shah et al., arXiv:2601.08823](#)

Net Baryon to positive baryon fluctuations

The maximum entropy freeze-out

Collision event

Baryon number
fluctuations

Proton number
fluctuations
in acceptance

Charge
conservation

The Max Ent method provides the least-biased reconstruction of particle multiplicity fluctuations consistent with the hydrodynamic correlations.

$$\hat{\Delta}G_{A_1 \dots A_k} = \hat{\Delta}\mathcal{H}_{a_1 \dots a_k} \prod_{i=1}^k P_{A_i}^{a_i}$$

$\hat{\Delta}$ Irreducible relative cumulants
(IRC) operator

[M.S. Pradeep, M.A. Stephanov Phys. Rev. Lett. 130, 162301](#)

$$\hat{\Delta}\mathcal{H}_{a_1 \dots a_k}$$

IRC of the K-points correlator of hydrodynamic fields $a_i = \epsilon, n_B$

$$\hat{\Delta}G_{A_1 \dots A_k}$$

IRC of the K-points correlator of particles $A_i \in \text{Particles}$

$$P_{A_i}^{a_i}$$

Hydro matching
conditions projectors

Net Baryon to positive baryon fluctuations

The maximum entropy freeze-out



Assumptions

- $B^+ B^-$ joint cumulants

$$B^\pm = \left\{ \hat{A}_i \mid q_{A_i} = \pm 1 \right\}_{i \in \text{Particles}}$$

- Boltzmann approximation

Particle IRCs = Particle factorial cumulants

- Cells are statistically independent

No correlation between cells

- Net-baryon density field only

$$a_i = n_B \forall i$$

Maximum Entropy $B^+ B^-$ factorial cumulants

$$\tilde{\kappa}_{nm}^{+-}(x) = \tilde{\tilde{\kappa}}_{nm}^{+-}(x) + \hat{\Delta} \mathcal{H}_{n+m}(x) (P_+)^n (P_-)^m$$

Net Baryon to positive baryon fluctuations

The maximum entropy freeze-out

Collision event

Baryon number
fluctuations

Proton number
fluctuations
in acceptance

Charge
conservation

Assumptions

- $B^+ B^-$ joint cumulants

$$B^\pm = \left\{ \hat{A}_i \mid q_{A_i} = \pm 1 \right\}_{i \in \text{Particles}}$$

- Boltzmann approximation

Particle IRCs = Particle factorial cumulants

- Cells are statistically independent

No correlation between cells

- Net-baryon density field only

$$a_i = n_B \forall i$$

Maximum Entropy $B^+ B^-$ factorial cumulants

$$\tilde{\kappa}_{nm}^{+-}(x) = \tilde{\tilde{\kappa}}_{nm}^{+-}(x) + \hat{\Delta} \mathcal{H}_{n+m}(x) (P_+)^n (P_-)^m$$

The ideal-HRG factorial cumulants

$$\tilde{\tilde{\kappa}}_{nm}^{+-}(x) = \delta_{n,1} \delta_{m,0} \langle N_{B^+} \rangle(x) + \delta_{n,0} \delta_{m,1} \langle N_{B^-} \rangle(x)$$

Net Baryon to positive baryon fluctuations

The maximum entropy freeze-out

Collision event

Baryon number
fluctuations

Proton number
fluctuations
in acceptance

Charge
conservation

Assumptions

- $B^+ B^-$ joint cumulants

$$B^\pm = \left\{ \hat{A}_i \mid q_{A_i} = \pm 1 \right\}_{i \in \text{Particles}}$$

- Boltzmann approximation

Particle IRCs = Particle factorial cumulants

- Cells are statistically independent

No correlation between cells

- Net-baryon density field only

$$a_i = n_B \forall i$$

Maximum Entropy $B^+ B^-$ -factorial cumulants

$$\tilde{\kappa}_{nm}^{+-}(x) = \tilde{\tilde{\kappa}}_{nm}^{+-}(x) + \hat{\Delta} \mathcal{H}_{n+m}(x) (P_+)^n (P_-)^m$$

The hydrodynamics correlators IRCs

$$\hat{\Delta} \mathcal{H}_{n+m}(x) = \delta V(x) T(x)^3 \bar{\chi}_{n+m}^B F_{nm}(\chi_2^B, \chi_3^B, \chi_4^B)$$

Net Baryon to positive baryon fluctuations

The maximum entropy freeze-out



Assumptions

- | | |
|---------------------------------------|--|
| ➤ $B^+ B^-$ joint cumulants | $B^\pm = \left\{ \hat{A}_i \mid q_{A_i} = \pm 1 \right\}_{i \in \text{Particles}}$ |
| ➤ Boltzmann approximation | Particle IRCs = Particle factorial cumulants |
| ➤ Cells are statistically independent | No correlation between cells |
| ➤ Net-baryon density field only | $a_i = n_B \forall i$ |

Maximum Entropy $B^+ B^-$ -factorial cumulants

$$\tilde{\kappa}_{nm}^{+-}(x) = \tilde{\tilde{\kappa}}_{nm}^{+-}(x) + \hat{\Delta} \mathcal{H}_{n+m}(x) (P_+)^n (P_-)^m$$

The hydrodynamic matching condition projectors

$$P_\pm(x) = \pm \frac{\langle \delta N^\pm \rangle^{\text{HRG}}}{\langle \delta N^+ \rangle^{\text{HRG}} + \langle \delta N^- \rangle^{\text{HRG}}}$$

Net Baryon to positive baryon fluctuations

The maximum entropy freeze-out



Assumptions

- | | |
|---------------------------------------|--|
| ➤ $B^+ B^-$ joint cumulants | $B^\pm = \left\{ \hat{A}_i \mid q_{A_i} = \pm 1 \right\}_{i \in \text{Particles}}$ |
| ➤ Boltzmann approximation | Particle IRCs = Particle factorial cumulants |
| ➤ Cells are statistically independent | No correlation between cells |
| ➤ Net-baryon density field only | $a_i = n_B \forall i$ |

Maximum Entropy $B^+ B^-$ factorial cumulants and cumulants

$$\tilde{\kappa}_{nm}^{+-}(x) = \tilde{\tilde{\kappa}}_{nm}^{+-}(x) + \hat{\Delta} \mathcal{H}_{n+m}(x) (P_+)^n (P_-)^m$$

$$\tilde{\kappa}_{nm}^{+-}(x) \longrightarrow \kappa_{nm}^{+-}(x)$$

Positive Baryon to proton cumulants in the acceptance

Binomial protons in binomial acceptance

Baryon to proton

Isospin randomization

$$N + \pi \leftrightarrow \Delta$$

At equilibrium in the hadronic phase

$$P(N_p | N_B^\pm) = \text{Binomial}(N_N, q_\pm)$$



$$q_\pm(x) = \frac{\bar{n}_p^{\text{FD}}}{\bar{n}_{B^\pm}}$$

[M. Kitazawa, M. Asakawa Phys. Rev. C 86, 024904](#)

Collision event

Baryon number
fluctuations

Proton number
fluctuations
in acceptance

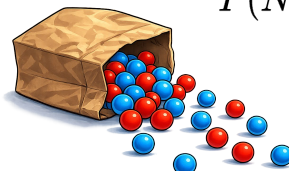
Charge
conservation

Protons to measured protons

Cooper-Frye ratio

Estimating the ratio of Acc/full

$$p(x_i) = \frac{\int_{\text{acc}} \frac{d^3p}{E_p} d\Sigma_\mu(x_i) p^\mu(x_i) f_p[u^\nu p_\nu, T(x_i), \mu_p(x_i)]}{\int \frac{d^3p}{E_p} d\Sigma_\mu(x_i) p^\mu(x_i) f_p[u^\nu p_\nu, T(x_i), \mu_p(x_i)]}$$



$$P(N_{p,\text{Acc}} | N_p) = \text{Binomial}(N_p, p)$$

[Bzdak, Koch, Phys. Rev. C 86, 044904](#)

Positive Baryon to proton cumulants in the acceptance

Binomial protons in binomial acceptance



Baryon to proton

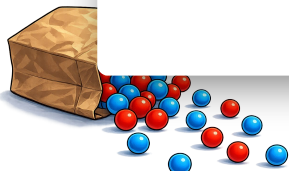
Isospin randomization

Total binomial distribution

At e

$$\alpha_{\pm}(x) = q_{\pm}(x)p(x)$$

$$P(N_{p,\text{Acc}}|N_B^{\pm}) = \text{Binomial}(N_B^{\pm}, \alpha_{\pm})$$



[M. Kitazawa, M. Asakawa Phys. Rev. C 86, 024904](#)

Protons to measured protons

Cooper-Frye ratio



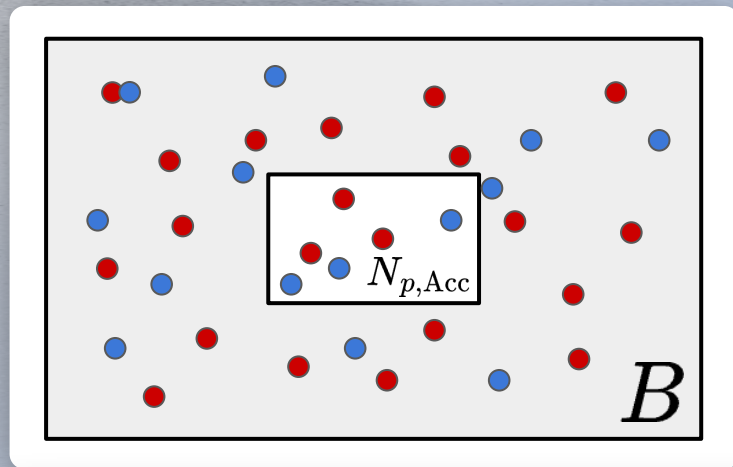
[Bzdak, Koch, Phys. Rev. C 86, 044904](#)

$$\frac{u_p(x_i)]}{p(x_i)]}$$

$$N_p, p)$$

Positive Baryon to proton cumulants in the acceptance

Towards the global B conservation



$$G(t_+, t_-) = \ln \left\langle e^{t_+ B_+ + t_- B_-} \right\rangle$$

$$\tilde{G}(t, t_B) = \ln \left\langle e^{t N_{p,Acc} + t_B B} \right\rangle$$

$$B = B^+ - B^-$$

$$\tilde{G}(t, t_B) = G(t_B + \theta(t, \alpha_+), -t_B)$$

GCE proton, total Baryon number joint cumulants

$$\kappa_{n,m}^{\text{GCE}}(N_{p,Acc}, B) = \sum_{k=1}^n B_{n,k}(c_1, c_2, \dots, c_{n-k+1}) \sum_{q=0}^m \binom{m}{q} (-1)^{m-q} \kappa_{k+q, m-q}^{+-}$$

$B_{n,k}$ Partial bell polynomials

$$c_k \equiv \partial_t^k \theta(t, \alpha_+) \big|_{t=0}$$

$$\theta(t, \alpha_+) \equiv \ln(1 - \alpha_+ + \alpha_+ e^t)$$

Sub ensemble Acceptance Method (SAM 3.0)

Collision event

Baryon number
fluctuations

Proton number
fluctuations
in acceptance

Charge
conservation

Connecting CE and GCE cumulants

$$\kappa_n^{\text{CE}}(N_{p,\text{Acc}}) = \text{SAM} \left[\kappa_{l,m}^{\text{GCE}}(N_{p,\text{Acc}}, B) \right]$$

R. Poberezhniuk, V. A. Kuznietsov, GP, V. Vochenko in prep

Two first orders

$$\kappa_1^{\text{CE}}(N_{p,\text{Acc}}) = \kappa_{1,0}^{\text{GCE}}(N_{p,\text{Acc}}, B)$$

$$\kappa_2^{\text{CE}}(N_{p,\text{Acc}}) = \kappa_{2,0}^{\text{GCE}}(N_{p,\text{Acc}}, B) - \frac{\kappa_{1,1}^{\text{GCE}}(N_{p,\text{Acc}}, B)^2}{\kappa_{0,2}^{\text{GCE}}(N_{p,\text{Acc}}, B)}$$

QCD Phase diagram setups

Acceptance probabilities and EoS

Proton acceptance probability

- Freeze-out = constant energy density
- Hydro cells contribution = distribution

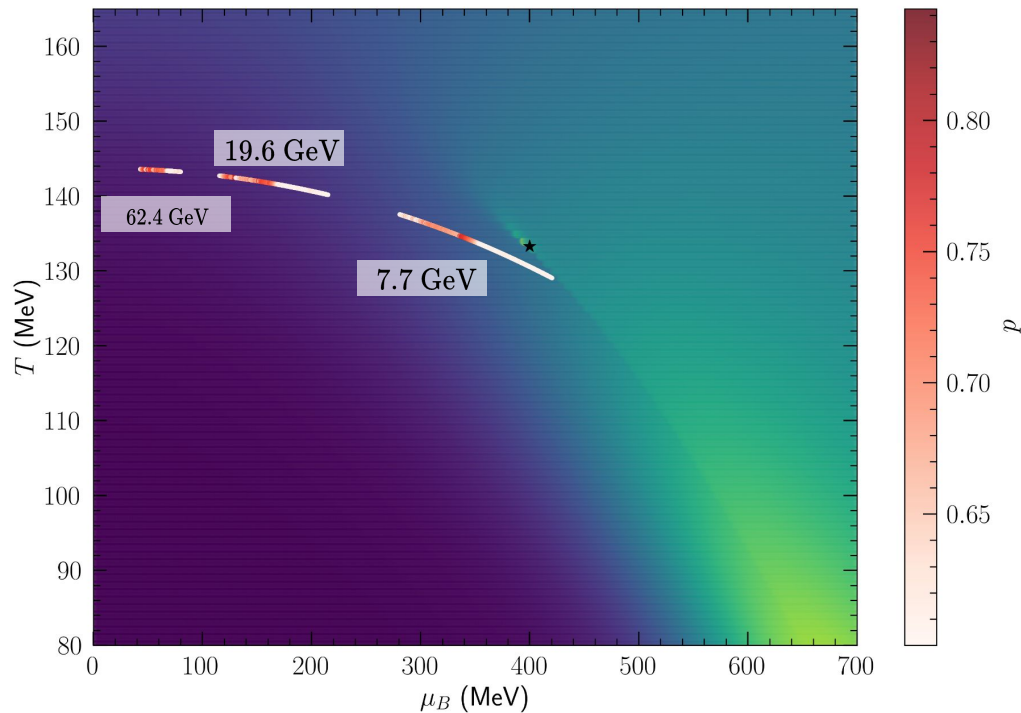
2D Ising T' Expansion 400

3D Ising model mapped

Parameters:

$$T_c, \mu_c = 133.3, 400 \text{ MeV}$$

No strangeness neutrality



QCD Phase diagram setups

Acceptance probabilities and EoS

Proton acceptance probability

- Freeze-out = constant energy density
- Hydro cells contribution = distribution

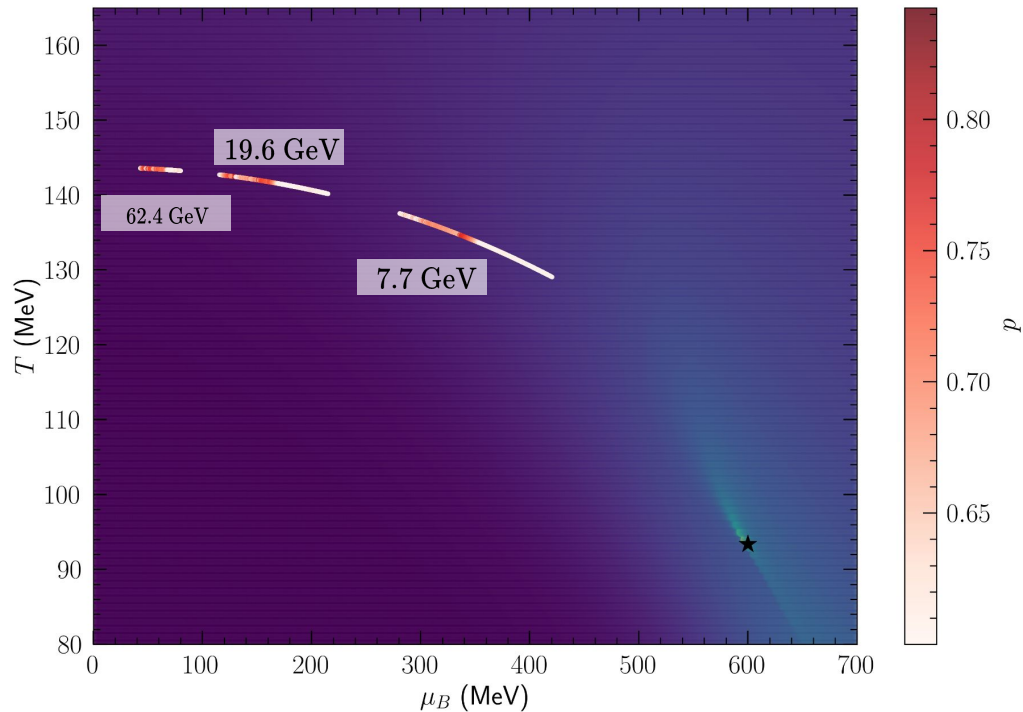
2D Ising T' Expansion 600

3D Ising model mapped

Parameters:

$$T_c, \mu_c = 93.4, 600 \text{ MeV}$$

No strangeness neutrality



QCD Phase diagram setups

Acceptance probabilities and EoS

Proton acceptance probability

- Freeze-out = constant energy density
- Hydro cells contribution = distribution

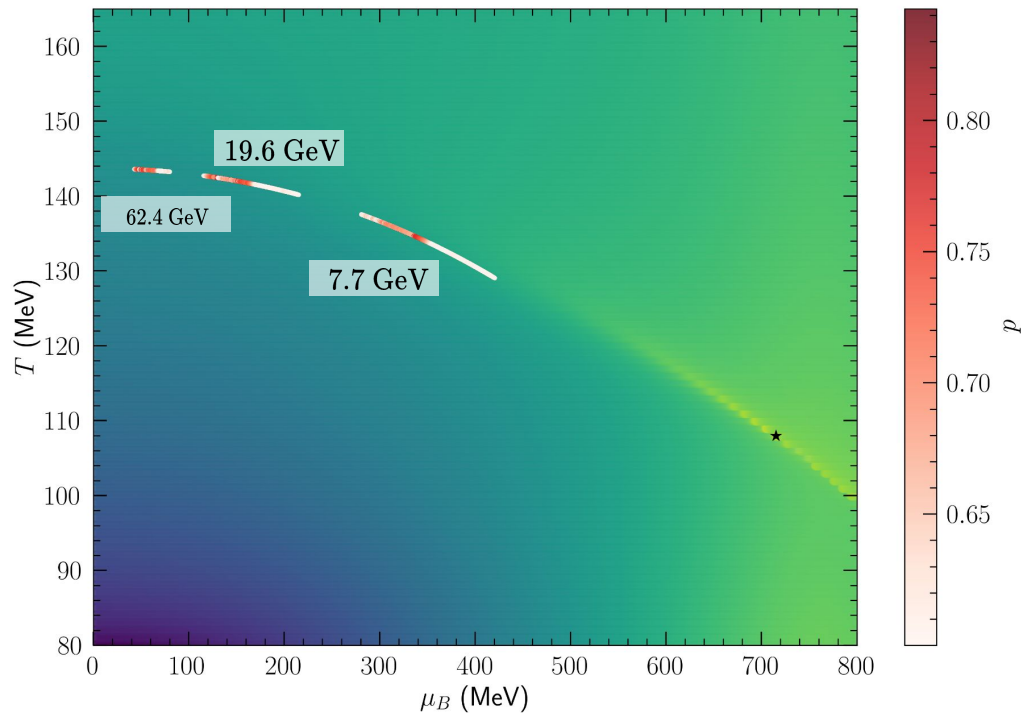
Constant Entropy contour

From lattice Taylor expansion

Parameters:

$$T_c, \mu_c = 108, 715 \text{ MeV}$$

Strangeness neutral



EoS comparison

EoS to data comparison

Close to experimental measurement

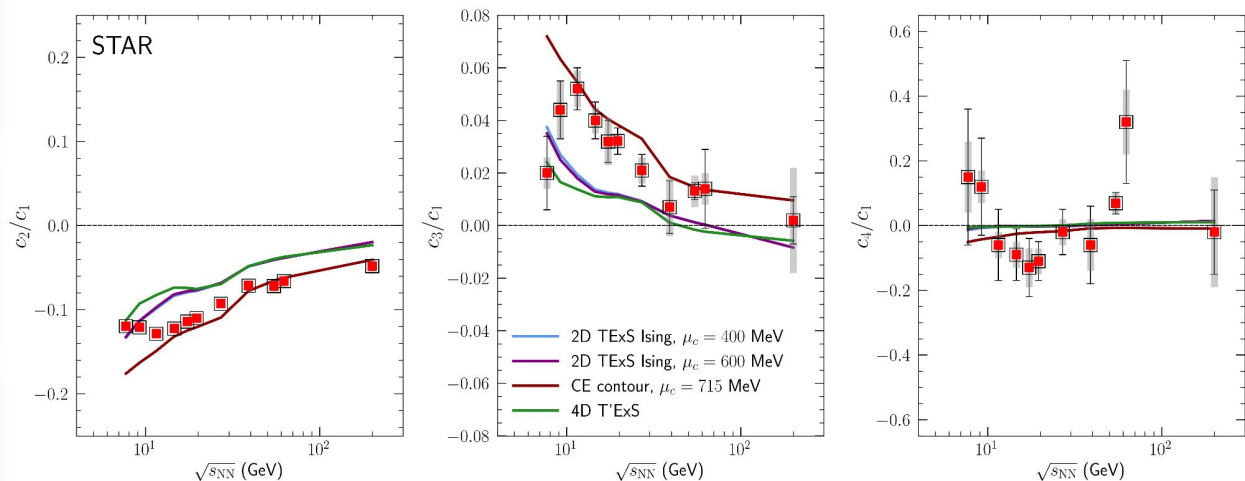
- Scale: Canonical correction
- Different EoS

Baseline?

$$\hat{\Delta}\mathcal{H}_{n+m}(x) = \delta V(x)T(x)^3 \bar{\chi}_{n+m}^B F_{nm}(\chi_2^B, \chi_3^B, \chi_4^B)$$

EoS consistency

NEOS-BQS $T_{fo} \mu_{B,fo}$ $\begin{matrix} \nearrow \text{EoS} \\ \searrow \text{iHRG} \end{matrix}$



GP, M. Kahangirwe, H. Shah, R. Poberezhniuk C. Ratti, V. Vovchenko in prep

EoS comparison

EoS to data comparison

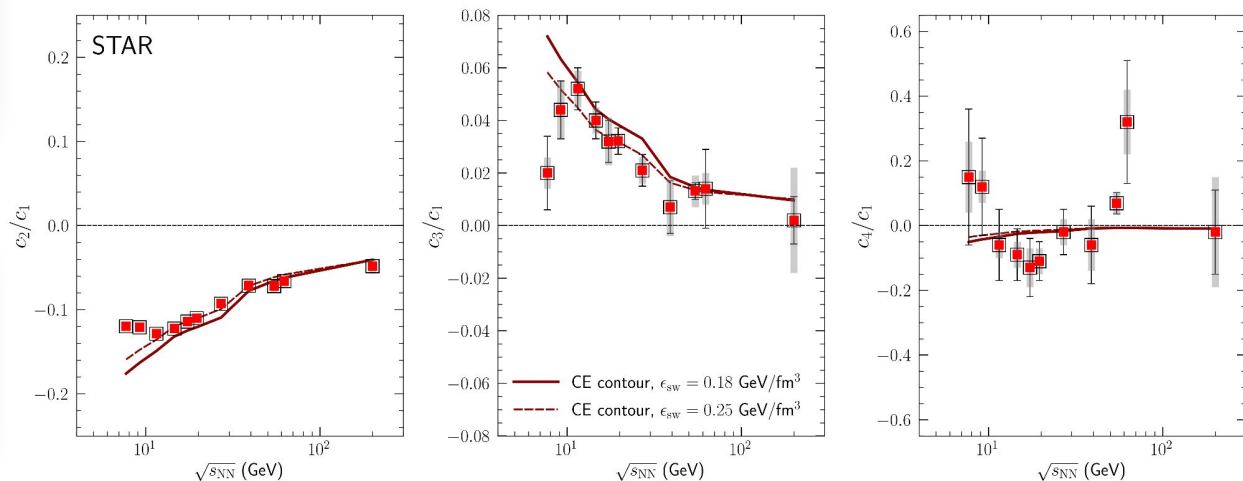
Close to experimental measurement

- Scale: Canonical correction
- Different EoS

F0 energy density?

$$e_{\text{sw}} = 0.18 \text{ GeV/fm}^3 \quad e_{\text{sw}} = 0.25 \text{ GeV/fm}^3$$

$$e_{\text{sw}} \sim 0.34 - 0.38 \text{ GeV/fm}^3$$



GP, M. Kahangirwe, H. Shah, R. Poberezhniuk C. Ratti, V. Vovchenko in prep

EoS comparison

EoS to data comparison

Close to experimental measurement

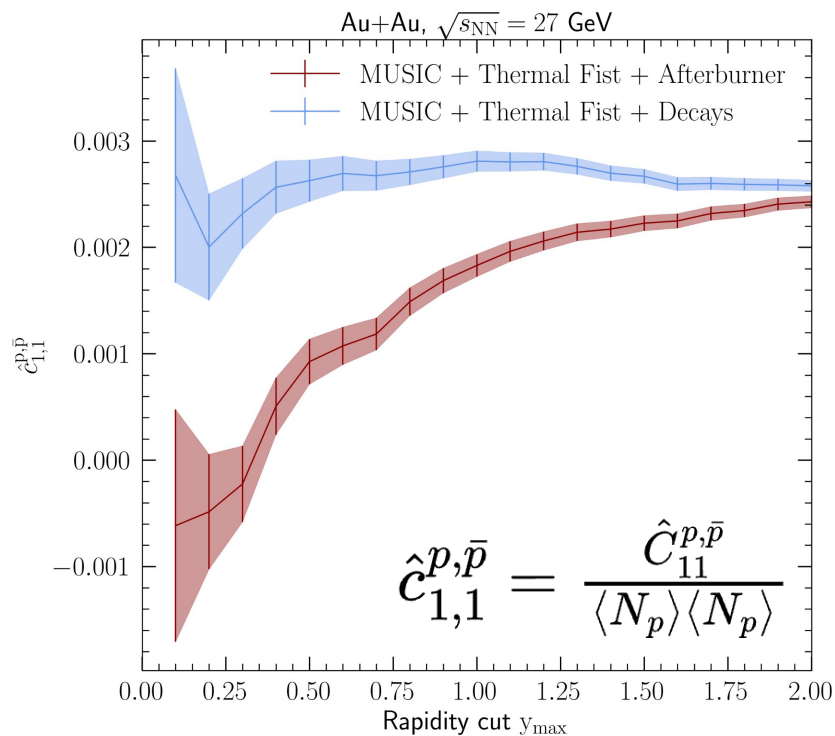
- Scale: Canonical correction
- Different EoS

F0 energy density?

$$e_{\text{sw}} = 0.18 \text{ GeV/fm}^3 \quad e_{\text{sw}} = 0.25 \text{ GeV/fm}^3$$
$$e_{\text{sw}} \sim 0.34 - 0.38 \text{ GeV/fm}^3$$

Late stage interactions

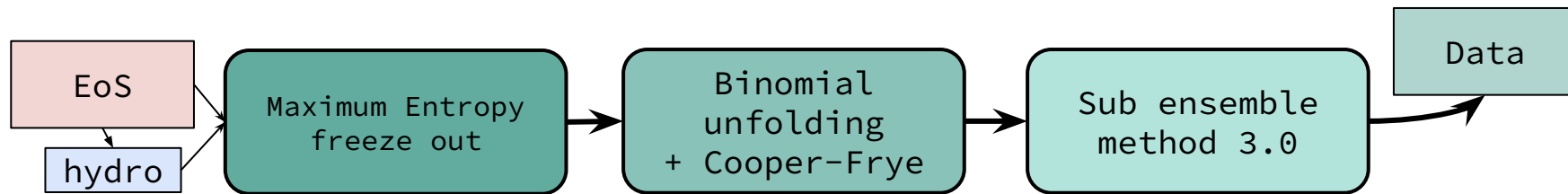
- Long range correlation
- Annihilations



J. Parra, GP, V. Koch, V. Vovchenko in prep

Conclusion

- Measuring fluctuations in heavy-ion experiments is not straightforward
- Corrections on the cumulants can be evaluated using the pipeline:



- Calculation of the factorial cumulants based on the literature susceptibilities, relatively good agreement.

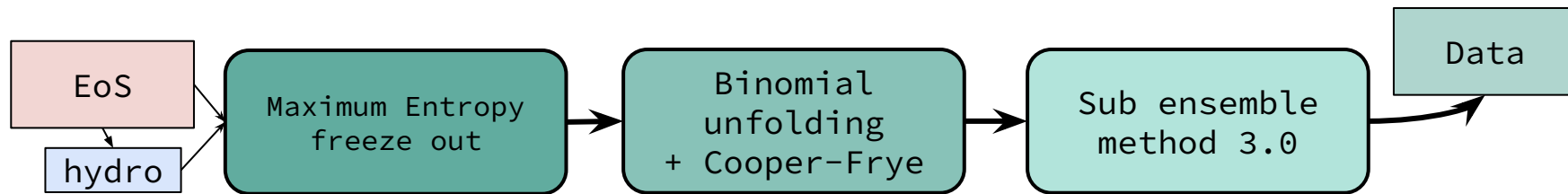
HRG baseline?

Freeze-out energy density

EoS Mixing

Conclusion

- Measuring fluctuations in heavy-ion experiments is not straightforward
- Corrections on the cumulants can be evaluated using the pipeline:



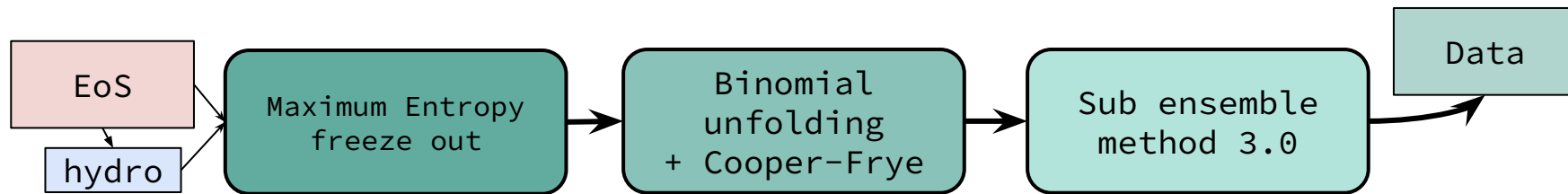
- Calculation of the factorial cumulants based on the literature susceptibilities, relatively good agreement.

HRG baseline?

Can we do more?

~~Conclusion~~

- Measuring fluctuations in heavy-ion experiments is not straightforward
- Corrections on the cumulants can be evaluated using the pipeline:



- Calculation of the factorial cumulants based on the literature susceptibilities, relatively good agreement.

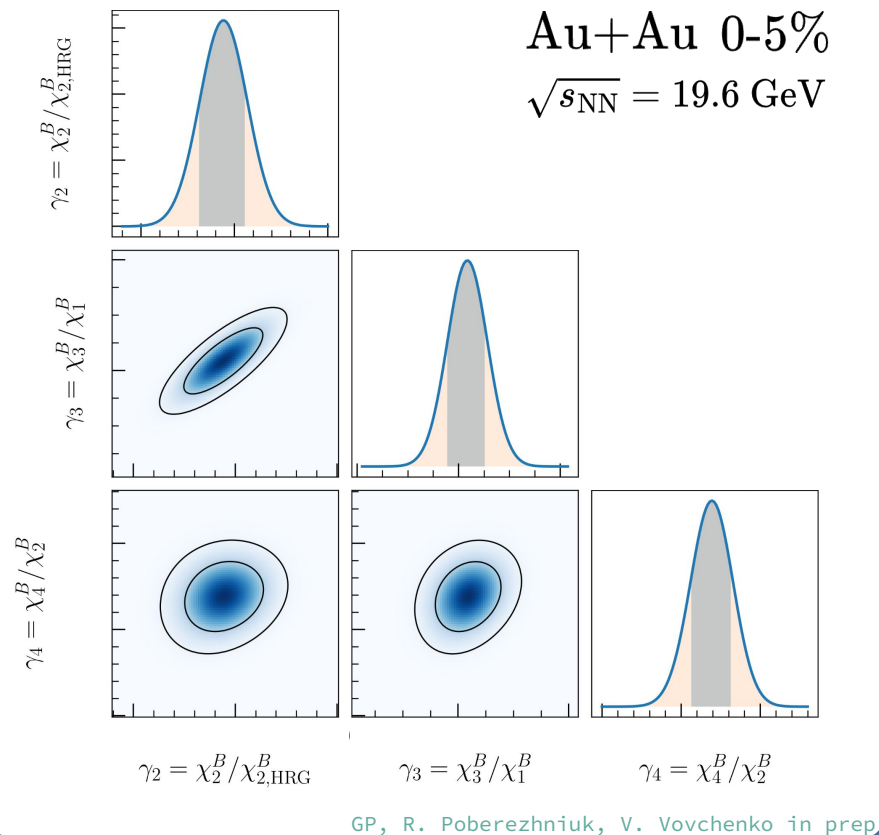
HRG baseline?

Can we do more?

YES!

Extracting QCD susceptibilities

Bayesian analysis on STAR data

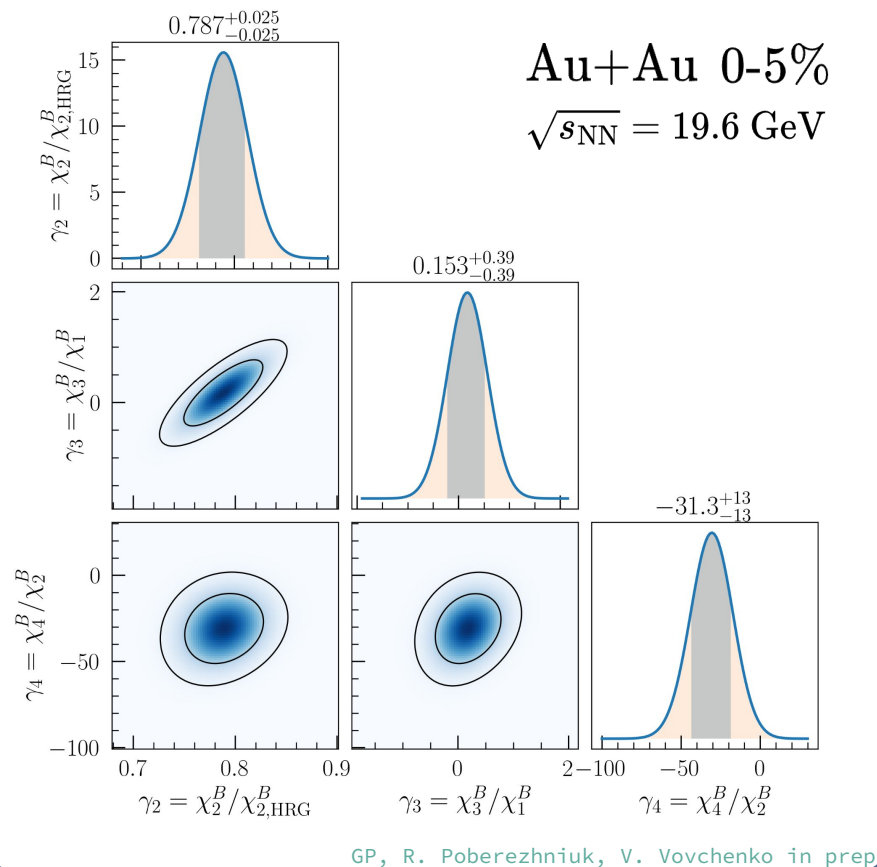


Extracting QCD susceptibilities

Bayesian analysis on STAR data

Extraction of QCD static response

Data driven approach to constrain QCD susceptibilities!



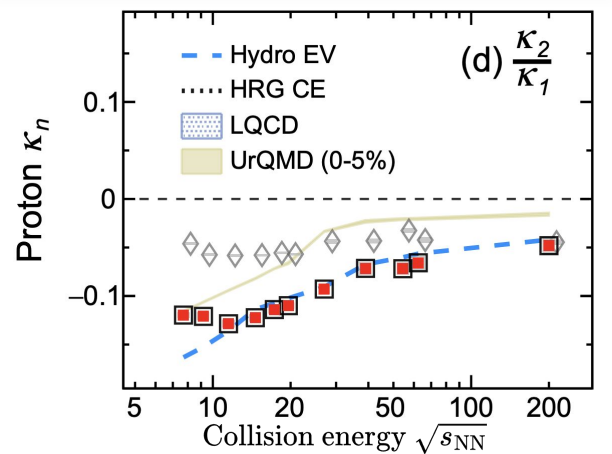
Extracting QCD susceptibilities

Bayesian analysis on STAR data

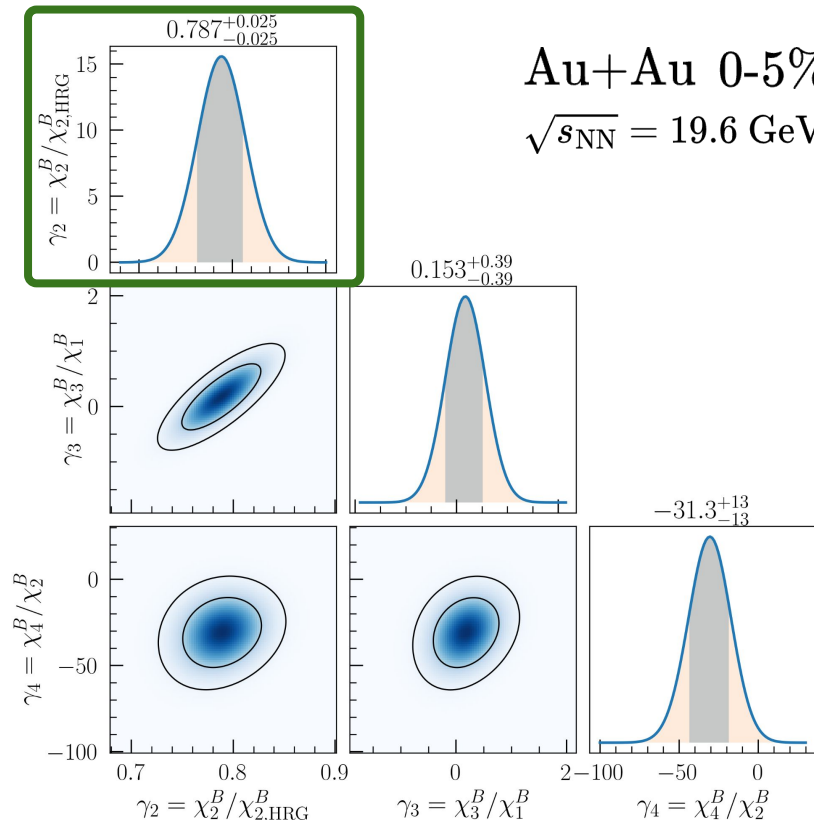
Extraction of QCD static response

Data driven approach to constrain QCD susceptibilities!

Possible at the 2nd order!



STAR Collaboration



GP, R. Poberezhniuk, V. Vovchenko in prep

Extracting QCD susceptibilities

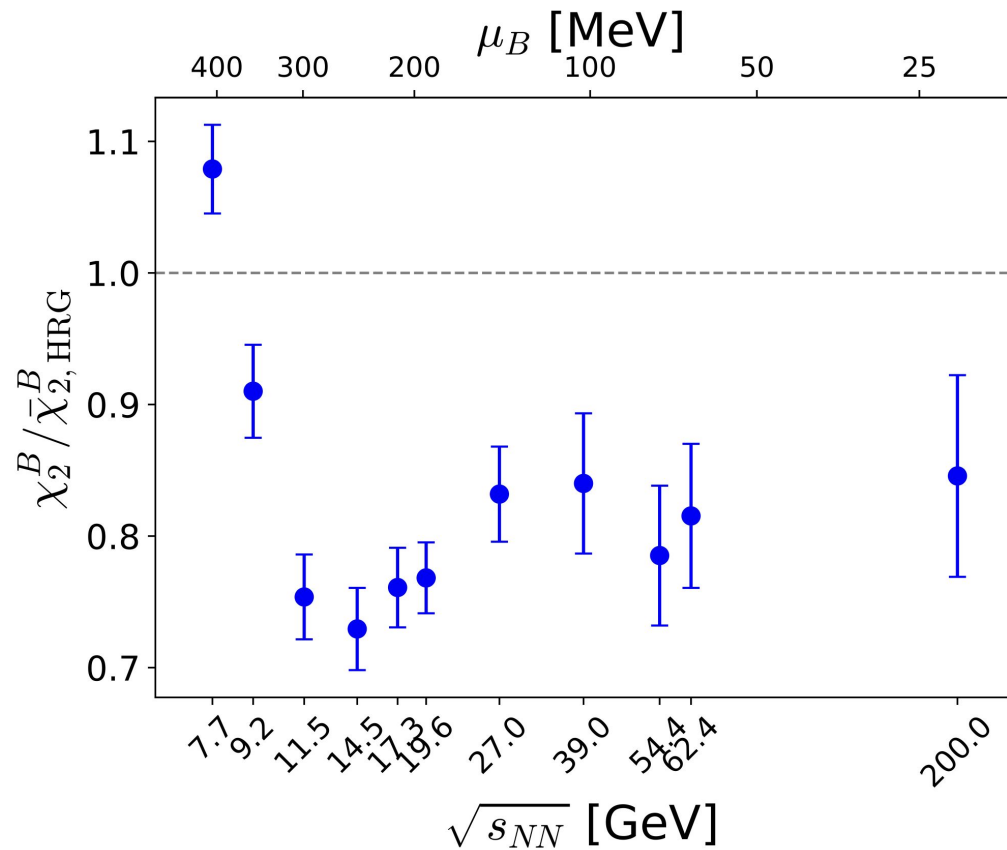
Deviations energy dependence

Deviations from Poisson baseline

Significant!

Non-monotonic structure

A change from repulsive to attractive interactions at small energies



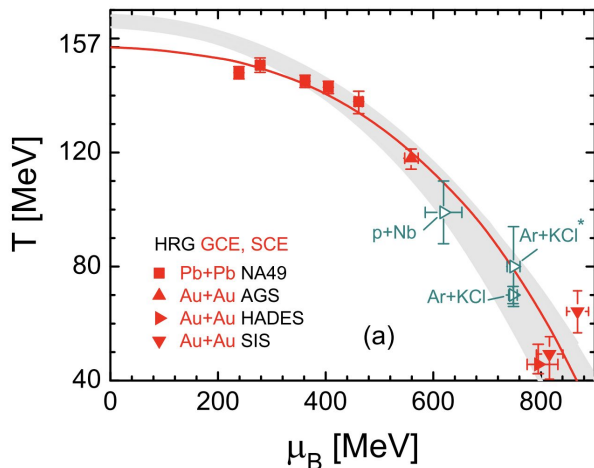
GP, R. Poberezhniuk, V. Vovchenko in prep

Extracting QCD susceptibilities

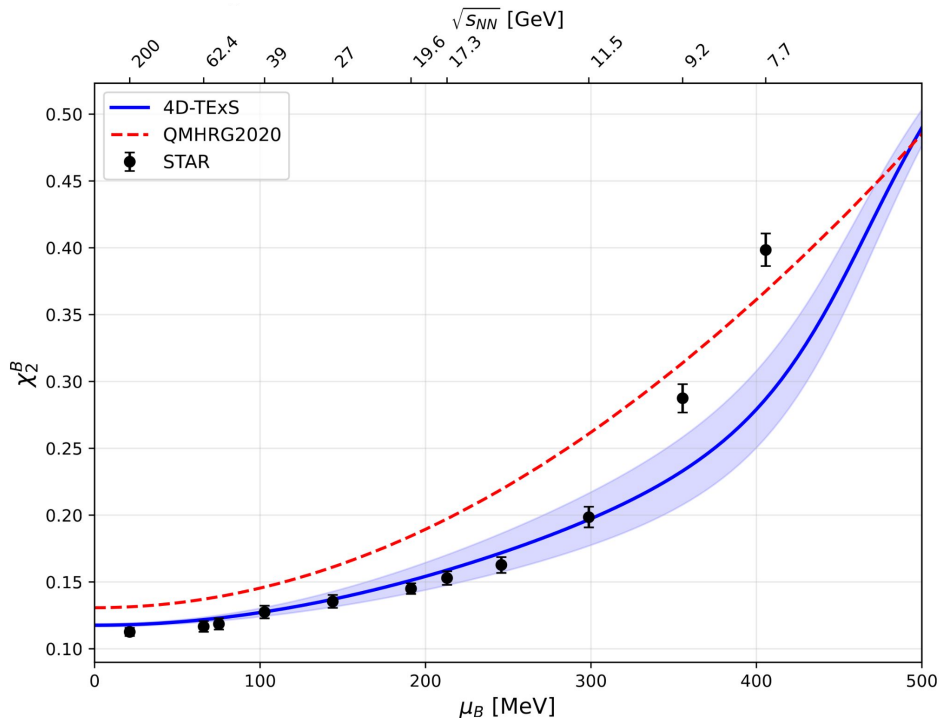
Comparison with 4D T'ExS EoS

Hadron Resonance Gas with Quark Model
particle list

Strangeness neutral freeze-out line



[V. Vovchenko et al Phys. Rev. C 93, 064906](#)



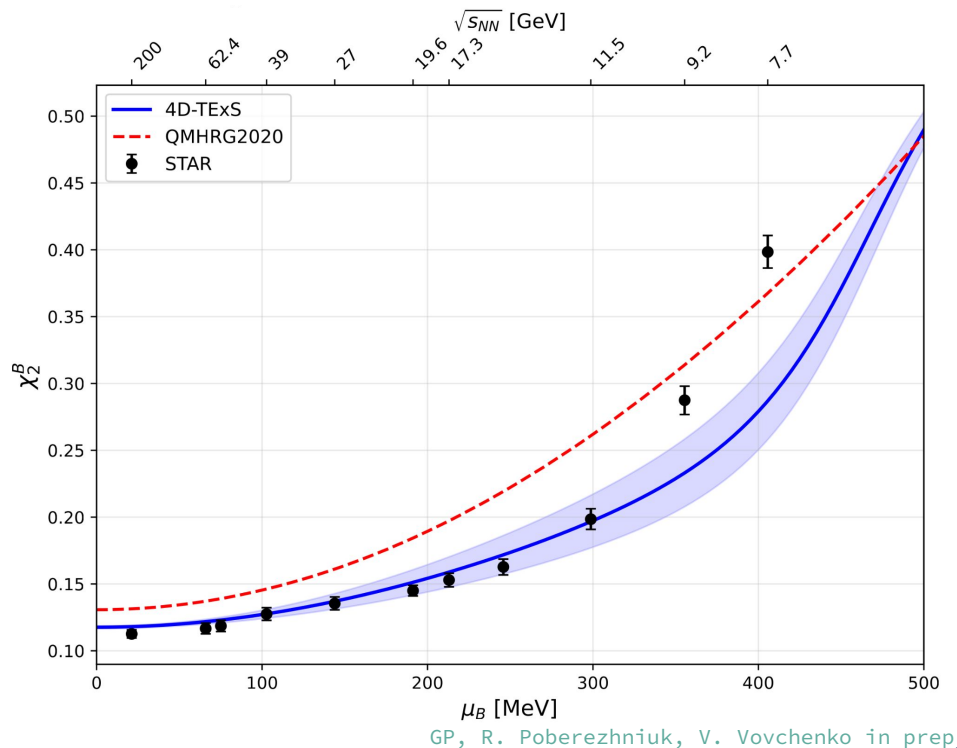
GP, R. Poberezhniuk, V. Vovchenko in prep

Extracting QCD susceptibilities

Comparison with 4D T'ExS EoS

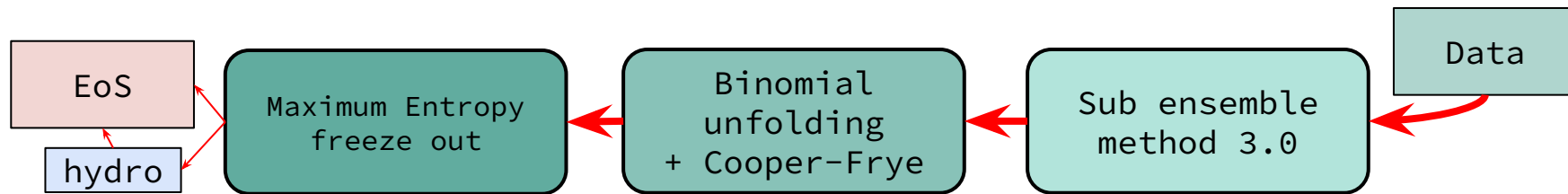
Agreement with lattice QCD based
equation of state at low μ_B

Deviation at large baryon chemical
potential



Conclusion II

- Measuring fluctuations in heavy-ion experiments is not straightforward
- Data driven constraints on the QCD susceptibilities:



Possible extensions?

- Inclusion of hydrodynamics energy density correlators and mixed correlators
- Consider B, Q and S as conserved charges for canonical corrections?
- Extend the bayesian analysis to other observables?

Thank you for your attention!

Simulation of the collision event

Smooth collision event: [iEBE-MUSIC](#)

Collision event

Baryon number
fluctuations

Proton number
fluctuations
in acceptance

Charge
conservation

Transverse: Glauber model

$$\epsilon(x, y, \eta_s) \propto e(\eta_s) T_{\text{Proj}}(x, y) T_{\text{Targ}}(x, y)$$

$$n_B(x, y, \eta_s) \propto f_{\text{Proj}}^{n_B}(\eta_s) T_{\text{Proj}}(x, y) + f_{\text{Targ}}^{n_B}(\eta_s) T_{\text{Targ}}(x, y)$$

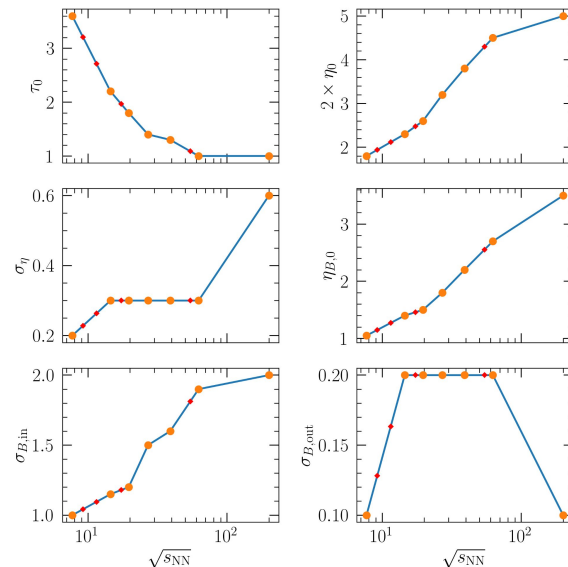
Longitudinal: Density envelopes

$$e(\eta_s, y_{CM}) \propto \exp\left[-\frac{(|\eta_s - y_{CM}| - \eta_0)^2}{2\sigma_\eta^2}\right] \theta(|\eta_s - y_{CM}| - \eta_0)$$

$$f_{\text{Proj}}^{n_B} \propto \theta(\eta_s - \eta_{B,0}) \exp\left[-\frac{(\eta_s - \eta_{B,0})^2}{2\sigma_{B,\text{out}}^2}\right] + \theta(\eta_s + \eta_{B,0}) \exp\left[-\frac{(\eta_s + \eta_{B,0})^2}{2\sigma_{B,\text{in}}^2}\right]$$

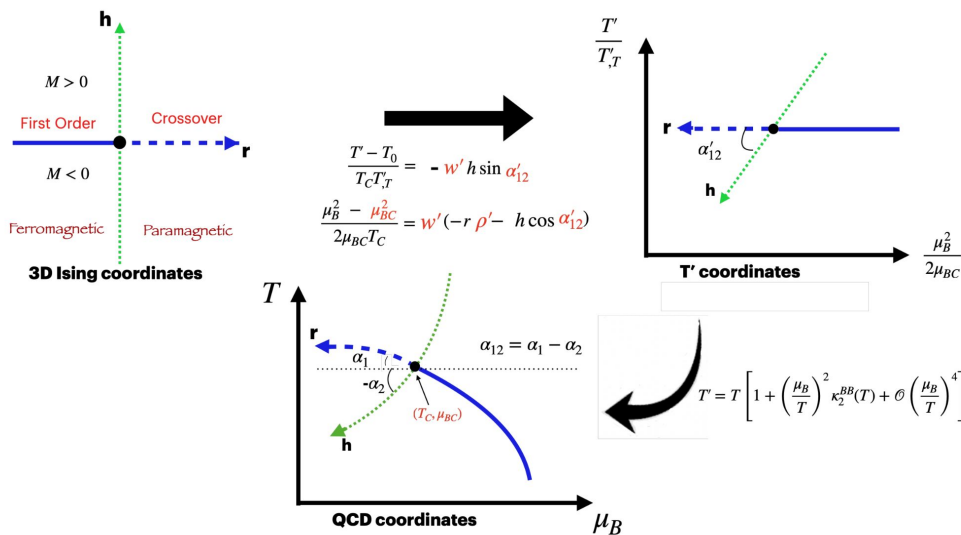
$$f_{\text{Targ}}^{n_B} \propto \theta(\eta_s + \eta_{B,0}) \exp\left[-\frac{(\eta_s + \eta_{B,0})^2}{2\sigma_{B,\text{in}}^2}\right] + \theta(-\eta_s - \eta_{B,0}) \exp\left[-\frac{(\eta_s + \eta_{B,0})^2}{2\sigma_{B,\text{out}}^2}\right]$$

Parameters tuned on the BES Au+Au
charged hadron and net-proton rapidity
distributions



Simulation of the collision event

3D Ising mapping



Parameters

$$T_c = 93.4 \text{ MeV}$$

$$\mu_{B,c} = 600 \text{ MeV}$$

$$\alpha_{12} = 90^\circ$$

$$\omega = 2.25$$

$$\rho = 1.0$$

Simulation of the collision event

Maximum Entropy F0 details

Maximum Entropy cumulants

General definition of the particle cumulants

$$\kappa_{\hat{A}_1 \dots \hat{A}_k} = \left\langle \prod_{i=1}^k \delta N_{\hat{A}_i} \right\rangle$$
$$\hat{A}_i \equiv (x_A, p_A, q_A, s_A, g_A, m_A, \dots)_i$$

Cumulants from single particle distributions correlators

$$\kappa_{\hat{A}_1 \dots \hat{A}_k} = \int_{\hat{A}_1 \dots \hat{A}_k} G_{A_1 \dots A_k}$$
$$G_{A_1 \dots A_k} = \langle \delta f_{A_1} \dots f_{A_k} \rangle$$

IRCs from single particle distributions IRCs

$$\hat{\Delta} \kappa_{\hat{A}_1 \dots \hat{A}_k} = \int_{\hat{A}_1 \dots \hat{A}_k} \hat{\Delta} G_{A_1 \dots A_k}$$

[M.S. Pradeep, M.A. Stephanov Phys. Rev. Lett. 130, 162301](#)

Particle IRCs from hydrodynamics

$$\hat{\Delta} \kappa_{\hat{A}_1 \dots \hat{A}_k} = \hat{\Delta} \mathcal{H}_{a_1 \dots a_k} \prod_{i=1}^k P_{\hat{A}_i}^{a_i}$$