2025 RHIC/AGS ANNUAL USERS' MEETING

RHIC 25:

A quarter century of discovery

May 20–23, 2025

Cumulants and fluctuations measurement at the BES II

Volodymyr Vovchenko (University of Houston)

2025 RHIC/AGS Annual Users' Meeting

May 20, 2025

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QCD under extreme conditions

$$\mathcal{L} = \sum_{q=u,d,s,\ldots} \bar{q}\left[i\gamma^{\mu}(\partial_{\mu} - igA^{a}_{\mu}\lambda_{a}) - m_{q}\right] q - \frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a}$$

What we know

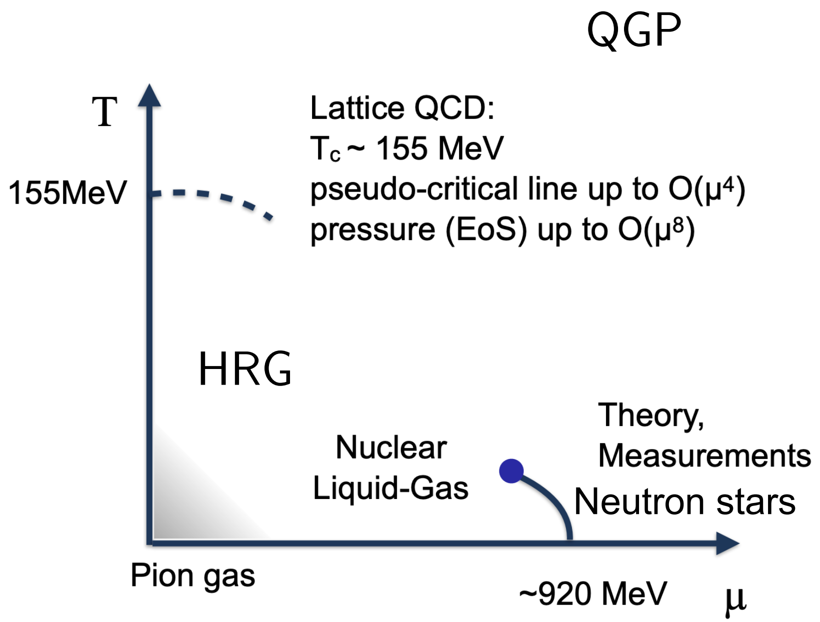
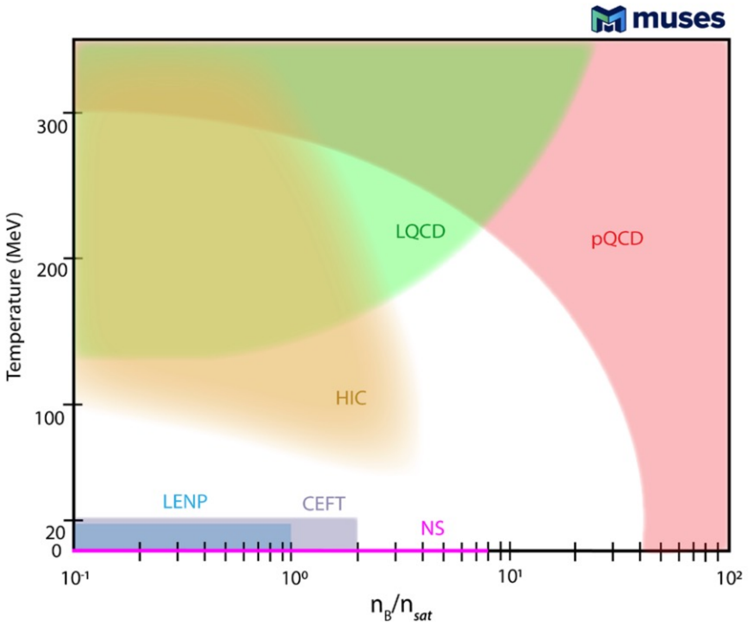


Figure courtesy of V. Koch

What we think we know



MUSES Collaboration, LRR 27 (2024)

What we hope to know

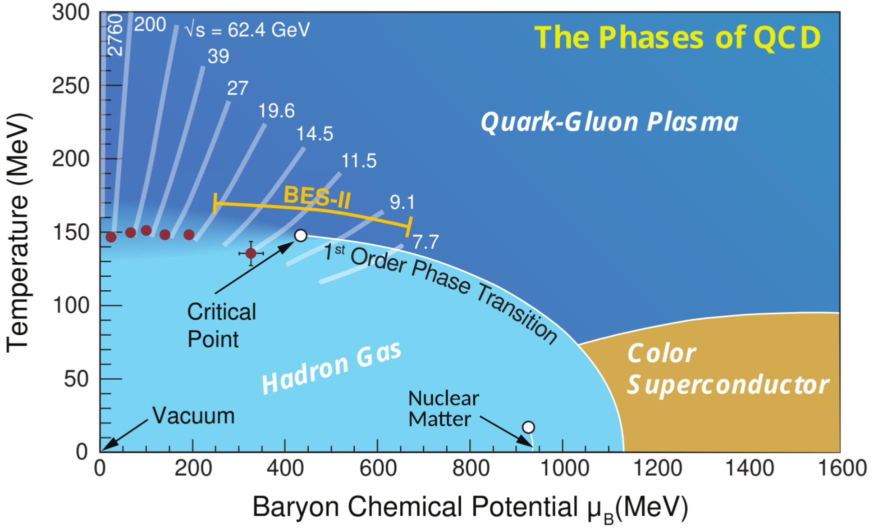


Figure from Bzdak et al., Phys. Rept. '20 & 2015 US Nuclear LRP

"The location of the transition from a gas of hadrons to QGP and the exact nature of this transition is of fundamental interest"

2023 Long Range Plan for Nuclear Science

Critical point predictions as of a some years ago

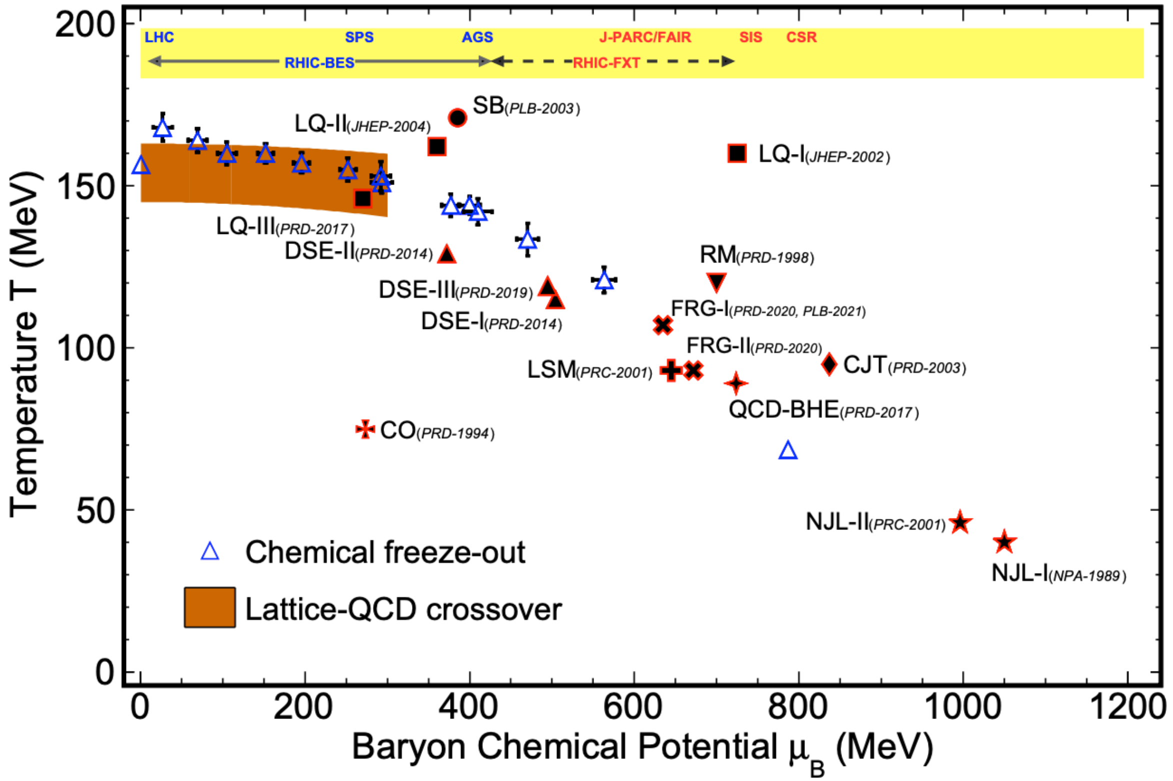


Figure adapted from A. Pandav, D. Mallick, B. Mohanty, Prog. Part. Nucl. Phys. 125 (2022)

- Including the possibility that the QCD critical point does not exist at all

de Forcrand, Philipsen, JHEP 01, 077 (2007); VV, Steinheimer, Philipsen, Stoecker, PRD 97, 114030 (2018)

Critical point predictions as of a some years ago

Lattice QCD

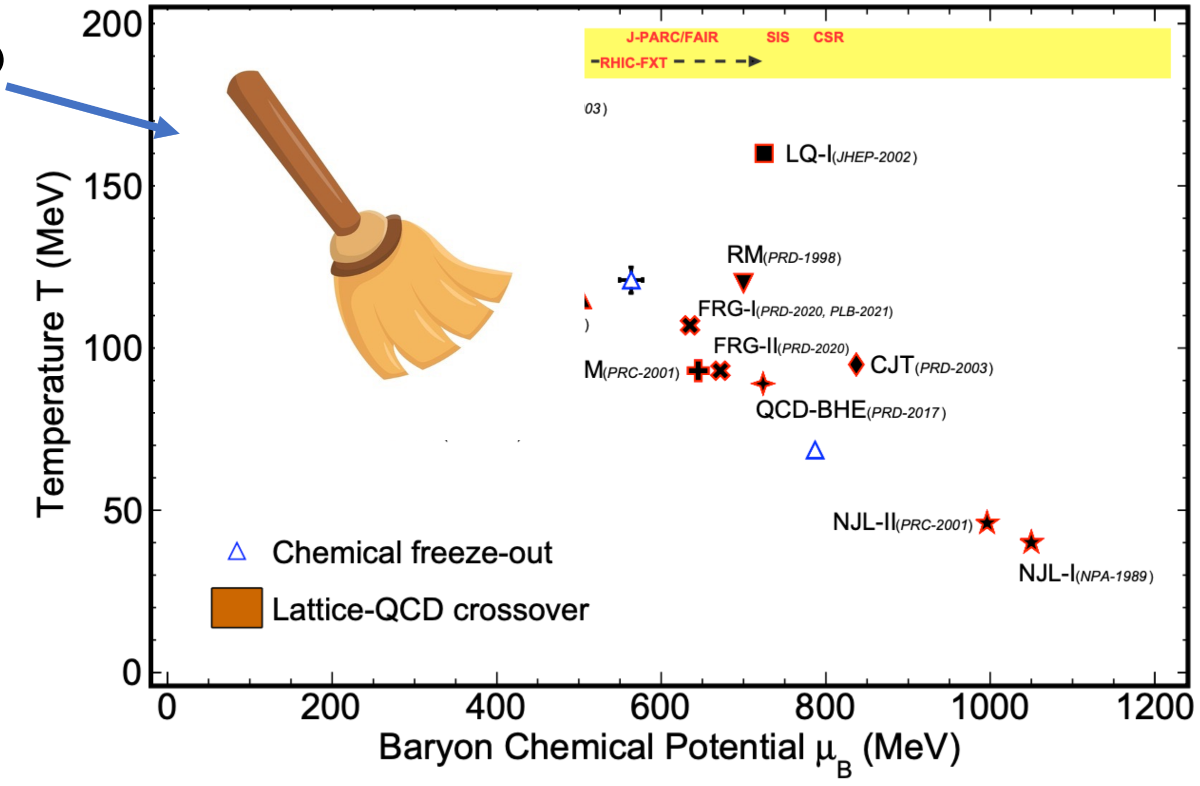


Figure adapted from A. Pandav, D. Mallick, B. Mohanty, Prog. Part. Nucl. Phys. 125 (2022)

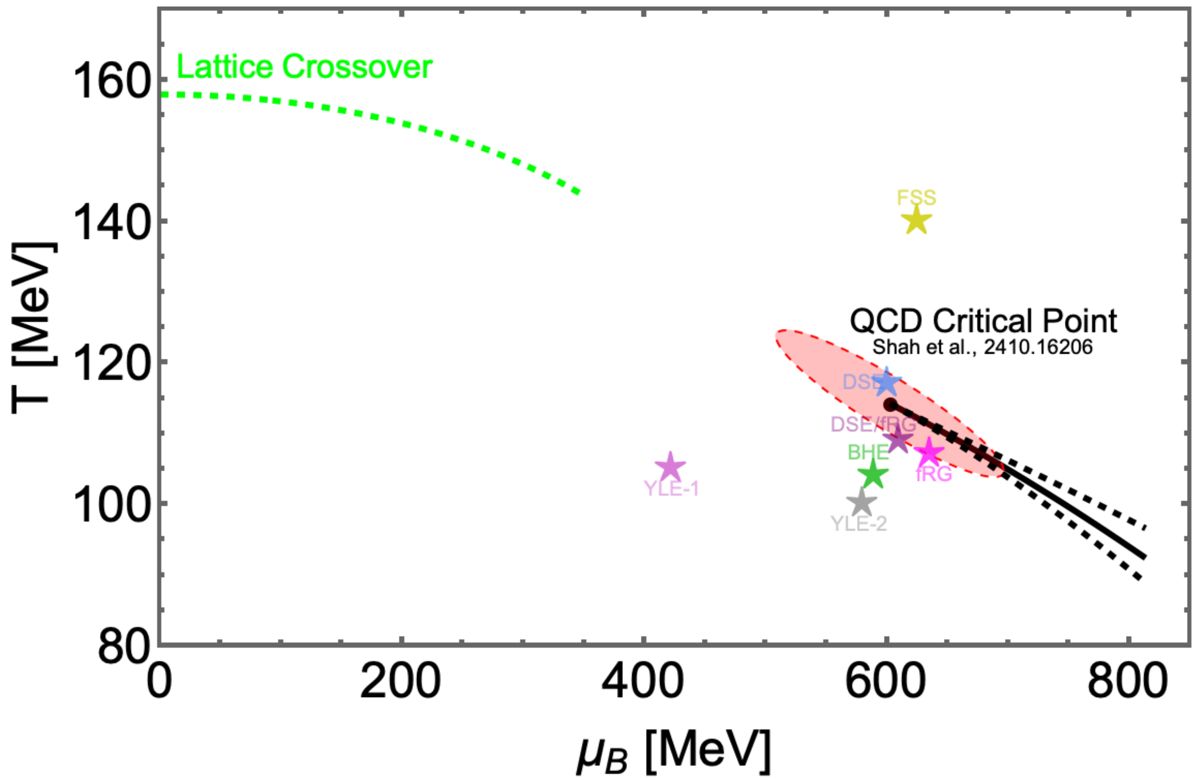
- Including the possibility that the QCD critical point does not exist at all

  de Forcrand, Philipsen, JHEP 01, 077 (2007); VV, Steinheimer, Philipsen, Stoecker, PRD 97, 114030 (2018)

- Lattice QCD excludes the CP at $\mu_B < 450$ MeV on (one-sided) 2σ level

  Borsanyi et al., arXiv:2502.10267

Critical point estimates



Critical point estimate at $O(\mu_B^2)$:

$T_c = 114 \pm 7$ MeV, $\mu_B = 602 \pm 62$ MeV

Estimates from recent literature:

YLE-1: D.A. Clarke et al. (Bielefeld-Parma), arXiv:2405.10196

YLE-2: G. Basar, PRC 110, 015203 (2024)

BHE: M. Hippert et al., arXiv:2309.00579

fRG: W-J. Fu et al., PRD 101, 054032 (2020)

DSE/fRG: Gao, Pawlowski., PLB 820, 136584 (2021)

DSE: P.J. Gunkel et al., PRD 104, 052022 (2021)

FSS: A. Sorensen et al., arXiv:2405.10278

Optimist's view: Different estimates converge onto the same region because QCD CP is likely there

Pessimist's view: Different estimates converge onto the same region because it's the closest not yet ruled out by LQCD

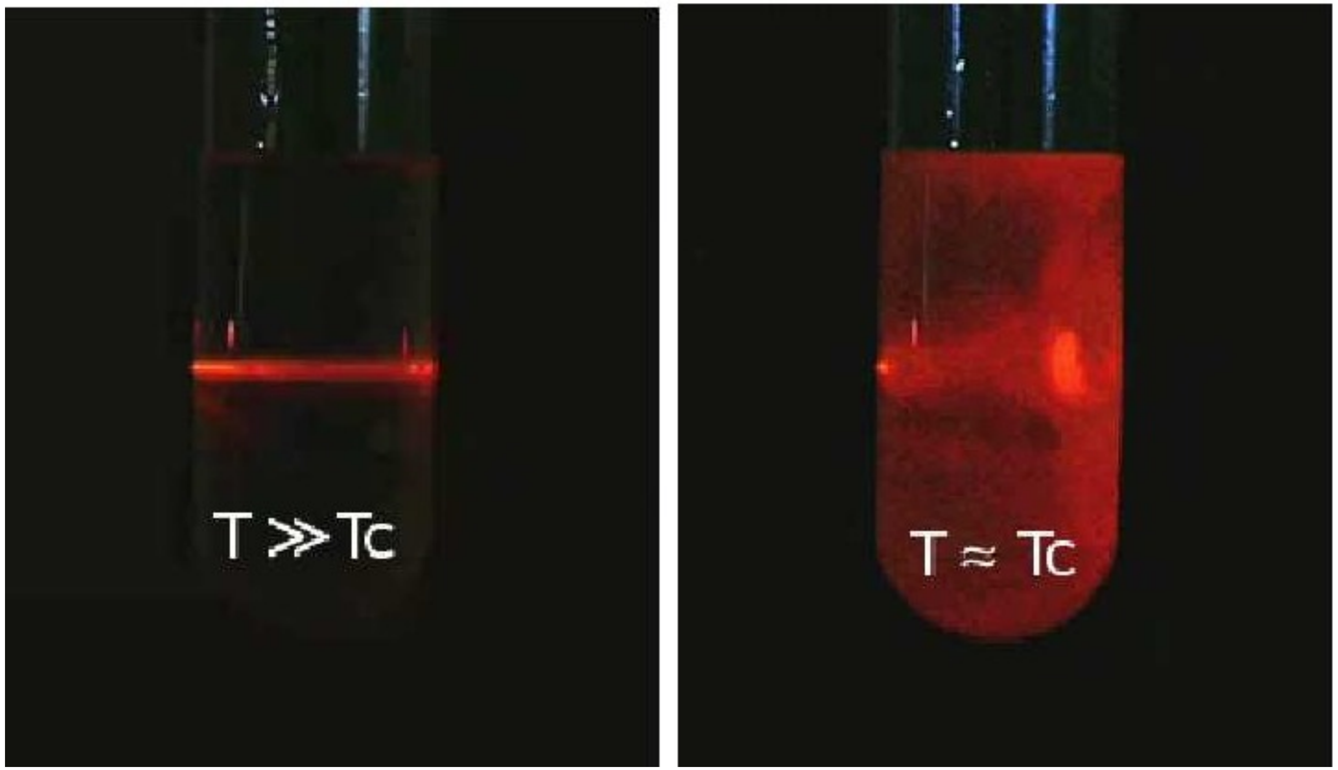
"…experimental measurements are essential to determine whether a QCD critical point exists."

2023 Long Range Plan for Nuclear Science

Critical point and fluctuations

Density fluctuations at macroscopic length scales

Critical opalescence



Unfortunately, we cannot do this in heavy-ion collisions

Event-by-event fluctuations and statistical mechanics

Consider a fluctuating number N

Cumulants: $G_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$

variance $\kappa_2 = \langle (\Delta N)^2 \rangle = \sigma^2$  width

skewness $\kappa_3 = \langle (\Delta N)^3 \rangle$  asymmetry

kurtosis $\kappa_4 = \langle (\Delta N)^4 \rangle - 3\langle (\Delta N^2) \rangle^2$  peak shape

Experiment:

$$P(N) \sim \frac{N_{events}(N)}{N_{events}^{total}}$$

Statistical mechanics:

Grand partition function $$ln\, Z^{gce}(T, V, \mu) = ln\left[\sum_N e^{\mu N} Z^{ce}(T, V, N)\right],$$

$$\kappa_n \propto \frac{\partial^n (\ln\, Z^{gce})}{\partial(\mu_N)^n}$$

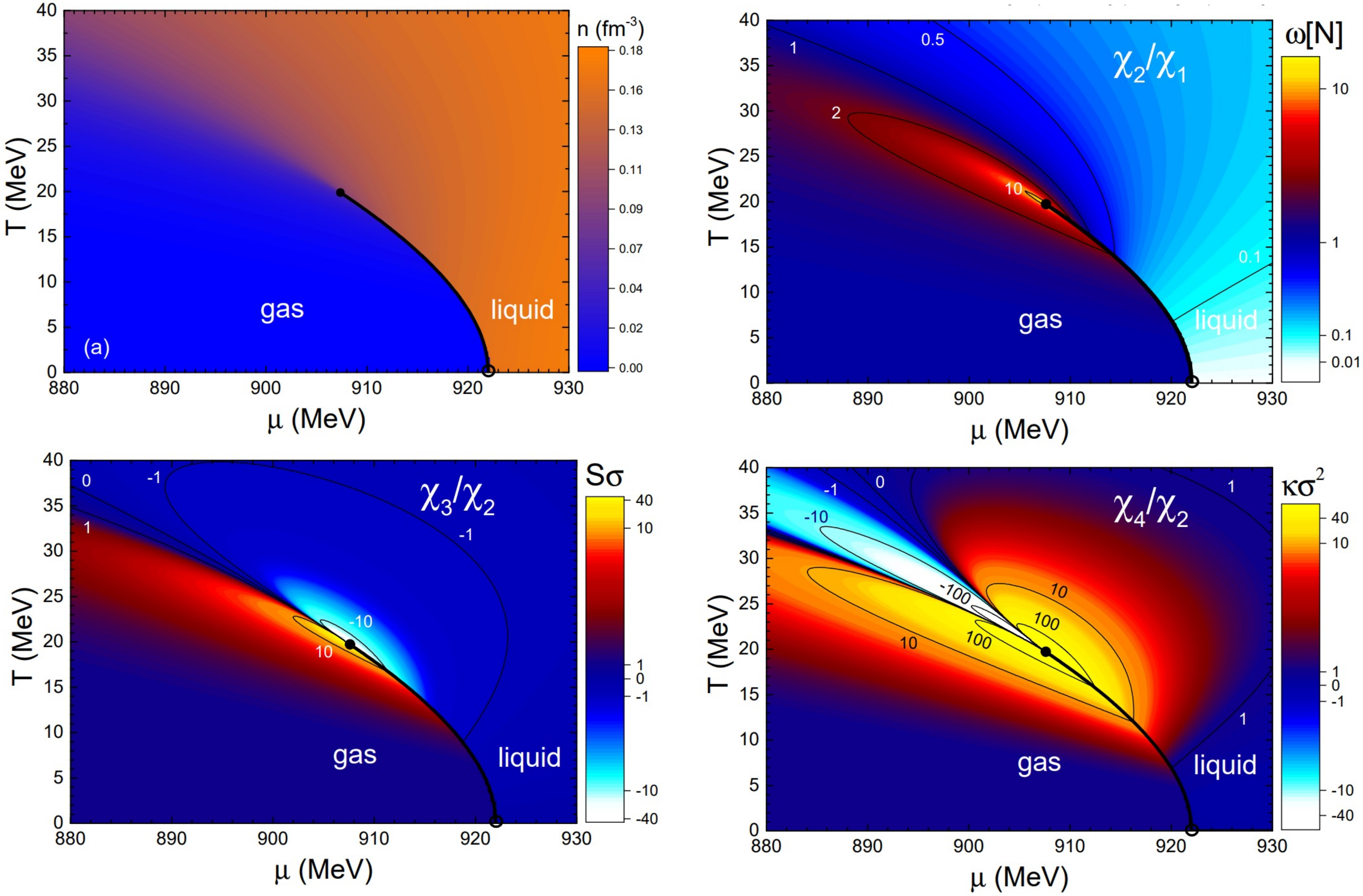
Cumulants measure chemical potential derivatives of the (QCD) equation of state

Example: (Nuclear) Liquid-gas transition

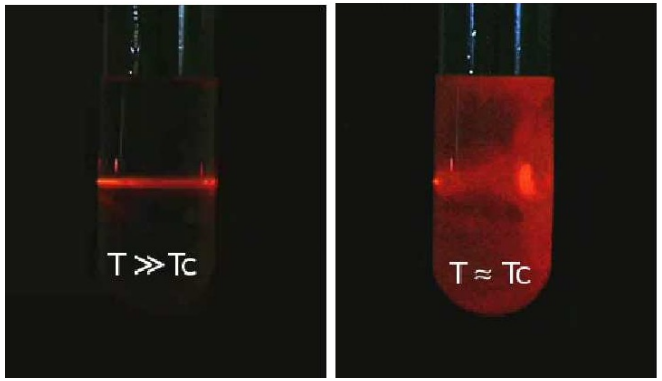
- **(QCD) critical point:** large correlation length and fluctuations $\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$

$$\xi \to \infty$$

M. Stephanov, PRL '09, '11



Critical opalescence



$$\langle N^2 \rangle - \langle N \rangle^2 \sim \langle N \rangle \sim 10^{23}$$

in equilibrium

VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

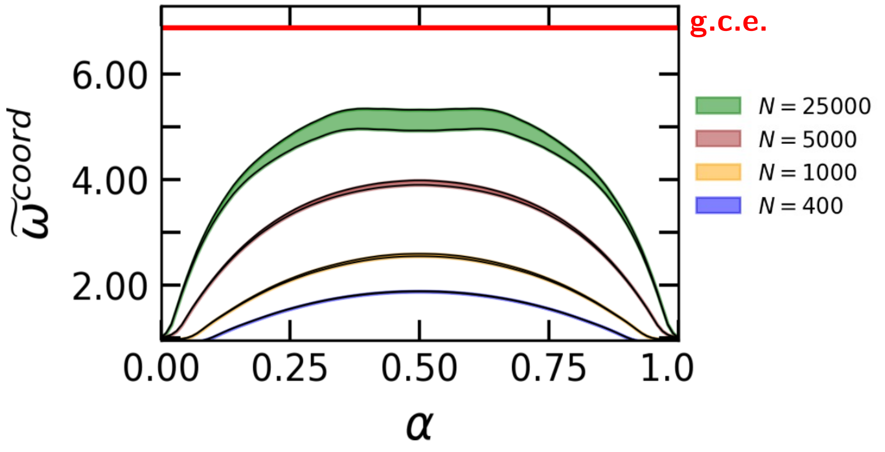
Example: Critical fluctuations in microscopic simulation

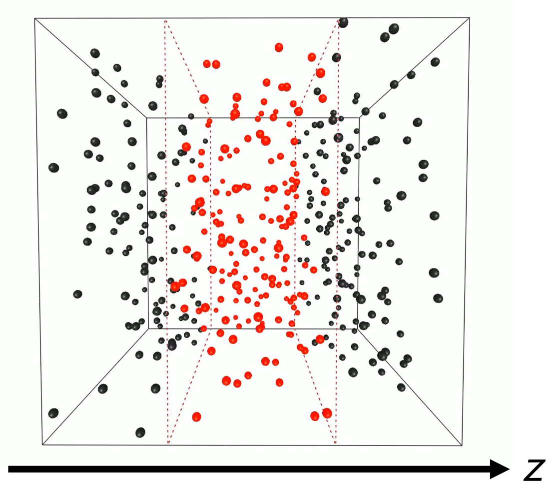
V. Kuznietsov et al., Phys. Rev. C 105, 044903 (2022)

Classical molecular dynamics simulations of the **Lennard-Jones fluid** near Z(2) critical point ($T \approx 1.06T_c$, $n \approx n_c$) of the liquid-gas transition

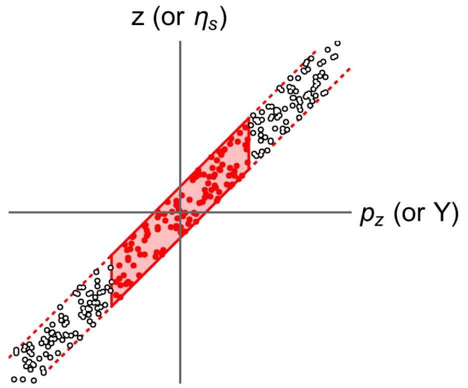
Scaled variance in coordinate space acceptance $|z| < z^{max}$

$$\tilde{\omega}^{\rm coord} = \frac{1}{1-\alpha} \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$





Heavy-ion collisions:
flow correlates p_z and z cuts

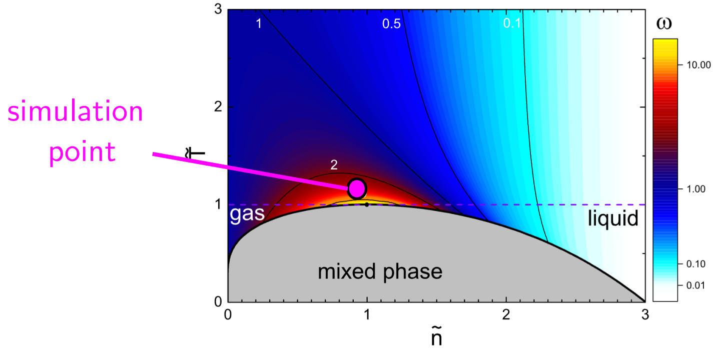


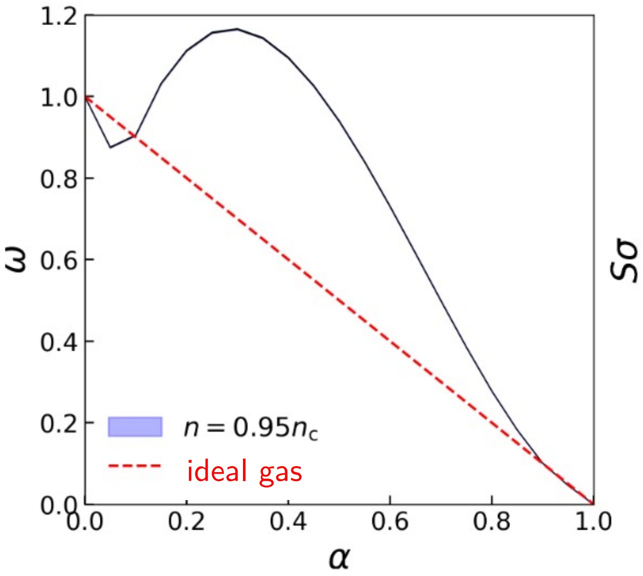
- Large fluctuations survive despite strong finite-size effects
- Need coordinate space cuts (collective flow helps)
- Here no finite-time effects

Non-Gaussian fluctuations from molecular dynamics

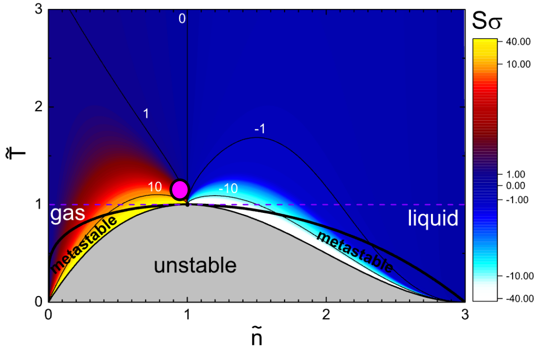
V. Kuznietsov, Gorenstein, Koch, VV, to appear

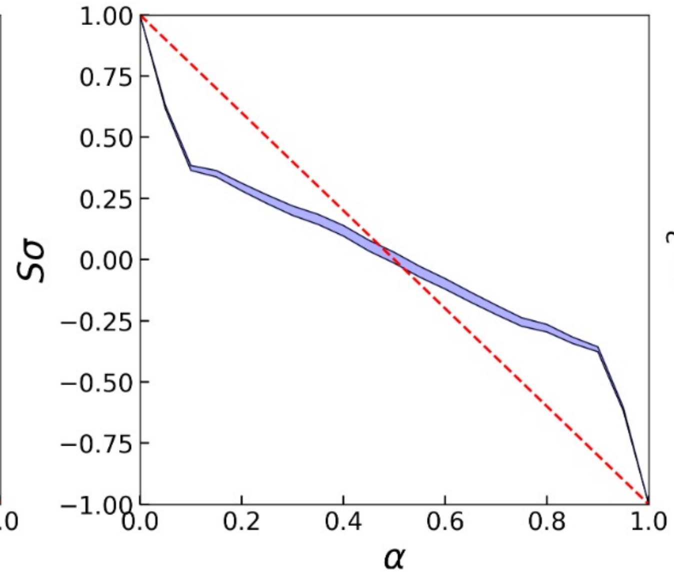
Scaled variance κ_2/κ_1



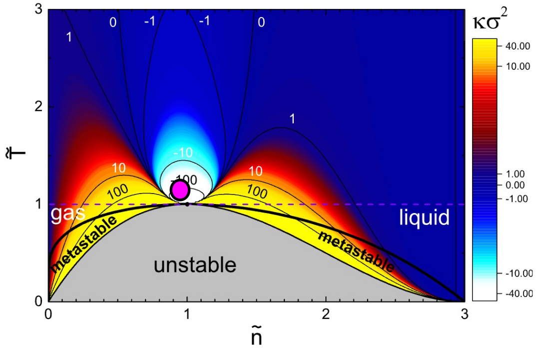


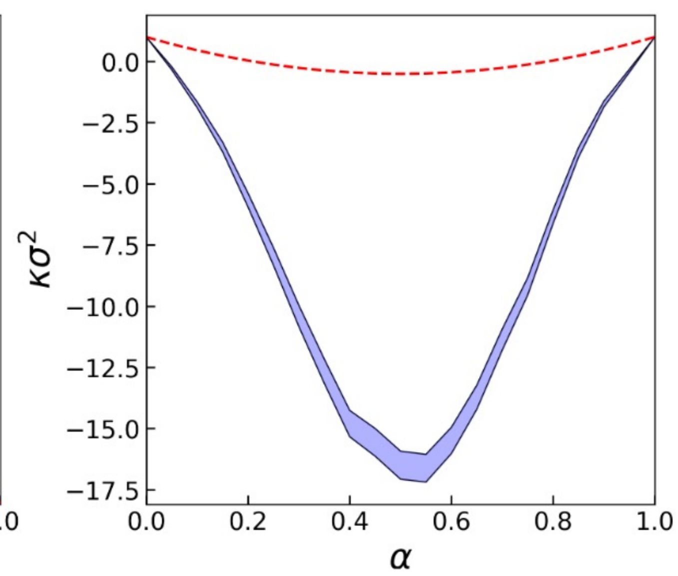
Skewness κ_3/κ_2



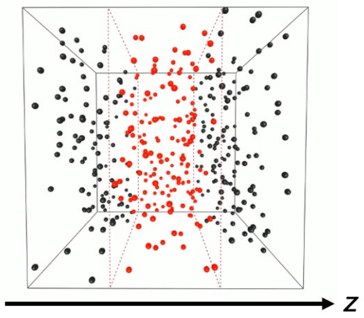


Kurtosis κ_4/κ_2





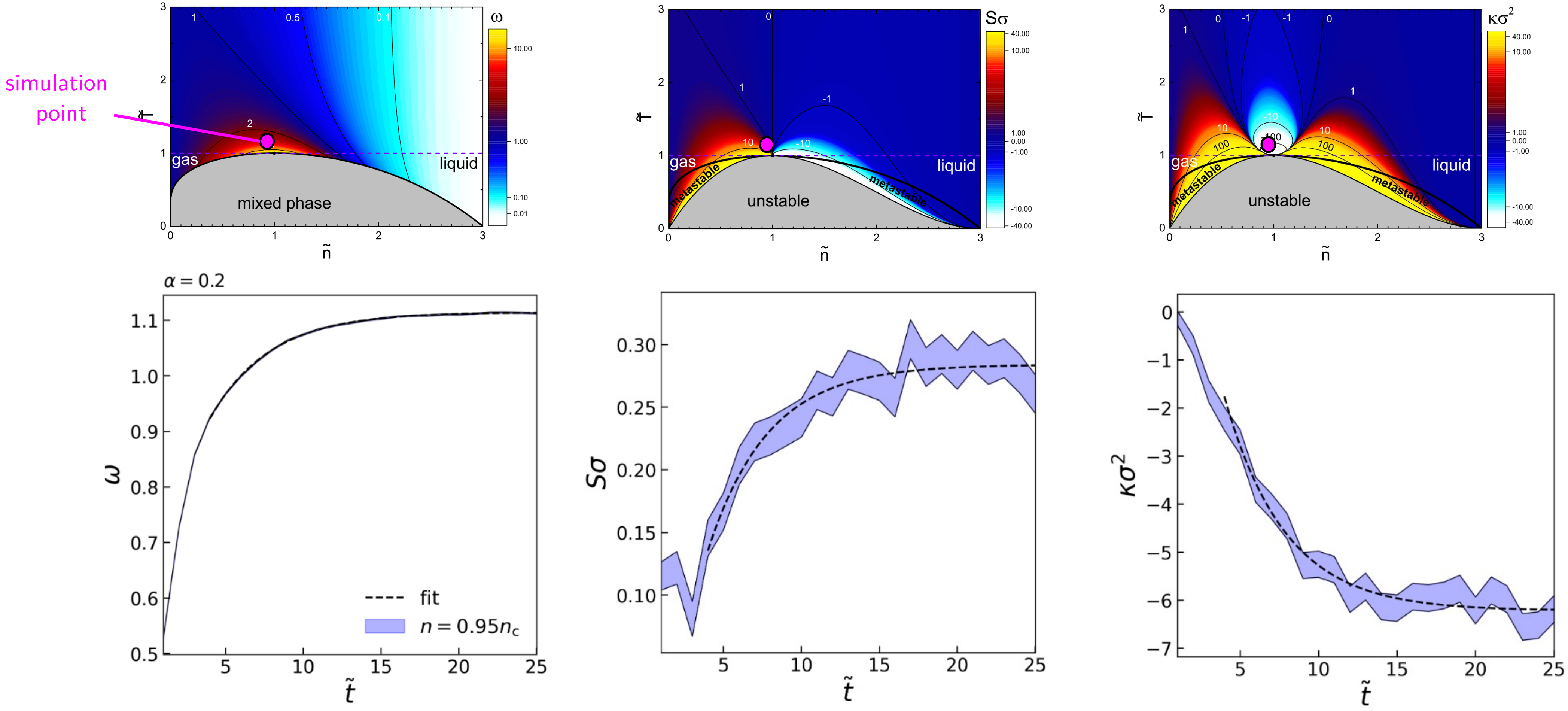
400 nucleons in a box



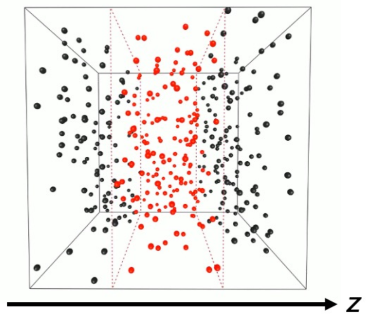
Non-Gaussian fluctuations from molecular dynamics

V. Kuznietsov, Gorenstein, Koch, VV, to appear

Scaled variance κ_2/κ_1 | Skewness κ_3/κ_2 | Kurtosis κ_4/κ_2



400 nucleons in a box



- (Non-)Gaussian cumulants equilibrate on comparable time scales

see also X. An et al., PRL 127, 072301 (2021); C. Chattopadhyay et al., PRL 133, 032301 (2024)

Equilibrium Expectations and Beam Energy Scan

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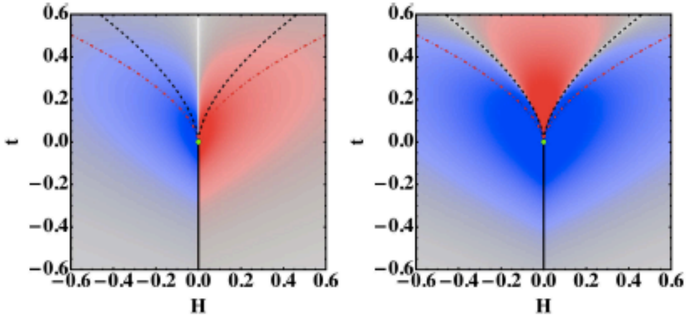
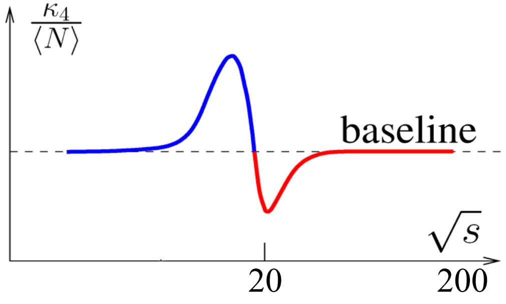
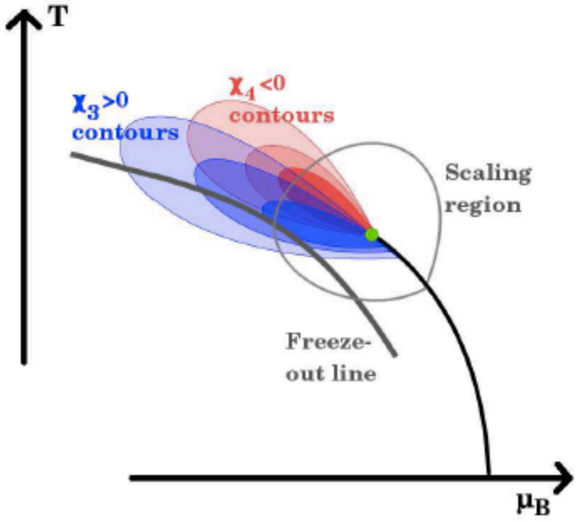
Expectation from Calculations



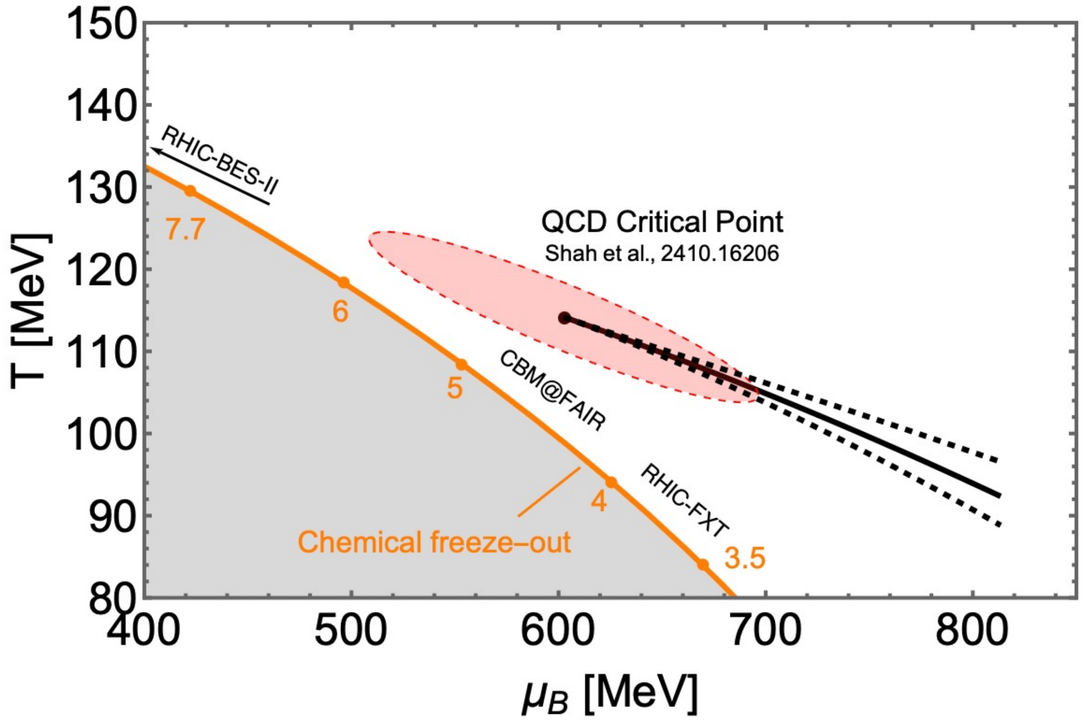




Characteristic "Oscillating pattern" is expected for the QCD critical point but *the exact shape depends on the location of freeze-out with respect to the location of CP*

- M. Stephanov, ***PRL107***, 052301(2011)
- V. Skokov, Quark Matter 2012
- J.W. Chen, J. Deng, H. Kohyyama, arXiv: 1603.05198, Phys. Rev. **D93** (2016) 034037

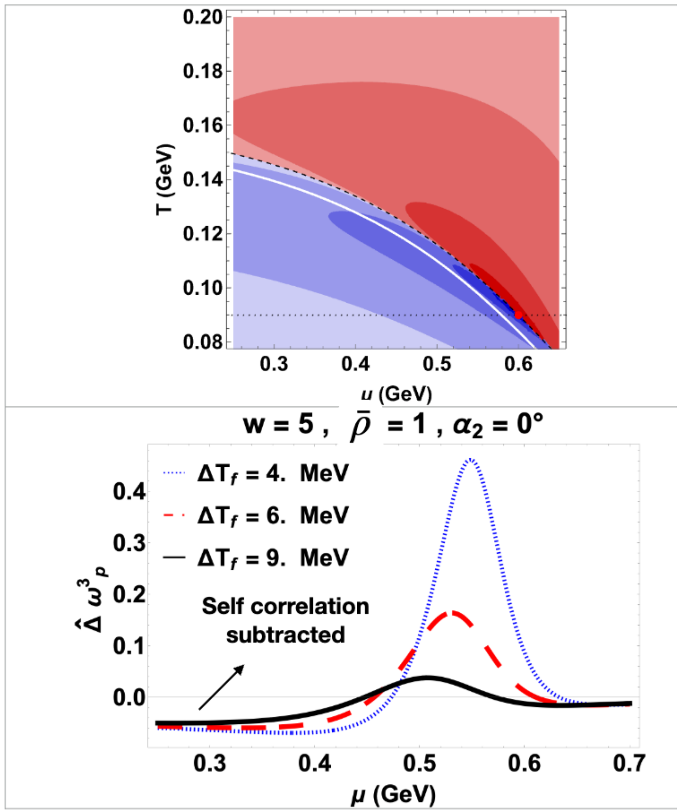
Recalling recent CP estimates and the freeze-out curve



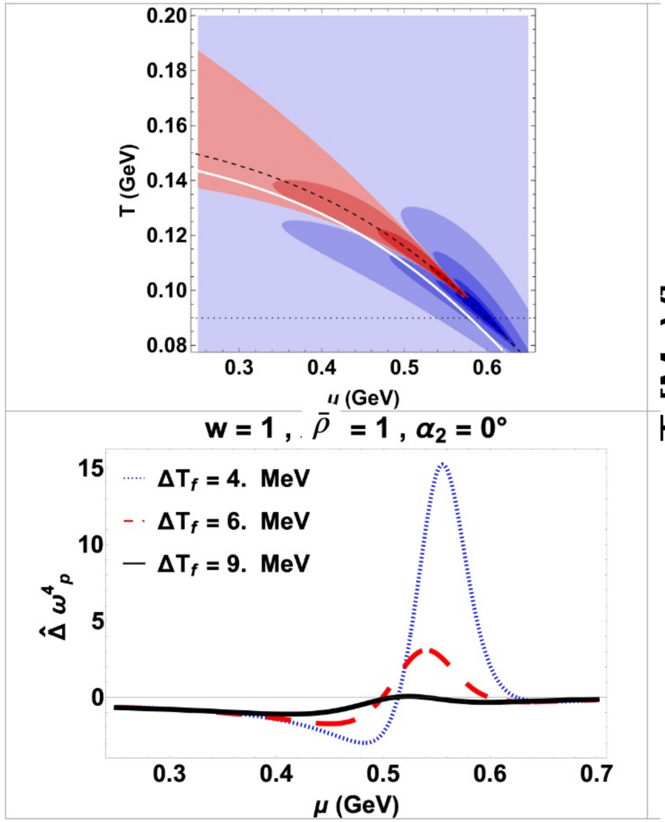
N. Xu, CPOD 2016

Equilibrium Expectations and Beam Energy Scan

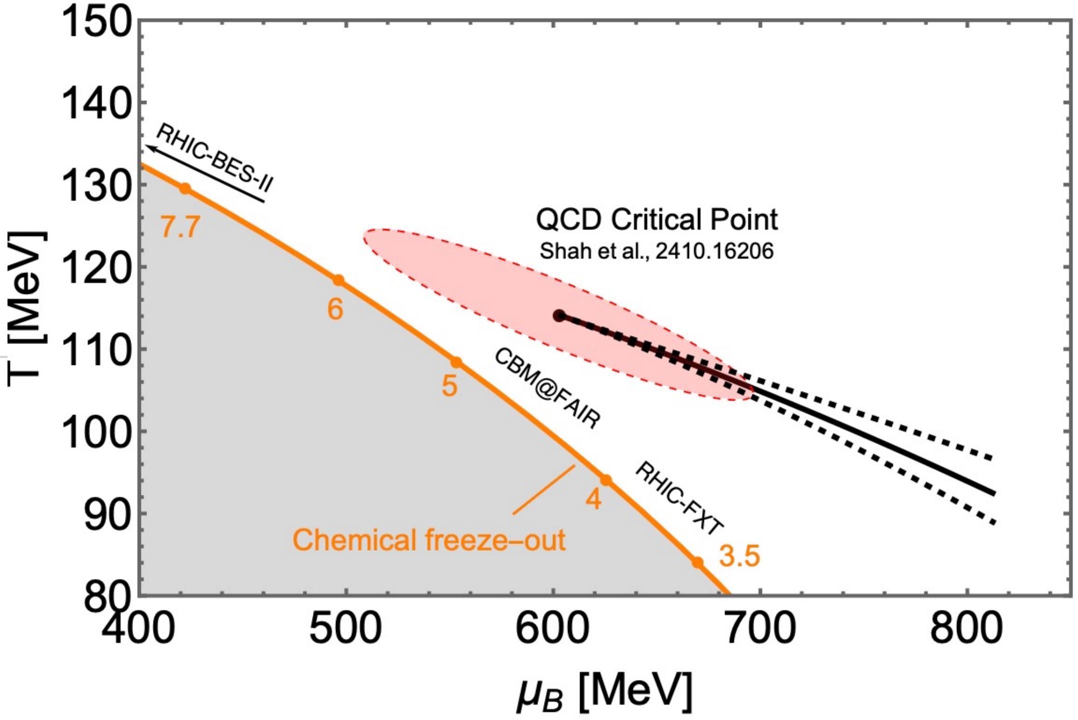
3rd order



4th order



Recalling recent CP estimates and the freeze-out curve



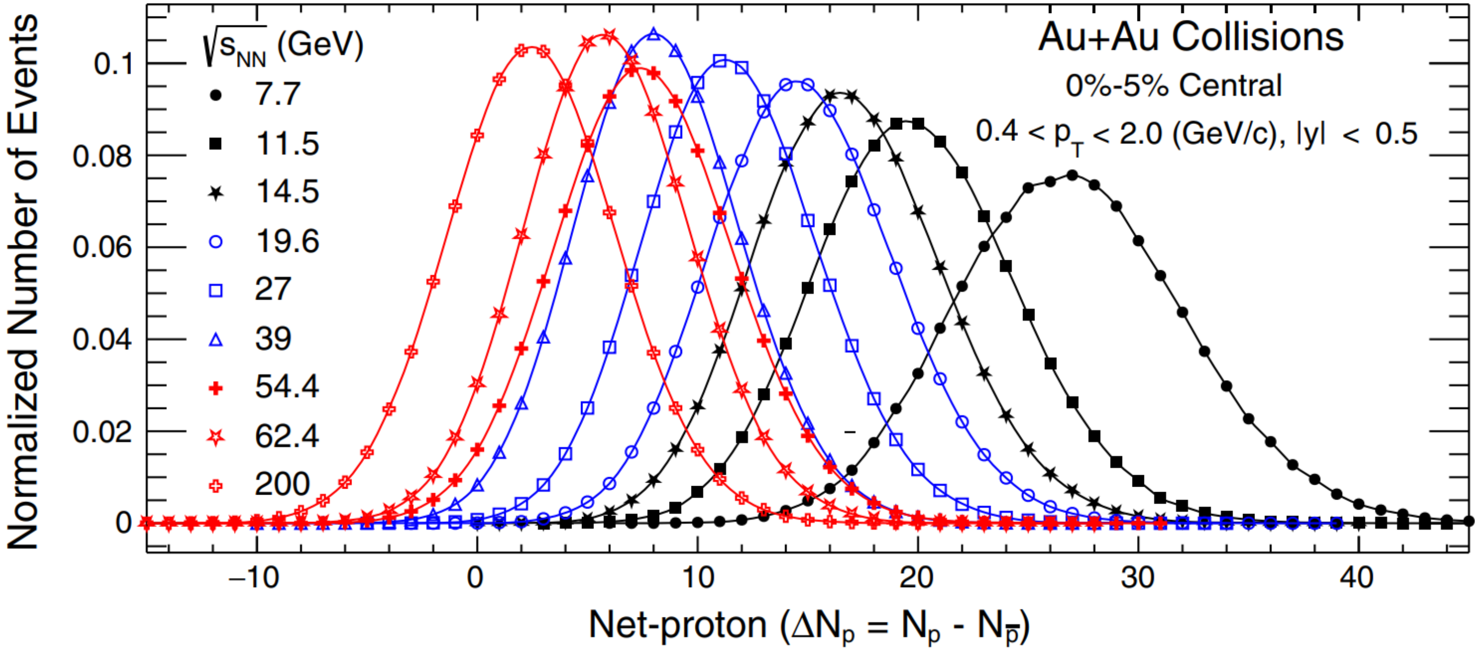
Ising-T EoS + maximum entropy freeze-out [M. Pradeep et al. (QM2025)]

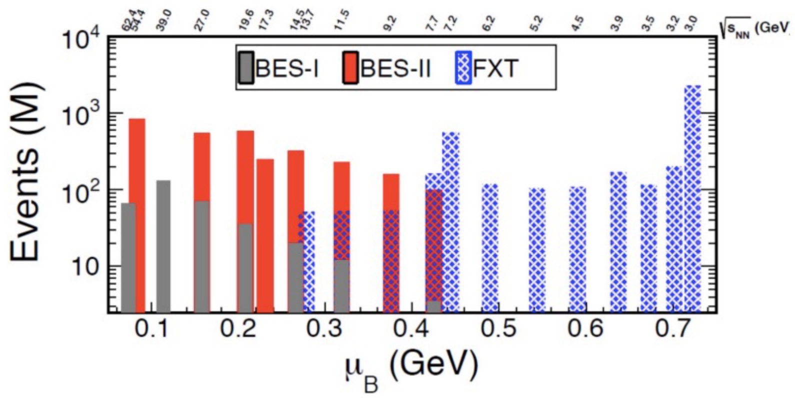
Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons

$$P(\Delta N_p) \sim \frac{N_{events}(\Delta N_p)}{N_{events}^{total}}$$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)





Statistics-hungry observables

Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

$$\frac{\kappa_2}{\langle N \rangle}, \quad \frac{\kappa_3}{\kappa_2}, \quad \frac{\kappa_4}{\kappa_2}$$

Look for subtle critical point signals

History of proton cumulants at RHIC

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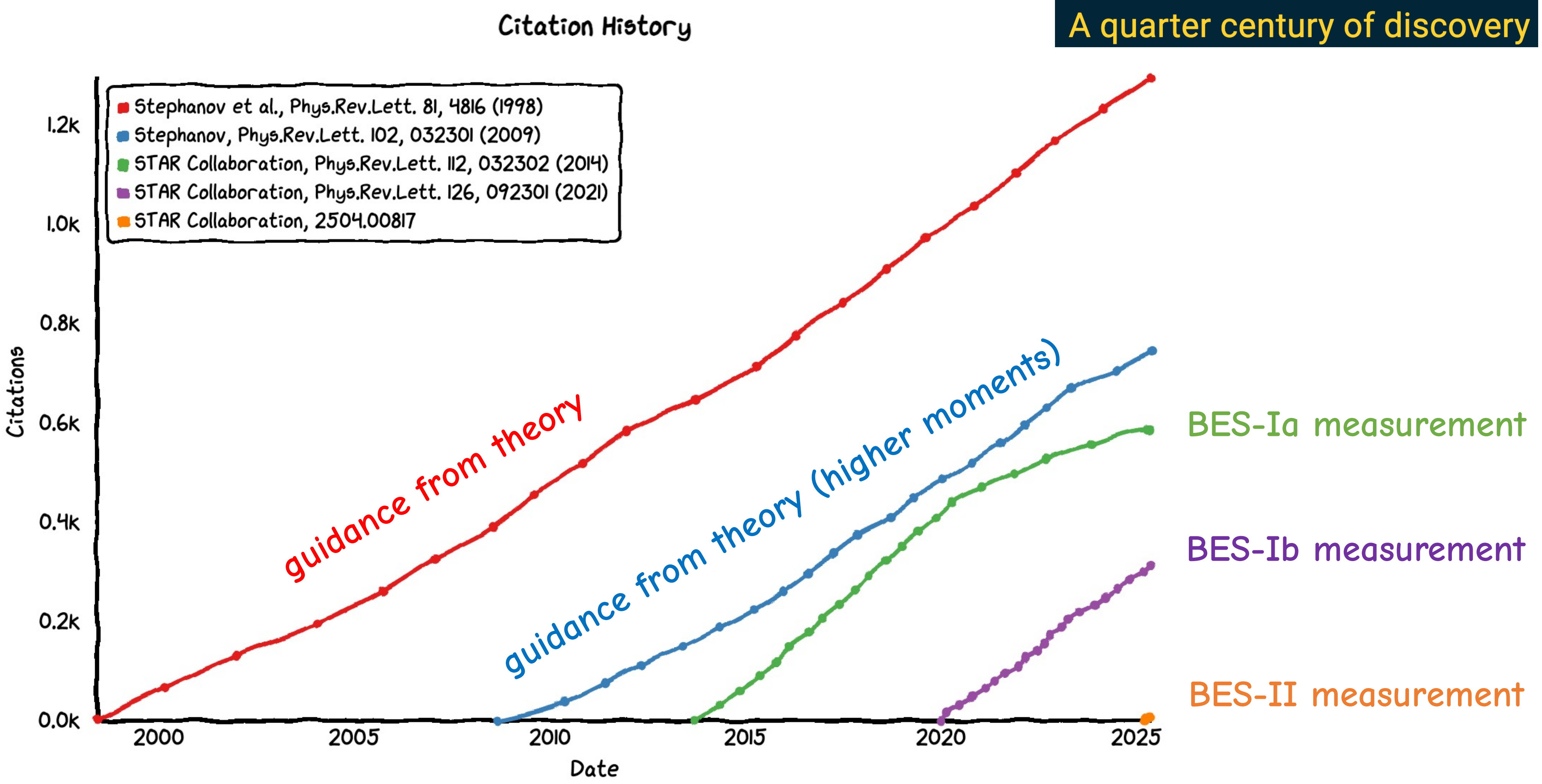
RHIC 25:
A quarter century of discovery



https://vovchenko.net/inspire-citation-history/?recids=471221-797125-1255072-1850675-2906592

Theory vs experiment: Challenges for fluctuations

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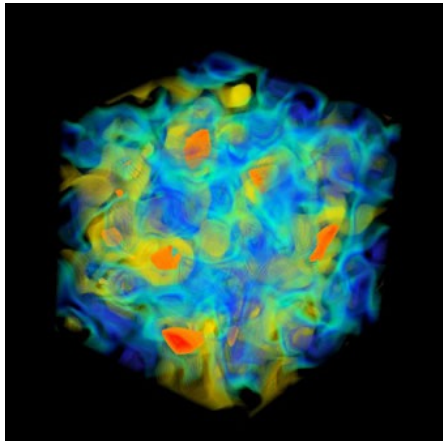
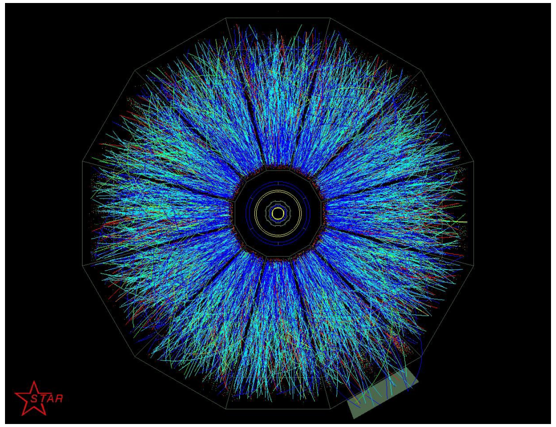
Theory



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

Comparing theory and experiment should be done very carefully

Theory vs experiment

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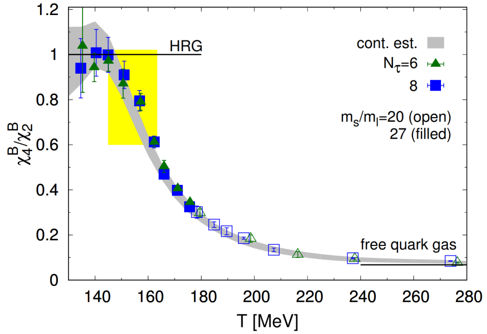
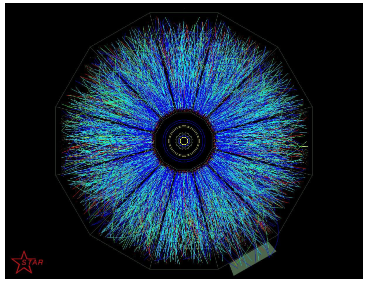
guidance from theory (e.g. lattice)

experiment (the real thing)





hydro simulation

hadronic phase

canonical effects

kinematic cuts

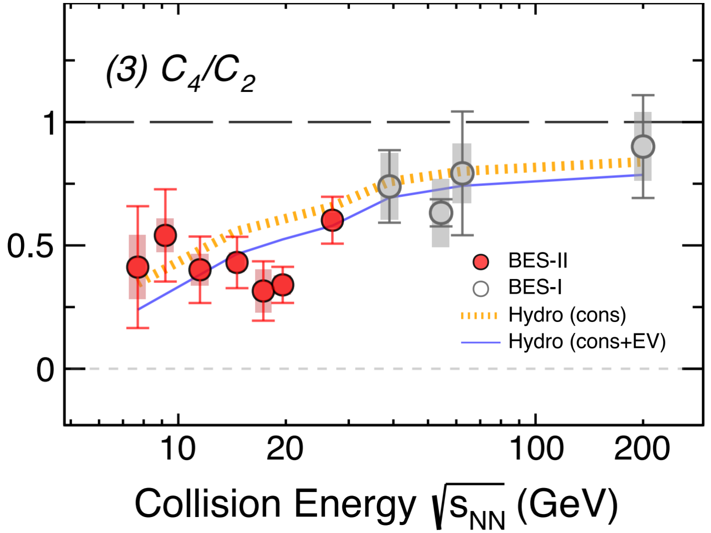
event selection

track reconstruction

efficiency correction

CBWC

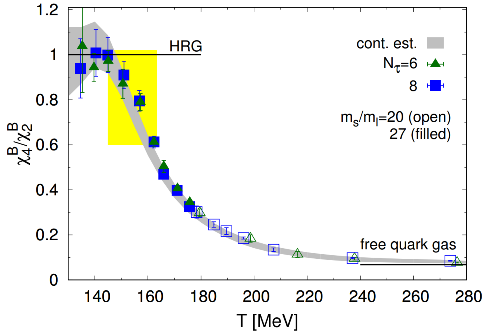
purity

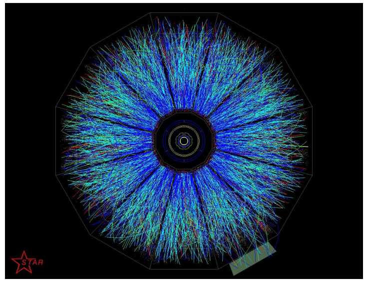


Theory vs experiment

guidance from theory (e.g. lattice)

experiment (the real thing)





This was done in [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)]

- Full hydro simulation
- Lattice QCD-like baryon susceptibilities (interacting HRG)
- Global baryon conservation (SAM)
- Experimental kinematic cuts

Non-critical baseline (hydro EV) **prediction**

hydro simulation

hadronic phase

canonical effects

kinematic cuts

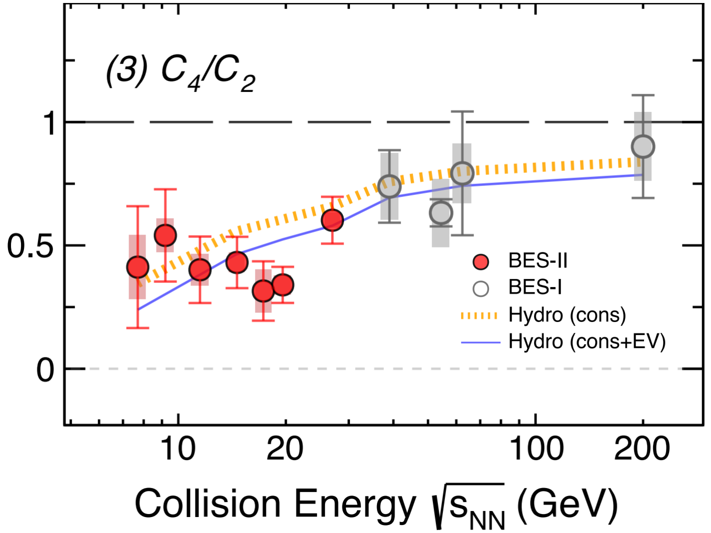
event selection

track reconstruction

efficiency correction

CBWC

purity



RHIC-BES-II data: Ordinary cumulants

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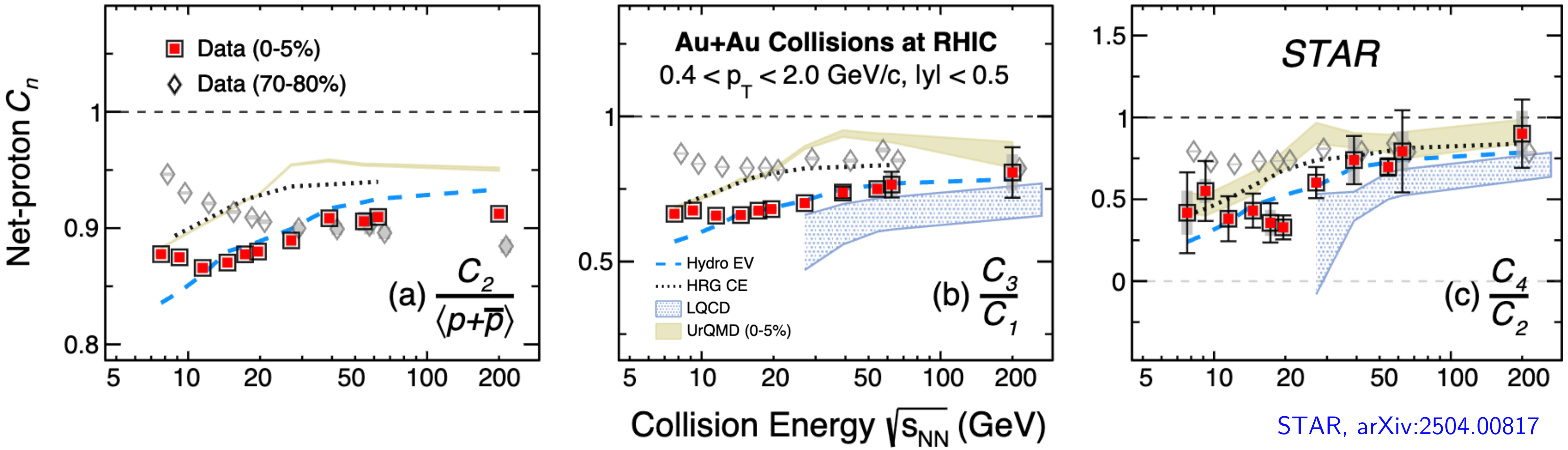
Net-proton cumulant ratios



STAR, arXiv:2504.00817

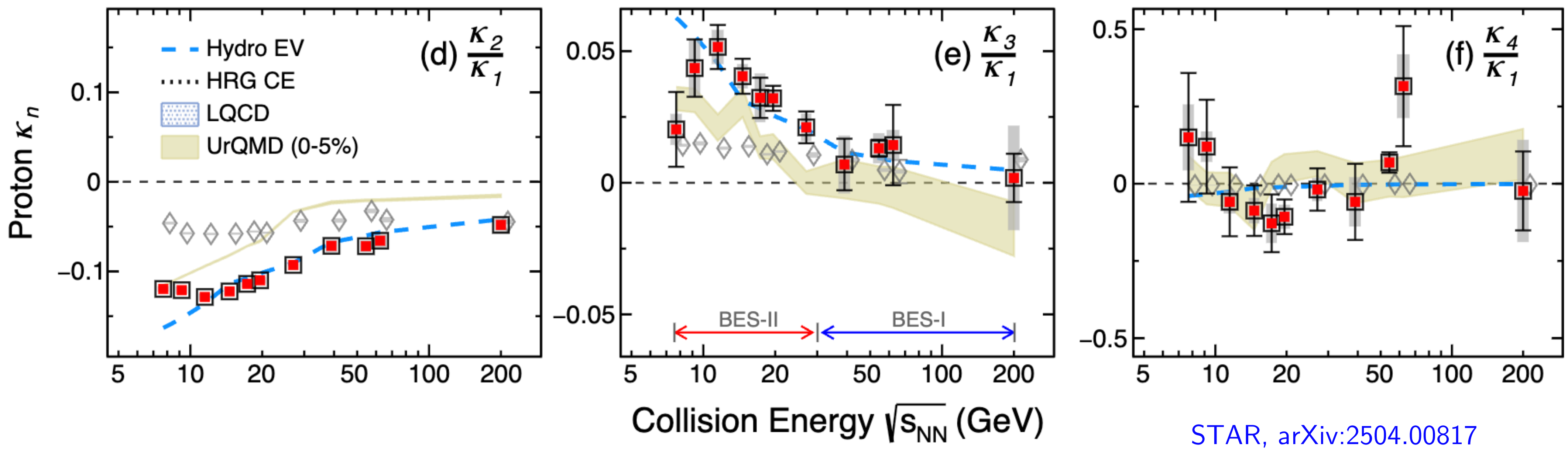
Hydro EV: VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Agreement with the baseline above $\sqrt{s_{NN}}{\sim}10-20$ GeV

But otherwise mostly boring. What else is there?

RHIC-BES-II data: Factorial cumulants

Proton factorial cumulant ratios



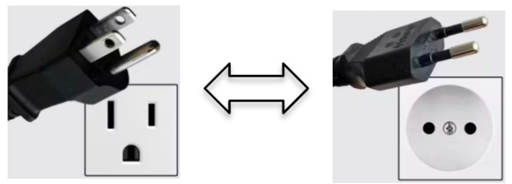
STAR, arXiv:2504.00817

Hydro EV: VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

More structure seen in factorial cumulants

- Non-monotonic κ_2/κ_1, κ_3/κ_1, and possibly κ_4/κ_1

STAR	**Others**
Cumulants (C)	Cumulants (κ)
Factorial cumulants (κ)	Factorial cumulants (C)



From M. Arslandok, QM2025

Factorial cumulants $\hat{C}_n$ vs ordinary cumulants C_n

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Factorial cumulants: ~irreducible n-particle correlations

$$\hat{C}_n \sim \langle N(N-1)(N-2)\ldots\rangle_c$$

$$\hat{C}_1 = C_1$$
$$\hat{C}_2 = C_2 - C_1$$
$$\hat{C}_3 = C_3 - 3C_2 + 2C_1$$
$$\hat{C}_4 = C_4 - 6C_3 + 11C_2 - 6C_1$$

Ordinary cumulants: mix correlations of different orders

$$C_n \sim \langle \delta N^n \rangle_c$$

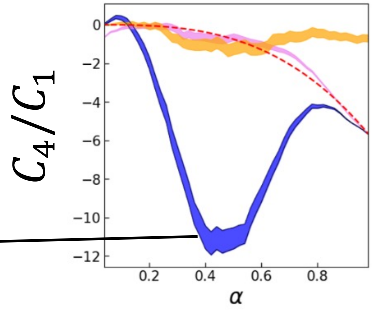
$$C_1 = \hat{C}_1$$
$$C_2 = \hat{C}_2 + \hat{C}_1$$
$$C_3 = \hat{C}_3 + 3\hat{C}_2 + \hat{C}_1$$
$$C_4 = \hat{C}_4 + 6\hat{C}_3 + 7\hat{C}_2 + \hat{C}_1$$

[Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017); Kitazawa, Luo, PRC 96, 024910 (2017); C. Pruneau, PRC 100, 034905 (2019)]

Factorial cumulants and different effects

- Baryon conservation [Bzdak, Koch, Skokov, EPJC '17] — $\hat{C}_n^{\text{cons}} \propto (\hat{C}_1)^n / \langle N_{\text{tot}} \rangle^{n-1}$ — *small*
- Excluded volume [VV et al, PLB '17] — $\hat{C}_n^{\text{EV}} \propto b^n$ — *small*
- Volume fluctuations [Holzman et al., arXiv:2403.03598] — $\hat{C}_n^{CF} \sim (\hat{C}_1)^n \kappa_n[V]$ — *depends on volume cumulants*
- **Critical point** [Ling, Stephanov, PRC '16] — $\hat{C}_2^{CP} \sim \xi^2, \quad \hat{C}_3^{CP} \sim \xi^{4.5}, \quad \hat{C}_4^{CP} \sim \xi^7$ — ***large***

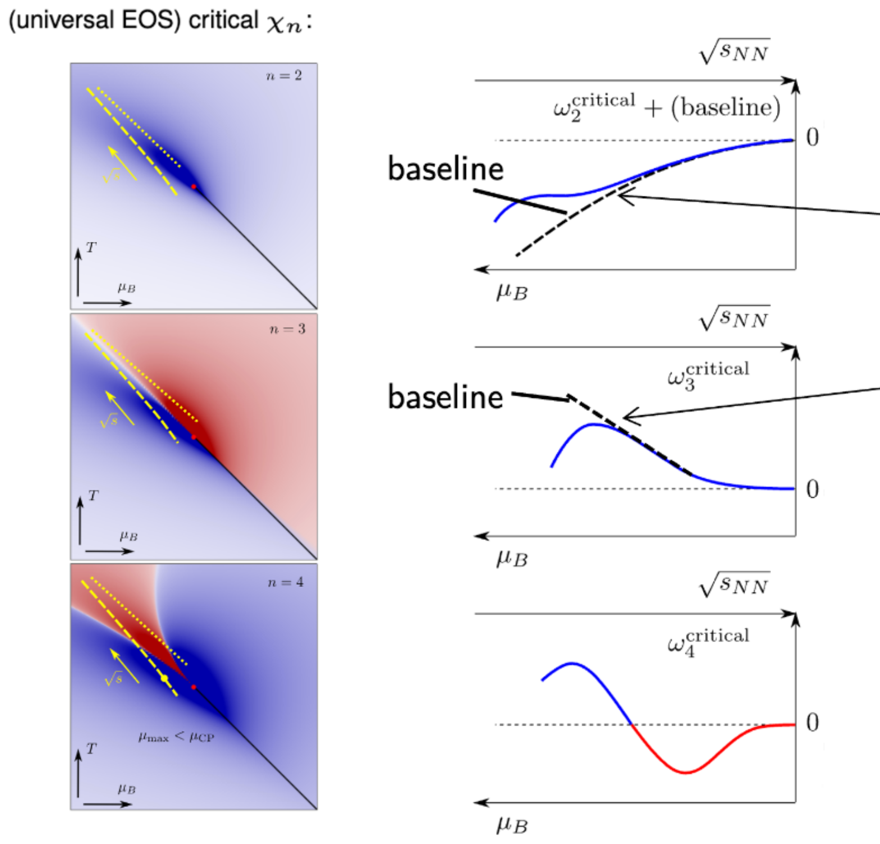
- proton vs baryon [Kitazawa, Asakawa, PRC '12] — $\hat{C}_n^B \sim 2^n \times \hat{C}_n^p$ — **same sign!**



RHIC-BES-II data and CP

VV, Koch, arXiv:2504.01368, plot adapted from M. Stephanov, arXiv:2410.02861

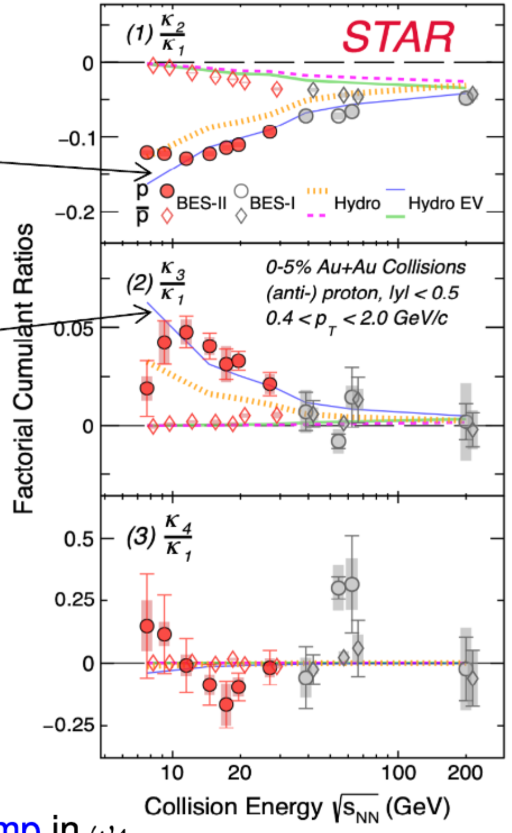
$\omega_n = \hat{C}_n/\hat{C}_1$



Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

BES-II data:

plot from A. Pandav, CPOD2024

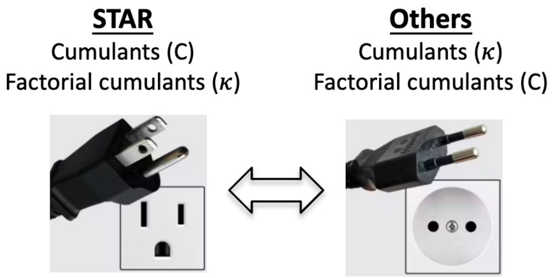


Non-critical baseline (hydro EV):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

- describes right side of the peak in $\hat{C}_3$
- **signal relative to baseline:**
  - *positive* $\hat{C}_2 - \hat{C}_2^{baseline} > 0$
  - *negative* $\hat{C}_3 - \hat{C}_3^{baseline} < 0$

Controlling the non-critical baseline is essential



From M. Arslandok, QM2025

If deviations from the baseline are driven by CP

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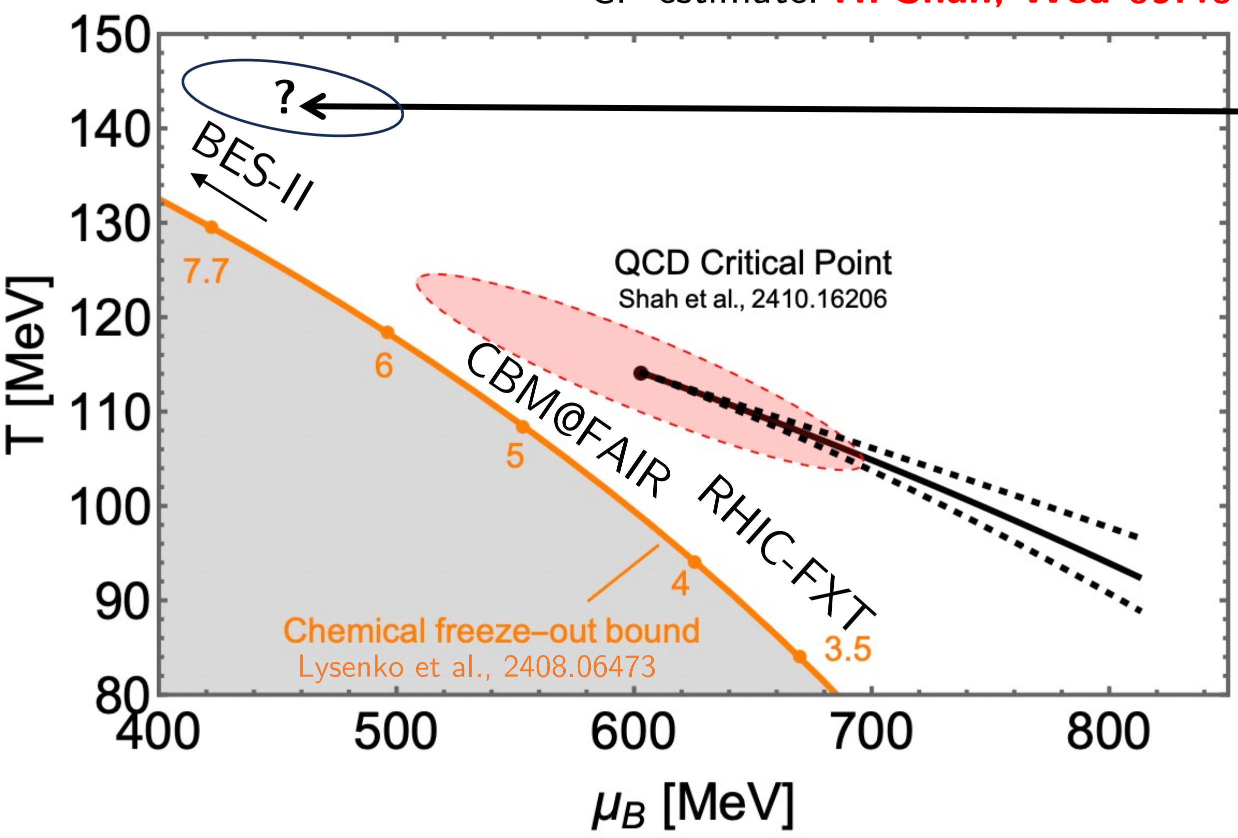
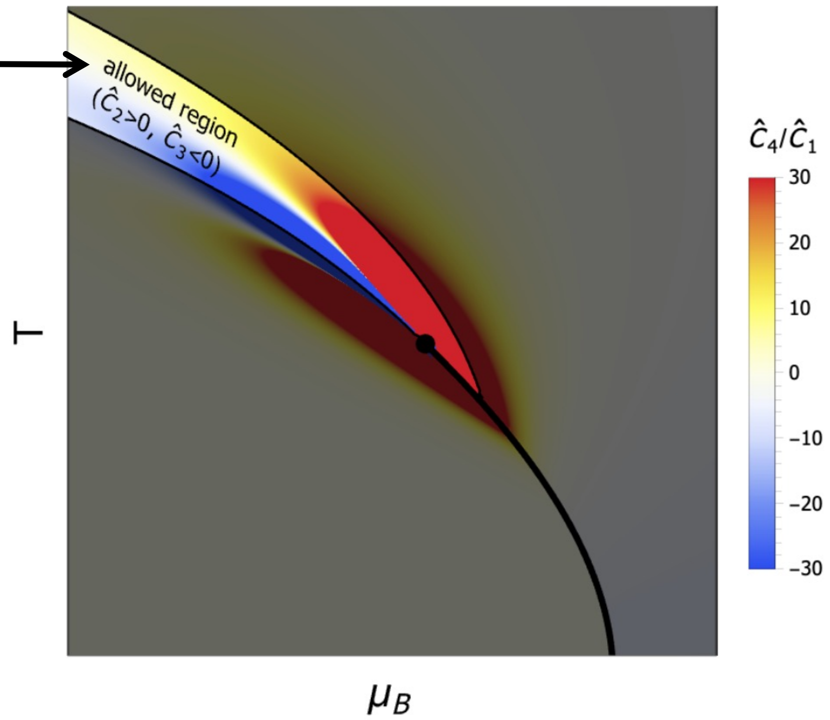
Equilibrium expectation

CP estimate: **H. Shah, Wed 09:40**



Exclusion plots

Exclude $\hat{C}_2<0$ & $\hat{C}_3>0$ regions on the phase diagram near CP



Analysis adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)

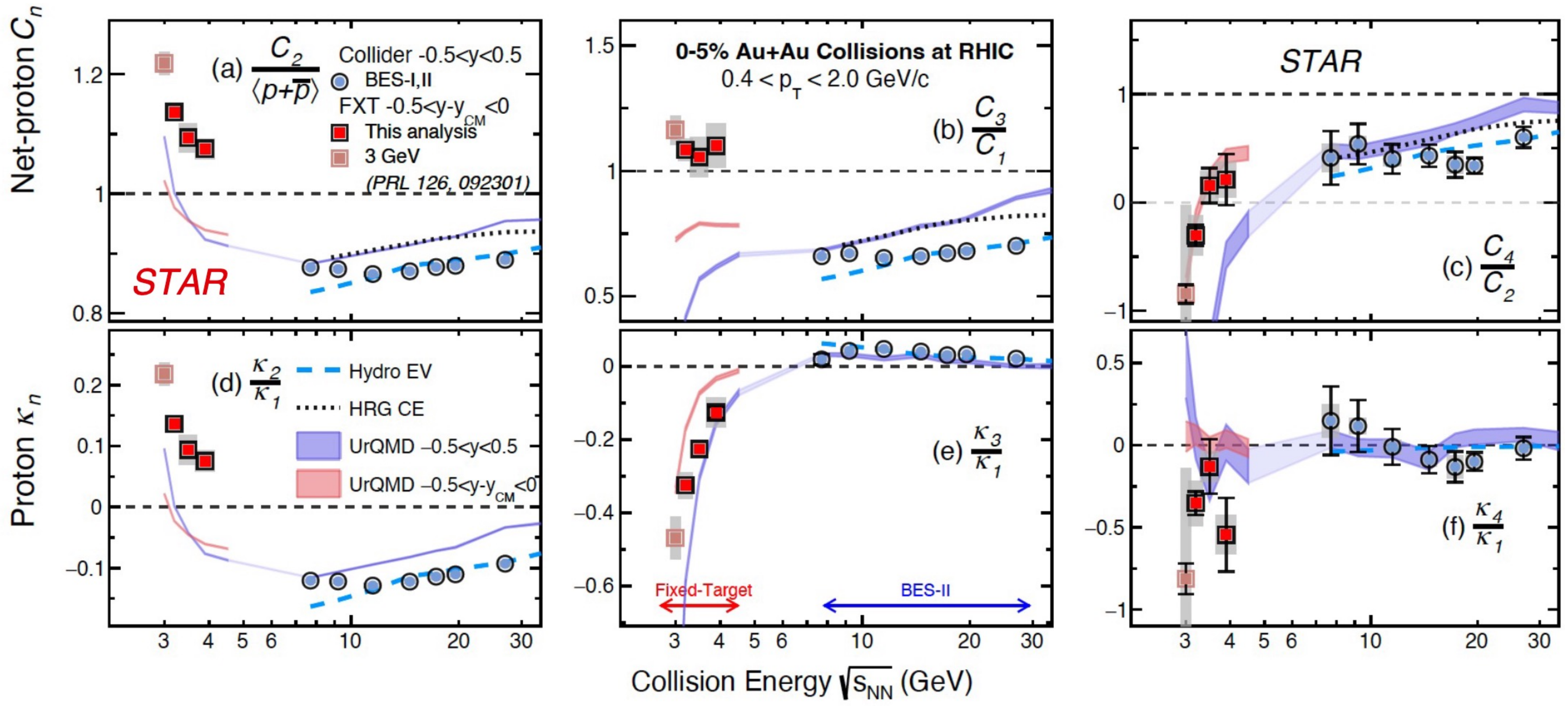
Freeze-out of fluctuations on the QGP side of the crossover?

Due to memory effect the sign of $\hat{C}_3$ may differ from equilibrium expectation

Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)

RHIC-BES-II data including STAR-FXT

Z. Sweger (STAR), QM2025



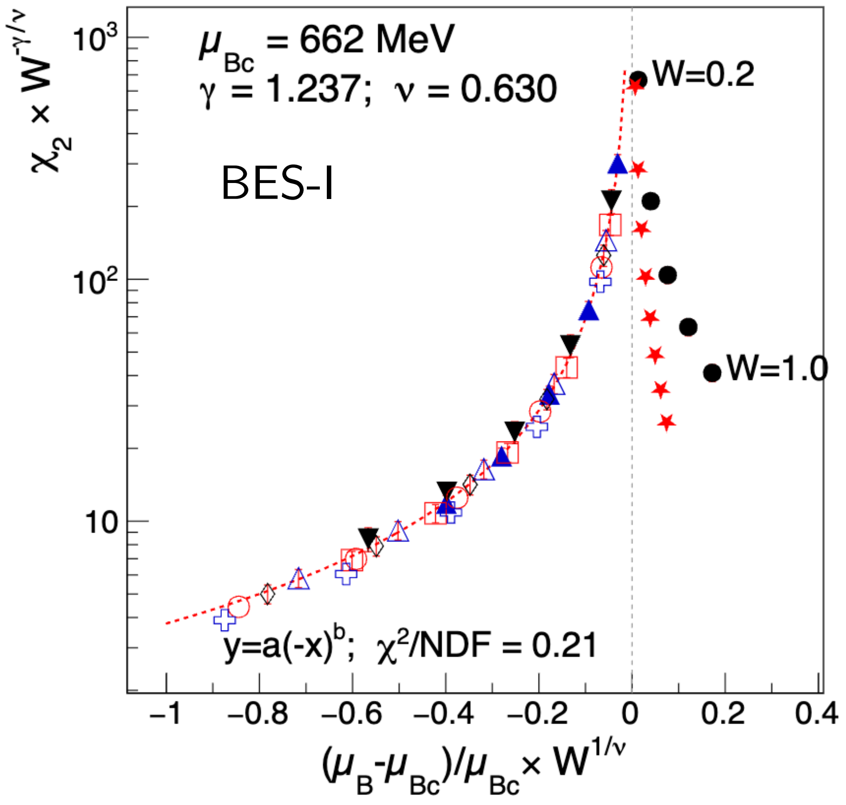
- Continues the trends seen at lowest collider energies, in a fairly dramatic fashion
- **UrQMD (cascade) describes reasonably well the qualitative features**
  - Dominance of non-critical effects (centrality selection and spectators)?

Finite Size Scaling

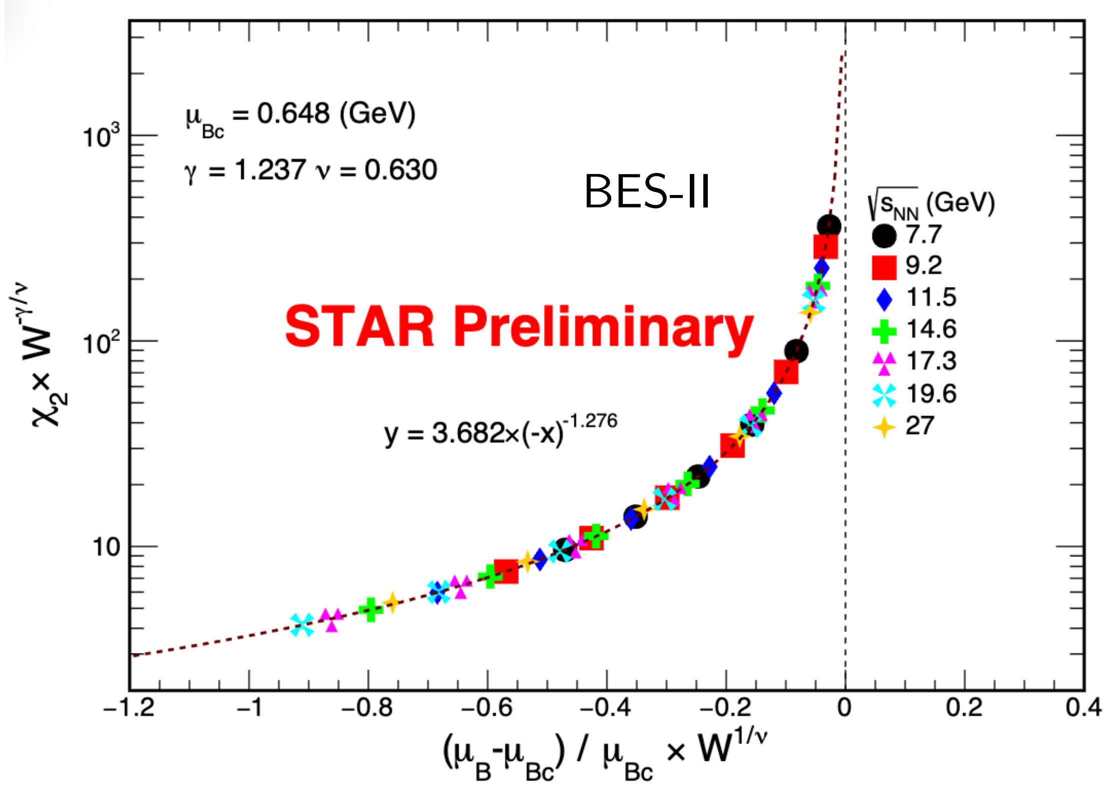
Near the CP: $\chi(L,t) = L^{\gamma/\nu}\Phi(tL^{1/\nu})$

Rapidity bin width W to vary size: $\chi_2(W,\mu_{\rm fo}) = \frac{C_2(W,\mu_{\rm fo})}{T_{\rm fo}^3 W dV_{\rm fo}/dy}$

A. Sorensen, P. Sorensen, arXiv:2405.10278



Y. Huang (STAR), QM2025



Related analysis using centralities in lieu of bin width, yields CP at $\sqrt{s_{NN}}$~ 33 GeV (μ_B~130 MeV)

R. Lacey, arXiv:2411.09139

Mean p_T fluctuations

Mean p_T fluctuations:

$$\langle \Delta p_{T,i} \Delta p_{T,i} \rangle \sim \langle \Delta \langle p_T \rangle^2 \rangle$$

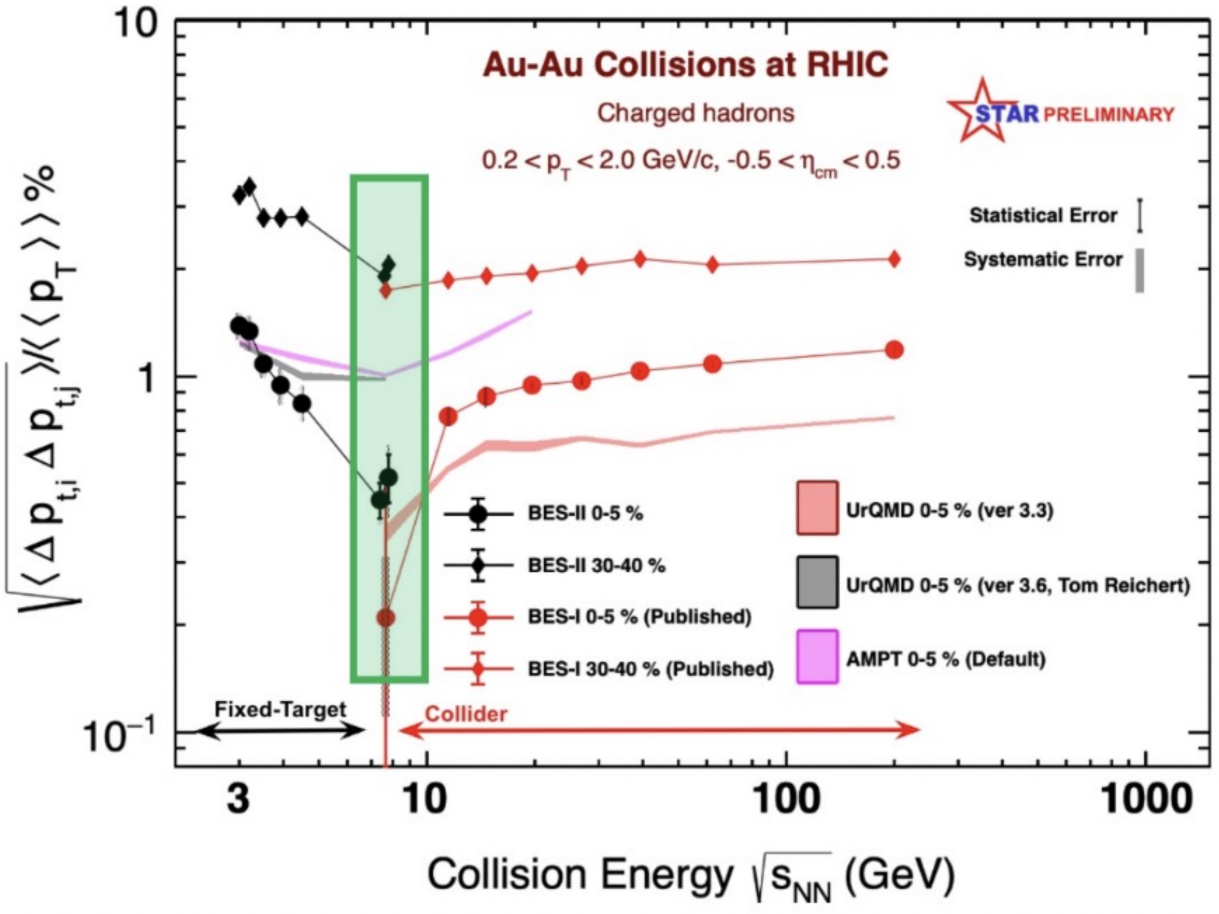
Mean p_T probes the temperature

Gardim et al, Nature Phys. (2020)

$$\langle p_T \rangle \propto T_{eff}$$

$$\langle \Delta p_{T,i} \Delta p_{T,i} \rangle \sim \langle \Delta T^2 \rangle$$

In equilibrium: $\langle \Delta T^2 \rangle = \dfrac{T^2}{V c_V}$



R. Manikandhan (STAR), QM2025

At the critical point $c_V \to \infty$ ➡ Minimum in $\sqrt{s_{NN}}$ dependence?

Scaled factorial cumulants, long-range correlations, and the “**antiproton puzzle**”

A. Bzdak, V. Koch, VV, arXiv:2503.16405

Scaled factorial cumulants

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Bzdak et al. introduced reduced correlation functions – “couplings” [Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)]

$$\hat{c}_k = \frac{\hat{C}_k}{\langle N \rangle^k}$$

$$c_k = \frac{\int \rho_1(y_1)\cdots\rho_1(y_k)c_k(y_1,\ldots,y_k)dy_1\cdots dy_k}{\int \rho_1(y_1)\cdots\rho_1(y_k)dy_1\cdots dy_k}$$

integrated correlation function in rapidity

A. Bzdak, V. Koch, VV, arXiv:2503.16405

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integrated correlation function in rapidity

Long-range correlations lead to acceptance-independent couplings, for example

- Global (not local) baryon conservation [Bzdak, Koch, Skokov, EPJC 77, 288 (2017); Bzdak, Koch, PRC 96, 054905 (2017)]
- $+$ volume fluctuations [Holzmann, Koch, Rustamov, Stroth, arXiv:2403.03598]
- $+$ (uniform) efficiency [Pruneau, Gavin, Voloshin, PRC 66, 044904 (2002)]

$$c_2 = -\frac{1}{B}, \qquad c_3 = \frac{2}{B^2}, \qquad c_4 = -\frac{6}{B^3}$$

$$\hat{\hat{c}}_{i,j} = \hat{c}_{i,j} + \frac{\kappa_2[V]}{\langle V \rangle^2}, \quad \text{for} \quad i + j = 2.$$

A. Bzdak, V. Koch, VV, arXiv:2503.16405

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$$c_2 = -\frac{1}{B}, \qquad c_3 = \frac{2}{B^2}, \qquad c_4 = -\frac{6}{B^3}$$

$$\hat{\tilde{c}}_{i,j} = \hat{c}_{i,j} + \frac{\kappa_2[V]}{\langle V \rangle^2}, \quad \text{for} \quad i+j=2.$$

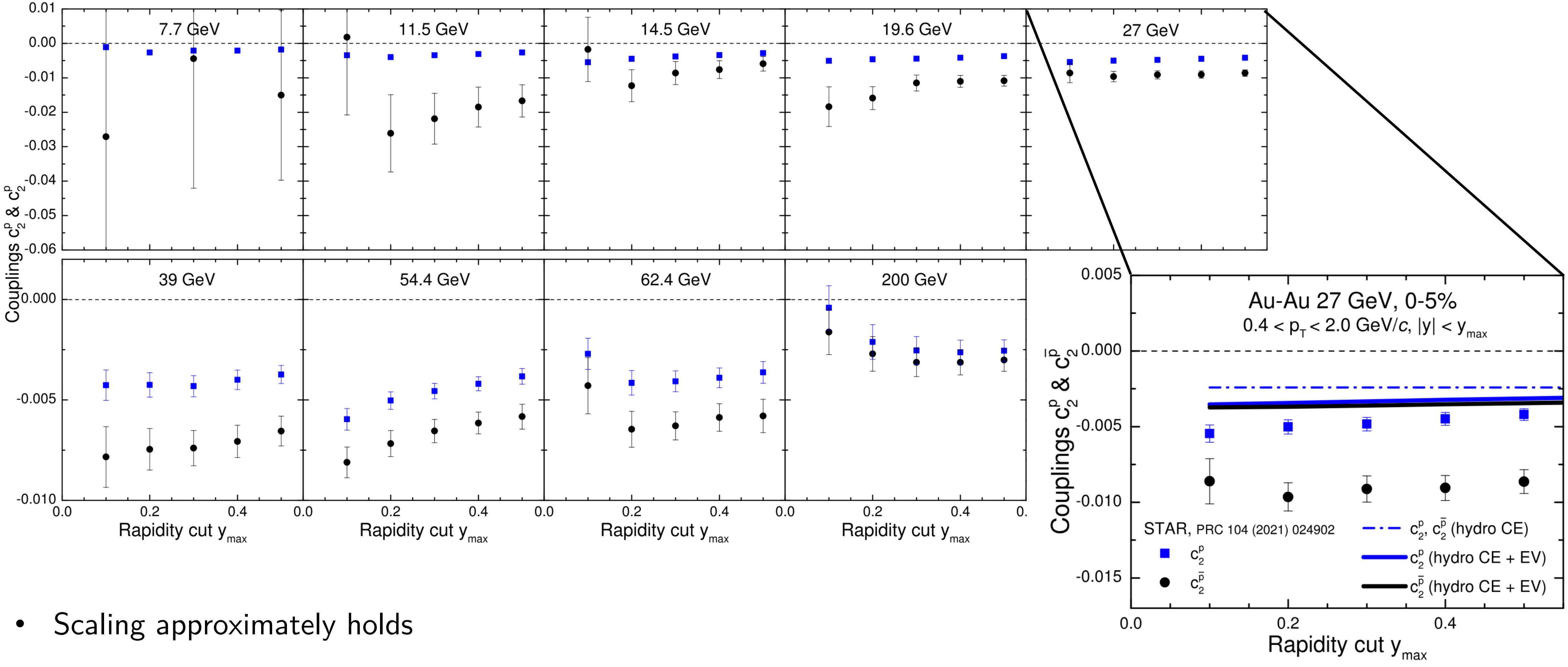
all lead to

$$\frac{\hat{C}_k}{\langle N \rangle^k} = const.$$ at a given $\sqrt{s_{NN}}$ and $$\frac{\hat{C}_2^p}{\langle N_p \rangle^2} \approx \frac{\hat{C}_2^{\bar{p}}}{\langle N_{\bar{p}} \rangle^2} = const.$$ at a given $\sqrt{s_{NN}}$

Can be tested *without* CBWC/volume fluctuations correction A. Bzdak, V. Koch, VV, arXiv:2503.16405

Scaled factorial cumulants from RHIC-BES-I

A. Bzdak, V. Koch, VV, arXiv:2503.16405



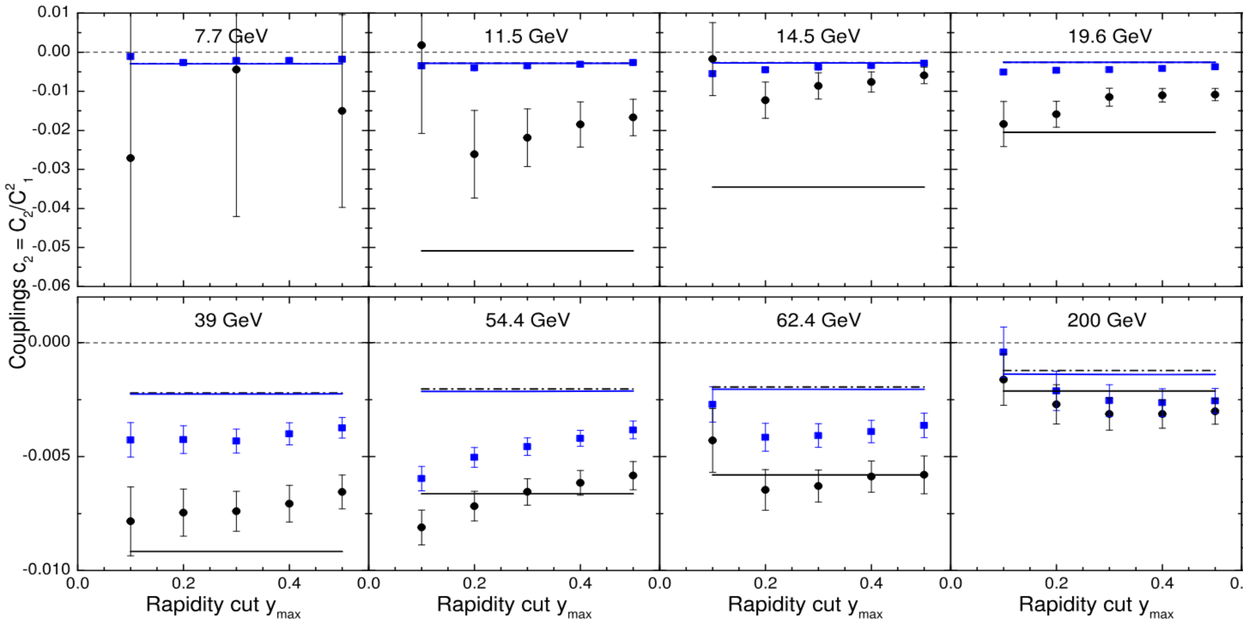
- Scaling approximately holds
- But significant difference between p and $\bar{p}$ in BES-I and hydro fails – **the antiproton puzzle**

no single thermalized fireball?

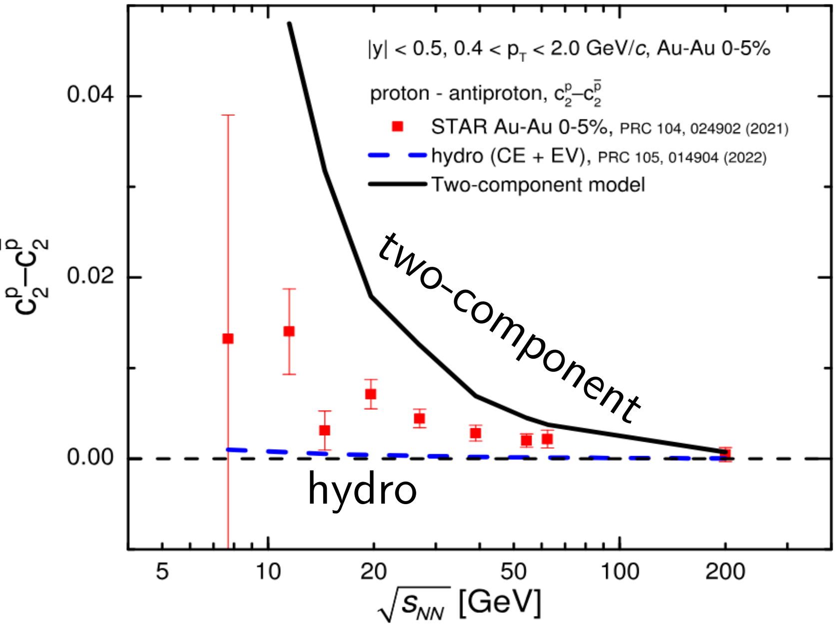
The antiproton puzzle and the two-component model

Two-component model: produced ($p\bar{p}$ pairs) and stopped protons comprise from two independent sources

The data lie in-between single and two-fireball models



Difference between p and $\bar{p}$



A. Bzdak, V. Koch, VV, arXiv:2503.16405

Opportunities for BES-II:

- Further tests of the splitting between p and $\bar{p}$ in 2nd order cumulants with extended y coverage
- Critical point signal expected to break the scaling

$$\frac{\hat{C}_n}{(\hat{C}_1)^n} = \text{const.}$$

[Ling. Stephanov, PRC 93, 034915 (2016)]

Scaled factorial cumulants and baryon annihilation

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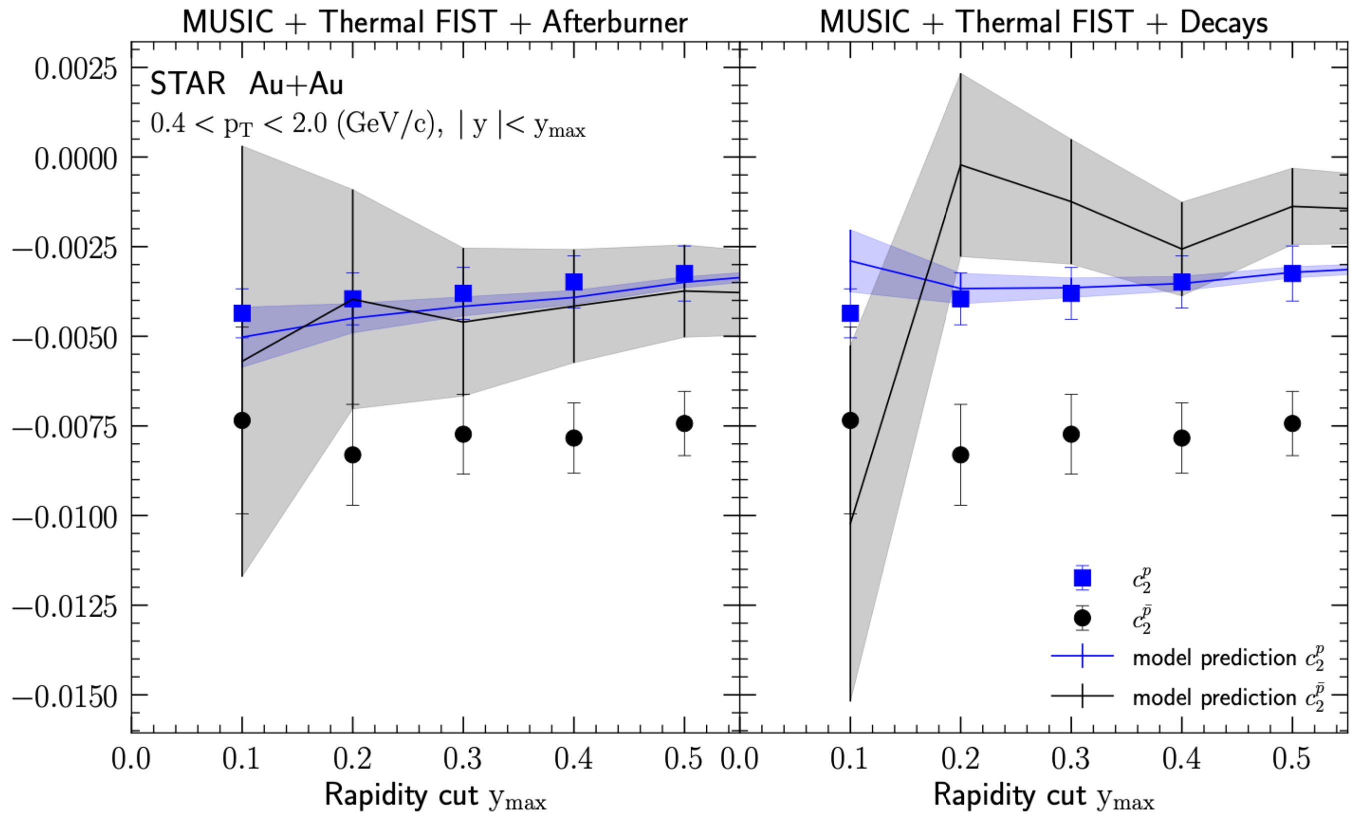
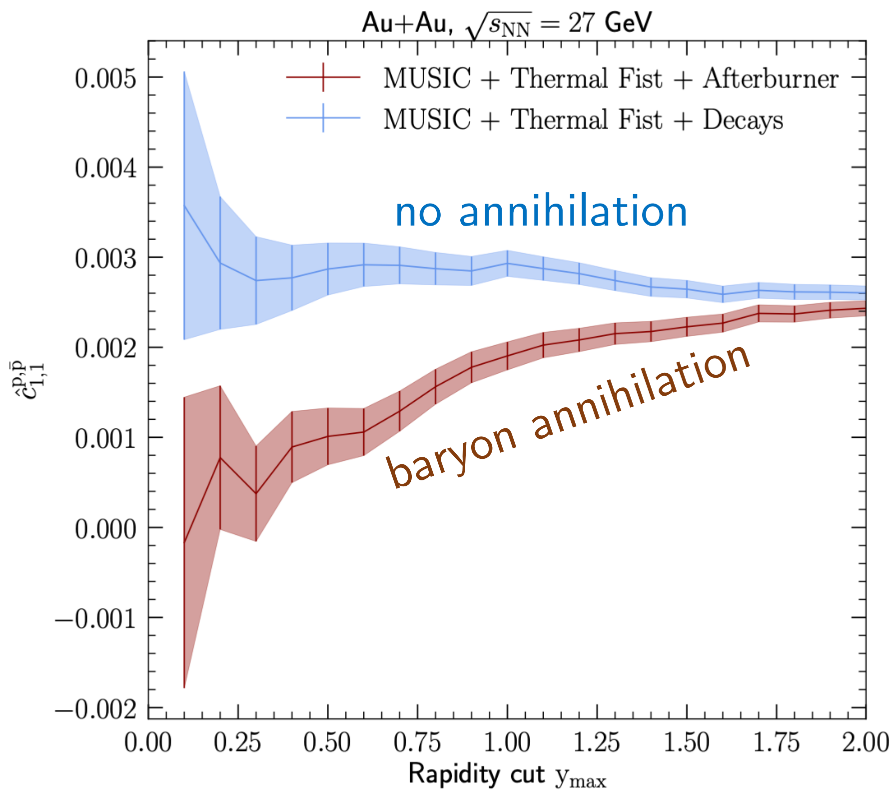
Extending Hydro EV to incorporate hadronic phase (UrQMD)

Au-Au, $\sqrt{s_{NN}} = 27$ GeV G. Pihan, VV, in preparation



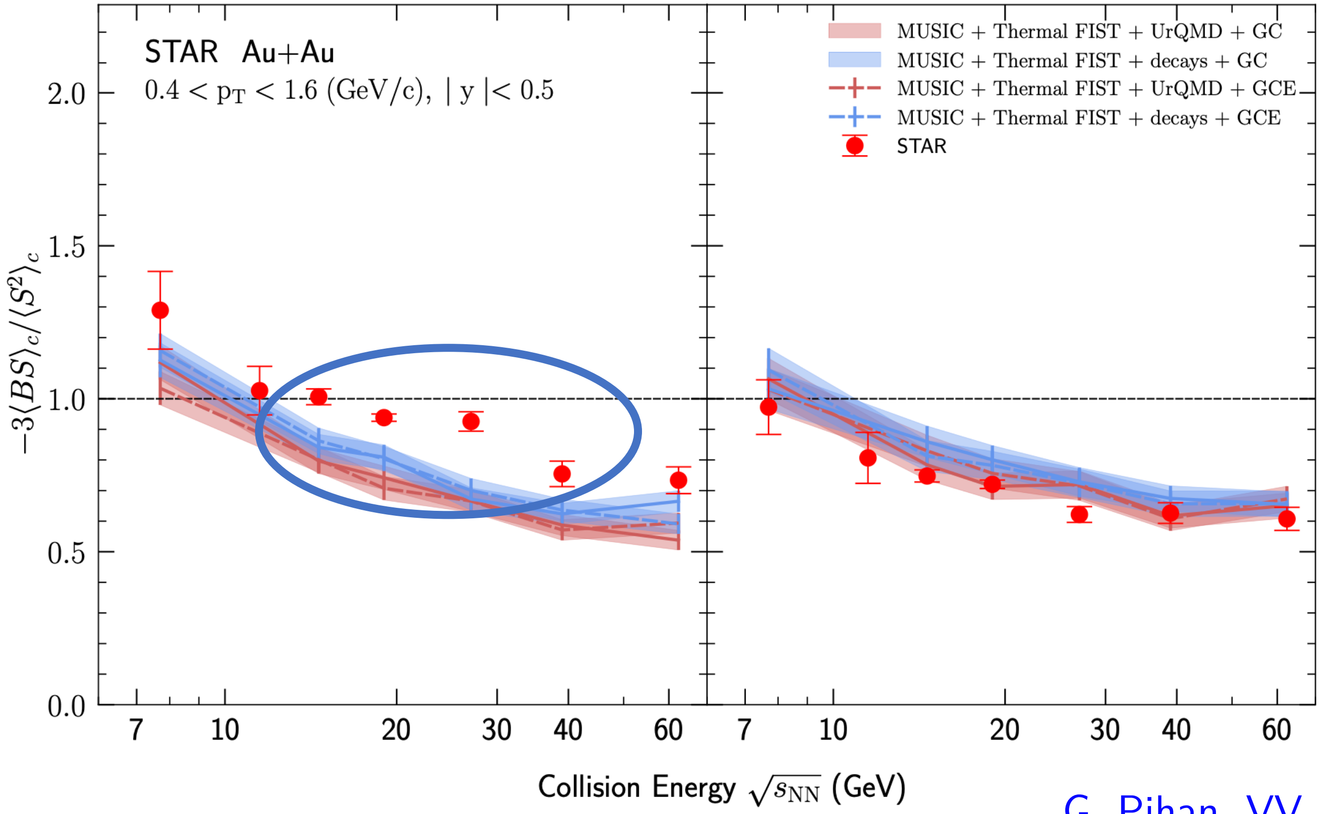
Covariance $c_{11}^{p\bar{p}}$



- Hadronic phase appears unlikely to resolve the antiproton puzzle (more statistics needed)
- Acceptance dependence of proton-antiproton covariance shows clear signature of hadronic phase

Baryon-strangeness correlator

Baryon-strangeness correlator is a diagnostic of QCD matter Koch, Majumder, Randrup, PRL (2005)



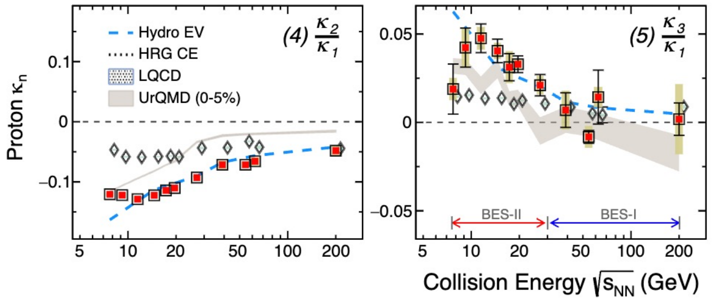
G. Pihan, VV, in preparation

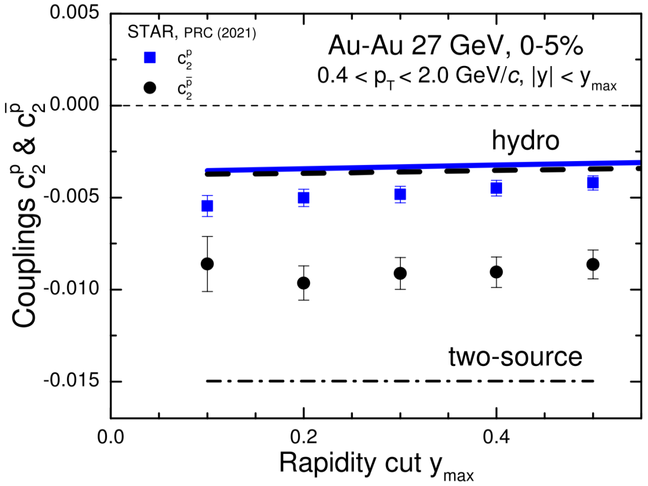
- Hadronic phase appears unlikely to resolve the antiproton puzzle (more statistics needed)
- Acceptance dependence of proton-antiproton covariance shows clear signature of hadronic phase

Other observables: light nuclei production, balance functions, HBT,…

Summary and Outlook

- Non-critical physics describe proton cumulants at $\sqrt{s_{NN}} \geq 20$ GeV
- A clear change of trend occurs at $\sqrt{s_{NN}} \sim 10$ GeV in all cumulants
  - $\hat{C}_2 - \hat{C}_2^{baseline} > 0$ and $\hat{C}_3 - \hat{C}_3^{baseline} < 0$ at $\sqrt{s_{NN}} < 10$ GeV
  - Presence of the CP is one possible explanation
  - However, UrQMD show qualitatively similar result
- Acceptance dependence of scaled factorial cumulants
  - Distinguishes short- vs long-range correlation, no need for CBWC
  - **Antiproton puzzle:** $|\hat{c}_2^{\bar{p}}| > |\hat{c}_2^{p}|$ not explained by standard hydro





Outlook and opportunities:

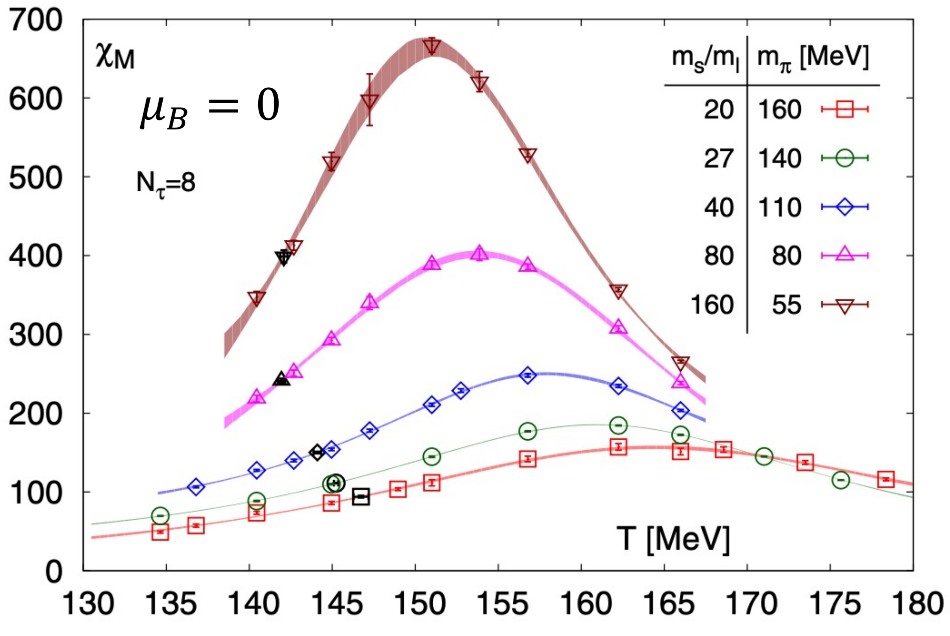
- Improved description of non-critical baselines ($\sqrt{s_{NN}} < 10$ GeV)
- Quantitative predictions of critical fluctuations
- Acceptance dependence of factorial cumulants, understanding antiprotons and baryon annihilation
- Mean p_T fluctuations and other observables

Thanks for your attention!

Additional slides

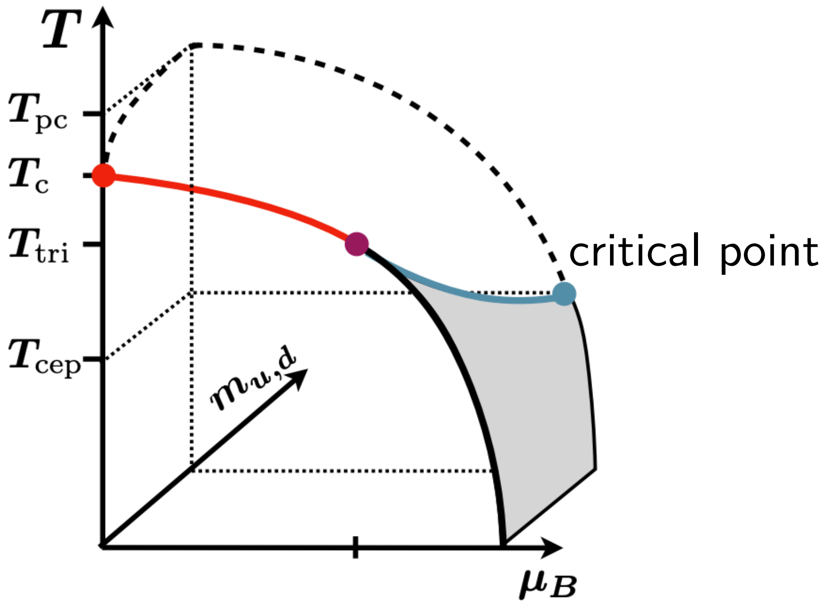
QCD critical point from chiral criticality

Remnants of O(4) chiral criticality at $\mu_B = 0$ quite well established with lattice QCD



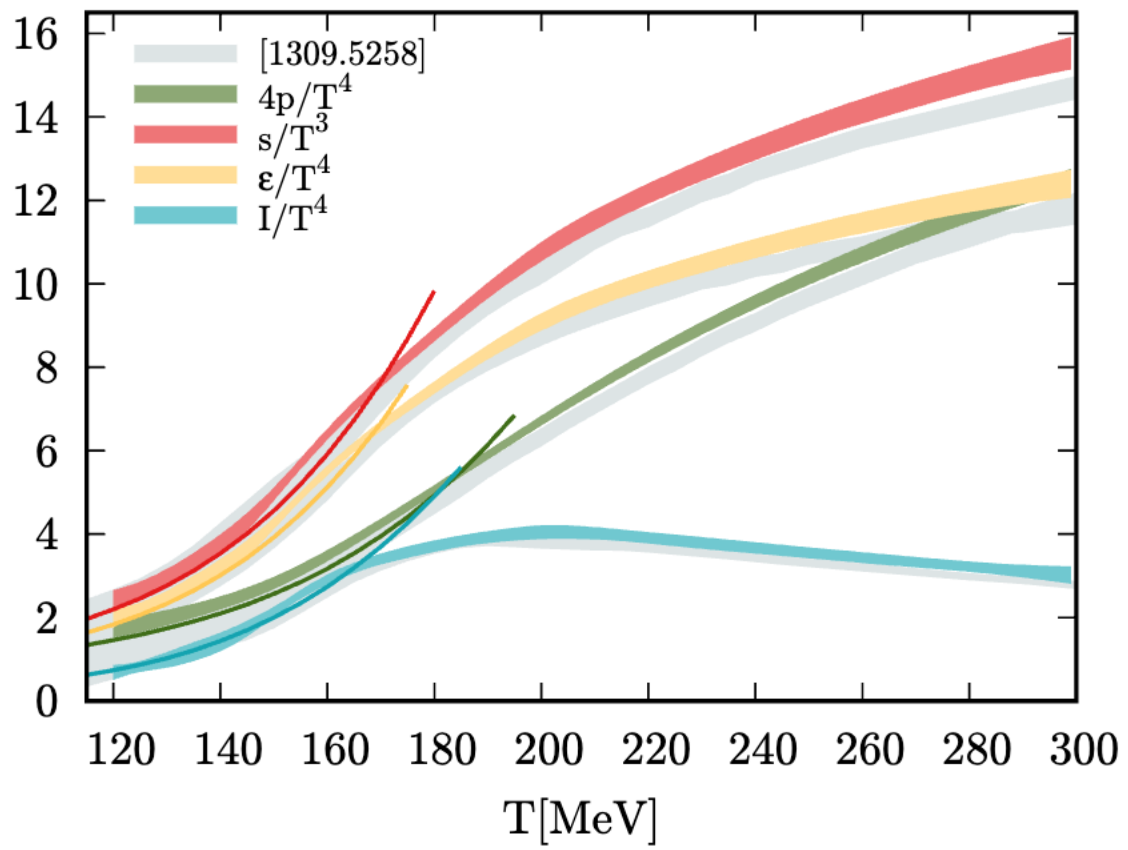
HotQCD Collaboration, PRL 123, 062002 (2019)

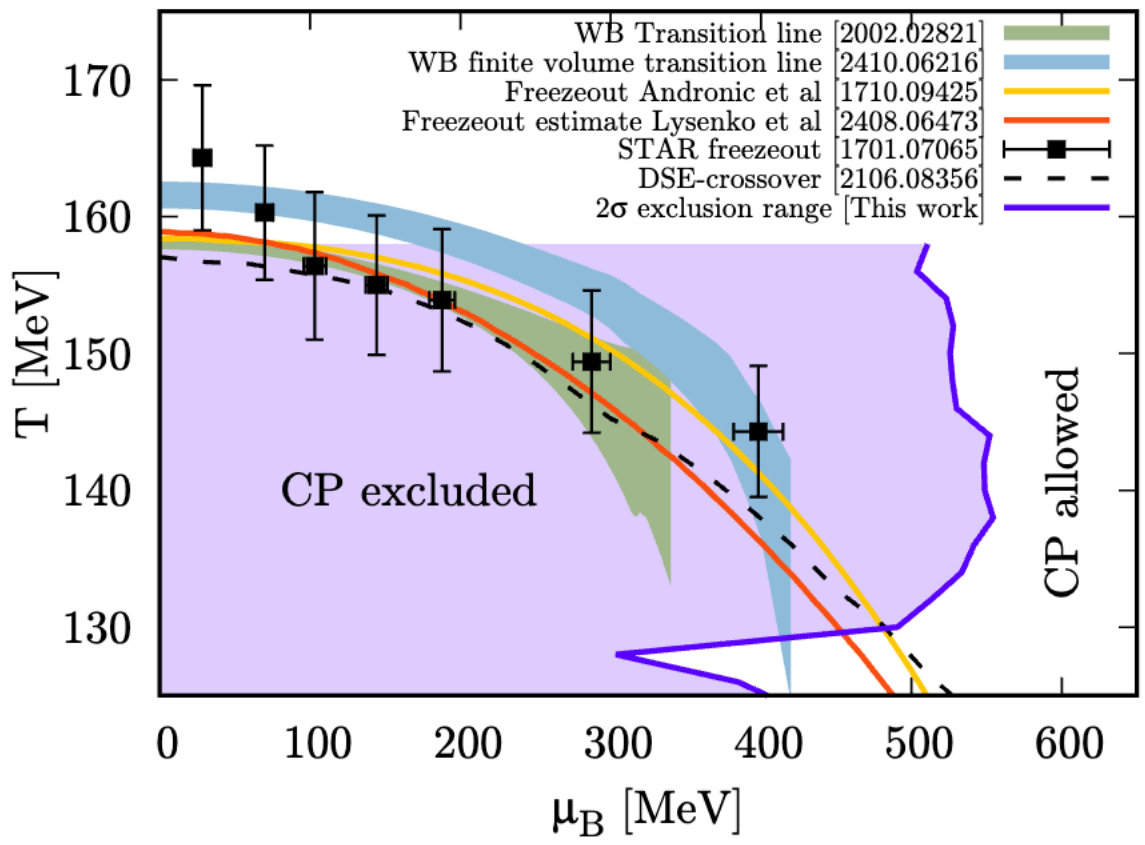
Physical quark masses away the chiral limit: Expect a Z(2) critical point at finite μ_B



C. Schmidt

New CP constraints from lattice QCD

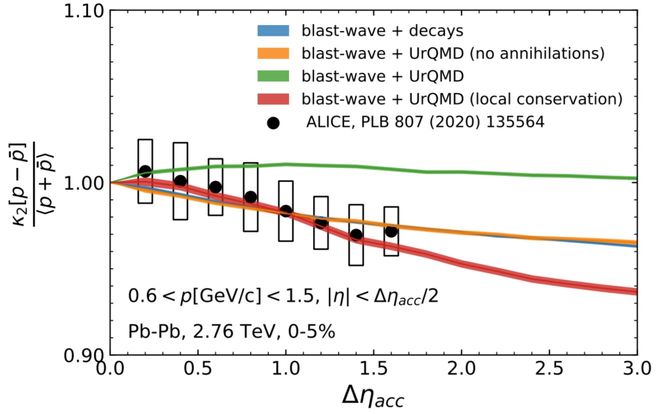




Proton cumulants at high energy

Second-order cumulants such as $\kappa_2[p-\bar{p}]/\langle p+\bar{p}\rangle$:

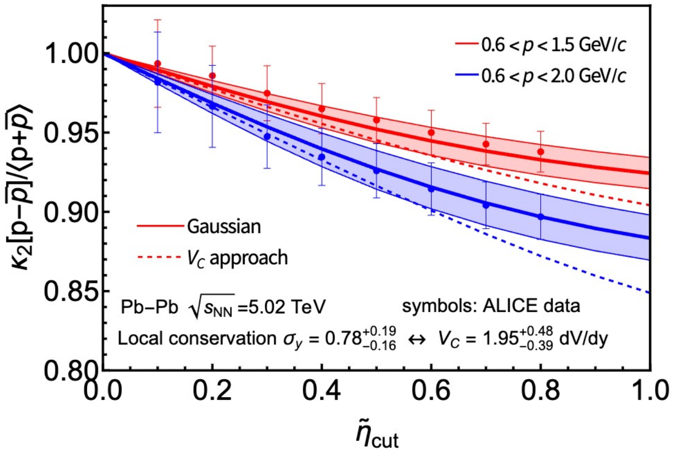
Pb-Pb 2.76 TeV



O. Savchuk et al., PLB 827, 136983 (2022)

- Largely understood as driven by baryon conservation
- baryon annihilation(↗) vs local conservation(↘)
  - Additional measurement of $\kappa_2[p+\bar{p}]$ can resolve it
- For some quantities like net-charge (or net-pion/net-kaon) fluctuations, resonance decays are improtant

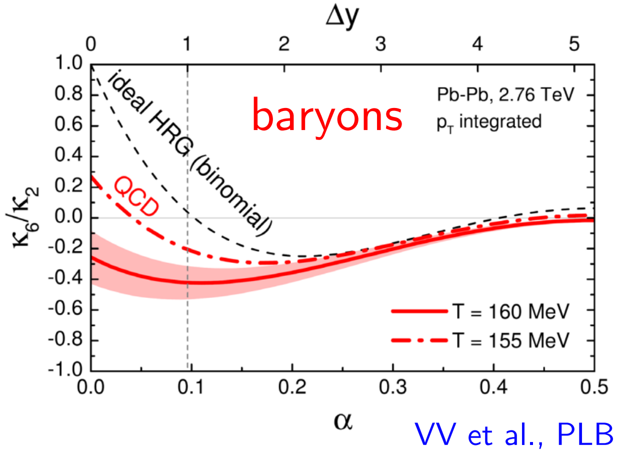
Pb-Pb 5.02 TeV



VV, arXiv:2409.01397

High-order cumulants: probe remnants of chiral criticality

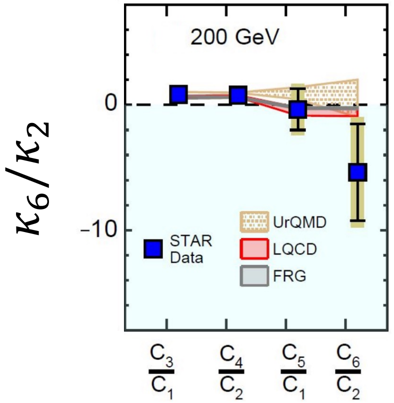
Friman et al., EPJC 71, 1694 (2011)



- negative κ_6 of baryons

VV et al., PLB 811, 135868 (2020)

RHIC 200 GeV: hints of negative $\kappa_6 < 0$ (protons)



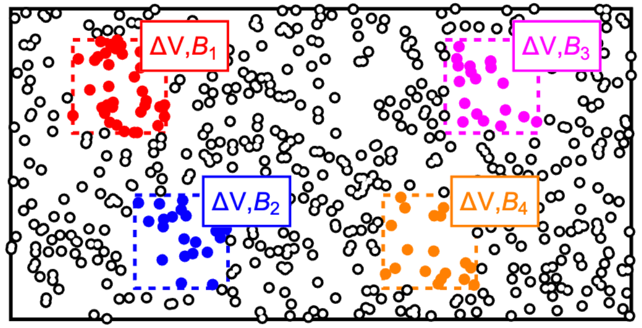
- are baryons even more negative?

STAR Collaboration, PRL 130, 082301 (2023)

Exact charge conservation

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020); VV, arXiv:2409.01397

Utilizing the canonical partition function in thermodynamic limit compute **n-point density correlators**



$$\mathcal{C}_1(\mathbf{r}_1) \; = \; \rho(\mathbf{r}_1)$$

$$\mathcal{C}_2(\mathbf{r}_1,\mathbf{r}_2) = \chi_2 \delta(\mathbf{r}_1 - \mathbf{r}_2) - \frac{\chi_2}{V}$$

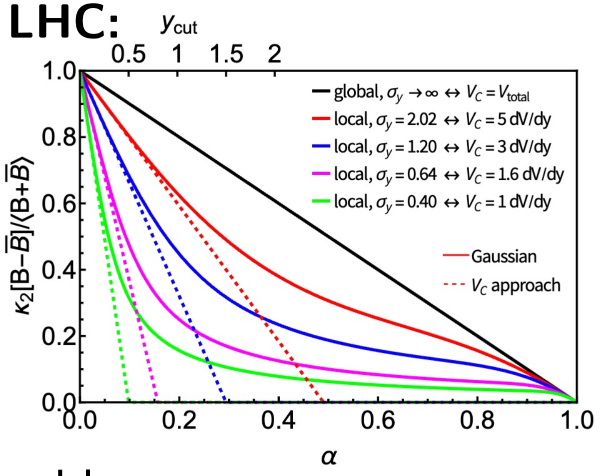
local correlation — balancing contribution (e.g. baryon conservation)

$$\mathcal{C}_3(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = \chi_3 \delta_{1,2,3} - \frac{\chi_3}{V}[\delta_{1,2} + \delta_{1,3} + \delta_{2,3}] + 2\frac{\chi_3}{V^2} \qquad \delta_{1,\ldots,n} = \prod_{i=2}^n \delta(\mathbf{r}_1 - \mathbf{r}_i)$$

local correlation — balancing contributions

$$\mathcal{C}_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = \chi_4 \delta_{1,2,3,4} - \frac{\chi_4}{V}[\delta_{1,2,3} + \delta_{1,2,4} + \delta_{1,3,4} + \delta_{2,3,4}] - \frac{(\chi_3)^2}{\chi_2 V}[\delta_{1,2}\delta_{3,4} + \delta_{1,3}\delta_{2,4} + \delta_{1,4}\delta_{2,3}]$$
$$+ \frac{1}{V^2}\left[\chi_4 + \frac{(\chi_3)^2}{\chi_2}\right][\delta_{1,2} + \delta_{1,3} + \delta_{1,4} + \delta_{2,3} + \delta_{2,4} + \delta_{3,4}] - \frac{3}{V^3}\left[\chi_4 + \frac{(\chi_3)^2}{\chi_2}\right] .$$

local correlation — balancing contributions



Integrating the correlator yields cumulant inside a subsystem of the canonical ensemble

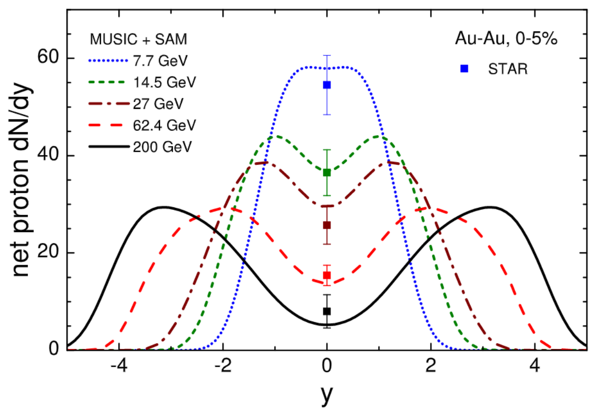
$$\kappa_n[B_{V_s}] = \int_{\mathbf{r}_1 \in V_s} d\mathbf{r}_1 \ldots \int_{\mathbf{r}_n \in V_s} d\mathbf{r}_n \, \mathcal{C}_n(\{\mathbf{r}_i\})$$

Momentum space: Fold with Maxwell-Boltzmann in LR frame and integrate out the coordinates

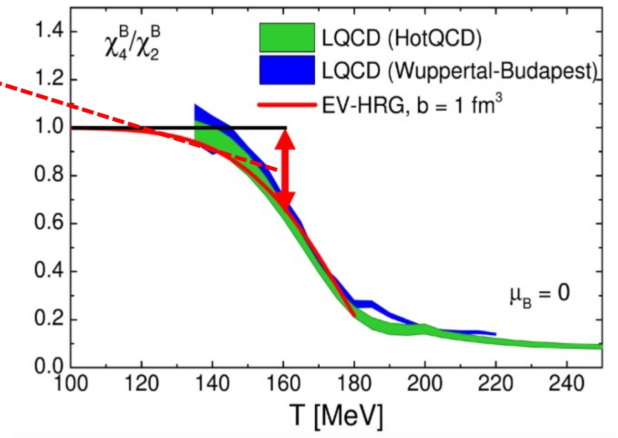
Hydro EV: Non-critical hydro baseline at RHIC-BES

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
  - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
  - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]



- Non-critical contributions computed at particlization ($\epsilon_{sw} = 0.26$ GeV/fm^3)
  - QCD-like baryon number distribution (χ_n^B) via **excluded volume** b = 1 fm^3 [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
  - **Exact global baryon conservation*** (and other charges)
    - Subensemble acceptance method 2.0 (analytic) [VV, Phys. Rev. C 105, 014903 (2022)]
    - or FIST sampler (Monte Carlo) [VV, Phys. Rev. C 106, 064906 (2022)] https://github.com/vlvovch/fist-sampler



- Included: baryon conservation, repulsion, kinematical cuts
- Absent: critical point, local conservation, initial-state/volume fluctuations, hadronic phase

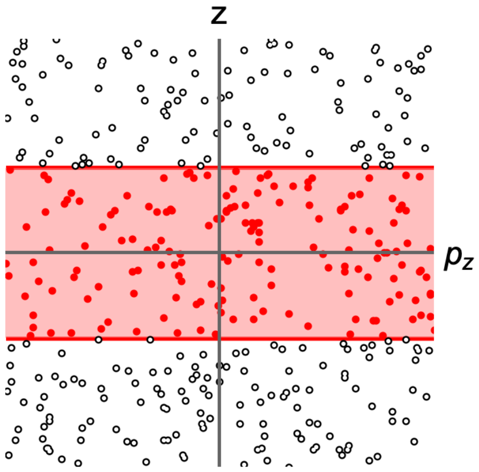
*If baryon conservation is the only effect (no other correlations), non-critical baseline can be computed without hydro
Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, NPA 1008, 122141 (2021)

Coordinate vs Momentum space

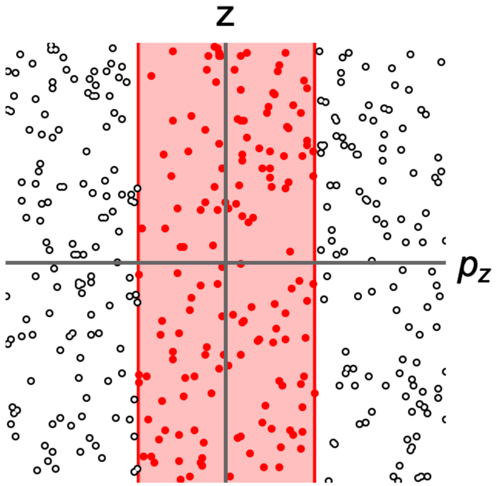
Box setup: Coordinates and momenta are uncorrelated

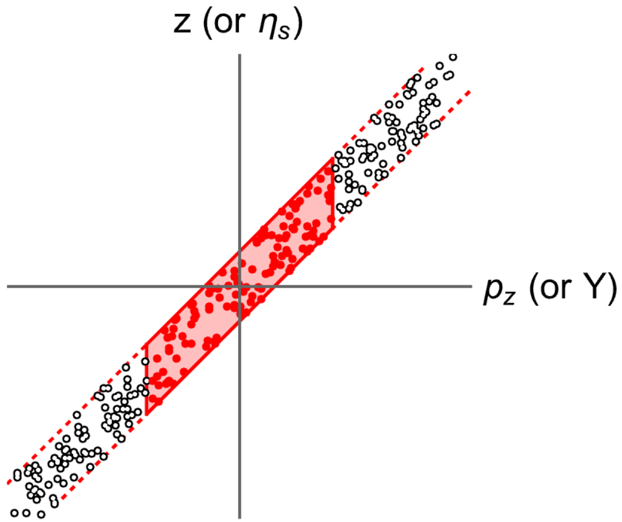
HICs: Flow (e.g. Bjorken)

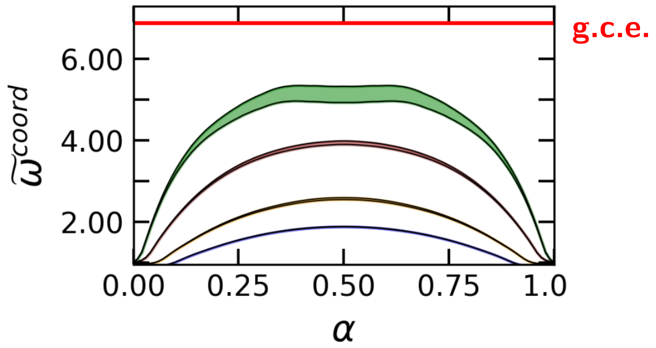
Coordinate space cut



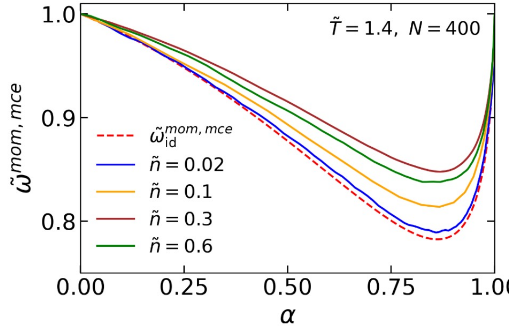
Momentum space cut



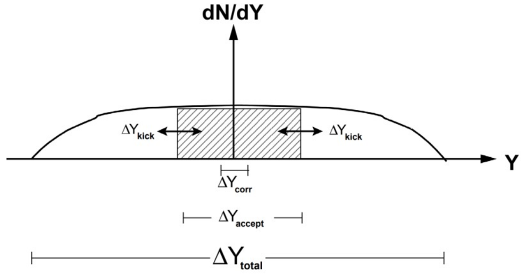




Large correlations



Nothing left

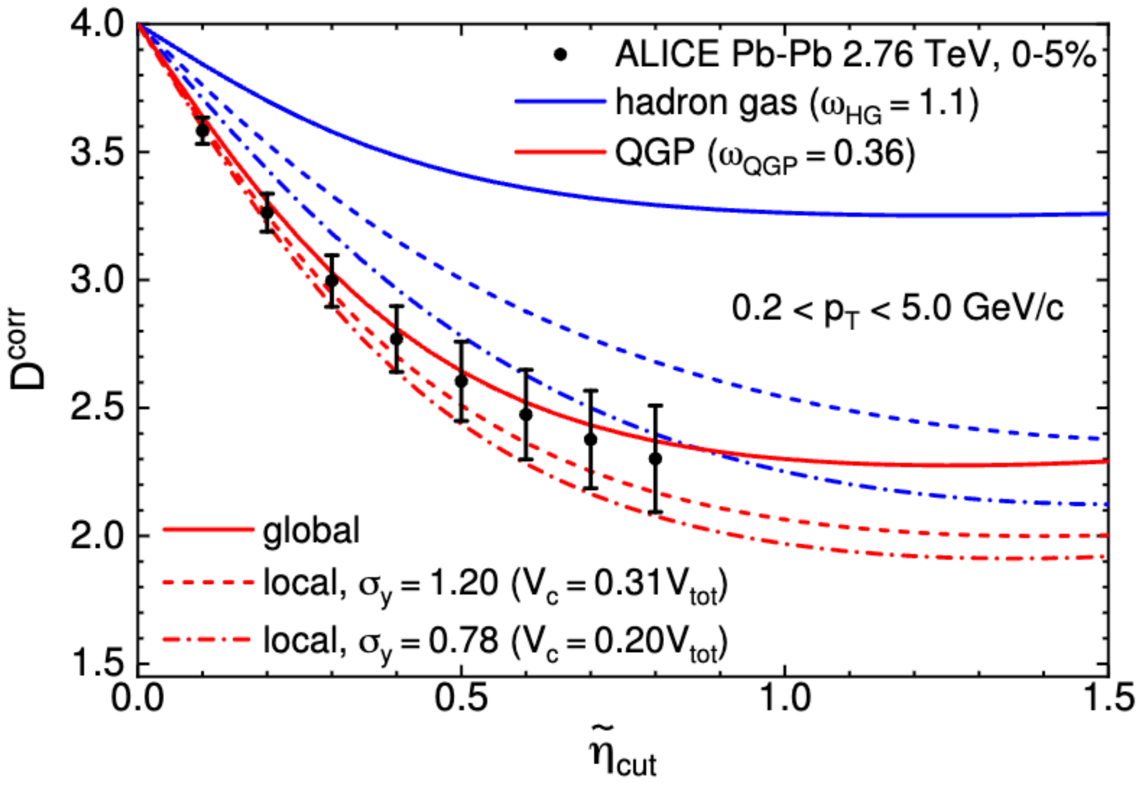


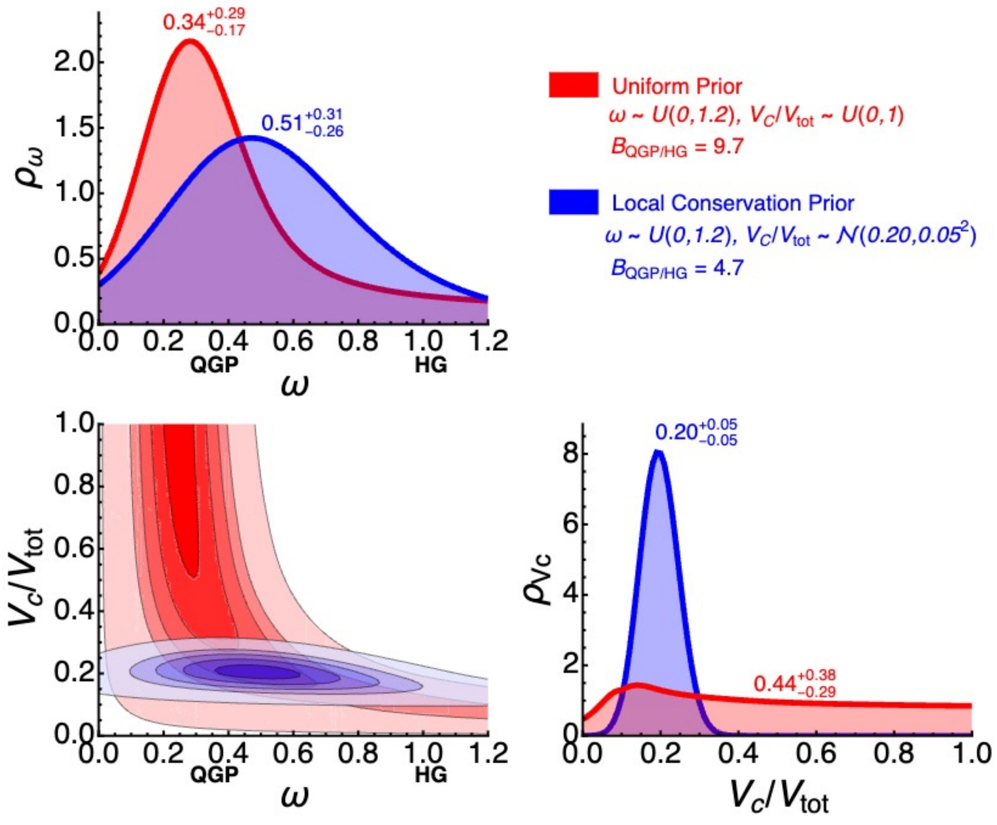
V. Koch, arXiv:0810.2520

momentum cut $\sim$ coordinate cut + smearing

D-measure of charge fluctuations

$$D = 4\frac{\kappa_2[N_+ - N_-]}{\langle N_{ch}\rangle} = 4\frac{\kappa_2[Q]}{\langle Q^+ + Q^-\rangle} = 4\left\{1 - \left(1 - \frac{\omega}{\gamma_Q}\right)\frac{\langle p^2(\eta)\rangle}{\langle p(\eta)\rangle} - \frac{\omega}{\gamma_Q}\frac{\langle p(\eta_1)p(\eta_2)\rangle_{\varkappa}}{\langle p(\eta)\rangle}\right\}$$





J. Parra, R. Poberezhniuk, V. Koch, C. Ratti, arXiv:2504.02085

Dynamical approaches to the QCD critical point search

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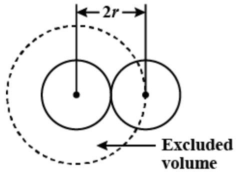
1. Dynamical model calculations of critical fluctuations — BEST COLLABORATION [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]
   - Fluctuating hydrodynamics (hydro+) and (non-equilibrium) evolution of fluctuations
   - Equation of state with a tunable critical point [P. Parotto et al, PRC 101, 034901 (2020); J. Karthein et al., EPJ Plus 136, 621 (2021)]
   - Generalized Cooper-Frye particlization [M. Pradeep, et al., PRD 106, 036017 (2022); PRL 130, 162301 (2023)]

   Alternatives at high μ_B: hadronic transport/molecular dynamics with a critical point

   [A. Sorensen, V. Koch, PRC 104, 034904 (2021); V. Kuznietsov et al., PRC 105, 044903 (2022)]



2. **Deviations from precision calculations of non-critical fluctuations**
   - Non-critical baseline is not flat [Braun-Munzinger et al., NPA 1008, 122141 (2021)]
   - Include essential non-critical contributions to (net-)proton number cumulants
   - Exact baryon conservation + hadronic interactions (hard core repulsion)
   - Based on realistic hydrodynamic simulations tuned to bulk data

   [VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

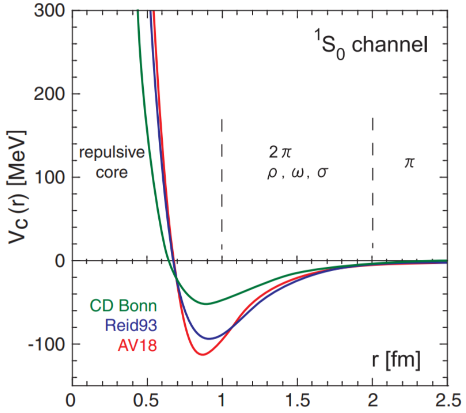


Figure from Ishii et al., PRL '07

Factorial cumulants $\hat{C}_n$ vs ordinary cumulants C_n

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Factorial cumulants: ~irreducible n-particle correlations

$$\hat{C}_n \sim \langle N(N-1)(N-2)\dots\rangle_c$$

$$\hat{C}_1 = C_1$$
$$\hat{C}_2 = C_2 - C_1$$
$$\hat{C}_3 = C_3 - 3C_2 + 2C_1$$
$$\hat{C}_4 = C_4 - 6C_3 + 11C_2 - 6C_1$$

Ordinary cumulants: mix correls. of different orders

$$C_n \sim \langle \delta N^n \rangle_c$$

$$C_1 = \hat{C}_1$$
$$C_2 = \hat{C}_2 + \hat{C}_1$$
$$C_3 = \hat{C}_3 + 3\hat{C}_2 + \hat{C}_1$$
$$C_4 = \hat{C}_4 + 6\hat{C}_3 + 7\hat{C}_2 + \hat{C}_1$$

[Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017); Kitazawa, Luo, PRC 96, 024910 (2017); C. Pruneau, PRC 100, 034905 (2019)]

Factorial cumulants and different effects

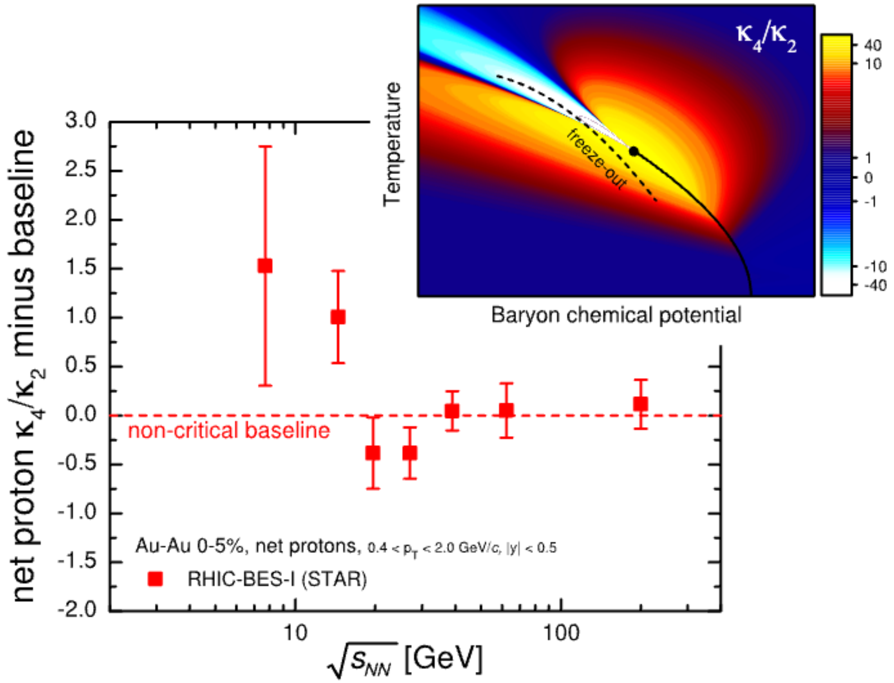
- Baryon conservation [Bzdak, Koch, Skokov, EPJC '17] — $\hat{C}_n^{\text{cons}} \propto (\hat{C}_1)^n / \langle N_{\text{tot}} \rangle^{n-1}$ — *small*
- Excluded volume [VV et al, PLB '17] — $\hat{C}_n^{\text{EV}} \propto b^n$ — *small*
- Volume fluctuations [Holzman et al., arXiv:2403.03598] — $\hat{C}_n^{CF} \sim (\hat{C}_1)^n \kappa_n[V]$ — *depends on volume cumulants*
- **Critical point** [Ling, Stephanov, PRC '16] — $\hat{C}_2^{CP} \sim \xi^2, \quad \hat{C}_3^{CP} \sim \xi^{4.5}, \quad \hat{C}_4^{CP} \sim \xi^7$ — ***large***

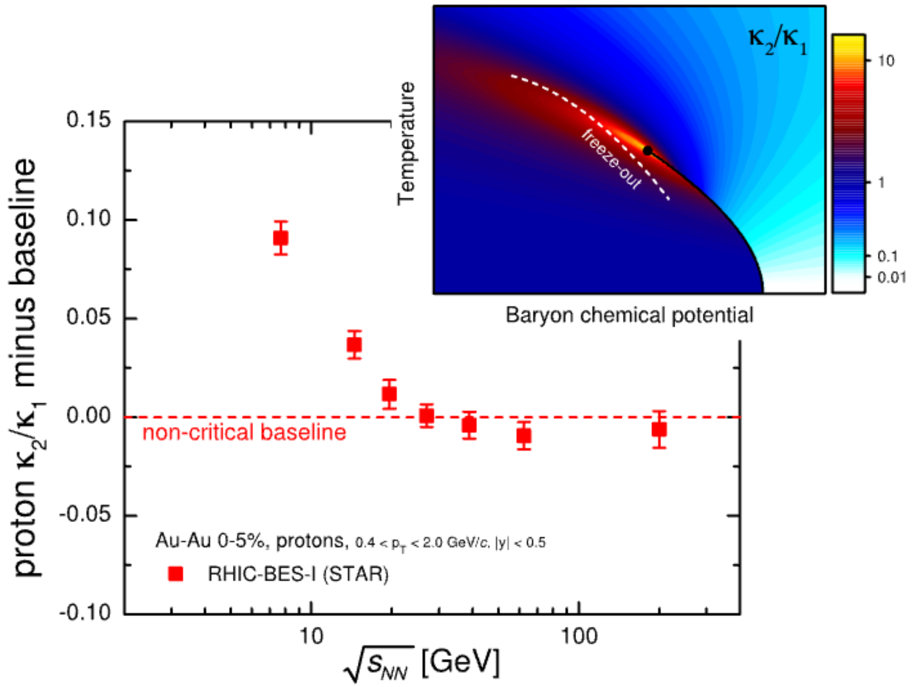
- proton vs baryon [Kitazawa, Asakawa, PRC '12] — $\hat{C}_n^{B} \sim 2^n \times \hat{C}_n^{p}$ — **same sign!**

Hints from RHIC-BES-I

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Subtracting the hydrodynamic non-critical baseline

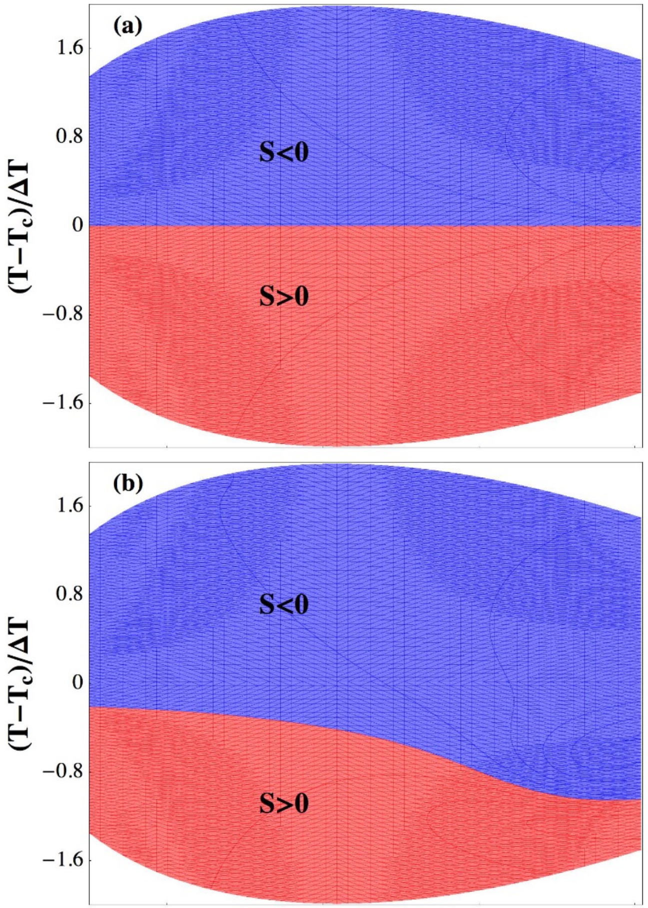




⚠️ **Notation:** Here we use κ_n for cumulants and $\hat{C}_n$ for factorial cumulants, STAR Collaboration uses the opposite

Factorial cumulants from RHIC-BES-II and CP

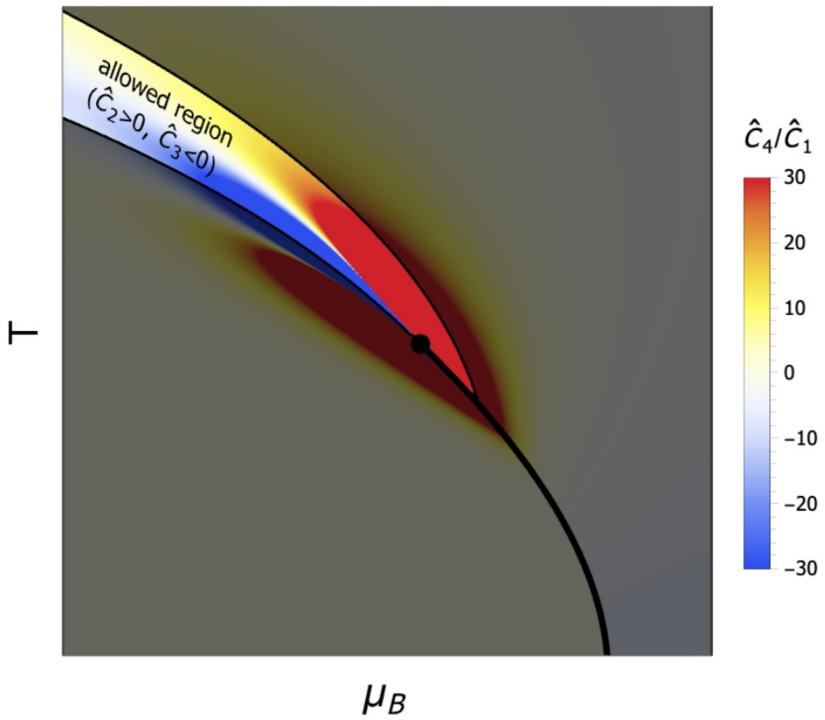
Memory effect



Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)

Exclusion plots

Exclude $\hat{C}_2<0$ & $\hat{C}_3>0$ regions on the phase diagram near CP

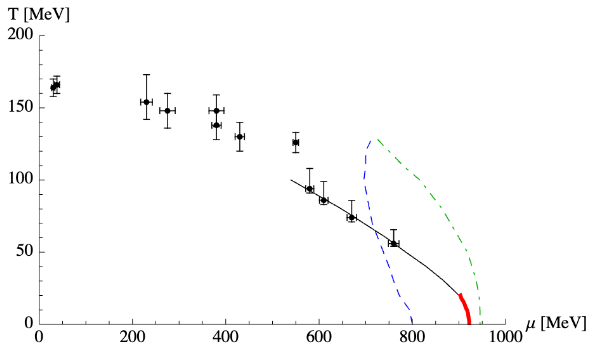


Adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)
and based on the model from
VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

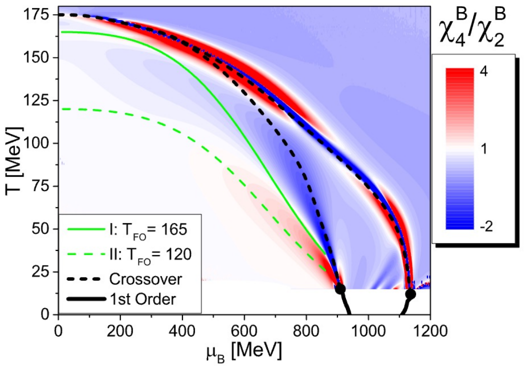
Freeze-out of fluctuations on the QGP side of the crossover?

Interplay with nuclear liquid-gas transition

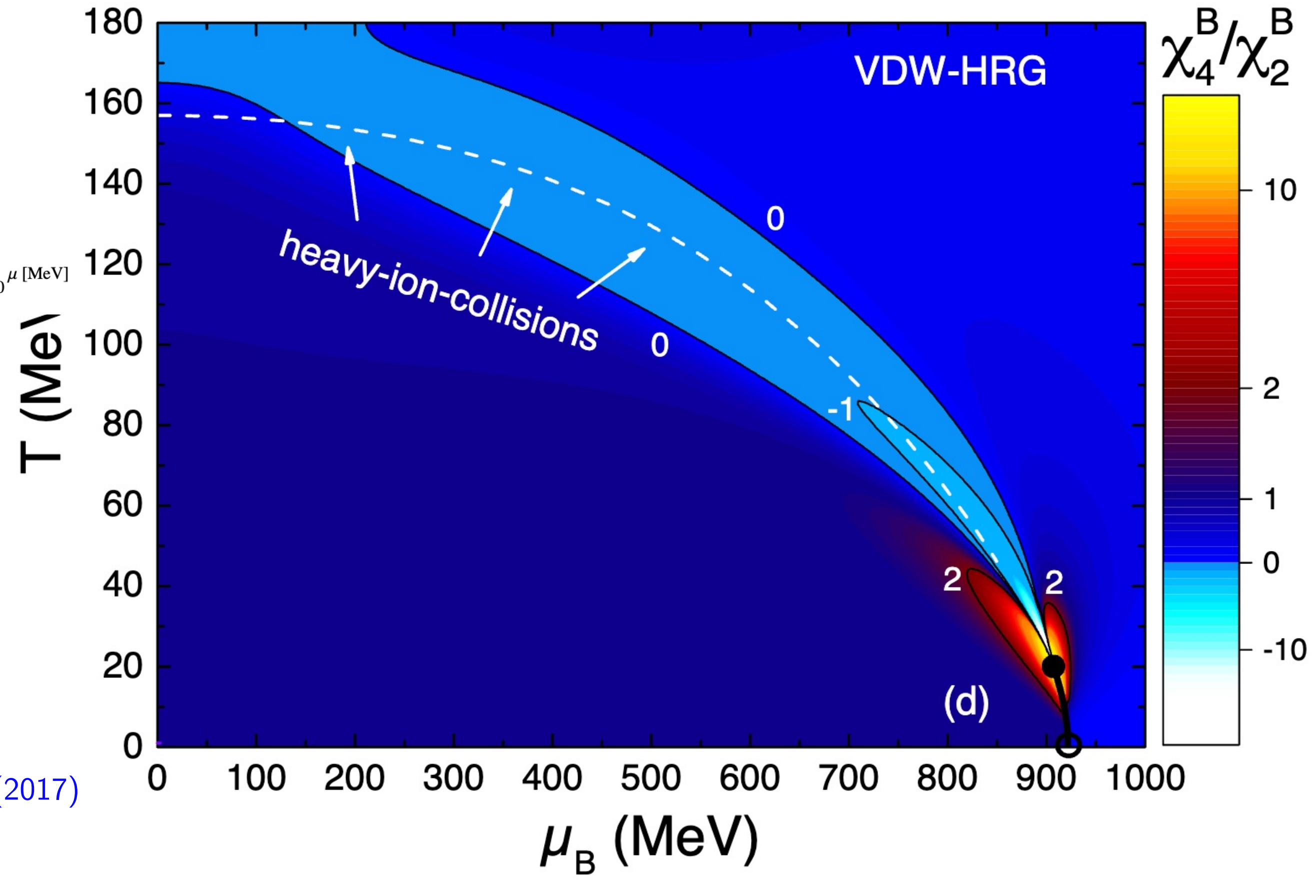
HRG with attractive and repulsive interactions among baryons



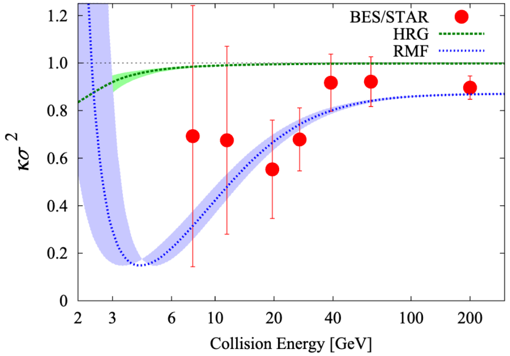
Floerchinger, Wetterich, NPA (2012)



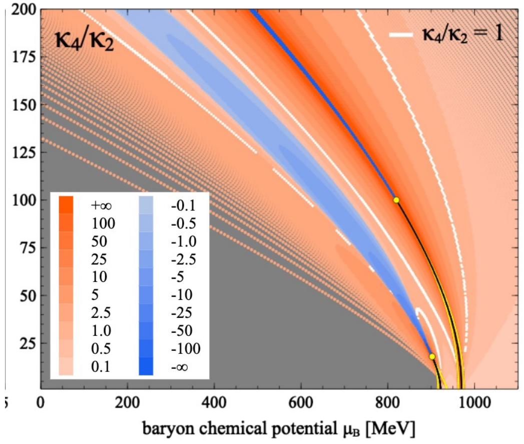
Mukherjee, Steinheimer, Schramm, PRC (2017)



VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)



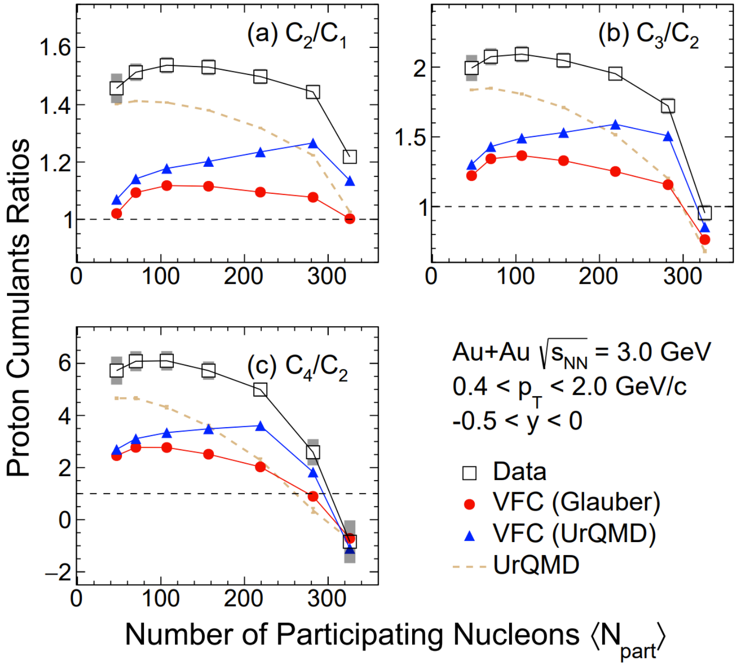
Fukushima, PRC (2014)



Sorensen, Koch, PRC (2020)

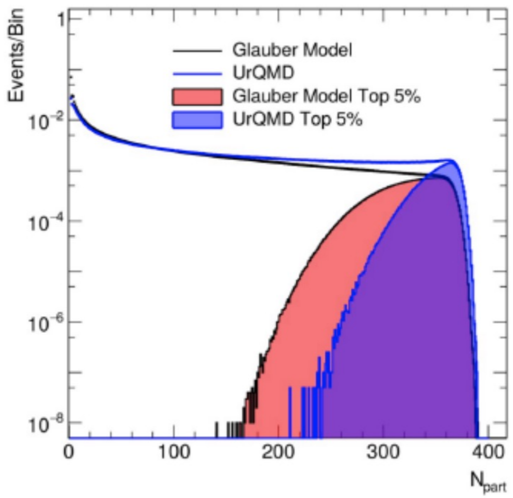
Increasingly relevant at lower energies probed through RHIC-FXT

Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV



STAR Collaboration, Phys. Rev. Lett. 128 (2022) 202303

STAR-FXT



HADES

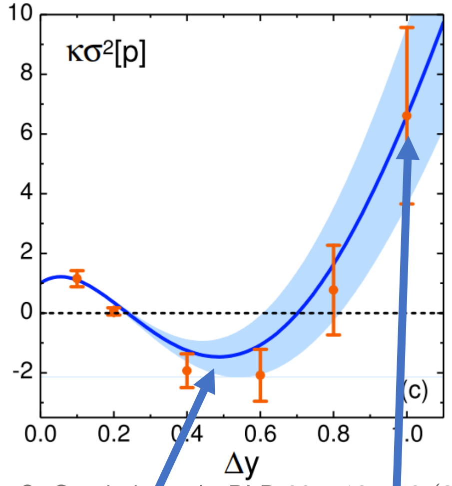


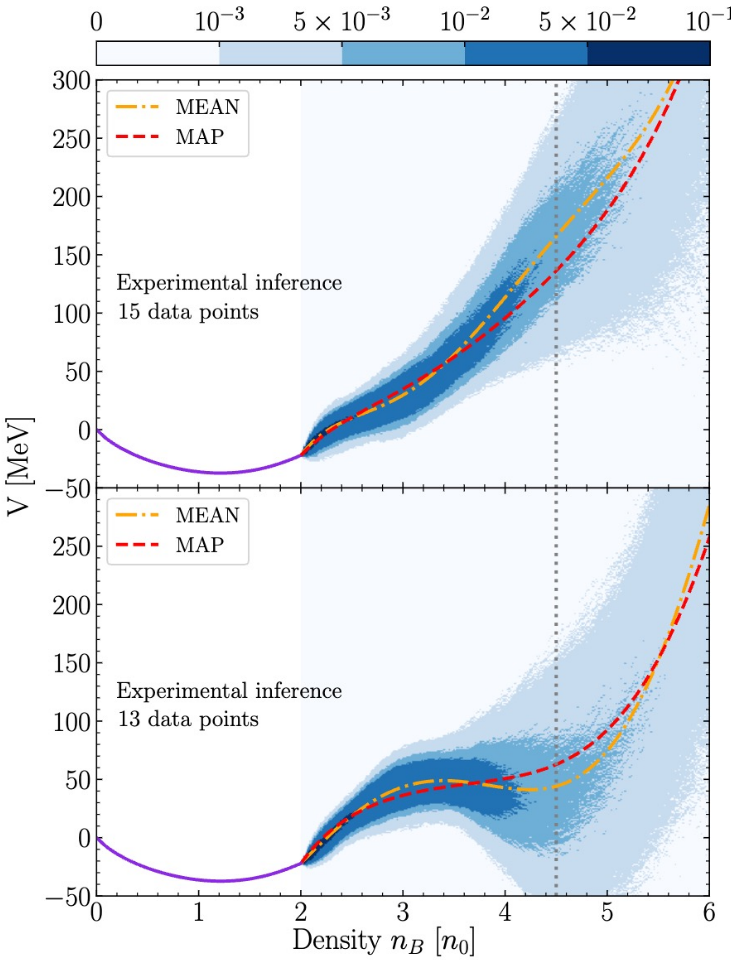
Figure from O. Savchuk et al., PLB 835, 137540 (2022)

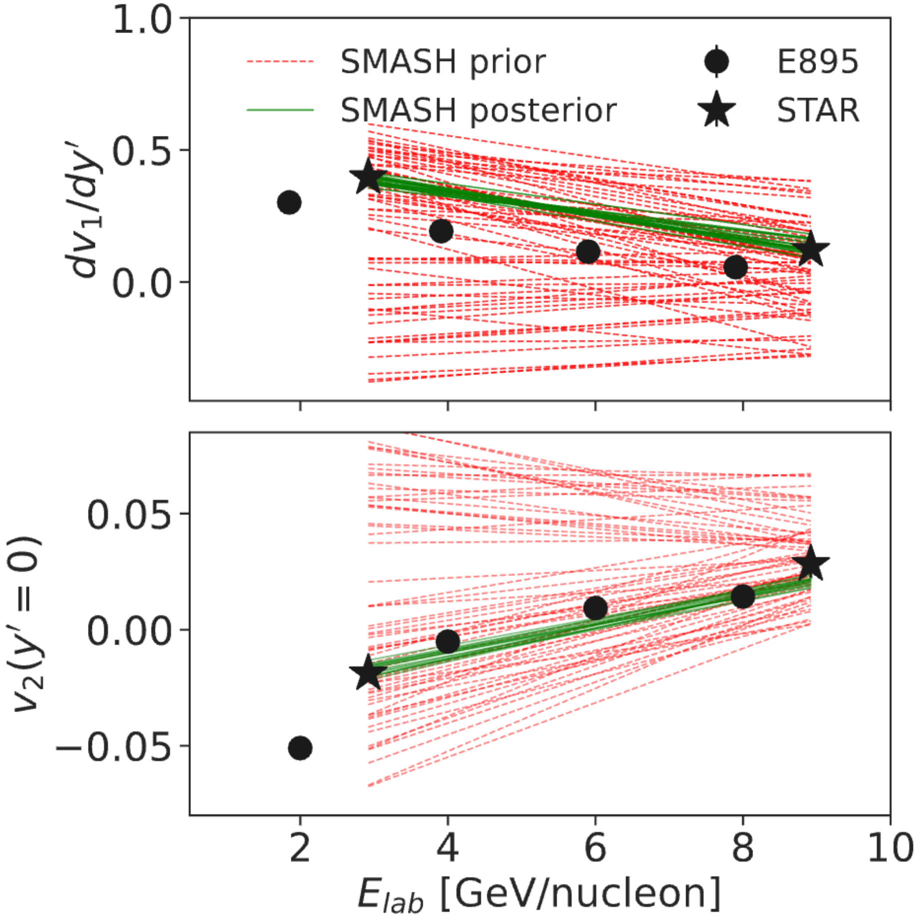
- Volume fluctuations/centrality selection appear to play an important role
  - UrQMD is useful for understanding basic systematics associated with it
- Indications for enhanced scaled variance, $\kappa_2/\kappa_1 > 1$
- κ_4/κ_2 negative and described by UrQMD (purely hadronic?), note -0.5<y<0 instead of |y|<0.5

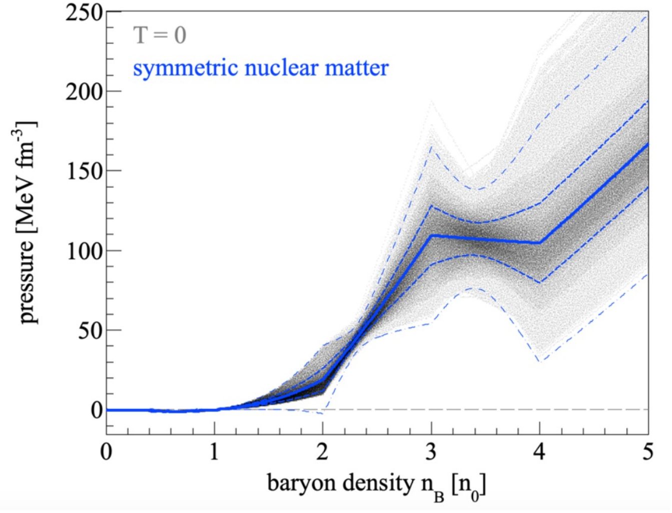
Proper understanding of $\kappa_2/\kappa_1 > 1$ in both HADES and STAR-FXT is missing

Dense matter EoS from flow measurements

- Use hadronic transport (UrQMD and SMASH) with adjustable mean field to use a flexible EoS
- Extract the EoS from proton flow measurements





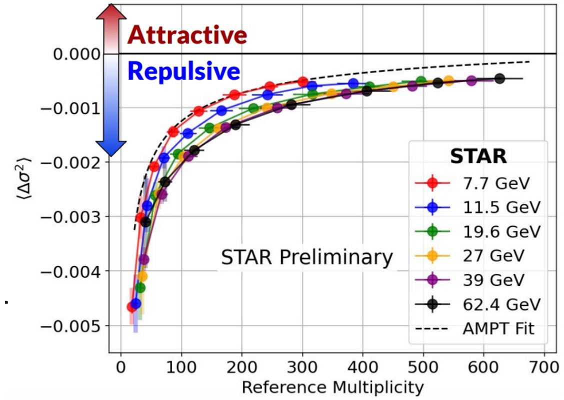


M. Kuttan, Steinheimer, Zhou, Stoecker, PRL 131, 202303 (2023)

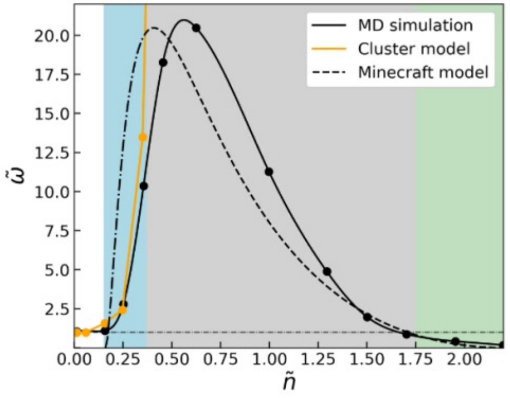
Oliinychenko, Sorensen, Koch, McLerran, PRC 108, 034908 (2023)

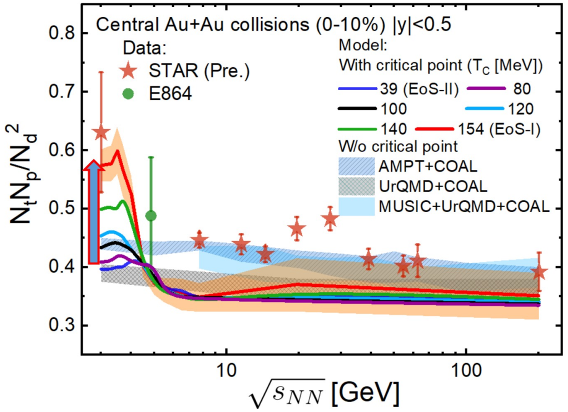
Other observables

- Azimuthal correlations of protons
  - points to repulsion at RHIC-BES



- Light nuclei
  - Spinodal/critical point enhancement of density fluctuations and light nuclei production





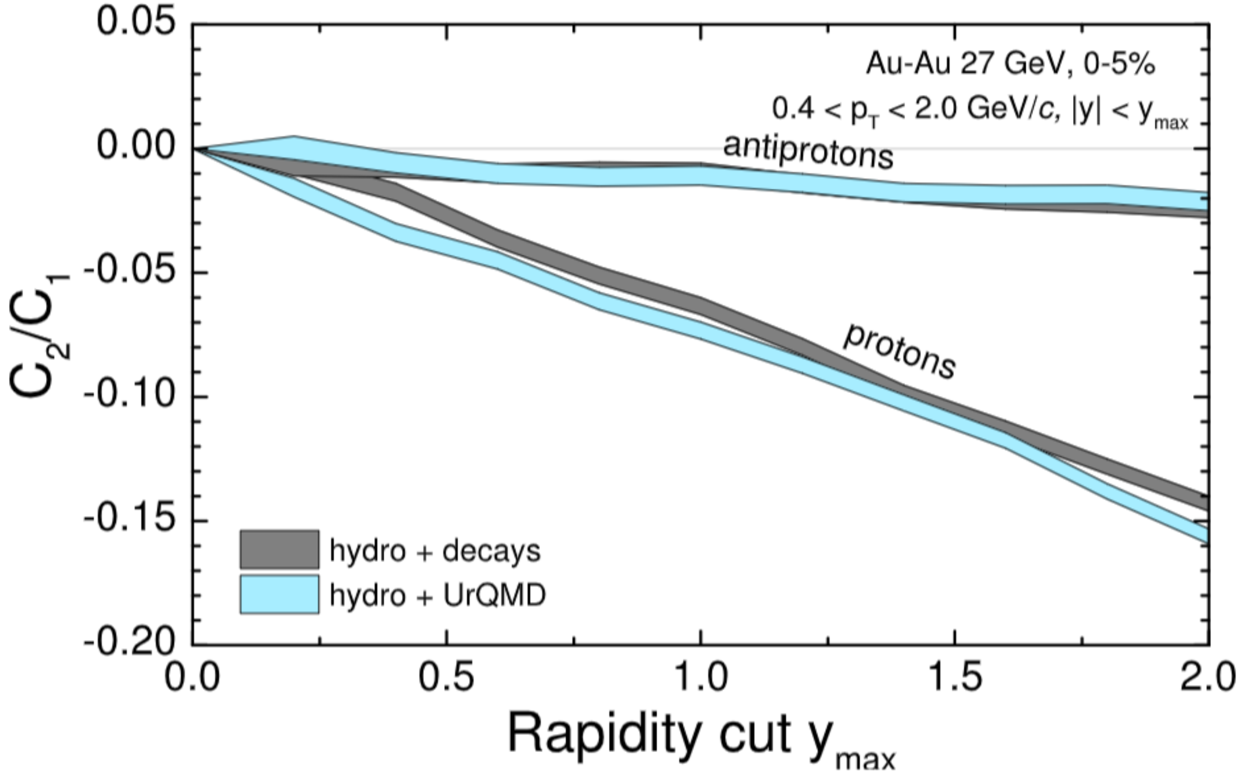
- Proton intermittency
  - No structure indicating power-law seen by NA61/SHINE

- Directed flow, speed of sound

Consistency in understanding all the observables is required

Effect of the hadronic phase

Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD



Dependence on the switching energy density