QCD Phase Diagram at Finite Density

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Early-Career Researcher Day, Quark Matter 2025

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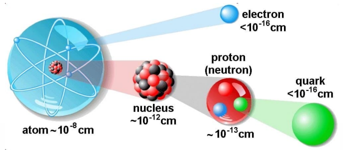
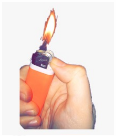
UNIVERSITY OF
HOUSTON

Extreme states of matter

Fermi 1953

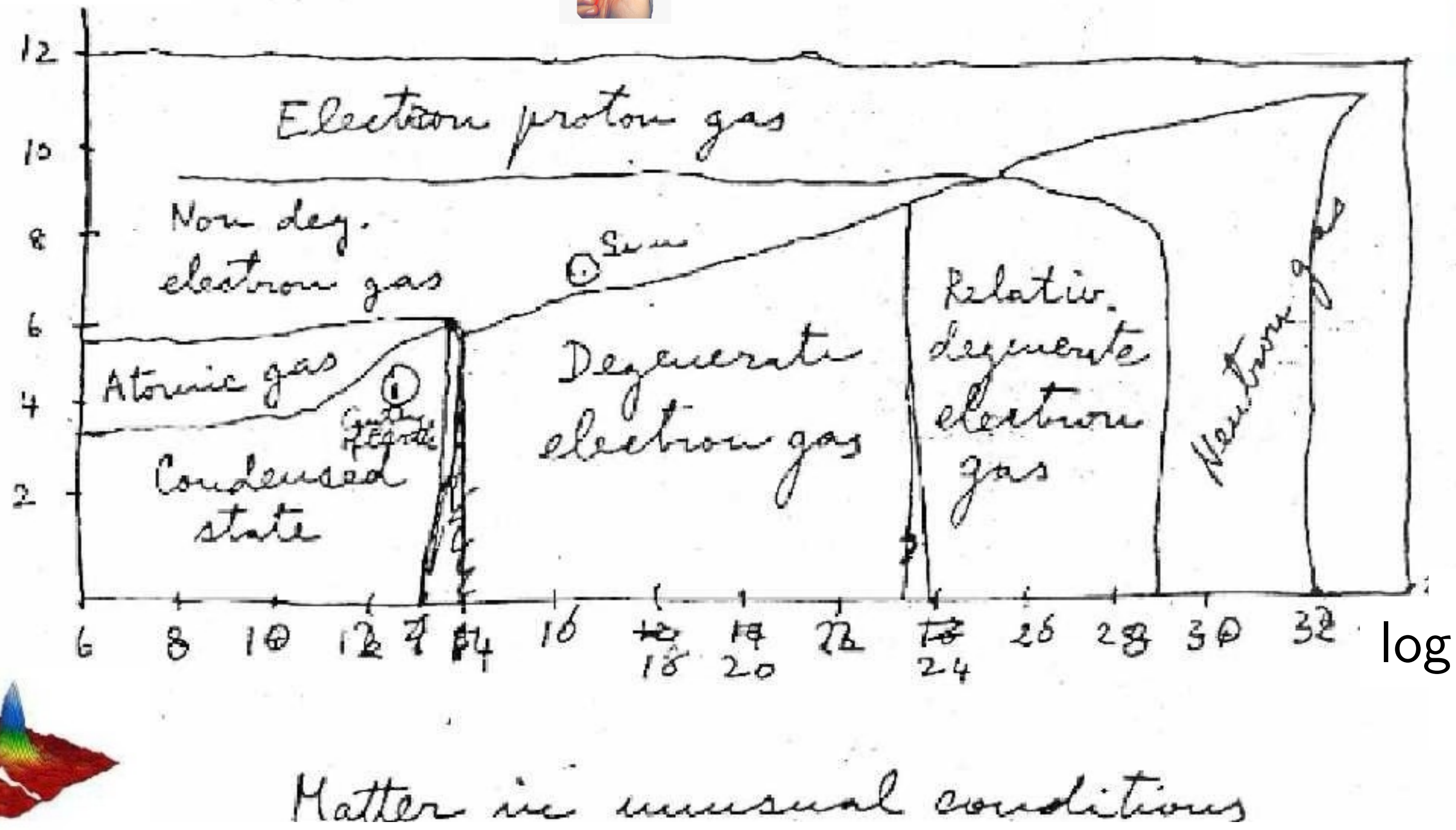




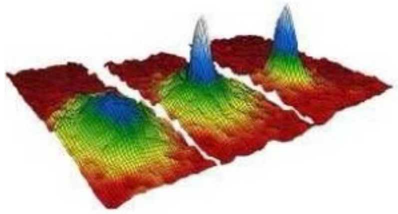
log T

tera Kelvin

1 MeV ~ 10^{10} K



log p



QCD under extreme conditions

$$\mathcal{L} = \sum_{q=u,d,s,\dots} \bar{q}\left[i\gamma^\mu(\partial_\mu - igA^a_\mu\lambda_a) - m_q\right] q - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

What we know

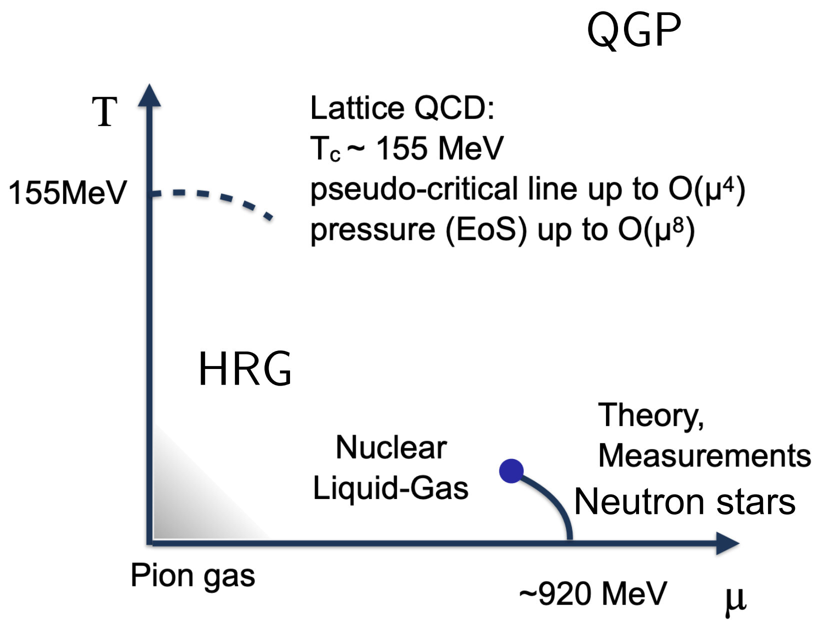


Figure courtesy of V. Koch

- Dilute hadron gas at low T & μ_B due to confinement, quark-gluon plasma high T & μ_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured

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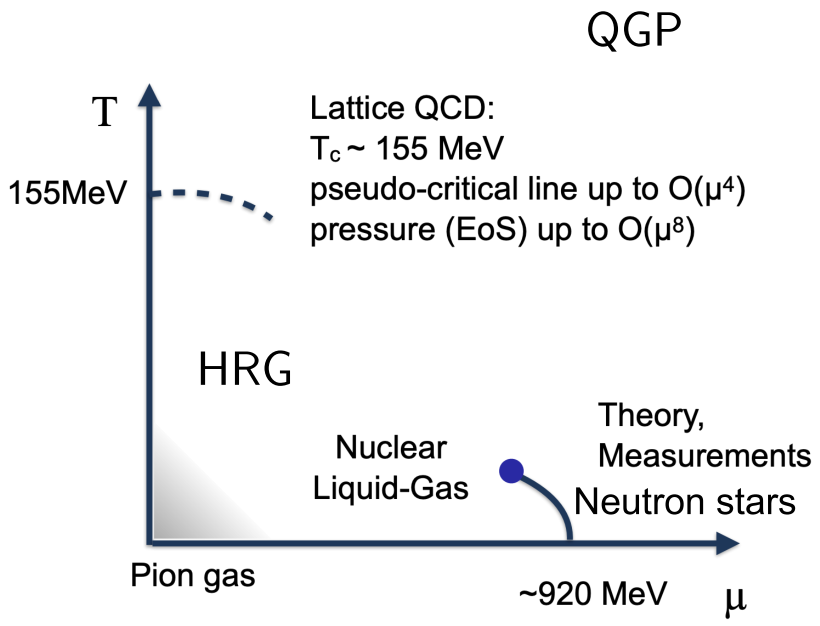

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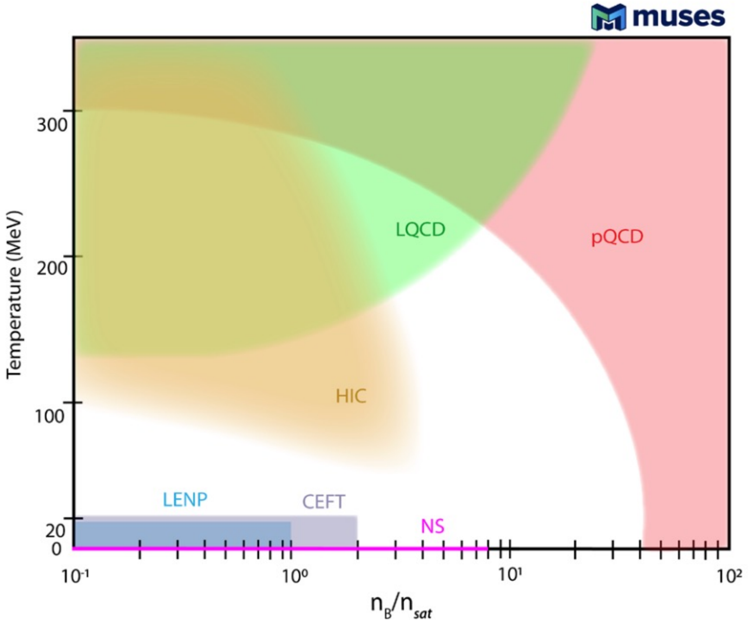
What we think we know



MUSES Collaboration, LRR 27 (2024)

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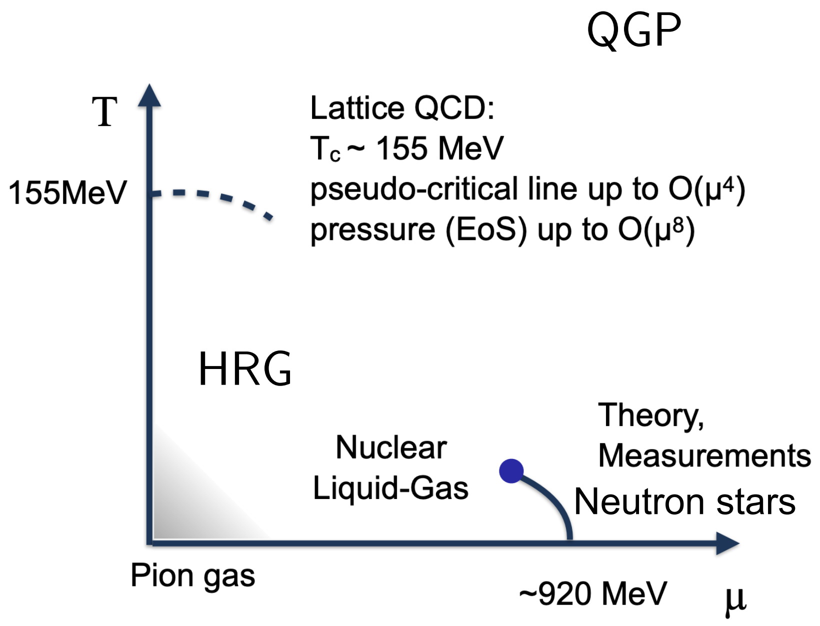

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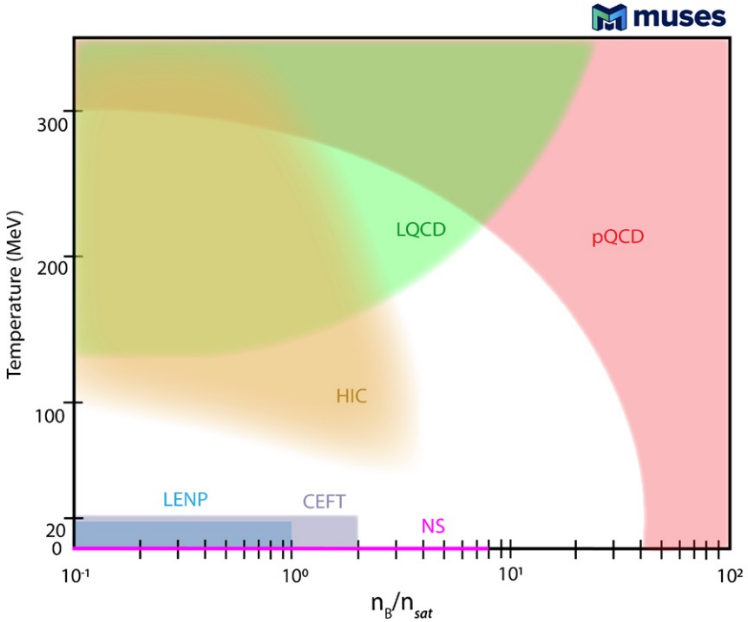


MUSES Collaboration, LRR 27 (2024)

What we hope to know

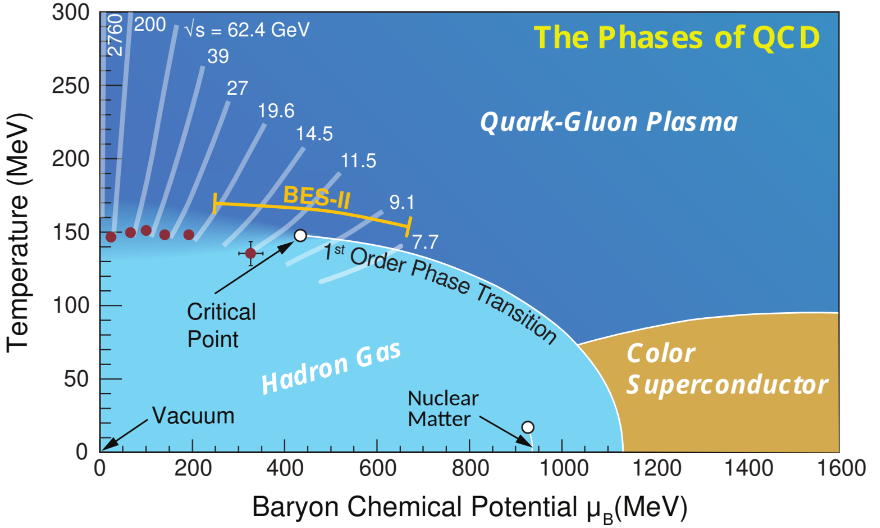


Figure from Bzdak et al., Phys. Rept. ’20 & 2015 US Nuclear LRP

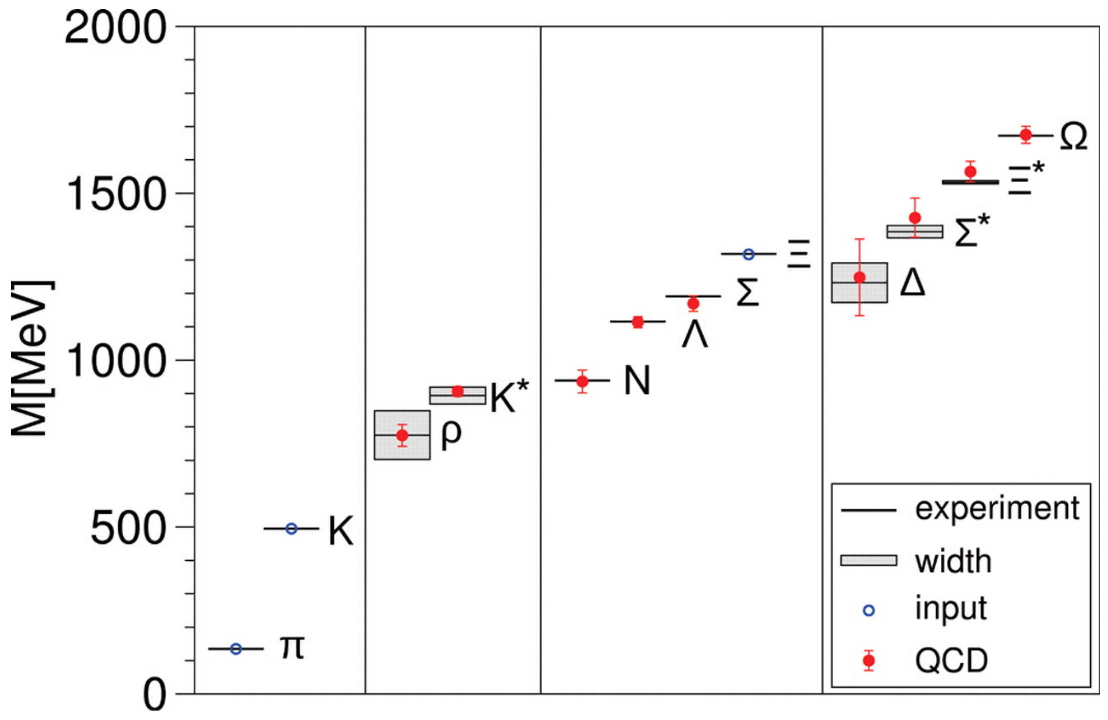
- Dilute hadron gas at low T & μ_B due to confinement, quark-gluon plasma high T & μ_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured

QCD Phase Diagram:
From zero to non-zero density

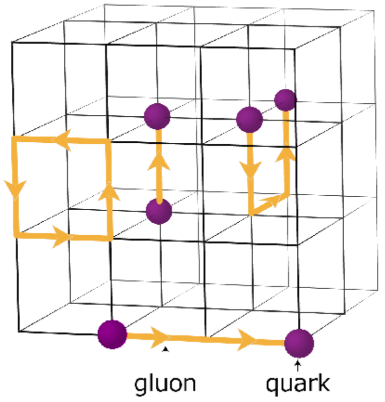
Non-perturbative methods

First-principle tool: Lattice QCD

Ab-initio calculation of hadron masses



BMW Collaboration, Science 322, 1224 (2008)



Remarkable agreement of QCD with the experiment

QCD transition from lattice QCD

$$Z = \mathrm{Tr}(e^{-(\hat{H}-\mu\hat{N})/T}) = \int DU \det M[U,\mu]\, e^{-S_{YM}} \rightarrow P = P(T,\mu)$$

equation of state

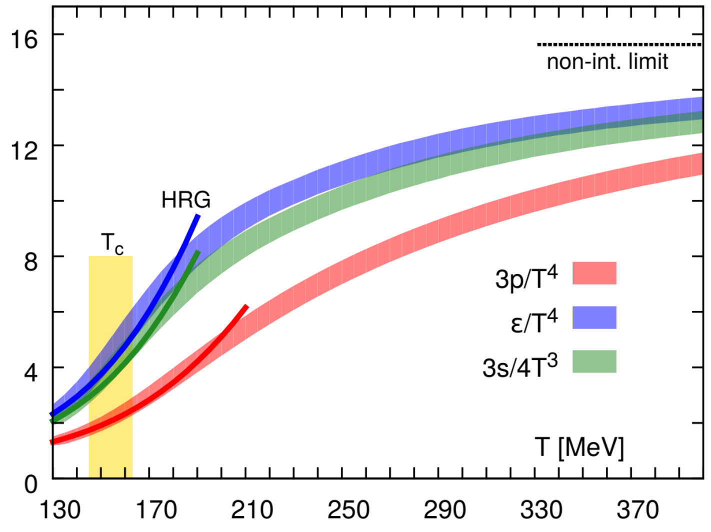


Figure from HotQCD Collaboration, PRD '14

lattice QCD

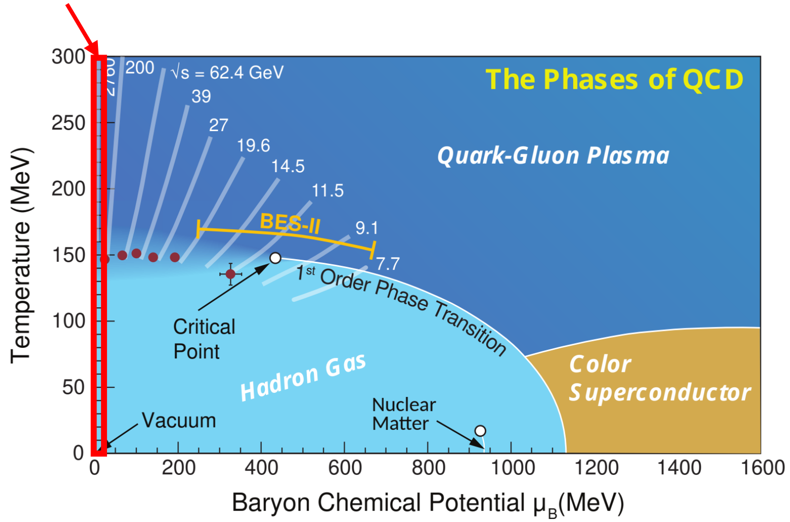


Figure from Bzdak et al., Phys. Rept. '20

- Analytic crossover at vanishing net baryon density at $T_{pc} \approx 155$ MeV – a first-principle result
  [Y. Aoki et al., Nature 443, 675 (2006)]

- **Finite density:** $\mu_B > 0$ (excess of baryons over antibaryons) encounters the **sign problem**

$$\det M[U,\mu] = |\det M[U,\mu]|\, e^{i\theta}$$

Extrapolations toward $\mu_B > 0$

Taylor expansion + various resummations and extrapolation schemes from $\mu_B = 0$

(Pseudo-)critical line: $\frac{T_c(\mu_B)}{T_c(\mu_B=0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c(\mu_B)}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c(\mu_B)}\right)^4 \ldots$ $\quad \kappa_2 = 0.0153(18), \quad \kappa_4 = 0.00032(67)$

[Borsanyi et al. (WB), PRL '20; Bazavov et al. (HotQCD), PLB '19]

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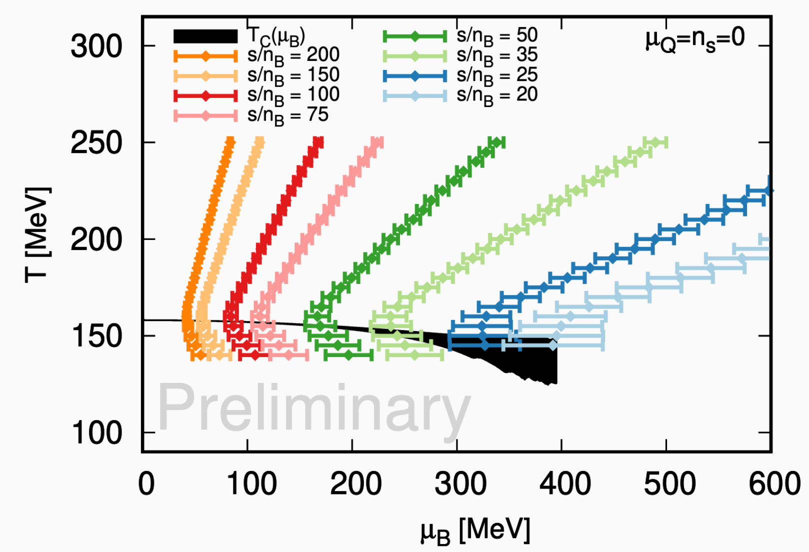
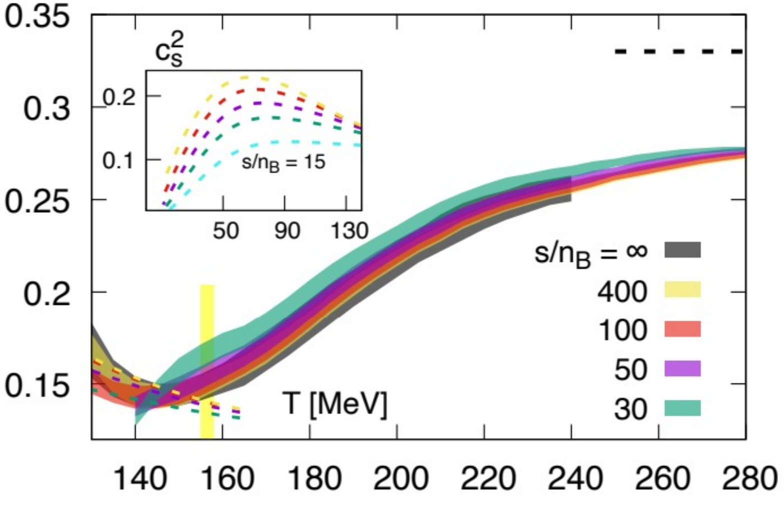
[Borsanyi et al. (WB), PRL '20; Bazavov et al. (HotQCD), PLB '19]

T' expansion scheme



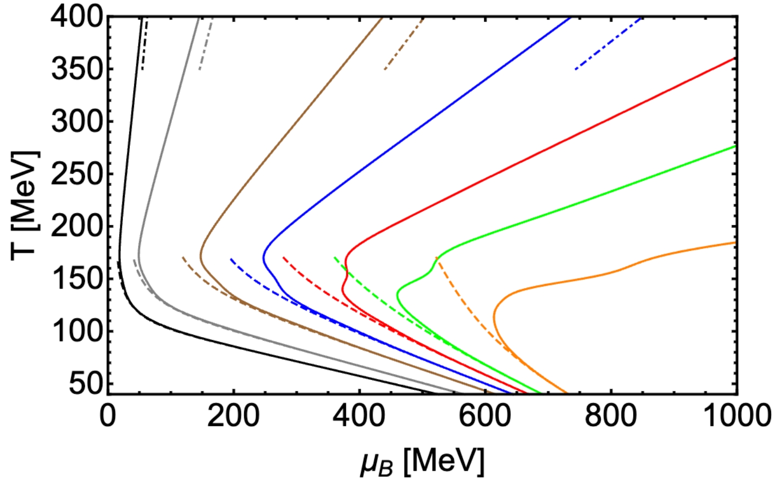
[Borsanyi et al. (WB), Phys. Rev. D 105, 114504 (2022)]

Padé approximants



[Bollweg et al. (HotQCD), Phys. Rev. D 108, 014510 (2023)]

Cluster expansion in fugacities



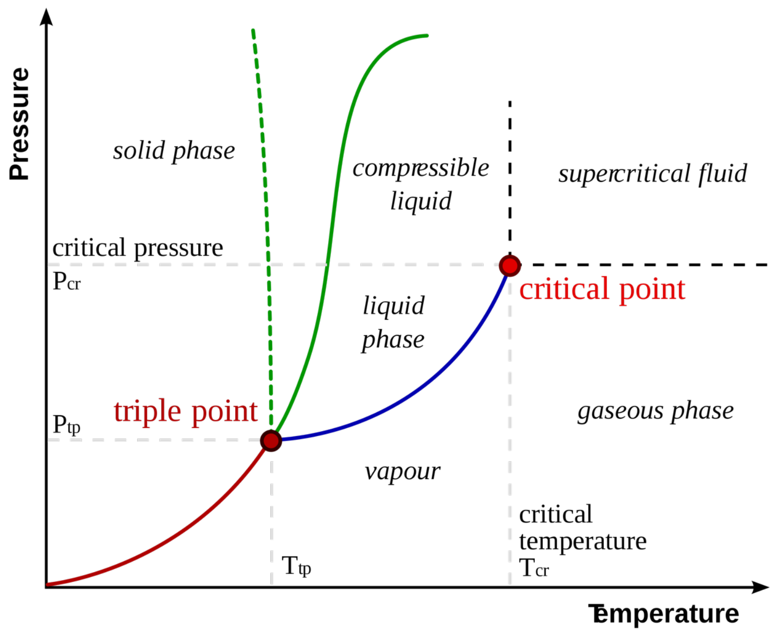
[VV et al., Phys. Rev. D 97, 114030 (2028)]

4D-T'ExS: T' expansion scheme in (μ_B, μ_Q, μ_S) [Abuali, Borsanyi et al. (WB), arXiv:2504.01881]

All work at small/moderate densities where there is only crossover (no phase transition or CP)

QCD critical point

Ordinary fluid (e.g. water)



QCD

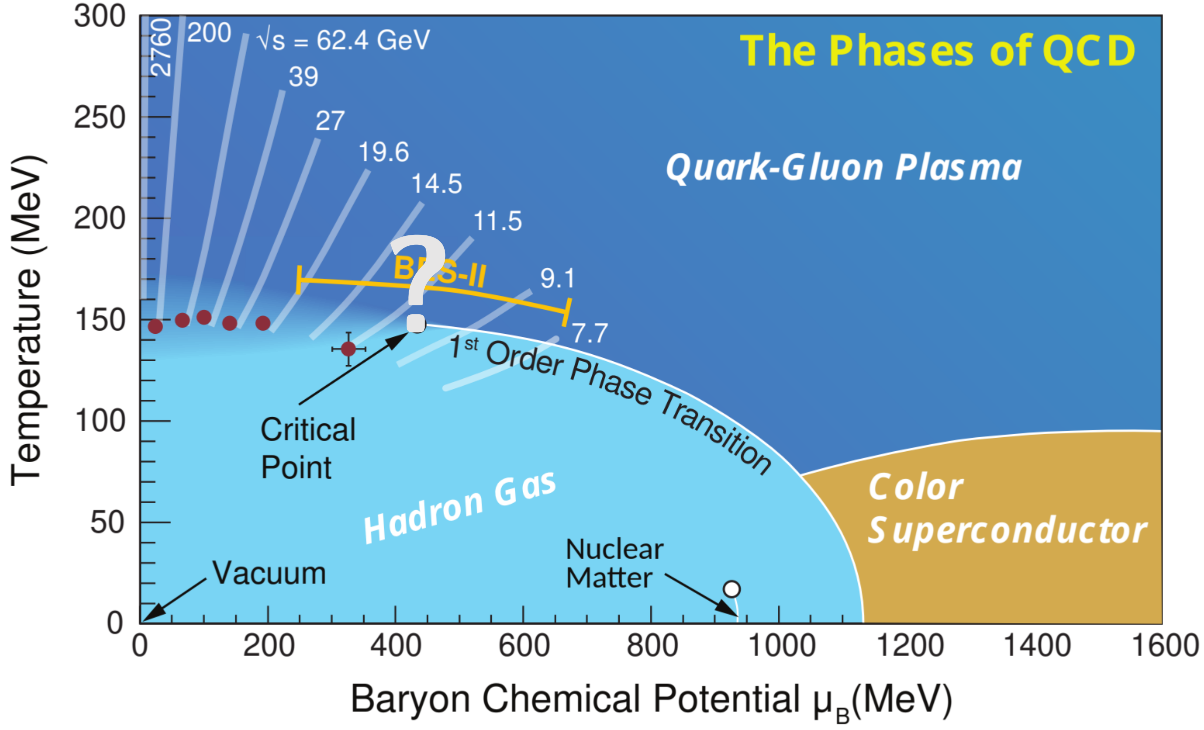


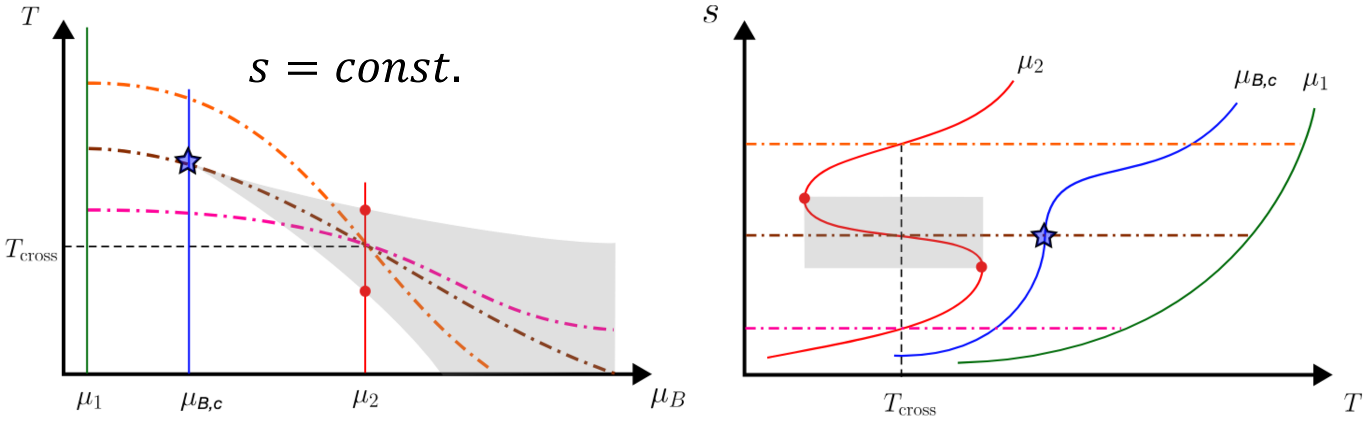
Figure from Bzdak et al., Phys. Rept. '20

What is the nature of the quark-hadron transition at finite baryon density?

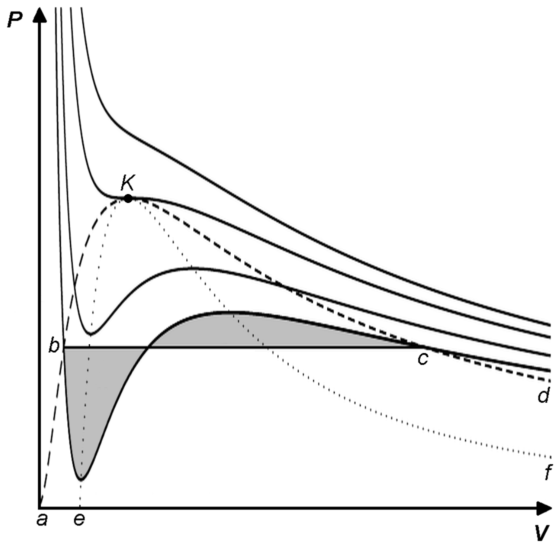
*Is there a QCD phase transition and **critical point**? Where?*

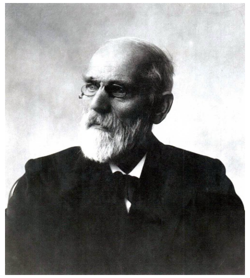
Extrapolating critical point from lattice

- Entropy density s becomes multi-valued function of T and μ_B for a first-order phase transition
- It develops a distinctive S-shape as a function of T at $\mu_B = const > \mu_{B,c}$



Recall the textbook:





Critical Point:

$$\left(\frac{\partial T}{\partial s}\right)_{\mu_B} = 0, \qquad \left(\frac{\partial^2 T}{\partial s^2}\right)_{\mu_B} = 0.$$

$$\left(\frac{\partial P}{\partial \rho_B}\right)_{T} = 0, \qquad \left(\frac{\partial^2 P}{\partial \rho_B^2}\right)_{T} = 0.$$

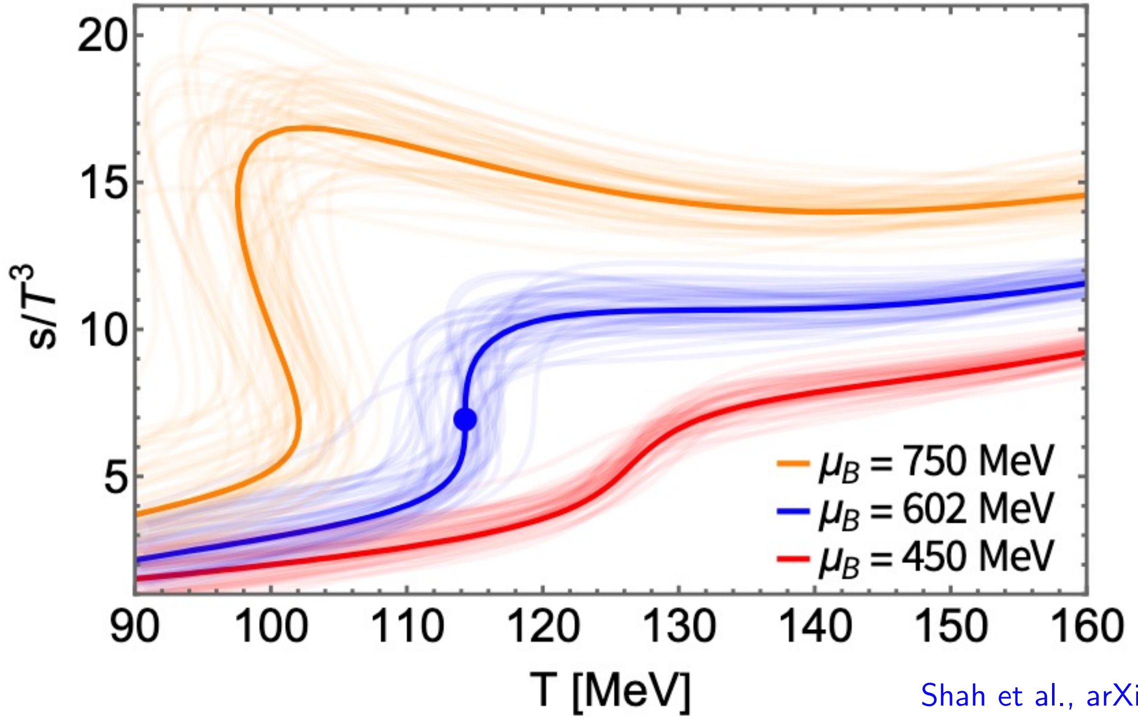
Shah et al., arXiv:2410.16026

Looking for entropy crossings

- Critical point ruled out (2σ level) at $\mu_B < 400$ MeV (no crossings)

Borsanyi et al., arXiv:2502.10267

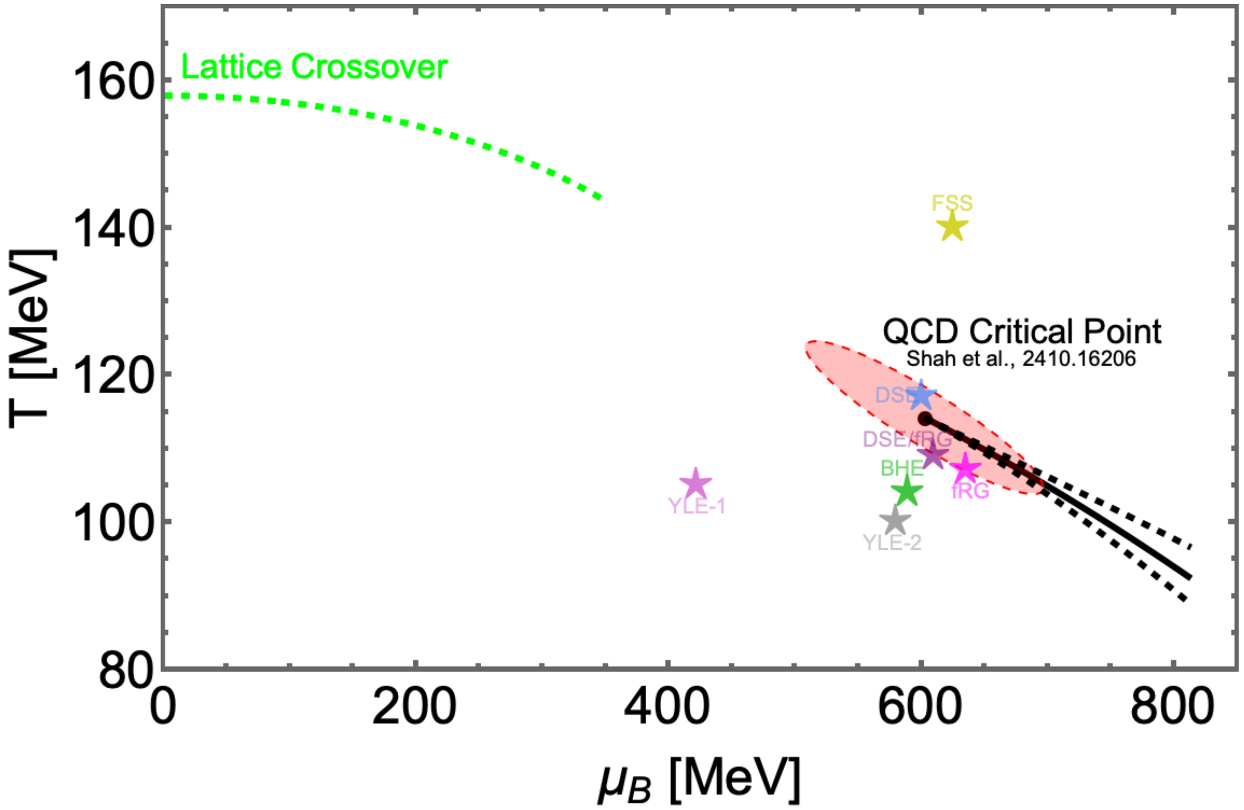
- Try going further $$T_s(\mu_B; T_0) = T_0 + \alpha_2(T_0)\frac{\mu_B^2}{2}$$



Shah et al., arXiv:2410.16026

- First-order phase transition emerges at $\mu_B > 600$ MeV

Critical point estimates



Critical point estimate at $O(\mu_B^2)$:

$T_c = 114 \pm 7$ MeV, $\mu_B = 602 \pm 62$ MeV

Estimates from recent literature:

YLE-1: D.A. Clarke et al. (Bielefeld-Parma), arXiv:2405.10196

YLE-2: G. Basar, PRC 110, 015203 (2024)

BHE: M. Hippert et al., arXiv:2309.00579

fRG: W-J. Fu et al., PRD 101, 054032 (2020)

DSE/fRG: Gao, Pawlowski., PLB 820, 136584 (2021)

DSE: P.J. Gunkel et al., PRD 104, 052022 (2021)

FSS: A. Sorensen et al., arXiv:2405.10278

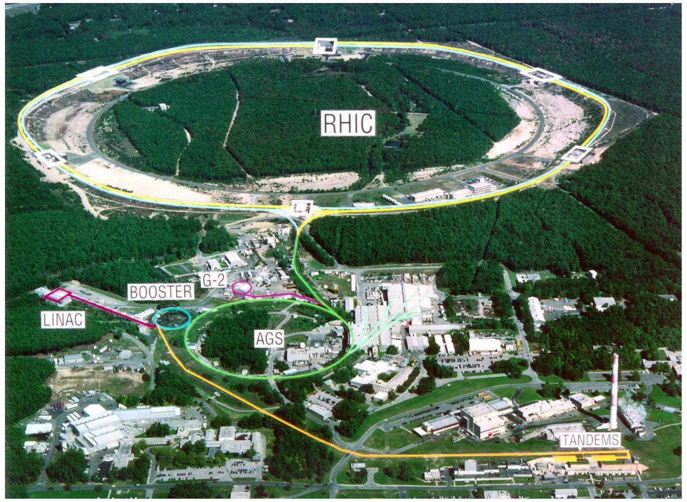
Optimist's view: Different estimates converge onto the same region because QCD CP is likely there

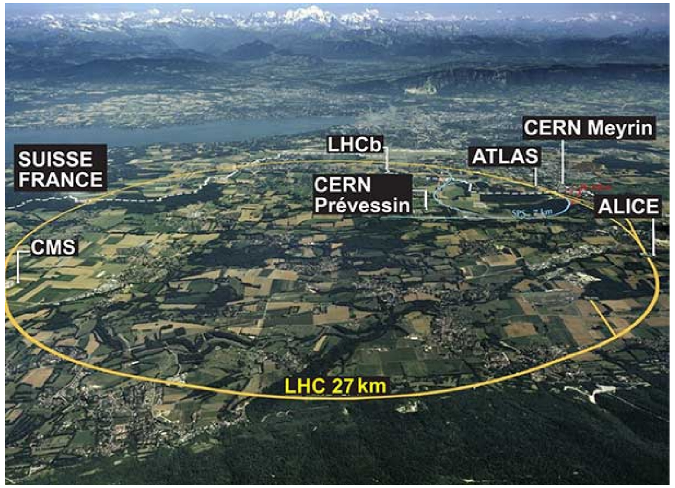
Pessimist's view: Different estimates converge onto the same region because it's the closest not yet ruled out by LQCD

Either way, can be tested in laboratory with **heavy-ion collisions**

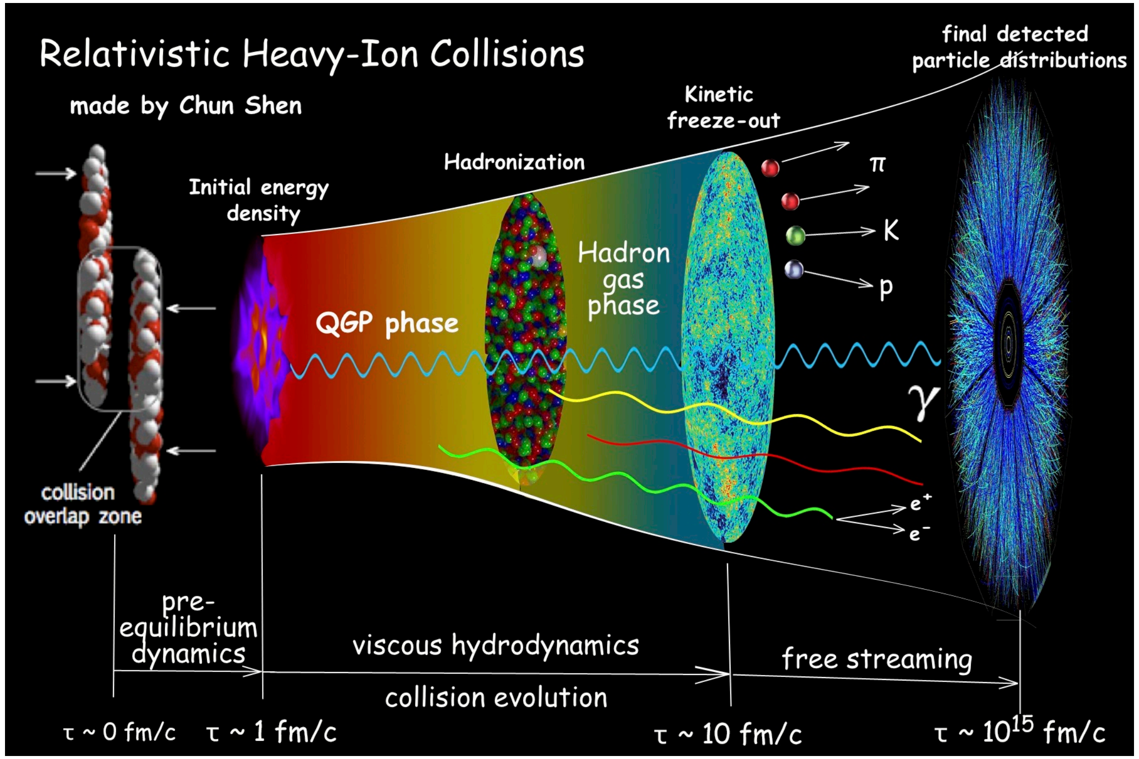
QCD Phase Diagram and Heavy-Ion Collisions

QCD laboratories (∼1980-...)





Relativistic heavy-ion collisions – "Little Bangs"



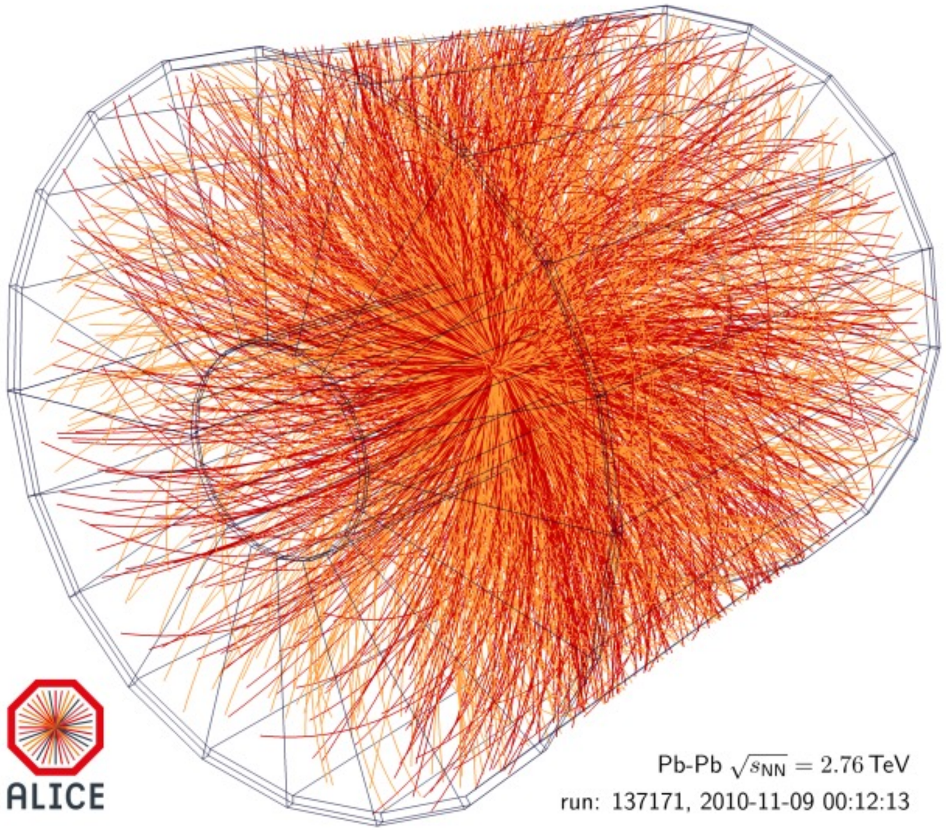
Control parameters

- Collision energy $\sqrt{s_{NN}} = 2.4 - 5020$ GeV
- Size of the collision region

Measurements

- Final hadron abundances and momentum distributions **event by event**

QCD phase diagram with heavy-ion collisions



Event display of a Pb-Pb collision in ALICE at the LHC

Thousands of particles created in relativistic heavy-ion collisions

⬇

Apply concepts of statistical mechanics

Particle production in heavy-ion collisions

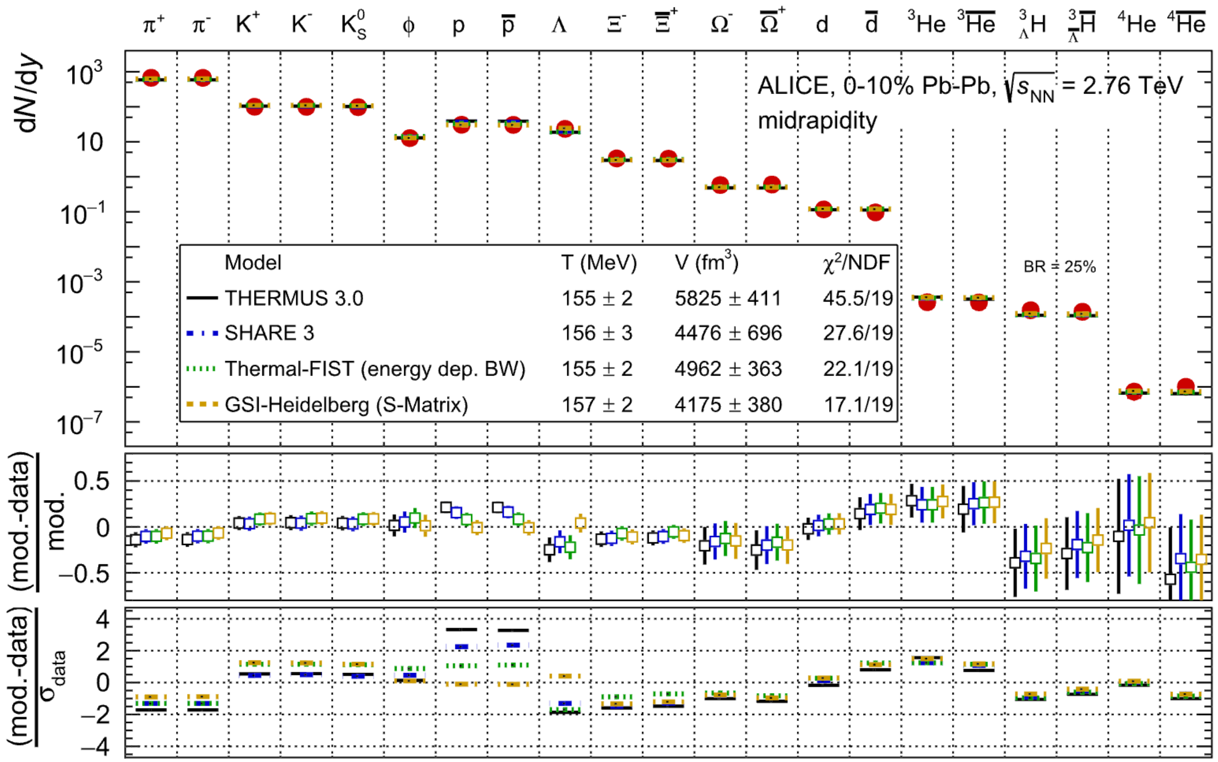
Ideal gas law (E. Clapeyron, 1834)

$$P_i V = N_i k_B T \quad (+\text{ feeddown})$$

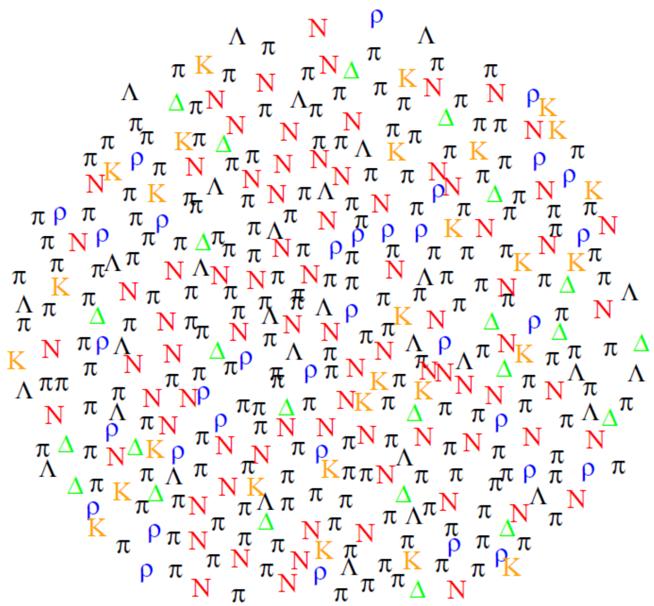
$$N_i = \frac{d_i V}{2\pi^2} \int dk\, k^2 \left[1 \pm \exp\left(\frac{\sqrt{k^2 + m_i^2} - \mu_i}{T} \right) \right]^{-1}$$

Bose-Einstein & Fermi-Dirac, 1924-1926

is the simplest model of particle production



ALICE Collaboration, EPJC 84, 813 (2024)



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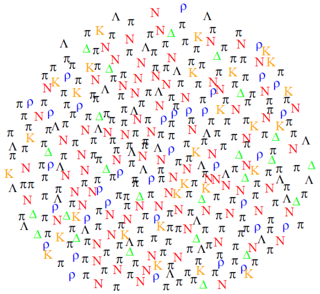
Hadron resonance gas (HRG) model

- **HRG model:** free gas of known hadrons and resonances



$$p(T,\mu_B) = \underbrace{T\,\phi_M(T)}_{\text{mesons}} + \underbrace{2\,T\,\phi_B(T)}_{\text{baryons}} \cosh(\mu_B/T)$$

$$\phi_{M(B)}(T) = \sum_{i \in M(B)} \frac{d_i}{2\pi^2} \int dk\, k^2 \exp\left(-\frac{\sqrt{m_i^2+k^2}}{T}\right)$$

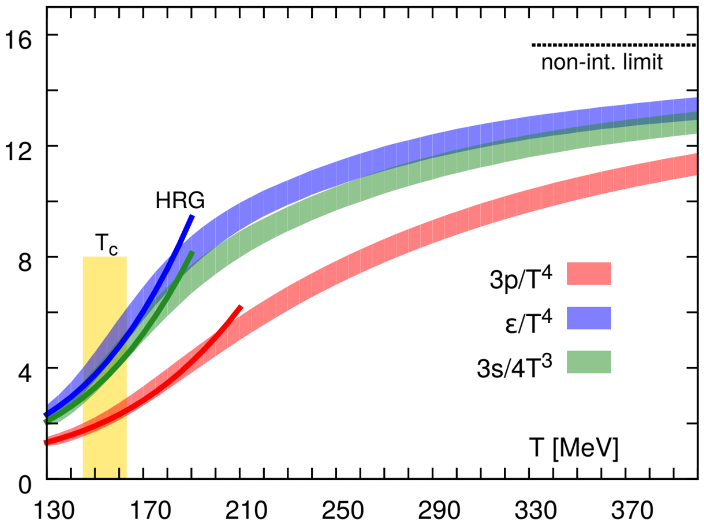


Figure from HotQCD coll., PRD ‘14

- Hadronic interactions dominated by resonance formation*
- Leading order in relativistic virial expansion
- Matches well with lattice QCD below T_{pc}
- Non-resonant interactions incorporated in extended descriptions (e.g. van der Waals HRG)

HRG model and heavy-ion collisions:

- Is the basis for the thermal model of particle production

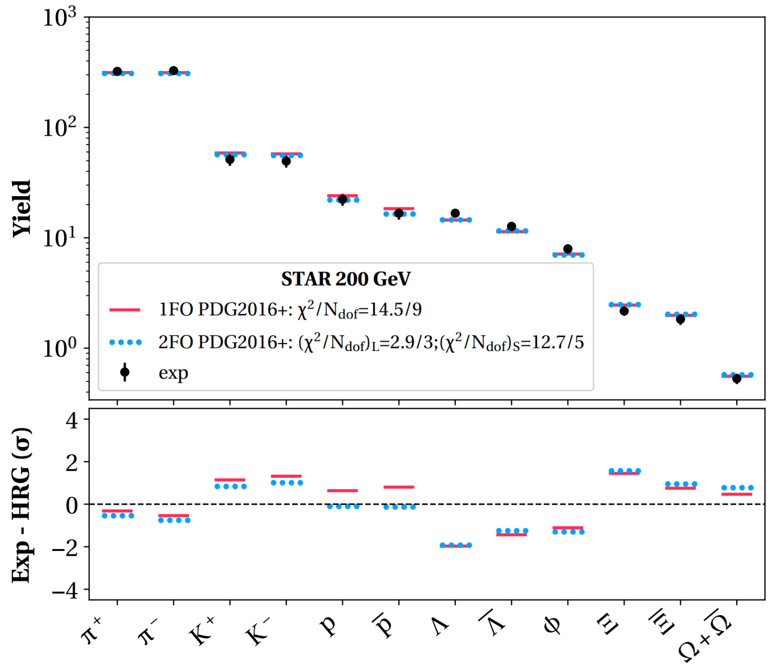
All bells and whistles implemented in open source codes, e.g. Thermal-FIST [VV, Stoecker, CPC 244, 295 (2019)]

Try it out! https://github.com/vlvovch/Thermal-FIST (latest release: version 1.5 on Mar 22, 2024)

* Dashen, Ma, Bernstein, “S-matrix formulation of statistical mechanics”, Phys. Rev. (1969); Prakash, Venugopalan, Nucl. Phys. A (1992)

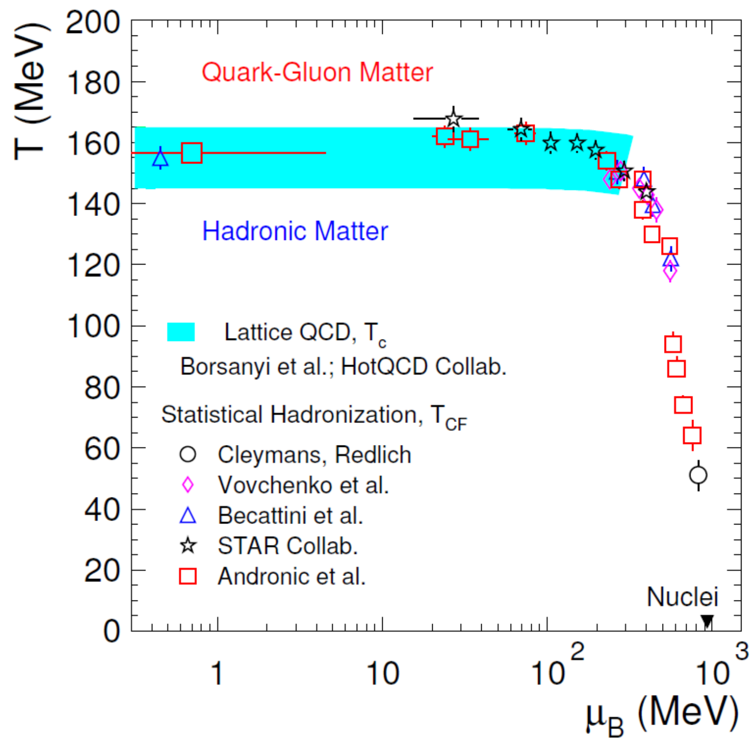
Mapping heavy-ion collisions onto QCD phase diagram

Fit hadron yields with the HRG model



P. Alba et al. (UH group), Phys. Rev. C 101, 054905 (2020)

$T(\sqrt{s_{NN}})$
$\mu_B(\sqrt{s_{NN}})$



A. Andronic et al., Nature 561, 321 (2018)

$\sqrt{s_{NN}} \searrow$ → $\mu_B \nearrow$

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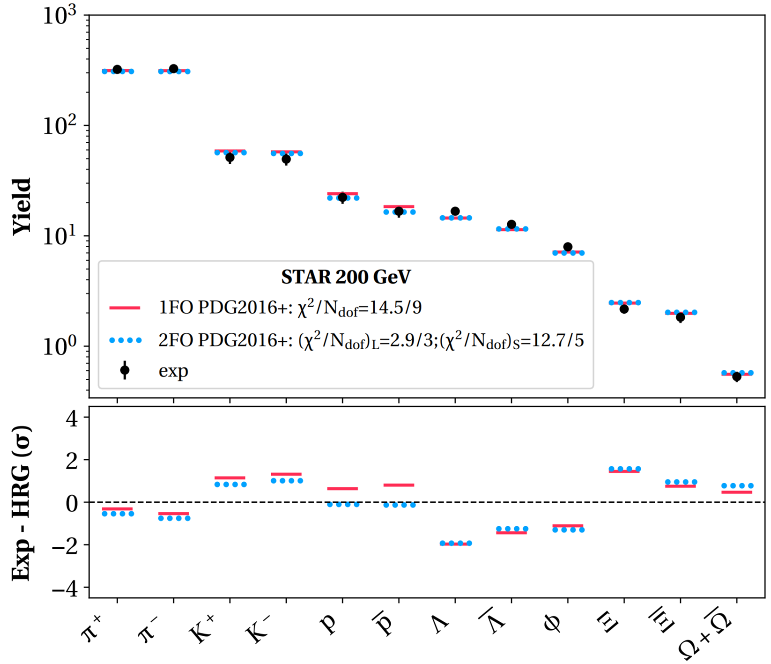

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$T(\sqrt{s_{NN}})$
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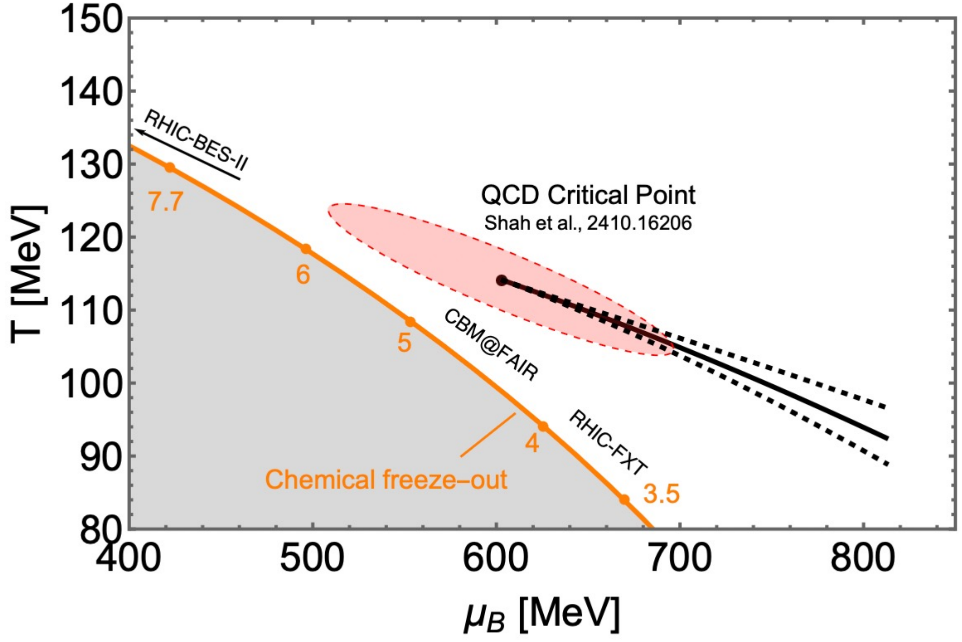


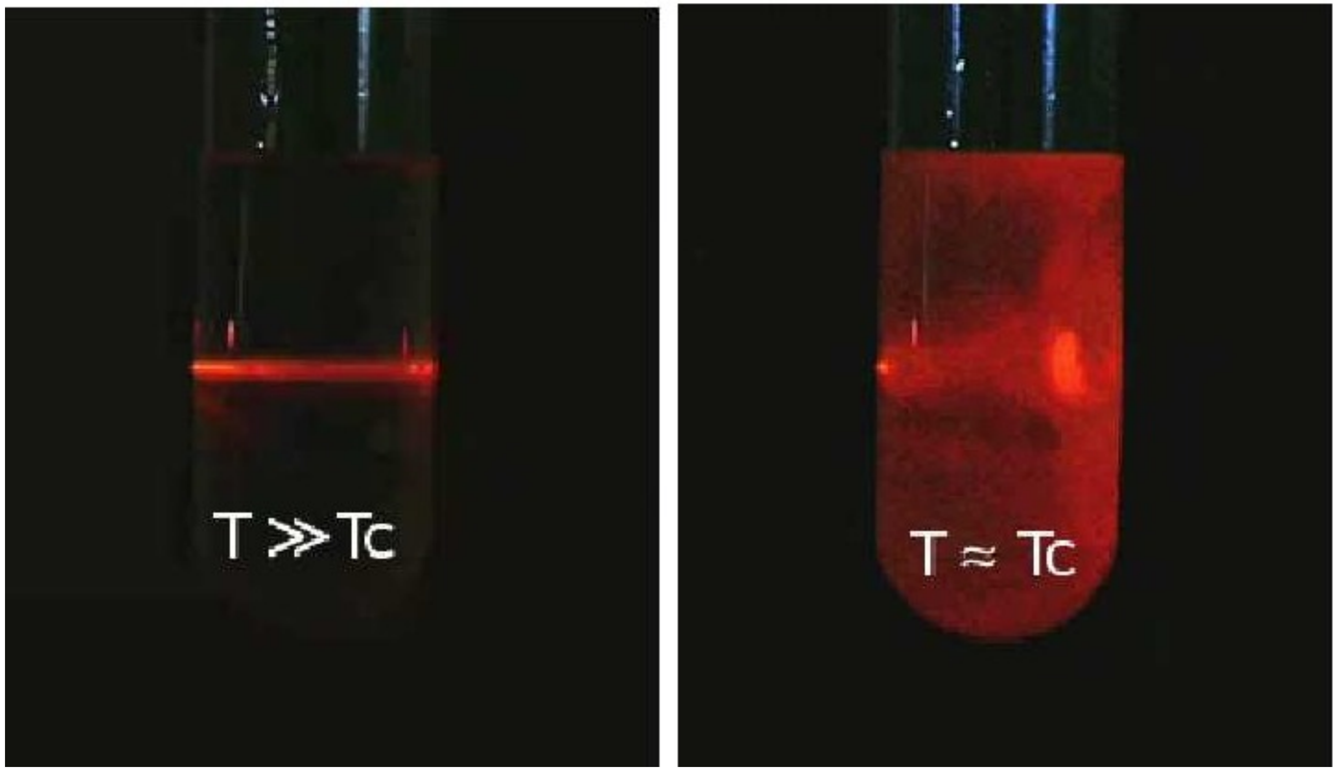
$\sqrt{s_{NN}} \searrow$ → $\mu_B \nearrow$

Critical point, cumulants, and heavy-ion collisions

Critical point and fluctuations

Density fluctuations at macroscopic length scales

Critical opalescence



Unfortunately, we cannot do this in heavy-ion collisions

Event-by-event fluctuations and statistical mechanics

Consider a fluctuating number N

Cumulants: $G_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$

variance $\kappa_2 = \langle(\Delta N)^2\rangle = \sigma^2$  width

skewness $\kappa_3 = \langle(\Delta N)^3\rangle$  asymmetry

kurtosis $\kappa_4 = \langle(\Delta N)^4\rangle - 3\langle(\Delta N^2)\rangle^2$  peak shape

Experiment:

$$P(N) \sim \frac{N_{events}(N)}{N_{events}^{total}}$$

Statistical mechanics:

Grand partition function

$$ln\, Z^{\mathrm{gce}}(T, V, \mu) = ln\left[\sum_N e^{\mu N} Z^{\mathrm{ce}}(T, V, N)\right],$$

$$\kappa_n \propto \frac{\partial^n (\ln Z^{\mathrm{gce}})}{\partial(\mu_N)^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

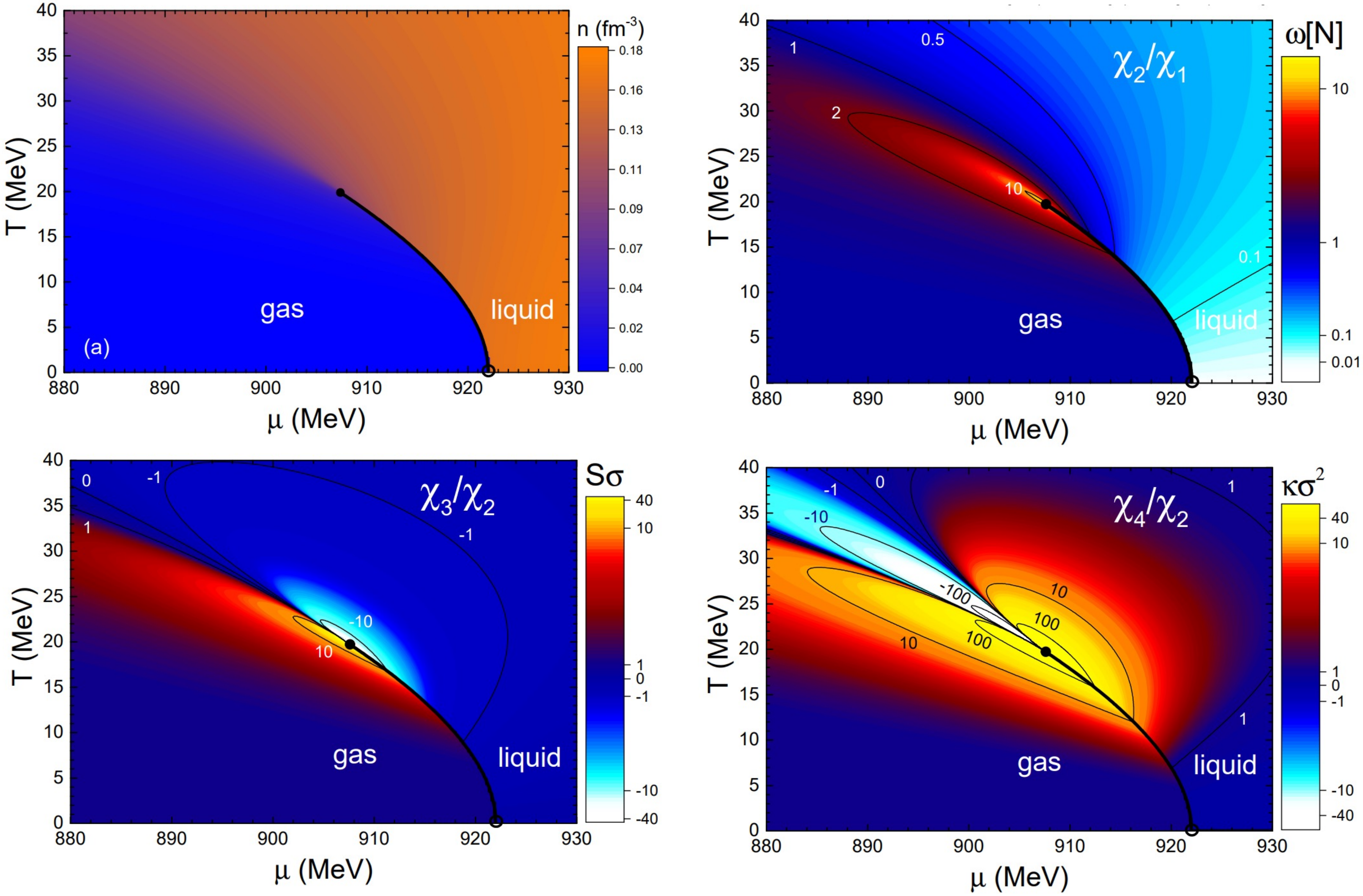
Example: (Nuclear) Liquid-gas transition

- **(QCD) critical point:** large correlation length and fluctuations

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

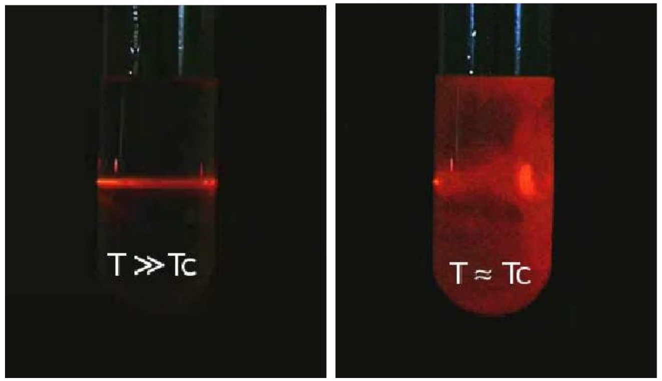
$$\xi \to \infty$$

M. Stephanov, PRL '09, '11



VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Critical opalescence



$$\langle N^2 \rangle - \langle N \rangle^2 \sim \langle N \rangle \sim 10^{23}$$

in equilibrium

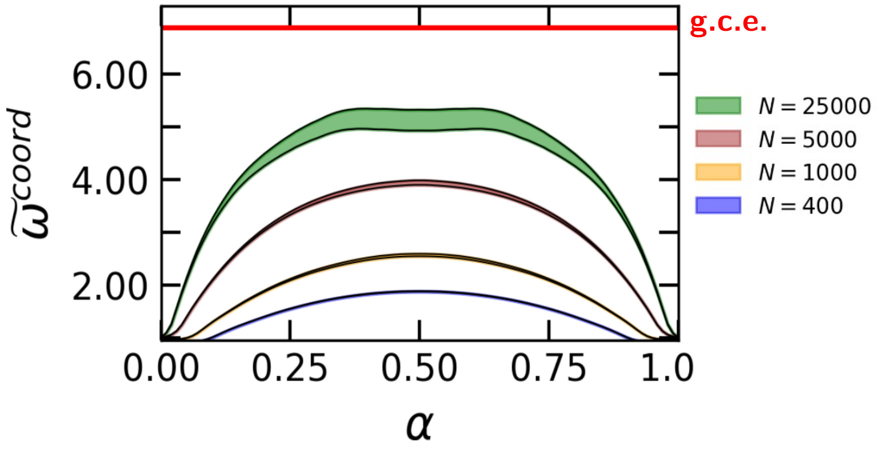
Example: Critical fluctuations in microscopic simulation

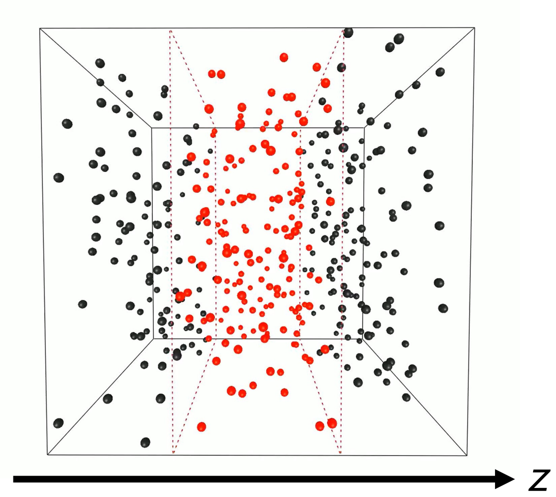
V. Kuznietsov et al., Phys. Rev. C 105, 044903 (2022)

Classical molecular dynamics simulations of the **Lennard-Jones fluid** near Z(2) critical point ($T \approx 1.06T_c$, $n \approx n_c$) of the liquid-gas transition

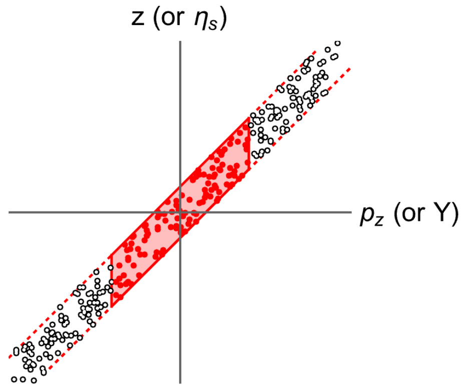
Scaled variance in coordinate space acceptance $|z| < z^{max}$

$$\tilde{\omega}^{\text{coord}} = \frac{1}{1-\alpha} \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$





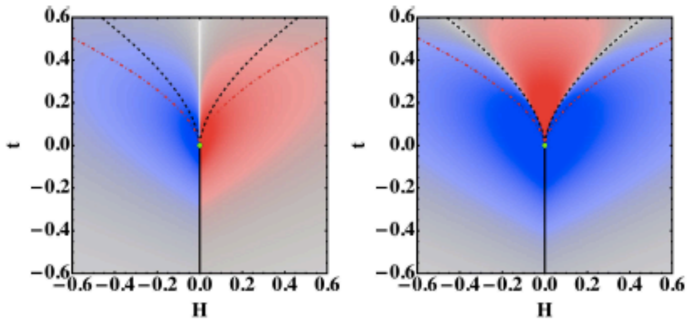
Heavy-ion collisions:
flow correlates p_z and z cuts

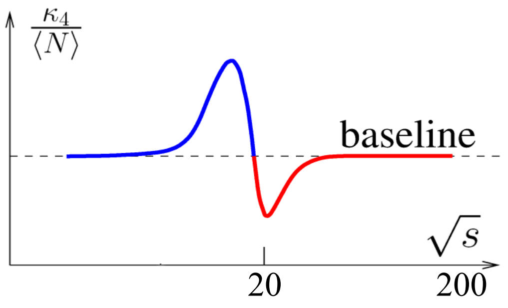


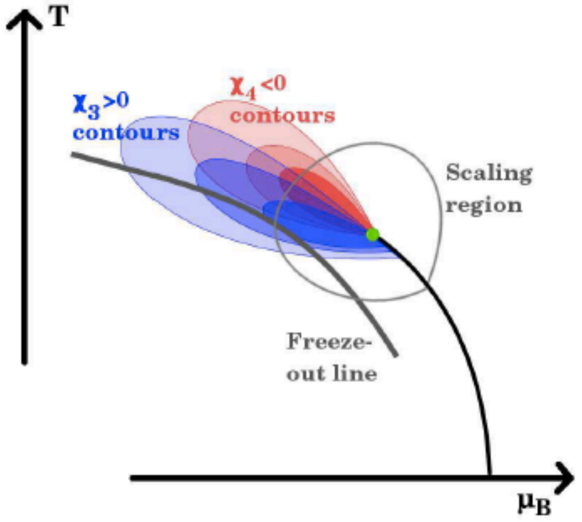
- Large fluctuations survive despite strong finite-size effects
- Need coordinate space cuts (collective flow helps)
- Here no finite-time effects

Equilibrium Expectations and Beam Energy Scan

Expectation from Calculations







Characteristic "Oscillating pattern" is expected for the QCD critical point but *the exact shape depends on the location of freeze-out with respect to the location of CP*

- M. Stephanov, ***PRL107***, 052301(2011)
- V. Skokov, Quark Matter 2012
- J.W. Chen, J. Deng, H. Kohyyama, arXiv: 1603.05198, Phys. Rev. **D93** (2016) 034037

N. Xu, CPOD 2016

Equilibrium Expectations and Beam Energy Scan

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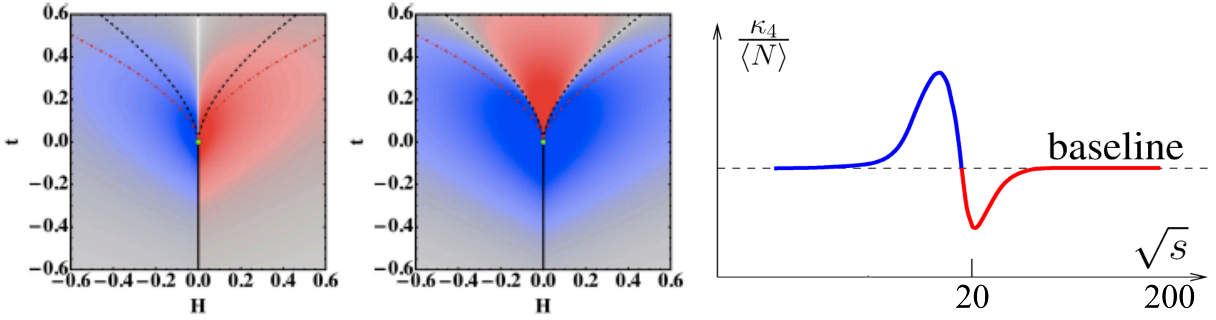

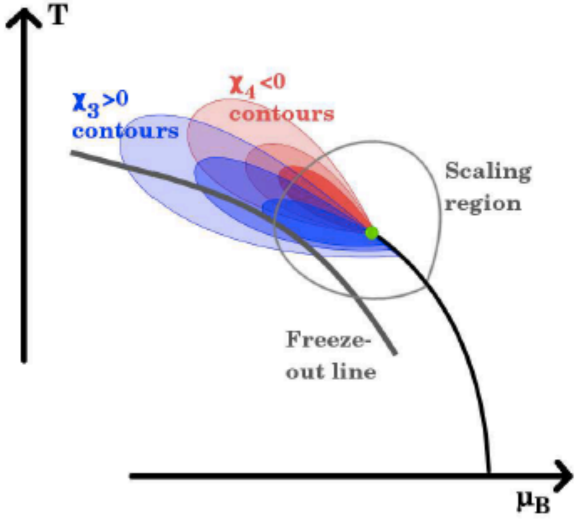

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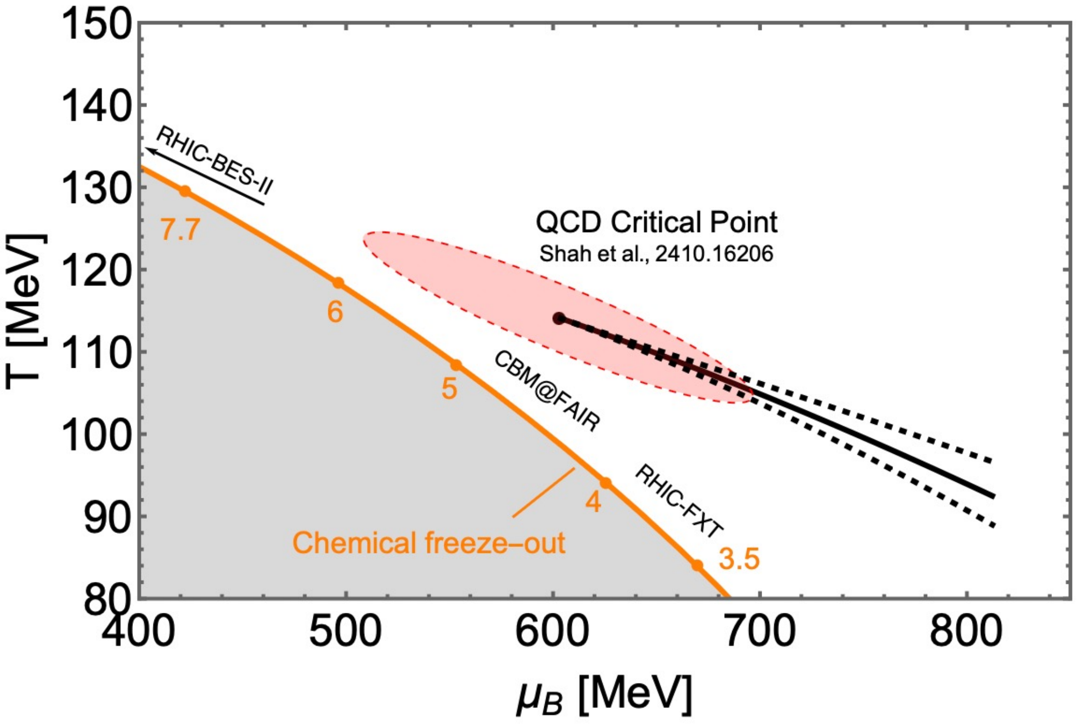
Recalling recent CP estimates and the freeze-out curve



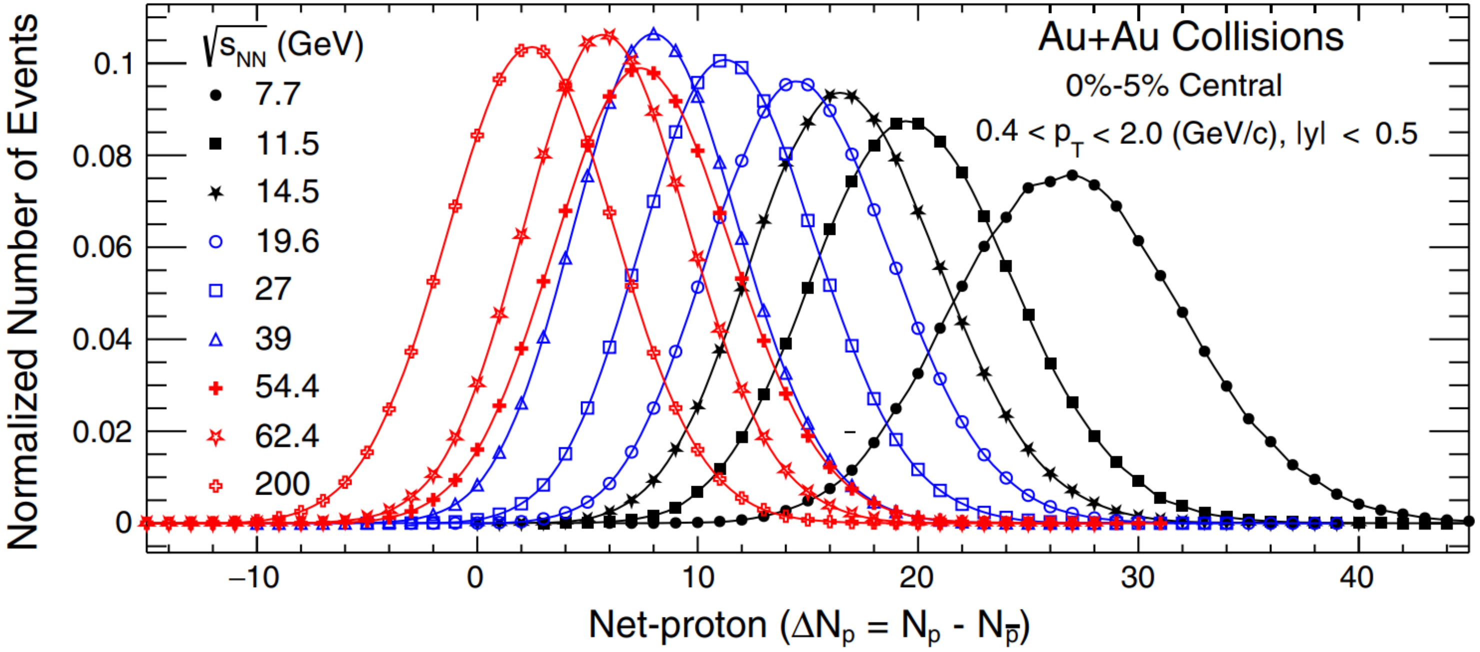
N. Xu, CPOD 2016

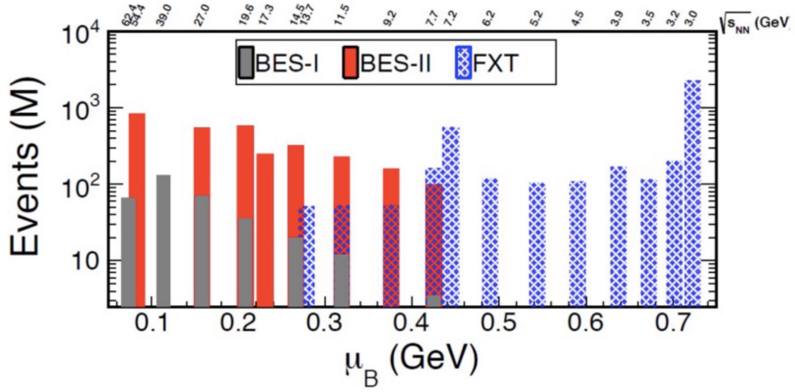
Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons

$$P(\Delta N_p) \sim \frac{N_{events}(\Delta N_p)}{N^{total}_{events}}$$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)





Statistics-hungry observables

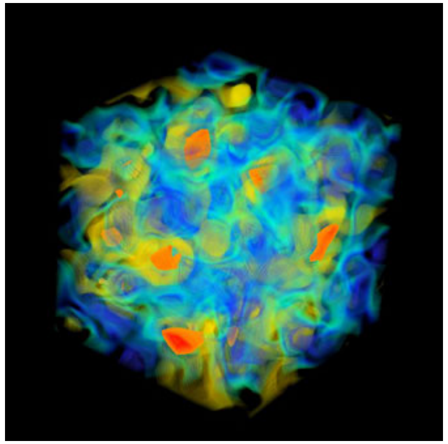
Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

$$\frac{\kappa_2}{\langle N \rangle}, \quad \frac{\kappa_3}{\kappa_2}, \quad \frac{\kappa_4}{\kappa_2}$$

Look for subtle critical point signals

Theory vs experiment: Challenges for fluctuations

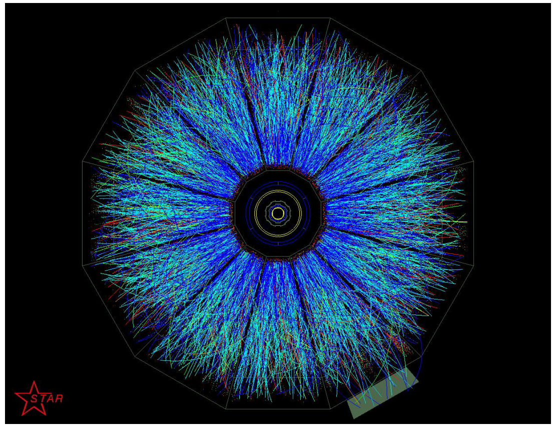
Theory



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



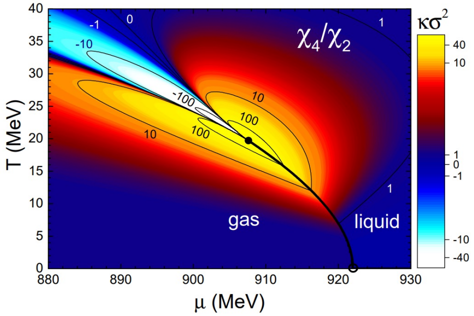
STAR event display

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

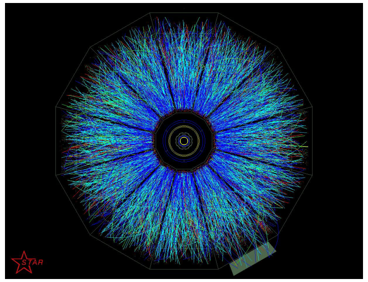
Comparing theory and experiment should be done very carefully

Theory vs experiment

guidance from theory



experiment (the real thing)



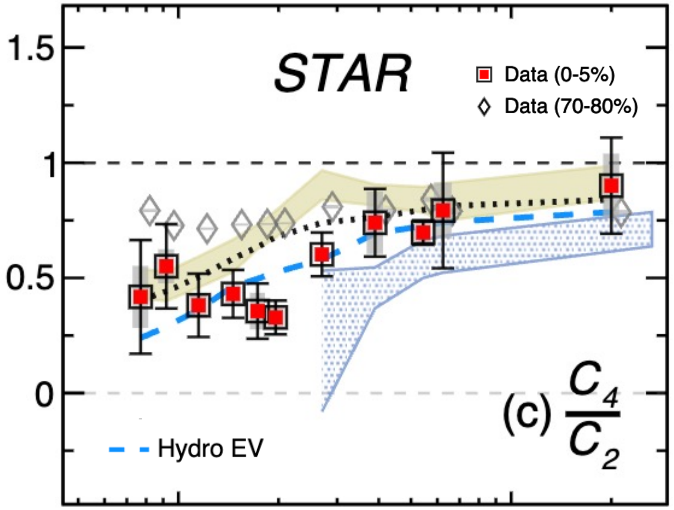
hydro simulation

critical slowing down

canonical effects

kinematic cuts

hadronic phase



track reconstruction

event selection

efficiency correction

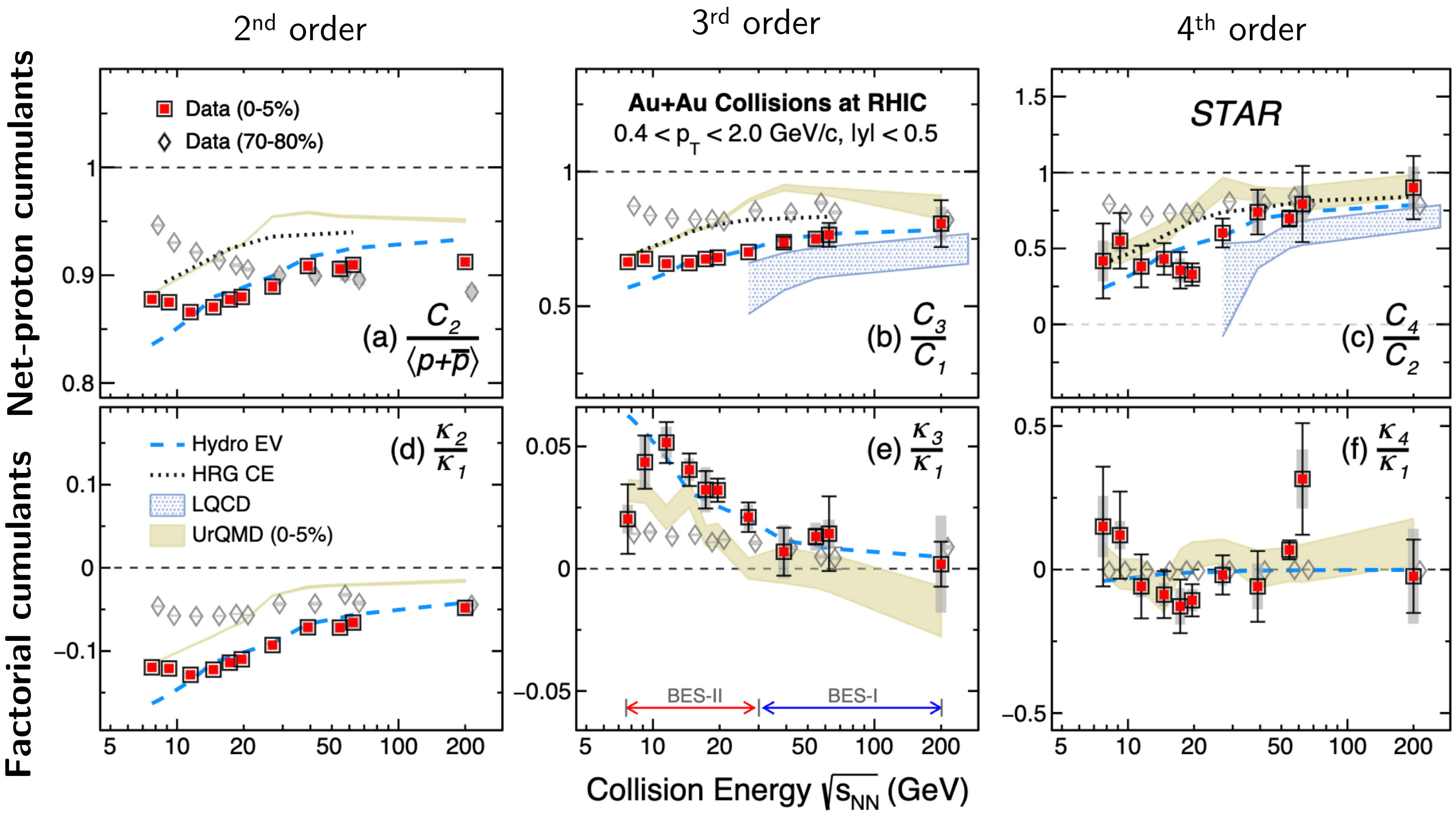
CBWC

purity

Proton cumulants and RHIC-BES-II

RHIC-BES-II data

STAR Collaboration, arXiv:2504.00817



Factorial cumulants: Linear combinations of cumulants that isolate multi-particle correlations

Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)

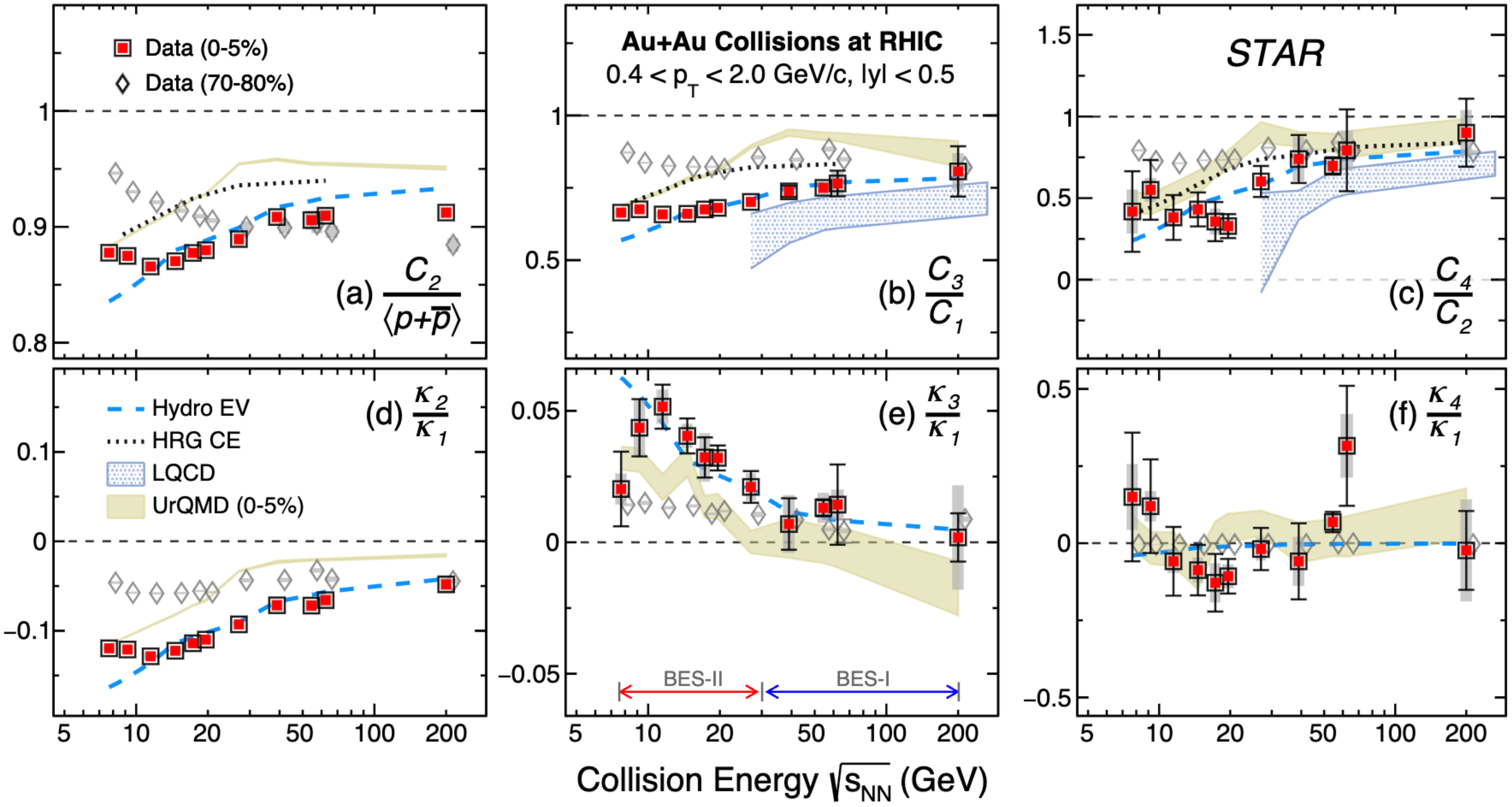
RHIC-BES-II data

STAR Collaboration, arXiv:2504.00817

2nd order | 3rd order | 4th order

Net-proton cumulants

Factorial cumulants



Ordinary cumulants

Factorial cumulants

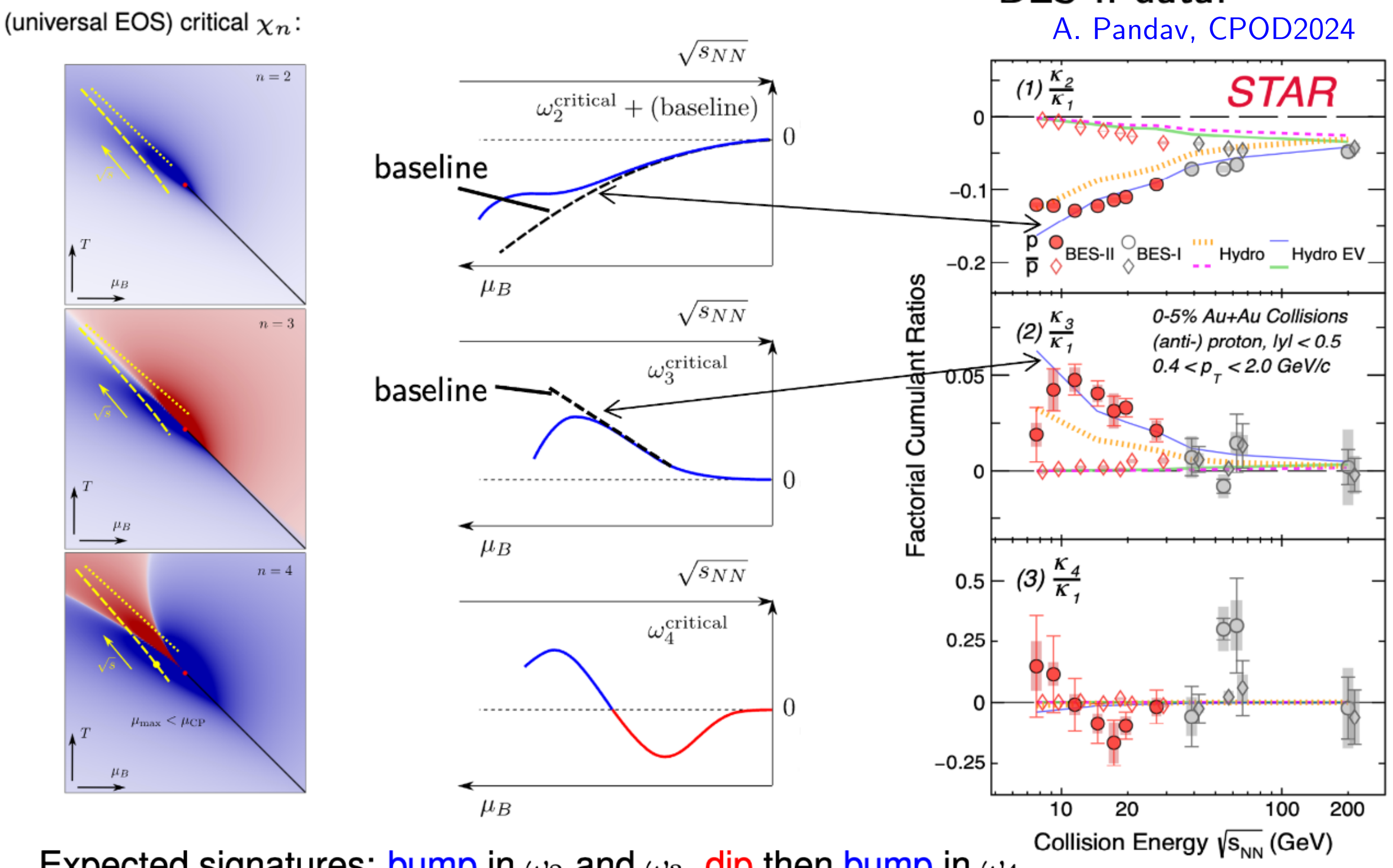
Factorial cumulants: Linear combinations of cumulants that isolate multi-particle correlations

Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)

RHIC-BES-II data

Based on M. Stephanov, arXiv:2410.02861

$\omega_n = \hat{C}_n / \hat{C}_1$



Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

Non-critical baseline (hydro EV):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

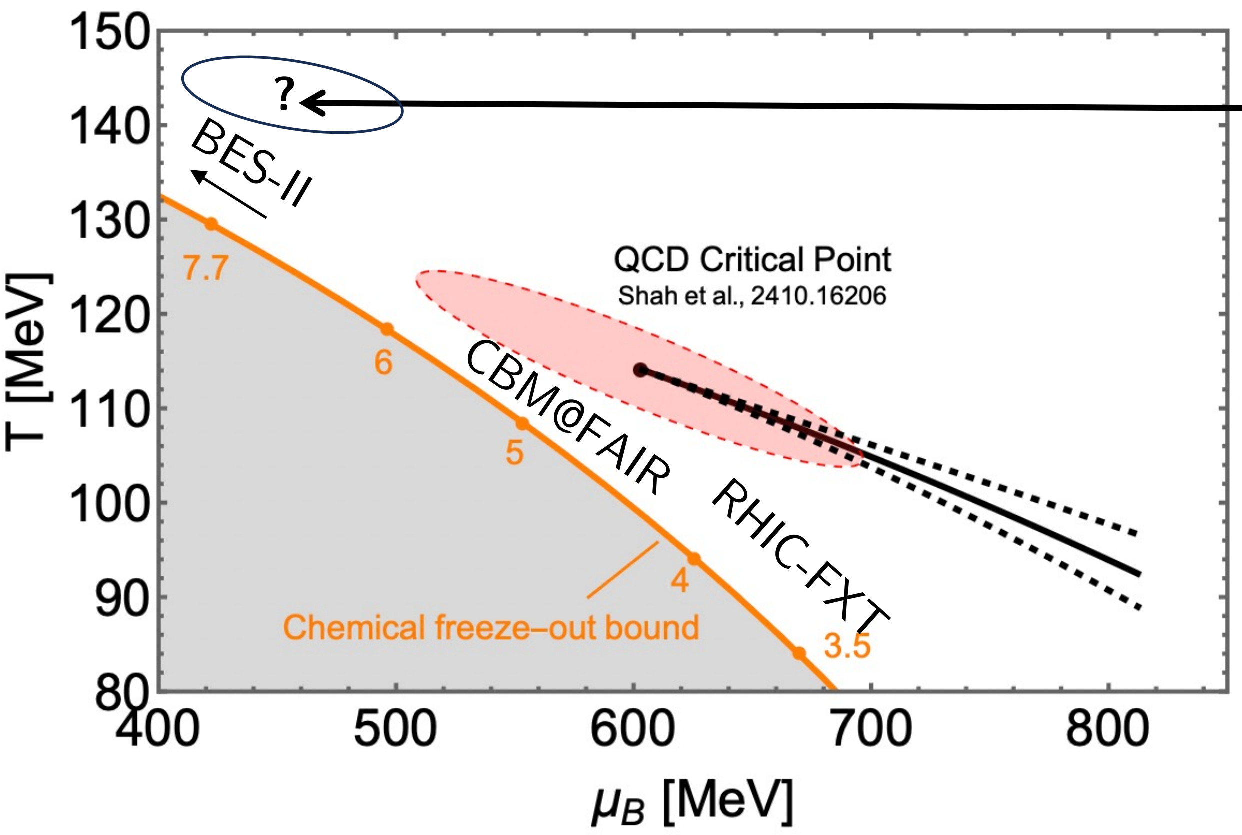
- describes right side of the peak in $\hat{C}_3$
- **signal relative to baseline:**
  - *positive* $\hat{C}_2 - \hat{C}_2^{baseline} > 0$
  - *negative* $\hat{C}_3 - \hat{C}_3^{baseline} < 0$

Controlling the non-critical baseline is essential

Notation: Here we use κ_n for cumulants and $\hat{C}_n$ for factorial cumulants, STAR uses the opposite

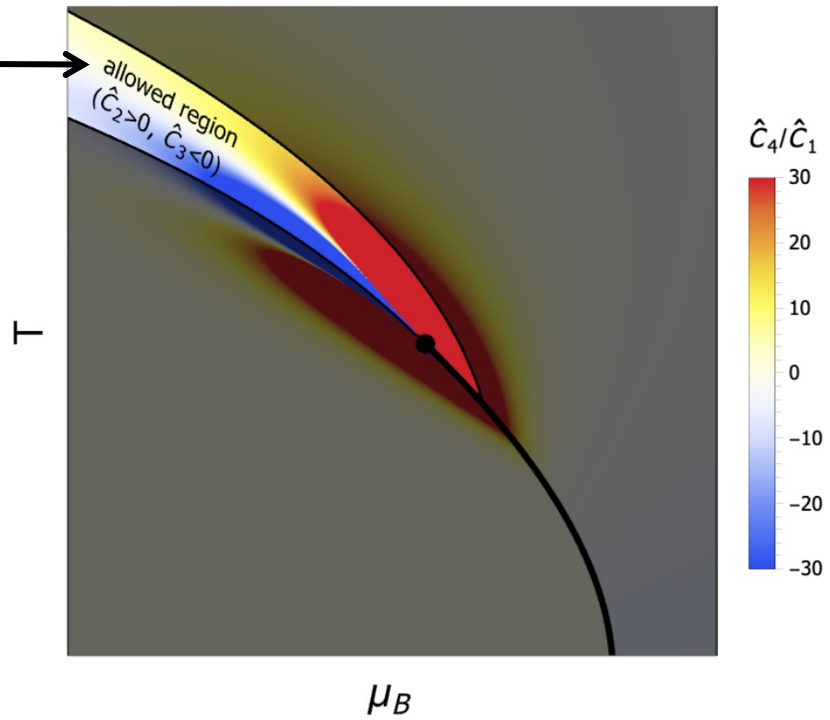
RHIC-BES-II data and CP

Equilibrium expectation



Exclusion plots

Exclude $\hat{C}_2<0$ & $\hat{C}_3>0$ regions on the phase diagram near CP

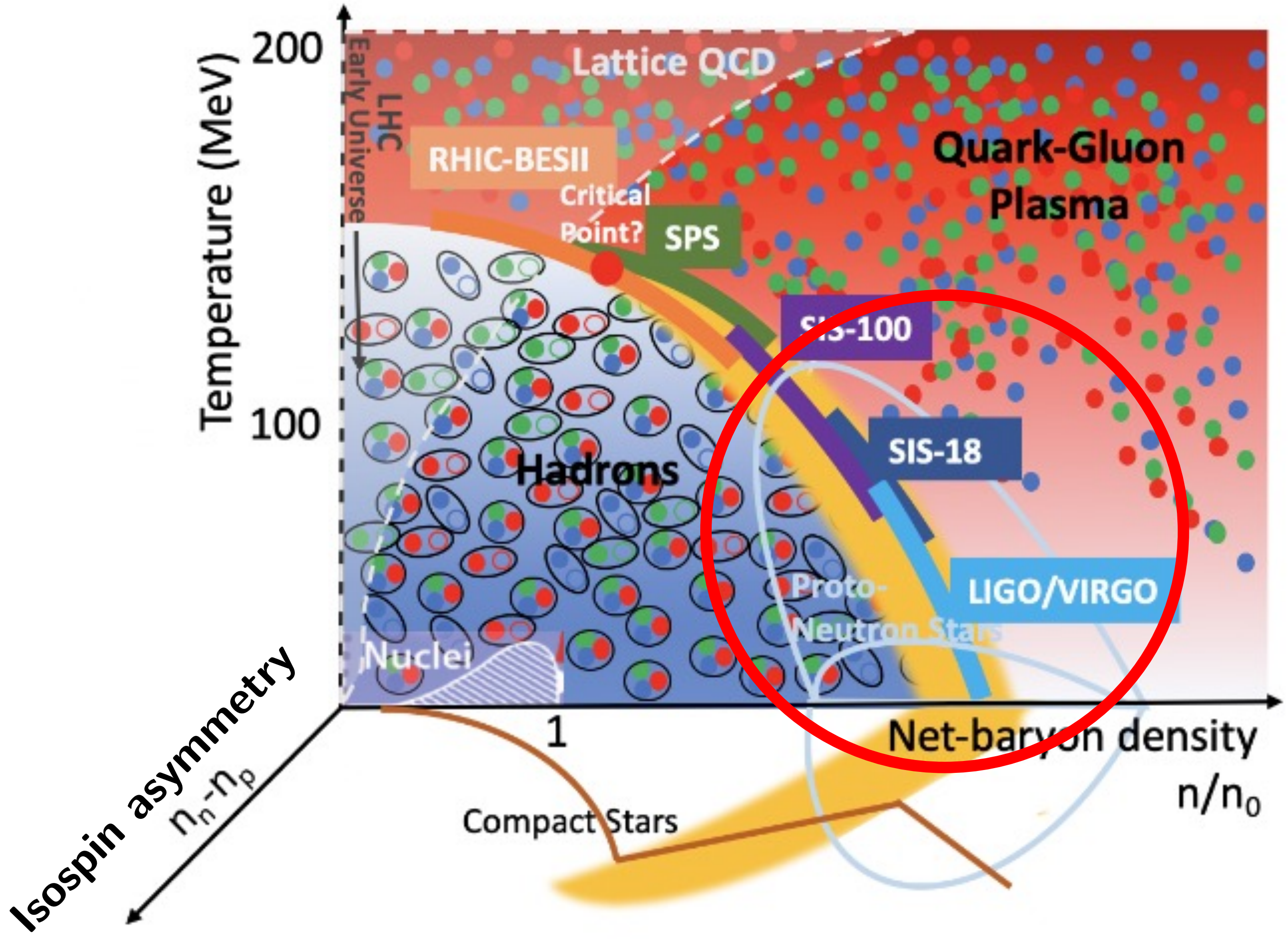


Adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)

Freeze-out in the QGP? Memory effects? Or not related to CP at all? We're here at QM to find out!

Present/Future: RHIC fixed target, NA61/SHINE, FAIR-CBM may be in the right spot!

Cold and Dense Matter EoS



Adapted from D. Blaschke

Dense Matter Equation of State

$$P = P(T, \mu_B, \mu_I, \mu_Y) \quad \leftrightarrow \quad P = P(\mathcal{E}, n_B, n_I, n_Y)$$

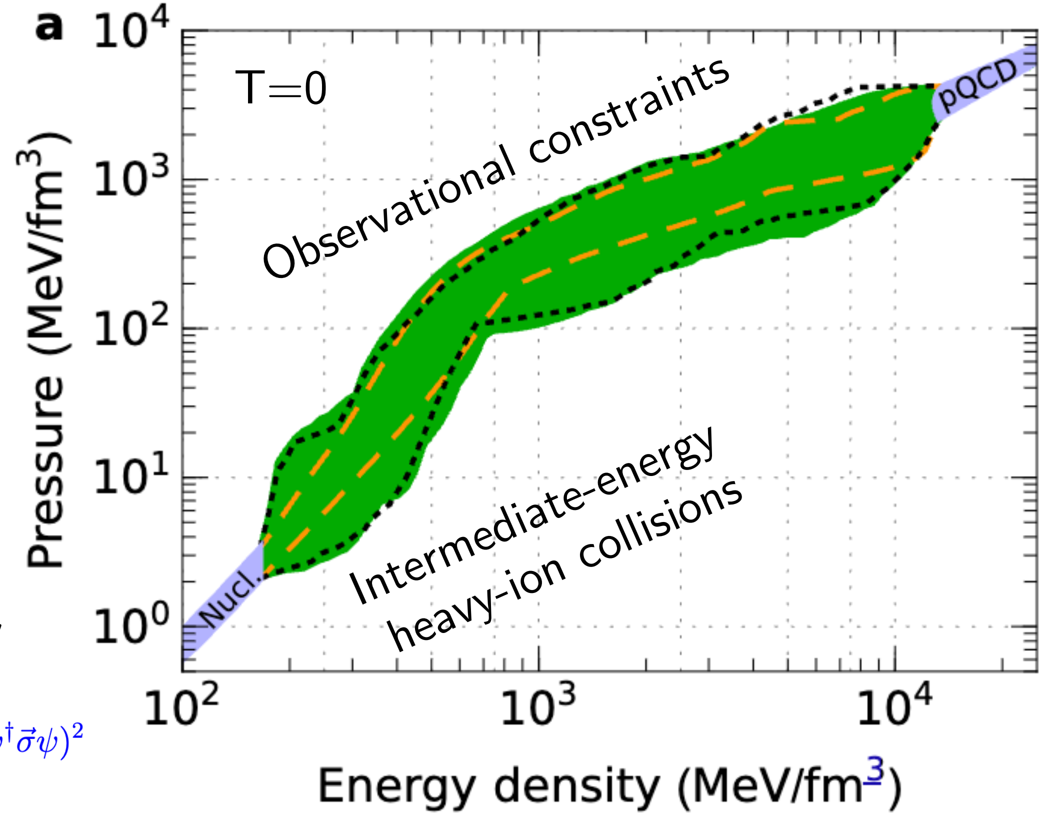
$$\frac{\mu_B}{T} \gg 1$$

Weak coupling QCD (pQCD)

$$\frac{p}{p_0} = 1 + a_1\alpha_s(\bar{\Lambda}) + a_2\alpha_s^2(\bar{\Lambda}) + a_3\alpha_s^3(\bar{\Lambda}) + \cdots$$

free quarks

$\rho_B \geq 40\rho_0$



Adapted from Annala et al., Nature Phys. 16, 907 (2020)

Chiral Effective Field Theory

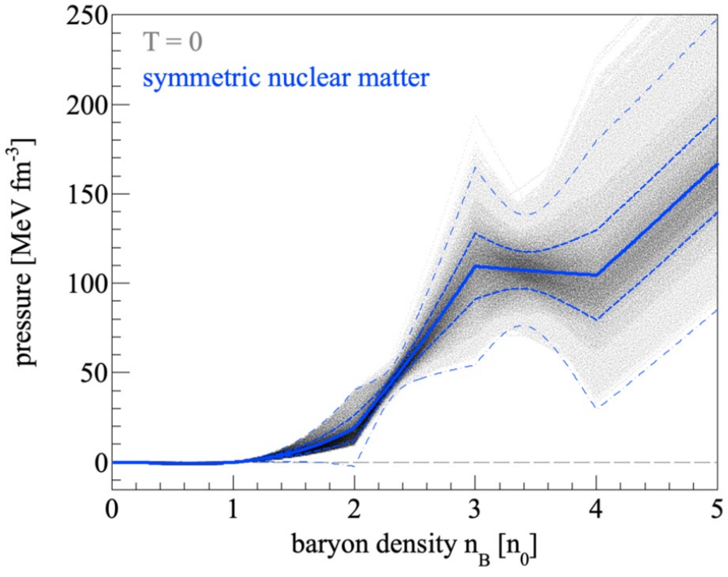
$$\mathcal{L}_{\rm eff} = \psi^\dagger\left(i\partial_0 + \frac{\nabla^2}{2M}\right)\psi - \frac{C_S}{2}(\psi^\dagger\psi)^2 - \frac{C_T}{2}(\psi^\dagger\vec{\sigma}\psi)^2 + \ldots$$

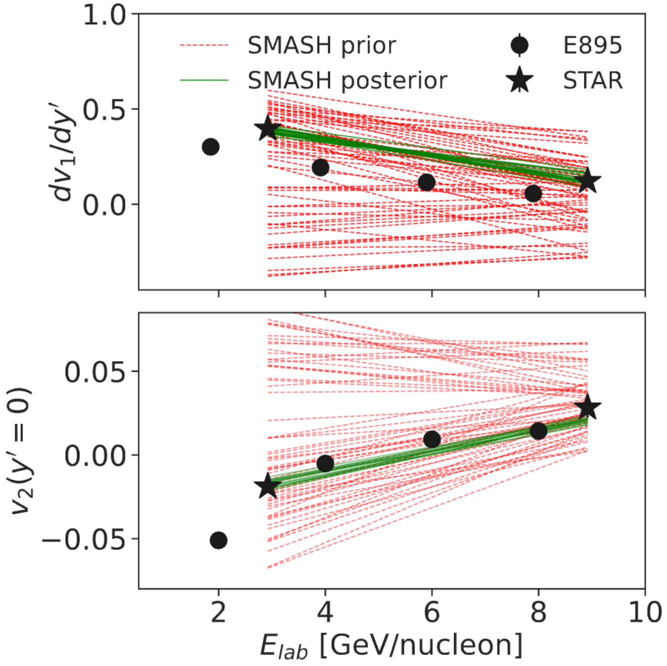
up to $\rho_B \sim 1 - 2\rho_0$

Isospin Symmetric EoS from Heavy-Ion Collisions

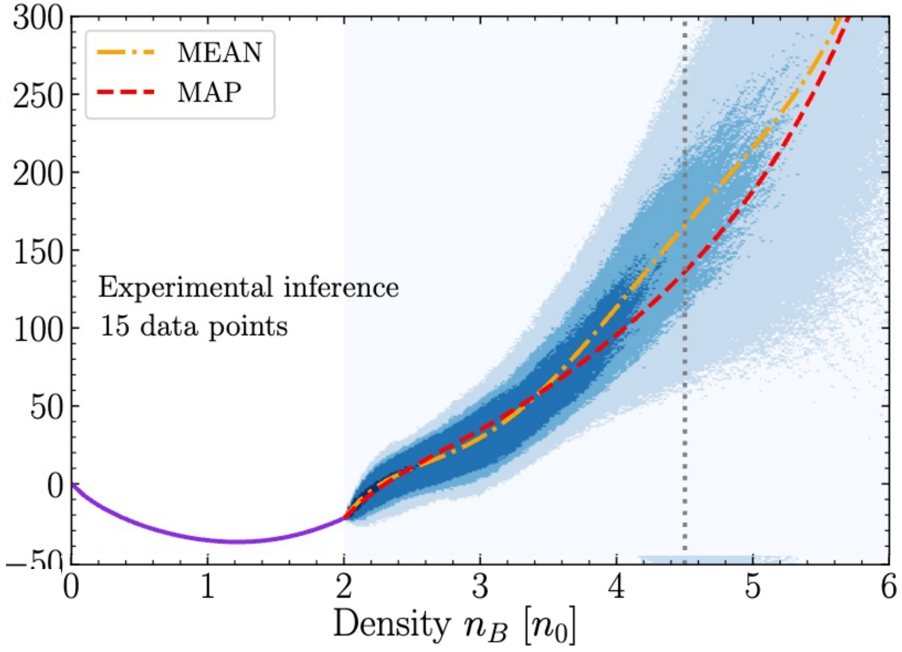
Transport simulations (UrQMD/SMASH) with adjustable potentials to describe any EoS

Bayesian inference from flow and mean pT measurements at $\sqrt{s_{NN}} = 2.7 - 4.5$ GeV





Oliinychenko et al., PRC 108, 034908 (2023)



M.O. Kuttan et al., PRL 131, 202303 (2023)

Overview on extracting EoS of Dense Matter from HICs: A. Sorensen et al, Prog.Part.Nucl.Phys. 134 (2024) 104080

Isospin Asymmetric EoS from Neutron Stars

Hydrostatic equilibrium of a neutron star (General Relativity)

$$\frac{dP}{dr} = -\frac{Gm(r)[\epsilon(r)+P(r)]}{r^2}\left(1+\frac{4\pi r^3 P(r)}{m(r)}\right)\left(1-\frac{2Gm(r)}{r}\right)^{-1}$$

Tolman-Oppenheimer-Volkov (TOV)

Mass: $m(r) = 4\pi \int_0^r d\bar{r}\, \bar{r}^2 \epsilon(\bar{r})$

$P(R) = 0$ → $M = m(R)$

Mass-radius relation

- Shapiro delay (maximum mass)
- NICER (M-R)
- LIGO (NS Mergers)

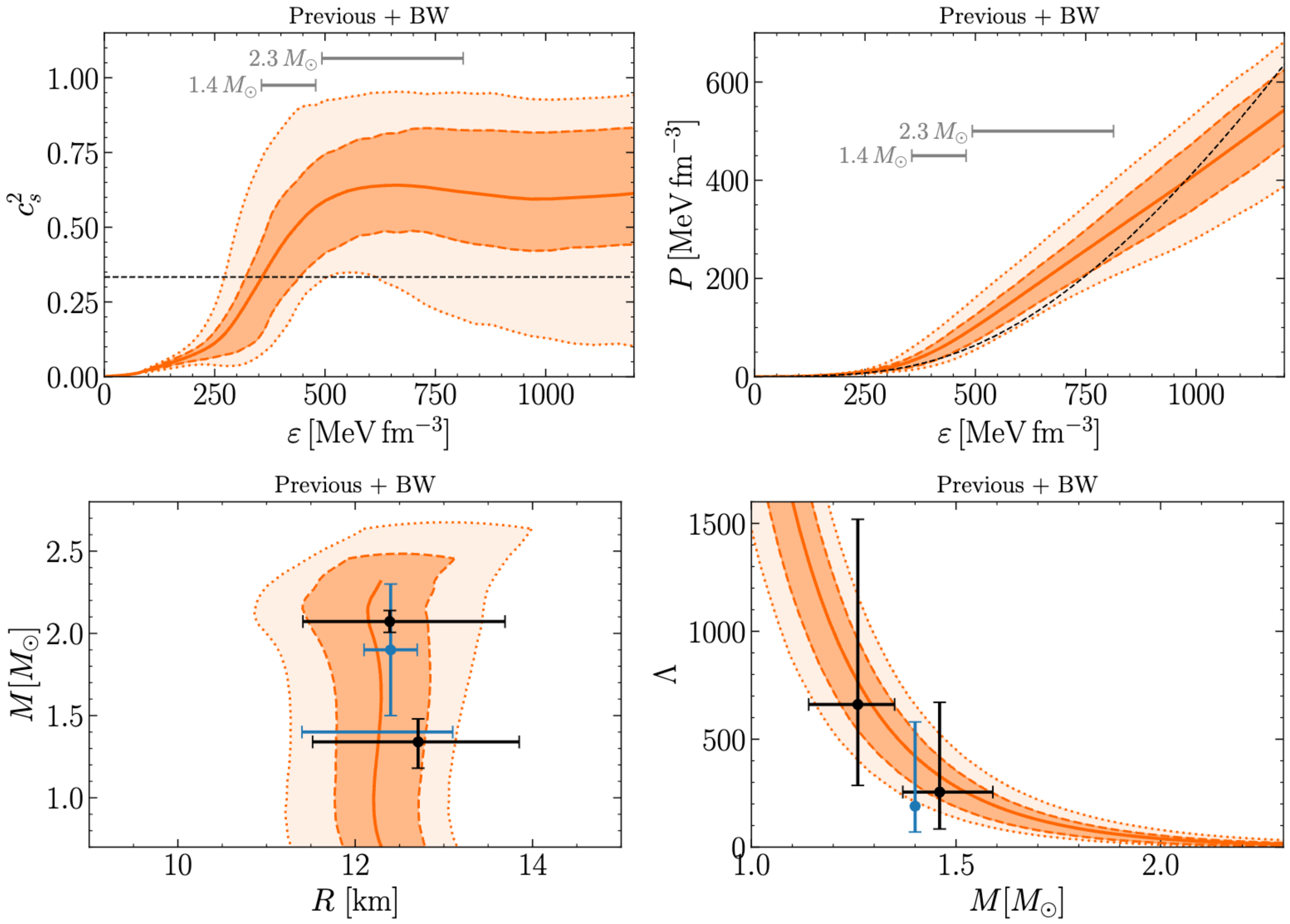
$\boldsymbol{P(\epsilon)}$**:** QCD + leptons (at T = 0)

charge neutrality: $\rho_Q = 0$ → more neutrons than protons

$$\boldsymbol{P(\epsilon)} \Leftrightarrow \mathbf{M(R)}$$

Additionally: tidal deformabilities (LIGO), gravitational waves from NS mergers

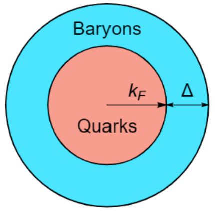
Bayesian Analysis



From Brandes, Weise, Kaiser, PRD 108, 094104 (2023)
and many more in the literature

Exotic States of Matter: Quarkyonic Phase

Quarkyonic matter: quarks fill low momenta (Fermi sea), baryons are on Fermi shell

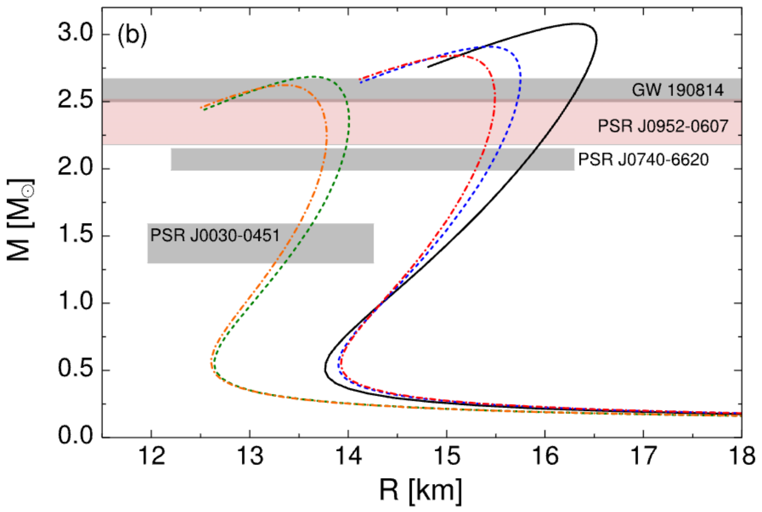


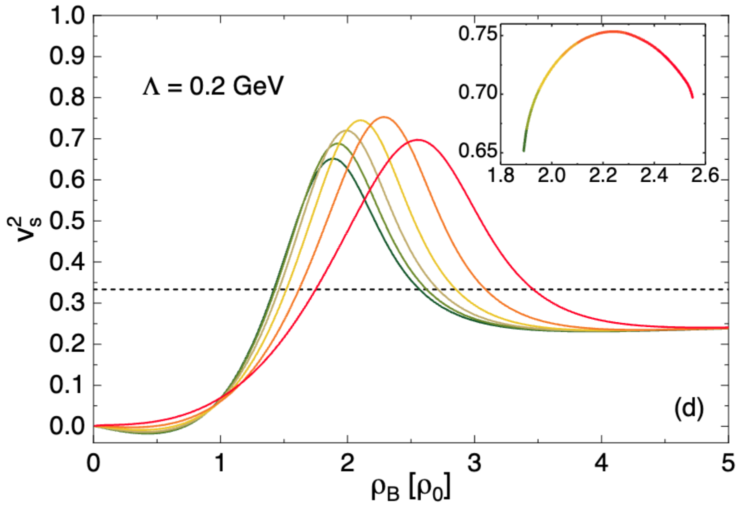
McLerran, Pisarski, NPA 796, 83 (2007)
McLerran, Reddy, PRL 122, 122701 (2019)

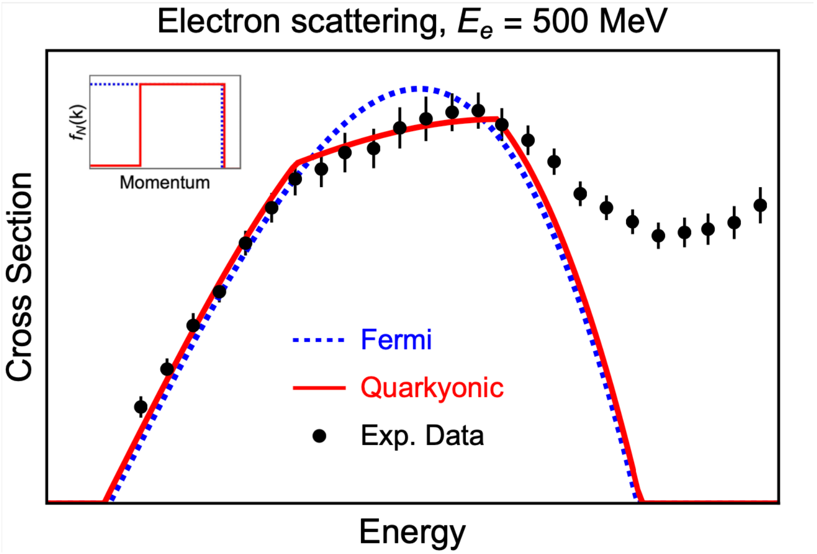
Can emerge as a consequence of Pauli exclusion principle for quarks

Fujimoto, Kojo, McLerran, PRL 132, 112701 (2024)

Gives stiff EoS needed to support heavy NSs and may emerge as early as at $\rho_B \sim \rho_0 = 0.16$ fm^{-3}







Moss, Poberezhniuk, VV, PRC 111, 035204 (2025)

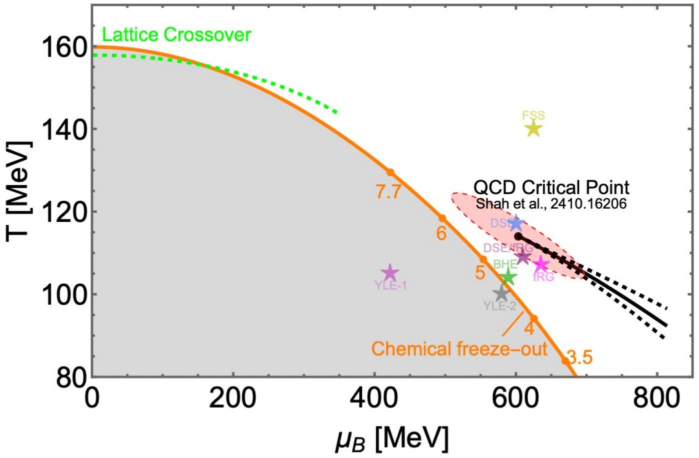
Koch et al., PRC 110, 025201 (2024)

Some open source tools

- Lattice-based equations of state
  - 4D Taylor: https://github.com/cratti/EoS_BQS
  - 4D Taylor expansion + HRG: https://sites.google.com/view/qcdneos4d/
- T'ExS + 3D-Ising: Lattice-based EoS with a tunable critical point
  - https://zenodo.org/records/14637802
- HRG (thermodynamics, hadron yields, thermal fits)
  - THERMUS-2.0: https://github.com/thermus-project/THERMUS
  - Thermal-FIST-1.5: https://github.com/vlvovch/Thermal-FIST
- MUSES – Modular Unified Solver of the Equation of State
  - https://musesframework.io/
  - Compute and merge different equations of state for
    - heavy ions (HRG, 4D Taylor, Ising-2DTExS, Holographic)
    - neutron stars (ChEFT, CMF, Crust DFT, Leptons)
  - Observables for neutron stars (mass-radius,…) and heavy ions (thermal fits,…)
  - First version of calculation engine released few weeks ago!
    - https://ce.musesframework.io/

Summary and Outlook

- QCD equation of state
  - Well-controlled at small/moderate baryon densities with lattice QCD where the transition is a chiral crossover
  - Recent developments: CP at $T \sim$ 90-120 MeV and $\mu_B \sim 500 - 650$ MeV?



- Proton cumulants are uniquely sensitive to the the CP but challenging to model dynamically in heavy-ion collisions
  - BES-II data are consistent with non-critical physics at $\sqrt{s_{NN}} \geq 20$ GeV but shows non-monotonic structures in factorial cumulants at $\sqrt{s_{NN}} < 10$ GeV

- Astrophysical observations increasingly constrain the EoS at $T = 0$

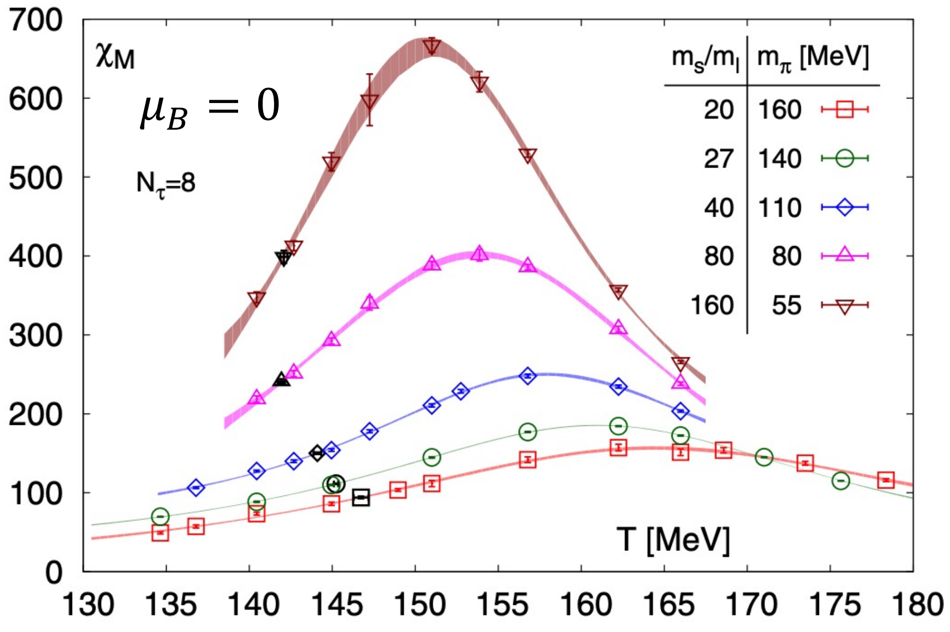
Outlook:
- Interesting times ahead with upcoming data in baryon-dense regime (BES-II, RHIC-FXT, CBM)
- Toward unified descriptions of the QCD EoS in hot, cold and dense regimes

Thanks for your attention and enjoy QM2025!

Additional slides

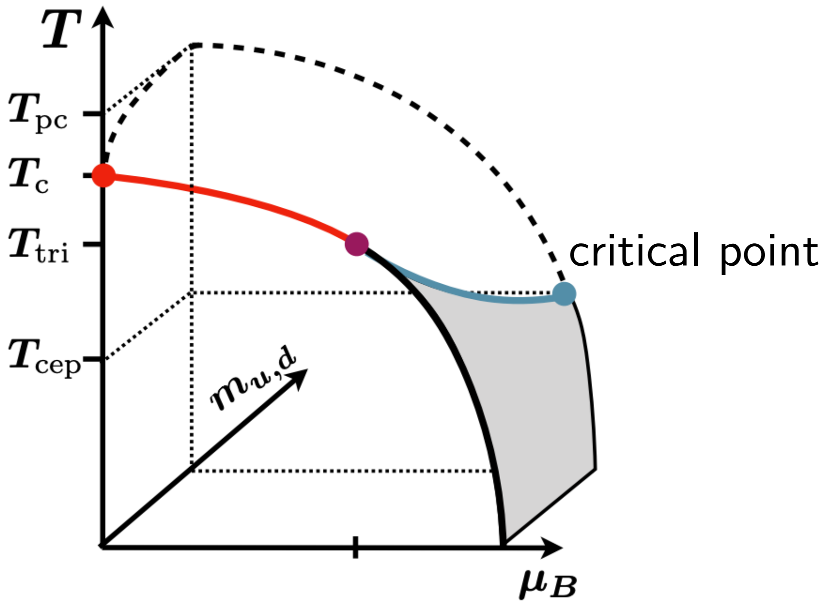
QCD critical point from chiral criticality

Remnants of O(4) chiral criticality at $\mu_B = 0$ quite well established with lattice QCD



HotQCD Collaboration, PRL 123, 062002 (2019)

Physical quark masses away the chiral limit: Expect a Z(2) critical point at finite μ_B



C. Schmidt

Searching for singularities in the complex plane

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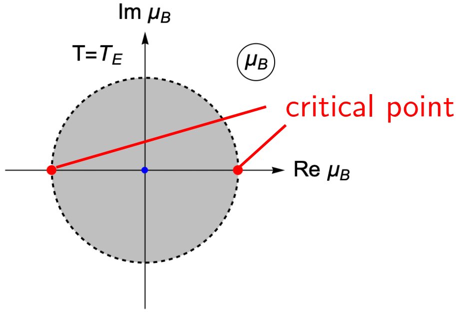
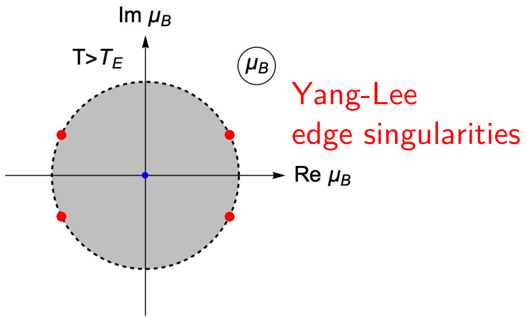
Critical point:

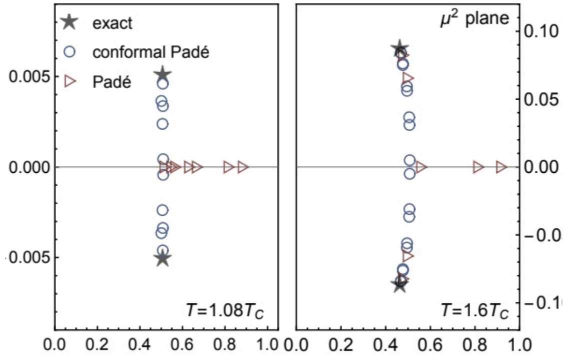
- *singularity in the partition function*
- *real μ_B axis*



Above the critical temperature:

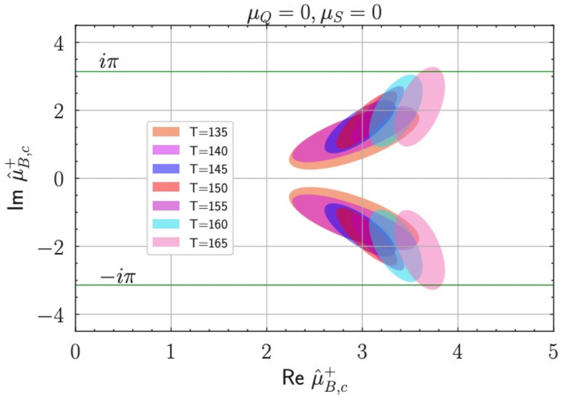
Yang-Lee edge singularities in the complex plane





G. Basar, PRC 110, 015203 (2024)

- Extract YL edge singularity through (multi-point) Pade fits
- See if it approaches the real axis as temperatures decreases



D.A. Clarke et al. (Bielefeld-Parma), arXiv:2405.10196

CP Z(2) scaling inspired fit:

$$\mathrm{Im}\,\mu_{LY} = c(T - T_{CEP})^{\Delta}$$

$$\mathrm{Re}\,\mu_{LY} = \mu_{CEP} + a(T - T_{CEP}) + b(T - T_{CEP})^2$$

Rough suggestion of CEP:
$T \sim 90$ MeV $\quad \mu_B \sim 600$ MeV

T~90-100 MeV, μ_B~500-600 MeV

NB: many things have to go right, systematic error still large

A. Adam et al. (Wuppertal-Budapest), LATTICE2024

Searching for singularities in the complex plane

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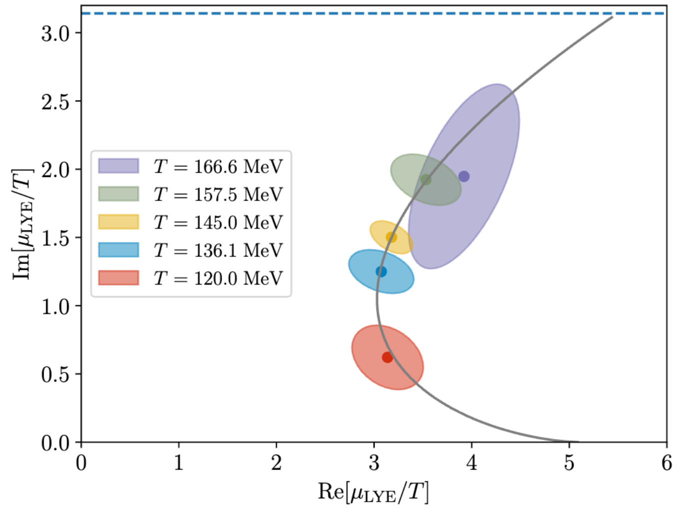
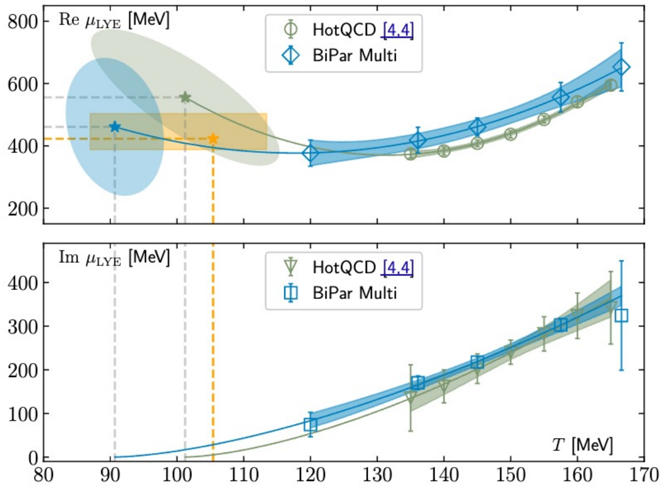
Critical point is a singularity on the real μ_B axis, which turns into **Yang-Lee edge singularities** above T_c **in the complex plane**

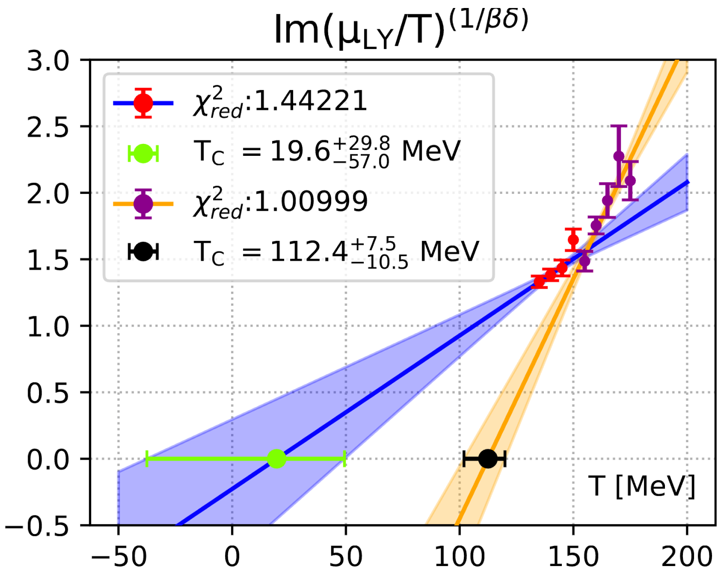
M. Stephanov, PRD 73, 094508 (2006)

Strategy: Extract YL edge singularity through (multi-point) Pade fits and see if it approaches the real axis as temperatures decreases





Variations in fit range etc.



A. Adam et al. (Wuppertal-Budapest), LATTICE2024

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$$\mathrm{Im}\,\mu_{LY} = c(T - T_{CEP})^{\Delta}$$

$$\mathrm{Re}\,\mu_{LY} = \mu_{CEP} + a(T - T_{CEP}) + b(T - T_{CEP})^2$$

Extrapolated CP estimate:

T~90-110 MeV, μ_B~400-600 MeV

D.A. Clarke et al. (Bielefeld-Parma), arXiv:2405.10196; G. Basar, PRC 110, 015203 (2024)

many things have to go right, systematic error still large (up to 100%)

Effective QCD theories anchored with lattice QCD

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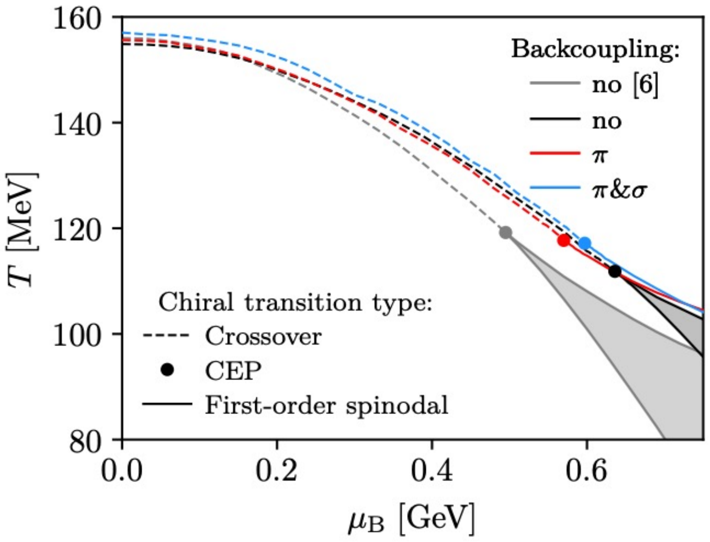
Dyson-Schwinger equations

truncations

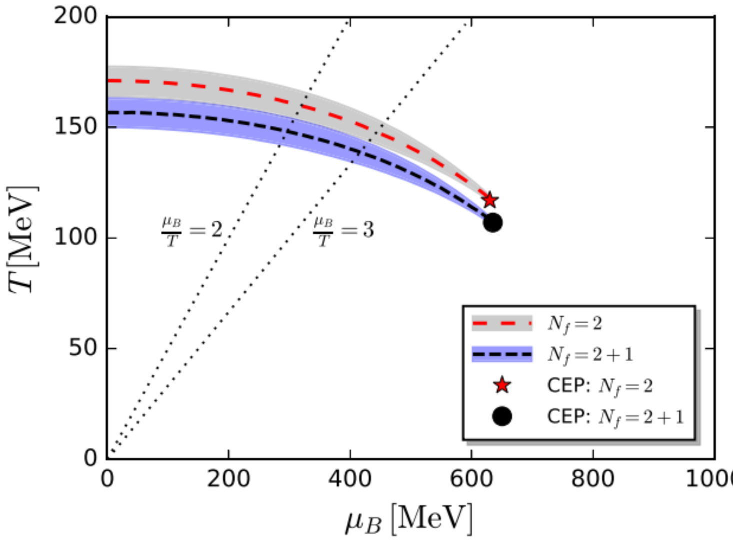


Gunkel, Fischer, PRD 104, 054202 (2021)

$T{\sim}120$ MeV $\mu_B{\sim}600$ MeV

Functional renormalization group

truncations

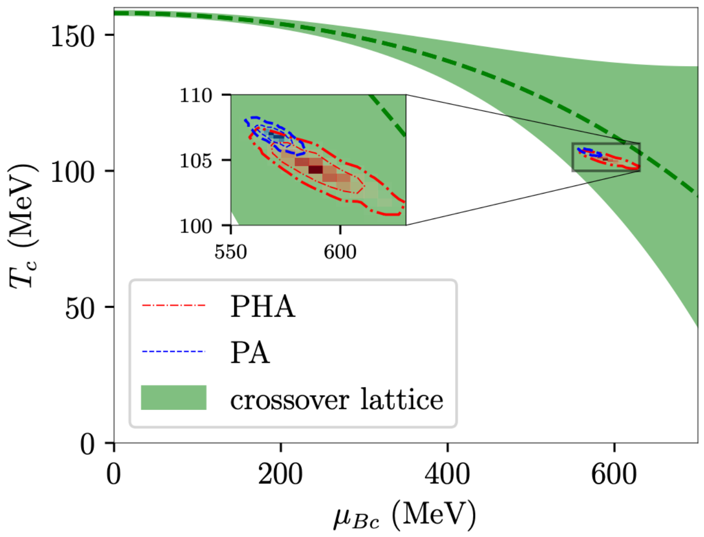


Fu, Pawlowski, Rennecke, PRD 101, 053032 (2020)

$T{\sim}100$ MeV $\mu_B{\sim}600-650$ MeV

Black-hole engineering

strongly-coupled only ($\eta/s=1/4\pi$)

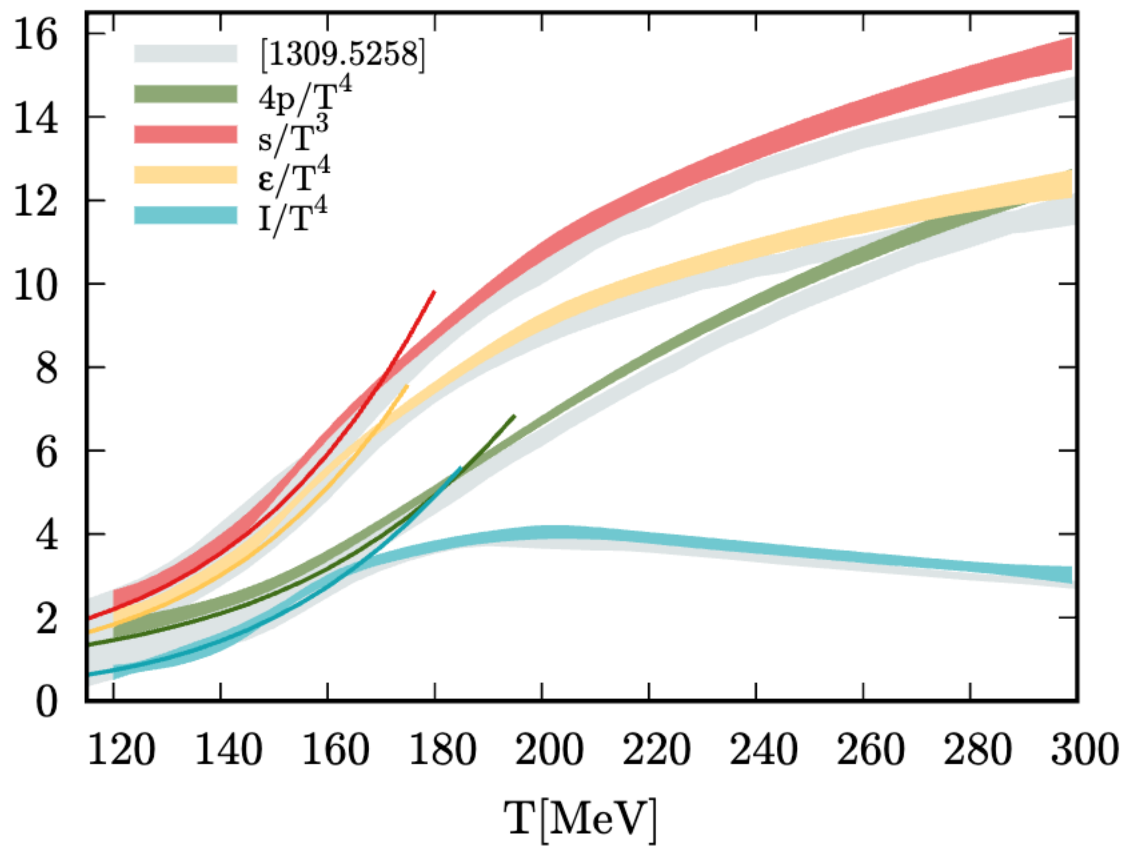


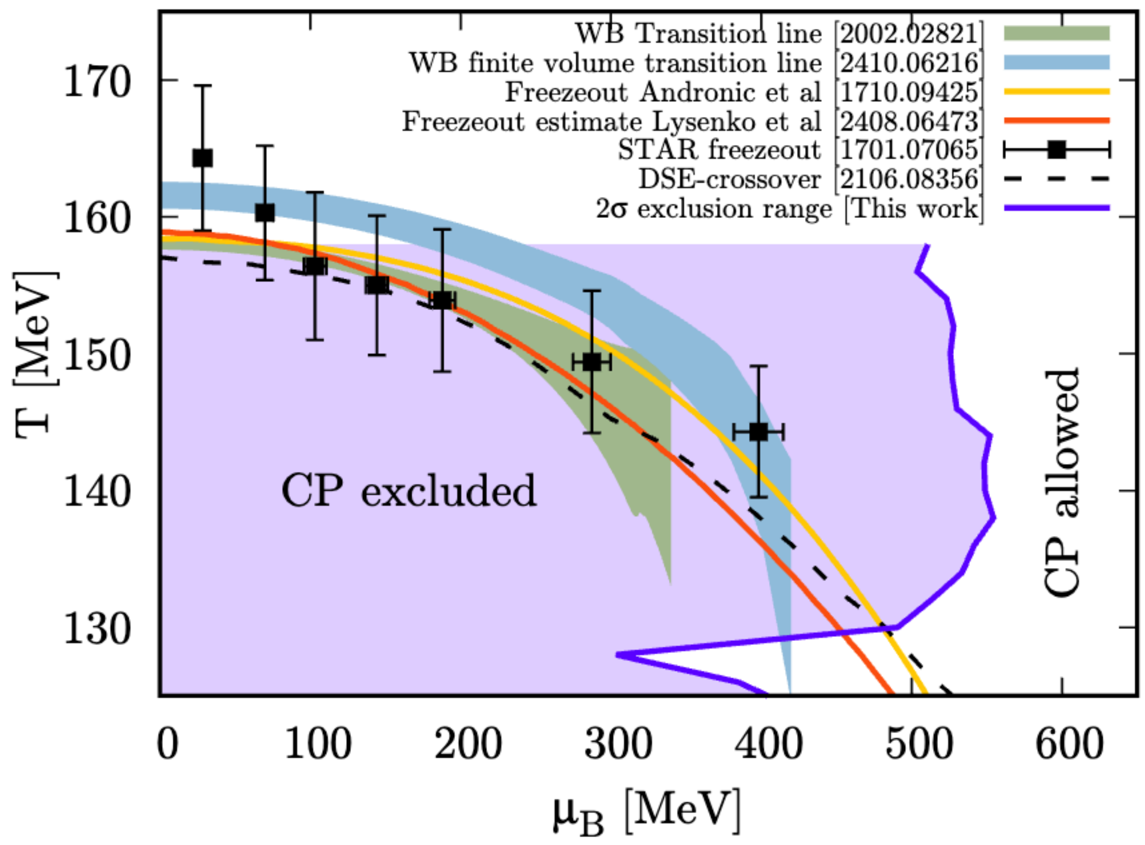
Hippert et al., arXiv:2309.00579

$T{\sim}105$ MeV $\mu_B{\sim}580$ MeV

- All in excellent agreement with lattice QCD at $\mu_B=0$ and predict QCD critical point in a similar ballpark of $\mu_B/T \sim$ 5-6
- Other estimates:
  - Finite-size scaling of heavy-ion observables [R. Lacey, PRL 114, 142301 (2015); A. Sorensen, P. Sorensen, arXiv:2405.10278]
  - Extrapolation of Yang-Lee edge singularities [D.A. Clarke et al. (Bielefeld-Parma), arXiv:2405.10196; G. Basar, PRC 110, 015203 (2024)]

New CP constraints from lattice QCD

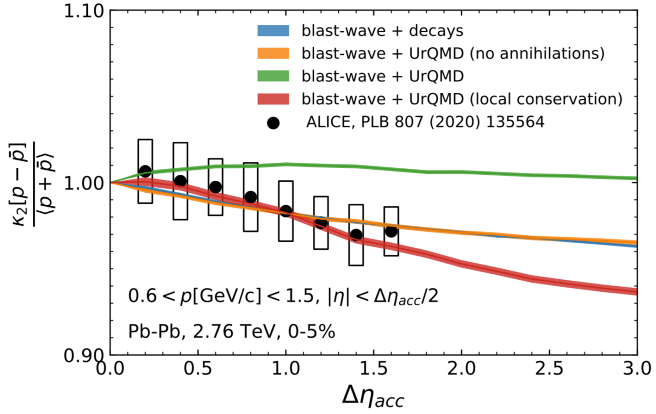




Proton cumulants at high energy

Second-order cumulants such as $\kappa_2[p-\bar{p}]/\langle p+\bar{p}\rangle$:

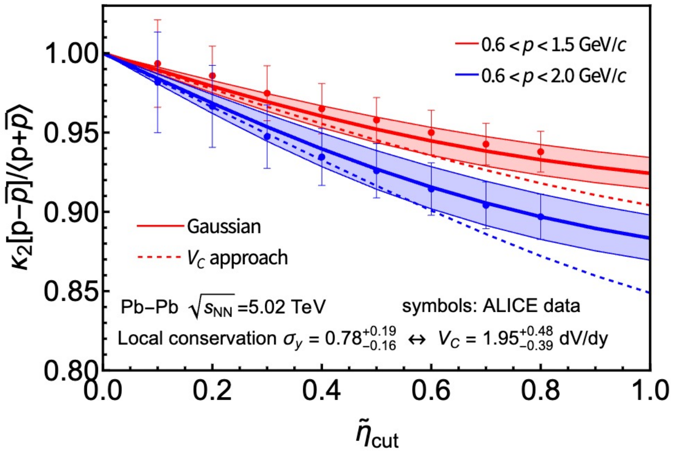
Pb-Pb 2.76 TeV



O. Savchuk et al., PLB 827, 136983 (2022)

- Largely understood as driven by baryon conservation
- baryon annihilation(↗) vs local conservation(↘)
  - Additional measurement of $\kappa_2[p+\bar{p}]$ can resolve it
- For some quantities like net-charge (or net-pion/net-kaon) fluctuations, resonance decays are improtant

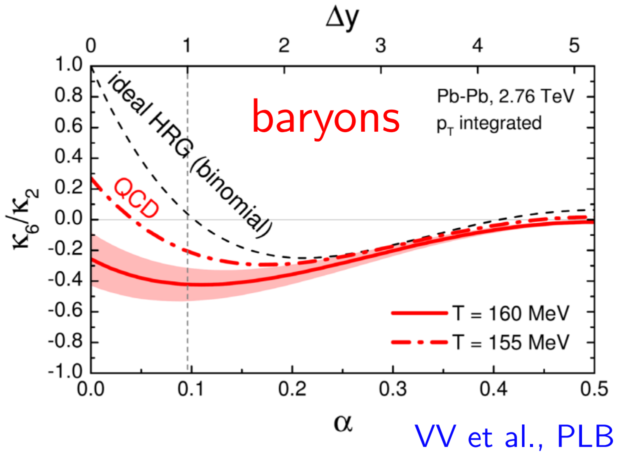
Pb-Pb 5.02 TeV



VV, arXiv:2409.01397

High-order cumulants: probe remnants of chiral criticality

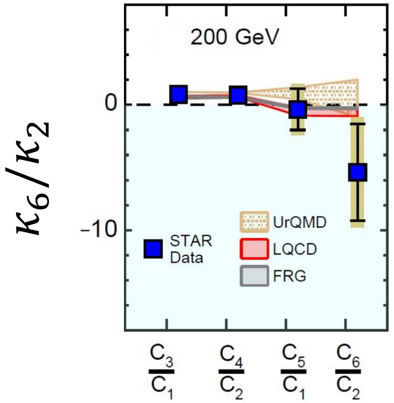
Friman et al., EPJC 71, 1694 (2011)



- negative κ_6 of baryons

VV et al., PLB 811, 135868 (2020)

RHIC 200 GeV: hints of negative $\kappa_6 < 0$ (protons)



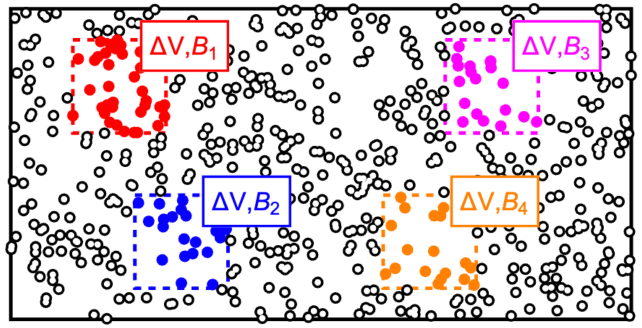
- are baryons even more negative?

STAR Collaboration, PRL 130, 082301 (2023)

Exact charge conservation

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020); VV, arXiv:2409.01397

Utilizing the canonical partition function in thermodynamic limit
compute **n-point density correlators**



$$\mathcal{C}_1(\mathbf{r}_1) = \rho(\mathbf{r}_1)$$

$$\mathcal{C}_2(\mathbf{r}_1,\mathbf{r}_2) = \chi_2\delta(\mathbf{r}_1-\mathbf{r}_2) - \frac{\chi_2}{V}$$

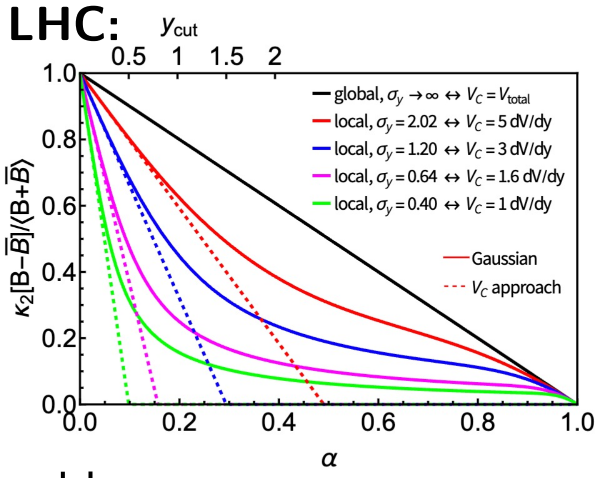
local correlation — balancing contribution (e.g. baryon conservation)

$$\mathcal{C}_3(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = \chi_3\delta_{1,2,3} - \frac{\chi_3}{V}[\delta_{1,2}+\delta_{1,3}+\delta_{2,3}] + 2\frac{\chi_3}{V^2} \qquad \delta_{1,\ldots,n} = \prod_{i=2}^n \delta(\mathbf{r}_1-\mathbf{r}_i)$$

local correlation — balancing contributions

$$\mathcal{C}_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = \chi_4\delta_{1,2,3,4} - \frac{\chi_4}{V}[\delta_{1,2,3}+\delta_{1,2,4}+\delta_{1,3,4}+\delta_{2,3,4}] - \frac{(\chi_3)^2}{\chi_2 V}[\delta_{1,2}\delta_{3,4}+\delta_{1,3}\delta_{2,4}+\delta_{1,4}\delta_{2,3}]$$
$$+\frac{1}{V^2}\left[\chi_4+\frac{(\chi_3)^2}{\chi_2}\right][\delta_{1,2}+\delta_{1,3}+\delta_{1,4}+\delta_{2,3}+\delta_{2,4}+\delta_{3,4}] - \frac{3}{V^3}\left[\chi_4+\frac{(\chi_3)^2}{\chi_2}\right].$$

local correlation — balancing contributions



Integrating the correlator yields cumulant inside a subsystem of the canonical ensemble

$$\kappa_n[B_{V_s}] = \int_{\mathbf{r}_1\in V_s} d\mathbf{r}_1 \ldots \int_{\mathbf{r}_n\in V_s} d\mathbf{r}_n\, \mathcal{C}_n(\{\mathbf{r}_i\})$$

Momentum space: Fold with Maxwell-Boltzmann in LR frame and integrate out the coordinates

Hydro EV: Non-critical hydro baseline at RHIC-BES

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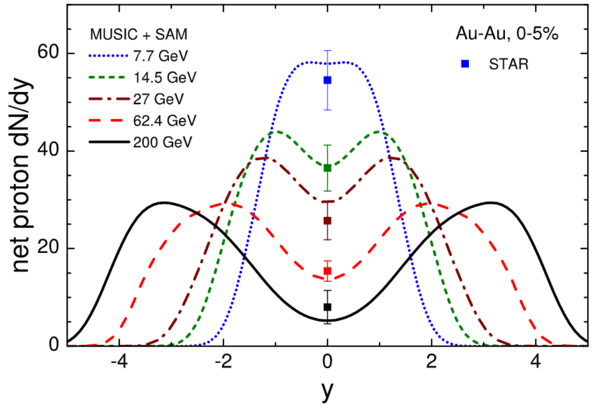
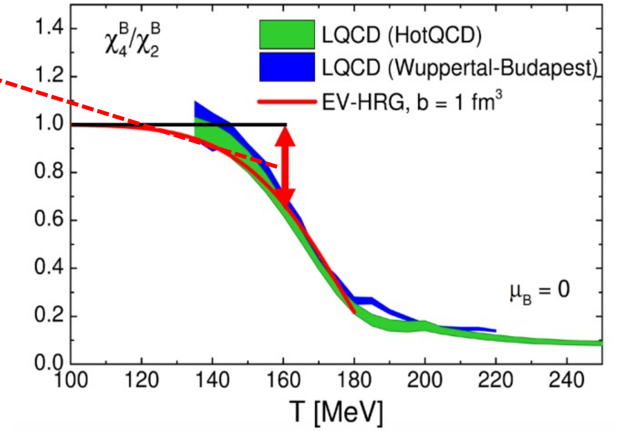
VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
  - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
  - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]



- Non-critical contributions computed at particlization (ϵ_{sw} = 0.26 GeV/fm^3)
  - QCD-like baryon number distribution (χ_n^B) via **excluded volume** b = 1 fm^3 [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
  - **Exact global baryon conservation*** (and other charges)
    - Subensemble acceptance method 2.0 (analytic) [VV, Phys. Rev. C 105, 014903 (2022)]
    - or FIST sampler (Monte Carlo) [VV, Phys. Rev. C 106, 064906 (2022)] https://github.com/vlvovch/fist-sampler



- Included: baryon conservation, repulsion, kinematical cuts
- Absent: critical point, local conservation, initial-state/volume fluctuations, hadronic phase

*If baryon conservation is the only effect (no other correlations), non-critical baseline can be computed without hydro
Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, NPA 1008, 122141 (2021)

Coordinate vs Momentum space

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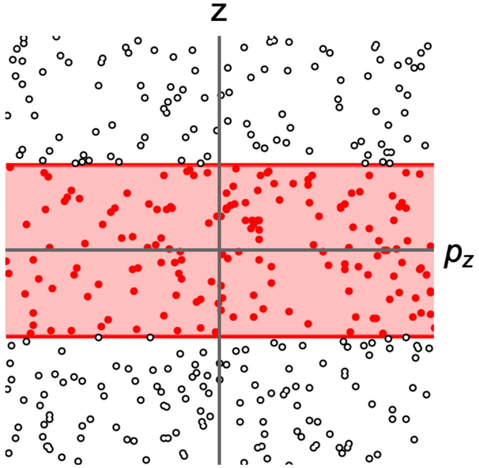
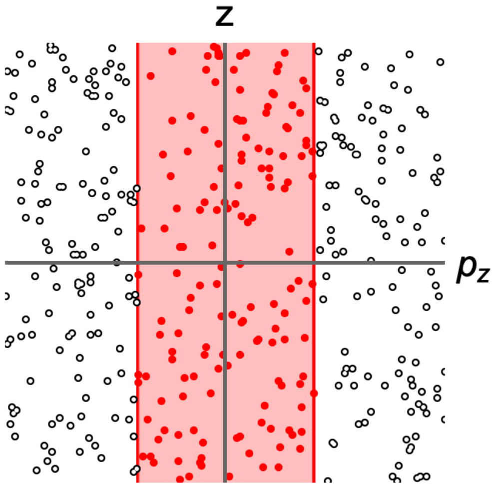
Box setup: Coordinates and momenta are uncorrelated

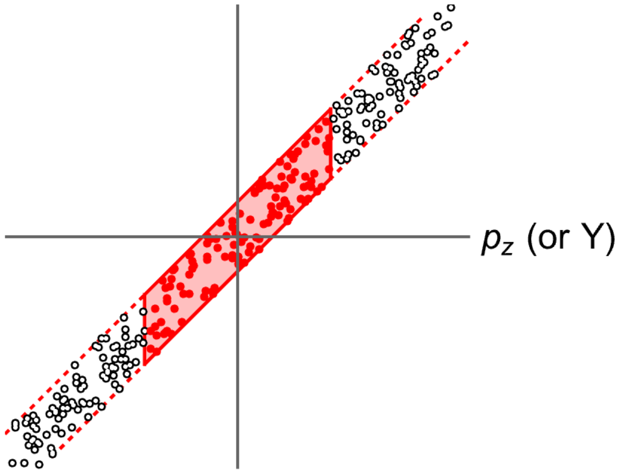
HICs: Flow (e.g. Bjorken)

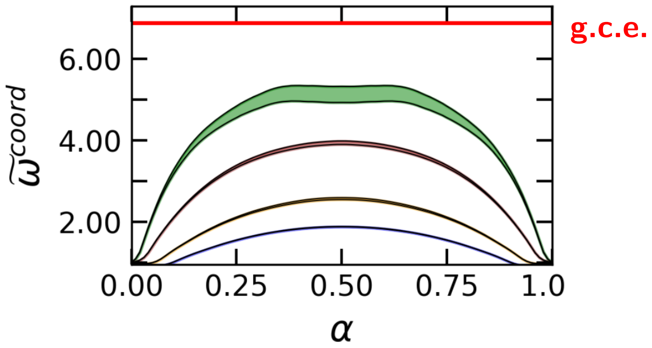
Coordinate space cut



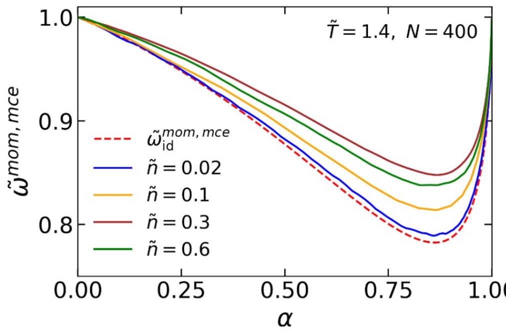
Momentum space cut



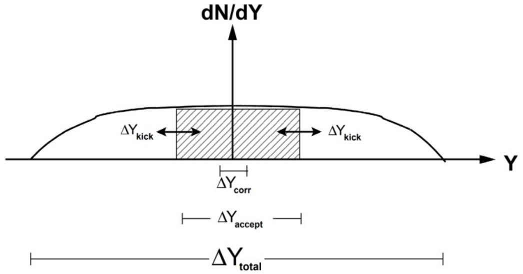




Large correlations



Nothing left



V. Koch, arXiv:0810.2520

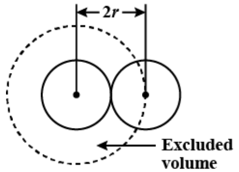
momentum cut ~ coordinate cut + smearing

Dynamical approaches to the QCD critical point search

1. Dynamical model calculations of critical fluctuations BEST COLLABORATION [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]
   - Fluctuating hydrodynamics (hydro+) and (non-equilibrium) evolution of fluctuations
   - Equation of state with a tunable critical point [P. Parotto et al, PRC 101, 034901 (2020); J. Karthein et al., EPJ Plus 136, 621 (2021)]
   - Generalized Cooper-Frye particlization [M. Pradeep, et al., PRD 106, 036017 (2022); PRL 130, 162301 (2023)]

   Alternatives at high μ_B: hadronic transport/molecular dynamics with a critical point

   [A. Sorensen, V. Koch, PRC 104, 034904 (2021); V. Kuznietsov et al., PRC 105, 044903 (2022)]



2. **Deviations from precision calculations of non-critical fluctuations**
   - Non-critical baseline is not flat [Braun-Munzinger et al., NPA 1008, 122141 (2021)]
   - Include essential non-critical contributions to (net-)proton number cumulants
   - Exact baryon conservation + hadronic interactions (hard core repulsion)
   - Based on realistic hydrodynamic simulations tuned to bulk data

   [VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

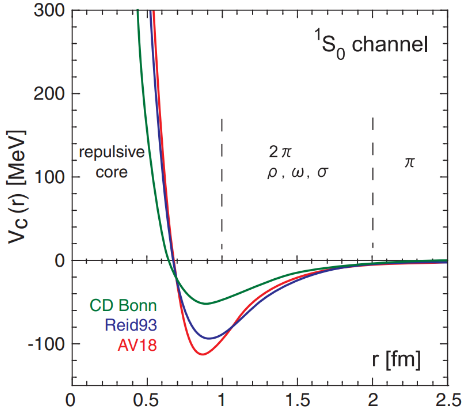


Figure from Ishii et al., PRL '07

Factorial cumulants $\hat{C}_n$ vs ordinary cumulants C_n

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Factorial cumulants: ~irreducible n-particle correlations

$$\hat{C}_n \sim \langle N(N-1)(N-2)\ldots\rangle_c$$

$$\hat{C}_1 = C_1$$
$$\hat{C}_2 = C_2 - C_1$$
$$\hat{C}_3 = C_3 - 3C_2 + 2C_1$$
$$\hat{C}_4 = C_4 - 6C_3 + 11C_2 - 6C_1$$

Ordinary cumulants: mix correls. of different orders

$$C_n \sim \langle \delta N^n \rangle_c$$

$$C_1 = \hat{C}_1$$
$$C_2 = \hat{C}_2 + \hat{C}_1$$
$$C_3 = \hat{C}_3 + 3\hat{C}_2 + \hat{C}_1$$
$$C_4 = \hat{C}_4 + 6\hat{C}_3 + 7\hat{C}_2 + \hat{C}_1$$

[Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017); Kitazawa, Luo, PRC 96, 024910 (2017); C. Pruneau, PRC 100, 034905 (2019)]

Factorial cumulants and different effects

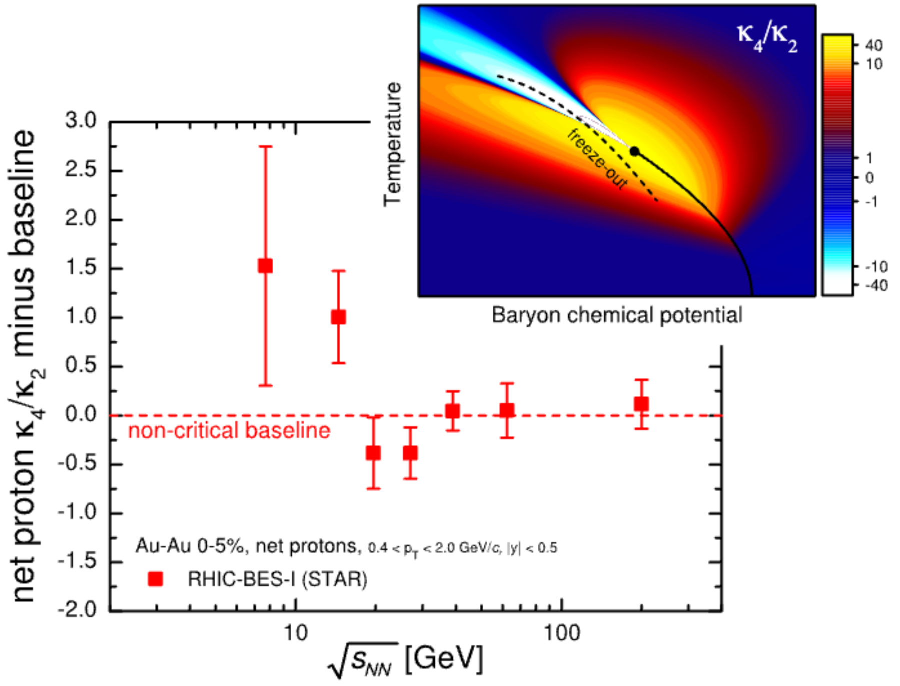
- Baryon conservation [Bzdak, Koch, Skokov, EPJC '17] — $\hat{C}_n^{\text{cons}} \propto (\hat{C}_1)^n / \langle N_{\text{tot}} \rangle^{n-1}$ *small*
- Excluded volume [VV et al, PLB '17] — $\hat{C}_n^{\text{EV}} \propto b^n$ *small*
- Volume fluctuations [Holzman et al., arXiv:2403.03598] — $\hat{C}_n^{CF} \sim (\hat{C}_1)^n \kappa_n[V]$ *depends on volume cumulants*
- **Critical point** [Ling, Stephanov, PRC '16] — $\hat{C}_2^{CP} \sim \xi^2, \quad \hat{C}_3^{CP} \sim \xi^{4.5}, \quad \hat{C}_4^{CP} \sim \xi^7$ ***large***

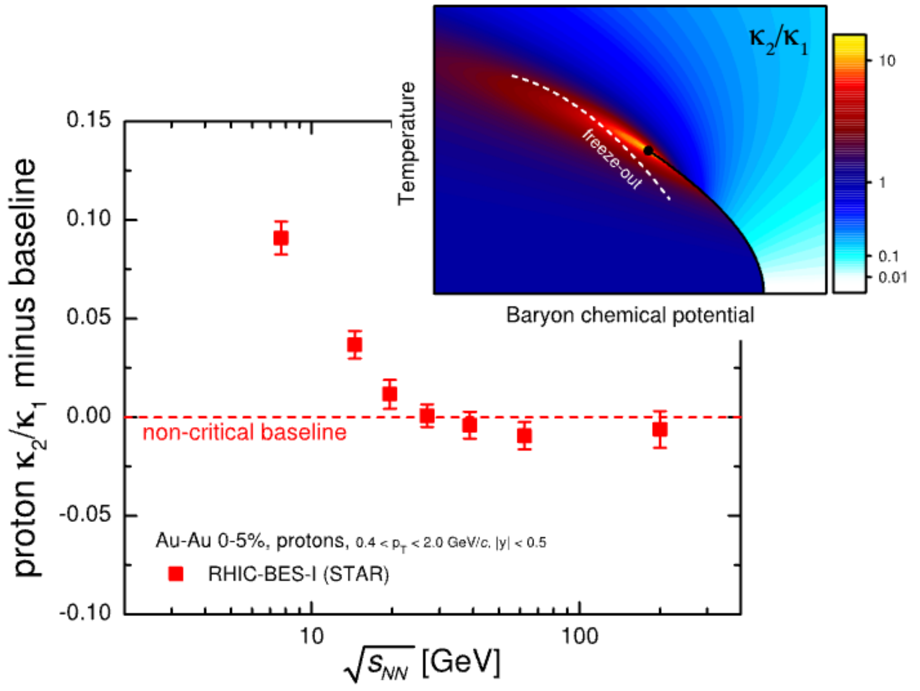
- proton vs baryon [Kitazawa, Asakawa, PRC '12] — $\hat{C}_n^B \sim 2^n \times \hat{C}_n^p$ **same sign!**

Hints from RHIC-BES-I

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Subtracting the hydrodynamic non-critical baseline

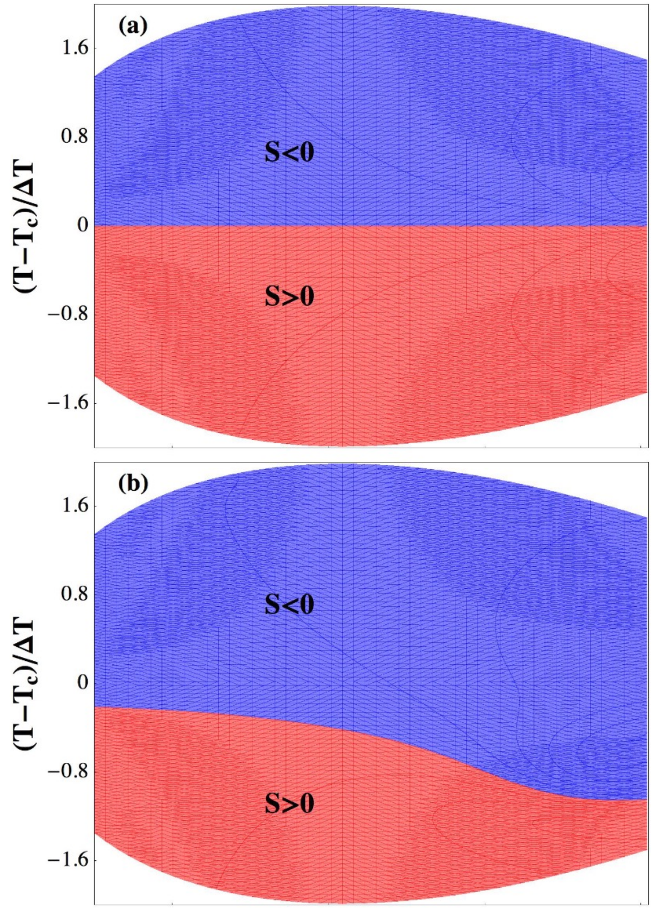




⚠️ **Notation:** Here we use κ_n for cumulants and $\hat{C}_n$ for factorial cumulants, STAR Collaboration uses the opposite

Factorial cumulants from RHIC-BES-II and CP

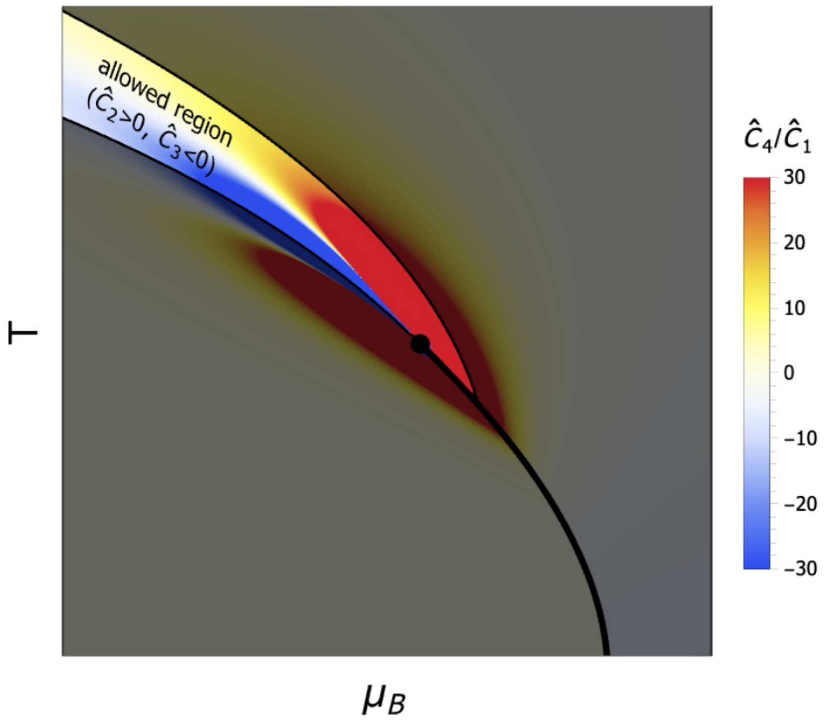
Memory effect



Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)

Exclusion plots

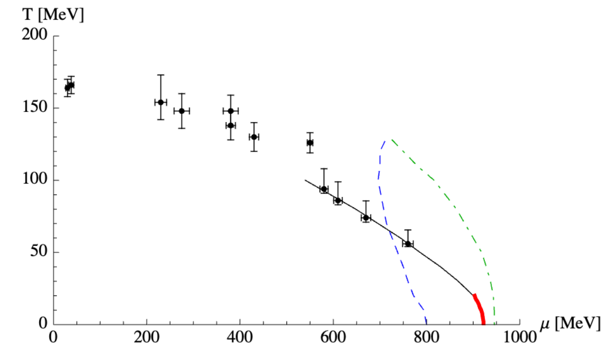
Exclude $\hat{C}_2<0$ & $\hat{C}_3>0$ regions on the phase diagram near CP



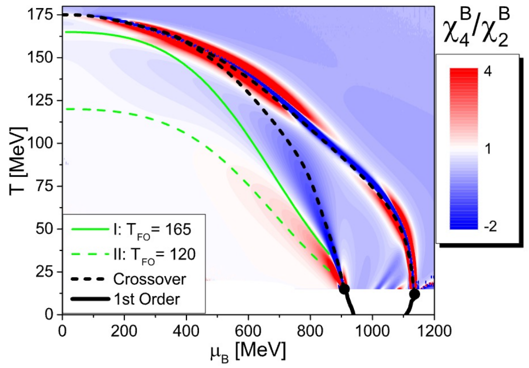
Adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017) and based on the model from VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Freeze-out of fluctuations on the QGP side of the crossover?

Interplay with nuclear liquid-gas transition

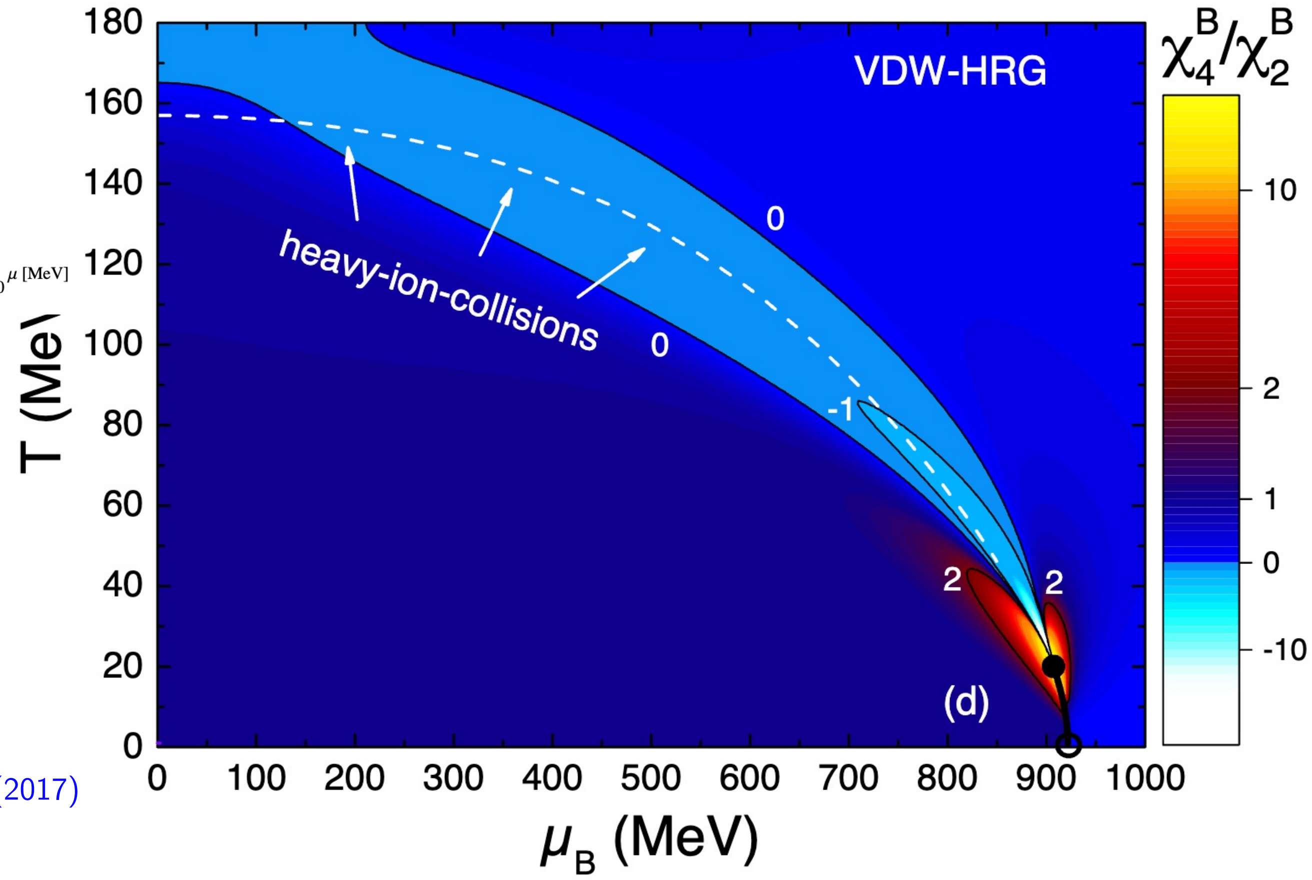


Floerchinger, Wetterich, NPA (2012)

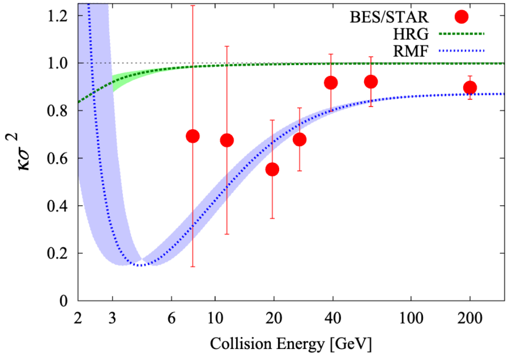


Mukherjee, Steinheimer, Schramm, PRC (2017)

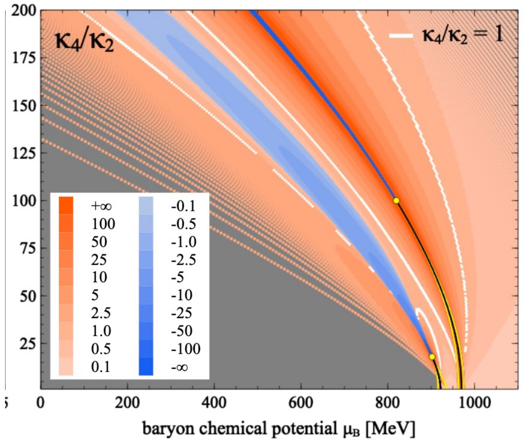
HRG with attractive and repulsive interactions among baryons



VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)



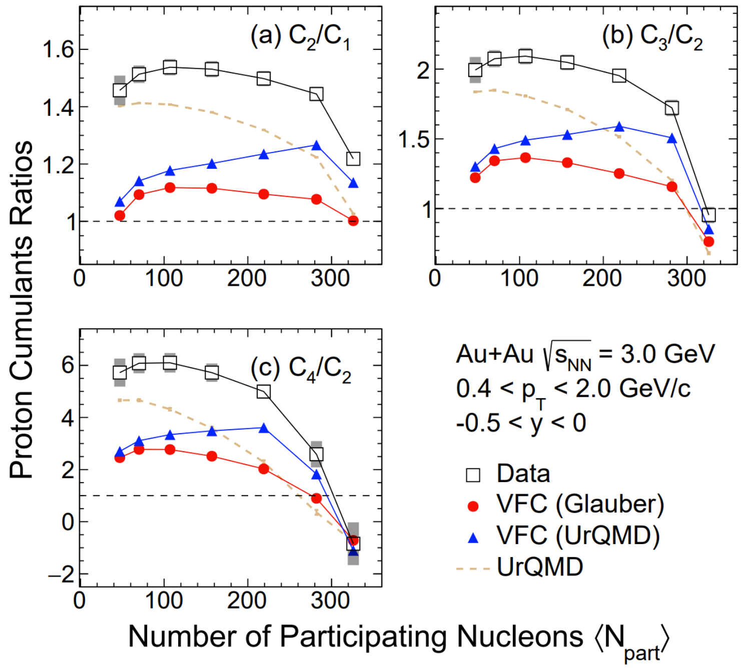
Fukushima, PRC (2014)



Sorensen, Koch, PRC (2020)

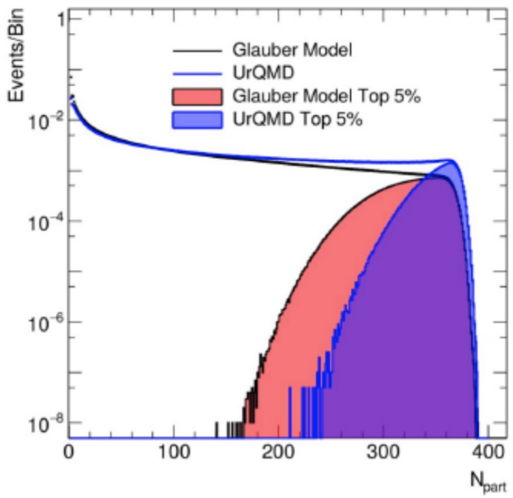
Increasingly relevant at lower energies probed through RHIC-FXT

Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV



STAR Collaboration, Phys. Rev. Lett. 128 (2022) 202303

STAR-FXT



HADES

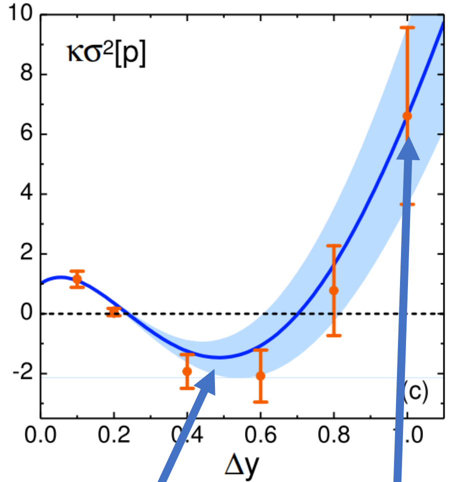


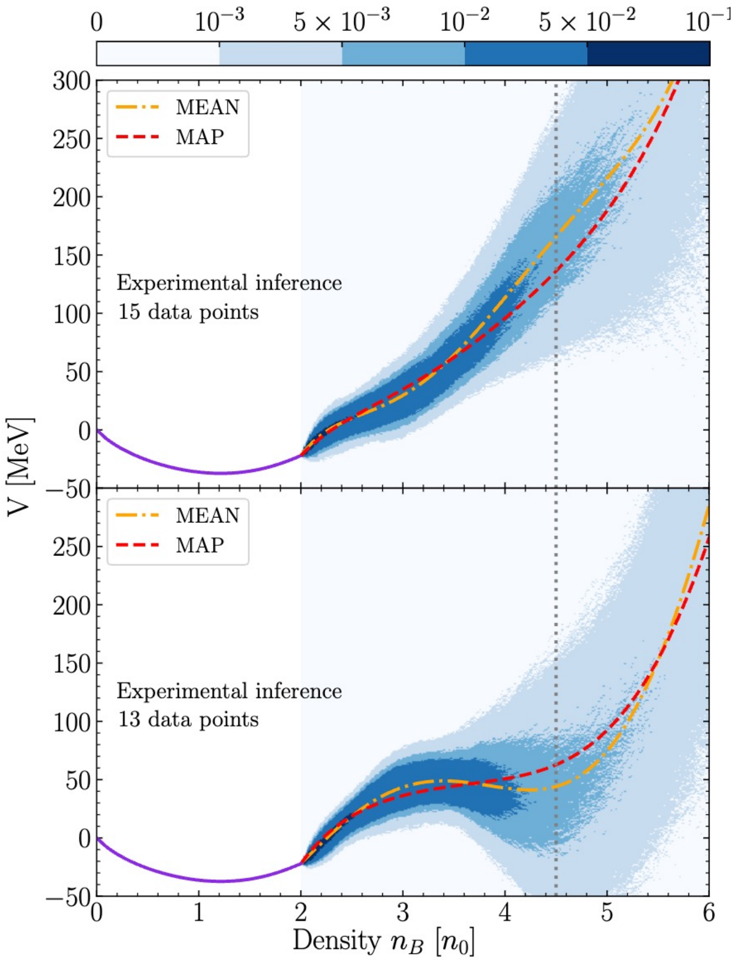
Figure from O. Savchuk et al., PLB 835, 137540 (2022)

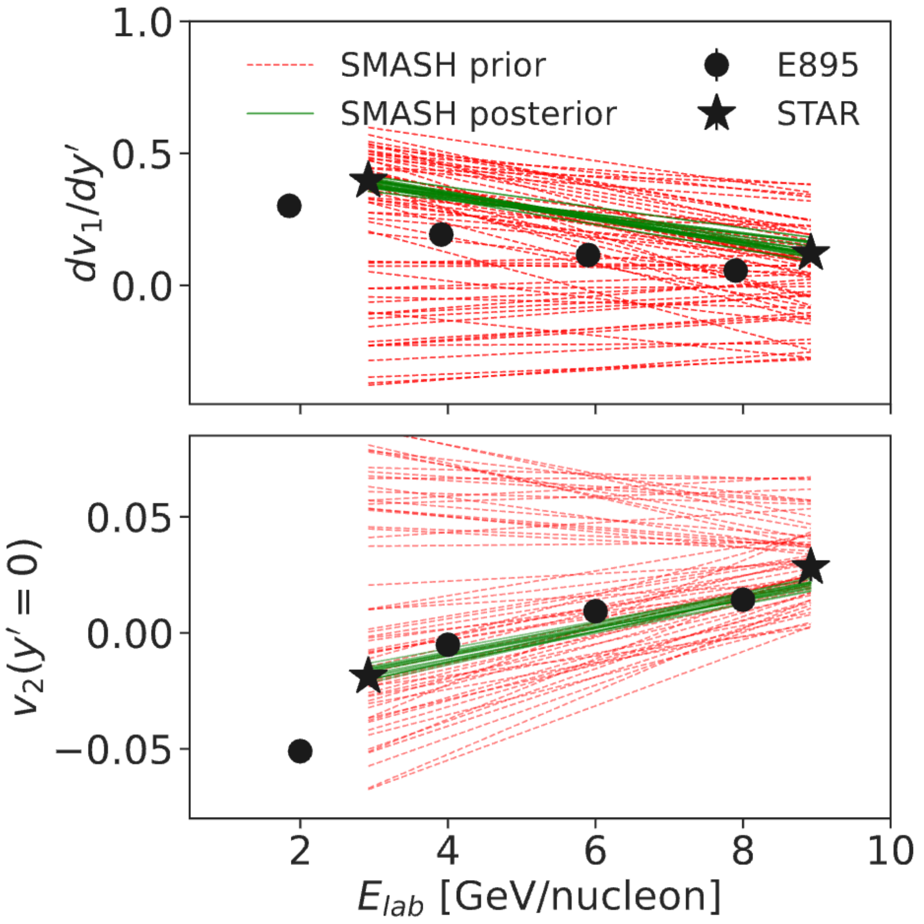
- Volume fluctuations/centrality selection appear to play an important role
  - UrQMD is useful for understanding basic systematics associated with it
- Indications for enhanced scaled variance, $\kappa_2/\kappa_1 > 1$
- κ_4/κ_2 negative and described by UrQMD (purely hadronic?), note -0.5<y<0 instead of |y|<0.5

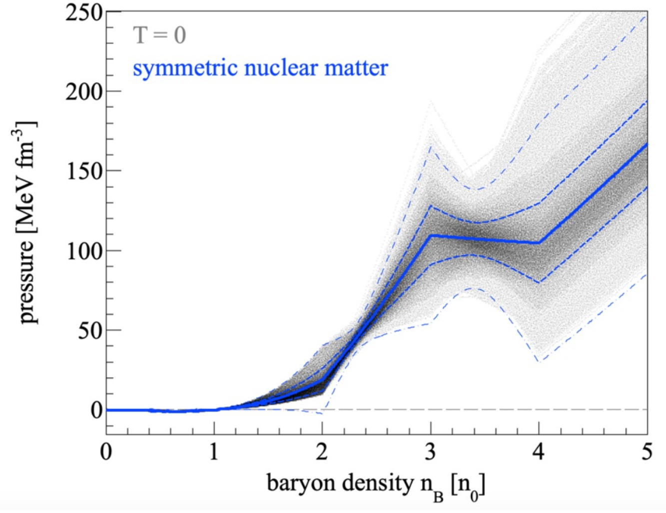
Proper understanding of $\kappa_2/\kappa_1 > 1$ in both HADES and STAR-FXT is missing

Dense matter EoS from flow measurements

- Use hadronic transport (UrQMD and SMASH) with adjustable mean field to use a flexible EoS
- Extract the EoS from proton flow measurements





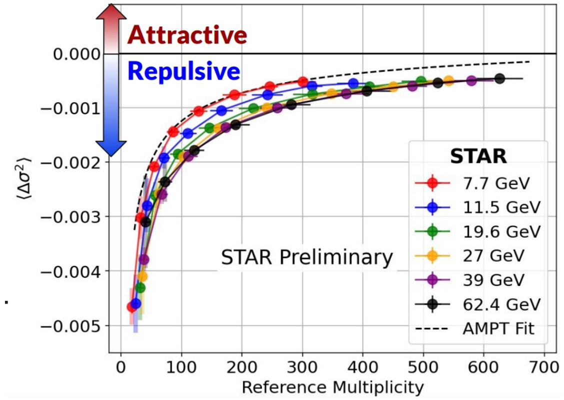


M. Kuttan, Steinheimer, Zhou, Stoecker, PRL 131, 202303 (2023)

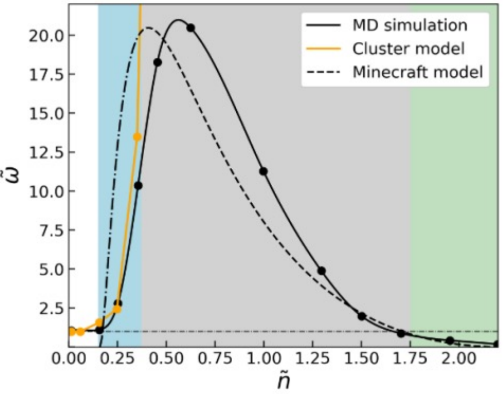
Oliinychenko, Sorensen, Koch, McLerran, PRC 108, 034908 (2023)

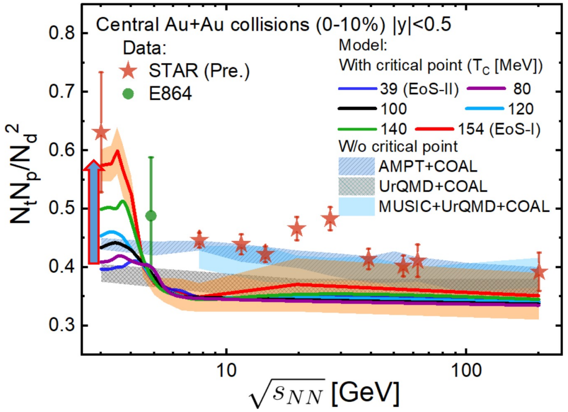
Other observables

- Azimuthal correlations of protons
  - points to repulsion at RHIC-BES



- Light nuclei
  - Spinodal/critical point enhancement of density fluctuations and light nuclei production





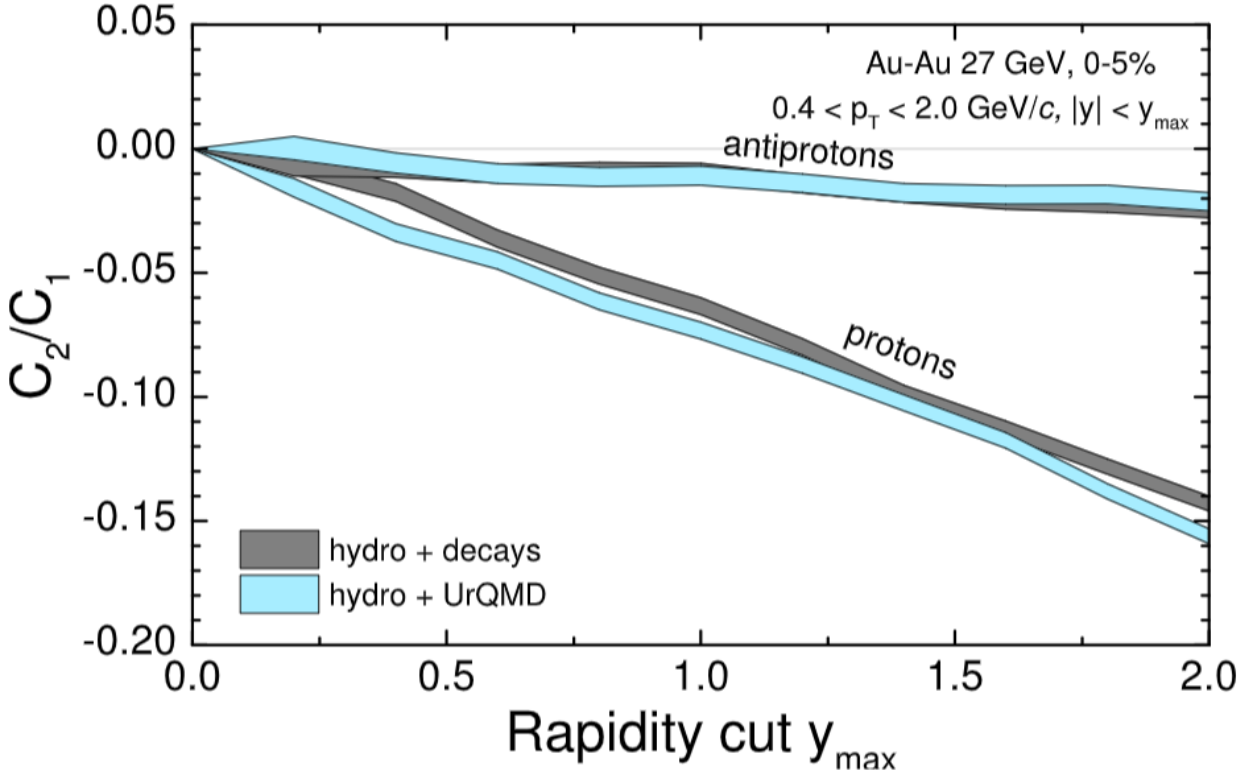
- Proton intermittency
  - No structure indicating power-law seen by NA61/SHINE

- Directed flow, speed of sound

Consistency in understanding all the observables is required

Effect of the hadronic phase

Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD



Dependence on the switching energy density