The antiproton puzzle, QCD critical point, and fireball properties in heavy-ion collisions

Volodymyr Vovchenko (University of Houston)

based on A. Bzdak, V. Koch, VV, arXiv:2503.16405

Quark Matter 2025

April 9, 2025

UNIVERSITY OF HOUSTON

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Scanning phase diagram with fluctuations

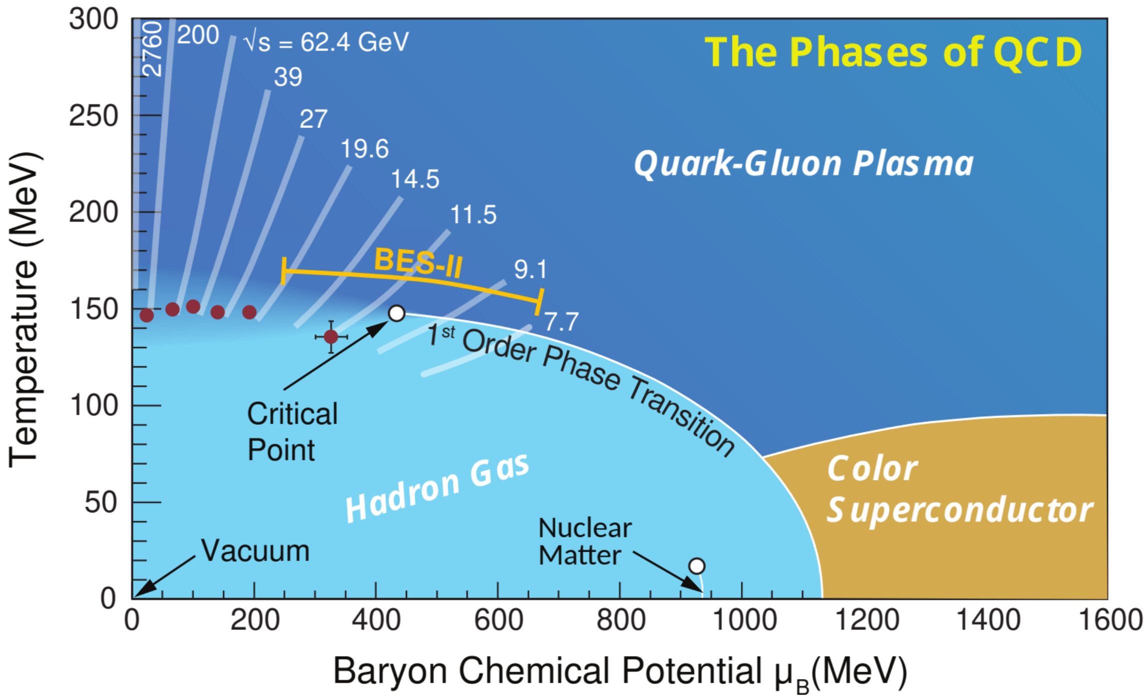
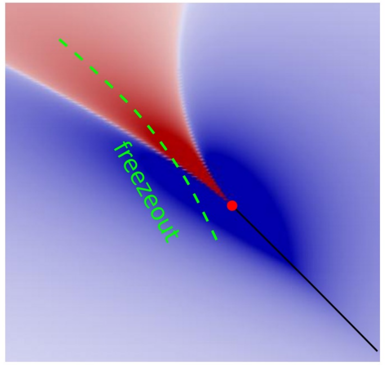
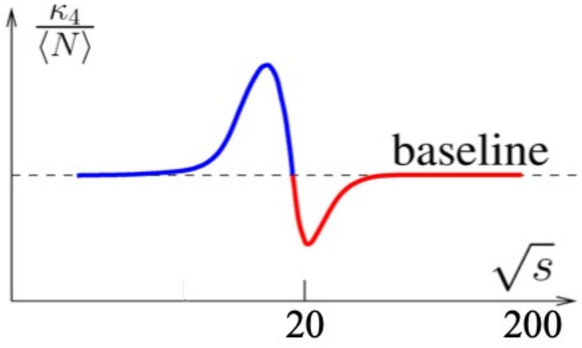


Figure from Bzdak et al., Phys. Rept. '20 & 2015 US Nuclear Long Range Plan

Critical point:

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7 \qquad \xi \to \infty$$



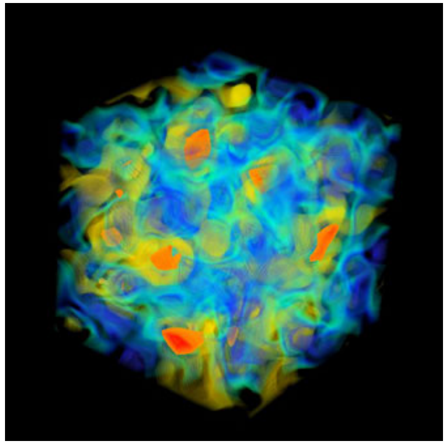


M. Stephanov, PRL '09, '11

Scan the phase diagram with heavy-ion collisions and look for non-monotonic dependence of proton fluctuations as a signature QCD CP

Theory vs experiment: Challenges for fluctuations

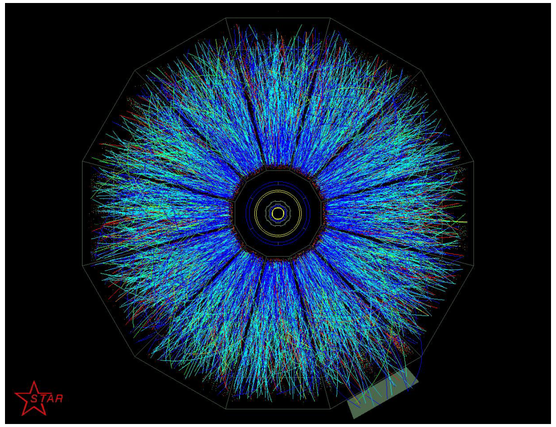
Theory



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

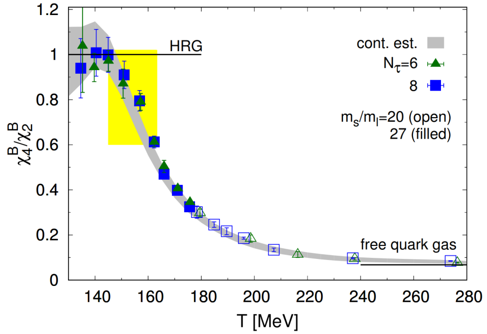
- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

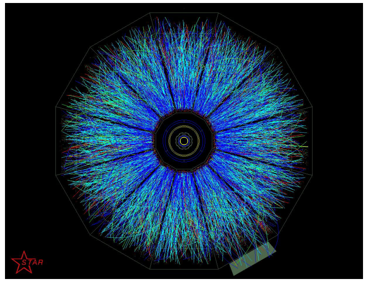
Direct comparison of GCE (e.g. lattice) with experiment is likely to yield misleading conclusions

Theory vs experiment

guidance from theory (e.g. lattice)

experiment (the real thing)





hydro simulation

hadronic phase

canonical effects

kinematic cuts

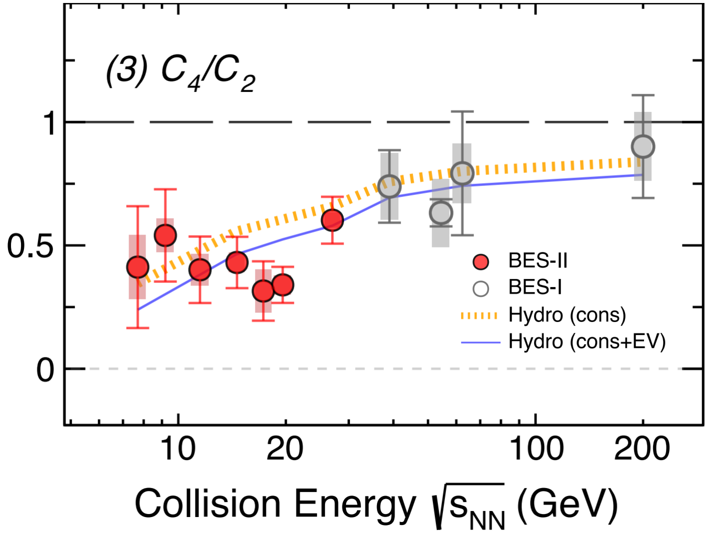
event selection

track reconstruction

efficiency correction

CBWC

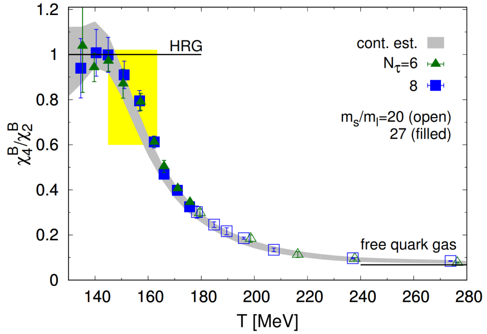
purity



Theory vs experiment

guidance from theory (e.g. lattice)

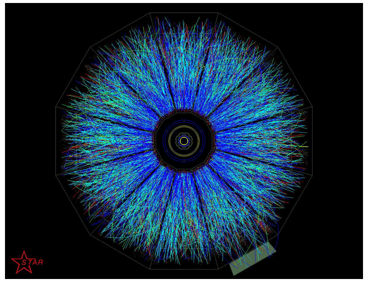
experiment (the real thing)



This was done in [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)]

- Full hydro simulation
- Lattice QCD-like baryon susceptibilities (interacting HRG)
- Global baryon conservation (SAM)
- Experimental kinematic cuts

Non-critical baseline (hydro EV) **prediction**



hydro simulation

hadronic phase

canonical effects

kinematic cuts

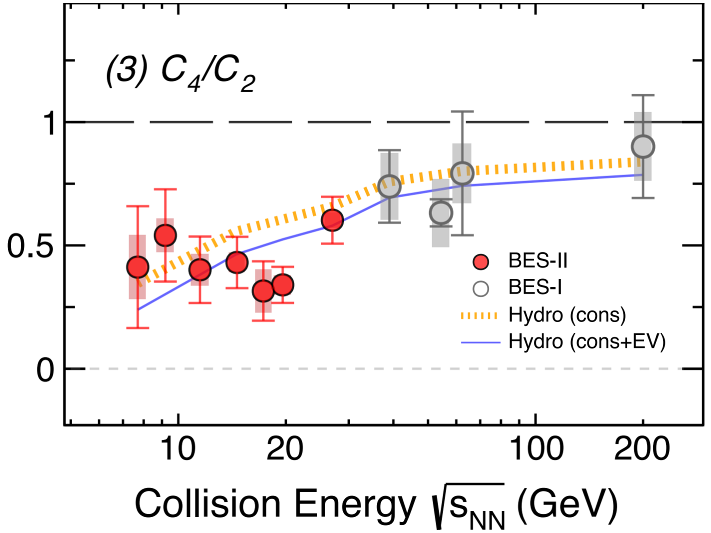
event selection

track reconstruction

efficiency correction

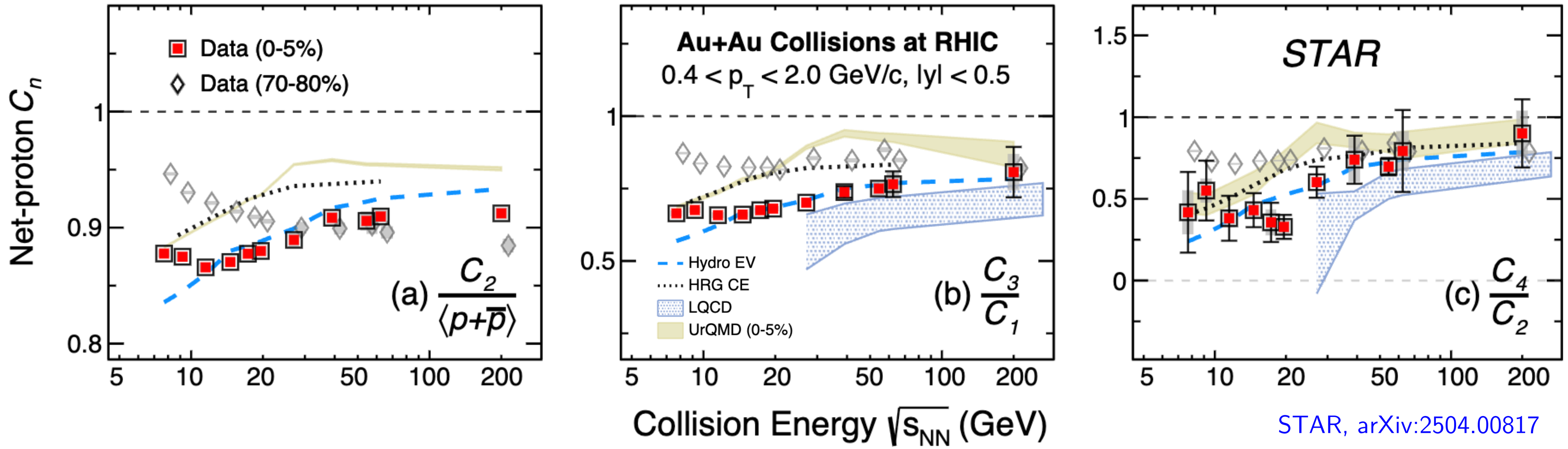
purity

CBWC



RHIC-BES-II data: Ordinary cumulants

Net-proton cumulant ratios



STAR, arXiv:2504.00817

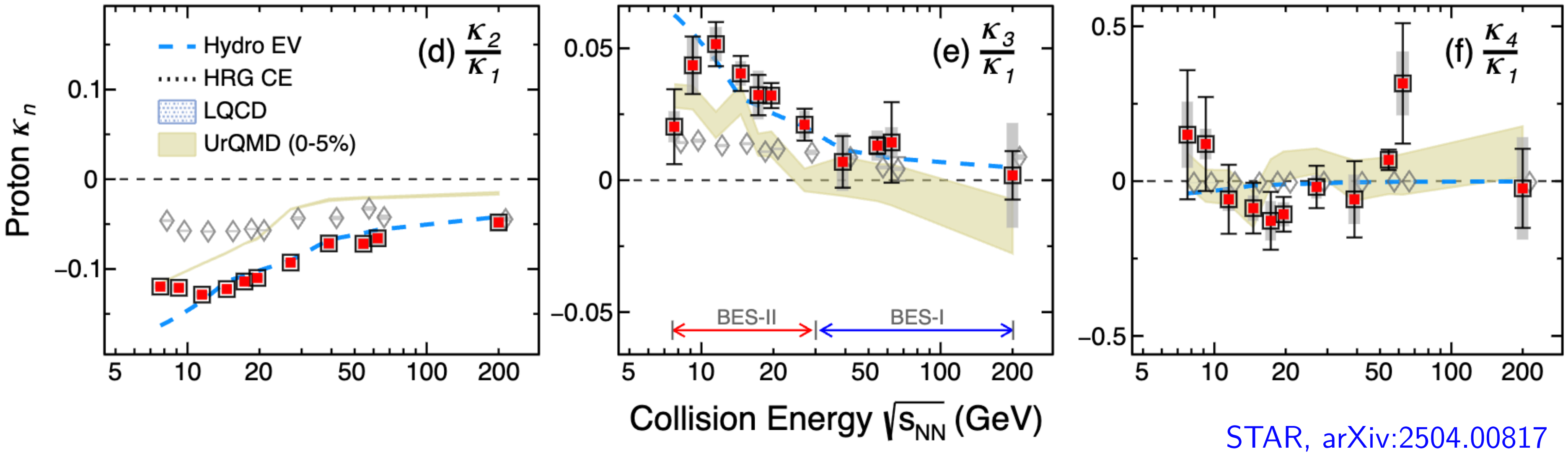
Hydro EV: VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Agreement with the baseline above $\sqrt{s_{NN}} \sim 10 - 20$ GeV

But otherwise mostly boring. What else is there?

RHIC-BES-II data: Factorial cumulants

Proton factorial cumulant ratios



STAR, arXiv:2504.00817

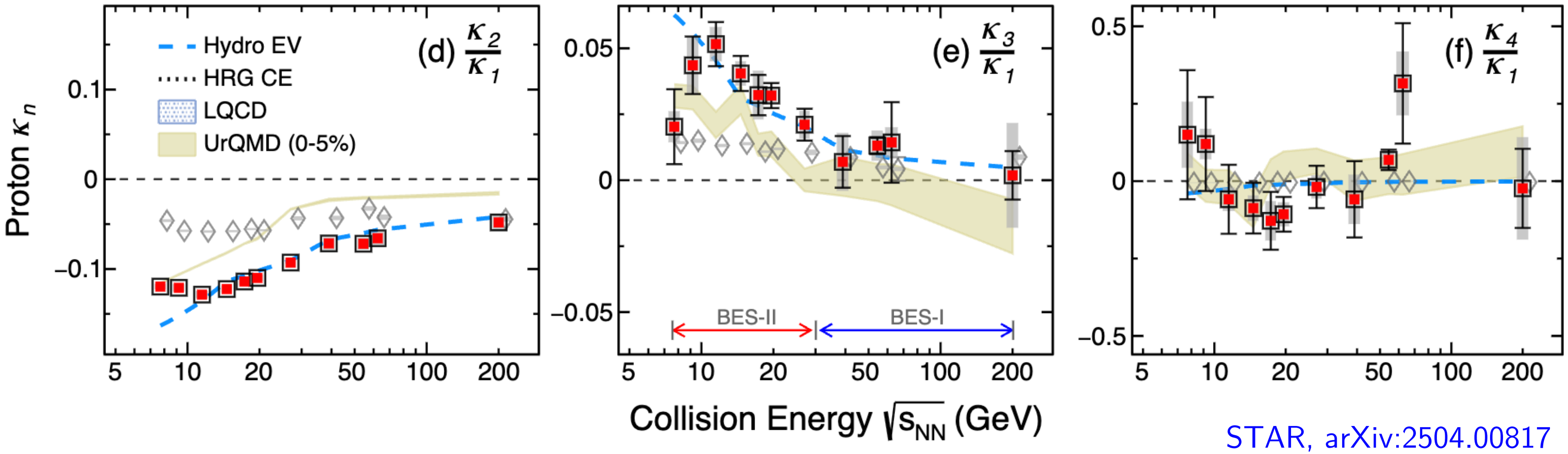
Hydro EV: VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

More structure seen in factorial cumulants

- Non-monotonic κ_2/κ_1, κ_3/κ_1, and possibly κ_4/κ_1

RHIC-BES-II data: Factorial cumulants

Proton factorial cumulant ratios



STAR, arXiv:2504.00817

Hydro EV: VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Conclusion 1:

Ordinary cumulants

Factorial cumulants

More structure seen in factorial cumulants

- Non-monotonic κ_2/κ_1, κ_3/κ_1, and possibly κ_4/κ_1

What are the factorial cumulants?

Factorial cumulants $\hat{C}_n$ vs ordinary cumulants C_n

Factorial cumulants: ~irreducible n-particle correlations

$$\hat{C}_n \sim \langle N(N-1)(N-2)\dots\rangle_c$$

$$\hat{C}_1 = C_1$$
$$\hat{C}_2 = C_2 - C_1$$
$$\hat{C}_3 = C_3 - 3C_2 + 2C_1$$
$$\hat{C}_4 = C_4 - 6C_3 + 11C_2 - 6C_1$$

Ordinary cumulants: mix correlations of different orders

$$C_n \sim \langle \delta N^n \rangle_c$$

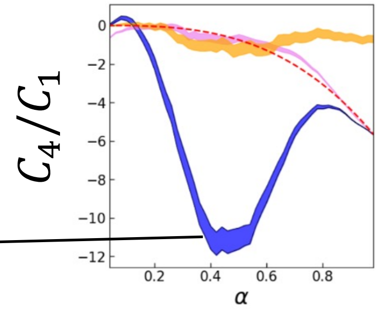
$$C_1 = \hat{C}_1$$
$$C_2 = \hat{C}_2 + \hat{C}_1$$
$$C_3 = \hat{C}_3 + 3\hat{C}_2 + \hat{C}_1$$
$$C_4 = \hat{C}_4 + 6\hat{C}_3 + 7\hat{C}_2 + \hat{C}_1$$

[Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017); Kitazawa, Luo, PRC 96, 024910 (2017); C. Pruneau, PRC 100, 034905 (2019)]

Factorial cumulants and different effects

- Baryon conservation [Bzdak, Koch, Skokov, EPJC '17] — $\hat{C}_n^{\rm cons} \propto (\hat{C}_1)^n / \langle N_{\rm tot}\rangle^{n-1}$ *small*
- Excluded volume [VV et al, PLB '17] — $\hat{C}_n^{\rm EV} \propto b^n$ *small*
- Volume fluctuations [Holzman et al., arXiv:2403.03598] — $\hat{C}_n^{CF} \sim (\hat{C}_1)^n \kappa_n[V]$ *depends on volume cumulants*
- **Critical point** [Ling, Stephanov, PRC '16] — $\hat{C}_2^{CP} \sim \xi^2, \quad \hat{C}_3^{CP} \sim \xi^{4.5}, \quad \hat{C}_4^{CP} \sim \xi^7$ ***large***

- proton vs baryon [Kitazawa, Asakawa, PRC '12] — $\hat{C}_n^B \sim 2^n \times \hat{C}_n^p$ **same sign!**



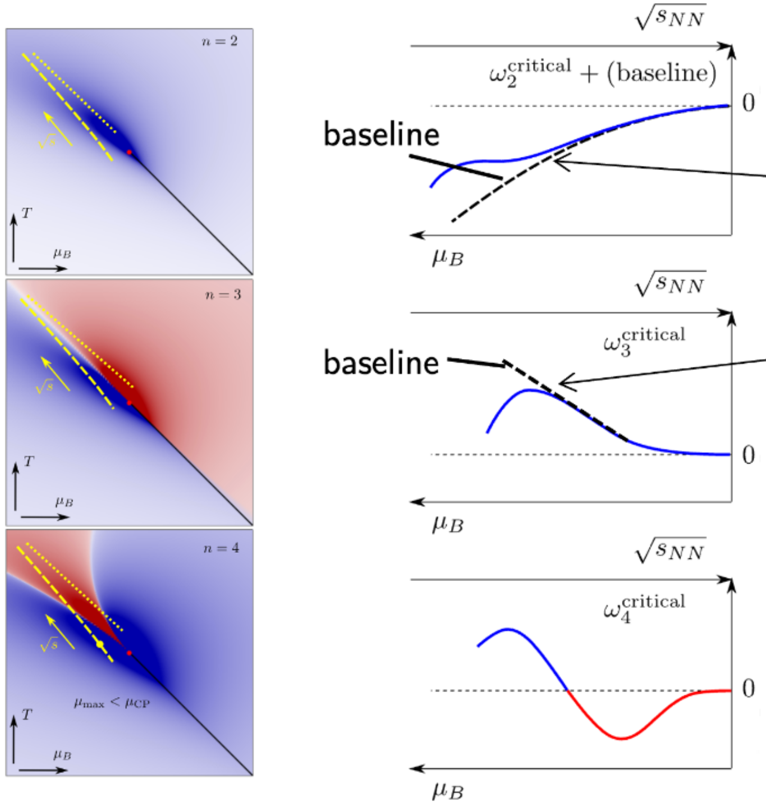
V. Kuznietsov, talk Tue 11:50

RHIC-BES-II data and CP

VV, Koch, arXiv:2504.01368, plot adapted from M. Stephanov, arXiv:2410.02861

$\omega_n = \hat{C}_n / \hat{C}_1$

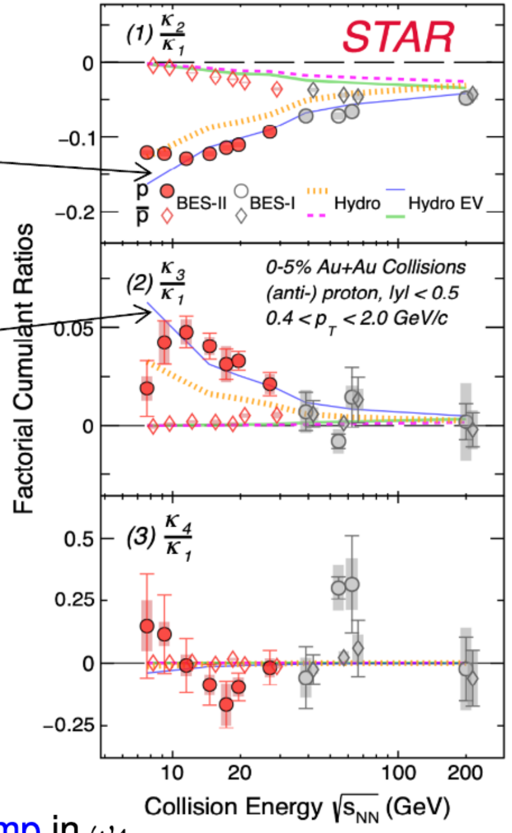
(universal EOS) critical χ_n:



Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

BES-II data:

plot from A. Pandav, CPOD2024



Non-critical baseline (hydro EV):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

- describes right side of the peak in $\hat{C}_3$
- **signal relative to baseline:**
  - *positive* $\hat{C}_2 - \hat{C}_2^{baseline} > 0$
  - *negative* $\hat{C}_3 - \hat{C}_3^{baseline} < 0$

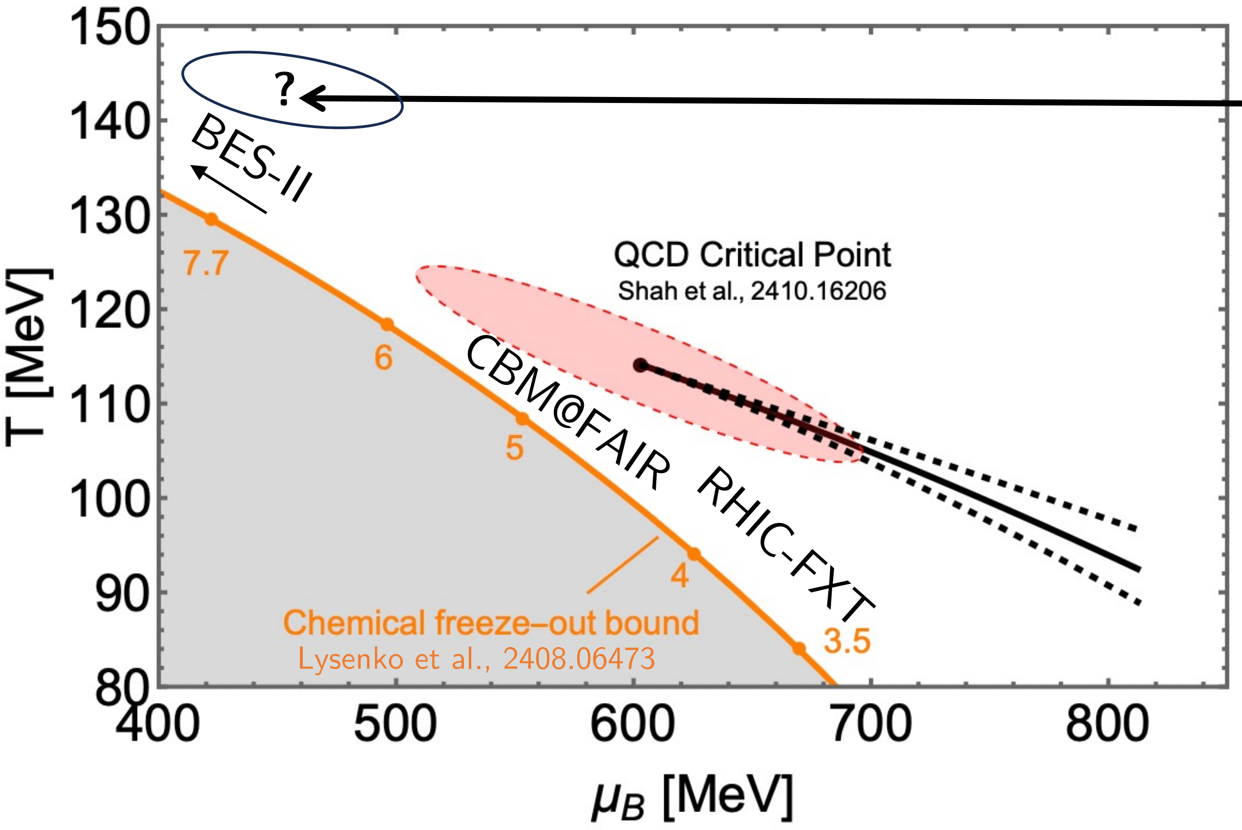
Controlling the non-critical baseline is essential

Notation: Here we use κ_n for cumulants and $\hat{C}_n$ for factorial cumulants, STAR uses the opposite

If deviations from the baseline are driven by CP

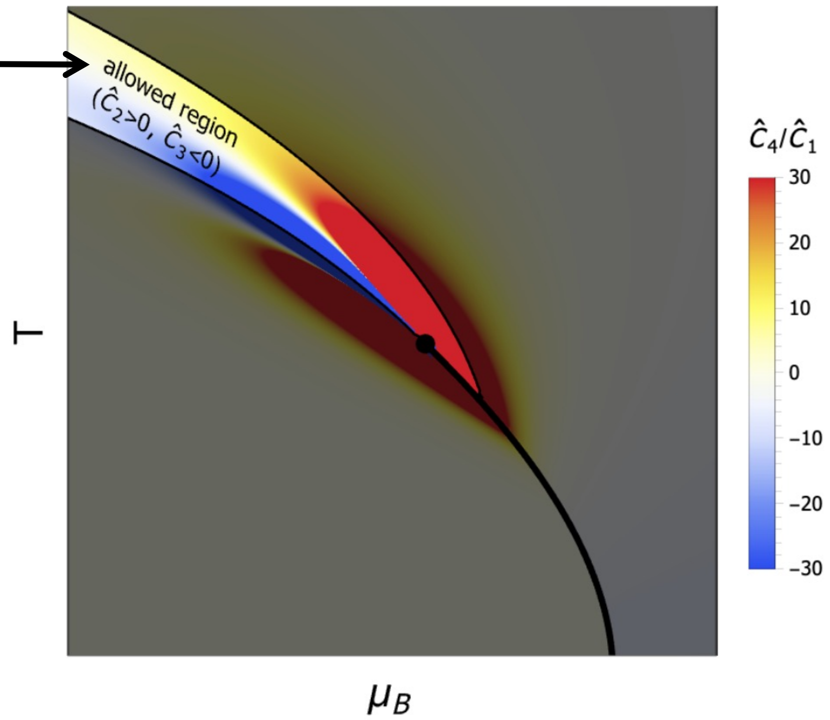
Equilibrium expectation

CP estimate: **H. Shah, Wed 09:40**



Exclusion plots

Exclude $\hat{C}_2<0$ & $\hat{C}_3>0$ regions on the phase diagram near CP



Analysis adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)

Freeze-out of fluctuations on the QGP side of the crossover?

Due to memory effect the sign of $\hat{C}_3$ may differ from equilibrium expectation

Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)

Scaled factorial cumulants, long-range correlations, and the antiproton puzzle

A. Bzdak, V. Koch, VV, arXiv:2503.16405

Scaled factorial cumulants

Bzdak et al. introduced reduced correlation functions – "couplings" [Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)]

$$\boxed{\hat{c}_k = \frac{\hat{C}_k}{\langle N \rangle^k}}$$

$$c_k = \frac{\int \rho_1(y_1)\cdots\rho_1(y_k)c_k(y_1,\ldots,y_k)dy_1\cdots dy_k}{\int \rho_1(y_1)\cdots\rho_1(y_k)dy_1\cdots dy_k}$$

integrated correlation function in rapidity

A. Bzdak, V. Koch, VV, arXiv:2503.16405

Scaled factorial cumulants

Bzdak et al. introduced reduced correlation functions – "couplings" [Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)]

$$\boxed{\hat{c}_k = \frac{\hat{C}_k}{\langle N \rangle^k}}$$

$$c_k = \frac{\int \rho_1(y_1)\cdots\rho_1(y_k)c_k(y_1,\ldots,y_k)dy_1\cdots dy_k}{\int \rho_1(y_1)\cdots\rho_1(y_k)dy_1\cdots dy_k}$$

integrated correlation function in rapidity

Long-range correlations lead to acceptance-independent couplings, for example

- Global (not local) baryon conservation
  [Bzdak, Koch, Skokov, EPJC 77, 288 (2017); Bzdak, Koch, PRC 96, 054905 (2017)]

$$c_2 = -\frac{1}{B}, \qquad c_3 = \frac{2}{B^2}, \qquad c_4 = -\frac{6}{B^3}$$

- $+$ volume fluctuations
  [Holzmann, Koch, Rustamov, Stroth, arXiv:2403.03598]

$$\hat{\hat{c}}_{i,j} = \hat{c}_{i,j} + \frac{\kappa_2[V]}{\langle V \rangle^2}, \quad \text{for} \quad i+j=2.$$

- $+$ (uniform) efficiency
  [Pruneau, Gavin, Voloshin, PRC 66, 044904 (2002)]

A. Bzdak, V. Koch, VV, arXiv:2503.16405

Scaled factorial cumulants

Bzdak et al. introduced reduced correlation functions – "couplings" [Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)]

$$\boxed{\hat{c}_k = \frac{\hat{C}_k}{\langle N \rangle^k}}$$

$$c_k = \frac{\int \rho_1(y_1)\cdots\rho_1(y_k)c_k(y_1,\ldots,y_k)dy_1\cdots dy_k}{\int \rho_1(y_1)\cdots\rho_1(y_k)dy_1\cdots dy_k}$$

integrated correlation function in rapidity

Long-range correlations lead to acceptance-independent couplings, for example

- Global (not local) baryon conservation
  [Bzdak, Koch, Skokov, EPJC 77, 288 (2017); Bzdak, Koch, PRC 96, 054905 (2017)]
- $+$ volume fluctuations
  [Holzmann, Koch, Rustamov, Stroth, arXiv:2403.03598]
- $+$ (uniform) efficiency
  [Pruneau, Gavin, Voloshin, PRC 66, 044904 (2002)]

$$c_2 = -\frac{1}{B}, \qquad c_3 = \frac{2}{B^2}, \qquad c_4 = -\frac{6}{B^3}$$

$$\hat{\hat{c}}_{i,j} = \hat{c}_{i,j} + \frac{\kappa_2[V]}{\langle V \rangle^2}, \quad \text{for} \quad i+j=2.$$

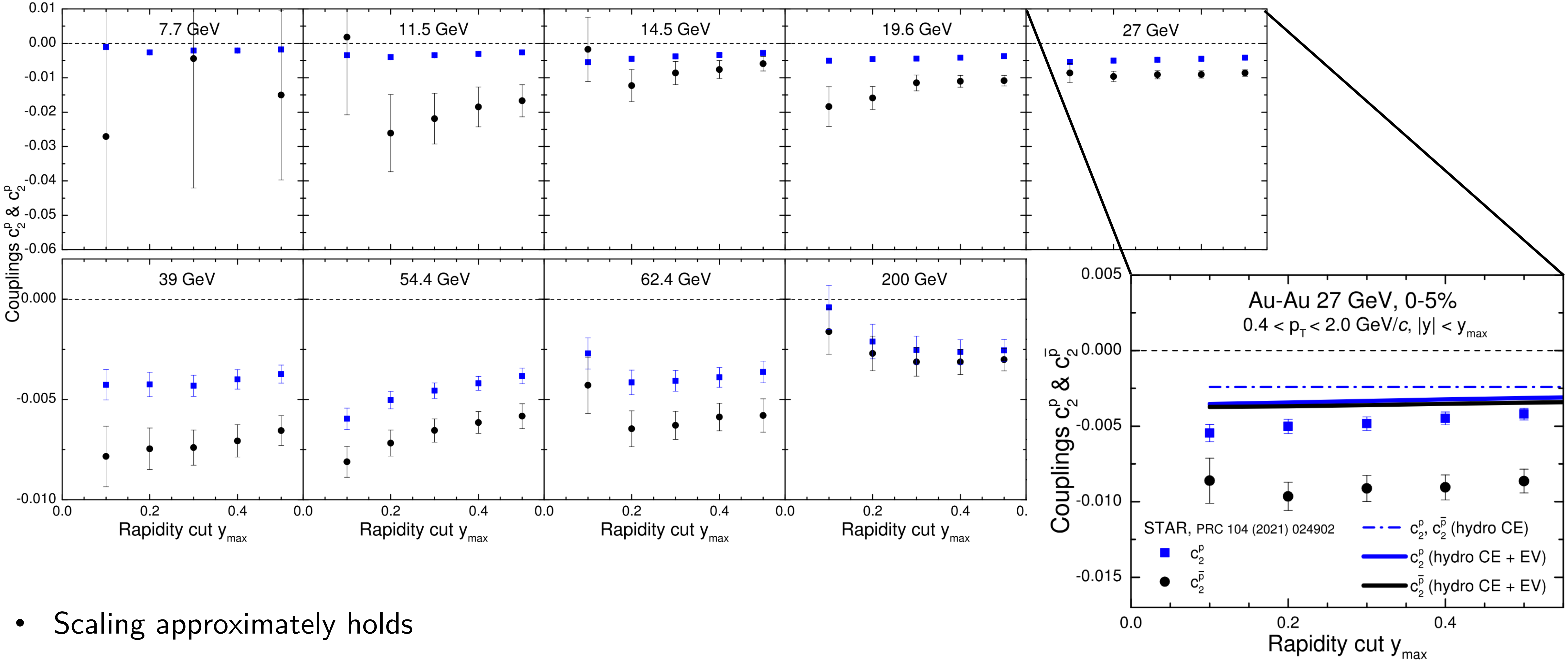
all lead to

$\boxed{\frac{\hat{C}_k}{\langle N \rangle^k} = const.}$ at a given $\sqrt{s_{NN}}$ and $\boxed{\frac{\hat{C}_2^p}{\langle N_p \rangle^2} \approx \frac{\hat{C}_2^{\bar{p}}}{\langle N_{\bar{p}} \rangle^2} = const.}$ at a given $\sqrt{s_{NN}}$

Can be tested *without* CBWC/volume fluctuations correction A. Bzdak, V. Koch, VV, arXiv:2503.16405

Scaled factorial cumulants from RHIC-BES-I

A. Bzdak, V. Koch, VV, arXiv:2503.16405



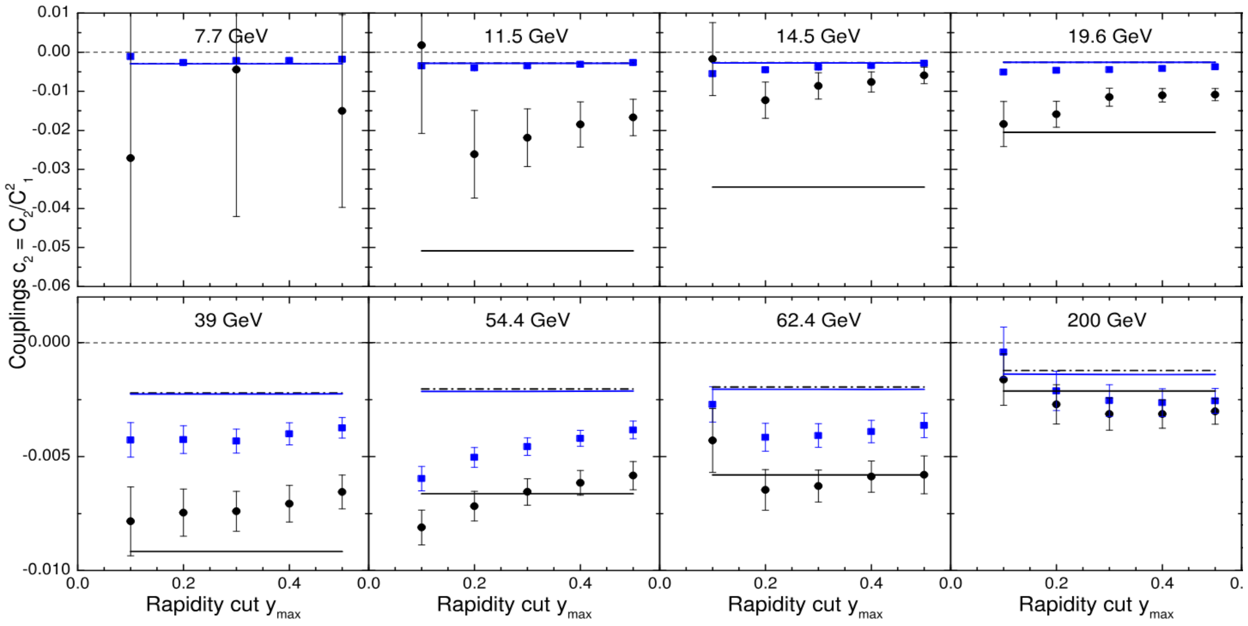
- Scaling approximately holds
- But significant difference between p and $\bar{p}$ in BES-I and hydro fails – **the antiproton puzzle**

no single thermalized fireball?

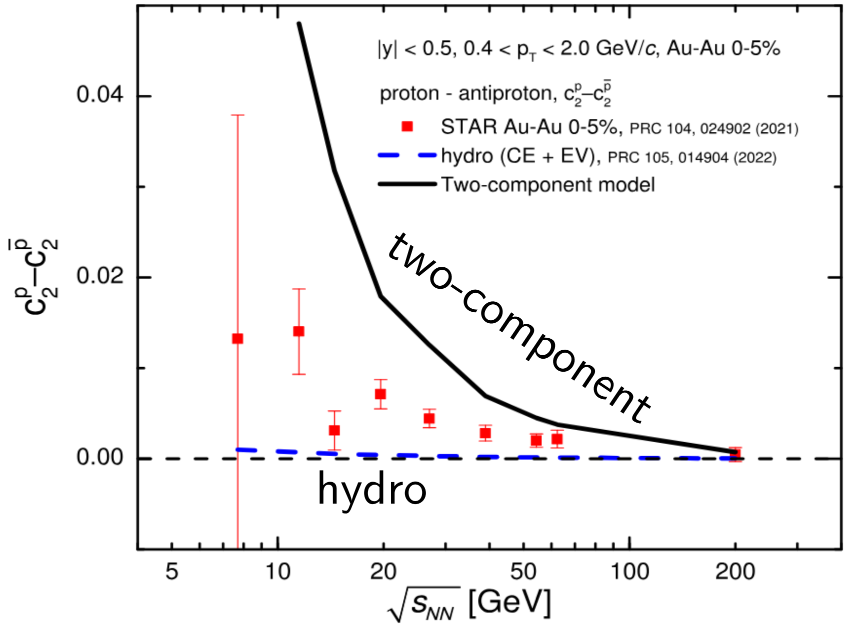
The antiproton puzzle and the two-component model

Two-component model: produced ($p\bar{p}$ pairs) and stopped protons comprise from two independent sources

The data lie in-between single and two-fireball models



Difference between p and $\bar{p}$



A. Bzdak, V. Koch, VV, arXiv:2503.16405

Opportunities for BES-II:

- Further tests of the splitting between p and $\bar{p}$ in 2nd order cumulants with extended y coverage
- Critical point signal expected to break the scaling

$$\frac{\hat{C}_n}{(\hat{C}_1)^n} = \text{const.}$$

[Ling. Stephanov, PRC 93, 034915 (2016)]

V. Kuznietsov, talk Tue 11:50

Opportunities at LHC

ALICE has mainly studied the ratio $\kappa_2[p-\bar{p}]/\langle p+\bar{p}\rangle$

[PLB 807, 135564 (2020), PLB 844, 137545 (2023)]

We advocate for measuring the acceptance dependence of:

- Scaled factorial cumulants

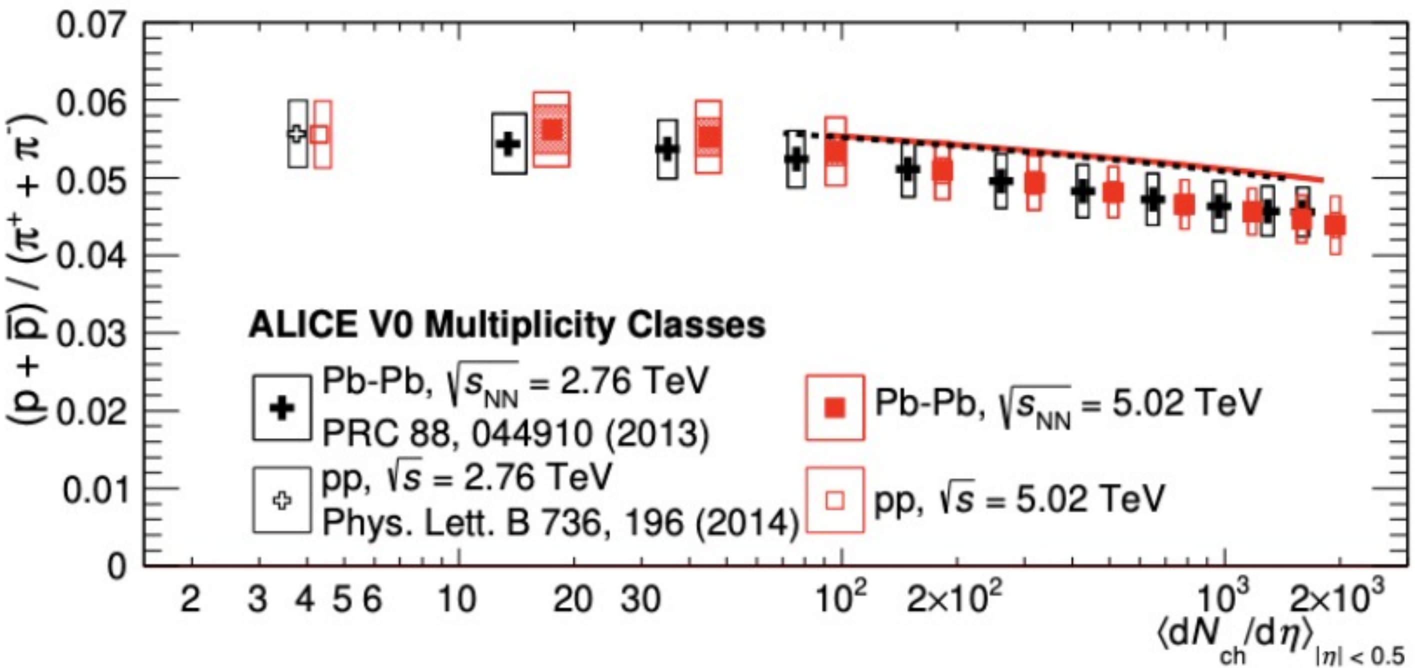
2nd order: $\hat{c}_2^{p} = \frac{\hat{C}_2^{p}}{\langle p\rangle^2}$ $\hat{c}_2^{\bar{p}} = \frac{\hat{C}_2^{\bar{p}}}{\langle \bar{p}\rangle^2}$ $\hat{c}_{11}^{p\bar{p}} = \frac{\hat{C}_{11}^{p\bar{p}}}{\langle p\rangle\langle \bar{p}\rangle}$ and more generally high-order $\hat{c}_{nm}^{p\bar{p}} = \frac{\hat{C}_{nm}^{p\bar{p}}}{\langle p\rangle^n\langle \bar{p}\rangle^m}$

- In addition, one can utilize the property $\mu_B \approx 0$ at LHC to construct

$$\frac{[\kappa_2[p-\bar{p}]-\langle p+\bar{p}\rangle]}{\langle p+\bar{p}\rangle^2} \approx \frac{1}{4}(\hat{c}_2^{p}+\hat{c}_2^{\bar{p}}-2\hat{c}_{11}^{p\bar{p}}) \quad \text{and} \quad \frac{[\kappa_2[p+\bar{p}]-\langle p+\bar{p}\rangle]}{\langle p+\bar{p}\rangle^2} \approx \frac{1}{4}\left(\hat{c}_2^{p}+\hat{c}_2^{\bar{p}}+2\hat{c}_{11}^{p\bar{p}}\right) = \hat{c}_2^{p+\bar{p}}$$

Unfortunately, acceptance dependence of $\kappa_2[p-\bar{p}]$, $\kappa_2[p+\bar{p}]$ and $\langle p+\bar{p}\rangle$ has not been published separately so far

Opportunities at LHC: Baryon annihilation



ALICE Collaboration, Phys. Rev. C 101 (2020) 044907

Evidence for suppression of p/pi ratio in central collisions (~20%, $>4\sigma$ level) measured by ALICE

Consistent with baryon annihilation but not a unique explanation

Can one obtain independent verification?

Opportunities at LHC: Baryon annihilation

At $\mu_B \approx 0$ (LHC) due to symmetry one has

$$\mathrm{cov}[p - \bar{p}, B + \bar{B}] = 0 \quad \Rightarrow \quad \frac{\hat{C}_2^{p+\bar{p}}}{\langle p + \bar{p} \rangle^2} = \hat{c}_2^{p+\bar{p}}$$ is unaffected by baryon conservation

But affected by local correlations (such as baryon annihilation).
Using density correlator one can derive:
[VV, PRC 110, L061902 (2024)]

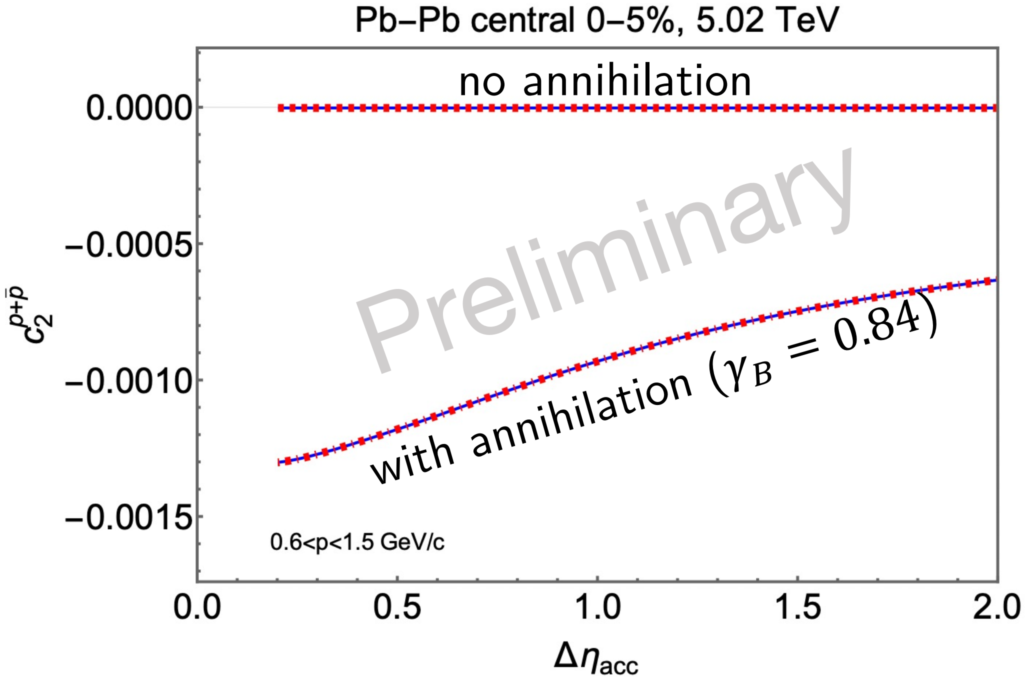
$$\hat{c}_2^{p+\bar{p}} \propto \underbrace{-(1-\gamma_B)}_{\text{correlation}} \overbrace{\langle p^2 \rangle}^{\text{pair acceptance}} + \underbrace{\frac{\kappa_2[V]}{V}}_{\text{vol.fluct.: shift by a constant factor}}$$

$\gamma_B = 1$: no correlation

$\gamma_B < 1$: correlations (e.g. baryon annihilation)

UrQMD gives $\gamma_B = 0.84$

[Savchuk et al., PLB 827, 136983 (2022)]

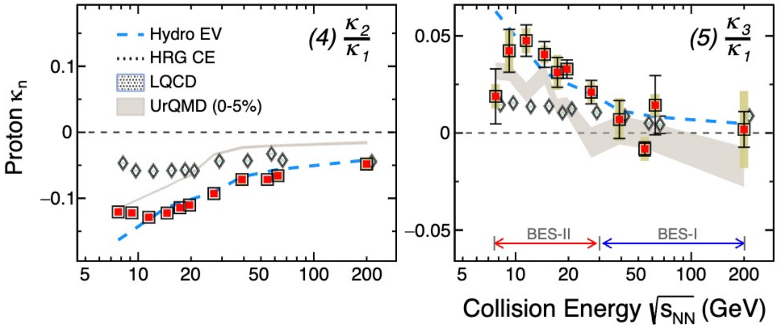


Clear signal and independent of baryon conservation

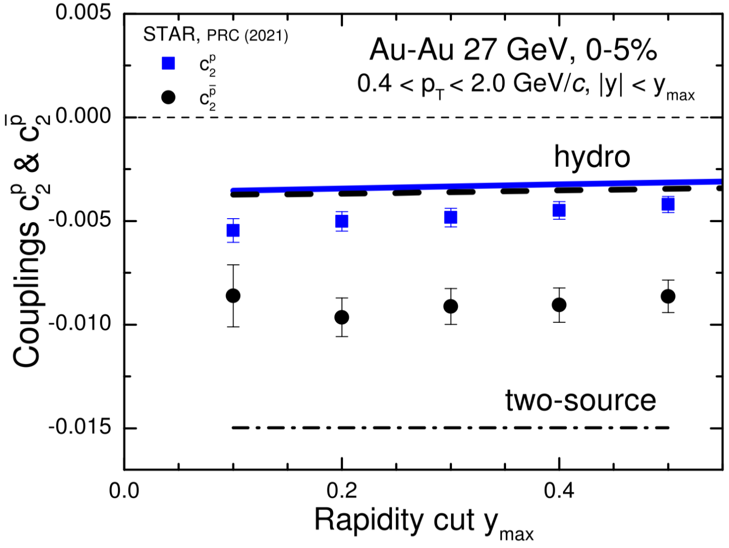
Summary

- **Proton cumulants at RHIC-BES-II**
  - Non-critical physics describe the proton data at $\sqrt{s_{NN}} \geq 20$ GeV
  - $\hat{C}_2 - \hat{C}_2^{baseline} > 0$ and $\hat{C}_3 - \hat{C}_3^{baseline} < 0$ at $\sqrt{s_{NN}} < 10$ GeV

$$\boxed{\hat{c}_k = \frac{\hat{C}_k}{\langle N \rangle^k}}$$



- **Acceptance dependence of scaled factorial cumulants**
  - Distinguishes short- vs long-range correlation, no need for CBWC
  - **Antiproton puzzle:** $|\hat{c}_2^{\bar{p}}| > |\hat{c}_2^{p}|$ not explained by standard hydro
    - Two-source model (stopped + produced) overshoots the difference
    - Possible evidence for incomplete equilibration of the fireball
  - May serve as a clear probe of baryon annihilation



Outlook:

- Improved description of non-critical baselines (lower energies), quantitative predictions of critical fluctuations
- Acceptance dependence of factorial cumulants, understanding antiprotons and baryon annihilation

Thanks for your attention!

Backup slides

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

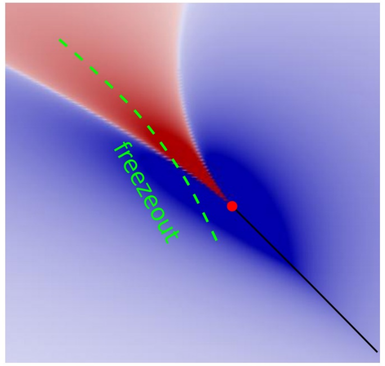
Grand partition function

$$\ln Z^{\mathrm{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\mathrm{ce}}(T,V,N)\right]$$

$$\boxed{\kappa_n \propto \frac{\partial^n(\ln Z^{\mathrm{gce}})}{\partial\mu^n}}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- **(QCD) critical point:** large correlation length and fluctuations



M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

$$\xi \to \infty$$

Looking for enhanced fluctuations
and non-monotonicities

Other uses of cumulants:

- **QCD degrees of freedom**
  Jeon, Koch, PRL 85, 2076 (2000)
  Asakawa, Heinz, Muller, PRL 85, 2072 (2000)
- Extracting the speed of sound
  A. Sorensen et al., PRL 127, 042303 (2021)
- Conservation volume V_C
  VV, Donigus, Stoecker, PRC 100, 054906 (2019)

Scaled factorial cumulants and baryon conservation

Ideal gas in the canonical ensemble: $$P(N_B, N_{\bar{B}}) \propto \frac{\langle N_B \rangle}{N_B!} \frac{\langle N_{\bar{B}} \rangle}{N_{\bar{B}}!} \delta_{B, N_B - N_{\bar{B}}},$$

Cumulant generating function: $$G_{N_B,N_{\bar{B}}}(t_B, t_{\bar{B}}) = \ln \frac{\sum_{N_B,N_{\bar{B}}} P(N_B, N_{\bar{B}}) e^{t_B N_B + t_{\bar{B}} N_{\bar{B}}}}{\sum_{N_B,N_{\bar{B}}} P(N_B, N_{\bar{B}})} = \ln \left[e^{B \frac{t_B - t_{\bar{B}}}{2}} \frac{I_B(2z e^{\frac{t_B + t_{\bar{B}}}{2}})}{I_B(2z)} \right]$$

Large volume: $$G_{N_B,N_{\bar{B}}}(t_B, t_{\bar{B}}) = N f(t_B, t_{\bar{B}})$$

$$f(t_B, t_{\bar{B}}) = \frac{t_B - t_{\bar{B}}}{2} r_B - 1 + \sqrt{r_B^2 + (1 - r_B^2) e^{t_B + t_{\bar{B}}}} + |r_B| \ln \left[\frac{(1 + |r_B|) e^{\frac{t_B + t_{\bar{B}}}{2}}}{|r_B| + \sqrt{r_B^2 + (1 - r_B^2) e^{t_B + t_{\bar{B}}}}} \right]$$

$$N = \langle N_B \rangle + \langle N_{\bar{B}} \rangle$$

Factorial cumulants: $$\hat{C}_{n,m} = \frac{\partial^{n+m}}{\partial z_B^n \partial z_{\bar{B}}^m} \hat{H}_{N_B,N_{\bar{B}}}(z_B, z_{\bar{B}}) \Big|_{z_B = z_{\bar{B}} = 1}$$

$$\hat{H}_{N_B,N_{\bar{B}}}(z_B, z_{\bar{B}}) = G_{N_B,N_{\bar{B}}}(\ln z_B, \ln z_{\bar{B}})$$

$$\hat{c}_{2,0}^{B,\bar{B}} = -\hat{c}_{1,1}^{B,\bar{B}} = \hat{c}_{0,2}^{B,\bar{B}} = -\frac{1}{N}$$

$$\hat{c}_{3,0}^{B,\bar{B}} = \frac{3 - r_B}{N^2}, \quad \hat{c}_{0,3}^{B,\bar{B}} = \frac{3 + r_B}{N^2},$$
$$\hat{c}_{2,1}^{B,\bar{B}} = -\frac{1 - r_B}{N^2}, \quad \hat{c}_{1,2}^{B,\bar{B}} = -\frac{1 + r_B}{N^2}.$$

$$r_B = B/N$$

Scaled factorial cumulants and volume fluctuations

Cumulant generating function for fluctuating volume:

$$G_{\tilde{N}_B,\tilde{N}_{\bar{B}}}(t_A,t_B) = \ln\left\langle e^{t_B\tilde{N}_B + t_{\bar{B}}\tilde{N}_{\bar{B}}}\right\rangle$$
$$= \ln\int dV\,\rho_V(V)\left\langle e^{t_B N_B + t_{\bar{B}} N_{\bar{B}}}\right\rangle_V .$$

Factorial cumulants:

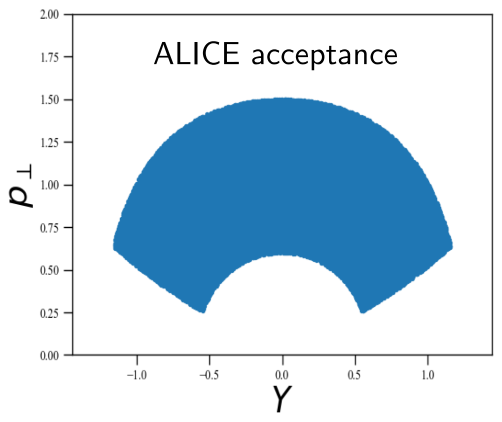
$$\hat{\hat{C}}_{n,m} = \underbrace{\langle N_B\rangle^n \langle N_{\bar{B}}\rangle^m}_{\textbf{scaling}} \underbrace{\sum_{\pi\in\Pi_{n+m}} \frac{\kappa_{|\pi|}[V]}{\langle V\rangle^{|\pi|}}}_{\textbf{volume cumulants}} \underbrace{\prod_{b\in\pi} \hat{c}_{b_n,|b|-b_n}}_{\textbf{fixed V couplings}}$$

$$\hat{\hat{c}}_{i,j} = \hat{c}_{i,j} + \frac{\kappa_2[V]}{\langle V\rangle^2}, \quad \text{for} \quad i+j=2.$$

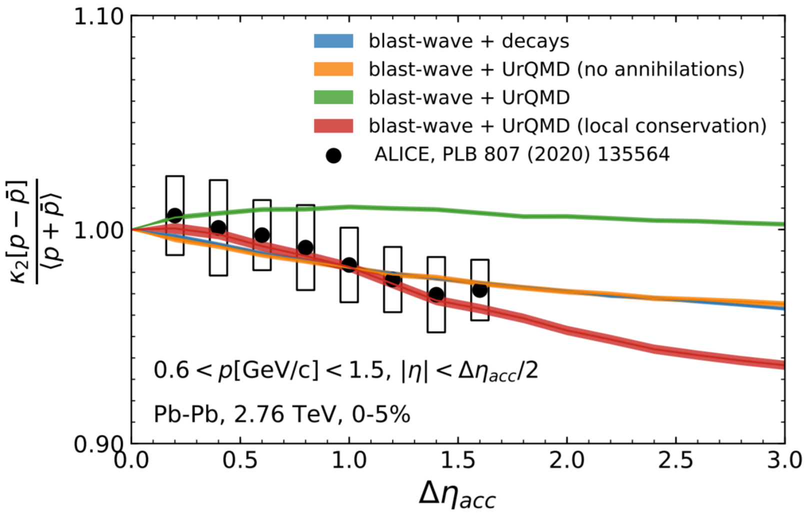
$$\hat{\hat{c}}_{2,0} - \hat{\hat{c}}_{0,2} = \hat{c}_{2,0} - \hat{c}_{0,2},$$
$$\underbrace{\hat{\hat{c}}_{2,0} + \hat{\hat{c}}_{0,2} - 2\hat{\hat{c}}_{1,1}}_{\text{with volume fluc}} = \underbrace{\hat{c}_{2,0} + \hat{c}_{0,2} - 2\hat{c}_{1,1}}_{\text{without volume fluc}} .$$

Net-particle fluctuations at the LHC (blast-wave model)

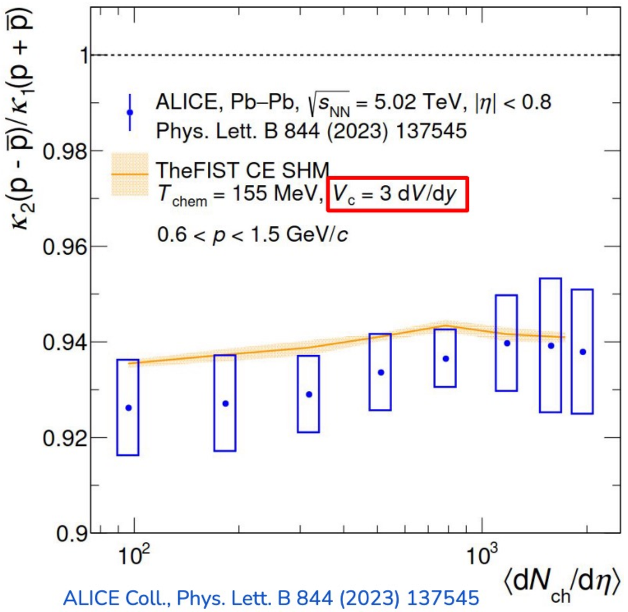
- Net protons described within errors and consistent with either
  - global baryon conservation only, without $B\bar{B}$ annihilations
    ALICE Collaboration, Phys. Lett. B 807, 135564 (2020)
  - or local baryon conservation with $B\bar{B}$ annihilations
    O. Savchuk et al., Phys. Lett. B 827, 136983 (2022)



$0.6 < p < 1.5$ GeV/c, $\Delta\eta_{acc} = 1.6$



multiplicity dependence



ALICE Coll., Phys. Lett. B 844 (2023) 137545

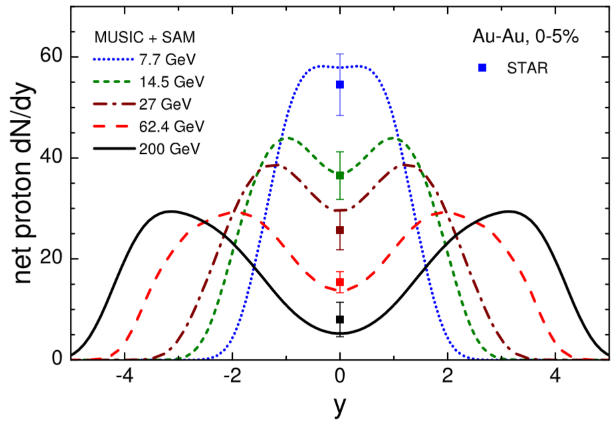
- Local conservation $V_C \sim 3dV/dy$ preferred by hadron yields* and several other cumulant measurements** (net-Xi, net-Lambda, pQK, …)

*VV, Donigus, Stoecker, PRC 100, 054906 (2019); **Talks by M. Ciacco & S. Saha (SQM2024)

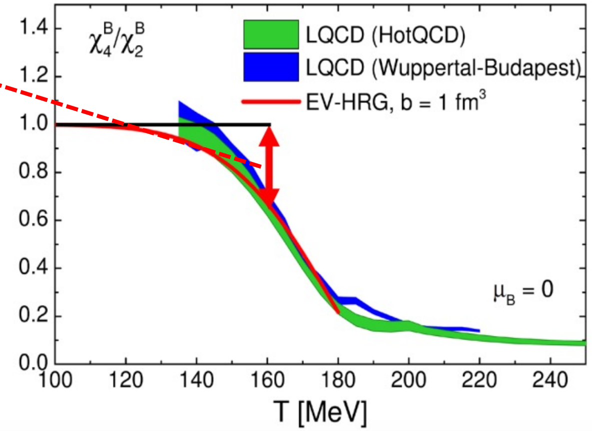
Hydro EV: Non-critical hydro baseline at RHIC-BES

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
  - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
  - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]



- Non-critical contributions computed at particlization ($\epsilon_{sw} = 0.26$ GeV/fm^3)
  - QCD-like baryon number distribution (χ_n^B) via **excluded volume** $b = 1$ fm^3 [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
  - **Exact global baryon conservation*** (and other charges)
    - Subensemble acceptance method 2.0 (analytic) [VV, Phys. Rev. C 105, 014903 (2022)]
    - or FIST sampler (Monte Carlo) [VV, Phys. Rev. C 106, 064906 (2022)] https://github.com/vlvovch/fist-sampler



- Included: baryon conservation, repulsion, kinematical cuts
- Absent: critical point, local conservation, initial-state/volume fluctuations, hadronic phase

*If baryon conservation is the only effect (no other correlations), non-critical baseline can be computed without hydro
Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, NPA 1008, 122141 (2021)

Excluded volume effect

Incorporate repulsive baryon (nucleon) hard core via excluded volume

VV, M.I. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

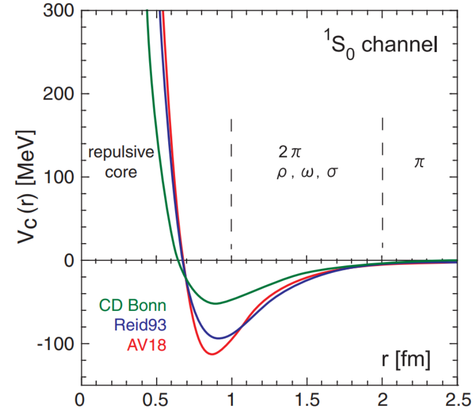
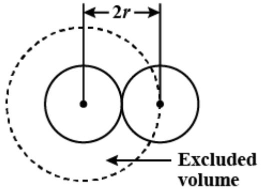


Figure from Ishii et al., PRL '07

Amounts to a van der Waals correction for baryons in the HRG model

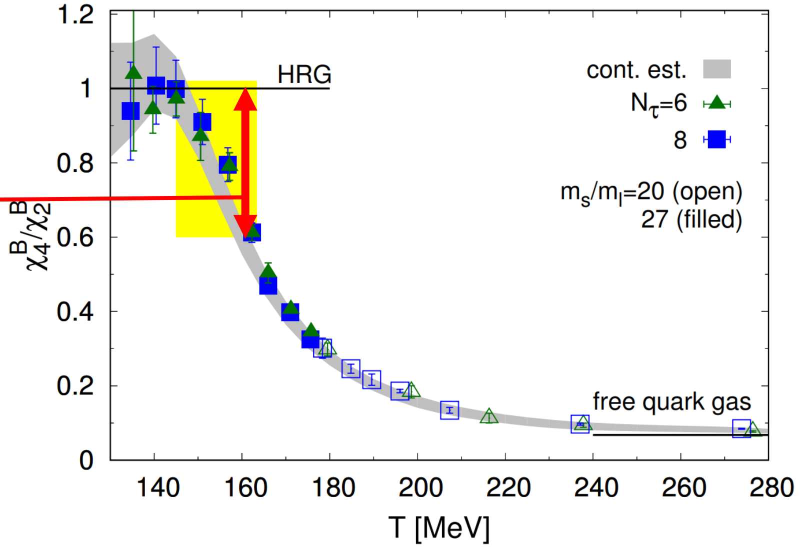
$$V \to V - bN$$



$$p^{\rm ev}_{B(\bar{B})} = p^{\rm id}_{B(\bar{B})}\, e^{-b p^{\rm ev}_{B(\bar{B})}/T}$$

- Net baryon kurtosis suppressed as in lattice QCD*

$$\frac{\chi_4^B}{\chi_2^B} \simeq 1 - 12 b \phi_B(T) + O(b^2)$$



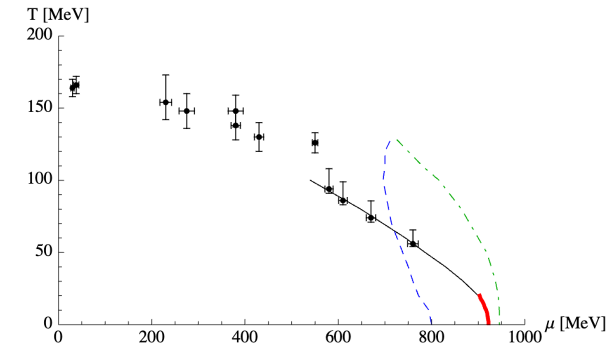
- Reproduces virial coefficients of baryon interaction from lattice QCD

VV, A. Pasztor, S. Katz, Z. Fodor, H. Stoecker, Phys. Lett. B 755, 71 (2017)

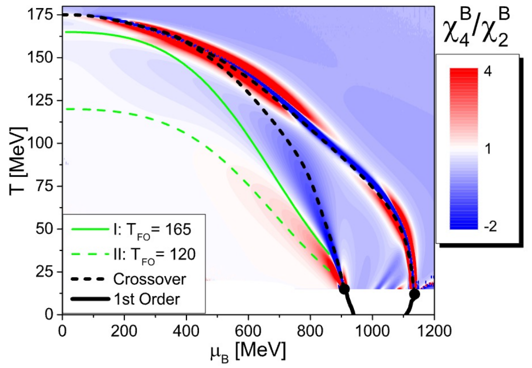
Excluded volume from lattice QCD: $\boxed{b \approx 1 \text{ fm}^3}$

*J.M. Karthein, V. Koch, C. Ratti, VV, PRD 104, 094009 (2021)

Interplay with nuclear liquid-gas transition

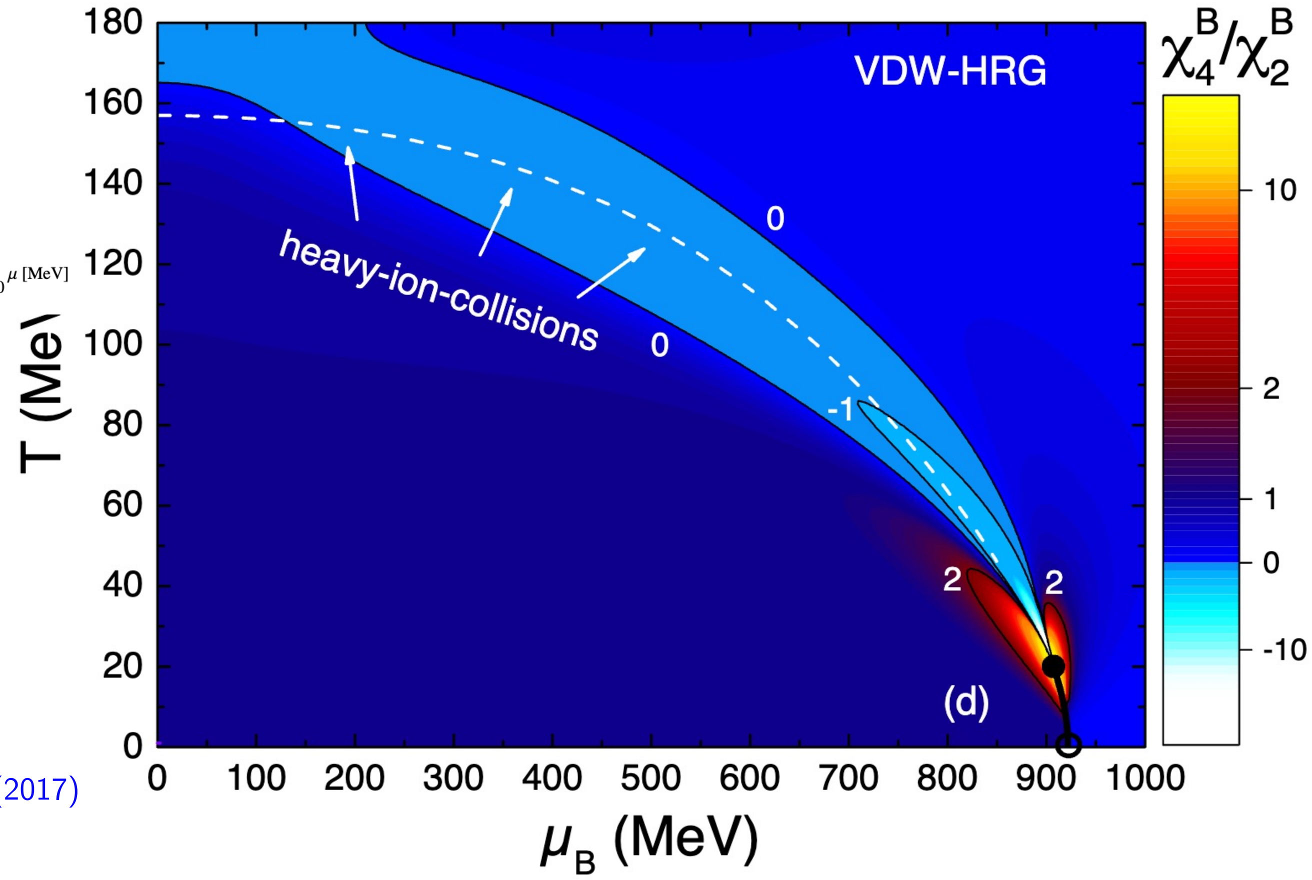


Floerchinger, Wetterich, NPA (2012)

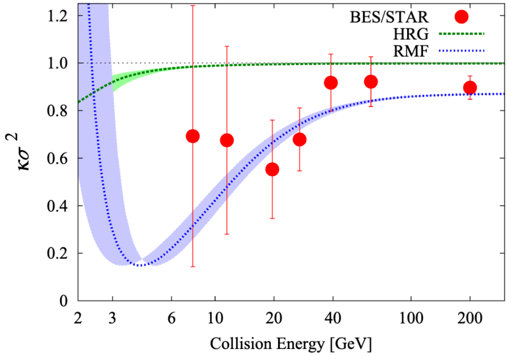


Mukherjee, Steinheimer, Schramm, PRC (2017)

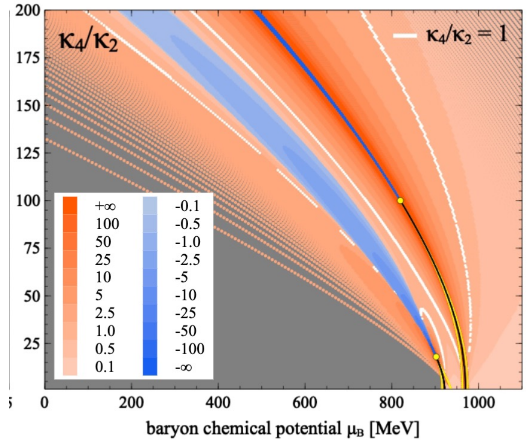
HRG with attractive and repulsive interactions among baryons



VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)



Fukushima, PRC (2014)



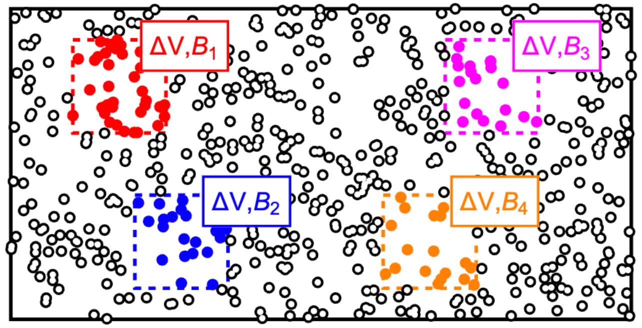
Sorensen, Koch, PRC (2020)

Increasingly relevant at lower energies probed through RHIC-FXT

Exact charge conservation

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020); VV, arXiv:2409.01397

Utilizing the canonical partition function in thermodynamic limit
compute **n-point density correlators**



$$\mathcal{C}_1(\mathbf{r}_1) = \rho(\mathbf{r}_1)$$

$$\mathcal{C}_2(\mathbf{r}_1,\mathbf{r}_2) = \chi_2\delta(\mathbf{r}_1-\mathbf{r}_2) - \frac{\chi_2}{V}$$

local correlation — balancing contribution (e.g. baryon conservation)

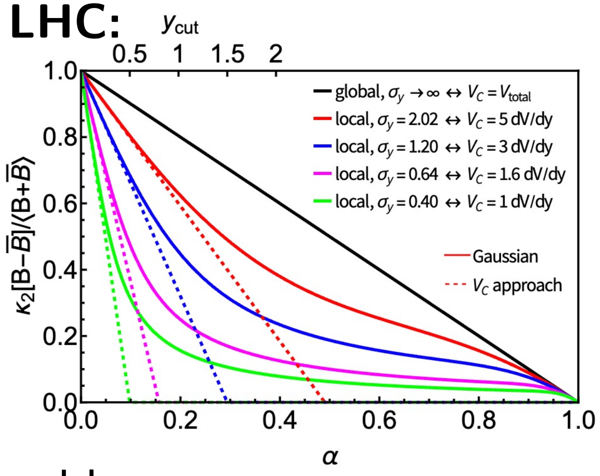
$$\mathcal{C}_3(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = \chi_3\delta_{1,2,3} - \frac{\chi_3}{V}[\delta_{1,2}+\delta_{1,3}+\delta_{2,3}] + 2\frac{\chi_3}{V^2} \qquad \delta_{1,\dots,n} = \prod_{i=2}^n \delta(\mathbf{r}_1-\mathbf{r}_i)$$

local correlation — balancing contributions

$$\mathcal{C}_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = \chi_4\delta_{1,2,3,4} - \frac{\chi_4}{V}[\delta_{1,2,3}+\delta_{1,2,4}+\delta_{1,3,4}+\delta_{2,3,4}] - \frac{(\chi_3)^2}{\chi_2 V}[\delta_{1,2}\delta_{3,4}+\delta_{1,3}\delta_{2,4}+\delta_{1,4}\delta_{2,3}]$$
$$+\frac{1}{V^2}\left[\chi_4+\frac{(\chi_3)^2}{\chi_2}\right][\delta_{1,2}+\delta_{1,3}+\delta_{1,4}+\delta_{2,3}+\delta_{2,4}+\delta_{3,4}] - \frac{3}{V^3}\left[\chi_4+\frac{(\chi_3)^2}{\chi_2}\right].$$

local correlation — balancing contributions

LHC:



Integrating the correlator yields cumulant inside a subsystem of the canonical ensemble

$$\kappa_n[B_{V_s}] = \int_{\mathbf{r}_1\in V_s} d\mathbf{r}_1 \dots \int_{\mathbf{r}_n\in V_s} d\mathbf{r}_n\, \mathcal{C}_n(\{\mathbf{r}_i\})$$

Momentum space: Fold with Maxwell-Boltzmann in LR frame and integrate out the coordinates

Local baryon conservation from density correlator

VV, arXiv:2409.01397

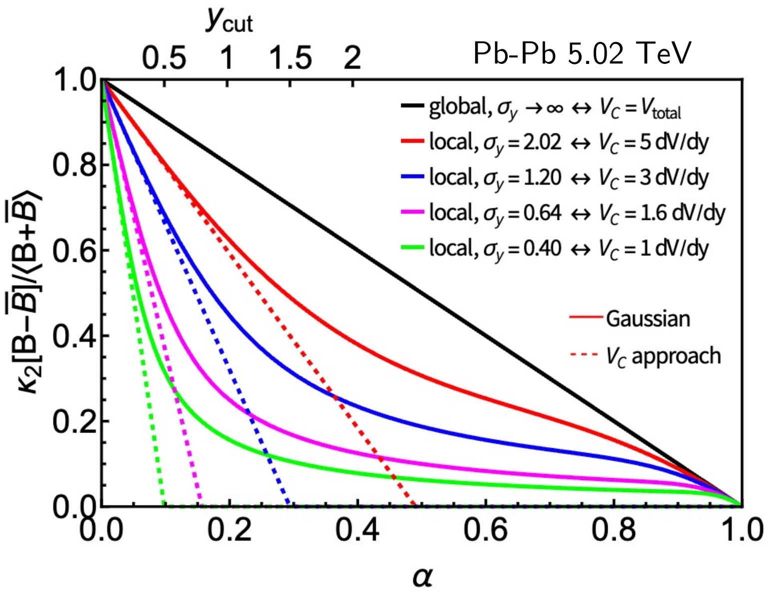
UNIVERSITY OF HOUSTON

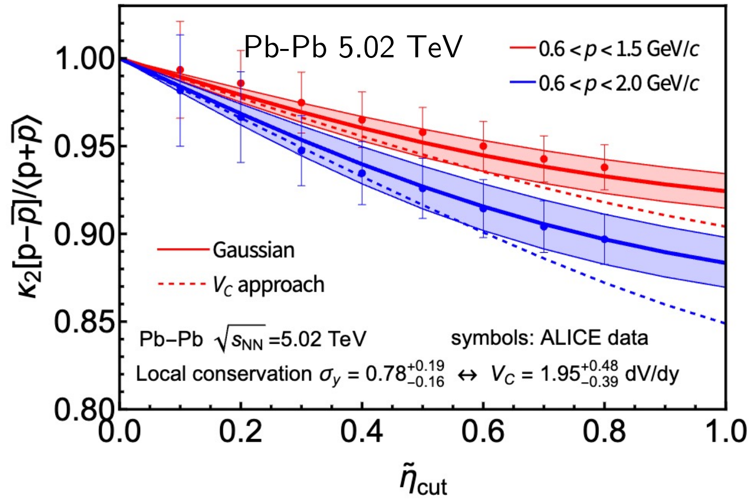
Introduce Gaussian (space-time) rapidity correlation into baryon-conservation balancing term

global conservation → **+ local conservation**

$$C_2^B(\eta_1,\eta_2) = \langle n_B + n_{\bar{B}} \rangle \left[\delta(\eta_1 - \eta_2) - \frac{1}{2\eta_{\max}} \right]$$

$$C_2^B(\eta_1,\eta_2) = \langle n_B + n_{\bar{B}} \rangle \left[\delta(\eta_1 - \eta_2) - \frac{\tilde{A}\, e^{-\frac{(\eta_1-\eta_2)^2}{2\sigma_\eta^2}}}{2\eta_{\max}} \right]$$





- Linear regime at small a establishes connection to the V_C approach ($V_C = kdV/dy$, $k \approx \sqrt{2\pi}\sigma_\eta$)
- V_C approach has limitations, likely provides upper bound on the conservation volume
- Evidence for local (not just global) baryon conservation for 5 TeV data (in contrast to 2.76 TeV data)

Improving local charge conservation

VV, arXiv:2409.01397

Utilize 2-point density correlation function in the canonical ensemble

Introduce Gaussian (space-time) rapidity correlation into baryon-conservation balancing term

global conservation

$$C_2^B(\eta_1,\eta_2) = \langle n_B + n_{\bar{B}} \rangle \left[\delta(\eta_1 - \eta_2) - \frac{1}{2\eta_{\max}} \right]$$

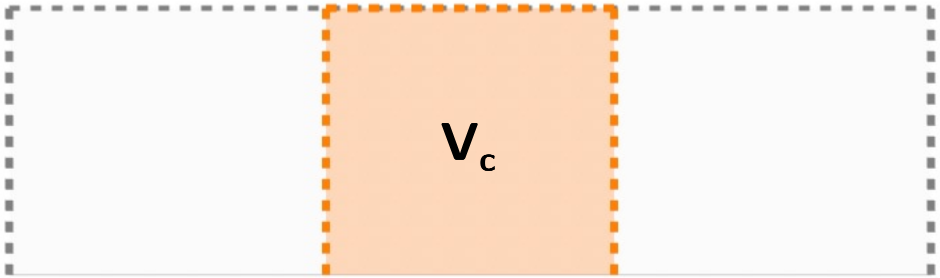
local correlation — balancing contribution (e.g. baryon conservation)

+ local conservation

$$C_2^B(\eta_1,\eta_2) = \langle n_B + n_{\bar{B}} \rangle \left[\delta(\eta_1 - \eta_2) - \frac{\tilde{A}\, e^{-\frac{(\eta_1-\eta_2)^2}{2\sigma_\eta^2}}}{2\eta_{\max}} \right]$$

local correlation — local balancing contribution

truncated fireball



vs

Gaussian correlation

With Gaussian correlation hadrons at forward/backward rapidities also contribute to the system

Calculating cumulants from MUSIC hydro

UNIVERSITY OF HOUSTON

Cooper-Frye formula:

$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x)\, p^\mu\, f_j[u^\mu(x)p_\mu; T(x), \mu_j(x)]$$

Calculation of the cumulants incorporates **balancing contributions from baryon conservation***

$$C_1^B(x_1) = \chi_1^B(x_1),$$

$$C_2^B(x_1, x_2) = \underbrace{\chi_2^B(x_1)\,\delta(x_1 - x_2)}_{\text{local correlation}} - \underbrace{\frac{\chi_2^B(x_1)\chi_2^B(x_2)}{\int_{\sigma(x)} d\sigma_\mu(x) u^\mu(x)\,\chi_2^B(x)}}_{\text{balancing contribution (baryon conservation)}},$$

$$\dots$$

global baryon conservation:

$$\int d\sigma_\mu(x_i) u^\mu(x_i) C_n^B(x_1, \dots, x_n) = 0 \quad \text{for} \quad n > 1$$

Generalized Cooper-Frye:

$$\kappa_n^B = \prod_{i=1}^{n} \int_{x_i \in \sigma(x)} d\sigma_\mu(x_i) \int_{|y_i|<0.5,\, 0.4<p_T<2} \frac{d^3p_i}{\omega_{p_i}}\, p_i^\mu \exp\left[-\frac{p_i^\mu u_\mu(x_i)}{T(x_i)}\right] C_n^B(x_1, \dots, x_n)$$

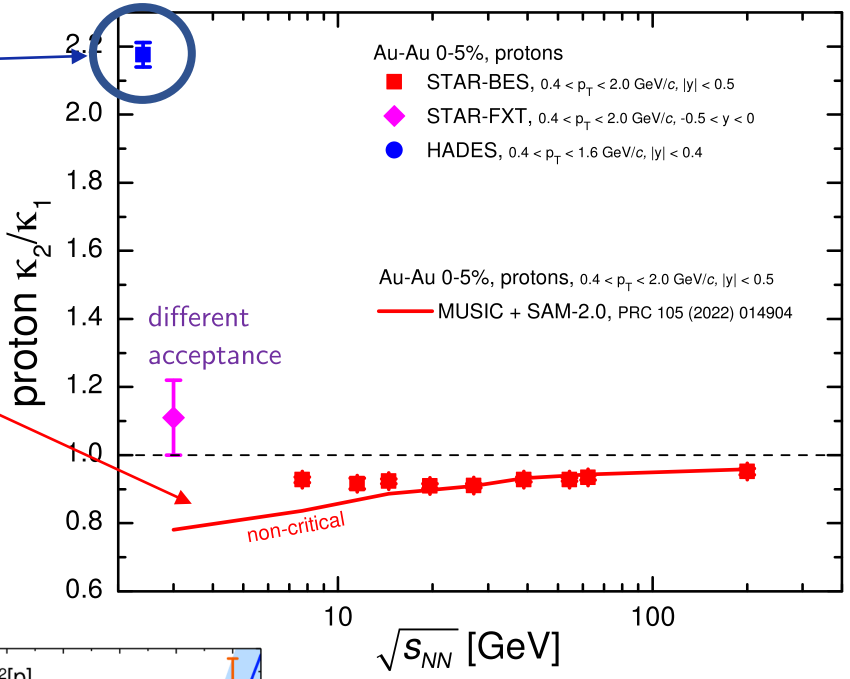
*based on SAM-2.0 method, VV, Phys. Rev. C 105, 014903 (2022)

Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV

- Intriguing hints from HADES@2.4 GeV and STAR-FXT@3GeV: huge excess of two-proton correlations!

  [HADES Collaboration, Phys. Rev. C 102, 024914 (2020)]

  [STAR Collaboration, Phys. Rev. Lett. 128, 202303 (2022)]
- No change of trend in the non-critical reference
- Additional mechanisms:
  - Nuclear liquid-gas transition (the other QCD critical point)
  - Light nuclei formation/fragmentation
  - Stronger initial state, volume, and baryon stopping fluctuations
- Difference in acceptance (-0.5<y<0 vs |y|<0.5)
- Improved modeling of lower energies required



VV, Phys. Rev. C 106, 064906 (2022)

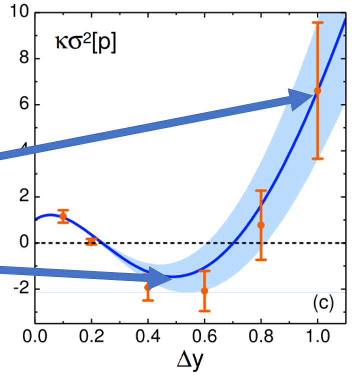
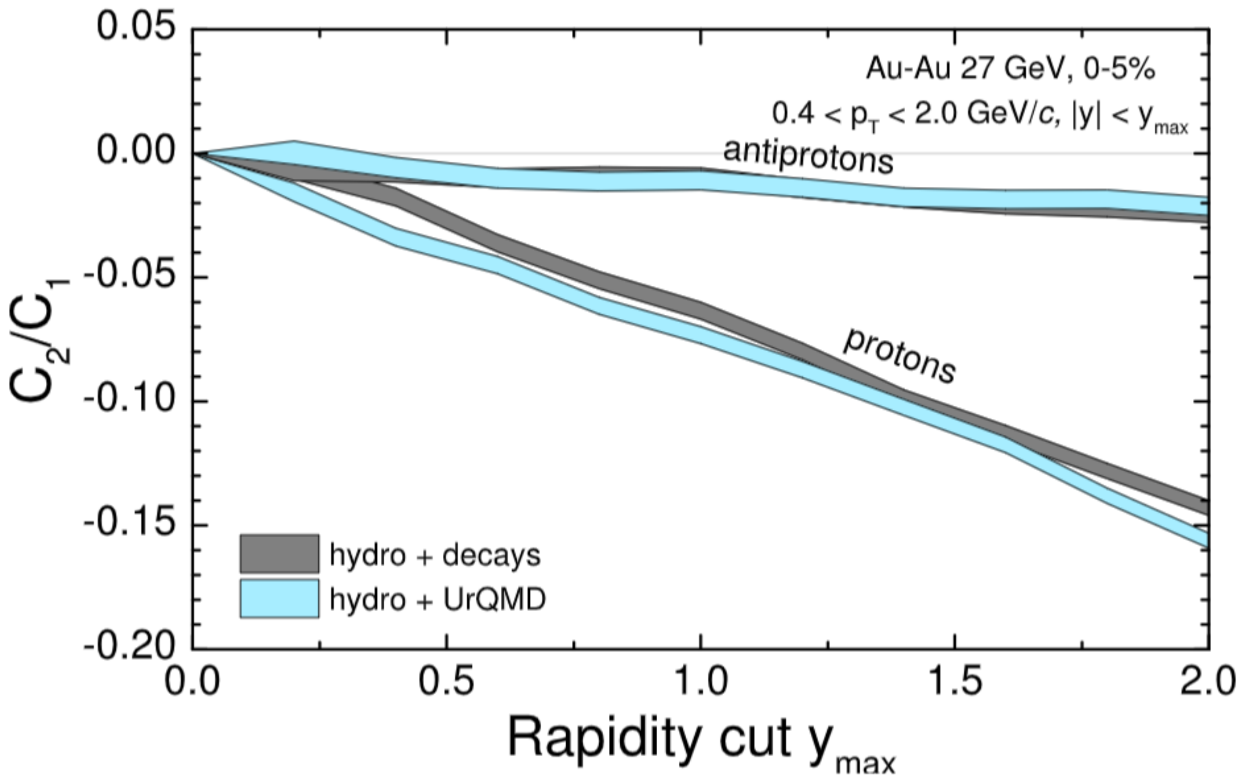


Figure from O. Savchuk et al., PLB 835, 137540 (2022)

We may want to understand κ_2 first

Effect of the hadronic phase

Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD



Dependence on the switching energy density