### Fluctuation Probes of QCD Matter in Heavy-Ion Collisions

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Thanks to:

A. Bzdak, V. Koch, V.A. Kuznietsov, R. Poberezhniuk, J. Parra, C. Ratti





# **QCD** under extreme conditions





- Dilute hadron gas at low T &  $\mu_{
  m B}$  due to confinement, quark-gluon plasma high T &  $\mu_{
  m B}$
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured
- Chiral crossover at  $\mu_B = 0$  which may turn into a *first-order phase transition* at finite  $\mu_B$

Key question: Is there a QCD critical point and how to find it?



- H. Shah, M. Hippert, J. Noronha, C. Ratti, VV, "Locating the QCD critical point from first principles through contours of constant entropy density", <u>arXiv:2410.16206</u>
- S. Borsanyi, Z. Fodor, J. Guenther, P. Parotto, A. Pasztor, C. Ratti, VV, C.-H. Wong, "Lattice QCD constraints on the critical point from an improved precision equation of state", <u>arXiv:2502.10267</u>

### Critical point and crossings of entropy density

- Entropy density s becomes multi-valued function of T and  $\mu_B$  for a first-order phase transition
- It develops a distinctive S-shape as a function of T at  $\mu_B = const$





### **Critical Point:**

$$\left(\frac{\partial T}{\partial s}\right)_{\mu_B} = 0, \qquad \left(\frac{\partial^2 T}{\partial s^2}\right)_{\mu_B} = 0.$$

H. Shah, M. Hippert, J. Noronha, C. Ratti, VV, arXiv:2410.16026

- Lattice QCD simulations at imaginary chemical potentials indicate that entropy contours are almost linear in  $\mu_B^2$ 

Idea: Follow contours of constant entropy density and look for crossings

$$T_s(\mu_B; T_0) \approx T_0 + \sum_{n=1}^N \alpha_{2n}(T_0) \frac{\mu_B^{2n}}{(2n)!} + \mathcal{O}\left(\mu_B^{2(N+1)}\right) \qquad \alpha_{2n}(T_0) = \left(\frac{\partial^{2n}T}{\partial\mu_B^{2n}}\right)_s \Big|_{T=T_0, \mu_B=0}$$

# Entropy density at finite $\mu_B$ at $O(\mu_B^2)$





• Excellent agreement at low  $\mu_B/T$  with available lattice QCD constraints

[Borsanyi et al., PRL 126, 232001 (2021)] First-order phase transition emerges at  $\mu_B > 600$  MeV

### Locating the critical point





Critical point location at  $O(\mu_B^2)$ :  $T_c = 114.3 \pm 6.9$  MeV,  $\mu_B = 602.1 \pm 62.1$  MeV

## Locating the critical point





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#### Similar to estimates from recent literature:

YLE-1: D.A. Clarke et al. (Bielefeld-Parma), arXiv:2405.10196
YLE-2: G. Basar, PRC 110, 015203 (2024)
BHE: M. Hippert et al., arXiv:2309.00579
fRG: W-J. Fu et al., PRD 101, 054032 (2020)
DSE/fRG: Gao, Pawlowski., PLB 820, 136584 (2021)
DSE: P.J. Gunkel et al., PRD 104, 052022 (2021)
FSS: A. Sorensen et al., arXiv:2405.10278
FSS-2: R. Lacey, arXiv:2411.09139

**Recent development:** Extrapolations of s = const contours from imaginary  $\mu_B$  with strangeness neutrality

 $\hookrightarrow$  CP excluded at  $\mu_B < 450$  MeV at a (one-sided)  $2\sigma$  level

S. Borsanyi, Z. Fodor, J. Guenther, P. Parotto, A. Pasztor, C. Ratti, VV, C.-H. Wong, arXiv:2502.10267

# Critical point and heavy-ion collisions



### Control parameters

- Collision energy  $\sqrt{s_{NN}} = 2.4 5020 \text{ GeV}$ 
  - Scan the QCD phase diagram
- Size of the collision region
  - Expect stronger signal in larger systems

### Measurements

 Final hadron abundances and momentum distributions event-by-event

### Chemical freeze-out curve and CP

- Sets the lower bound on the temperature of the CP [Lysenko, Poberezhnyuk, Gorenstein, VV, arXiv:2408.06473]
- **Caveats:** strangeness neutrality ( $\mu_S \neq 0$ ), uncertainty in the freeze-out curve
- CP may be close to freeze-out at  $\sqrt{s_{NN}}\sim 3.5-5~{\rm GeV}$



# Critical point, cumulants, and heavyion collisions





Cumulants measure chemical potential derivatives of the (QCD) equation of state

• (QCD) critical point: large correlation length and fluctuations



M. Stephanov, PRL '09, '11 Energy scans at RHIC (STAR) and CERN-SPS (NA61/SHINE)

$$\kappa_2 \sim \xi^2$$
,  $\kappa_3 \sim \xi^{4.5}$ ,  $\kappa_4 \sim \xi^7$ 

 $\xi o \infty$ 

Looking for enhanced fluctuations and non-monotonicities

Other uses of cumulants:

- QCD degrees of freedom Jeon, Koch, PRL 85, 2076 (2000) Asakawa, Heinz, Muller, PRL 85, 2072 (2000)
- Extracting the speed of sound A. Sorensen et al., PRL 127, 042303 (2021)
- Conservation volume V<sub>C</sub> VV, Donigus, Stoecker, PRC 100, 054906 (2019)

### **Example: (Nuclear) Liquid-gas transition**



VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

#### **Critical opalescence**



 $\langle N^2 \rangle - \langle N \rangle^2 \sim \langle N \rangle \sim 10^{23}$ in equilibrium



### **Example: Critical fluctuations in a microscopic simulation**

V. Kuznietsov et al., Phys. Rev. C 105, 044903 (2022)

0.50

α

0.75

1.0

g.c.e.

N = 400

Classical molecular dynamics simulations of the Lennard-Jones fluid near Z(2) critical point ( $T \approx 1.06T_c$ ,  $n \approx n_c$ ) of the liquid-gas transition

Scaled variance in coordinate space acceptance  $|z| < z^{max}$ 



- Large fluctuations survive despite strong finite-size effects
- Need coordinate space cuts (collective flow helps)
- Here no finite-time effects

Collective flow and finite-time effects explored in V. Kuznietsov et al., Phys. Rev. C 110, 015206 (2024)



Heavy-ion collisions: flow correlates  $p_z$  and z cuts z (or  $\eta_s$ )



### Non-Gaussian fluctuations from molecular dynamics





### Non-Gaussian fluctuations from molecular dynamics





• (Non-)Gaussian cumulants equilibrate on comparable time scales

see also X. An et al., PRL 127, 072301 (2021); C. Chattopadhyay et al., PRL 133, 032301 (2024)

### **Theory vs experiment: Challenges for fluctuations**



### Theory



 $\ensuremath{\mathbb{C}}$  Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

### **Experiment**



STAR event display

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogeneous
- Fluctuating volume

#### Need dynamical description

# Proton cumulants from RHIC-BES-II





Hydro EV: VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

What is hydro EV?

# Hydro EV: Non-critical hydro baseline at RHIC-BES



Au-Au, 0-5%

#### VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

MUSIC + SAM

- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
  - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
  - Crossover equation of state based on lattice QCD

[Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]

- Non-critical contributions computed at particlization ( $\epsilon_{sw} = 0.26 \text{ GeV/fm}^{3}$ )
  - QCD-like baryon number distribution  $(\chi_n^B)$  via **excluded volume** b = 1 fm<sup>3</sup> [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
  - **Exact global baryon conservation\*** (and other charges)
    - Subensemble acceptance method 2.0 (analytic) [VV, Phys. Rev. C 105, 014903 (2022)]
    - or FIST sampler (Monte Carlo) [VV, Phys. Rev. C 106, 064906 (2022)] https://github.com/vlvovch/fist-sampler
- Included: baryon conservation, repulsion, kinematical cuts
- Absent: critical point, local conservation, initial-state/volume fluctuations, hadronic phase

\*If baryon conservation is the only effect (no other correlations), non-critical baseline can be computed without hydro Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, NPA 1008, 122141 (2021)



# RHIC-BES-II (Factorial) cumulants A. Pandav, CPOD2024

Factorial Cumulant Ratios







 $(1)\frac{\kappa_2}{\kappa_1}$ -0.1 BES-II 🖉 BES-I Hydro Hydro EV -0.20-5% Au+Au Collisions (2) (anti-) proton, lyl < 0.5  $0.4 < p_{_{T}} < 2.0 \; GeV/c$ 0.05  $0.5 - (3) \frac{\kappa_4}{\kappa_4}$ 0.25 -0.25 20 200 10 100 Collision Energy  $\sqrt{s_{NN}}$  (GeV)

**Proton/antiproton** 

factorial cumulant ratios

More structure seen in factorial cumulants

#### **Conclusion 1:**



Ordinary cumulants

Factorial cumulants

What are the factorial cumulants?

# Factorial cumulants $\hat{C}_n$ vs ordinary cumulants $C_n$



**Factorial cumulants:** ~irreducible n-particle correlations

$$\begin{split} \hat{C}_n &\sim \langle N(N-1)(N-2) \dots \rangle_c & C_n \sim \langle \delta N^n \rangle_c \\ \hat{C}_1 &= C_1 & C_1 = \hat{C}_1 \\ \hat{C}_2 &= C_2 - C_1 & C_2 = \hat{C}_2 + \hat{C}_1 \\ \hat{C}_3 &= C_3 - 3C_2 + 2C_1 & C_3 = \hat{C}_3 + 3\hat{C}_2 + \hat{C}_1 \\ \hat{C}_4 &= C_4 - 6C_3 + 11C_2 - 6C_1 & C_4 = \hat{C}_4 + 6\hat{C}_3 + 7\hat{C}_2 + \hat{C}_1 \end{split}$$

**Ordinary cumulants:** mix correls. of different orders

[Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017); Kitazawa, Luo, PRC 96, 024910 (2017); C. Pruneau, PRC 100, 034905 (2019)]

#### Factorial cumulants and different effects

- Baryon conservation [Bzdak, Koch, Skokov, EPJC '17]
- Excluded volume [VV et al, PLB '17]
- Volume fluctuations [Holzman et al., arXiv:2403.03598]
- Critical point [Ling, Stephanov, PRC '16]
- $\hat{C}_n^{\mathrm{cons}} \propto (\hat{C}_1)^n / \langle N_{\mathrm{tot}} \rangle^{n-1}$ small  $\hat{C}_n^{\sf EV} \propto b^n$ small
- proton vs baryon  $\hat{C}_n^B \sim 2^n \times \hat{C}_n^p$ same sign! [Kitazawa, Asakawa, PRC '12]

 $+\hat{C}_{1}$ 

- $\hat{C}_{n}^{CF} \sim (\hat{C}_{1})^{n} \kappa_{n}[V]$  depends on volume cumulants
- $\hat{C}_2^{CP} \sim \xi^2$ ,  $\hat{C}_3^{CP} \sim \xi^{4.5}$ ,  $\hat{C}_4^{CP} \sim \xi^7$  large

### **Factorial cumulants from RHIC-BES-II**

#### From M. Stephanov, SQM2024 & arXiv:2410.02861



 $\mu_B$ 

 $\omega_n = \hat{C}_n / \hat{C}_1$ 

Bzdak et al review 1906.00936

Expected signatures: bump in  $\omega_2$  and  $\omega_3$ , dip then bump in  $\omega_4$  for CP at  $\mu_B > 420$  MeV

0



## **Factorial cumulants from RHIC-BES-II**



#### From M. Stephanov, SQM2024 & arXiv:2410.02861



### baseline (hydro EV):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

Bzdak et al review 1906.00936

Expected signatures: bump in  $\omega_2$  and  $\omega_3$ , dip then bump in  $\omega_4$ for CP at  $\mu_B > 420$  MeV

# **Factorial cumulants from RHIC-BES-II**



#### From M. Stephanov, SQM2024 & arXiv:2410.02861



baseline (hydro EV):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

- describes right side of the peak in  $\hat{C}_3$ •
- signal relative to baseline:
  - positive  $\hat{C}_2 \hat{C}_2^{baseline} > 0$
  - negative  $\hat{C}_3 \hat{C}_3^{baseline} < 0$

#### **Conclusion 2:**

Controlling the non-critical baseline is essential

Bzdak et al review 1906.00936

Expected signatures: bump in  $\omega_2$  and  $\omega_3$ , dip then bump in  $\omega_4$ for CP at  $\mu_B > 420$  MeV

# Factorial cumulants from RHIC-BES-II and CP





#### Equilibrium expectation

#### **Exclusion plots**

Exclude  $\hat{\mathcal{C}}_2{<}0$  &  $\hat{\mathcal{C}}_3{>}0$  regions on the phase diagram near CP



Adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017) and based on the model from VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Freeze-out of fluctuations on the QGP side of the crossover?

# Factorial cumulants from RHIC-BES-II and CP





#### **Exclusion plots**

Exclude  $\hat{C}_2 < 0$  &  $\hat{C}_3 > 0$  regions on the phase diagram near CP



 $\mu_B$ 

Adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017) and based on the model from VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)

Freeze-out of fluctuations on the QGP side of the crossover?

### Interplay with nuclear liquid-gas transition







VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

### Interplay with nuclear liquid-gas transition





VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

Increasingly relevant at lower energies probed through RHIC-FXT

# Scaled factorial cumulants, long-range correlations, and the antiproton puzzle A. Bzdak, V. Koch, VV, in preparation

### **Scaled factorial cumulants**

Bzdak et al. introduced reduced correlation functions – "couplings" [Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)]

Bzdak ratios

 $\hat{c}_k = \frac{\sigma_k}{\langle N \rangle^k}$ 

$$c_k = \frac{\int \rho_1(y_1) \cdots \rho_1(y_k) c_k(y_1, \dots, y_k) dy_1 \cdots dy_k}{\int \rho_1(y_1) \cdots \rho_1(y_k) dy_1 \cdots dy_k}$$

integrated correlation function in rapidity

Long-range correlations lead to acceptance-independent couplings, for example

- Global (not local) baryon conservation
   [Bzdak, Koch, Skokov, EPJC 77, 288 (2017); Bzdak, Koch, PRC 96, 054905 (2017)]
- + volume fluctuations

[Holzmann, Koch, Rustamov, Stroth, arXiv:2403.03598]

$$c_2 = -\frac{1}{B},$$
  $c_3 = \frac{2}{B^2},$   $c_4 = -\frac{6}{B^3},$   
 $\hat{c}_{i,j} = \hat{c}_{i,j} + \frac{\kappa_2[V]}{\langle V \rangle^2},$  for  $i+j=2.$ 

• + (uniform) efficiency

[Pruneau, Gavin, Voloshin, PRC 66, 044904 (2002)]

all lead to

$$\frac{\hat{C}_k}{\langle N \rangle^k} = const.$$
 at a given  $\sqrt{s_{NN}}$  and

$$\frac{\hat{C}_2^p}{\left\langle N_p \right\rangle^k} \approx \frac{\hat{C}_2^{\overline{p}}}{\left\langle N_{\overline{p}} \right\rangle^k} = const. \quad \text{at a given } \sqrt{S_{NN}}$$

Can be tested without CBWC/volume fluctuations correction A. Bzdak, V. Koch, VV, in preparation



# Scaled factorial cumulants from RHIC-BES-I



Significant difference between p and  $\bar{p}$  in BES-I not explained by hydro •

no single thermalized fireball?

#### A. Bzdak, V. Koch, VV, in preparation



**Two-component model:** produced ( $p\bar{p}$  pairs) and stopped protons come from two independent fireballs



Data lie in-between single and two-fireball models

A. Bzdak, V. Koch, VV, in preparation

- **Opportunities/wishlist for BES-II:** Acceptance dependence of scaled factorial cumulants
- Further tests of the splitting between p and  $\bar{p}$  in 2<sup>nd</sup> order cumulants with extended y coverage
- Covariance  $c_{1,1}^{p,\bar{p}}$  to probe baryon annihilation
- Critical point signal expected to break the scaling



# Charge fluctuations as a signature of QGP

• J. Parra, R. Poberezhniuk, V. Koch, C. Ratti, VV, in preparation



An old idea: Hadrons carry integer electric charges, quarks carry *fractional* electric charges.



Fluctuations depend on the square of the charges and are smaller in the QGP

### Quantified by:

$$D = 4 \frac{\kappa_2 [N_+ - N_-]}{\langle N_{\rm ch} \rangle} = 4 \frac{\kappa_2 [Q]}{\langle Q^+ + Q^- \rangle}$$

Naïve grand canonical ensemble (GCE) expectations:

- $D_{HG} \approx 2.8 4$
- $D_{QGP} \approx 1 1.5$

No quantitative calculations have been done for QGP beyond GCE





Measurements came and went, conclusions are ambiguous. What more can be done?

# **Charge fluctuations: stages**





Here we model these effects through novel charge density correlations formalism [VV, PRC '14]

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Define  $\omega$  as a measure of charge fluctuations at hadronization





P. Hanus, A. Mazeliauskas, K. Reygers, PRC (2019)



### 2. Hadronic phase (resonance decays)

- Decays are local and conserve charge but increase charged multiplicity,  $\langle N_{ch} \rangle = \langle N_{ch}^{prim} \rangle \gamma_Q$ , where  $\gamma_Q \approx 1.67$
- Total charge susceptibility does not change,  $\chi_2^{Q,final} = \chi_2^{Q,prim}$ , but balance between self-correlation and two-particle correlations does

### 3. Local charge conservation [VV, PRC 110, L061902 (2024)]

- 2-point charge density correlator with a balancing term
- Local charge conservation introduced through modulation of the balancing term

$$\mathcal{C}_{2}^{Q}(\mathbf{r}_{1},\mathbf{r}_{2}) = \chi_{2}^{Q} \left[ \delta(\mathbf{r}_{1}-\mathbf{r}_{2}) - \frac{\varkappa(\mathbf{r}_{1},\mathbf{r}_{2})}{V_{\text{tot}}} \right] \quad \mathbf{r} = \eta$$

$$\begin{array}{c} \mathbf{r} = \eta \end{array}$$

$$\mathcal{C}_2^Q(\mathbf{r}_1,\mathbf{r}_2)\equiv \langle\delta
ho_Q(\mathbf{r}_1)\delta
ho_Q(\mathbf{r}_2)
angle$$

$$\varkappa(\eta_1,\eta_2) \propto \exp\left[-\frac{(\eta_1-\eta_2)^2}{2\sigma_\eta^2}\right]$$

local charge conservation

### 4. Kinematical cuts

$$\kappa_{2}[Q]|_{|\eta| < \eta_{\rm cut}} \propto \int_{-\eta_{\rm cut}/2}^{\eta_{\rm cut}/2} d\eta_{1} \int_{-\eta_{\rm cut}/2}^{\eta_{\rm cut}/2} d\eta_{2} C_{2}^{Q}(\eta_{1},\eta_{2}) p(\eta_{1}) p(\eta_{2})$$

Acceptance probability  $p(\eta)$ from blast-wave model

### **Putting everything together**





 $\omega$  - Charge fluctuations at hadronization

$$\omega_{HG} = 1$$
  $\omega_{QGP} = 0.36$ 

 $\gamma_Q$  - Resonance decays

 $\langle p(\eta_1)p(\eta_2)
angle_{arkappa}$  - Pair acceptance weighted with Local Charge Conservation

 $\frac{\langle p^2(\eta) \rangle}{\langle p(\eta) \rangle} - \text{Momentum Acceptance Cuts}$   $\underset{\text{using } p(\eta) \text{ from the blast-wave model}}{\text{Womentum Acceptance Cuts}}$ 

### **D**-measure at LHC: comparison with experiment





Parameters used:  $\omega_{HG} = 1$   $\omega_{QGP} = 0.36$  $\gamma_0 = 1.67$ 

- Vary  $\sigma_v$  to accommodate global vs local charge conservation
  - Here values of  $\sigma_{v}$  are based on local baryon conservation estimates VV, PRC 110, L061902 (2024)

Hadron gas scenario requires a very local charge conservation range



Vary primordial fluctuation  $\omega$  (HG vs QGP) and correlation volume V<sub>C</sub> (local conservation) freely



- Uniform prior  $\omega \in U(0, 1.2), \quad V_c \in U(0, V_{tot}),$ Bayes factor (QGP : HG) = 8.74
- Local conservation prior  $\omega \in U(0, 1.2), V_c \in \text{Gaussian at } (0.20 \pm 0.05)V_{tot}$ Bayes factor (QGP : HG) = 4.93

Moderate evidence for freeze-out of charge fluctuations in QGP

Sensitivity to assumed prior currently under investigation

# **Summary**





#### **Outlook:**

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- Improved description of non-critical baselines and quantitative predictions of critical fluctuations
- Acceptance dependence of factorial cumulants, understanding antiprotons

### **Thanks for your attention!**

HG

QGP