Factorial cumulants as a probe of fireball properties in heavy-ion collisions

Volodymyr Vovchenko (University of Houston)

Mini-Symposium: Experimental and Theoretical Searches for the QCD Critical Point

March 17, 2025

Thanks to:

A. Bzdak, V. Koch, V.A. Kuznietsov, R. Poberezhniuk, J. Parra, C. Ratti

APS | JOINT MARCH MEETING AND APRIL MEETING
Global Physics Summit
March 16–21, 2025, Anaheim, CA and virtual

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QCD under extreme conditions

What we know

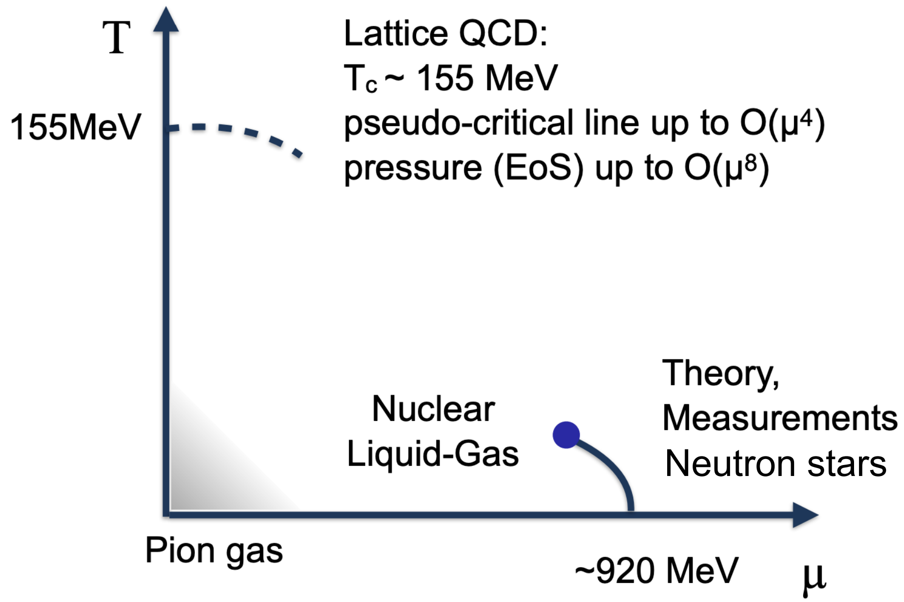


Figure courtesy of V. Koch

What we hope to know

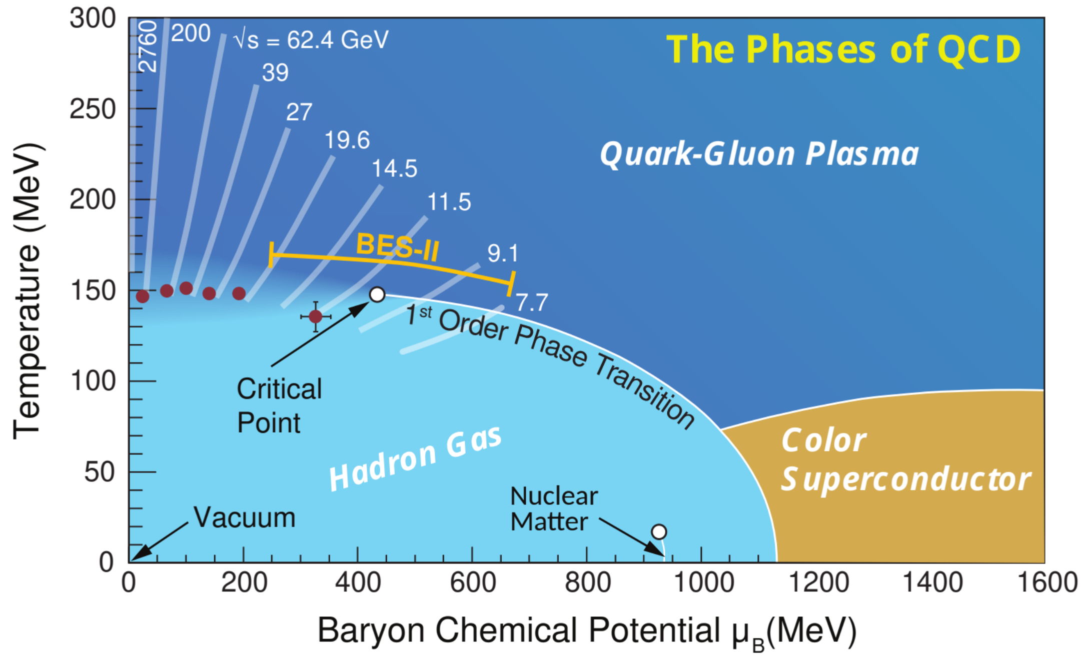


Figure from Bzdak et al., Phys. Rept. '20 & 2015 US Nuclear Long Range Plan

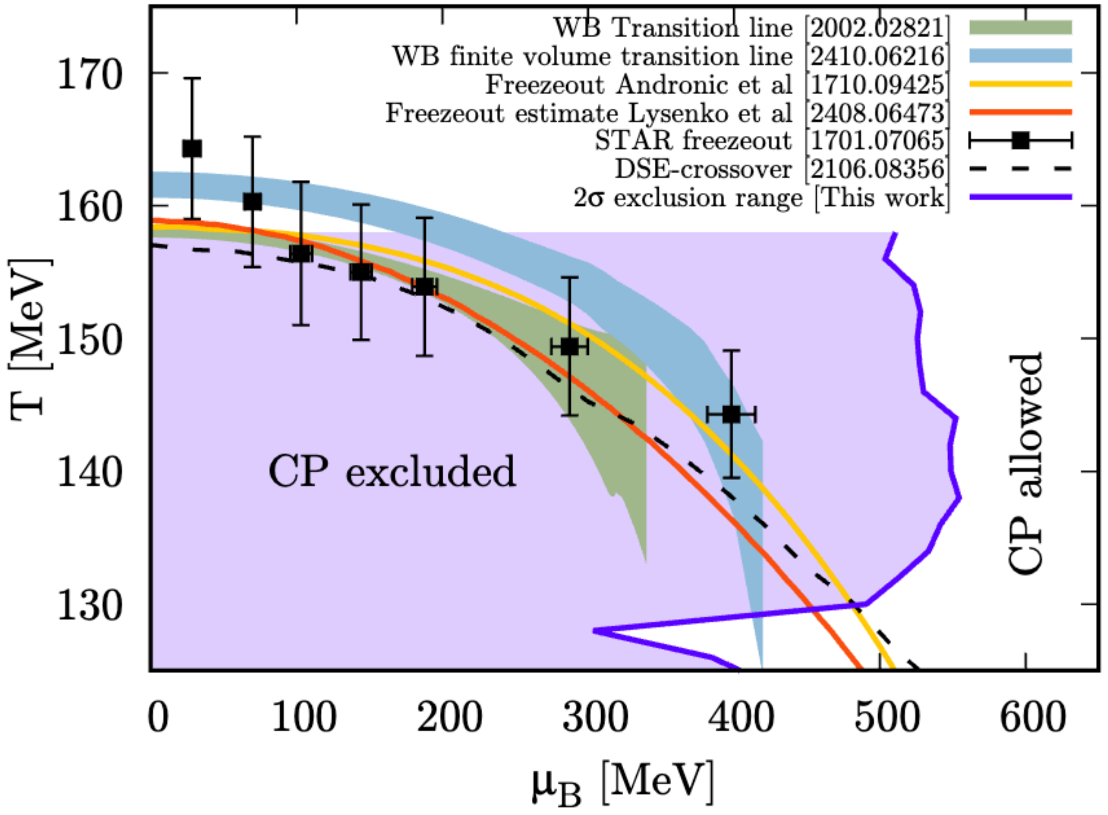
- Dilute hadron gas at low T & μ_B due to confinement, quark-gluon plasma high T & μ_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured
- Chiral crossover at $\mu_B = 0$ which may turn into a *first-order phase transition* at finite μ_B

Key question: *Is there a QCD critical point and how to find it?*

QCD CP theory estimates: new developments

Analysis of crossings of constant entropy contours

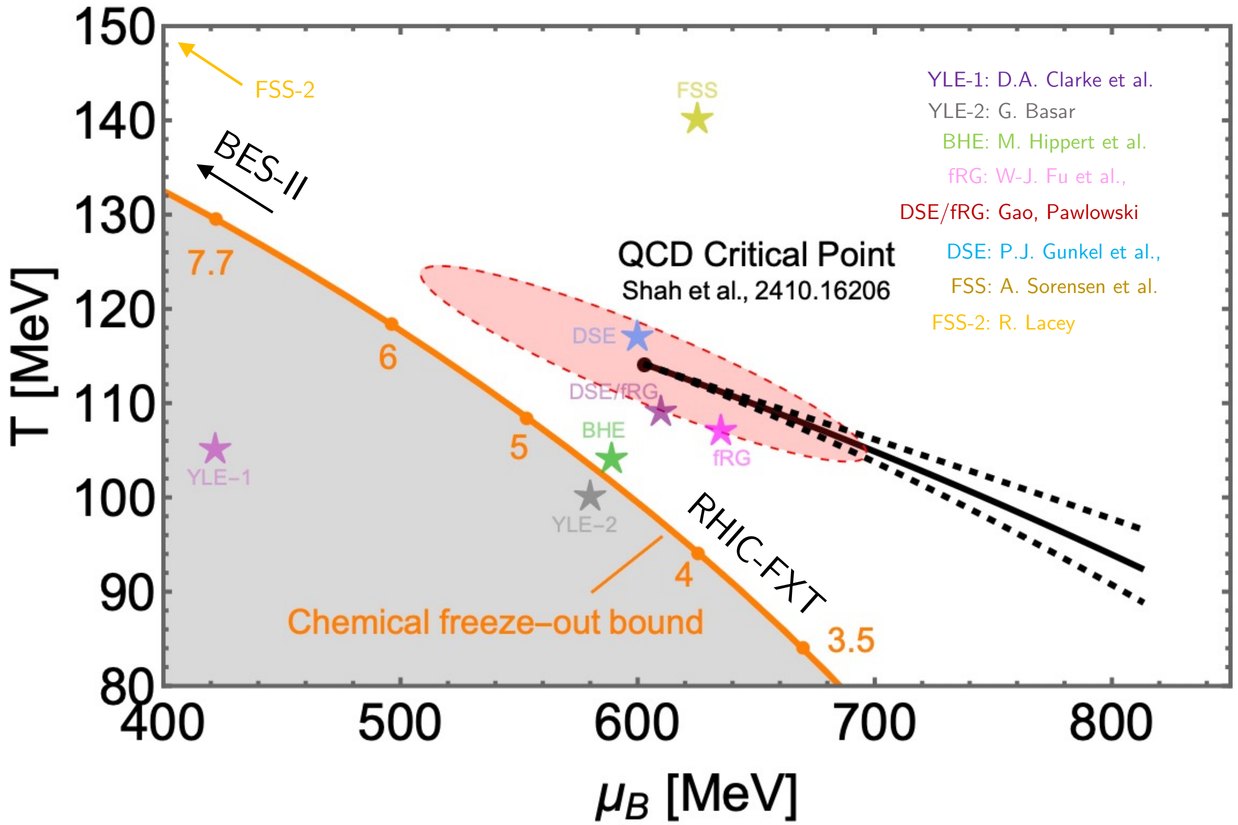
Borsanyi, Fodor, Guenther, Parotto, Pasztor, Ratti, VV, Wong, arXiv:2502.10267



- CP excluded at $\mu_B < 450$ MeV at a (one-sided) 2σ level

H. Shah, Tue 4:21 PM

H. Shah, M. Hippert, J. Noronha, C. Ratti, VV, arXiv:2410.16026



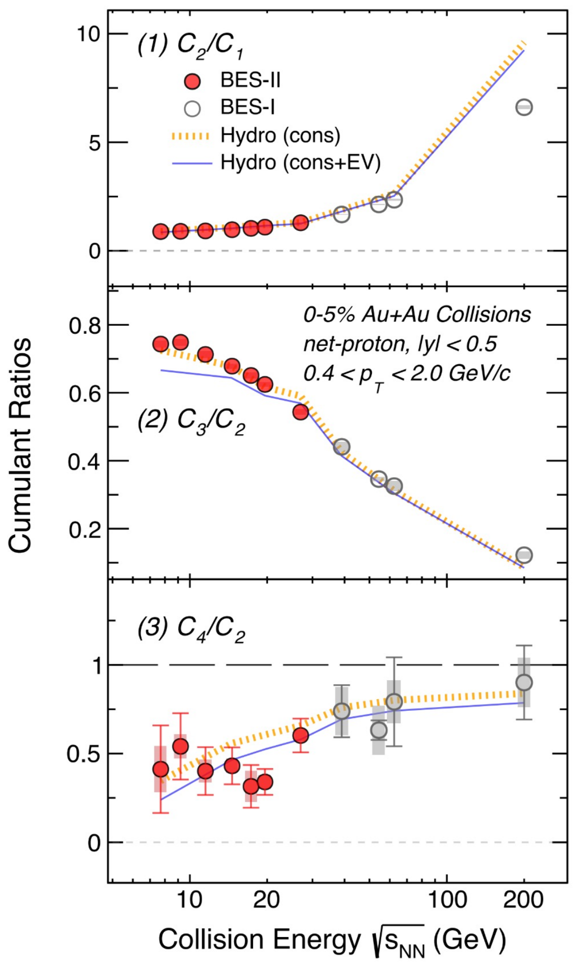
- Critical point location at $O(\mu_B^2)$:

$T_c = 114 \pm 7$ MeV, $\mu_B = 602 \pm 62$ MeV

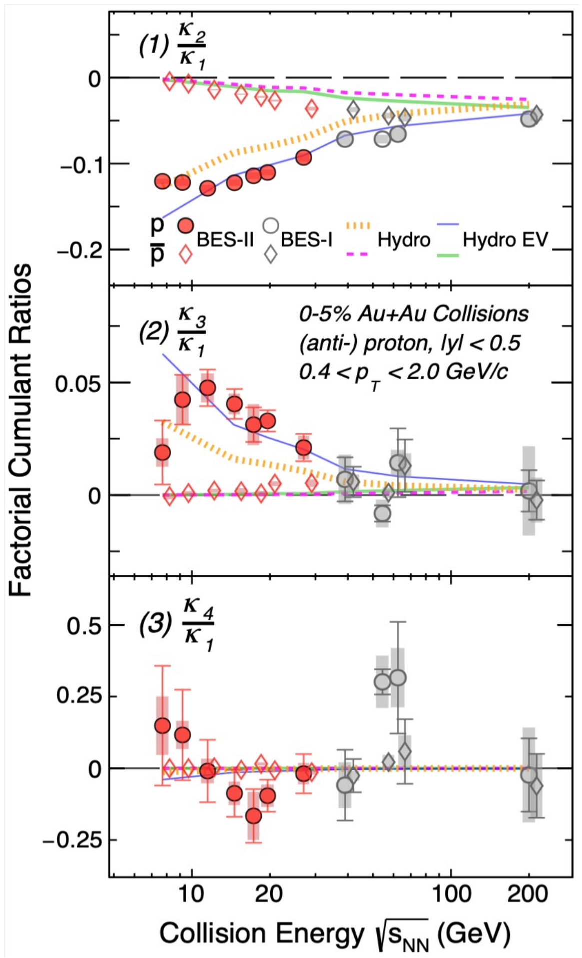
RHIC-BES-II (Factorial) cumulants

A. Pandav, CPOD2024

Net-proton cumulant ratios

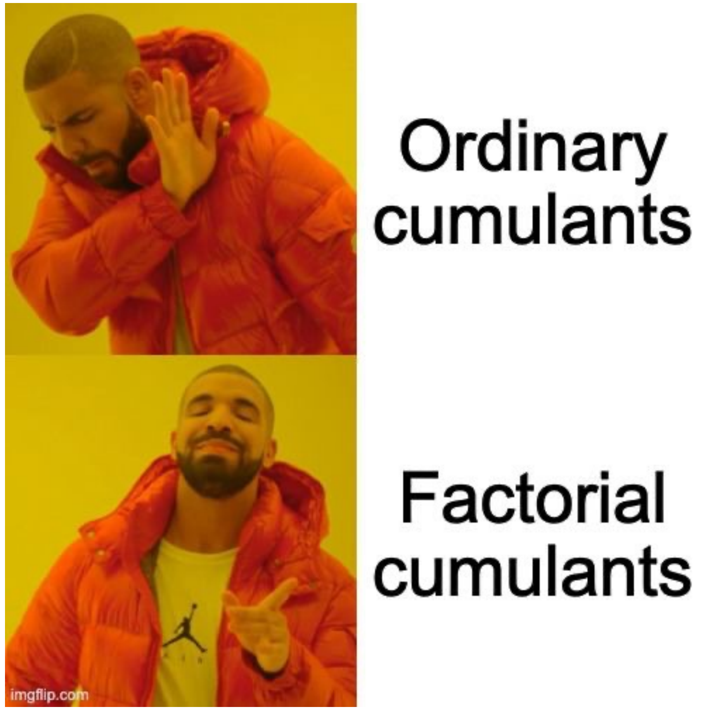


Proton/antiproton factorial cumulant ratios



- More structure seen in factorial cumulants

Conclusion 1:



What are the factorial cumulants?

Hydro EV: hydrodynamics with baryon conservation and excluded volume (non-critical baseline)

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Factorial cumulants $\hat{C}_n$ vs ordinary cumulants C_n

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Factorial cumulants: ~irreducible n-particle correlations

$$\hat{C}_n \sim \langle N(N-1)(N-2)\dots\rangle_c$$

$$\hat{C}_1 = C_1$$
$$\hat{C}_2 = C_2 - C_1$$
$$\hat{C}_3 = C_3 - 3C_2 + 2C_1$$
$$\hat{C}_4 = C_4 - 6C_3 + 11C_2 - 6C_1$$

Ordinary cumulants: mix correls. of different orders

$$C_n \sim \langle \delta N^n \rangle_c$$

$$C_1 = \hat{C}_1$$
$$C_2 = \hat{C}_2 + \hat{C}_1$$
$$C_3 = \hat{C}_3 + 3\hat{C}_2 + \hat{C}_1$$
$$C_4 = \hat{C}_4 + 6\hat{C}_3 + 7\hat{C}_2 + \hat{C}_1$$

[Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017); Kitazawa, Luo, PRC 96, 024910 (2017); C. Pruneau, PRC 100, 034905 (2019)]

Factorial cumulants and different effects

- Baryon conservation [Bzdak, Koch, Skokov, EPJC '17] — $\hat{C}_n^{\rm cons} \propto (\hat{C}_1)^n/\langle N_{\rm tot}\rangle^{n-1}$ *small*
- Excluded volume [VV et al, PLB '17] — $\hat{C}_n^{\rm EV} \propto b^n$ *small*
- Volume fluctuations [Holzman et al., arXiv:2403.03598] — $\hat{C}_n^{CF} \sim (\hat{C}_1)^n \kappa_n[V]$ *depends on volume cumulants*
- **Critical point** [Ling, Stephanov, PRC '16] — $\hat{C}_2^{CP} \sim \xi^2$, $\hat{C}_3^{CP} \sim \xi^{4.5}$, $\hat{C}_4^{CP} \sim \xi^7$ ***large***

- proton vs baryon [Kitazawa, Asakawa, PRC '12] — $\hat{C}_n^B \sim 2^n \times \hat{C}_n^p$ **same sign!**

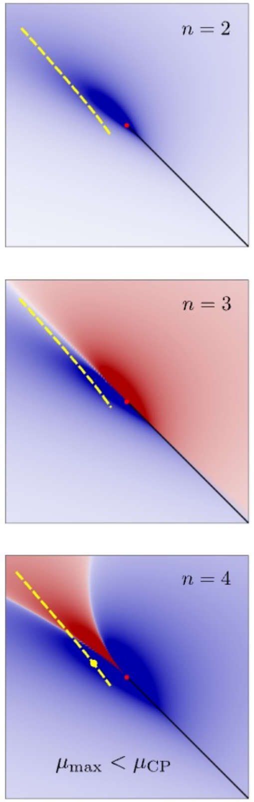
Cumulants from CP
in a microscopic model
V. Kuznietsov, Mon 5:09 PM

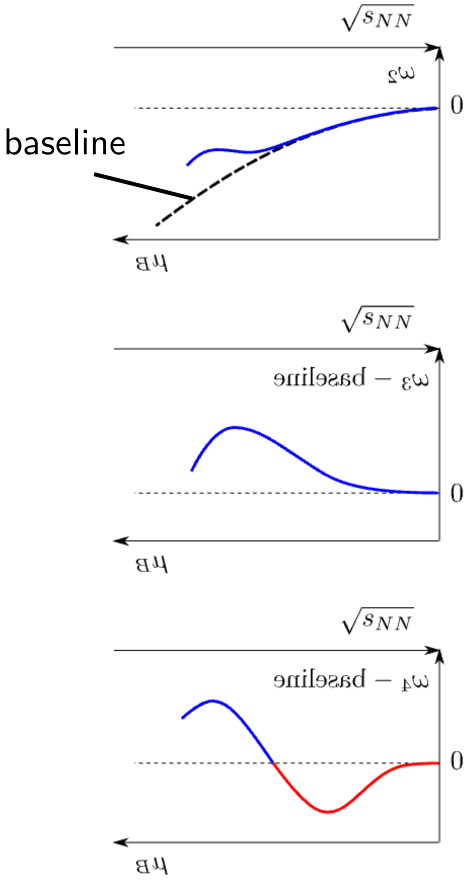
Factorial cumulants from RHIC-BES-II

From M. Stephanov, SQM2024 & arXiv:2410.02861

$$\omega_n = \hat{C}_n / \hat{C}_1$$

(universal EOS) critical χ_n:





Bzdak et al review 1906.00936

Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

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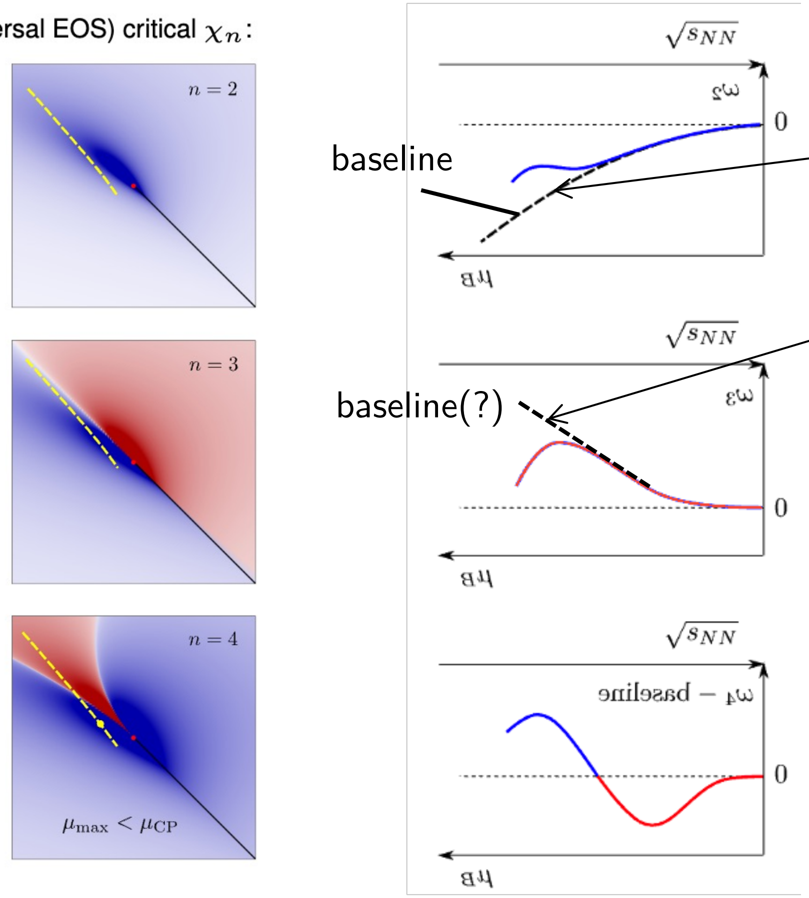

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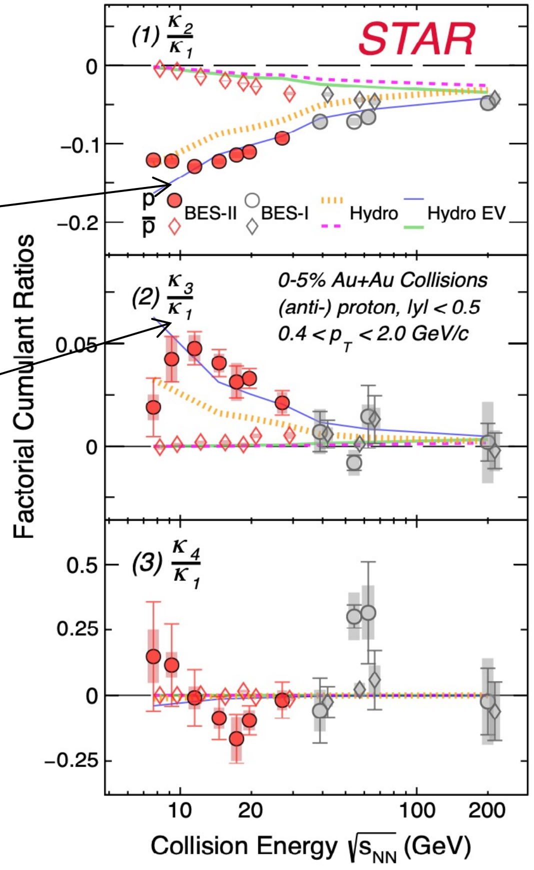
STAR data:



A. Pandav, CPOD2024

baseline (hydro EV):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

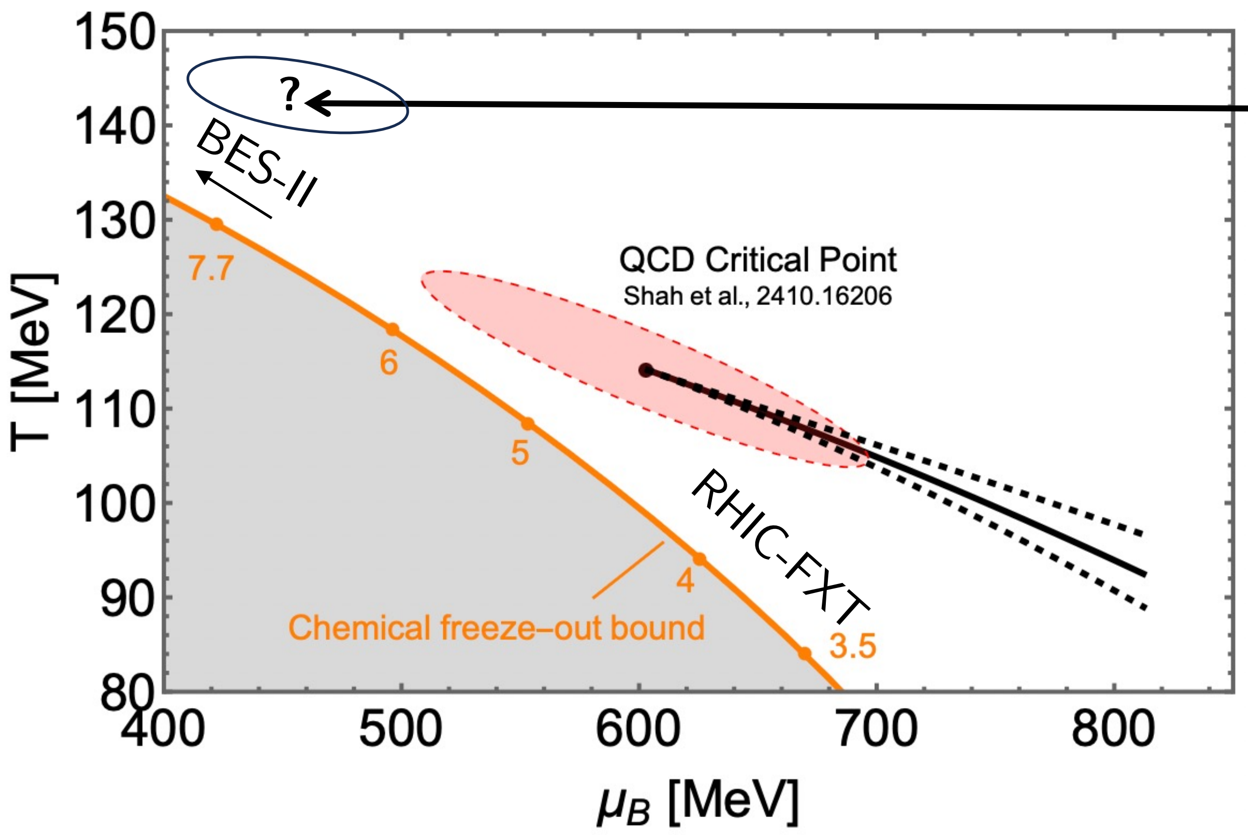
- describes right side of the peak in $\hat{C}_3$
- **signal relative to baseline:**
  - *positive* $\hat{C}_2 - \hat{C}_2^{baseline} > 0$
  - *negative* $\hat{C}_3 - \hat{C}_3^{baseline} < 0$

Conclusion 2:

Controlling the non-critical baseline is essential

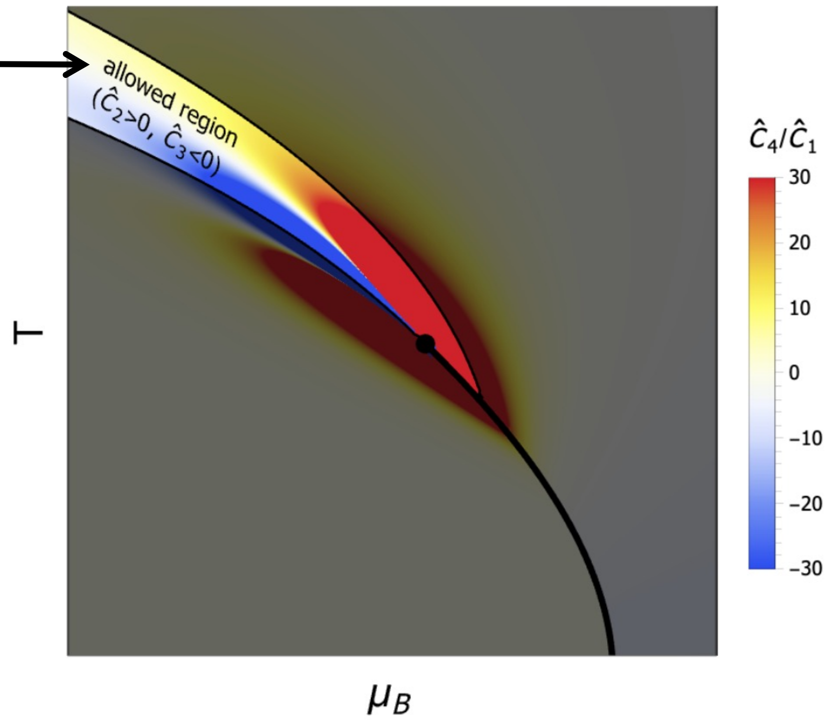
Factorial cumulants from RHIC-BES-II and CP

Equilibrium expectation



Exclusion plots

Exclude $\hat{C}_2<0$ & $\hat{C}_3>0$ regions on the phase diagram near CP

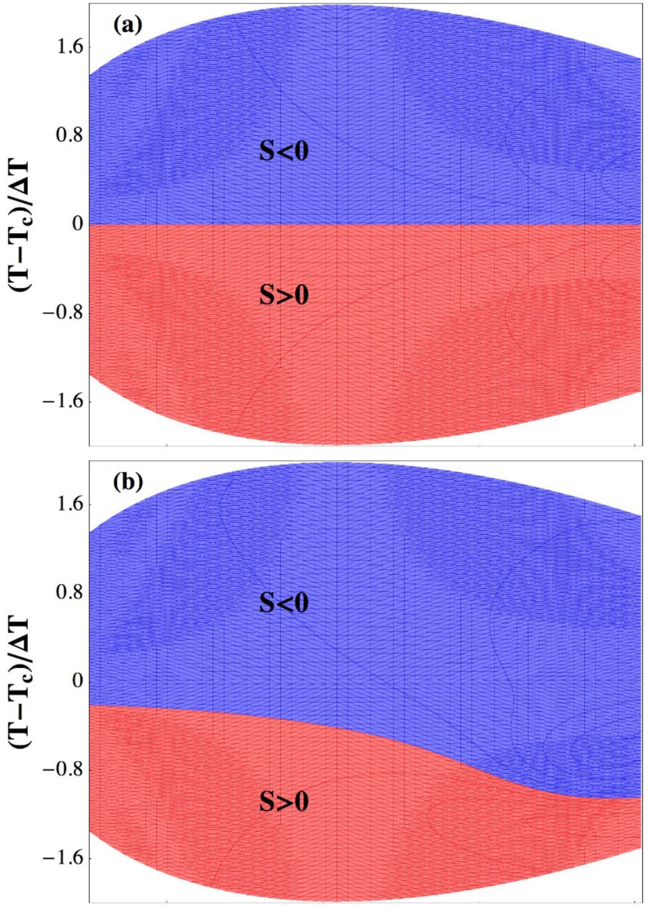


Adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017) and based on the model from VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Freeze-out of fluctuations on the QGP side of the crossover?

Factorial cumulants from RHIC-BES-II and CP

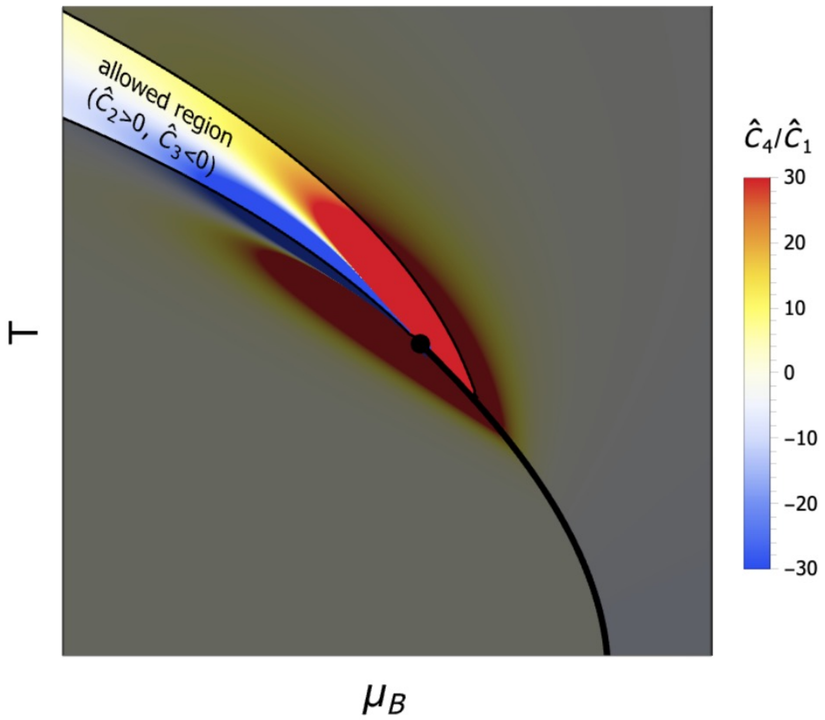
Memory effect



Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)

Exclusion plots

Exclude $\hat{C}_2 < 0$ & $\hat{C}_3 > 0$ regions on the phase diagram near CP



Adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)
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Freeze-out of fluctuations on the QGP side of the crossover?

Scaled factorial cumulants

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Bzdak et al. introduced reduced correlation functions – "couplings" [Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)]

Bzdak ratios $\hat{c}_k = \frac{\hat{C}_k}{\langle N \rangle^k}$

$$c_k = \frac{\int \rho_1(y_1)\cdots\rho_1(y_k)c_k(y_1,\ldots,y_k)dy_1\cdots dy_k}{\int \rho_1(y_1)\cdots\rho_1(y_k)dy_1\cdots dy_k}$$

integrated correlation function in rapidity

Long-range correlations lead to acceptance-independent couplings, for example

- Global (not local) baryon conservation
  [Bzdak, Koch, Skokov, EPJC 77, 288 (2017); Bzdak, Koch, PRC 96, 054905 (2017)]
- + volume fluctuations
  [Holzmann, Koch, Rustamov, Stroth, arXiv:2403.03598]
- + (uniform) efficiency
  [Pruneau, Gavin, Voloshin, PRC 66, 044904 (2002)]

$$c_2 = -\frac{1}{B}, \qquad c_3 = \frac{2}{B^2}, \qquad c_4 = -\frac{6}{B^3}$$

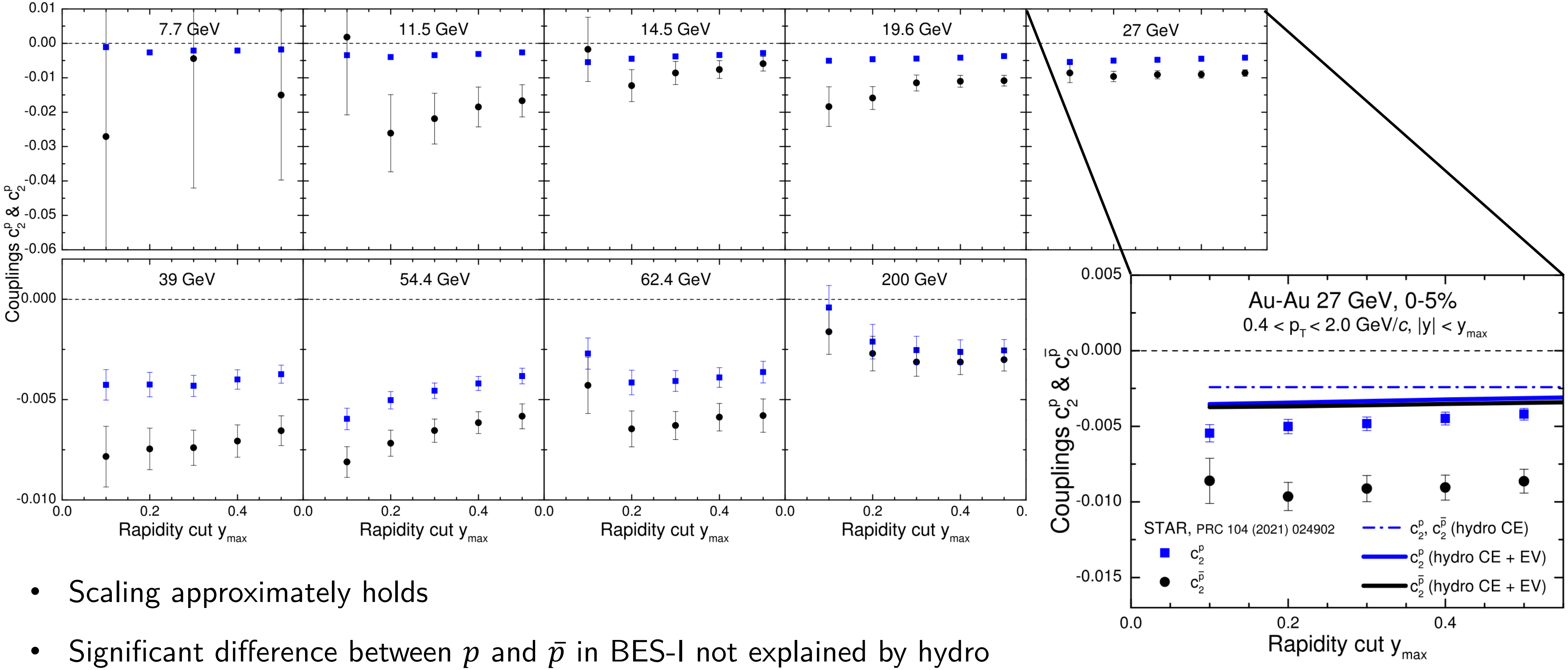
$$\hat{\hat{c}}_{i,j} = \hat{c}_{i,j} + \frac{\kappa_2[V]}{\langle V \rangle^2}, \quad \text{for} \quad i+j=2.$$

all lead to

$\frac{\hat{C}_k}{\langle N \rangle^k} = const.$ at a given $\sqrt{s_{NN}}$ and $\frac{\hat{C}_2^p}{\langle N_p \rangle^2} \approx \frac{\hat{C}_2^{\bar{p}}}{\langle N_{\bar{p}} \rangle^2} = const.$ at a given $\sqrt{s_{NN}}$

Can be tested without CBWC/volume fluctuations correction A. Bzdak, V. Koch, VV, in preparation

Scaled factorial cumulants from RHIC-BES-I



- Scaling approximately holds
- Significant difference between p and $\bar{p}$ in BES-I not explained by hydro

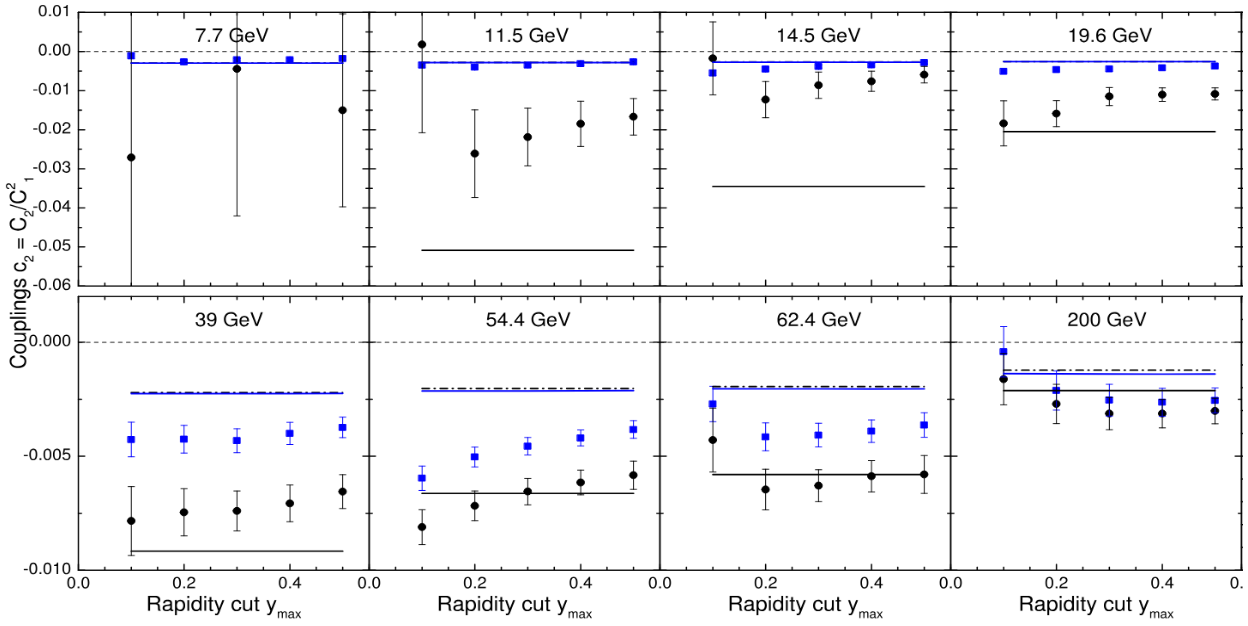
no single thermalized fireball?

A. Bzdak, V. Koch, VV, in preparation

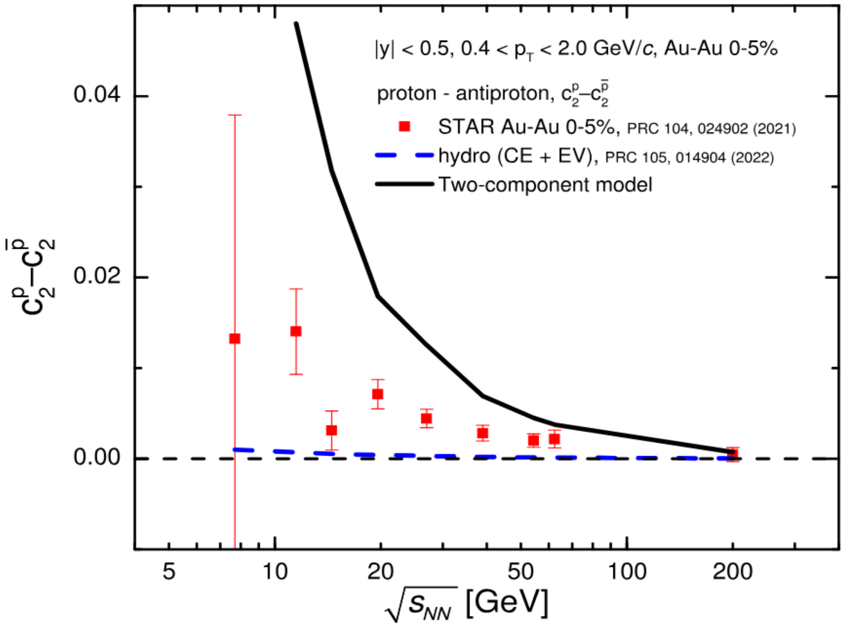
The antiproton puzzle and the two-component model

Two-component model: produced ($p\bar{p}$ pairs) and stopped protons come from two independent fireballs

Data lie in-between single and two-fireball models



Difference between p and $\bar{p}$



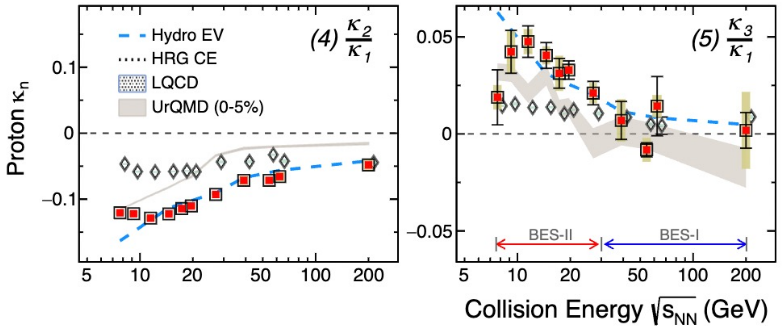
A. Bzdak, V. Koch, VV, in preparation

Opportunities for RHIC-BES-II: Acceptance dependence of scaled factorial cumulants

- Further tests of the splitting between p and $\bar{p}$ in 2nd order cumulants with extended y coverage
- Covariance $c_{1,1}^{p,\bar{p}}$ to probe baryon annihilation
- Critical point signal expected to break the scaling $\frac{\hat{C}_n}{(\hat{C}_1)^n} = \text{const.}$ [Ling. Stephanov, PRC 93, 034915 (2016)]

Summary

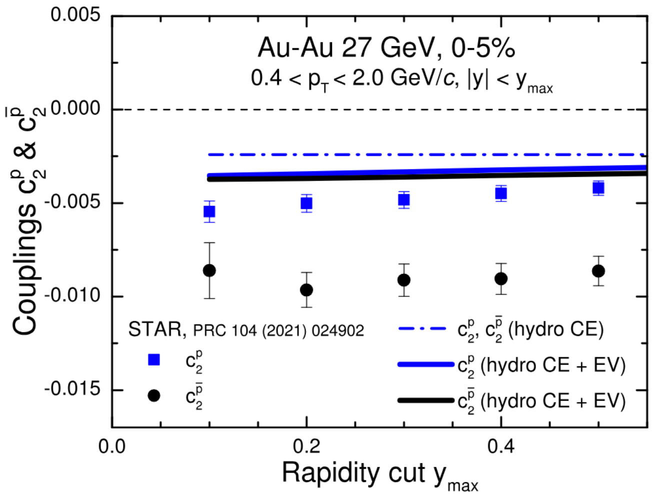
- **RHIC-BES-II data on proton number fluctuations is in**
  - Non-critical physics describe the proton data at $\sqrt{s_{NN}} \geq 20$ GeV
  - More structure seen in factorial cumulants
  - $\hat{C}_2 - \hat{C}_2^{baseline} > 0$ and $\hat{C}_3 - \hat{C}_3^{baseline} < 0$ at $\sqrt{s_{NN}} < 10$ GeV
    - May indicate freeze-out of fluctuations on the QGP side of crossover



- **Scaled factorial cumulants**
  - For long-range correlations only one has acceptance dependence

$$\frac{\hat{C}_k}{\langle N \rangle^k} = const. \quad \text{and} \quad \frac{\hat{c}_2^{p}}{\langle N_p \rangle^2} \approx \frac{\hat{c}_2^{\bar{p}}}{\langle N_{\bar{p}} \rangle^2} = const. \quad \text{at a given } \sqrt{s_{NN}}$$

  - The antiproton puzzle indicates incomplete thermalization of the system



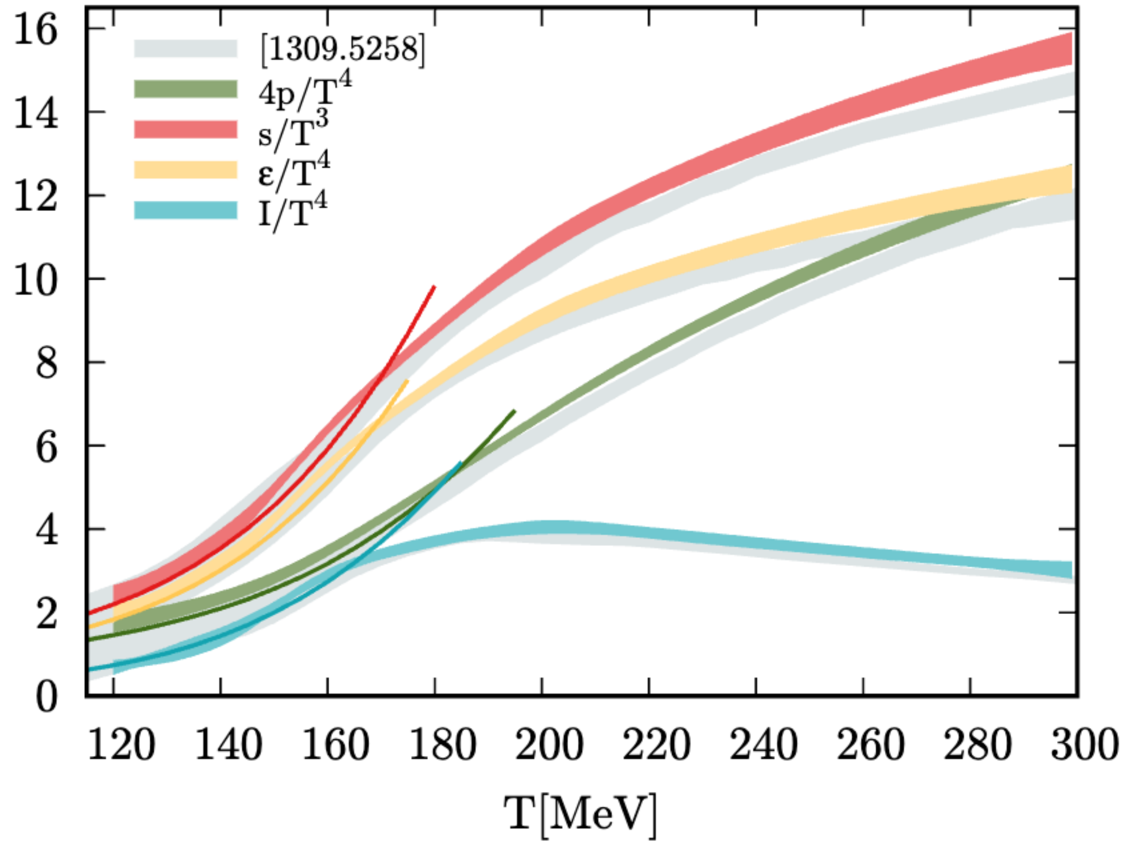
Outlook:

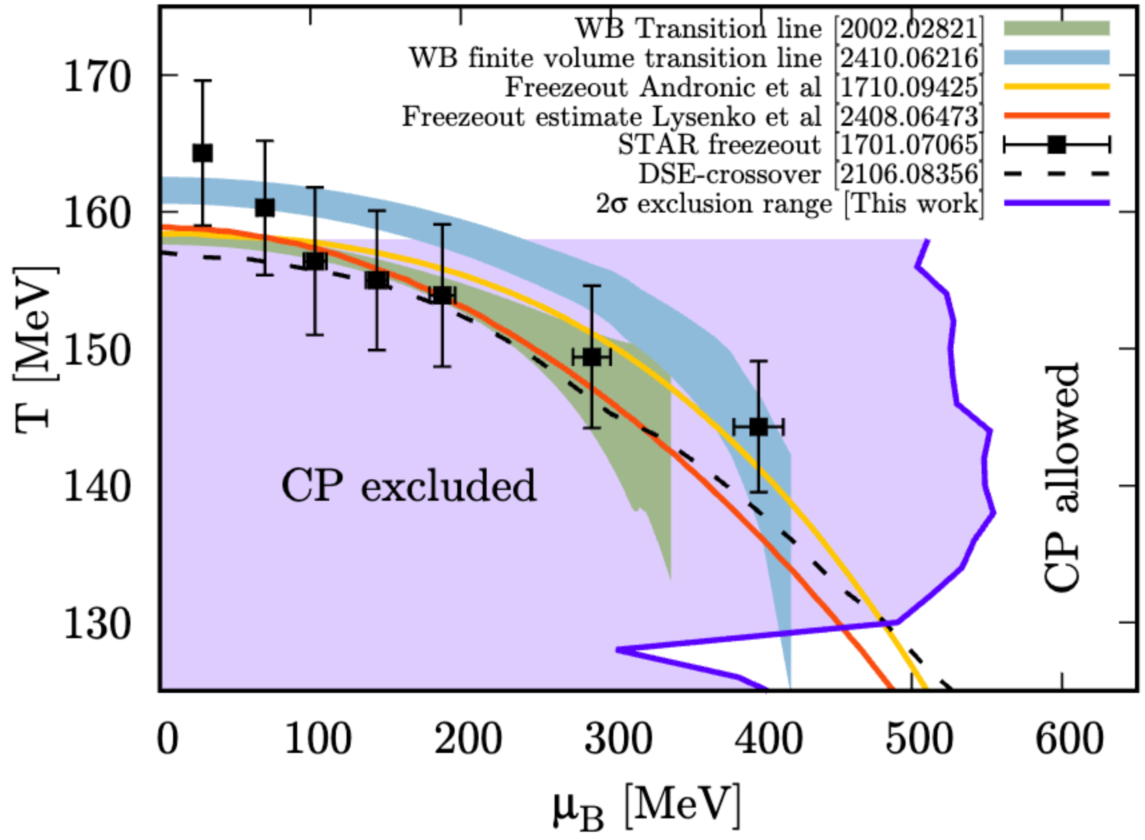
- Improved description of non-critical baselines and quantitative predictions of critical fluctuations
- Acceptance dependence of factorial cumulants, understanding antiprotons

Thanks for your attention!

Backup slides

New constraints from lattice QCD





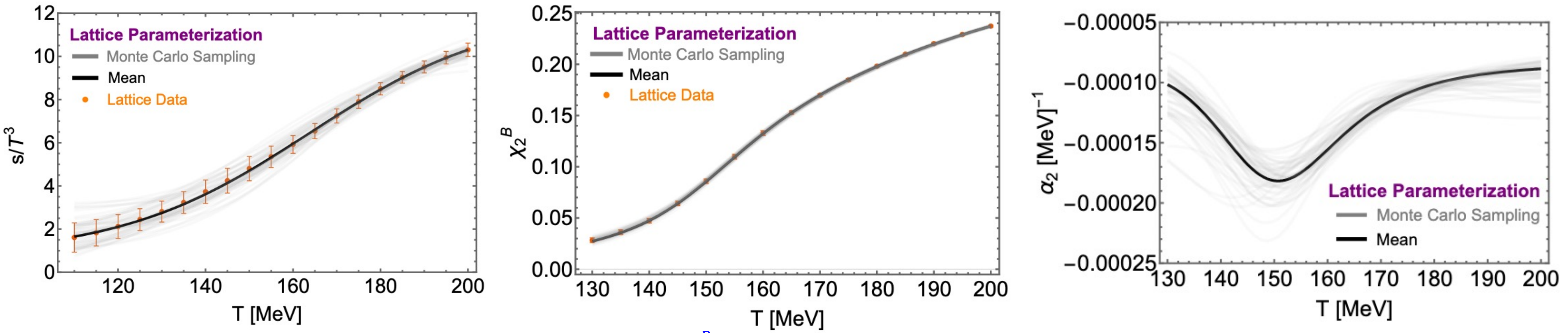
New expansion

Define the s = const. line in T-μ_B plane as an expansion around $\mu_B = 0$

$$T_s(\mu_B;T_0) \approx T_0 + \sum_{n=1}^{N} \alpha_{2n}(T_0)\frac{\mu_B^{2n}}{(2n)!} + \mathcal{O}\left(\mu_B^{2(N+1)}\right) \qquad \alpha_{2n}(T_0) = \left(\frac{\partial^{2n} T}{\partial \mu_B^{2n}}\right)_s\Bigg|_{T=T_0,\mu_B=0}$$

$$\alpha_2(T_0) = -\frac{2T_0\chi_2^B(T_0) + T_0^2\chi_2^{B\prime}(T_0)}{s'(T_0)}$$

Parametrized continuum-extrapolated lattice QCD input $[s(T), \chi_2^B(T)]$ from Bayesian analysis



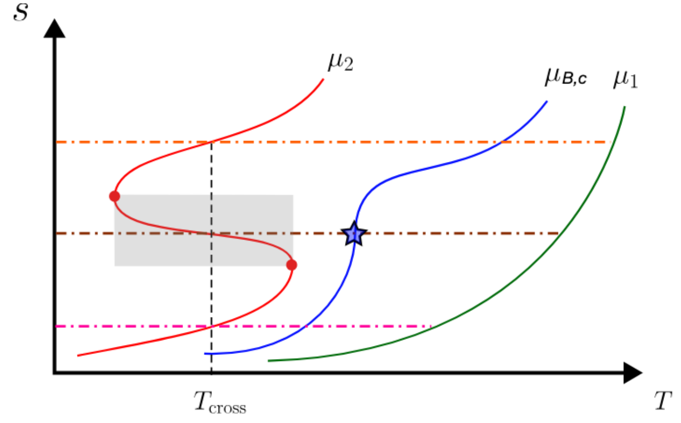
Lattice data for $s(T)$: Borsanyi et al., PLB 730, 99 (2014) Lattice data for $\chi_2^B(T)$: Borsanyi et al., PRL 126, 232001 (2021)

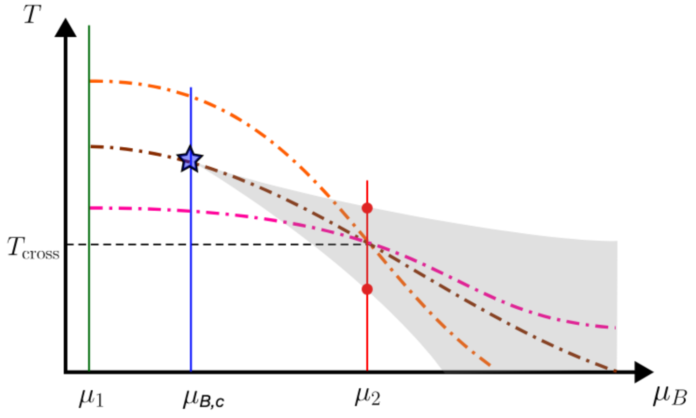
Propagation of the correlated lattice QCD uncertainties through Monte Carlo sampling of the parameter posterior distribution

H. Shah, M. Hippert, J. Noronha, C. Ratti, VV, arXiv:2410.16026

Critical point and crossings of entropy density

- Entropy density s becomes multi-valued function of T and μ_B for a first-order phase transition
- It develops a distinctive S-shape as a function of T at $\mu_B = const$

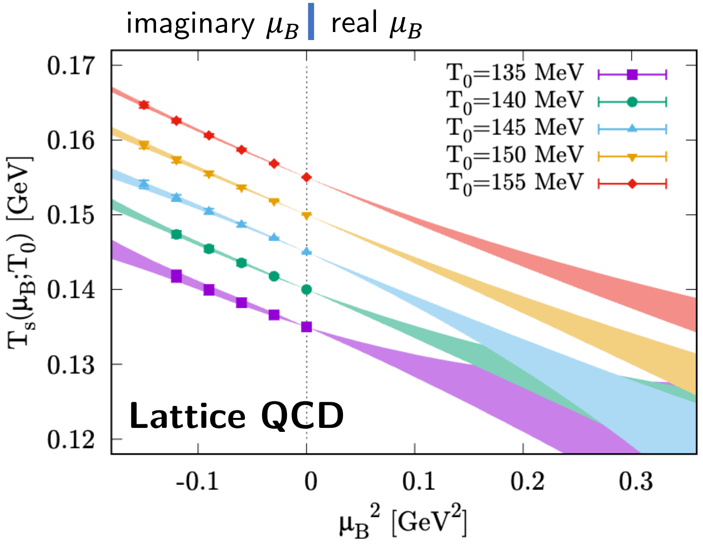




Critical Point:

$$\left(\frac{\partial T}{\partial s}\right)_{\mu_B} = 0, \qquad \left(\frac{\partial^2 T}{\partial s^2}\right)_{\mu_B} = 0.$$

H. Shah, M. Hippert, J. Noronha, C. Ratti, VV, arXiv:2410.16026



S. Borsanyi et al., arXiv:2502.10267

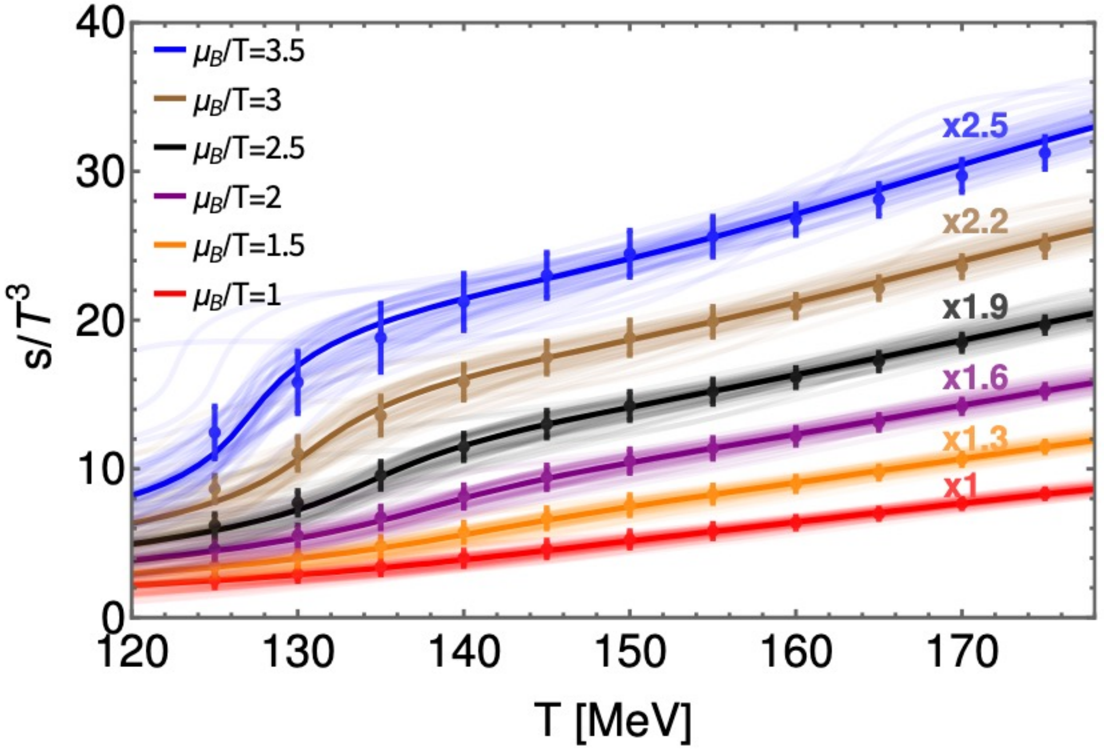
- Lattice QCD simulations at imaginary chemical potentials indicate that entropy contours are almost linear in μ_B^2

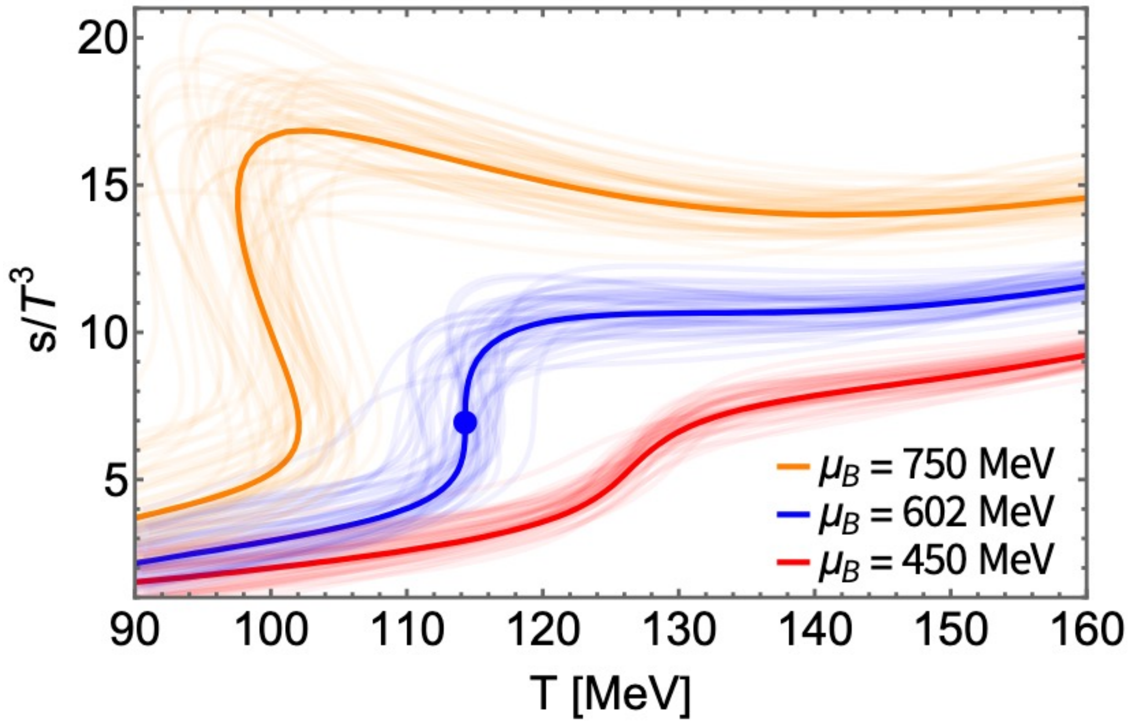
Idea: Follow contours of constant entropy density and look for crossings

$$T_s(\mu_B;T_0) \approx T_0 + \sum_{n=1}^{N} \alpha_{2n}(T_0)\,\frac{\mu_B^{2n}}{(2\,n)!} + \mathcal{O}\left(\mu_B^{2(N+1)}\right) \qquad \alpha_{2n}(T_0) = \left(\frac{\partial^{2n} T}{\partial \mu_B^{2n}}\right)_s \Bigg|_{T=T_0,\mu_B=0}$$

Entropy density at finite μ_B at $O(\mu_B^2)$

$$T_s(\mu_B; T_0) = T_0 + \alpha_2(T_0)\frac{\mu_B^2}{2} \qquad \alpha_2(T_0) = -\frac{2T_0\chi_2^B(T_0) + T_0^2\chi_2^{B'}(T_0)}{s'(T_0)}$$



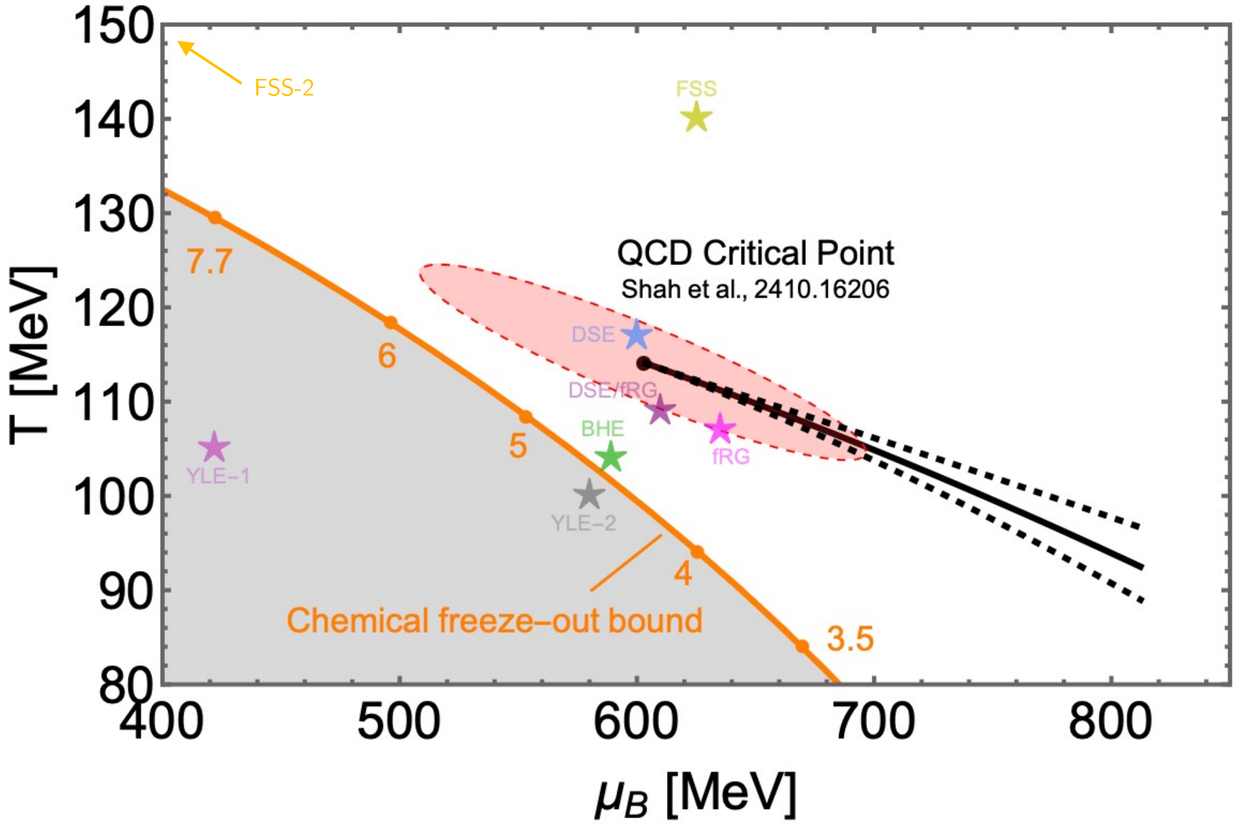


- Excellent agreement at low μ_B/T with available lattice QCD constraints [Borsanyi et al., PRL 126, 232001 (2021)]
- First-order phase transition emerges at $\mu_B > 600$ MeV

H. Shah, M. Hippert, J. Noronha, C. Ratti, VV, arXiv:2410.16026

Locating the critical point

H. Shah, M. Hippert, J. Noronha, C. Ratti, VV, arXiv:2410.16026



Critical point location at $O(\mu_B^2)$:

$T_c = 114.3 \pm 6.9$ MeV, $\mu_B = 602.1 \pm 62.1$ MeV

Similar to estimates from recent literature:

YLE-1: D.A. Clarke et al. (Bielefeld-Parma), arXiv:2405.10196

YLE-2: G. Basar, PRC 110, 015203 (2024)

BHE: M. Hippert et al., arXiv:2309.00579

fRG: W-J. Fu et al., PRD 101, 054032 (2020)

DSE/fRG: Gao, Pawlowski., PLB 820, 136584 (2021)

DSE: P.J. Gunkel et al., PRD 104, 052022 (2021)

FSS: A. Sorensen et al., arXiv:2405.10278

FSS-2: R. Lacey, arXiv:2411.09139

Recent development: Extrapolations of $s = const$ contours from imaginary μ_B with strangeness neutrality

↪ CP excluded at $\mu_B < 450$ MeV at a (one-sided) 2σ level

S. Borsanyi, Z. Fodor, J. Guenther, P. Parotto, A. Pasztor, C. Ratti, VV, C.-H. Wong, arXiv:2502.10267

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

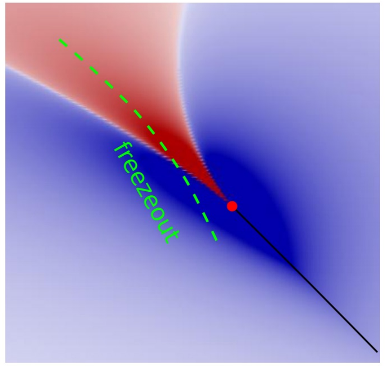
Grand partition function

$$\ln Z^{\rm gce}(T,V,\mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\rm ce}(T,V,N)\right]$$

$$\kappa_n \propto \frac{\partial^n(\ln Z^{\rm gce})}{\partial \mu^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- **(QCD) critical point:** large correlation length and fluctuations



M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR) and CERN-SPS (NA61/SHINE)

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

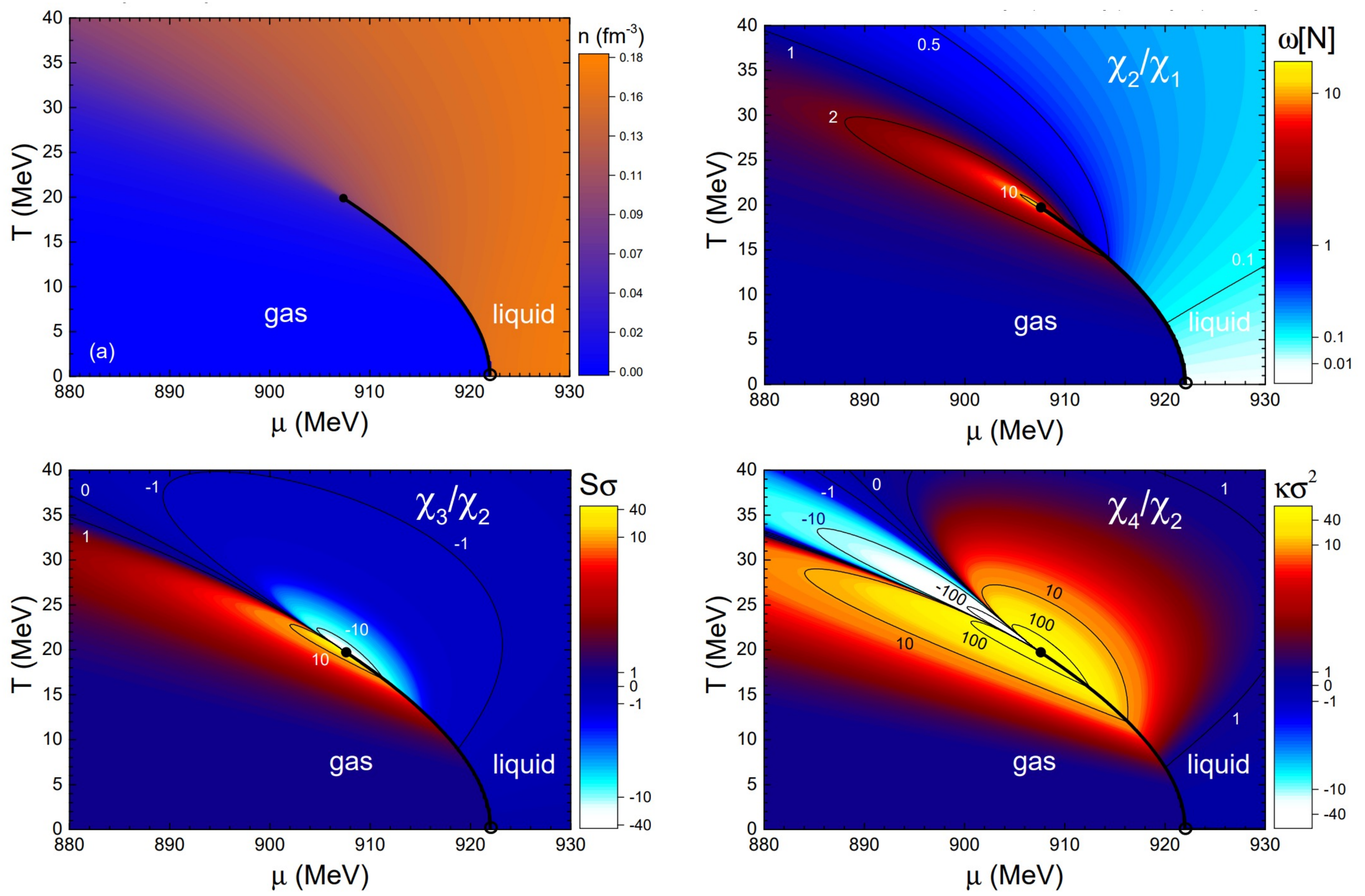
$$\xi \to \infty$$

Looking for enhanced fluctuations and non-monotonicities

Other uses of cumulants:

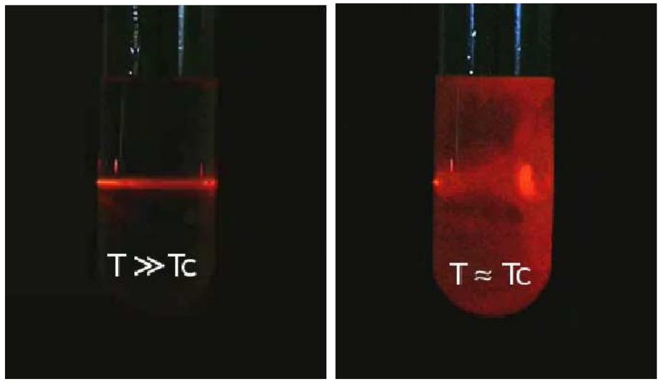
- **QCD degrees of freedom**
  Jeon, Koch, PRL 85, 2076 (2000)
  Asakawa, Heinz, Muller, PRL 85, 2072 (2000)
- Extracting the speed of sound
  A. Sorensen et al., PRL 127, 042303 (2021)
- Conservation volume V_C
  VV, Donigus, Stoecker, PRC 100, 054906 (2019)

Example: (Nuclear) Liquid-gas transition



VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Critical opalescence



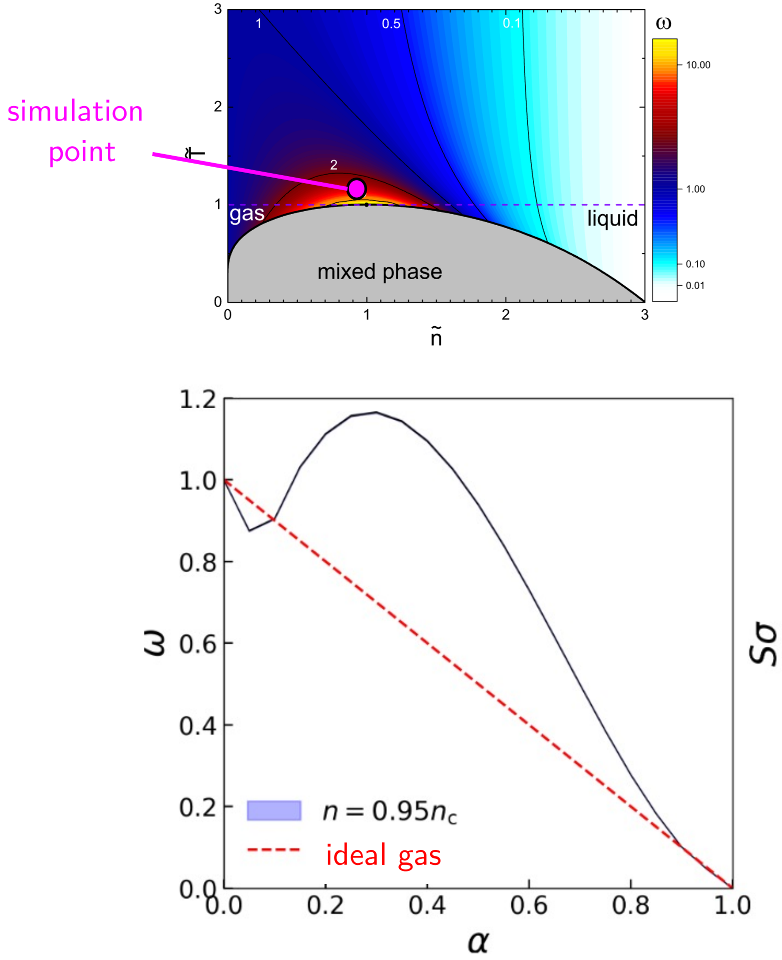
$$\langle N^2 \rangle - \langle N \rangle^2 \sim \langle N \rangle \sim 10^{23}$$

in equilibrium

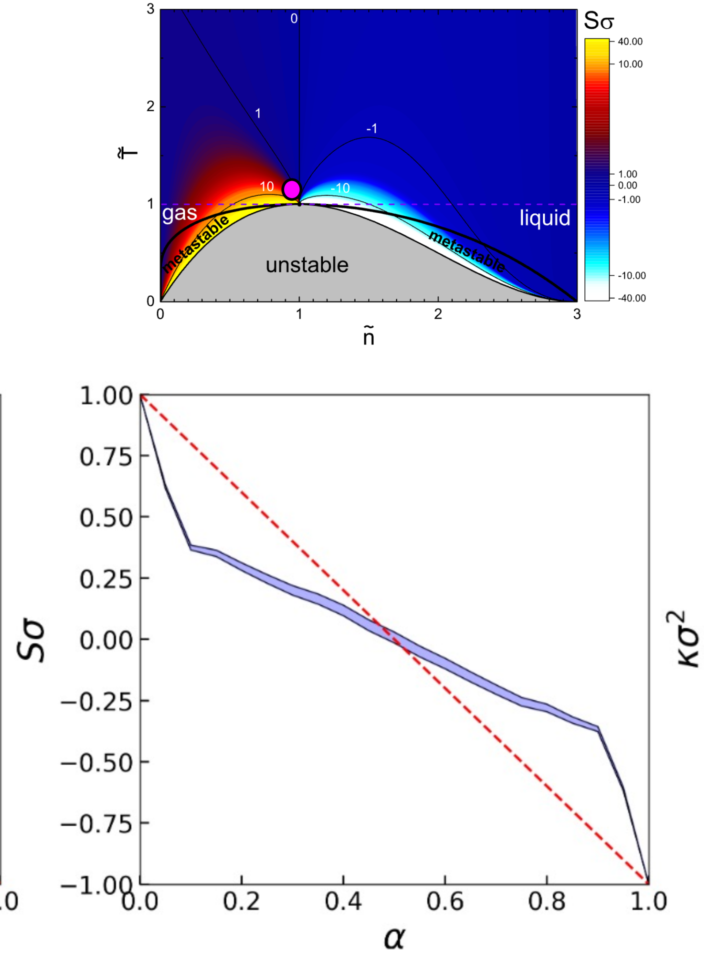
Non-Gaussian fluctuations from molecular dynamics

V. Kuznietsov, Gorenstein, Koch, VV, to appear

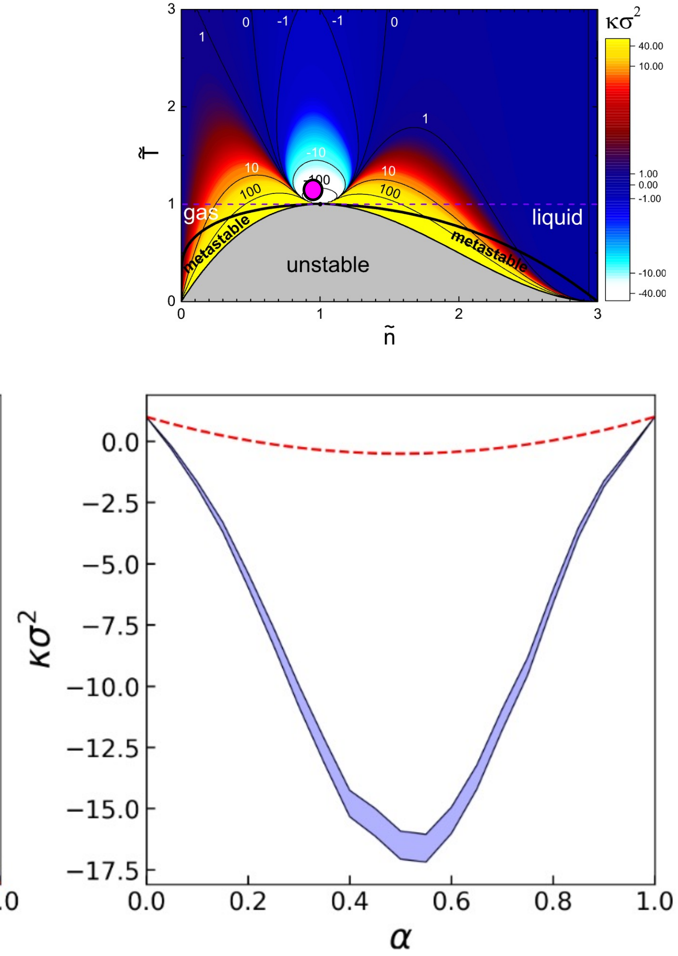
Scaled variance κ_2/κ_1



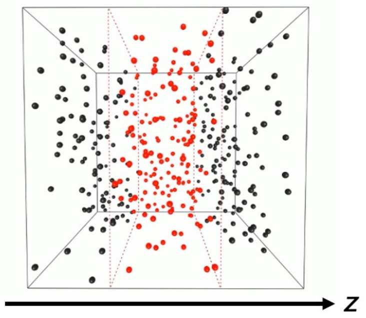
Skewness κ_3/κ_2



Kurtosis κ_4/κ_2



400 nucleons in a box



Non-Gaussian fluctuations from molecular dynamics

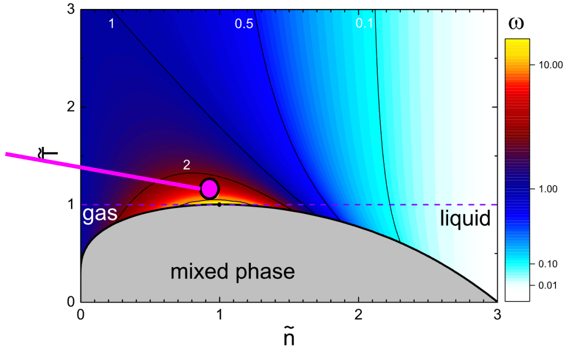
V. Kuznietsov, Gorenstein, Koch, VV, to appear

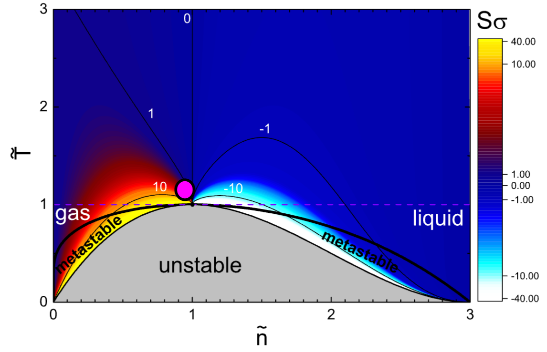
Scaled variance κ_2/κ_1

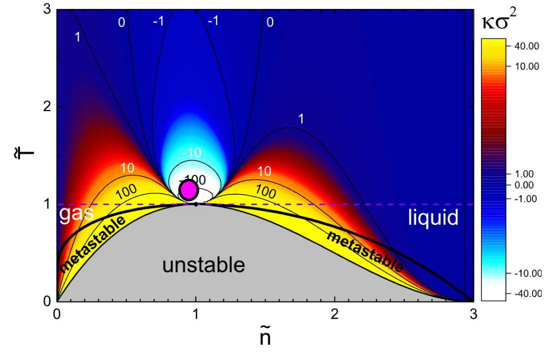
Skewness κ_3/κ_2

Kurtosis κ_4/κ_2

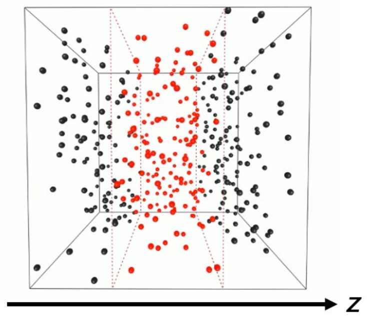
simulation point

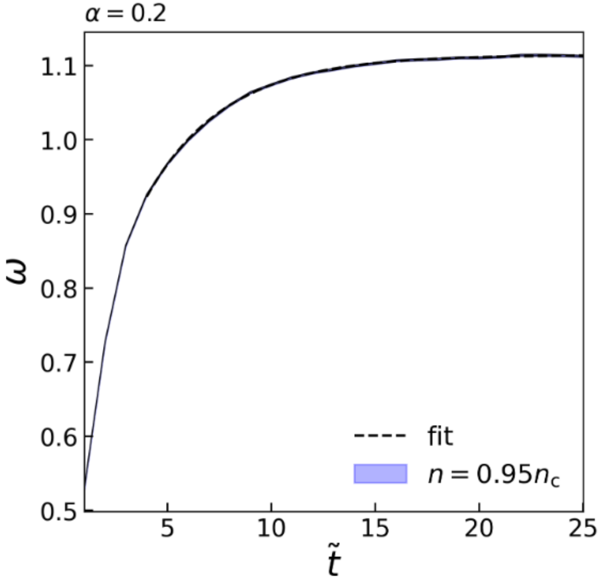


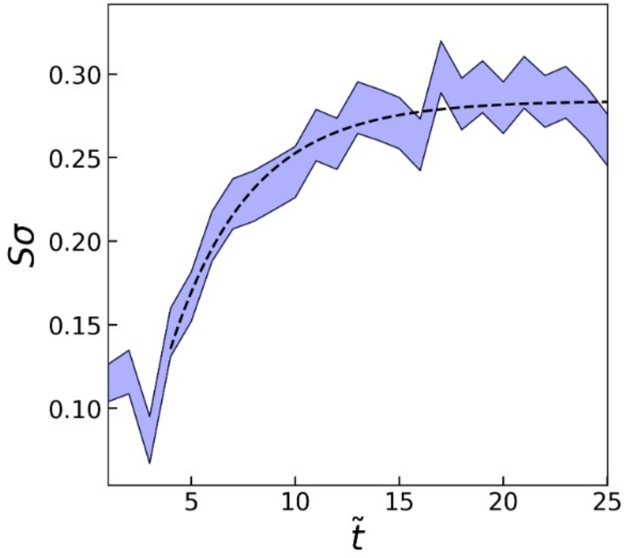


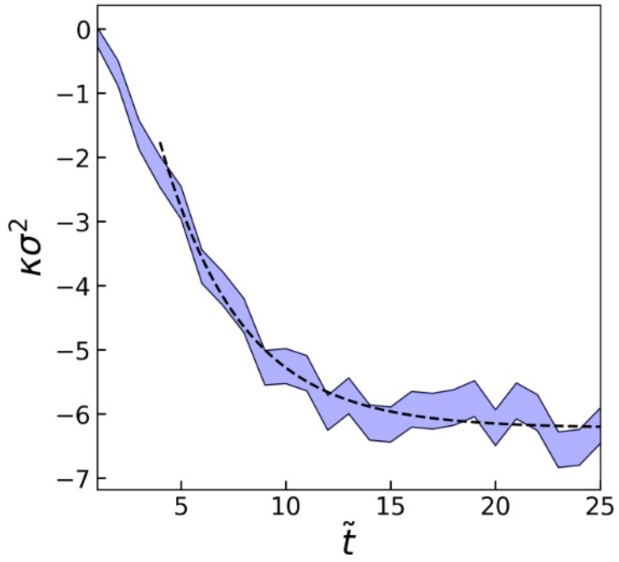


400 nucleons in a box









- (Non-)Gaussian cumulants equilibrate on comparable time scales

see also X. An et al., PRL 127, 072301 (2021); C. Chattopadhyay et al., PRL 133, 032301 (2024)

Theory vs experiment: Challenges for fluctuations

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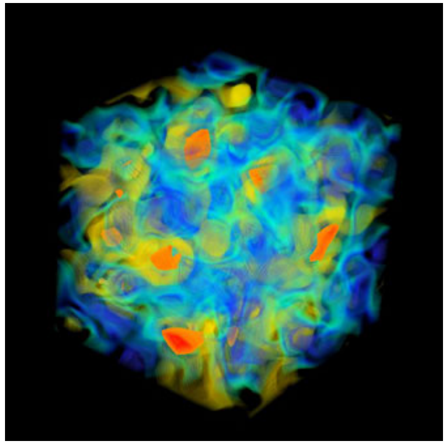
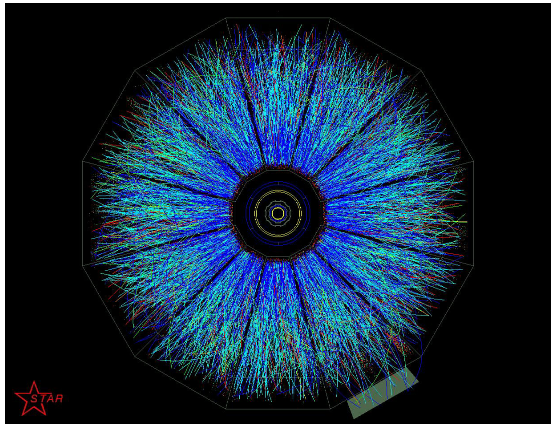
Theory



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

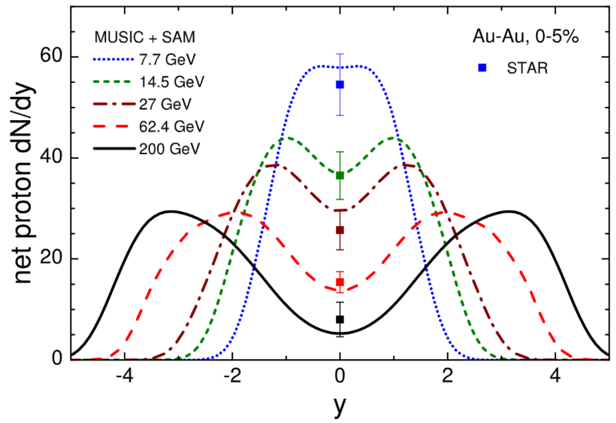
- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogeneous
- Fluctuating volume

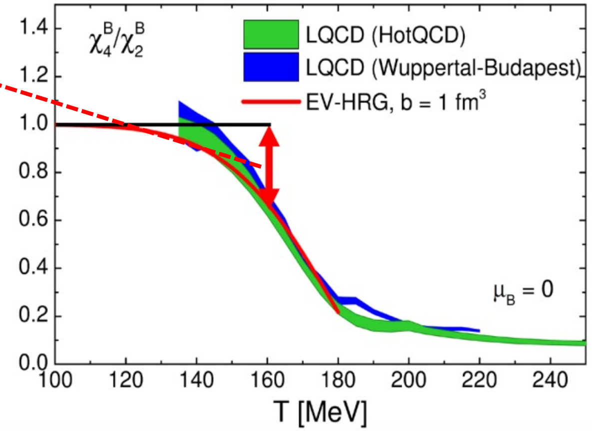
Need dynamical description

Hydro EV: Non-critical hydro baseline at RHIC-BES

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
  - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
  - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]
- Non-critical contributions computed at particlization ($\epsilon_{sw} = 0.26$ GeV/fm^3)
  - QCD-like baryon number distribution (χ_n^B) via **excluded volume** $b = 1$ fm^3 [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
  - **Exact global baryon conservation*** (and other charges)
    - Subensemble acceptance method 2.0 (analytic) [VV, Phys. Rev. C 105, 014903 (2022)]
    - or FIST sampler (Monte Carlo) [VV, Phys. Rev. C 106, 064906 (2022)] https://github.com/vlvovch/fist-sampler

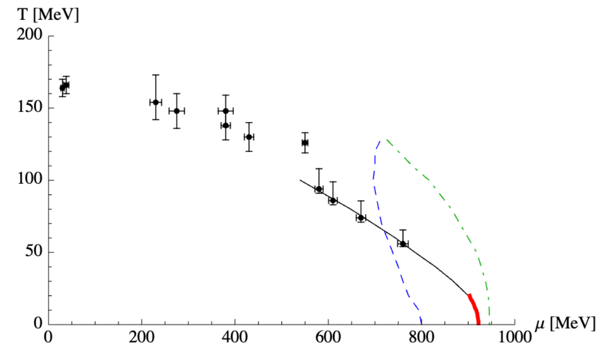




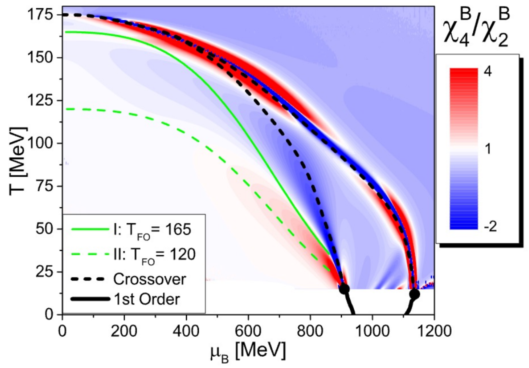
- Included: baryon conservation, repulsion, kinematical cuts
- Absent: critical point, local conservation, initial-state/volume fluctuations, hadronic phase

*If baryon conservation is the only effect (no other correlations), non-critical baseline can be computed without hydro
Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, NPA 1008, 122141 (2021)

Interplay with nuclear liquid-gas transition

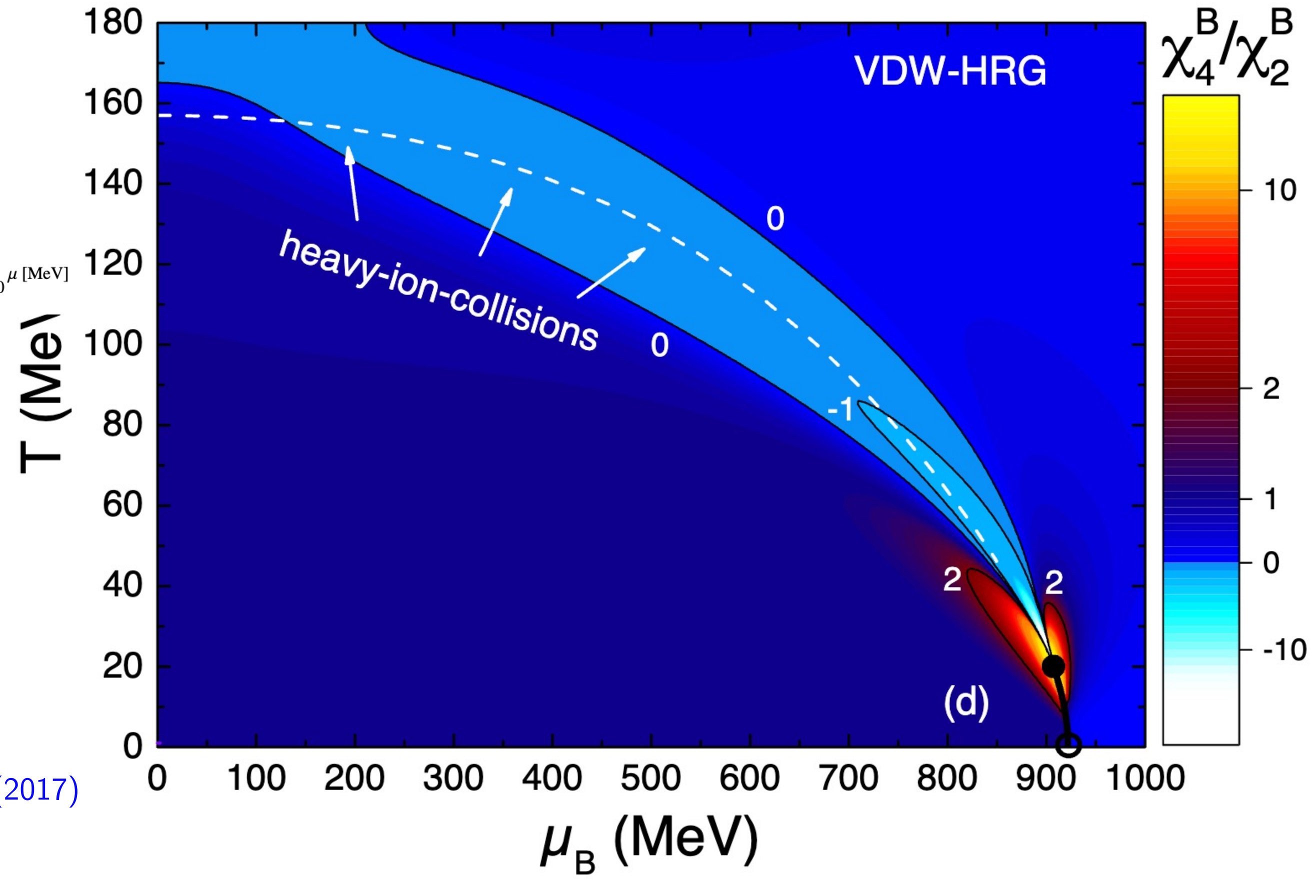


Floerchinger, Wetterich, NPA (2012)

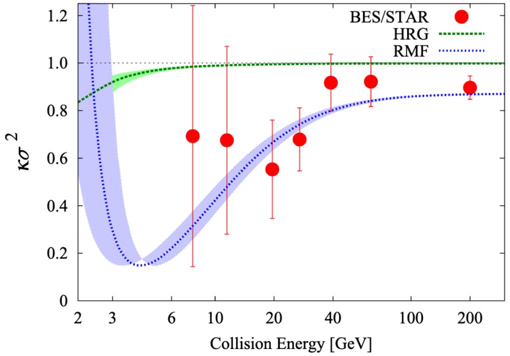


Mukherjee, Steinheimer, Schramm, PRC (2017)

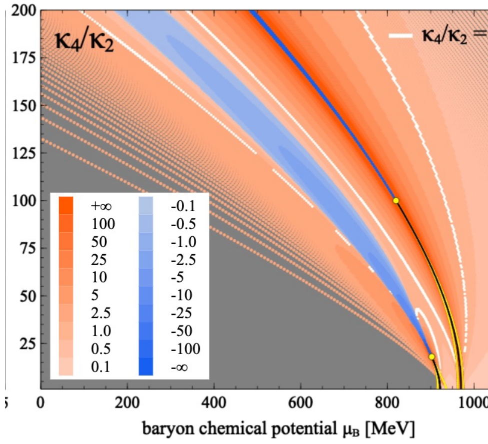
HRG with attractive and repulsive interactions among baryons



VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)



Fukushima, PRC (2014)



Sorensen, Koch, PRC (2020)

Increasingly relevant at lower energies probed through RHIC-FXT

Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV

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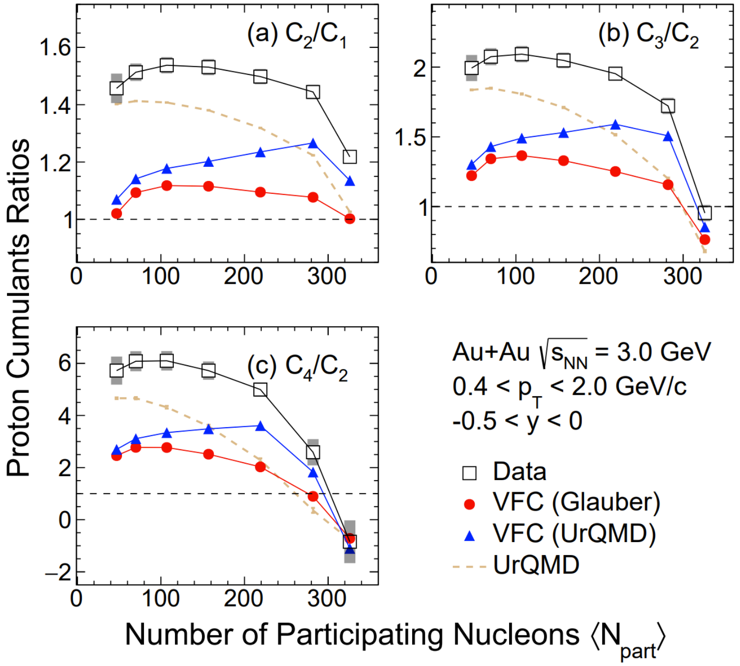
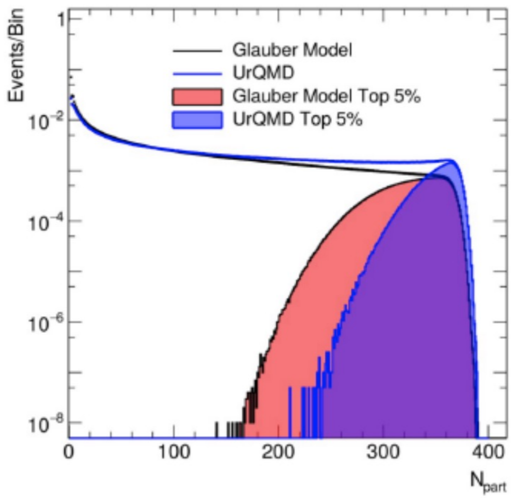


STAR Collaboration, Phys. Rev. Lett. 128 (2022) 202303

STAR-FXT



HADES

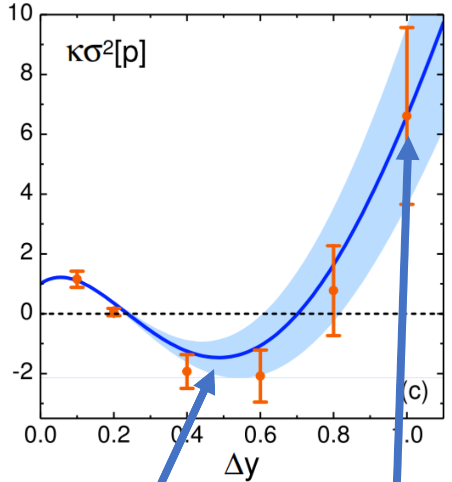


Figure from O. Savchuk et al., PLB 835, 137540 (2022)

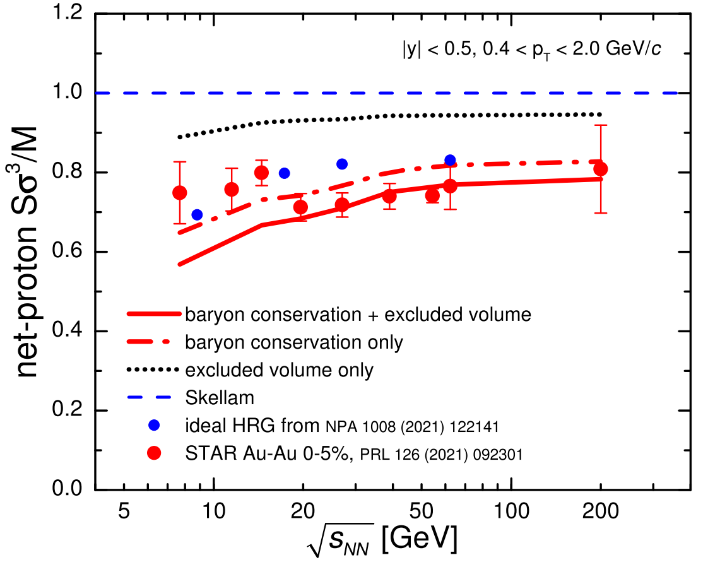
- Volume fluctuations/centrality selection appear to play an important role
  - UrQMD is useful for understanding basic systematics associated with it
- Indications for enhanced scaled variance, $\kappa_2/\kappa_1 > 1$
- κ_4/κ_2 negative and described by UrQMD (purely hadronic?), note -0.5<y<0 instead of |y|<0.5

Proper understanding of $\kappa_2/\kappa_1 > 1$ in both HADES and STAR-FXT is missing

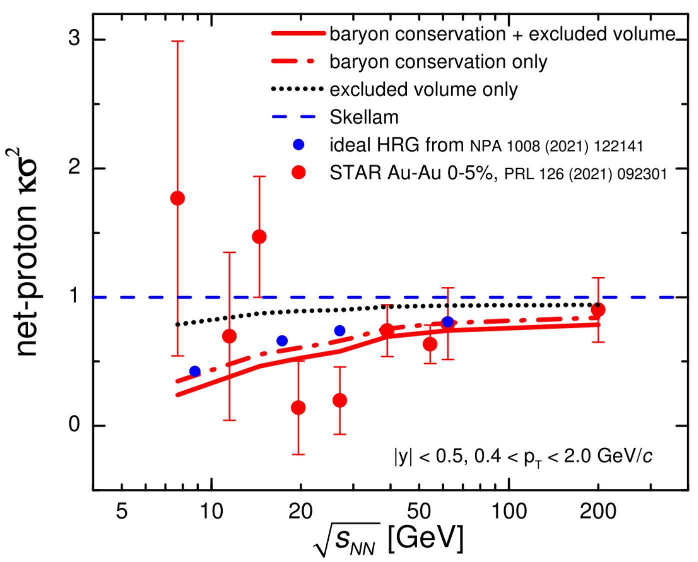
RHIC-BES-I: Net proton cumulant ratios (MUSIC)

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

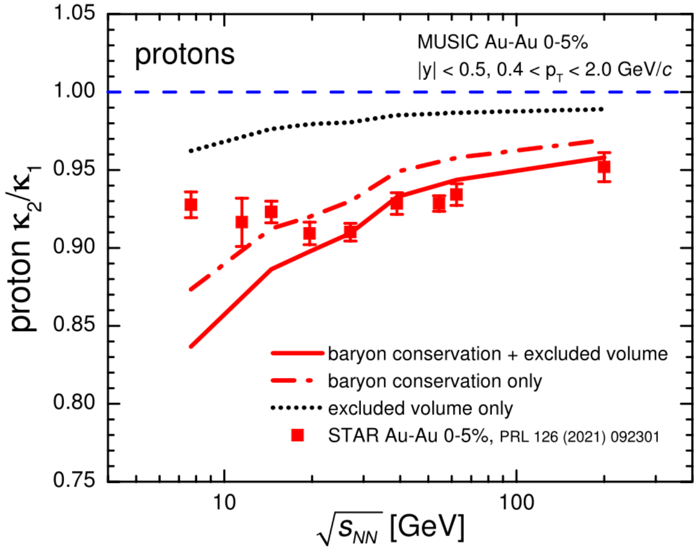
net proton C_3/C_1



net proton C_4/C_2



proton C_2/C_1

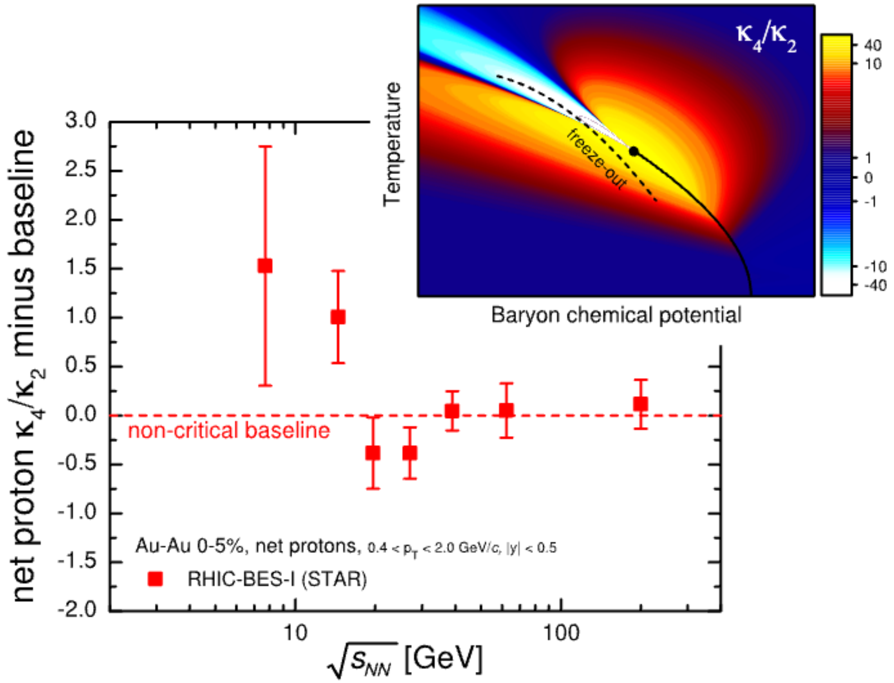


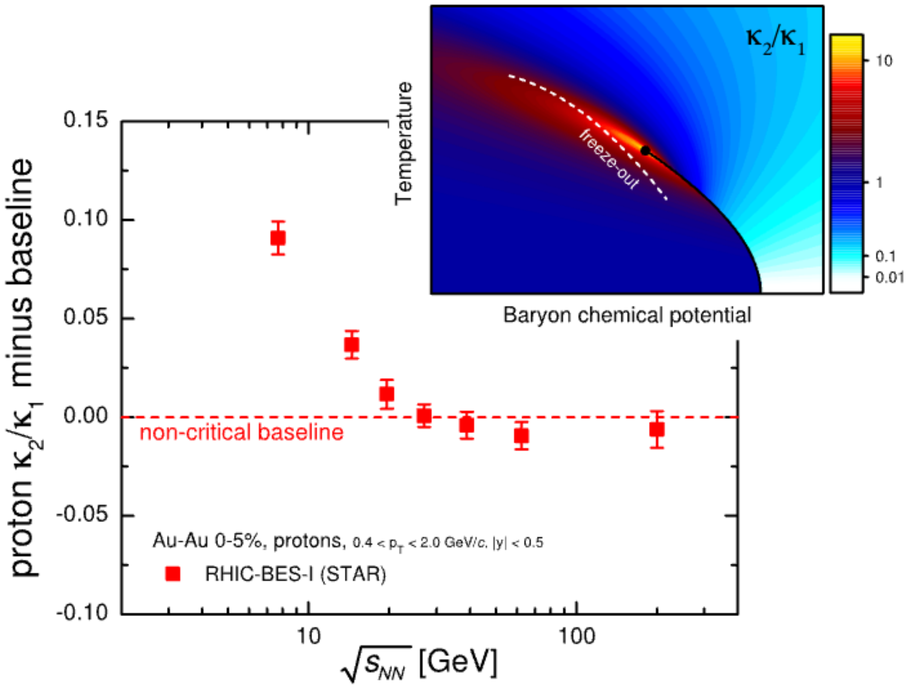
- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (BQS conservation and repulsion)
- Effect from baryon conservation is stronger than repulsion but both are required at $\sqrt{s_{NN}} \geq 20$ GeV
- Deviations from baseline at lower energies?

Hints from RHIC-BES-I

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Subtracting the hydrodynamic non-critical baseline





⚠️ **Notation:** Here we use κ_n for cumulants and $\hat{C}_n$ for factorial cumulants, STAR Collaboration uses the opposite

Non-critical cumulants

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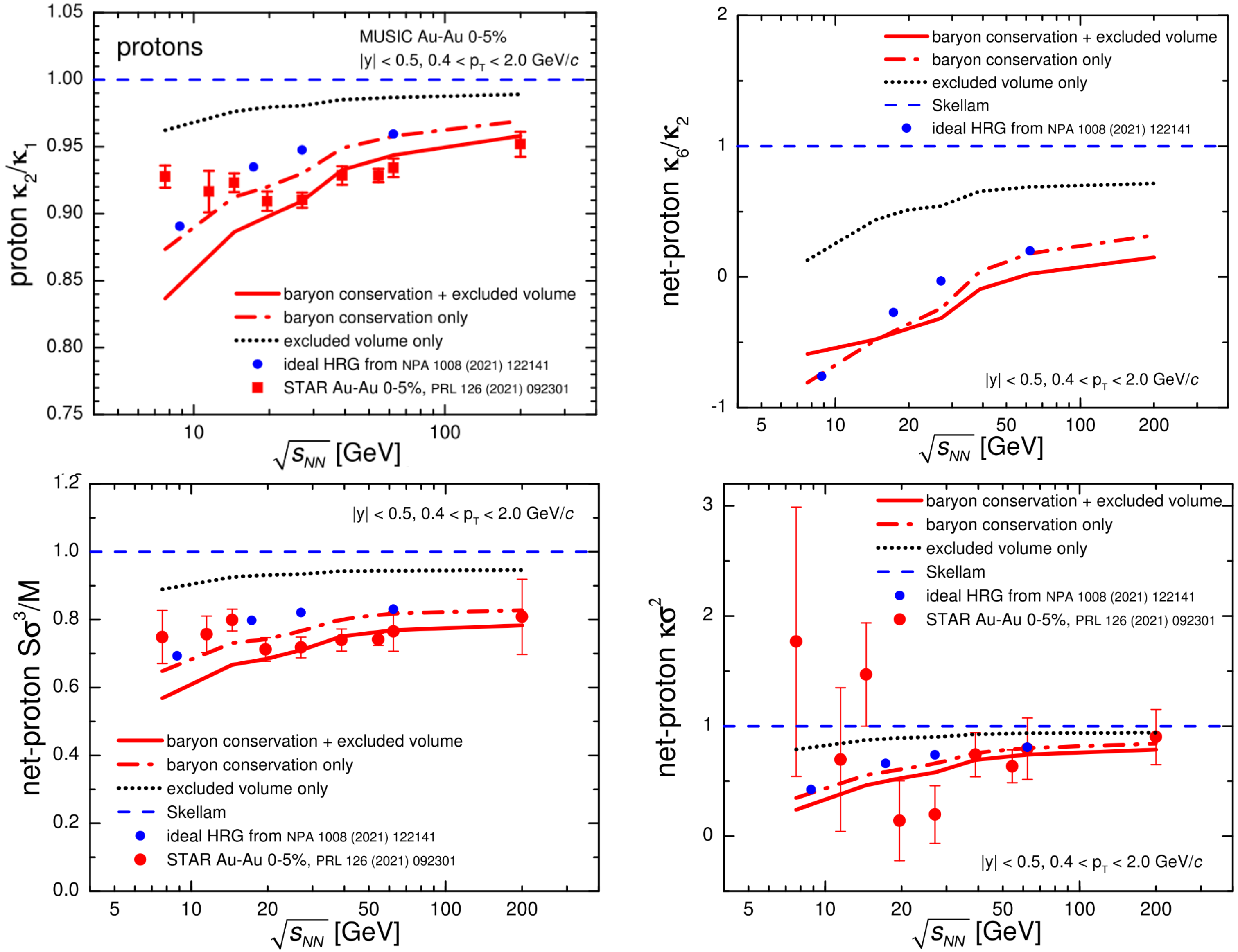
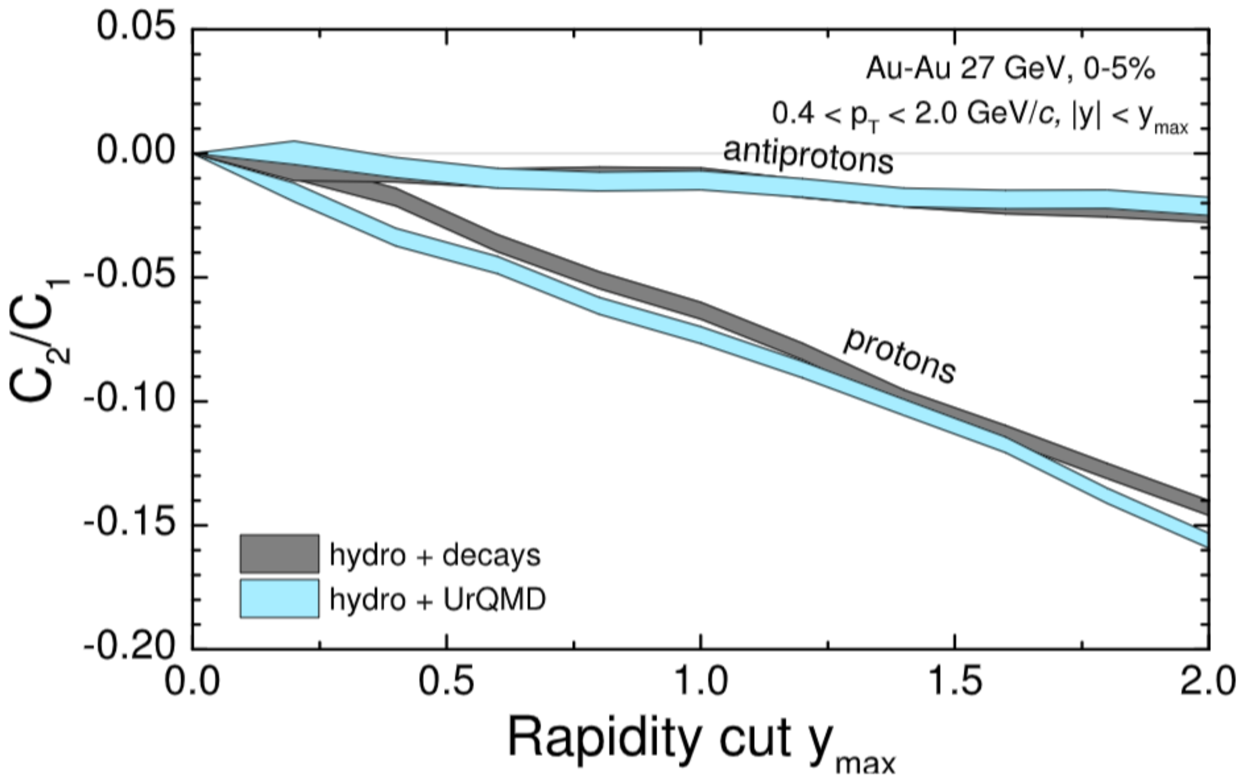
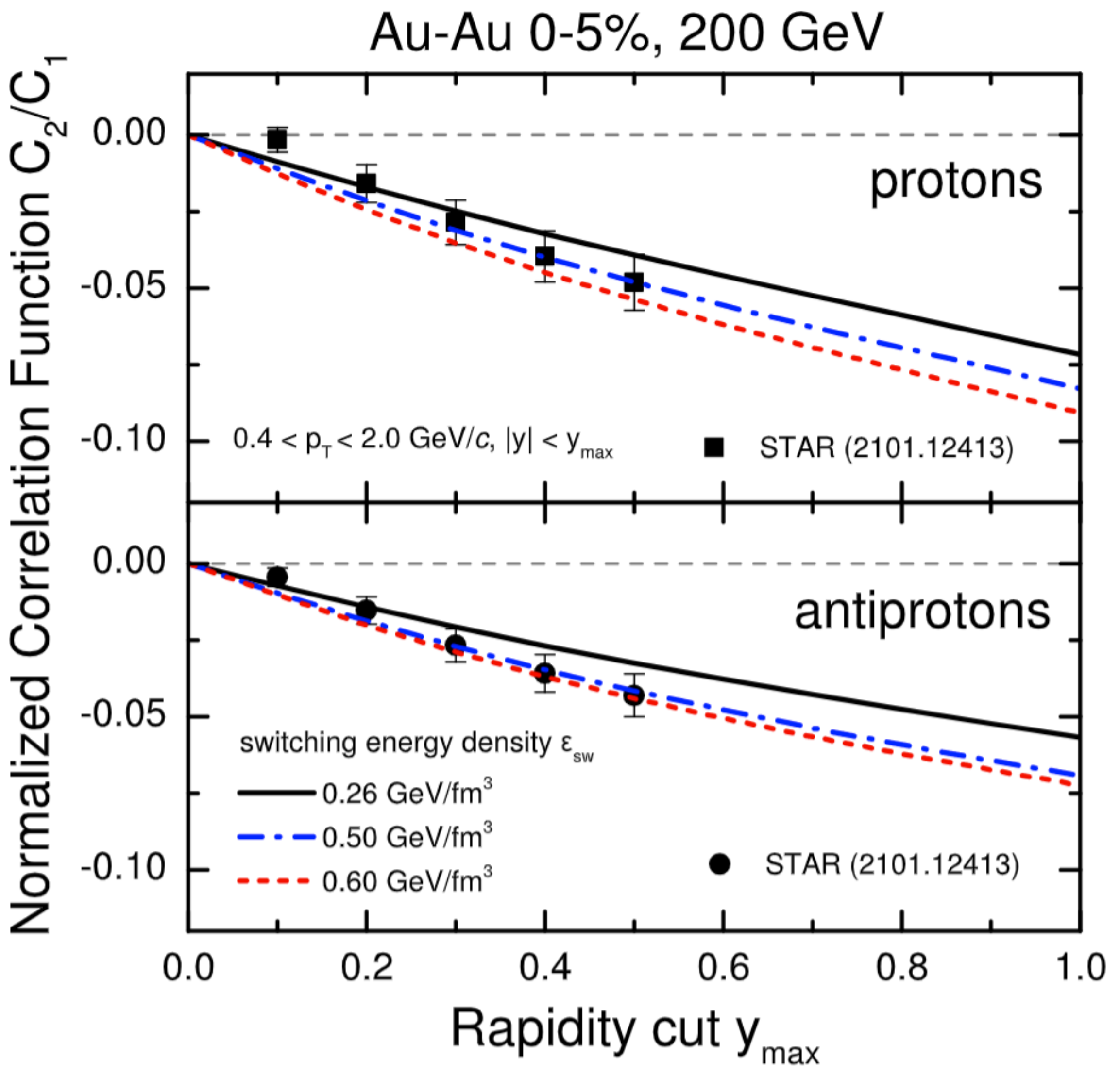


Effect of the hadronic phase

Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD



Dependence on the switching energy density

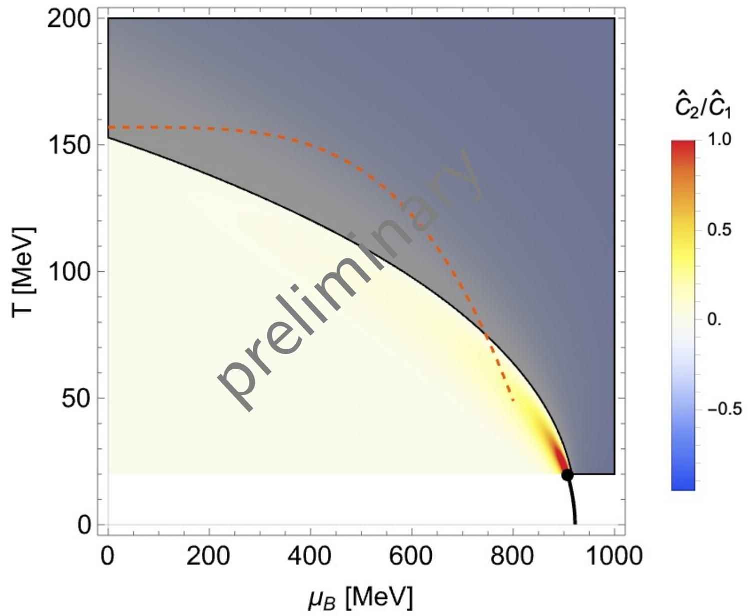


Factorial cumulants and nuclear liquid-gas transition

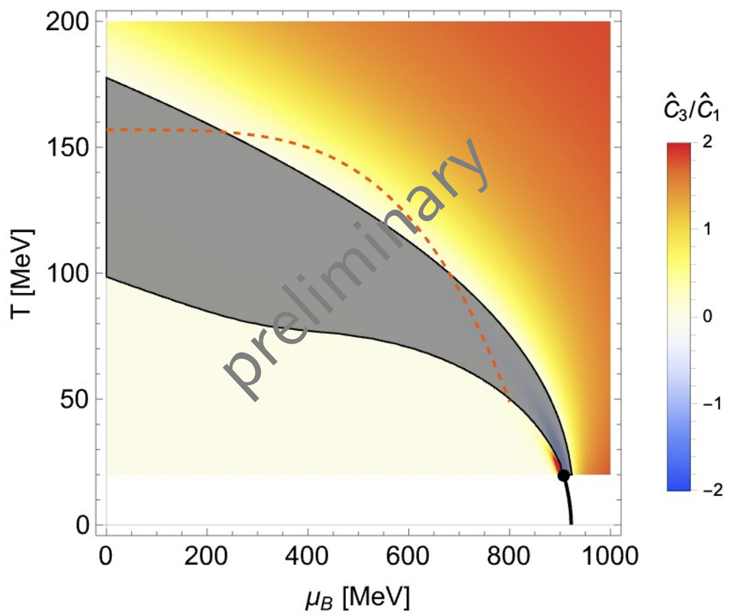
Calculation in a van der Waals-like HRG model

VV, Gorenstein, Stoecker, EPJA 54, 16 (2018)

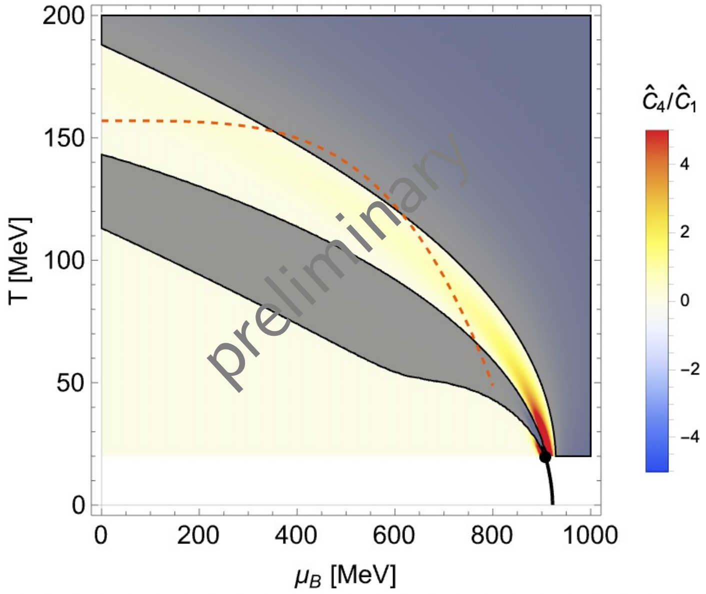
ω_2



ω_3



ω_4

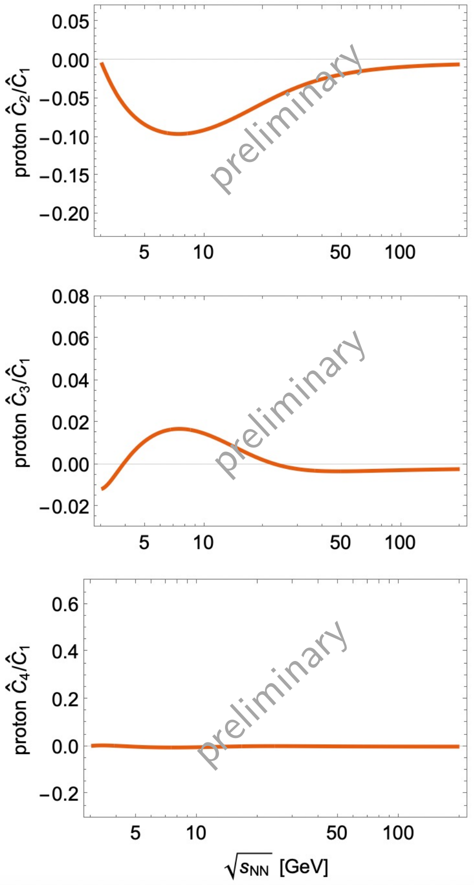


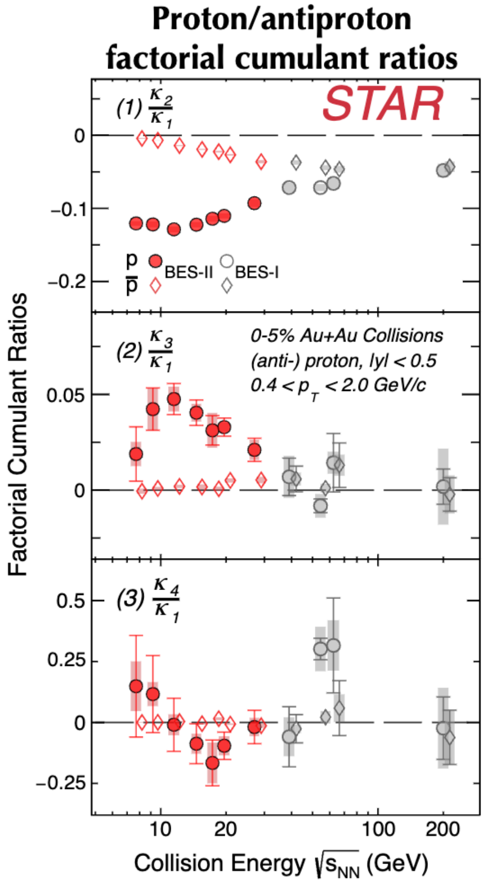
Shaded regions: negative values

Factorial cumulants and nuclear liquid-gas transition

Calculation in a van der Waals-like HRG model along the freeze-out curve*

VV, Gorenstein, Stoecker, EPJA 54, 16 (2018)





NB: The calculation is grand-canonical

*Poberezhnyuk et al., PRC 100, 054904 (2019)