Phase Structure of Strongly Interacting Matter under Extreme Conditions

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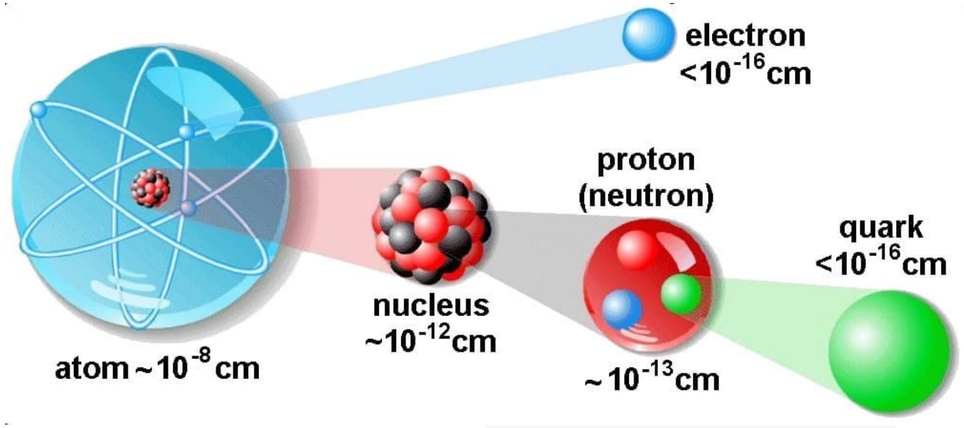
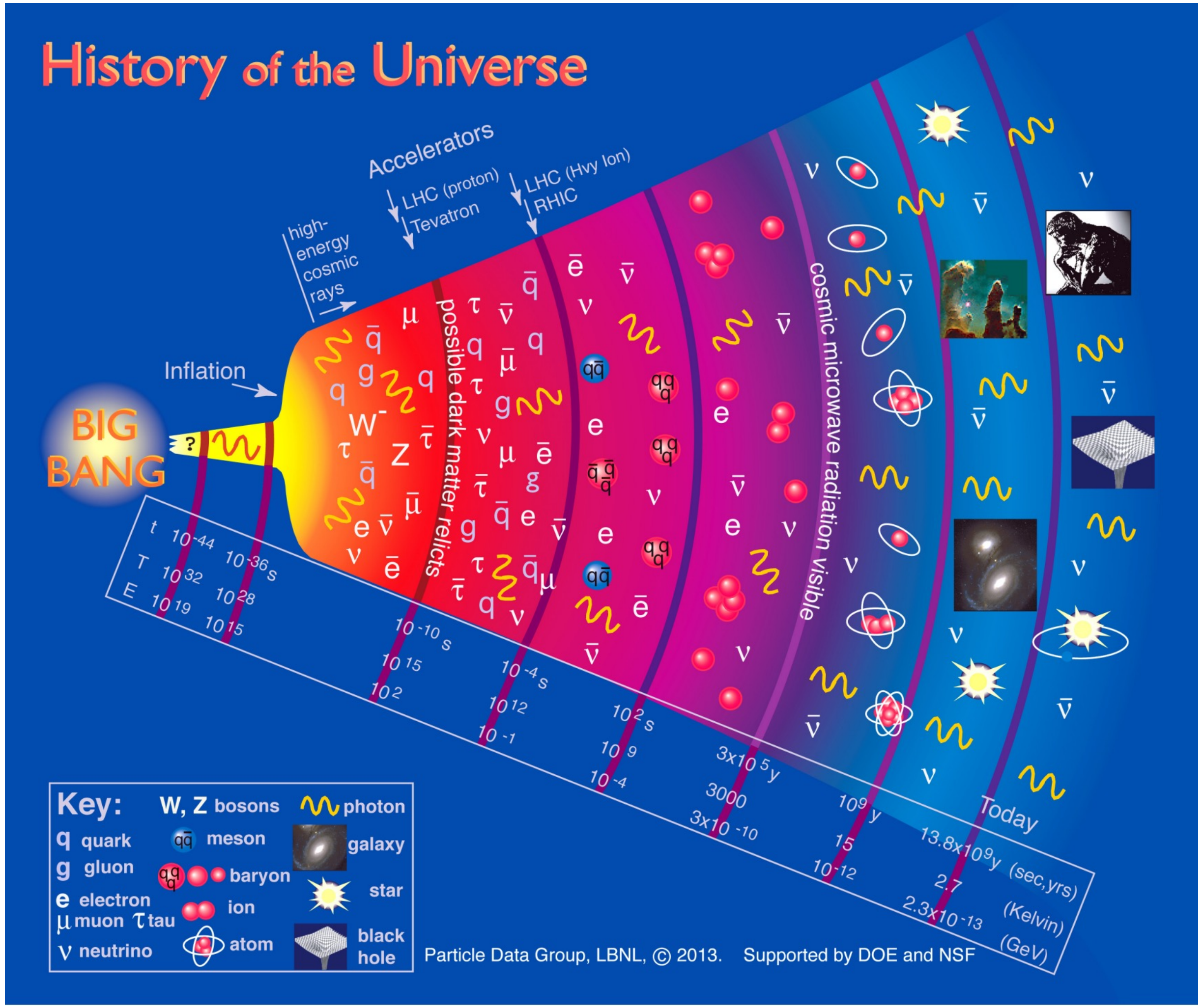
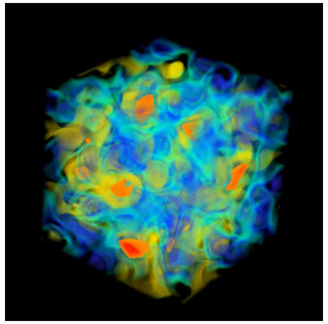
Structure of matter





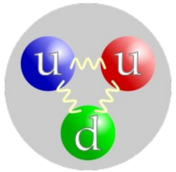
Strongly interacting matter

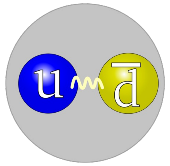


- Theory of strong interactions: *Quantum Chromodynamics* (QCD)

$$\mathcal{L} = \sum_{q=u,d,s,\dots} \bar{q}\left[i\gamma^{\mu}(\partial_{\mu} - igA^{a}_{\mu}\lambda_{a}) - m_{q}\right] q - \frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a}$$

- Basic degrees of freedom: quarks and gluons that carry color charge
- At smaller energies confined into baryons (qqq) and mesons $(q\bar{q})$



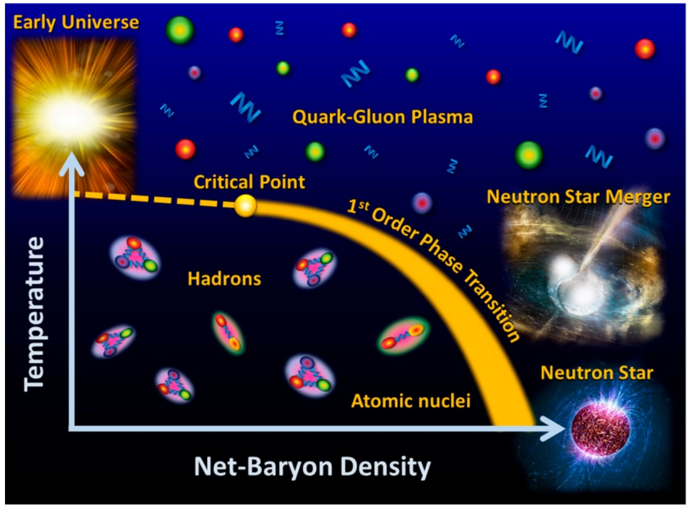


Scales

- Length: 1 femtometer $= 10^{-15}$ m
- Temperature: 100 MeV$/k_B = 10^{12}$ K

Where is it relevant?

- Early Universe
- Astrophysics: Neutron star (mergers)

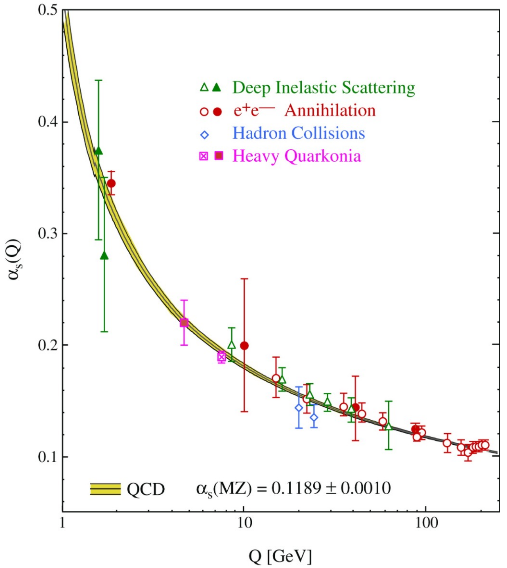


Studied in **laboratory** with heavy-ion collisions

QCD features and emergent phenomena

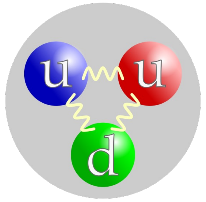
- Asymptotic freedom Gross, Politzer, Wilczek (1973)
  - Interaction becomes *weaker* at high energies/small distances
  - Theory is in perturbative regime at small distances

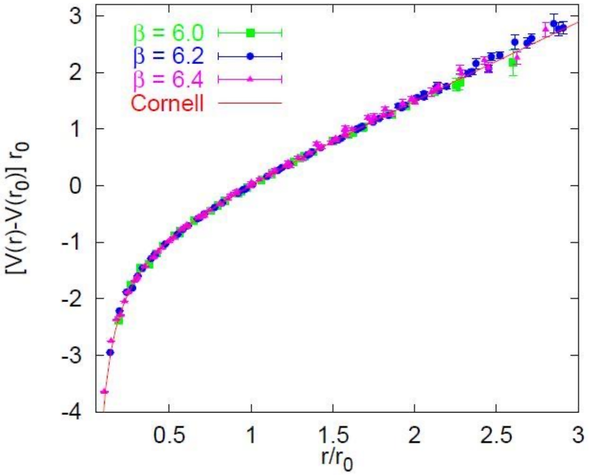
2004



S. Bethke

- Hadrons (confinement)
  - No free quarks or gluons ever observed
  - They must form composite, color-neutral objects – the hadrons
    - Proton (uud) and neutron (udd)
  - No small parameter makes the theory virtually untractable ☹





G. Bali

- Dynamical mass generation
  - Proton (uud) mass is $m_p = 938$ MeV/c^2 but $m_u + m_u + m_d \sim 15$ MeV/c^2
  - >95% of proton's mass from QCD, only <5% is from Higgs

QCD under extreme conditions

$$\mathcal{L} = \sum_{q=u,d,s,\ldots} \bar{q}\left[i\gamma^{\mu}(\partial_{\mu} - igA^{a}_{\mu}\lambda_{a}) - m_{q}\right] q - \frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a}$$

What we know

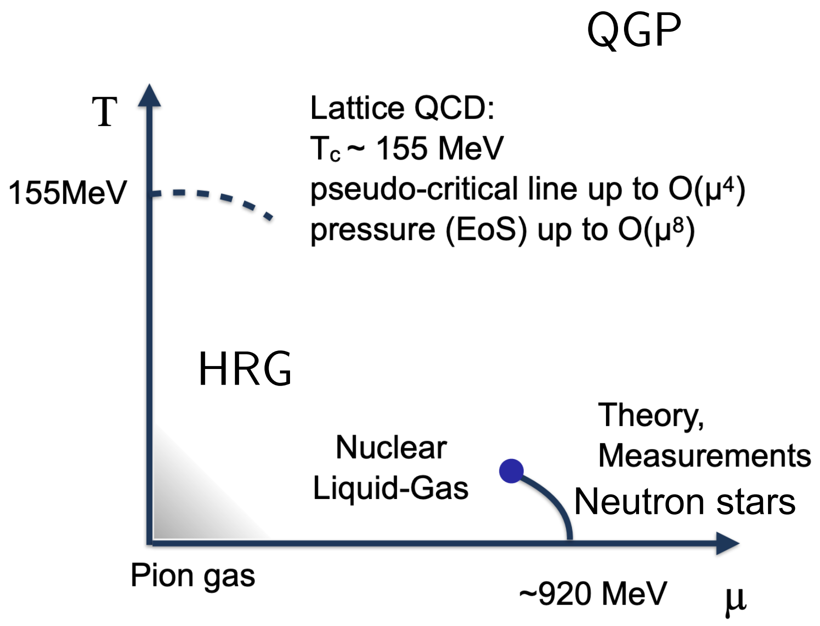
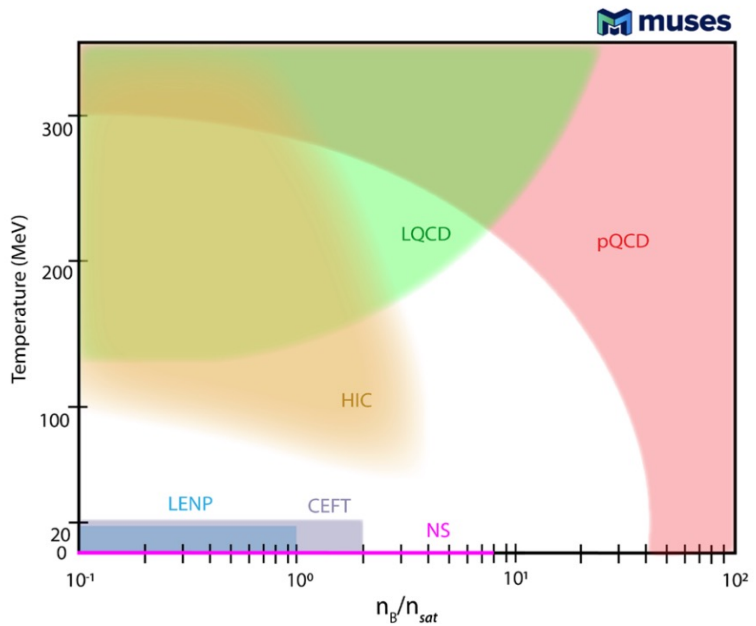


Figure courtesy of V. Koch

What we think we know



MUSES Collaboration, LRR 27 (2024)

What we hope to know

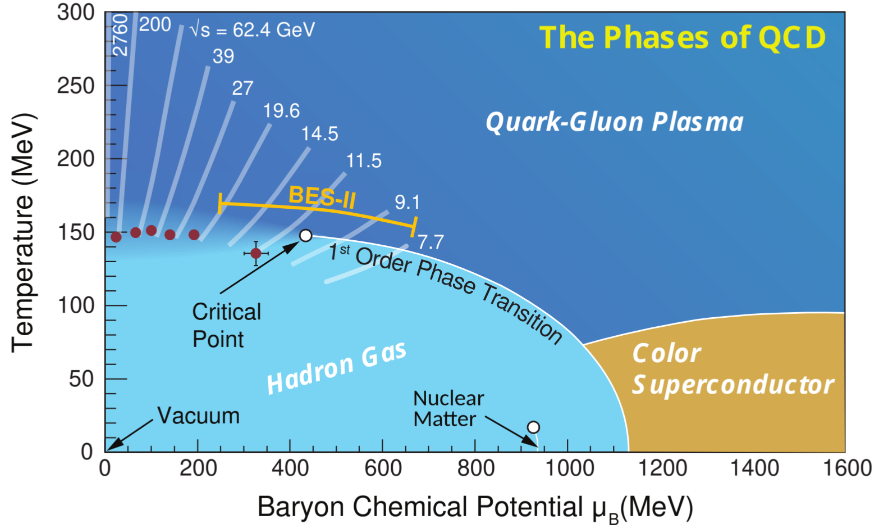


Figure from Bzdak et al., Phys. Rept. '20 & 2015 US Nuclear LRP

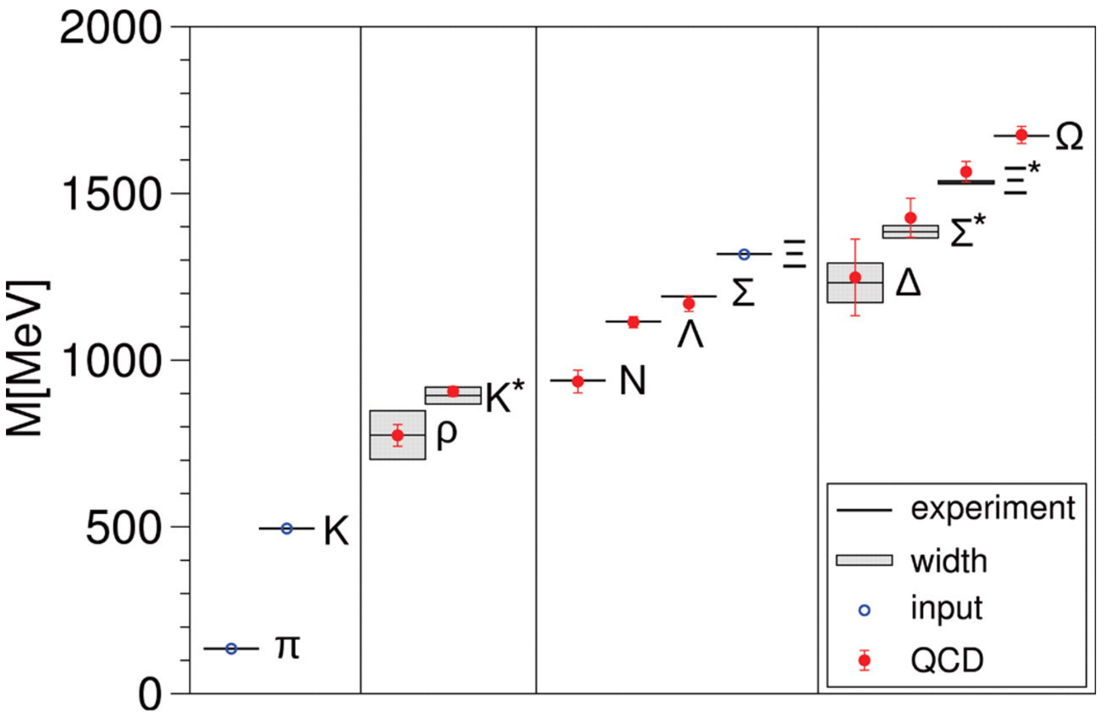
- Dilute hadron gas at low T & μ_{B} due to confinement, quark-gluon plasma high T & μ_{B}
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured

QCD Phase Diagram:
From zero to non-zero density

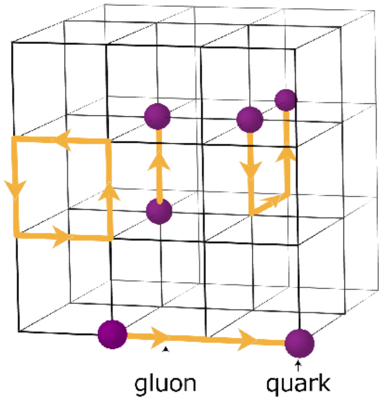
Non-perturbative methods

First-principle tool: **Lattice QCD**

Ab-initio calculation of hadron masses



BMW Collaboration, Science 322, 1224 (2008)



Remarkable agreement of QCD with the experiment

QCD transition from lattice QCD

$$Z = \mathrm{Tr}(e^{-(\hat{H}-\mu\hat{N})/T}) = \int DU \det M[U,\mu]\, e^{-S_{YM}} \quad \Longrightarrow \quad P = P(T,\mu)$$

equation of state

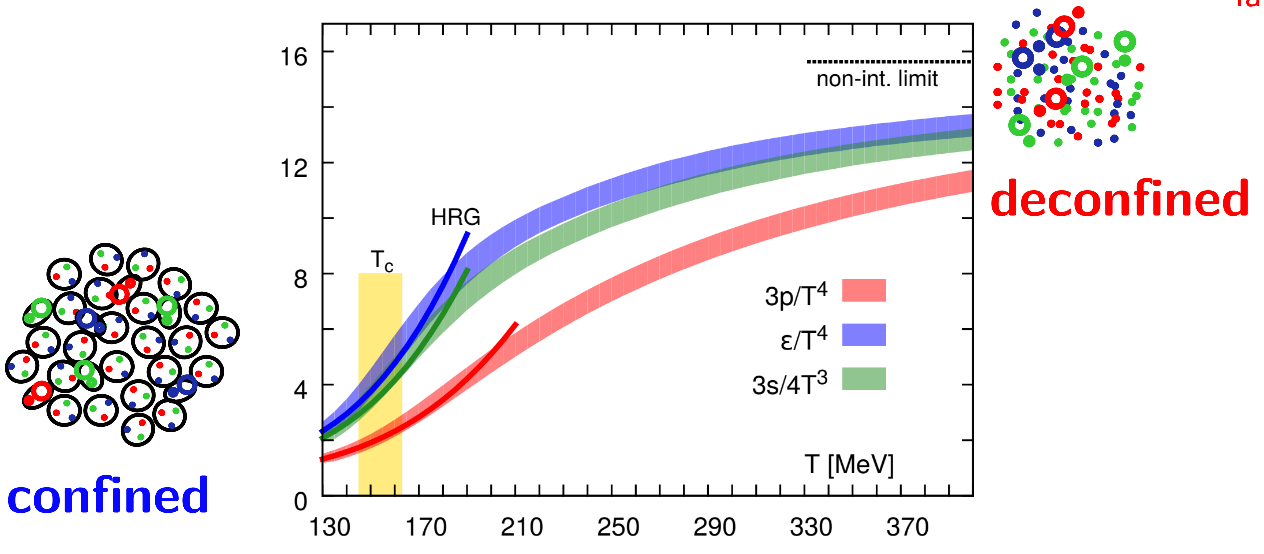


Figure from HotQCD Collaboration, PRD '14

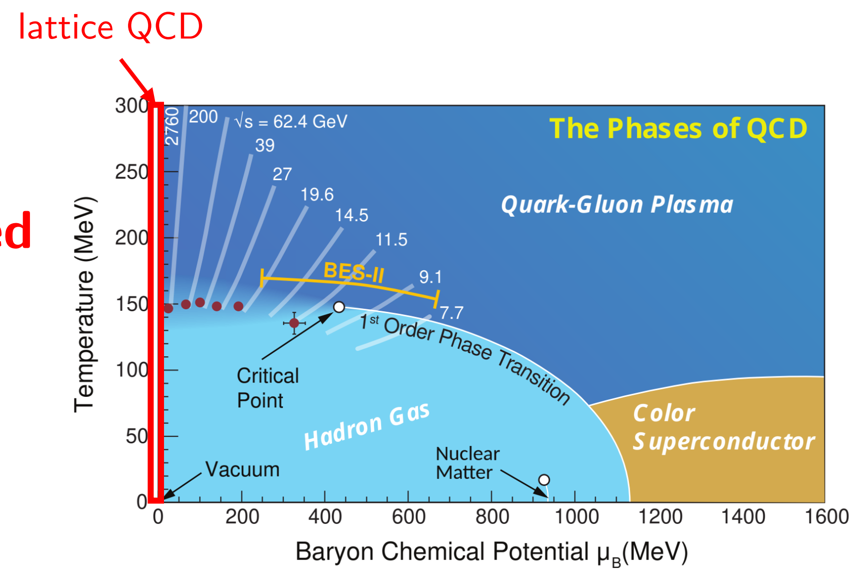


Figure from Bzdak et al., Phys. Rept. '20

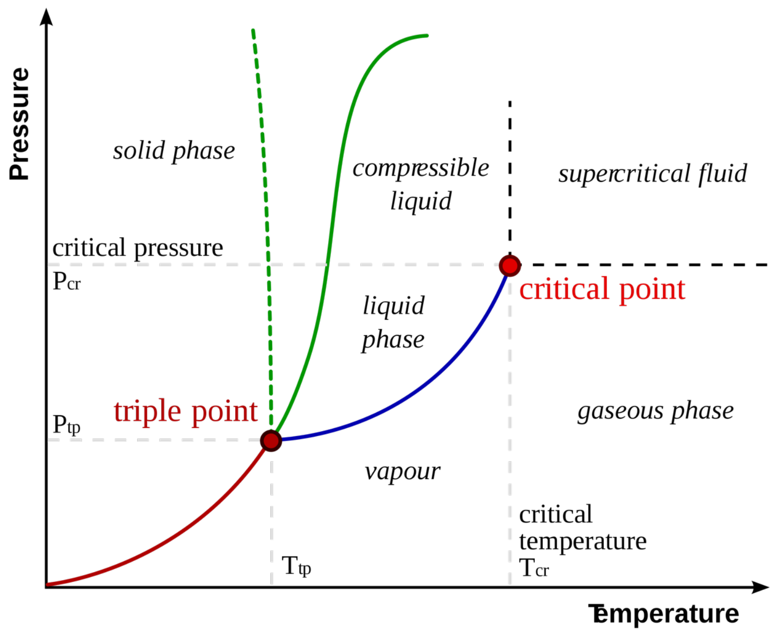
- Analytic crossover at vanishing net baryon density at $T_{pc} \approx 155$ MeV – a first-principle result [Y. Aoki et al., Nature 443, 675 (2006)]
- **Finite density:** $\mu_B > 0$ (excess of baryons over antibaryons) encounters the **sign problem**

$$\det M[U,\mu] = |\det M[U,\mu]|\, e^{i\theta}$$

The challenge of discovering the QCD critical point

QCD critical point

Ordinary fluid (e.g. water)



QCD

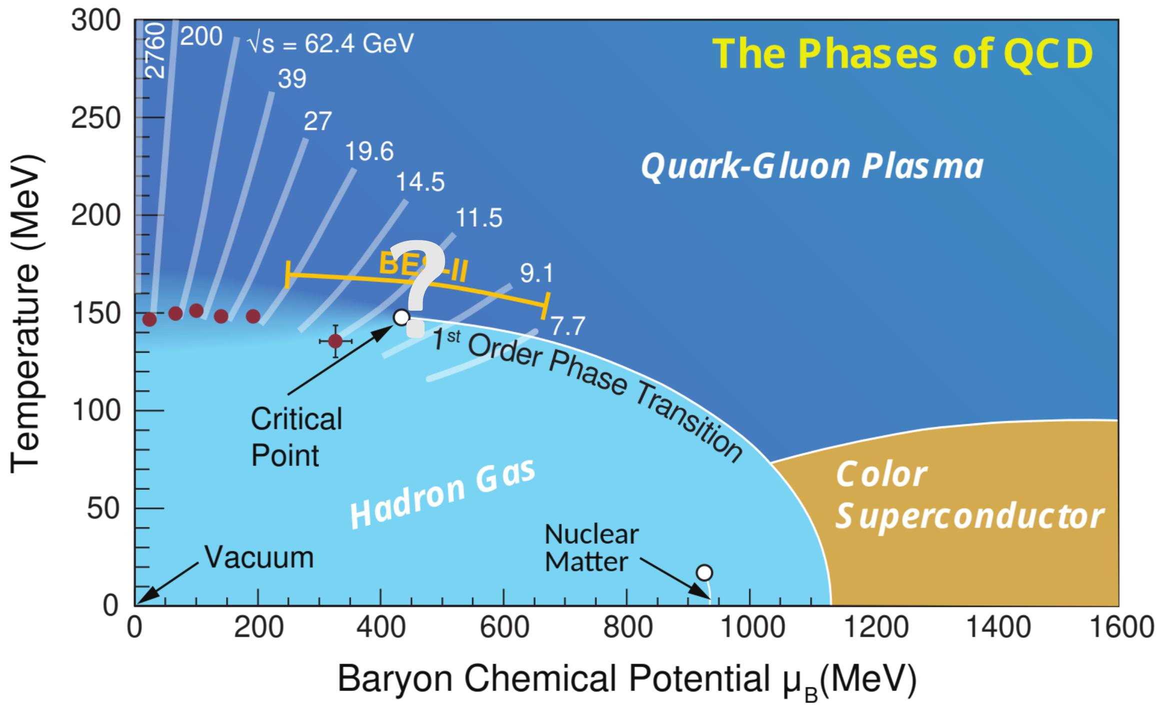


Figure from Bzdak et al., Phys. Rept. '20 and 2015 Nuclear Long Range Plan

What is the nature of the quark-hadron transition at finite baryon density?

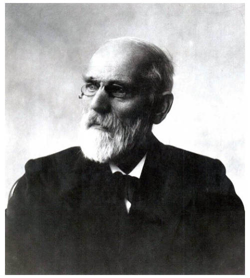
*Is there a QCD phase transition and **critical point**? Where?*

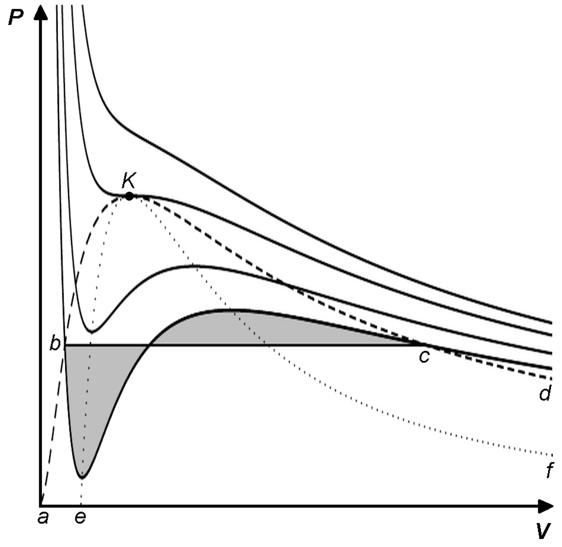
Lattice QCD: sign problem prevents simulations at non-zero baryon density

Heavy-ion collisions: access to finite density but might be too short-lived to observe a signal

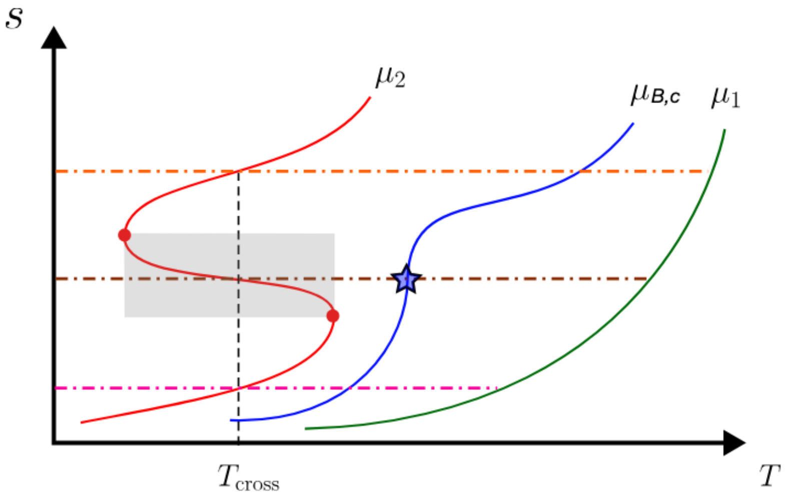
Extrapolating critical point from lattice QCD

van der Waals (1873)





change the variables



Critical Point:

$$\left(\frac{\partial P}{\partial \rho_B}\right)_T = 0, \qquad \left(\frac{\partial^2 P}{\partial \rho_B^2}\right)_T = 0.$$

$$\left(\frac{\partial T}{\partial s}\right)_{\mu_B} = 0, \qquad \left(\frac{\partial^2 T}{\partial s^2}\right)_{\mu_B} = 0.$$

Shah, Hippert, Noronha, Ratti, VV, arXiv:2410.16026

Extrapolate from $\mu_B = 0$!

Looking for entropy crossings

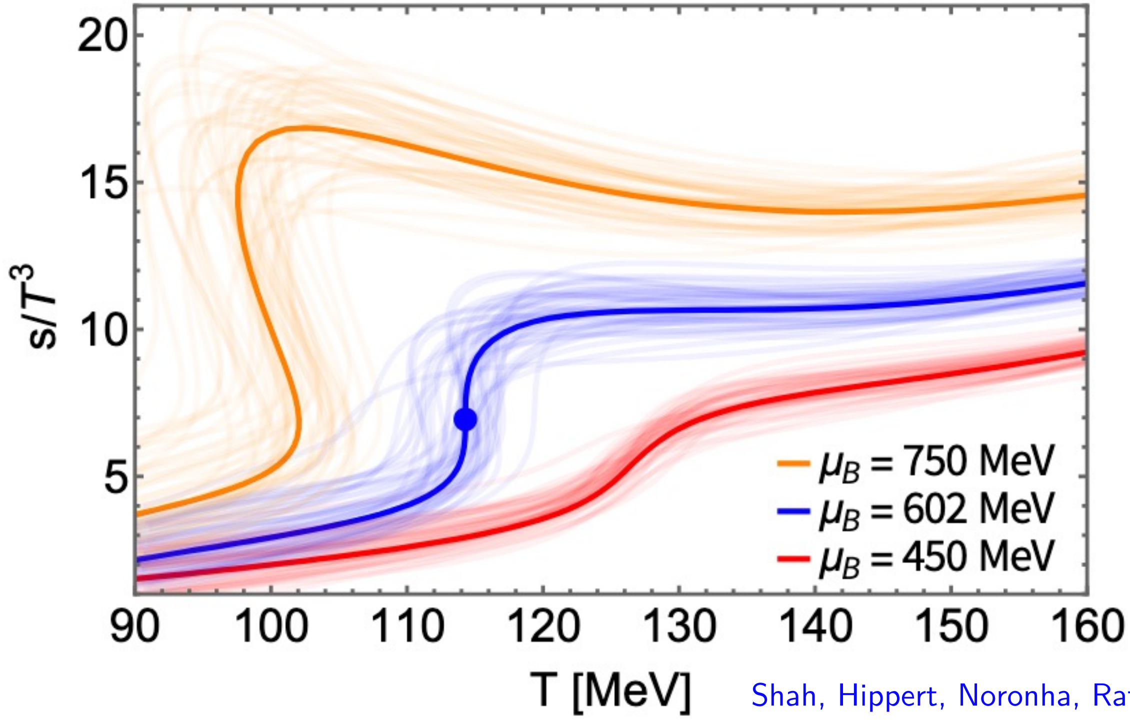
- Critical point ruled out (2σ level) at $\mu_B <$ 400 MeV

Borsanyi et al., arXiv:2502.10267

- Try going further

Expansion around $\mu_B = 0$

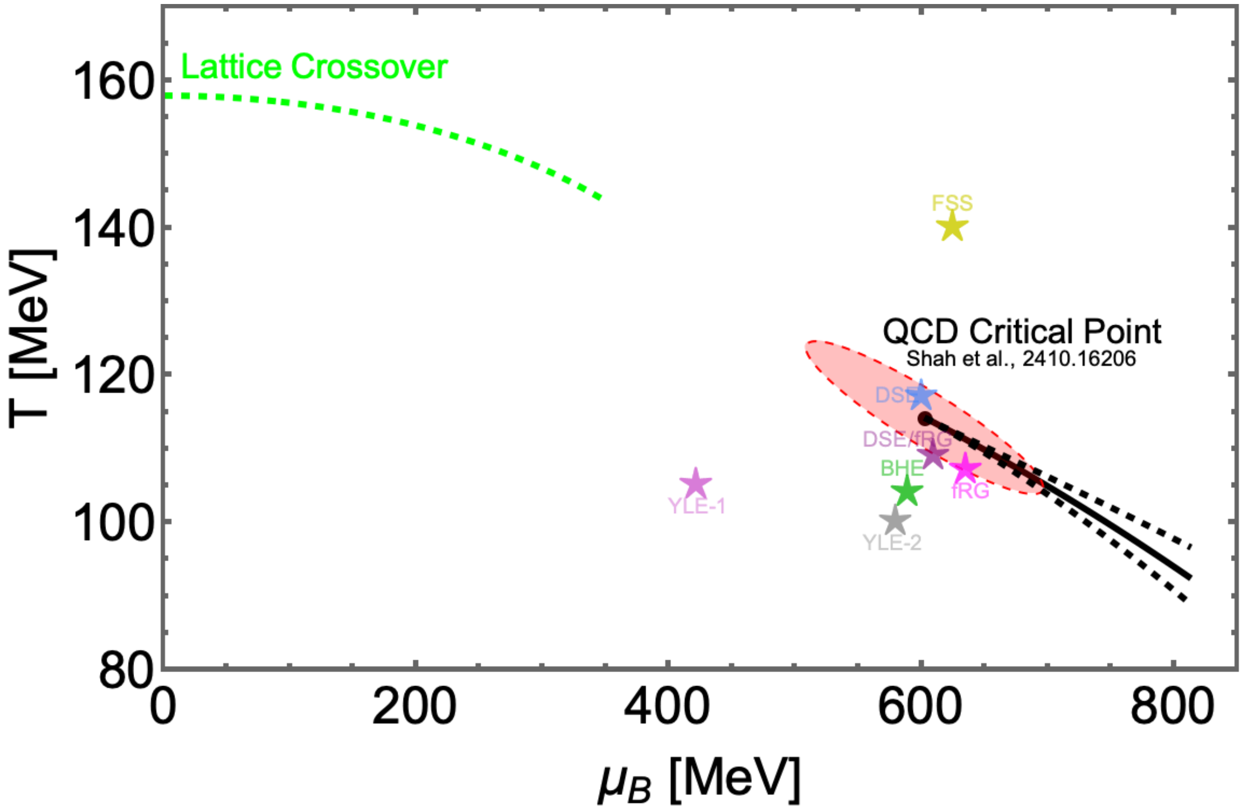
$$T_s(\mu_B; T_0) = T_0 + \alpha_2(T_0)\frac{\mu_B^2}{2}$$



Shah, Hippert, Noronha, Ratti, VV, arXiv:2410.16026

- First-order phase transition emerges at $\mu_B >$ 600 MeV

QCD critical point estimates



Critical point estimate at $O(\mu_B^2)$:

$T_c = 114 \pm 7$ MeV, $\mu_B = 602 \pm 62$ MeV

Estimates from recent literature:

YLE-1: D.A. Clarke et al. (Bielefeld-Parma), arXiv:2405.10196

YLE-2: G. Basar, PRC 110, 015203 (2024)

BHE: M. Hippert et al., arXiv:2309.00579

fRG: W-J. Fu et al., PRD 101, 054032 (2020)

DSE/fRG: Gao, Pawlowski., PLB 820, 136584 (2021)

DSE: P.J. Gunkel et al., PRD 104, 052022 (2021)

FSS: A. Sorensen et al., arXiv:2405.10278

Optimist's view: Different estimates converge onto the same region because QCD CP is likely there

Pessimist's view: Different estimates converge onto the same region because it's the closest not yet ruled out by LQCD

Can be tested in laboratory with **heavy-ion collisions**

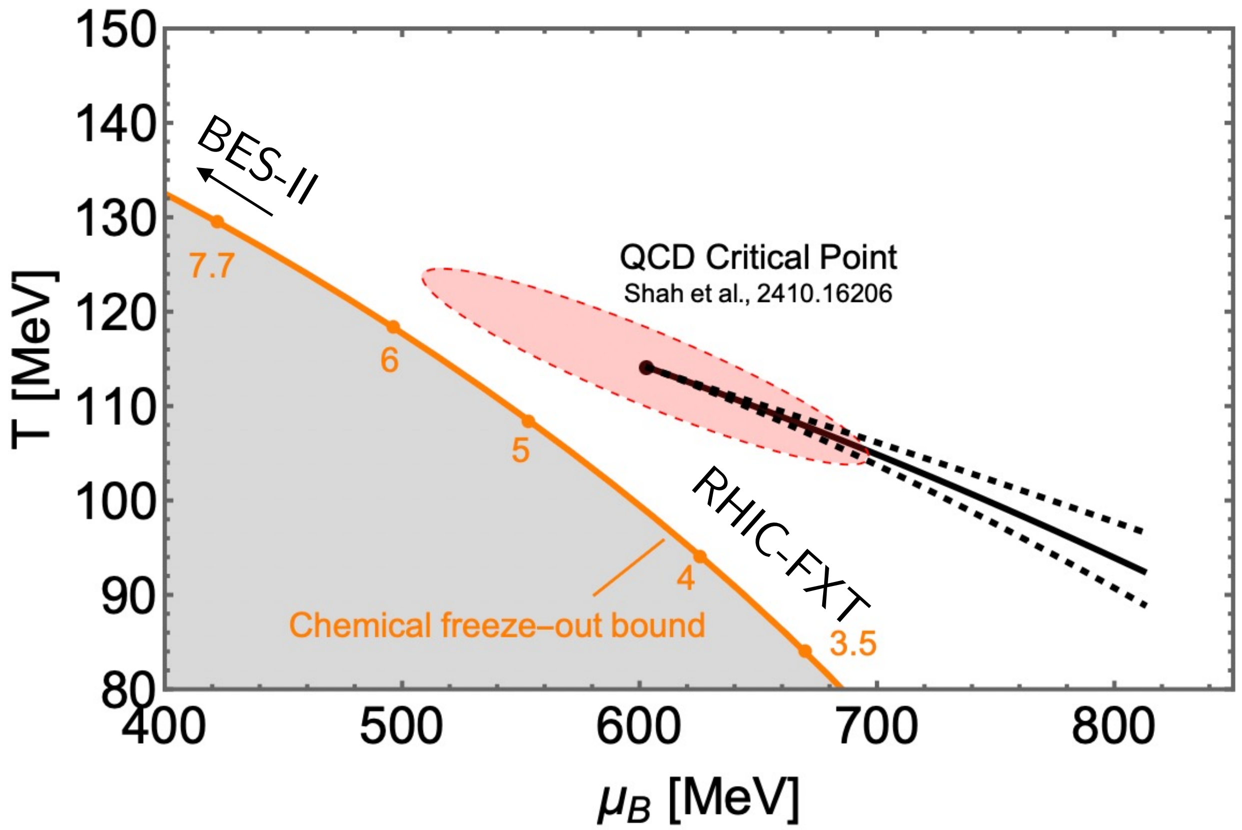
Critical point and heavy-ion collisions

Control parameters

- Collision energy $\sqrt{s_{NN}} = 2.4 - 5020$ GeV
  - Scan the QCD phase diagram
- Size of the collision region
  - Expect stronger signal in larger systems

Measurements

- Final hadron abundances and momentum distributions **event-by-event**



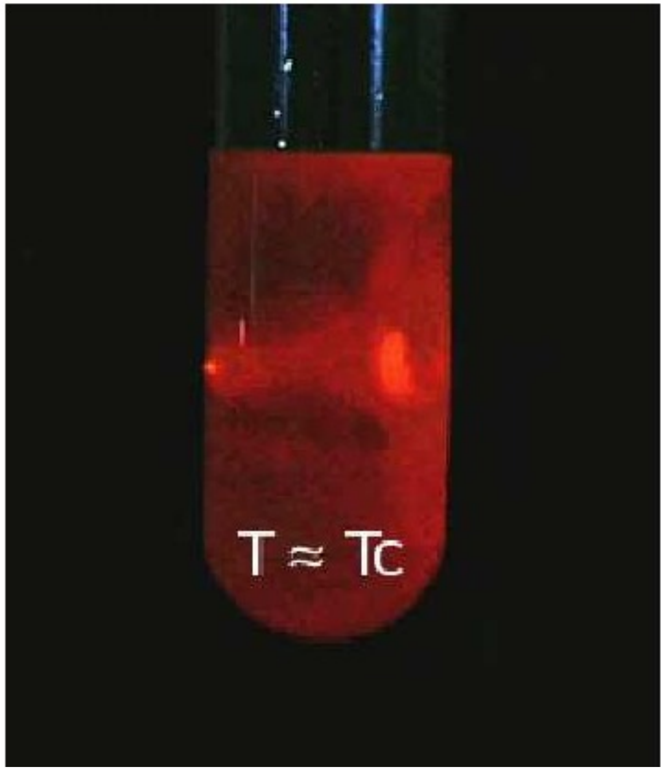
Chemical freeze-out curve and CP

- Sets the **lower bound** on the temperature of the CP [**Lysenko**, Poberezhnyuk, Gorenstein, VV, arXiv:2408.06473]
- **Caveats:** strangeness neutrality ($\mu_S \neq 0$), uncertainty in the freeze-out curve
- CP may be close to freeze-out at $\sqrt{s_{NN}} \sim 3.5 - 5$ GeV

Critical point and fluctuations

Density fluctuations at macroscopic length scales

Critical opalescence



Unfortunately, we cannot do this in heavy-ion collisions

Event-by-event fluctuations and statistical mechanics

Consider a fluctuating number N

Cumulants: $G_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$

variance $\kappa_2 = \langle(\Delta N)^2\rangle = \sigma^2$  width

skewness $\kappa_3 = \langle(\Delta N)^3\rangle$  asymmetry

kurtosis $\kappa_4 = \langle(\Delta N)^4\rangle - 3\langle(\Delta N^2)\rangle^2$  peak shape

Experiment:

$$P(N) \sim \frac{N_{events}(N)}{N_{events}^{total}}$$

Statistical mechanics:

Grand partition function $$\ln Z^{\text{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N} Z^{\text{ce}}(T,V,N)\right],$$

$$\boxed{\kappa_n \propto \frac{\partial^n(\ln Z^{\text{gce}})}{\partial(\mu_N)^n}}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

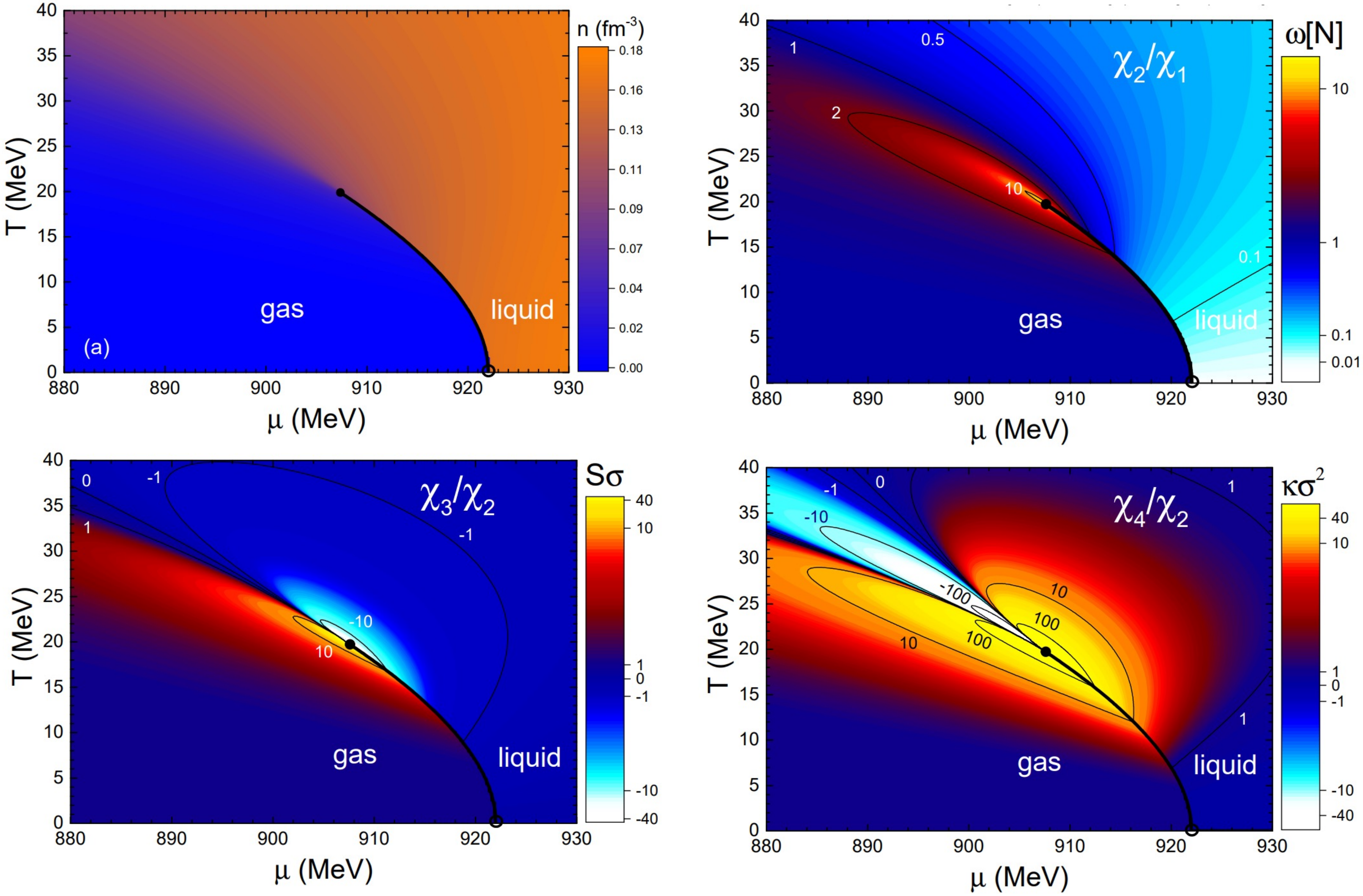
Example: (Nuclear) Liquid-gas transition

- **(QCD) critical point:** large correlation length and fluctuations

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

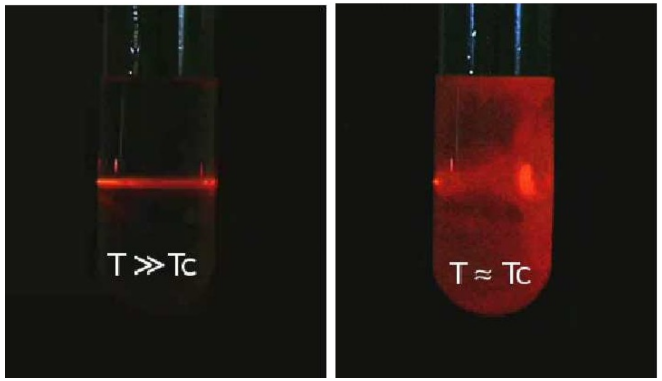
$$\xi \to \infty$$

M. Stephanov, PRL '09, '11



VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Critical opalescence



$$\langle N^2\rangle - \langle N\rangle^2 \sim \langle N\rangle \sim 10^{23}$$

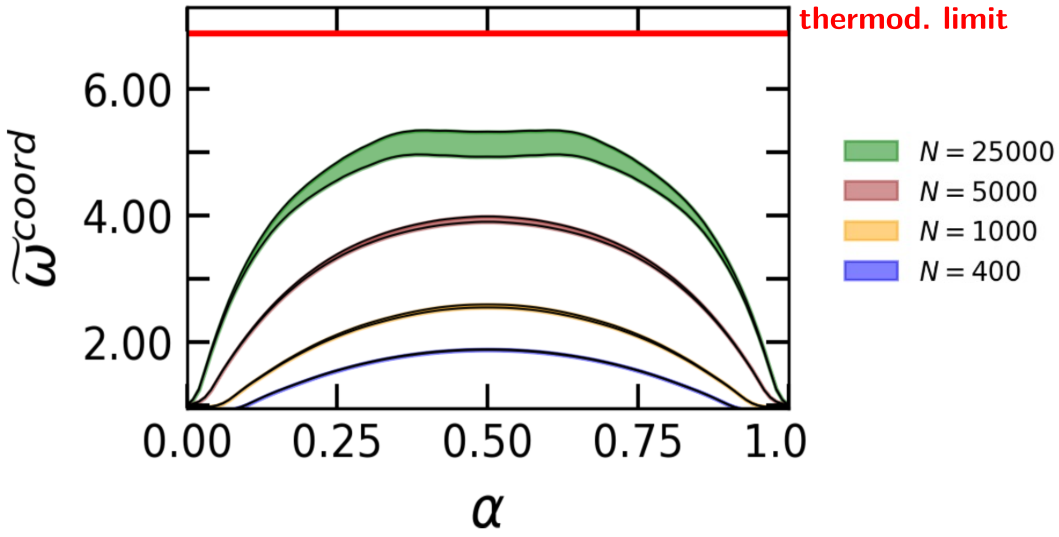
in equilibrium

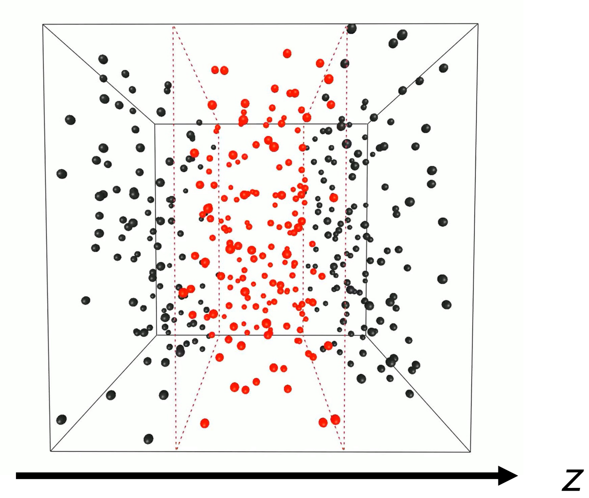
Example: Critical fluctuations in microscopic simulation

V. Kuznietsov (grad student @UH) et al., Phys. Rev. C 105, 044903 (2022)

Instead of observing system macroscopically, track each single particle

Classical molecular dynamics simulations of the **Lennard-Jones fluid** near critical point ($T \approx 1.06T_c$, $n \approx n_c$) of the liquid-gas transition



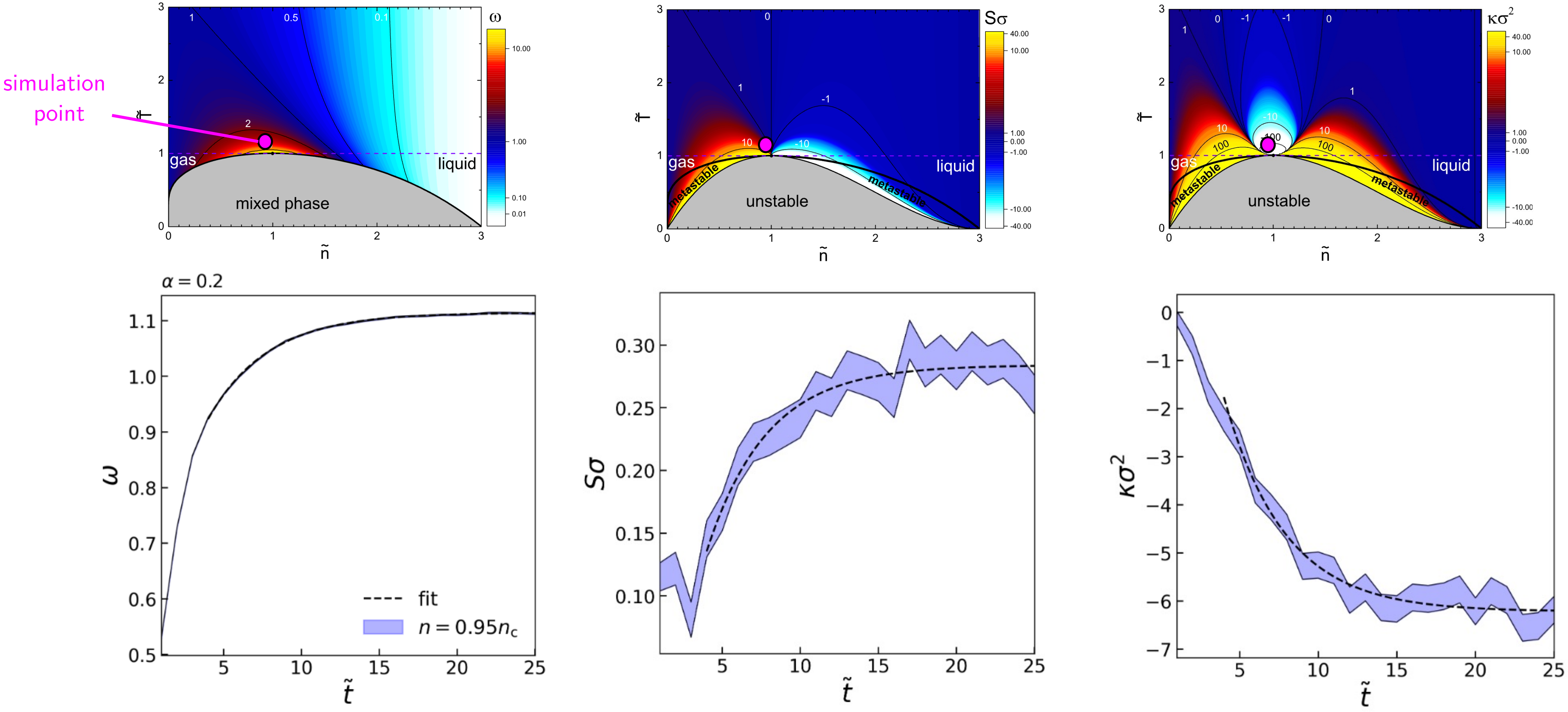


Large fluctuations survive despite strong finite-size effects and are large as advertised near the critical point

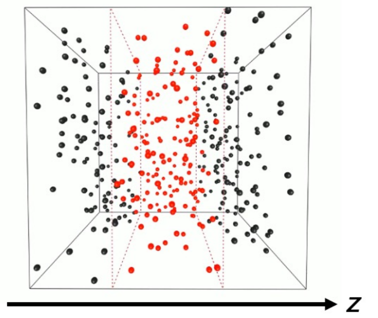
Non-Gaussian fluctuations from molecular dynamics

V. Kuznietsov, **R. Poberehzniuk**, Gorenstein, Koch, VV, arXiv:2511.00755

Scaled variance κ_2/κ_1 Skewness κ_3/κ_2 Kurtosis κ_4/κ_2



400 nucleons in a box

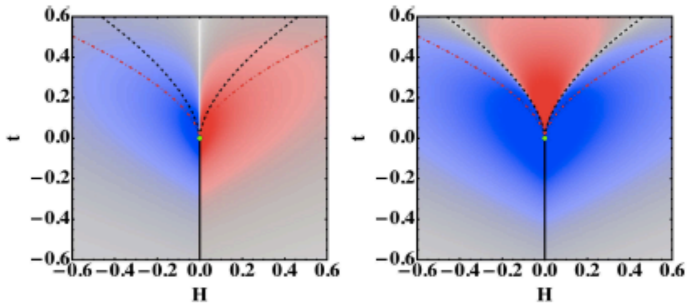


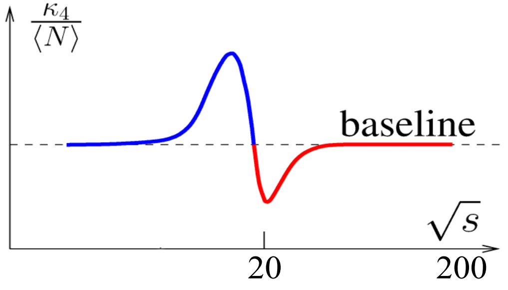
- (Non-)Gaussian cumulants equilibrate on comparable time scales

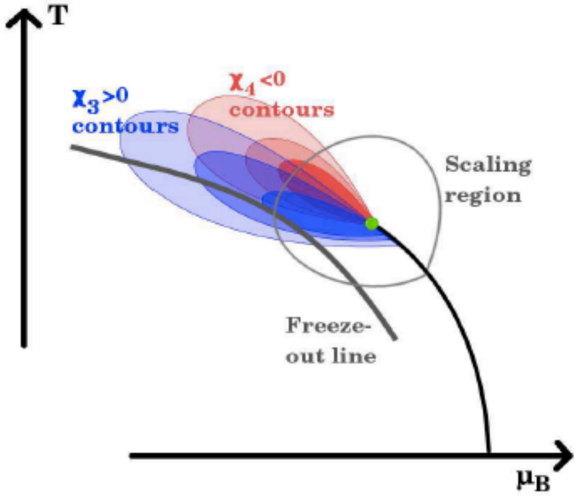
see also X. An et al., PRL 127, 072301 (2021); C. Chattopadhyay et al., PRL 133, 032301 (2024)

Equilibrium Expectations and Beam Energy Scan

Expectation from Calculations





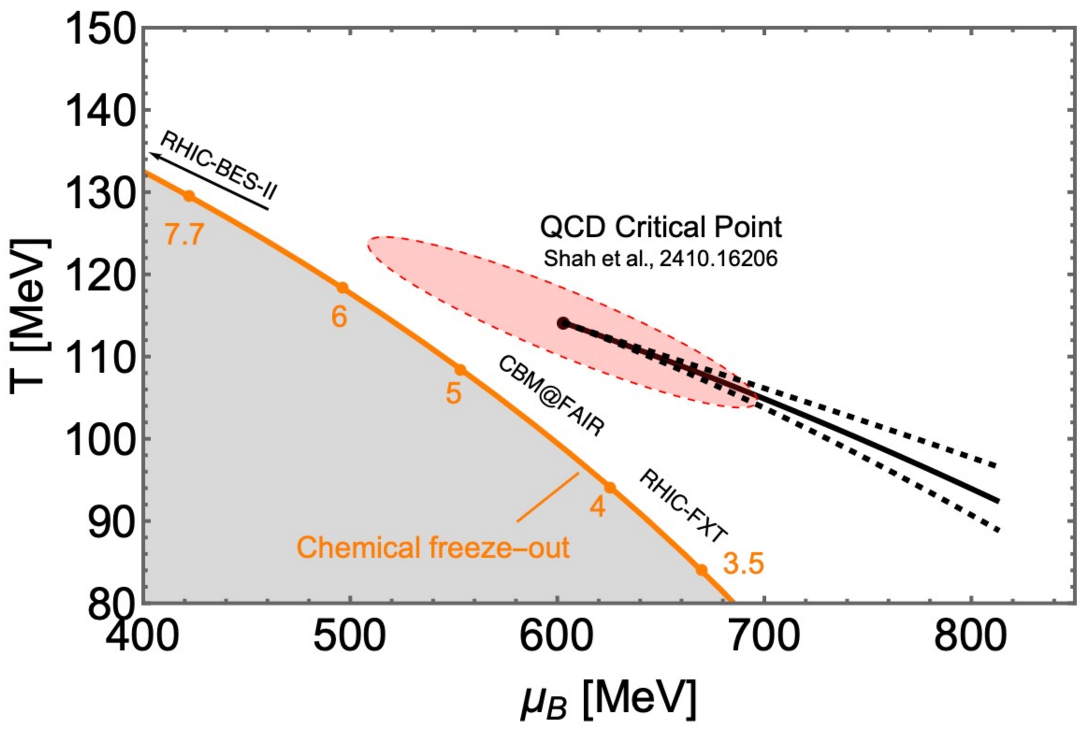


N. Xu, CPOD 2016

Characteristic "Oscillating pattern" is expected for the QCD critical point but *the exact shape depends on the location of freeze-out with respect to the location of CP*

- M. Stephanov, ***PRL107***, 052301(2011)
- V. Skokov, Quark Matter 2012
- J.W. Chen, J. Deng, H. Kohyyama, arXiv: 1603.05198, Phys. Rev. **D93** (2016) 034037

Compare recent CP estimates and the freeze-out curve



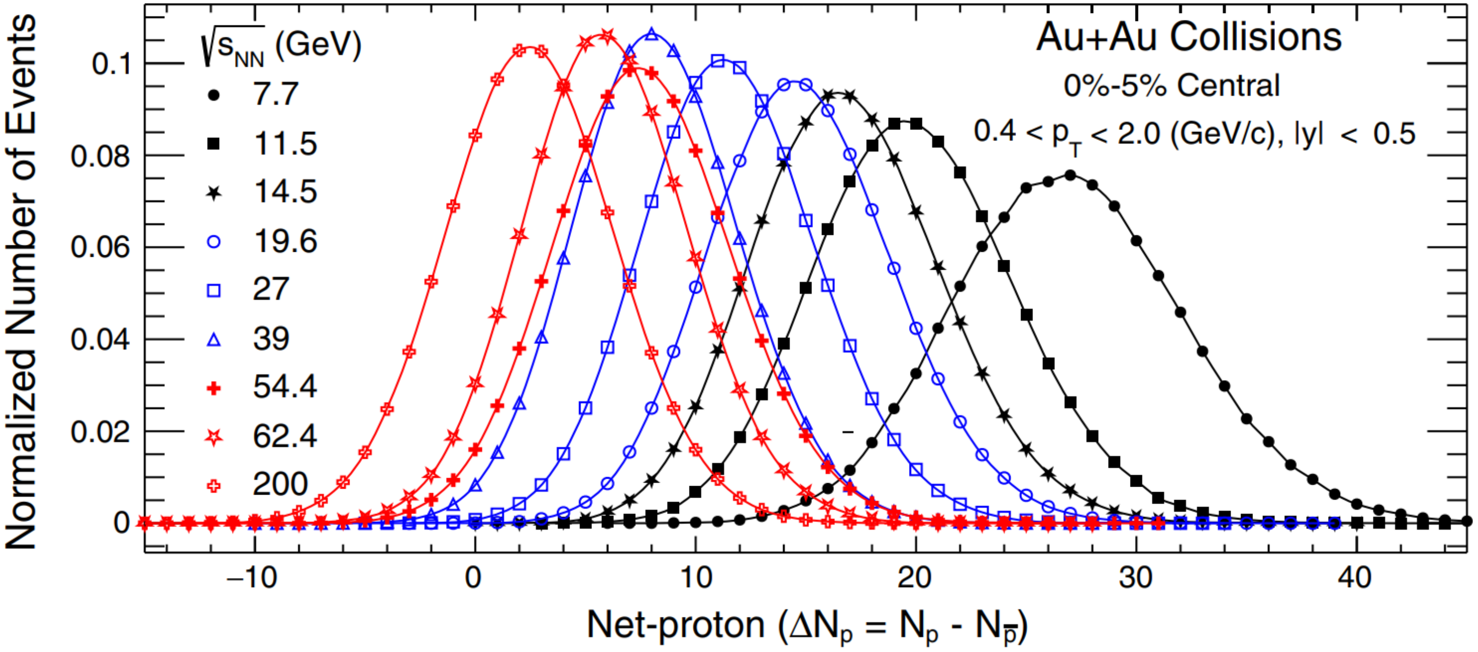
One of primary motivations for beam energy scan (BES) programs at RHIC BES-I (7.7-200 GeV) and BES-II (3-4.5 & 7.7-39 GeV)

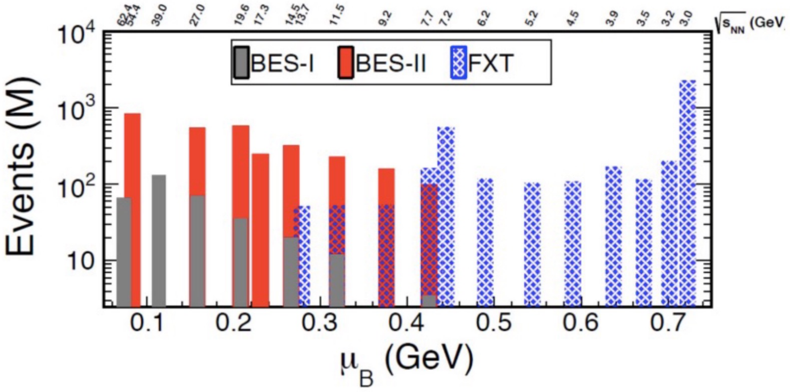
Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons

$$P(\Delta N_p) \sim \frac{N_{\text{events}}(\Delta N_p)}{N_{events}^{total}}$$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)





Statistics-hungry observables

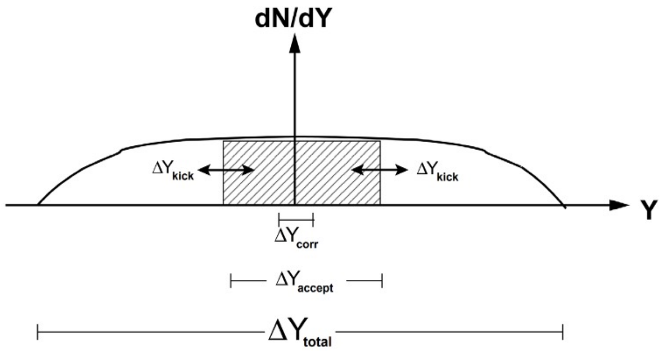
Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

$$\frac{\kappa_2}{\langle N \rangle}, \qquad \frac{\kappa_3}{\kappa_2}, \qquad \frac{\kappa_4}{\kappa_2}$$

Look for subtle critical point signals

Statistical ensemble in HICs is neither CE or GCE

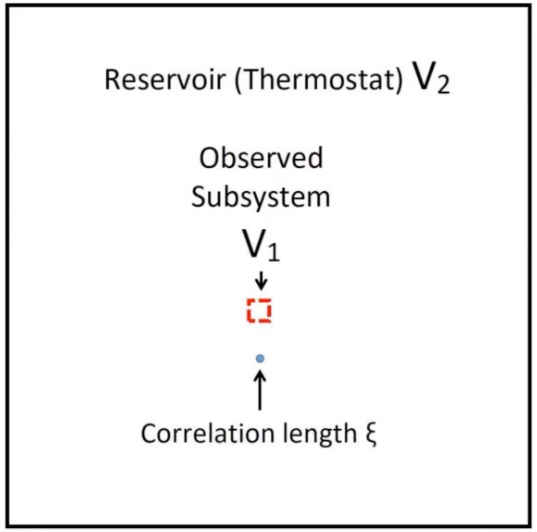
- Experimental measurements apply momentum cuts
  - The idea is to mimick GCE conditions
- However, in reality, the measured subsystem is of comparable size to total system where baryon number does not fluctuate and the canonical ensemble (CE) applies



V. Koch, arXiv:0810.2520

$$\frac{\kappa_2}{\kappa_1} = \frac{\chi_2}{\chi_1}$$

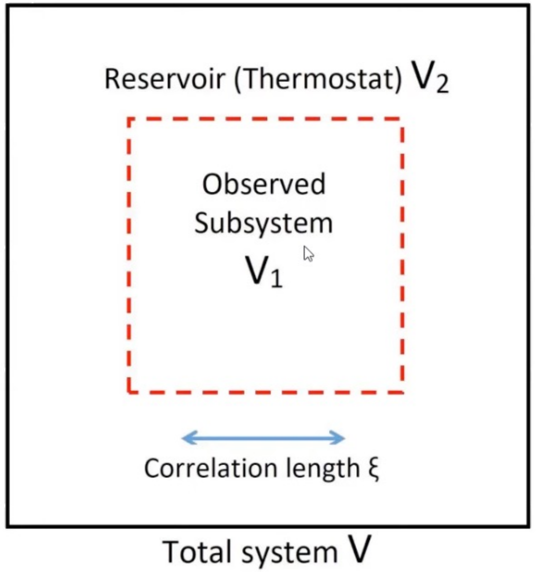
Grand Canonical Ensemble



VS

$\alpha \equiv V_1/V$
$(0 \ll \alpha \ll 1)$

Subensemble



$$\frac{\kappa_2}{\kappa_1} = (1-\alpha)\frac{\chi_2}{\chi_1}$$

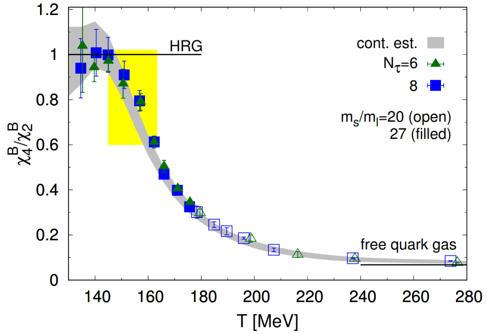
- Statistical ensemble relevant for heavy-ion collisions is something "in-between" GCE and CE

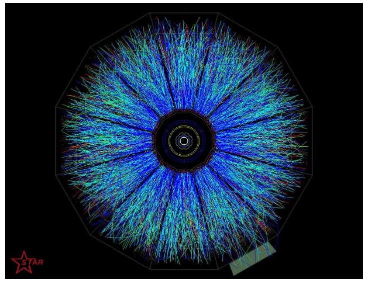
Solution: Subensemble acceptance method (SAM), VV, **Savchuk, Poberezhniuk**, Gorenstein, Koch, PLB 811, 135868 (2020)

Theory vs experiment

guidance from theory (e.g. lattice)

experiment (the real thing)





hydro simulation

hadronic phase

canonical effects

kinematic cuts

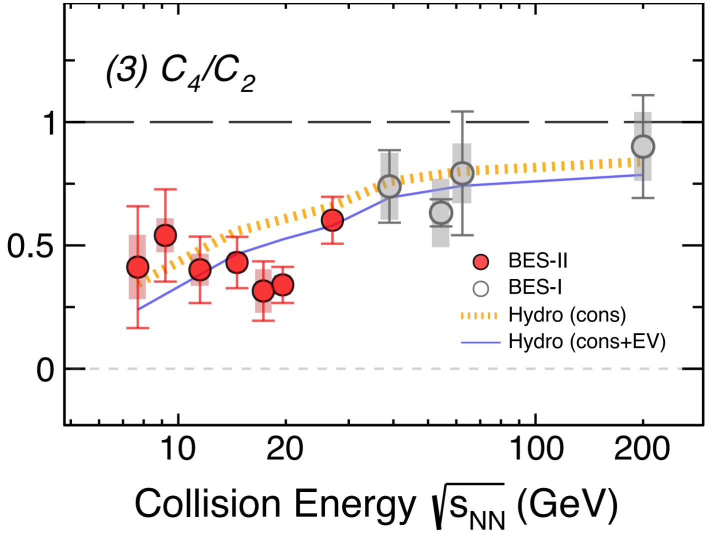
event selection

track reconstruction

efficiency correction

CBWC

purity



Theory vs experiment

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guidance from theory (e.g. lattice)

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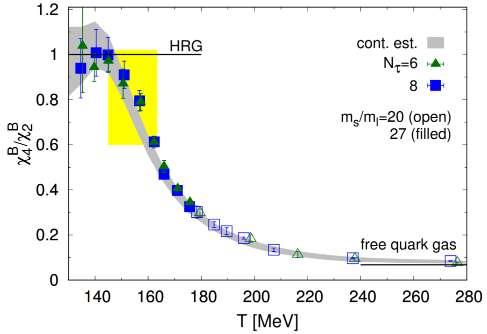
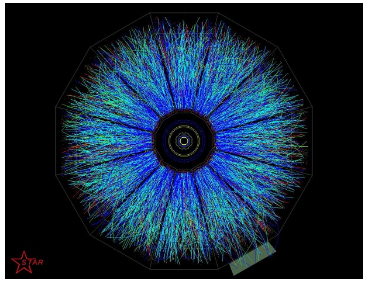




This was done in [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)]

- Full hydro simulation
- Lattice QCD-like baryon susceptibilities (interacting HRG)
- Global baryon conservation (SAM)
- Experimental kinematic cuts

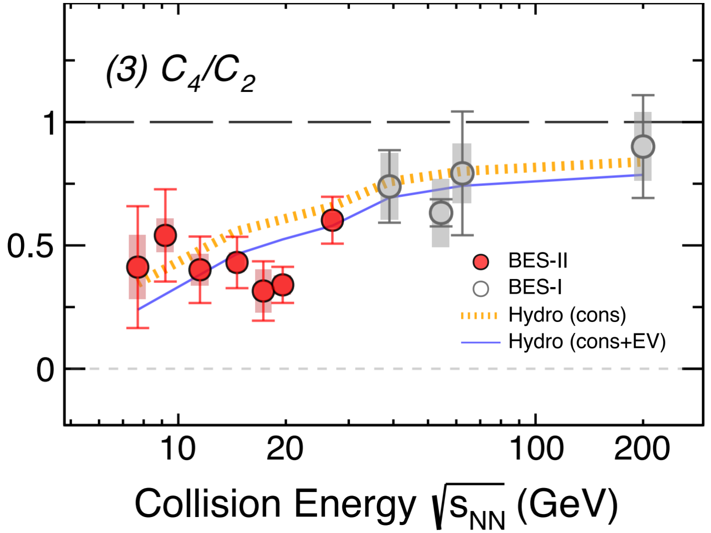
Non-critical baseline (hydro EV) **prediction**

hydro simulation

hadronic phase

canonical effects

kinematic cuts



event selection

track reconstruction

efficiency correction

CBWC

purity

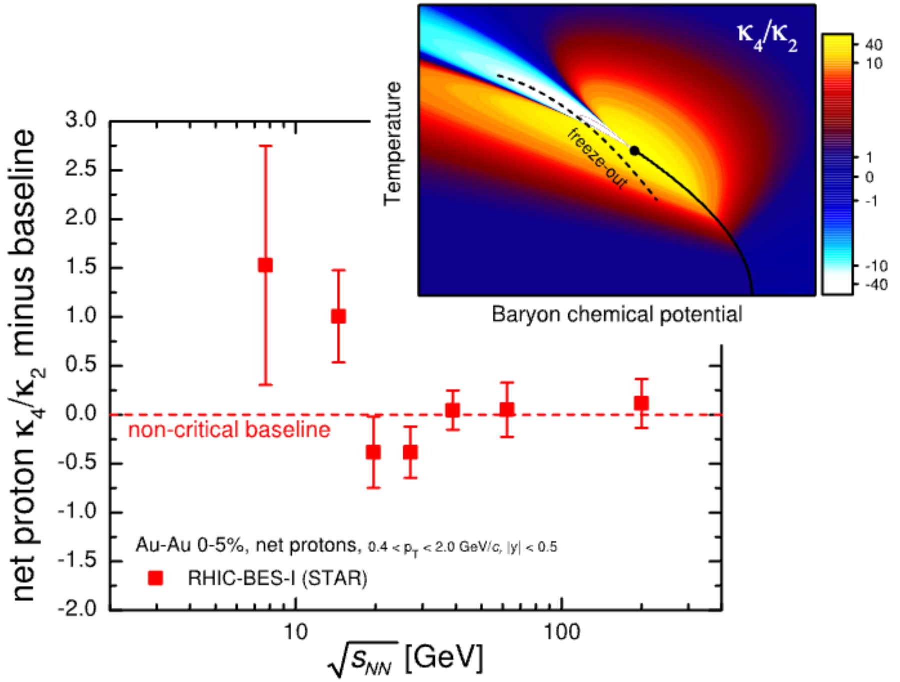
Hints from RHIC-BES-I

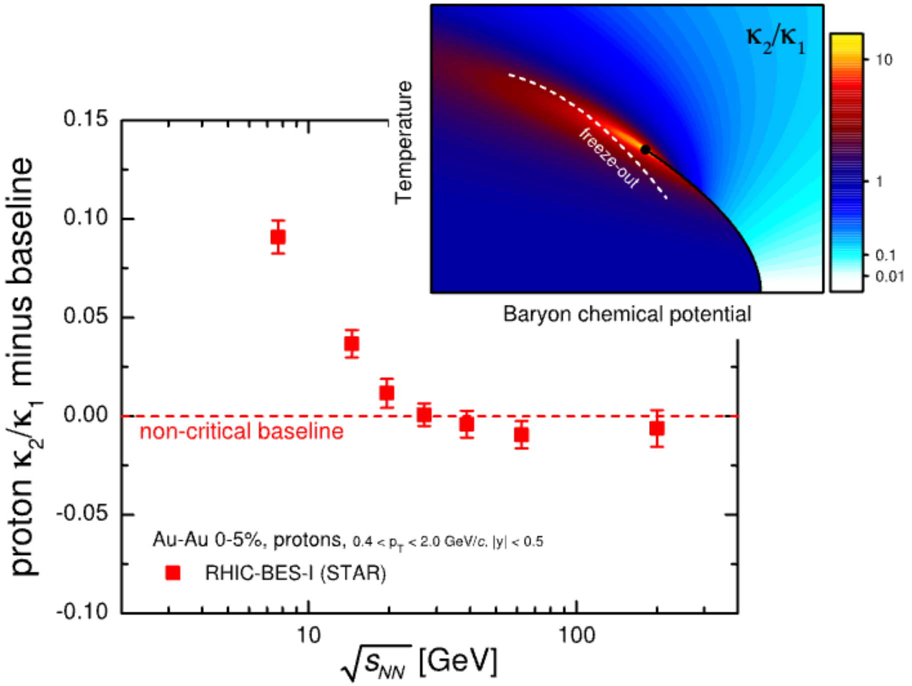
Quantitative calculations of critical fluctuations are still not available

State-of-the-art non-critical baseline computed using hydrodynamics

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Subtract it from the data and look for a possible signal of CP





Analysis of RHIC-BES-II data in progress

First data from RHIC-BES-II

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PHYSICAL REVIEW LETTERS **135,** 142301 (2025)

Editors' Suggestion | Featured in Physics

Precision Measurement of Net-Proton-Number Fluctuations in Au + Au Collisions at RHIC

(STAR Collaboration)

(Received 26 March 2025; accepted 18 July 2025; published 29 September 2025)

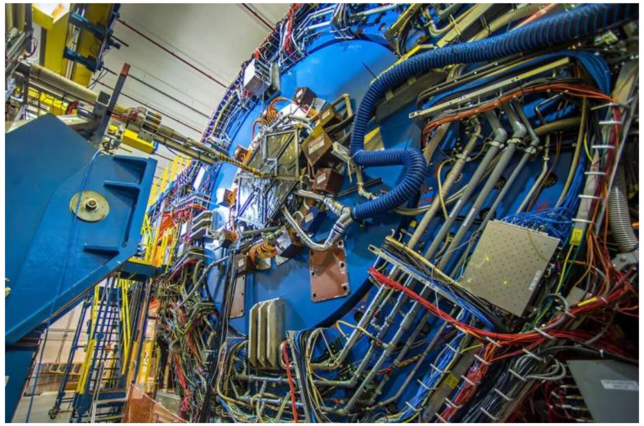
We report precision measurements on cumulants (C_n) and factorial cumulants (κ_n) of (net) proton number distributions up to fourth order in Au + Au collisions over center-of-mass energies $\sqrt{s_{NN}}$ = 7.7–27 GeV from phase II of the Beam Energy Scan program at RHIC. (Anti)protons are selected

SEPTEMBER 29, 2025 The GIST

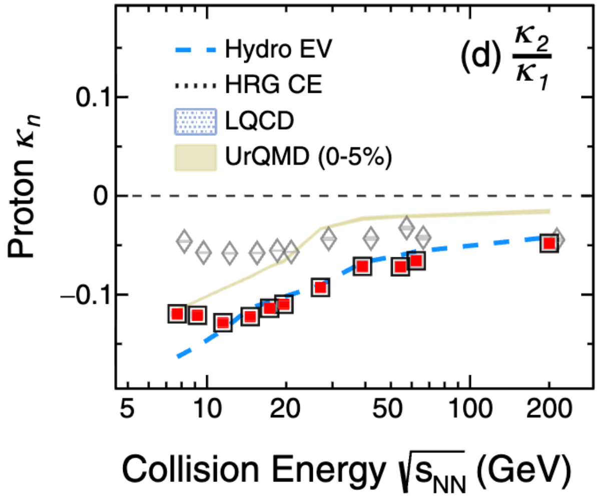
High-order analysis reveals more signs of phase-change 'turbulence' in nuclear matter

by Lawrence Berkeley National Laboratory

edited by Lisa Lock, reviewed by Robert Egan Editors' notes



The STAR detector at the U.S. Department of Energy's Brookhaven National C...



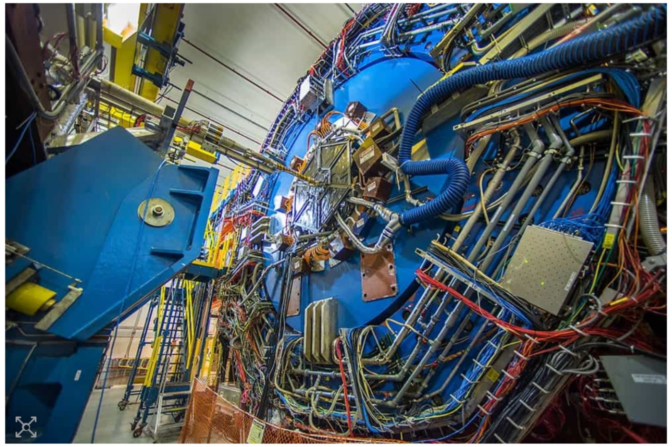
IOP Publishing

physicsworld

PARTICLE AND NUCLEAR | RESEARCH UPDATE

Hints of a boundary between phases of nuclear matter found at RHIC

09 Oct 2025



STAR at RHIC The experiment at Brookhaven National Laboratory has revealed hints of a critical point in the phase transition of nuclear matter. (Courtesy: BNL)

In a major advance for nuclear physics, scientists on the STAR Detector at the Relativistic Heavy Ion Collider (RHIC) in the US have spotted subtle but striking fluctuations in the number

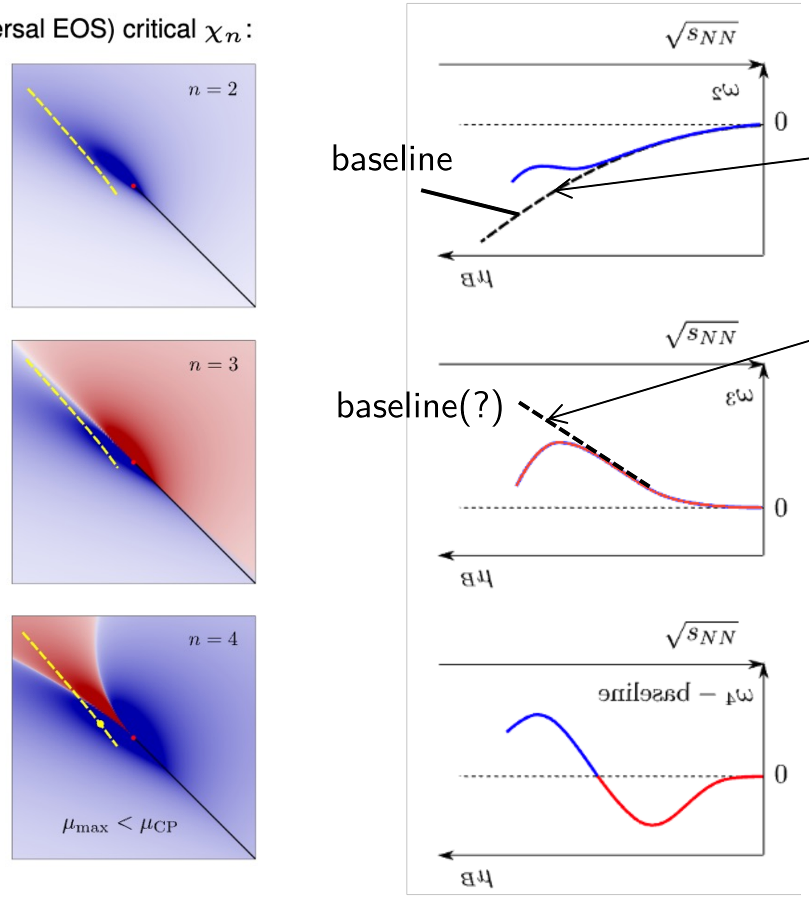
Meanwhile, theorists are racing to catch up. “The ball now moves largely to theory’s court,” Vovchenko says. He emphasizes the need for “quantitative predictions across energies and cumulants of various order that are appropriate for apples-to-apples comparisons with these data.”

Factorial cumulants from RHIC-BES-II

From M. Stephanov, SQM2024 & arXiv:2410.02861

$\omega_n = \hat{C}_n / \hat{C}_1$

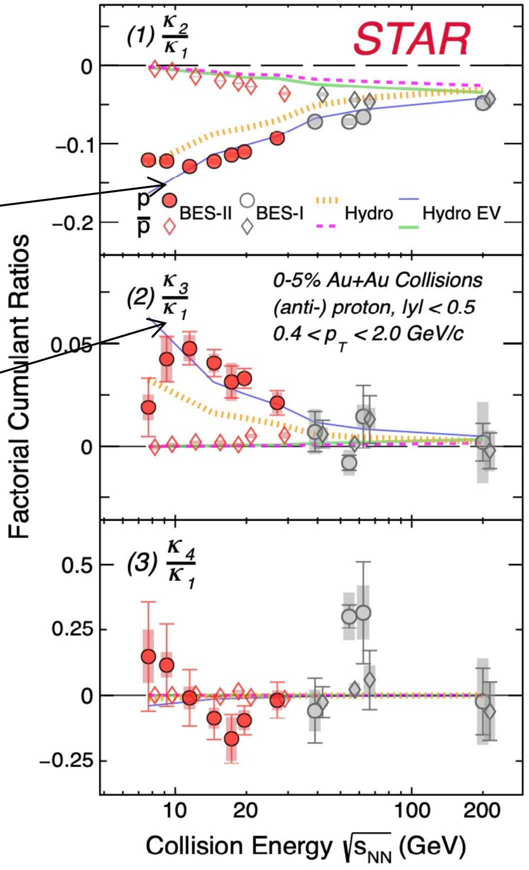
(universal EOS) critical χ_n:



Bzdak et al review 1906.00936

Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

STAR data:



A. Pandav, CPOD2024

baseline (hydro EV):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

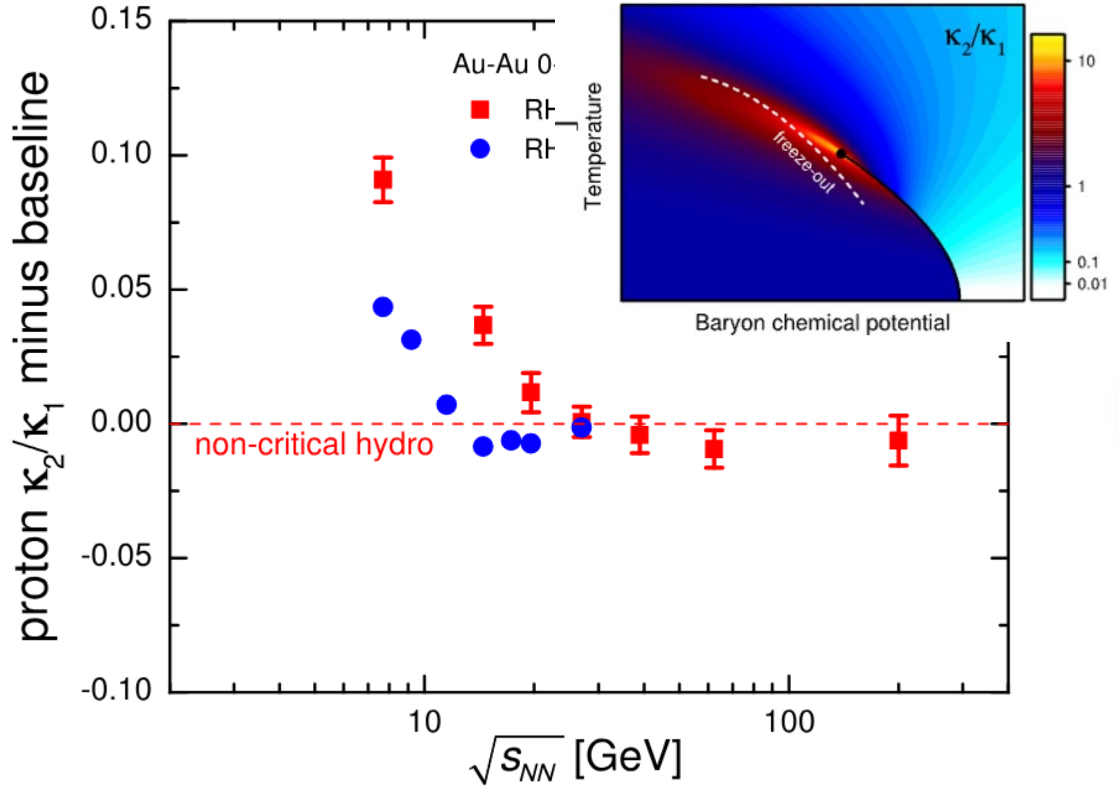
- describes right side of the peak in $\hat{C}_3$
- **signal relative to baseline:**
  - *positive* $\hat{C}_2 > 0$
  - *negative* $\hat{C}_3 < 0$

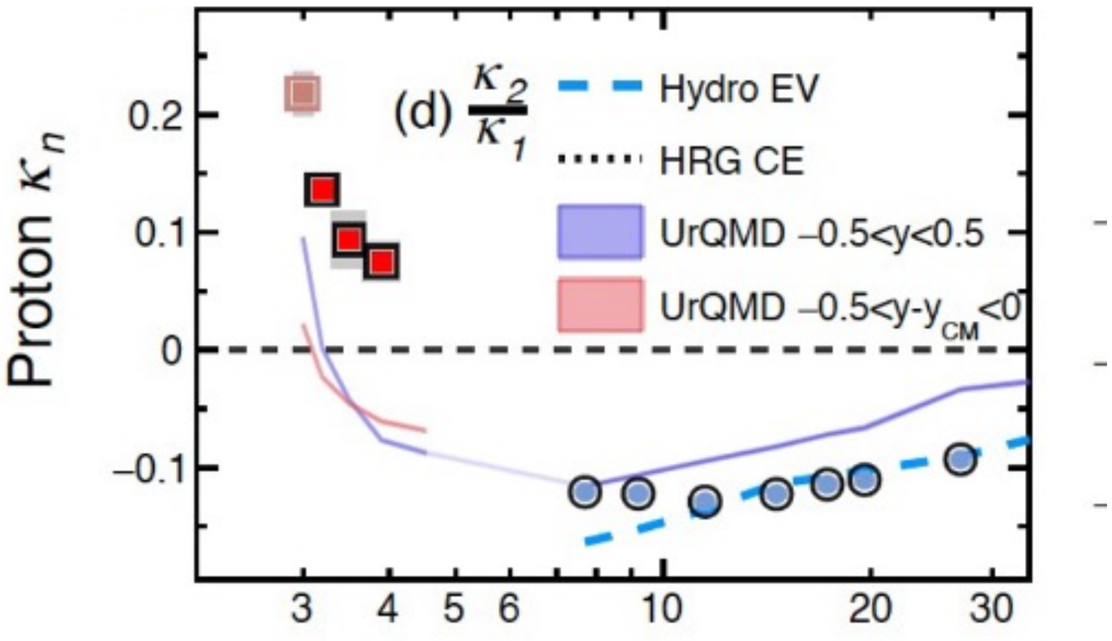
Conclusion:

Controlling the non-critical baseline is essential

Subtracting the baseline

If $\kappa_2^{tot} \approx \kappa_2^{crit} + \kappa_2^{reg}$ try to isolate the critical part* by subtracting the baseline (here hydro EV)





Enhancement relative to the baseline at lower $\sqrt{s_{NN}}$ which continues at fixed target energies

*May be a useful quantity for finite-size scaling analysis compared to the bare κ_2/κ_1

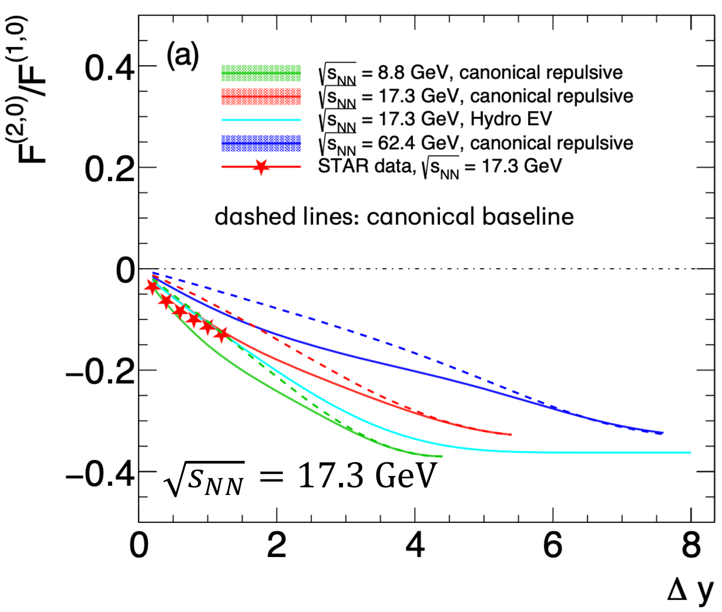
Attraction vs repulsion

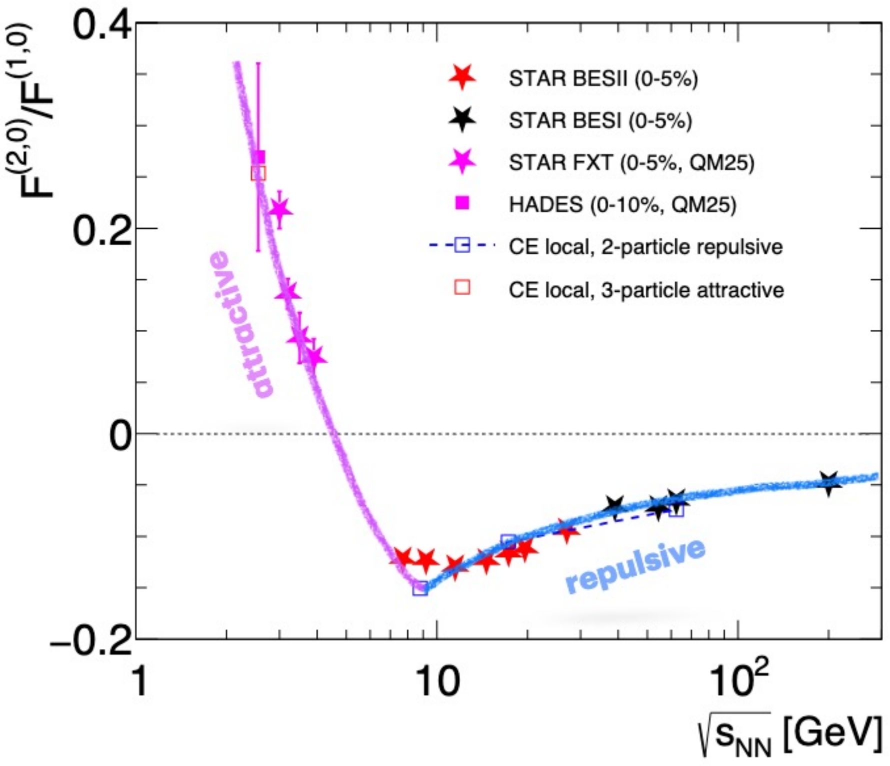
Friman, Redlich, Rustamov, arXiv:2508.18879

Recently, attractive and repulsive interactions implemented through a potential in rapidity

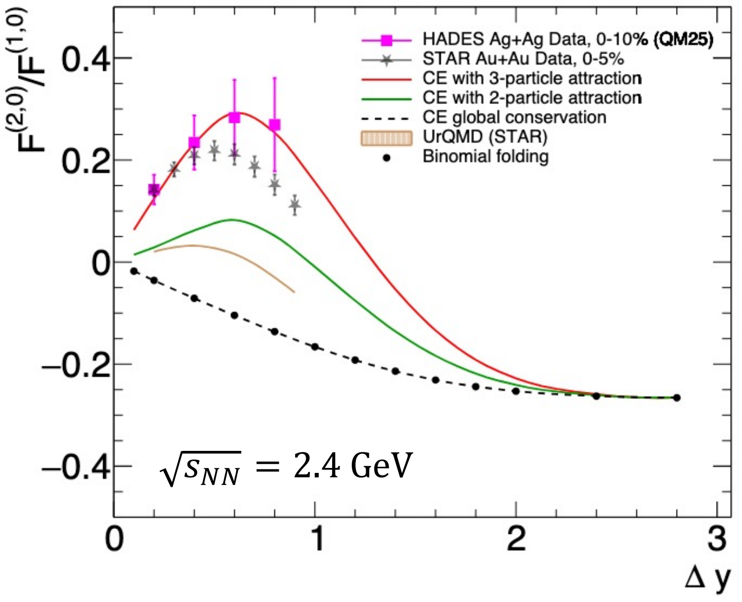
$$E_r(y_1, y_2) = \alpha_r e^{-|y_1 - y_2|/\rho_r} \qquad P(y_1, y_1) = \frac{e^{-E(y_1,y_2)}}{Z} \qquad E_a(y_1, y_2) = \alpha_a |y_1 - y_2|^{\beta_a}$$

repulsion





attraction

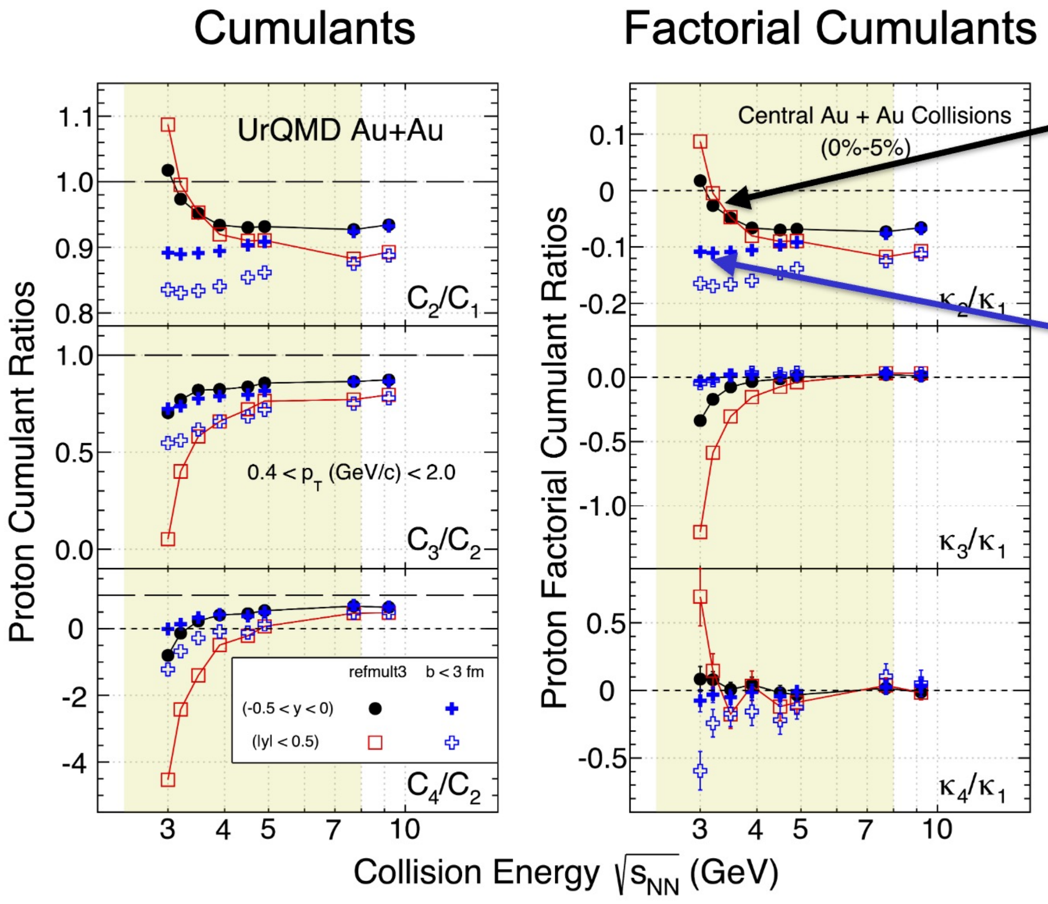


Interplay of repulsive (high $\sqrt{s_{NN}}$) and attractive (low $\sqrt{s_{NN}}$) interactions?

Time to celebrate!

The (possible) culprit

X. Zhang, Y. Zhang, X. Luo, N. Xu, arXiv: 2506.18832



Fluctuating impact parameter
STAR centrality selection

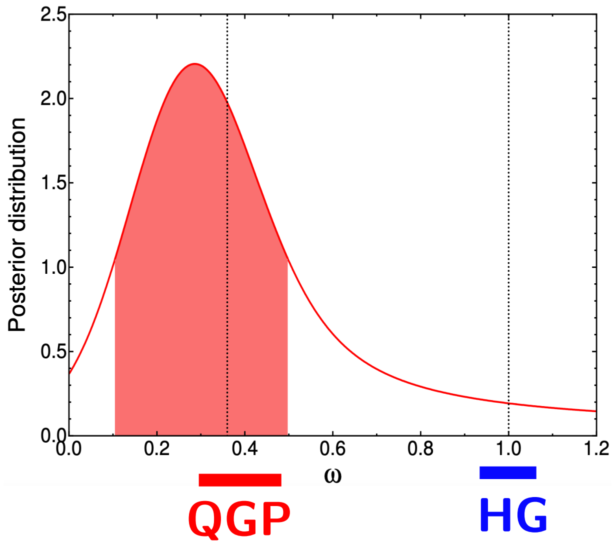
Fixed impact parameter $(b < 3\ \mathrm{fm})$
minimal volume fluctuations.

N.B.: Centrality Bin Width Corrections applied to both

Possible culprit:
volume fluctuations/centrality selection

Other challenges: Antiproton puzzle (not described by hydro)

Bzdak, Koch, VV, arXiv:2503.16405



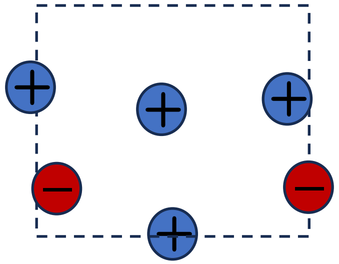
Charge fluctuations as a signature of QGP

- J. Parra, R. Poberezhniuk, V. Koch, C. Ratti, VV, arXiv:2504.02085 (Phys. Rev. Lett., in print)

Charge fluctuations

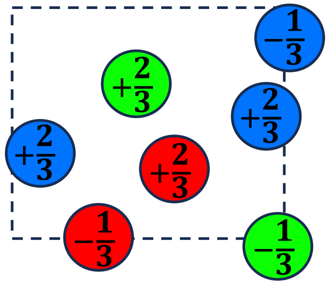
An old idea: Hadrons carry integer electric charges, quarks carry *fractional* electric charges.

Koch, Jeon, PRL (2000);
Asakawa, Muller, Heinz, PRL (2000)



Hadrons

vs



Quarks

HG: large fluctuations

QGP: small fluctuations

Fluctuations depend on the square of the charges
and are smaller in the QGP

Quantified by:

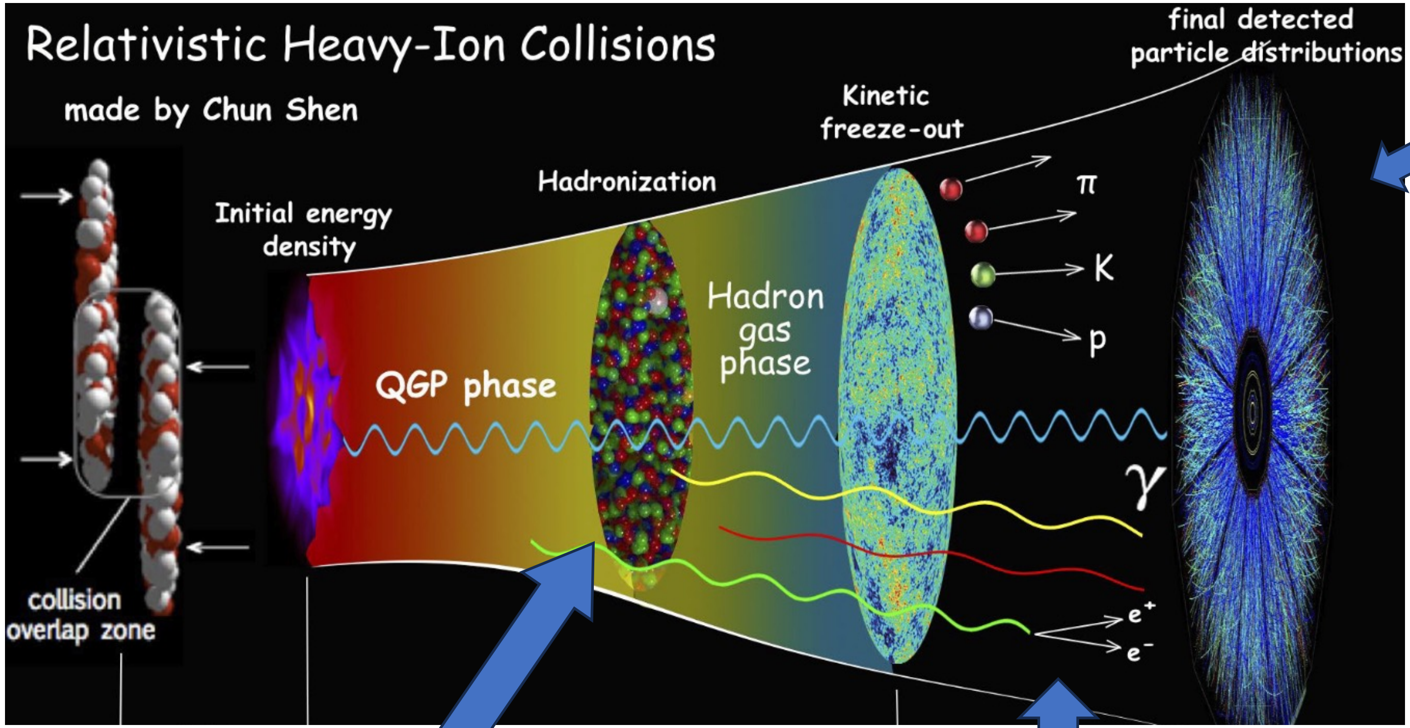
$$D = 4\frac{\kappa_2[N_+ - N_-]}{\langle N_{\rm ch}\rangle} = 4\frac{\kappa_2[Q]}{\langle Q^+ + Q^-\rangle}$$

Naïve grand canonical ensemble (GCE) expectations:

- $D_{HG} \approx 2.8 - 4$
- $D_{QGP} \approx 1 - 1.5$

No quantitative calculations have been done for QGP beyond GCE

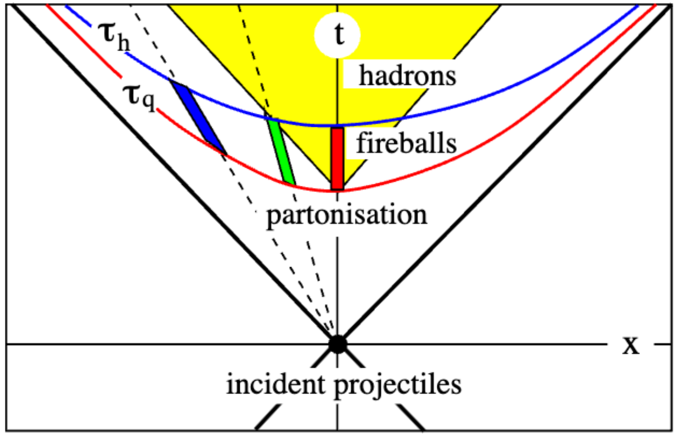
Charge fluctuations: stages



4. Kinematical cuts

never done for QGP

3. (Local) charge conservation



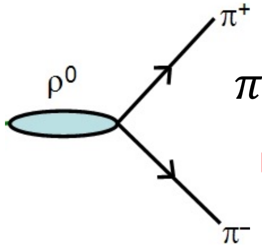
Castorina, Satz, IJMPE '14

never done both for HG and QGP

1. Fluctuations at hadronization (primordial charges)

$\boldsymbol{\omega}$ – Distinguishes hadron gas ($\boldsymbol{\omega \approx 1}$) from QGP ($\omega \approx 0.25 - 0.40$)

2. Resonance decays



$\pi^+\pi^-$ are correlated!

never done for QGP

Here we model these effects through novel charge density correlations formalism [VV, PRC '14]

Charge fluctuations: hadronization

Define ω as a measure of charge fluctuations at hadronization

$$\omega = \frac{\kappa_2[Q]}{\langle N_{\mathrm{ch}}^{\mathrm{prim}} \rangle}$$

variance

charged multiplicity

Hadron gas: $\omega_{HG} \approx 1$ (Poisson statistics)

Free QGP: $\omega_{QGP} \approx 0.36$ (Stefan-Boltzmann limit)

More generally:

$$\omega = \frac{\kappa_2[Q]}{\langle N_{\mathrm{ch}}^{\mathrm{prim}} \rangle} = \frac{VT^3\chi_2^Q}{S} \frac{S}{\langle N_{\mathrm{ch}}^{\mathrm{prim}} \rangle}$$

$$= \frac{\chi_2^Q}{s/T^3} \frac{S}{\langle N_{\mathrm{ch}} \rangle} \frac{\langle N_{\mathrm{ch}} \rangle}{\langle N_{\mathrm{ch}}^{\mathrm{prim}} \rangle}$$

The EoS

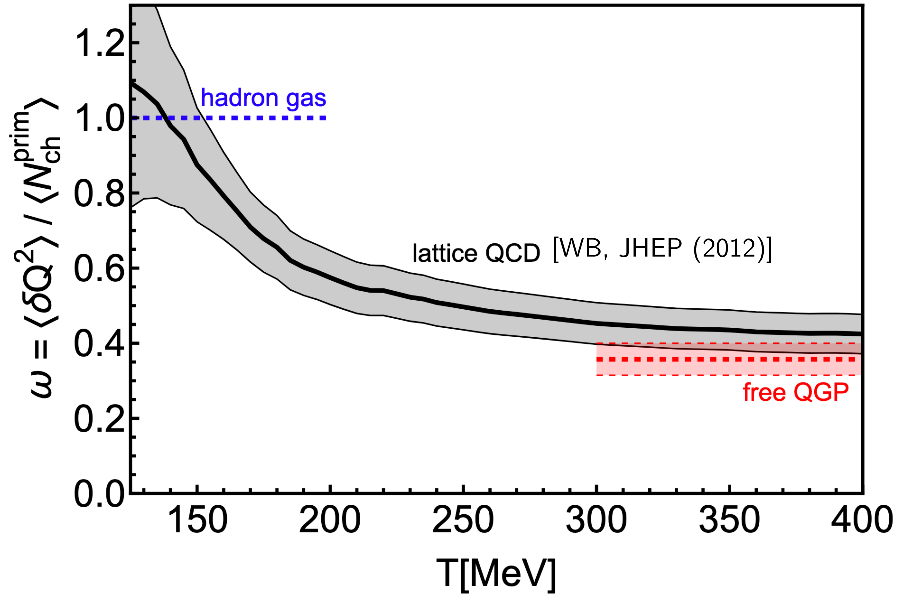
e.g. lattice QCD

$S/N_{\mathrm{ch}} = 6.7 \pm 0.8$ (LHC)

$\gamma_Q \approx 1.67$ (decays)

from thermal model

ω from lattice QCD



P. Hanus, A. Mazeliauskas, K. Reygers, PRC (2019)

Charge fluctuations: other stages

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2. Hadronic phase (resonance decays)

- Decays are local and conserve charge but increase charged multiplicity, $\langle N_{ch} \rangle = \left\langle N_{ch}^{prim} \right\rangle \gamma_Q$, where $\gamma_Q \approx 1.67$
- Total charge susceptibility does not change, $\chi_2^{Q,final} = \chi_2^{Q,prim}$, but balance between self-correlation and two-particle correlations does

3. Local charge conservation [VV, PRC 110, L061902 (2024)]

- 2-point charge density correlator with a balancing term
- Local charge conservation introduced through modulation of the balancing term

$$\mathcal{C}_2^Q(\mathbf{r}_1, \mathbf{r}_2) \equiv \langle \delta\rho_Q(\mathbf{r}_1) \delta\rho_Q(\mathbf{r}_2) \rangle$$

$$\mathcal{C}_2^Q(\mathbf{r}_1, \mathbf{r}_2) = \chi_2^Q \left[\underbrace{\delta(\mathbf{r}_1 - \mathbf{r}_2)}_{\text{local correlation}} - \underbrace{\frac{\varkappa(\mathbf{r}_1, \mathbf{r}_2)}{V_{\rm tot}}}_{\text{balancing contribution}} \right] \qquad \mathbf{r} = \eta$$

$$\underbrace{\varkappa(\eta_1, \eta_2) \propto \exp\left[-\frac{(\eta_1 - \eta_2)^2}{2\sigma_\eta^2} \right]}_{\text{local charge conservation}}$$

4. Kinematical cuts

$$\kappa_2[Q]|_{|\eta|<\eta_{\rm cut}} \propto \int_{-\eta_{\rm cut}/2}^{\eta_{\rm cut}/2} d\eta_1 \int_{-\eta_{\rm cut}/2}^{\eta_{\rm cut}/2} d\eta_2 \mathcal{C}_2^Q(\eta_1, \eta_2)\, p(\eta_1) p(\eta_2)$$

Acceptance probability $p(\eta)$ from blast-wave model

Putting everything together

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$$D = 4\left\{1 - \left(1 - \frac{\overset{\text{hadronization}}{\omega}}{\underset{\text{decays}}{\gamma_Q}}\right)\frac{\overset{\text{pair acceptance}}{\langle p^2(\eta)\rangle}}{\underset{\text{acceptance}}{\langle p(\eta)\rangle}} - \frac{\overset{\text{hadronization}}{\omega}}{\underset{\text{decays}}{\gamma_Q}}\frac{\overset{\text{local charge conservation}}{\langle p(\eta_1)p(\eta_2)\rangle_{\varkappa}}}{\underset{\text{acceptance}}{\langle p(\eta)\rangle}}\right\}$$

ω - Charge fluctuations at hadronization

$\omega_{HG} = 1$ $\qquad$ $\omega_{QGP} = 0.36$

γ_Q - Resonance decays

$\langle p(\eta_1)p(\eta_2)\rangle_{\varkappa}$ - Pair acceptance weighted with Local Charge Conservation

$\frac{\langle p^2(\eta)\rangle}{\langle p(\eta)\rangle}$ - Momentum Acceptance Cuts

using $p(\eta)$ from the blast-wave model

D-measure at LHC: comparison with experiment

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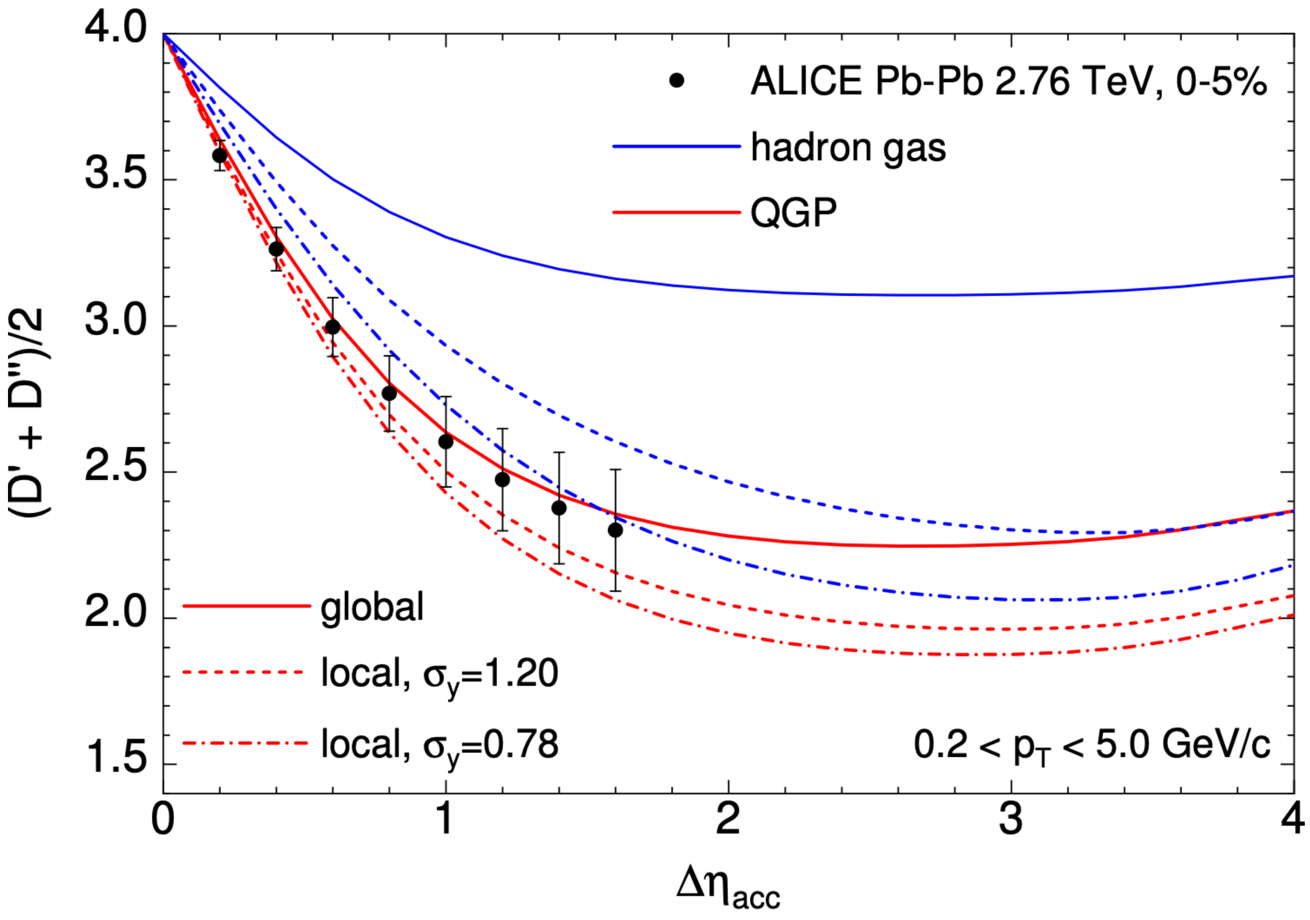
$$D = 4\left\{1 - \left(1 - \frac{\omega}{\gamma_Q}\right)\frac{\langle p^2(\eta)\rangle}{\langle p(\eta)\rangle} - \frac{\omega}{\gamma_Q}\frac{\langle p(\eta_1)p(\eta_2)\rangle_{\varkappa}}{\langle p(\eta)\rangle}\right\}$$



Parameters used:

$\omega_{HG} = 1$ $\omega_{QGP} = 0.36$

$\gamma_Q = 1.67$

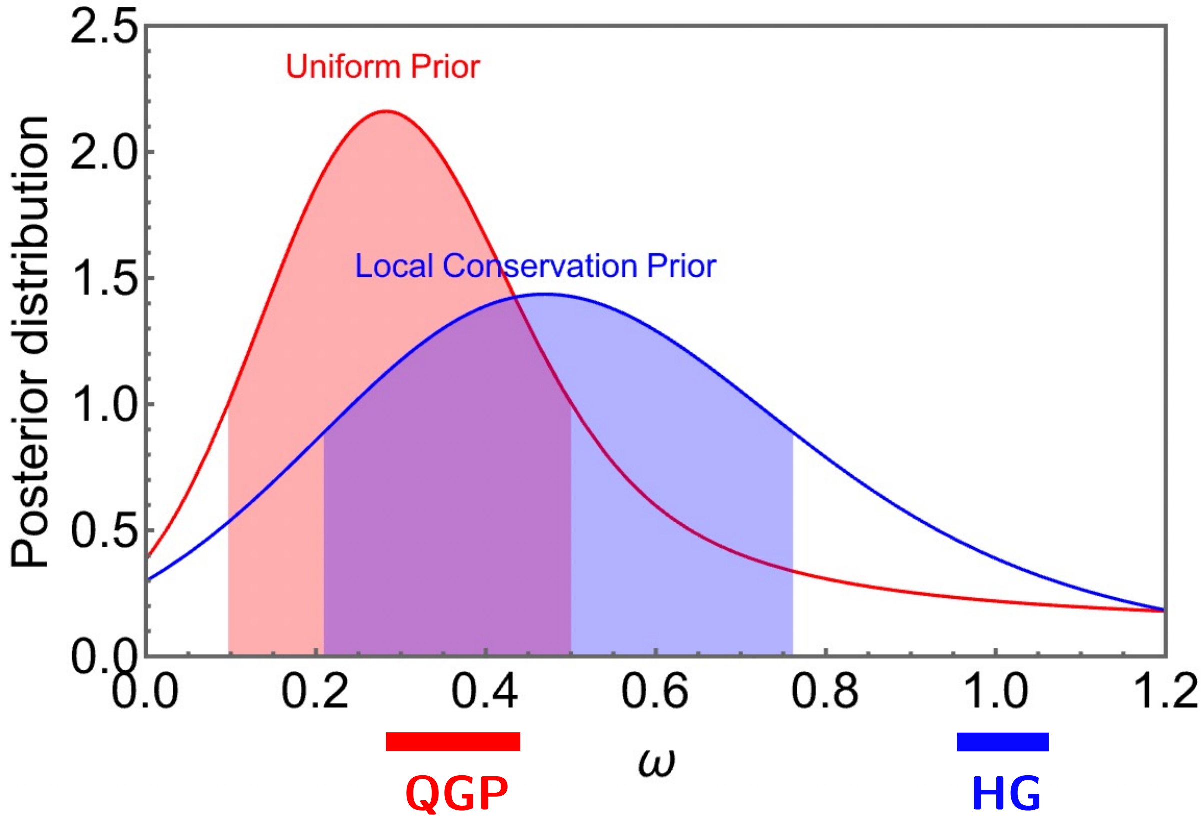
- Vary σ_y to accommodate global vs local charge conservation
  - Here values of σ_y are based on local baryon conservation estimates

VV, PRC 110, L061902 (2024)

Hadron gas scenario requires a very local charge conservation range

D-measure at LHC: Bayesian analysis

Vary primordial fluctuation ω (HG vs QGP) and correlation volume V_C (local conservation) freely



- Uniform prior

$\omega \in U(0, 1.2)$, $\quad V_c \in U(0, V_{tot})$,

Bayes factor (QGP : HG) $= 8.74$

- Local conservation prior

$\omega \in U(0, 1.2)$, $\quad V_c \in$ Gaussian at $(0.20 \pm 0.05) V_{tot}$

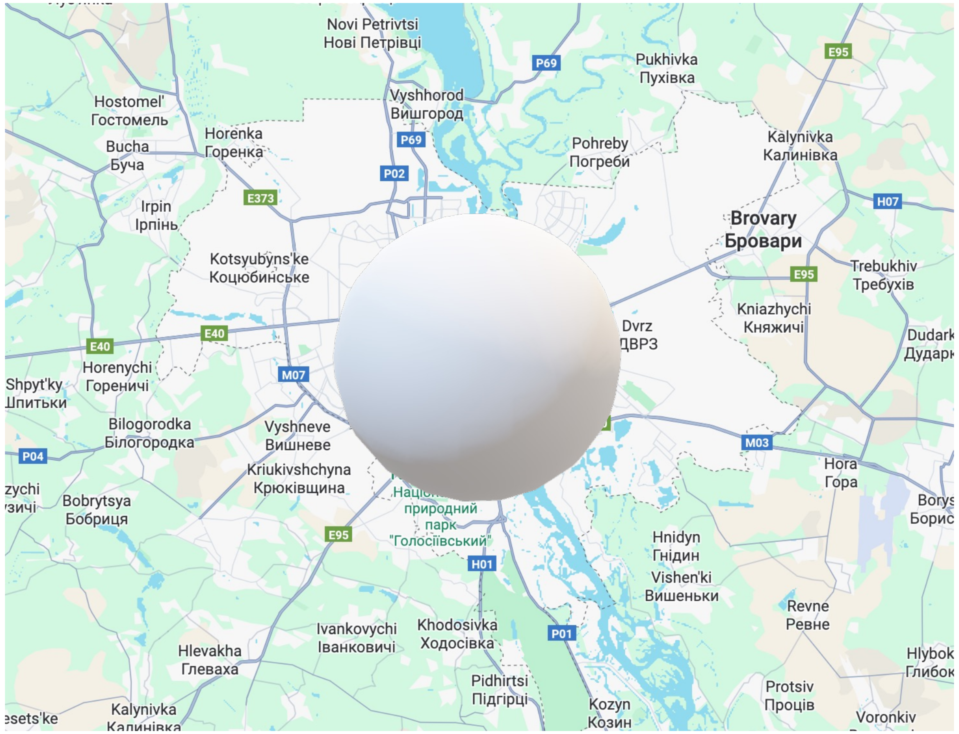
Bayes factor (QGP : HG) $= 4.93$

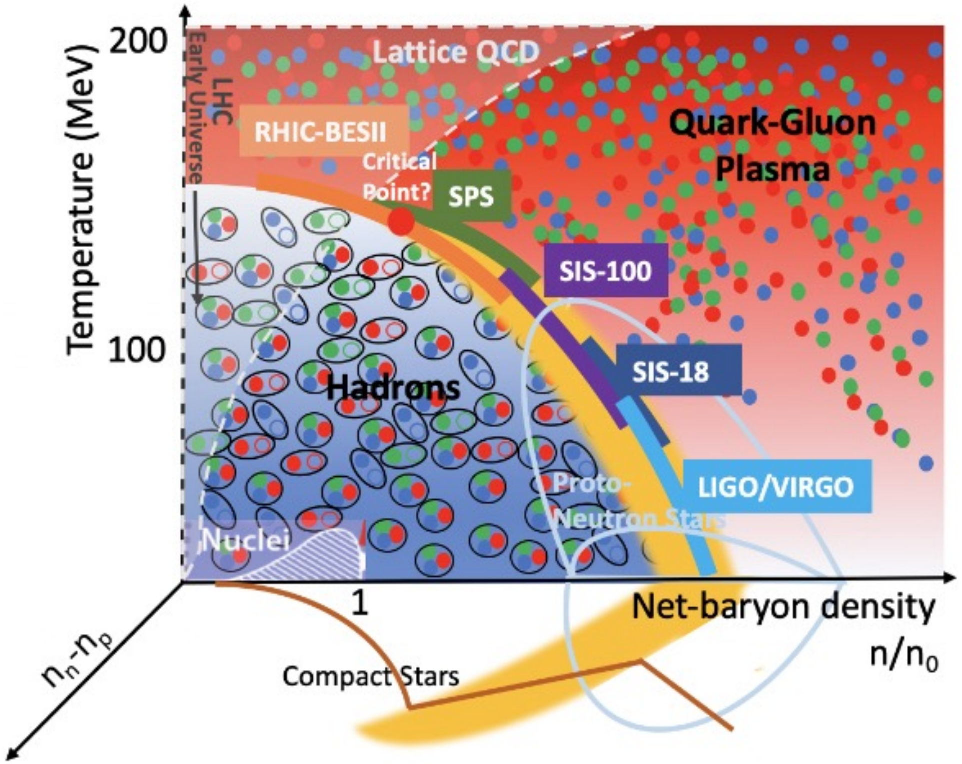
Moderate evidence for freeze-out of charge fluctuations in QGP

<u>Sensitivity to assumed prior currently under investigation</u>

QCD in astrophysics

Neutron stars are extremely compact objects (1-2 solar masses confined to an 8-mile sphere)

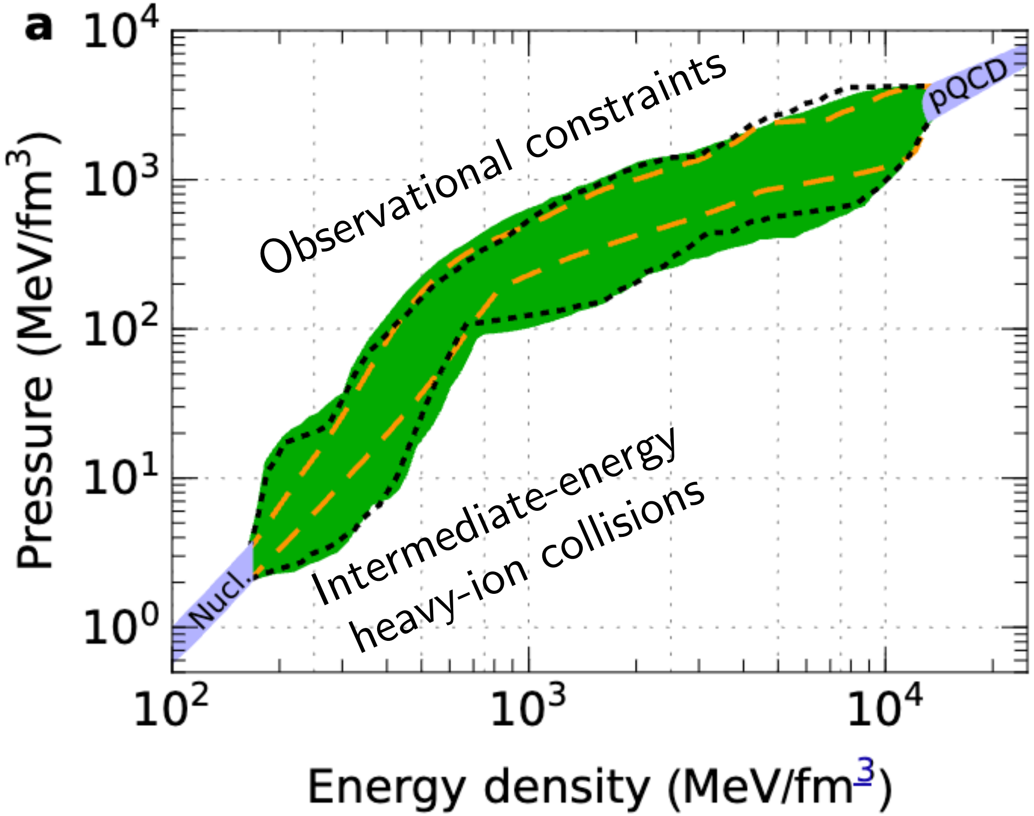


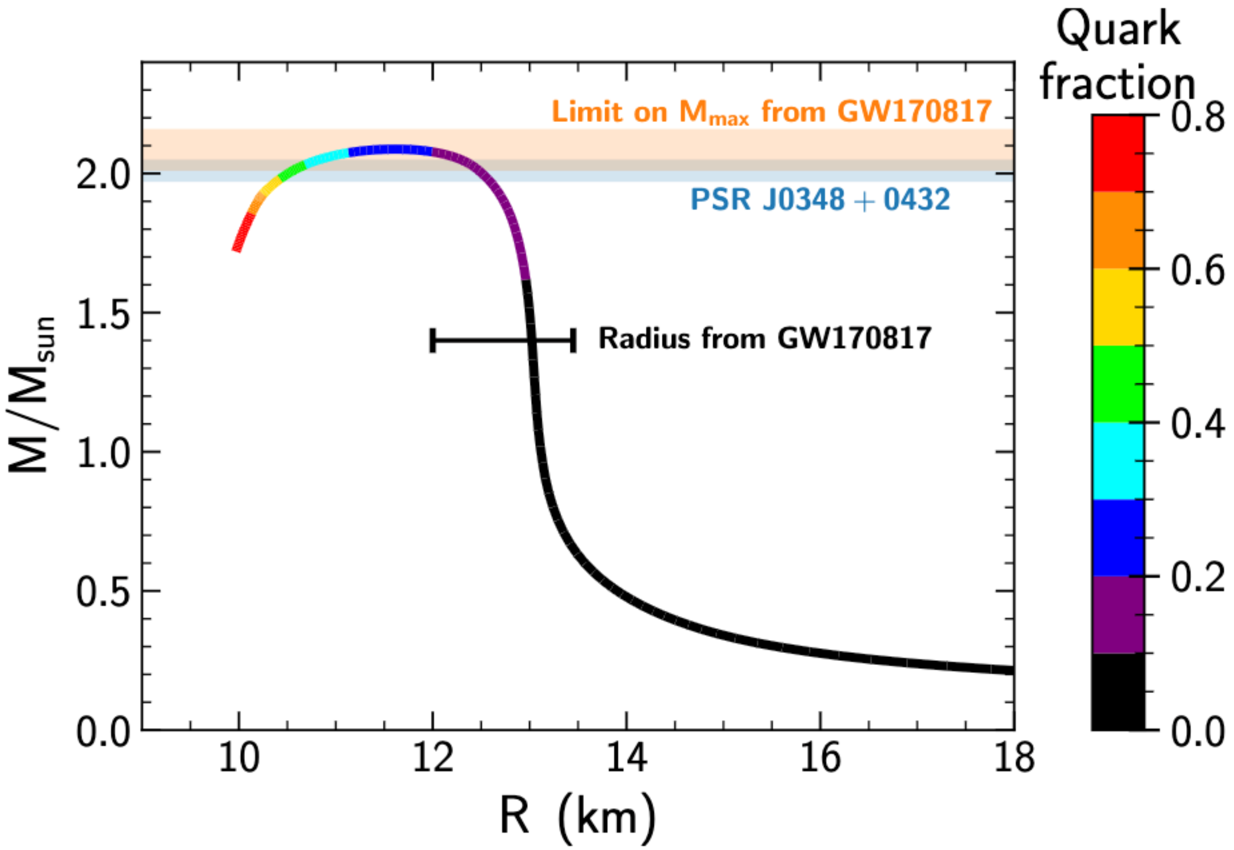


Pressure of dense nuclear matter balances the gravitational pull

QCD in astrophysics

Properties of dense nuclear matter define how heavy neutron stars can be and how large they are





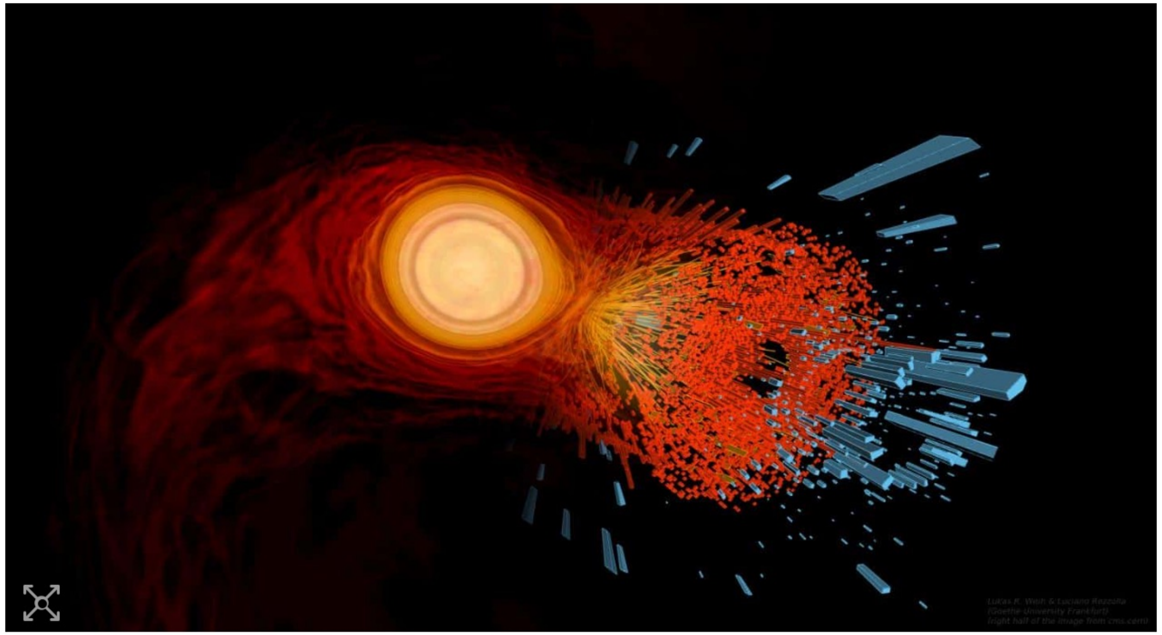
Intermediate energy heavy-ion collisions probe same dense nuclear matter

The ultimate “heavy-ion” collision

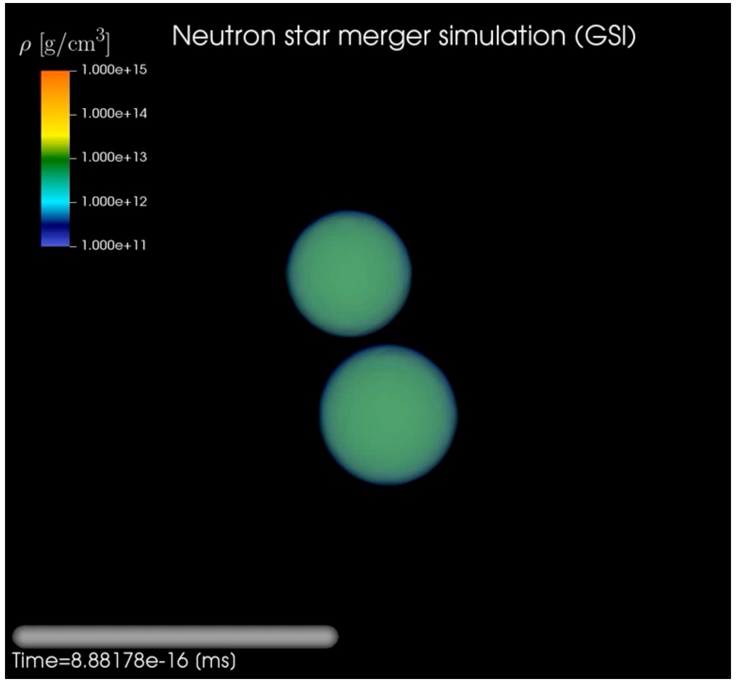
COSMOLOGY | **RESEARCH UPDATE**

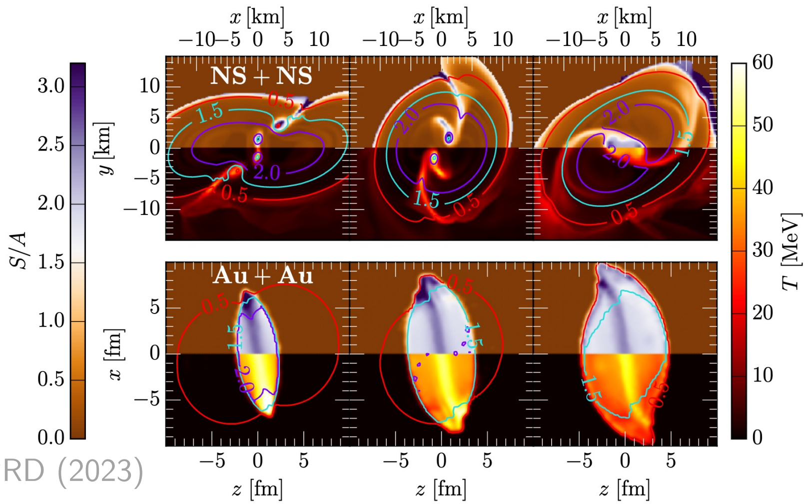
Gravitational waves from neutron-star mergers could reveal quark-gluon plasma

15 May 2020



Dark star crashes: the computer simulation of two merging neutron stars (left) blended with an image of heavy-ion collisions at CERN to highlight the connection of astrophysics with nuclear physics. Courtesy: Lukas R Weih and Luciano Rezzolla/Goethe University Frankfurt and CMS/CERN)

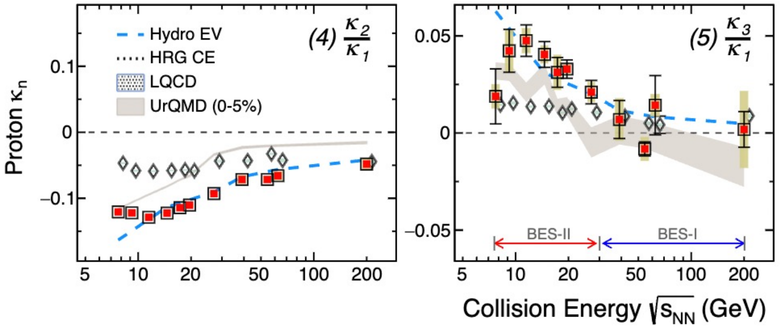


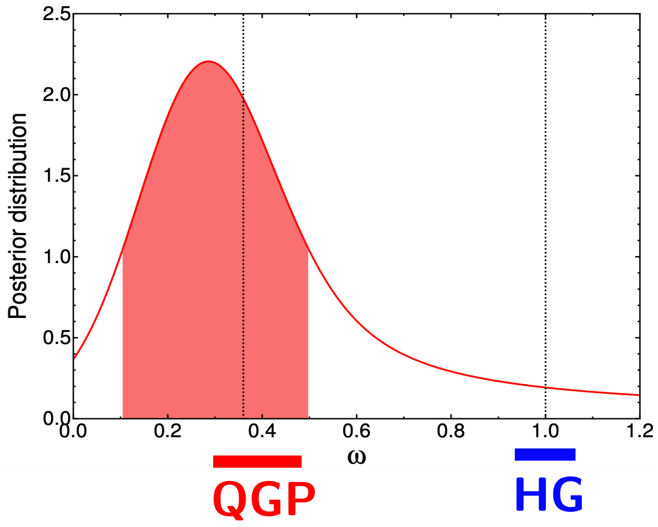


Most et al., PRD (2023)

Summary

- **Lattice-based constraints on the QCD critical point**
  - Rule out QCD CP at $\mu_B < 450$ MeV at a 2-sigma level
  - New method based on contours of constant entropy places QCD CP at $T_c = 114 \pm 7$ MeV, $\mu_B = 602 \pm 62$ MeV
- **RHIC-BES-II data is in**
  - Non-critical physics describe the proton data at $\sqrt{s_{NN}} \geq 20$ GeV
  - $\hat{C}_2 - \hat{C}_2^{baseline} > 0$ and $\hat{C}_3 - \hat{C}_3^{baseline} > 0$ at $\sqrt{s_{NN}} < 10$ GeV
    - May indicate freeze-out of fluctuations on the QGP side of crossover
- **Net-charge fluctuations at the LHC revisited**
  - Moderate evidence for charge fluctuations freeze-out in the QGP





Outlook:

- Improved description of non-critical baselines and quantitative predictions of critical fluctuations

Thanks for your attention!

Dynamical approaches to the QCD critical point search

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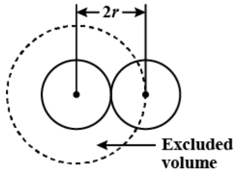
1. Dynamical model calculations of critical fluctuations   BEST COLLABORATION [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]
   - Fluctuating hydrodynamics (hydro+) and (non-equilibrium) evolution of fluctuations
   - Equation of state with a tunable critical point [P. Parotto et al, PRC 101, 034901 (2020); J. Karthein et al., EPJ Plus 136, 621 (2021)]
   - Generalized Cooper-Frye particlization [M. Pradeep, et al., PRD 106, 036017 (2022); PRL 130, 162301 (2023)]

   Alternatives at high μ_B: hadronic transport/molecular dynamics with a critical point

   [A. Sorensen, V. Koch, PRC 104, 034904 (2021); V. Kuznietsov et al., PRC 105, 044903 (2022)]



2. **Deviations from precision calculations of non-critical fluctuations**
   - Non-critical baseline is not flat [Braun-Munzinger et al., NPA 1008, 122141 (2021)]
   - Include essential non-critical contributions to (net-)proton number cumulants
   - Exact baryon conservation + hadronic interactions (hard core repulsion)
   - Based on realistic hydrodynamic simulations tuned to bulk data

   [VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

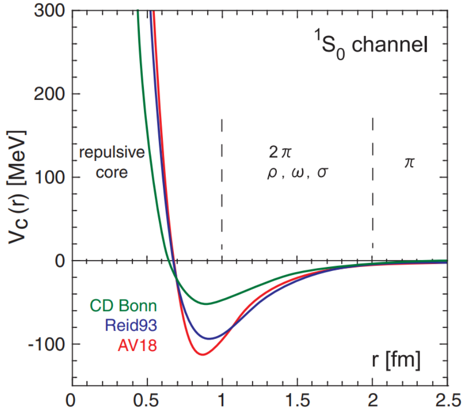


Figure from Ishii et al., PRL '07

Factorial cumulants $\hat{C}_n$ vs ordinary cumulants C_n

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Factorial cumulants: ~irreducible n-particle correlations

$$\hat{C}_n \sim \langle N(N-1)(N-2)\ldots\rangle_c$$

$$\hat{C}_1 = C_1$$
$$\hat{C}_2 = C_2 - C_1$$
$$\hat{C}_3 = C_3 - 3C_2 + 2C_1$$
$$\hat{C}_4 = C_4 - 6C_3 + 11C_2 - 6C_1$$

Ordinary cumulants: mix correls. of different orders

$$C_n \sim \langle \delta N^n \rangle_c$$

$$C_1 = \hat{C}_1$$
$$C_2 = \hat{C}_2 + \hat{C}_1$$
$$C_3 = \hat{C}_3 + 3\hat{C}_2 + \hat{C}_1$$
$$C_4 = \hat{C}_4 + 6\hat{C}_3 + 7\hat{C}_2 + \hat{C}_1$$

[Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017); Kitazawa, Luo, PRC 96, 024910 (2017); C. Pruneau, PRC 100, 034905 (2019)]

Factorial cumulants and different effects

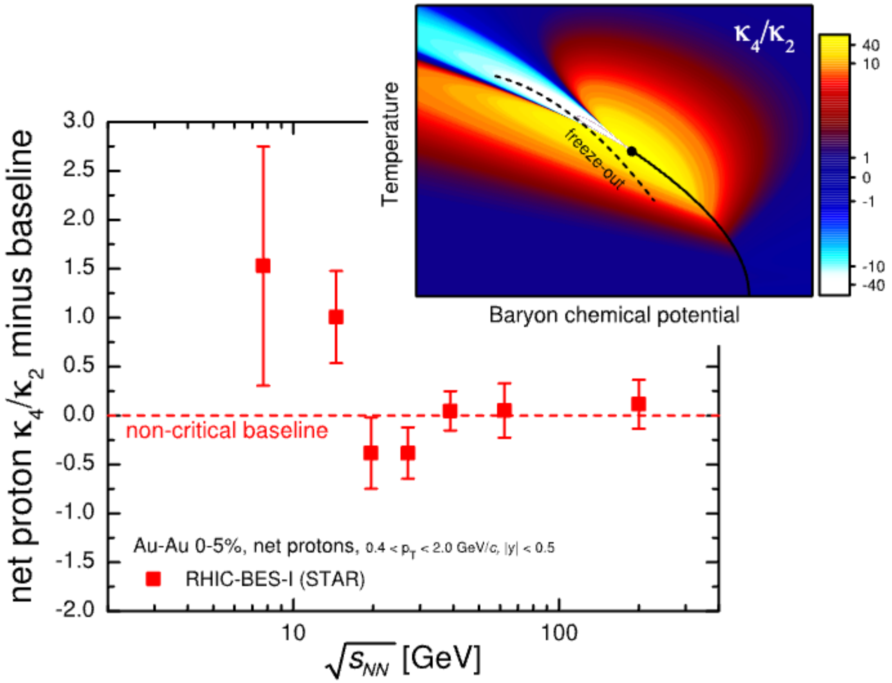
- Baryon conservation [Bzdak, Koch, Skokov, EPJC '17] — $\hat{C}_n^{\text{cons}} \propto (\hat{C}_1)^n / \langle N_{\text{tot}} \rangle^{n-1}$ *small*
- Excluded volume [VV et al, PLB '17] — $\hat{C}_n^{\text{EV}} \propto b^n$ *small*
- Volume fluctuations [Holzman et al., arXiv:2403.03598] — $\hat{C}_n^{CF} \sim (\hat{C}_1)^n \kappa_n[V]$ *depends on volume cumulants*
- **Critical point** [Ling, Stephanov, PRC '16] — $\hat{C}_2^{CP} \sim \xi^2$, $\hat{C}_3^{CP} \sim \xi^{4.5}$, $\hat{C}_4^{CP} \sim \xi^7$ ***large***

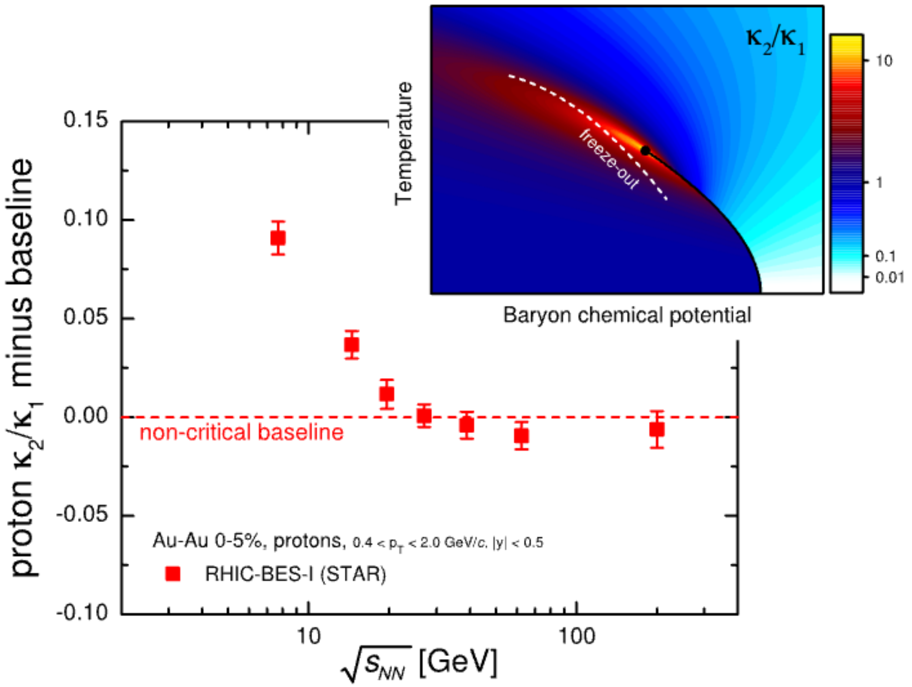
- proton vs baryon [Kitazawa, Asakawa, PRC '12] — $\hat{C}_n^B \sim 2^n \times \hat{C}_n^p$ **same sign!**

Hints from RHIC-BES-I

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Subtracting the hydrodynamic non-critical baseline

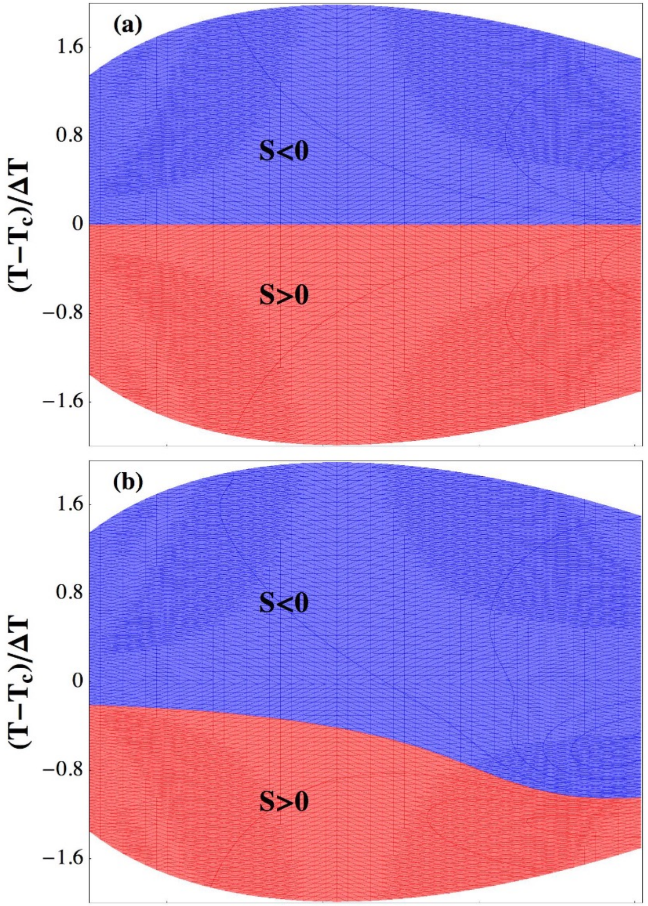




⚠️ **Notation:** Here we use κ_n for cumulants and $\hat{C}_n$ for factorial cumulants, STAR Collaboration uses the opposite

Factorial cumulants from RHIC-BES-II and CP

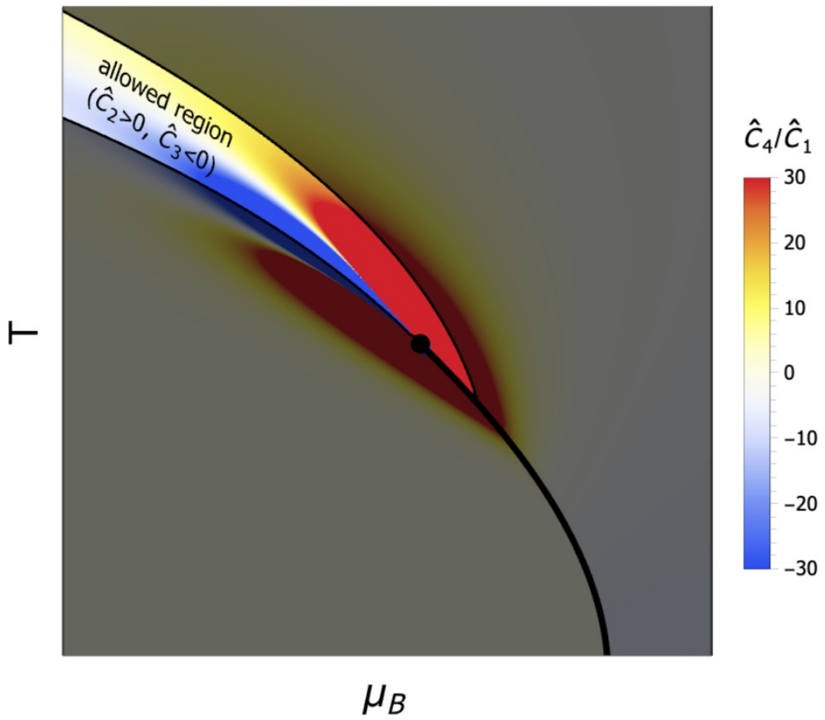
Memory effect



Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)

Exclusion plots

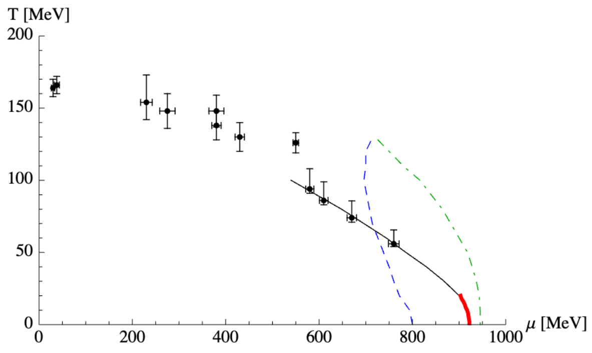
Exclude $\hat{C}_2<0$ & $\hat{C}_3>0$ regions on the phase diagram near CP



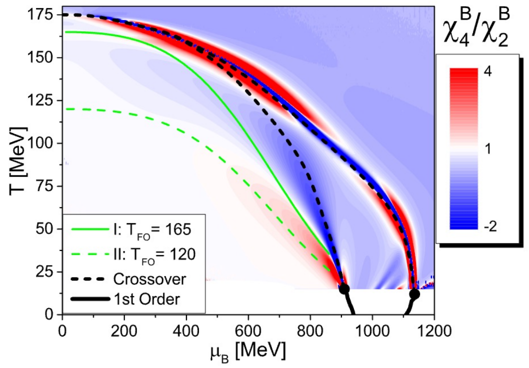
Adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)
and based on the model from
VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Freeze-out of fluctuations on the QGP side of the crossover?

Interplay with nuclear liquid-gas transition

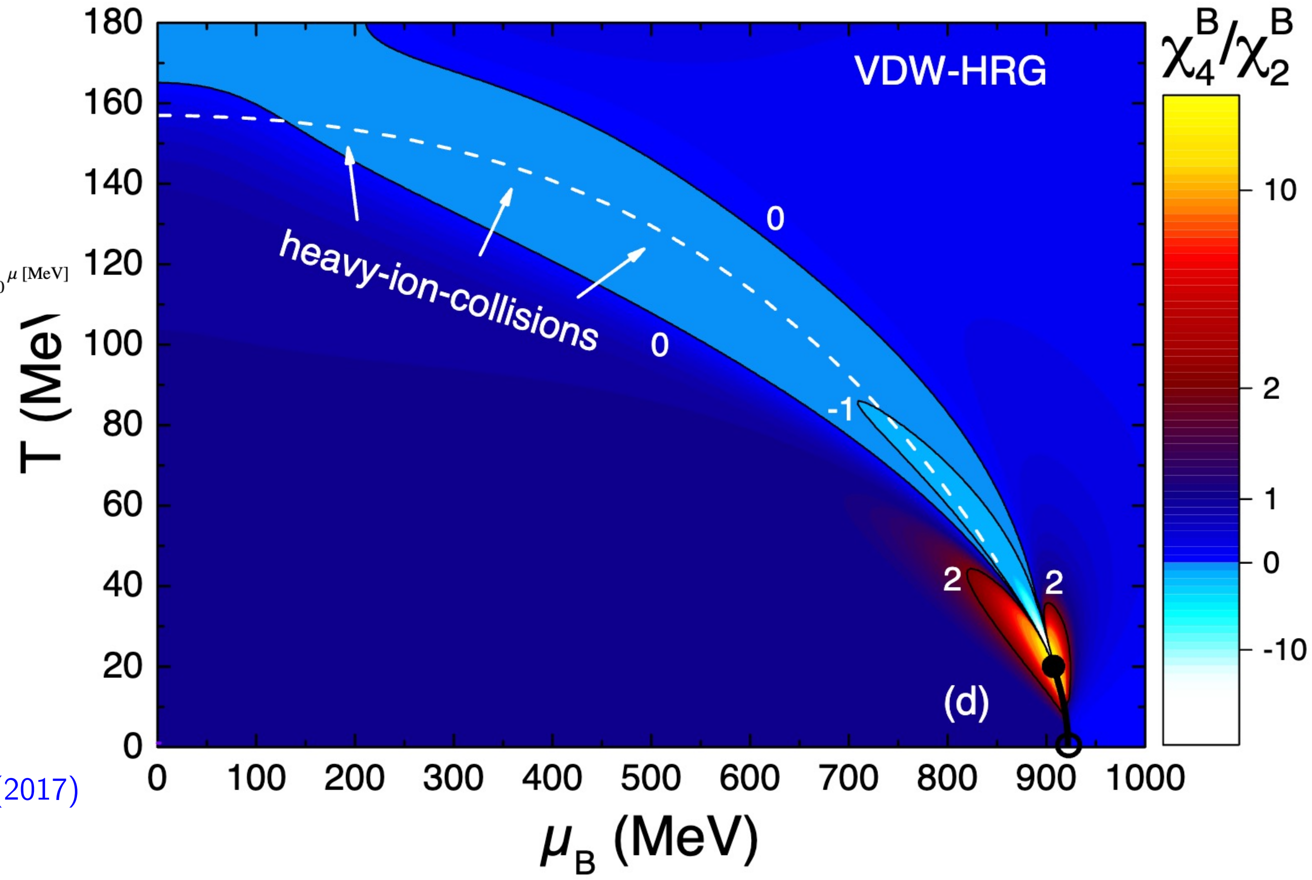


Floerchinger, Wetterich, NPA (2012)

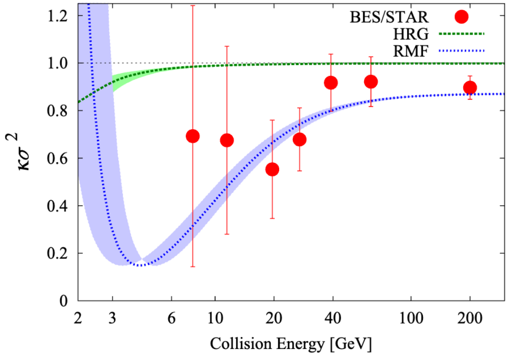


Mukherjee, Steinheimer, Schramm, PRC (2017)

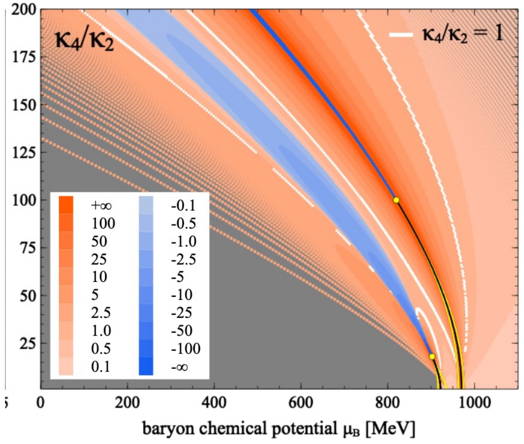
HRG with attractive and repulsive interactions among baryons



VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)



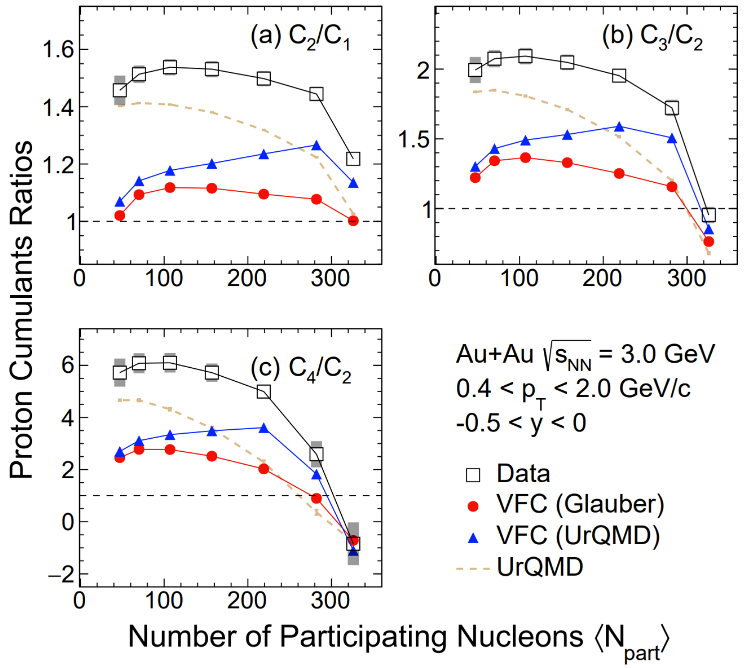
Fukushima, PRC (2014)



Sorensen, Koch, PRC (2020)

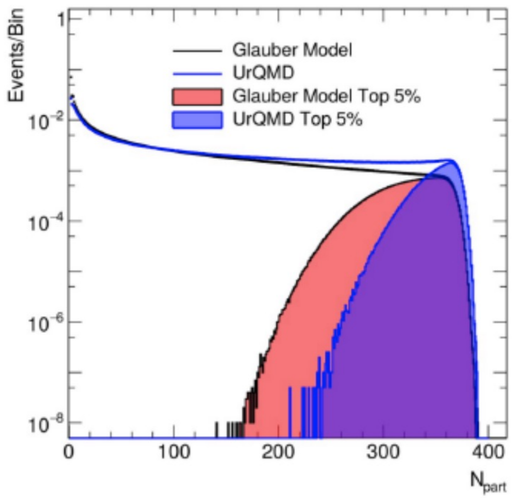
Increasingly relevant at lower energies probed through RHIC-FXT

Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV



STAR Collaboration, Phys. Rev. Lett. 128 (2022) 202303

STAR-FXT



HADES

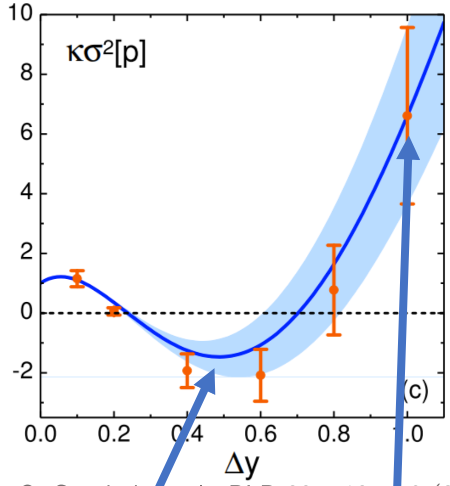


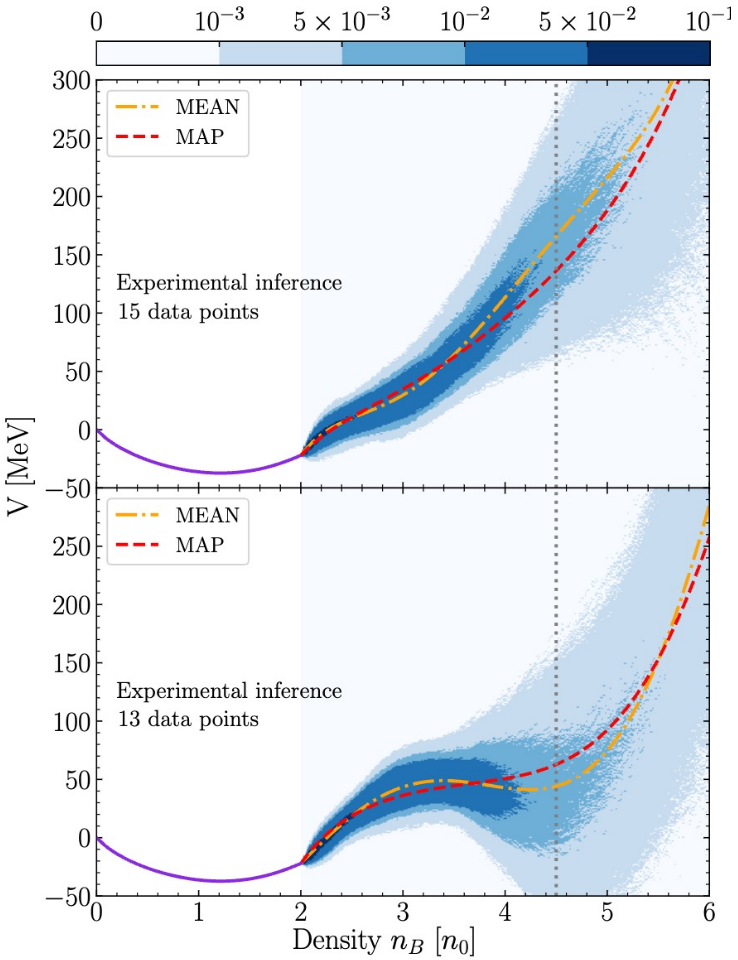
Figure from O. Savchuk et al., PLB 835, 137540 (2022)

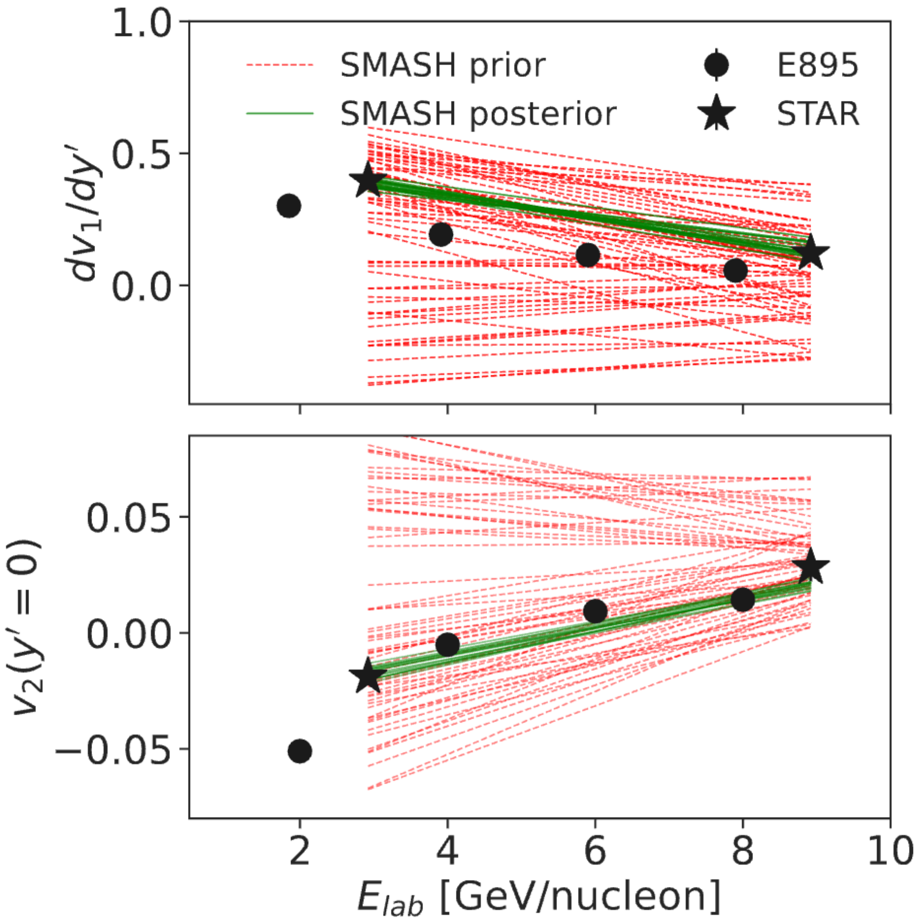
- Volume fluctuations/centrality selection appear to play an important role
  - UrQMD is useful for understanding basic systematics associated with it
- Indications for enhanced scaled variance, $\kappa_2/\kappa_1 > 1$
- κ_4/κ_2 negative and described by UrQMD (purely hadronic?), note -0.5<y<0 instead of |y|<0.5

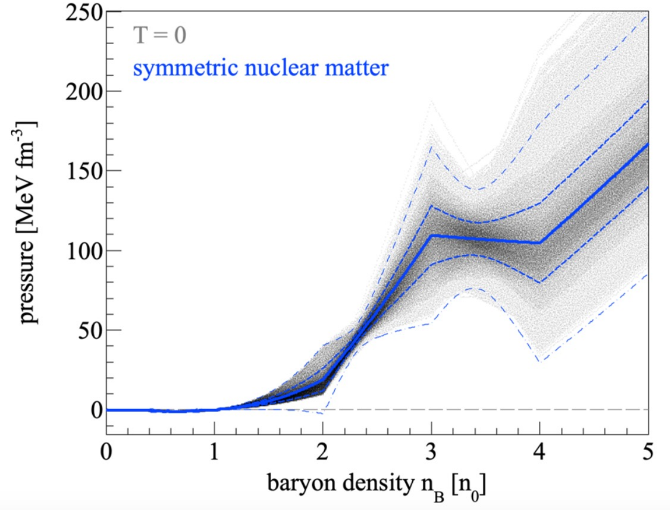
Proper understanding of $\kappa_2/\kappa_1 > 1$ in both HADES and STAR-FXT is missing

Dense matter EoS from flow measurements

- Use hadronic transport (UrQMD and SMASH) with adjustable mean field to use a flexible EoS
- Extract the EoS from proton flow measurements





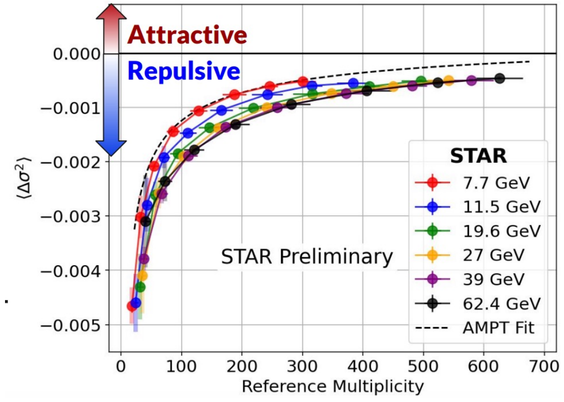


M. Kuttan, Steinheimer, Zhou, Stoecker, PRL 131, 202303 (2023)

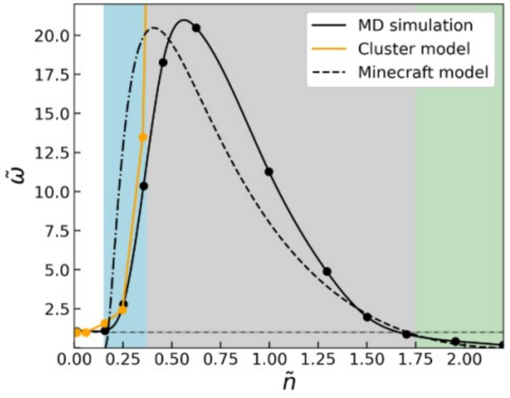
Oliinychenko, Sorensen, Koch, McLerran, PRC 108, 034908 (2023)

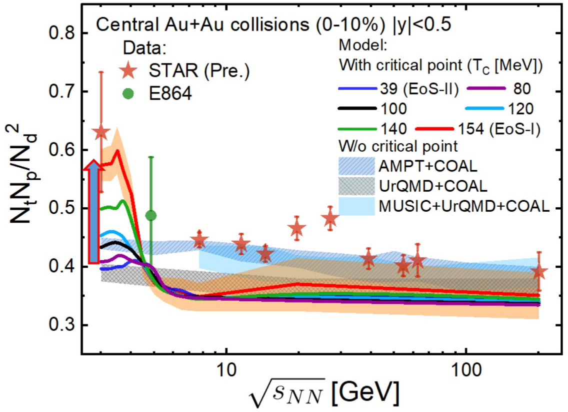
Other observables

- Azimuthal correlations of protons
  - points to repulsion at RHIC-BES



- Light nuclei
  - Spinodal/critical point enhancement of density fluctuations and light nuclei production





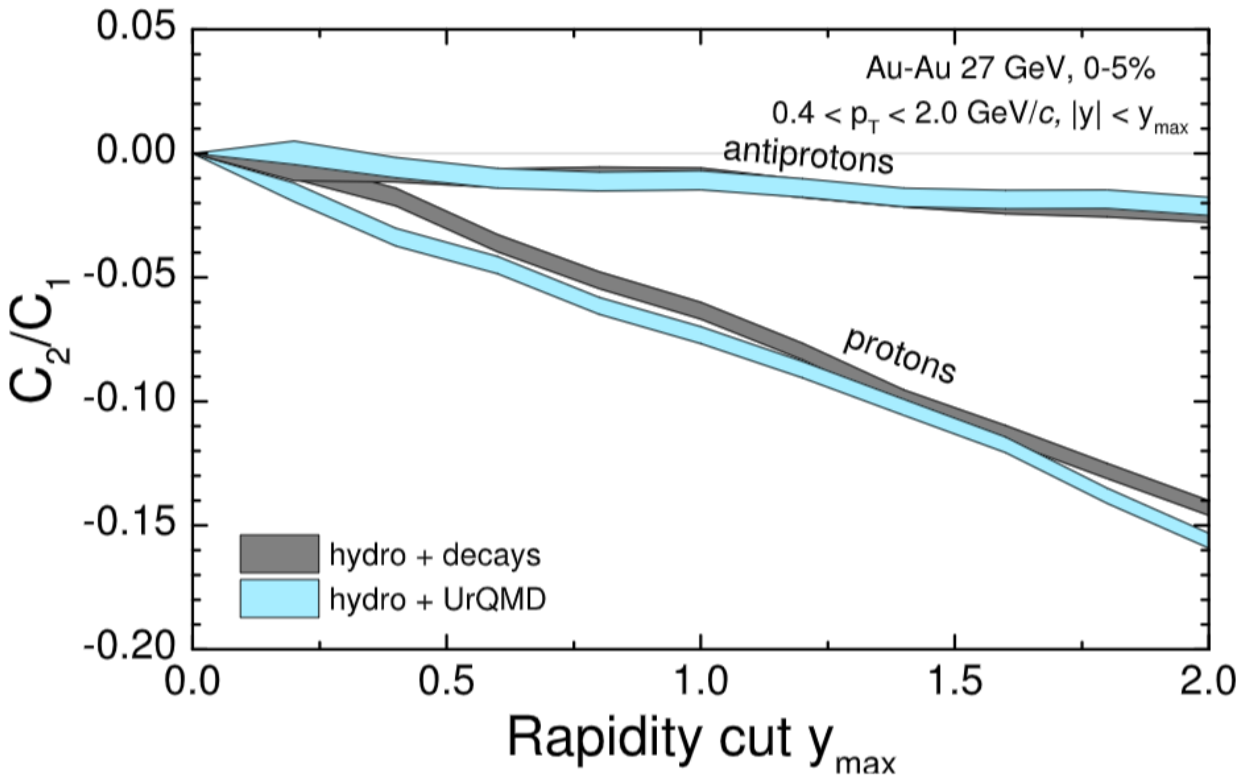
- Proton intermittency
  - No structure indicating power-law seen by NA61/SHINE

- Directed flow, speed of sound

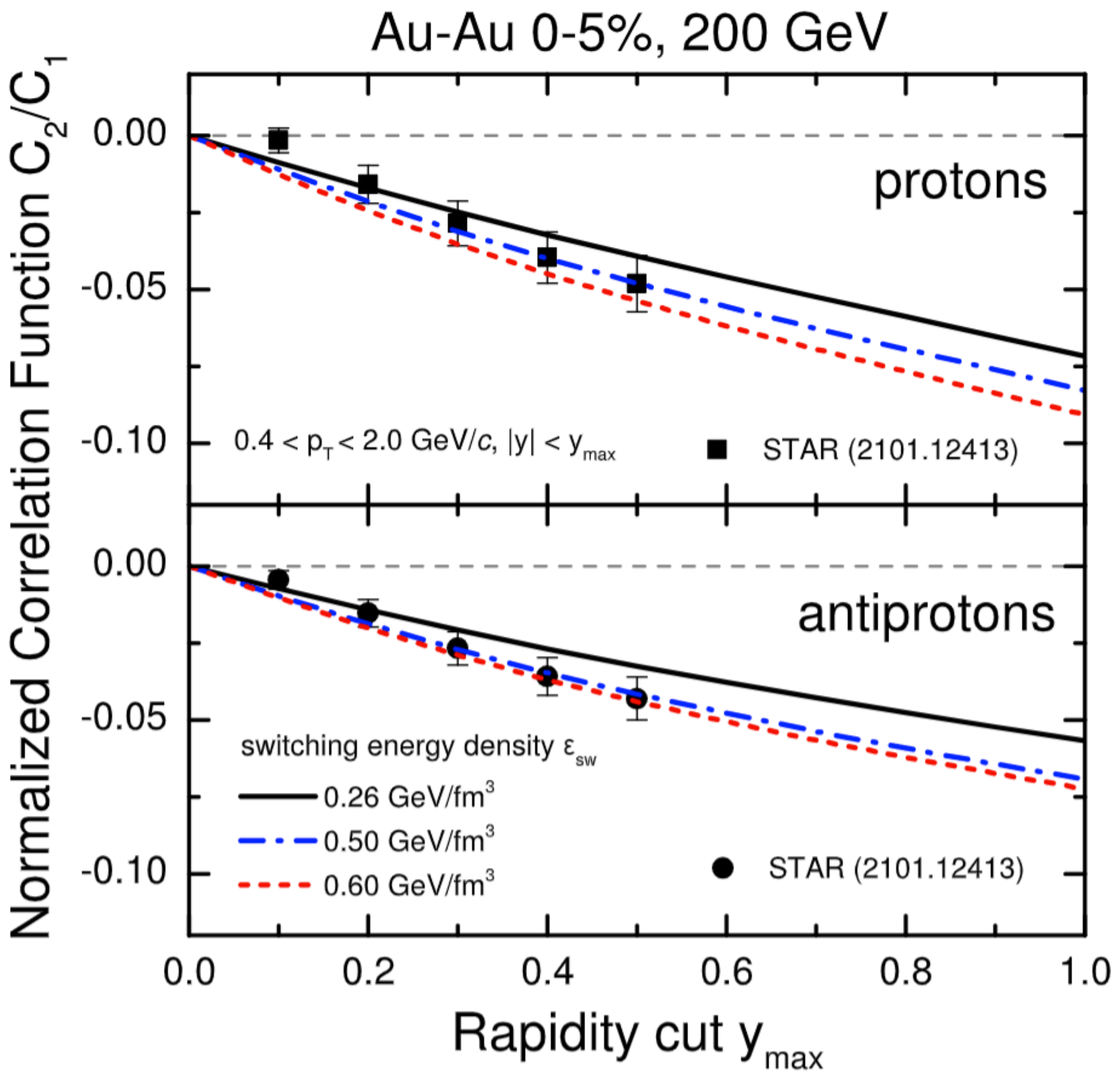
Consistency in understanding all the observables is required

Effect of the hadronic phase

Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD



Dependence on the switching energy density

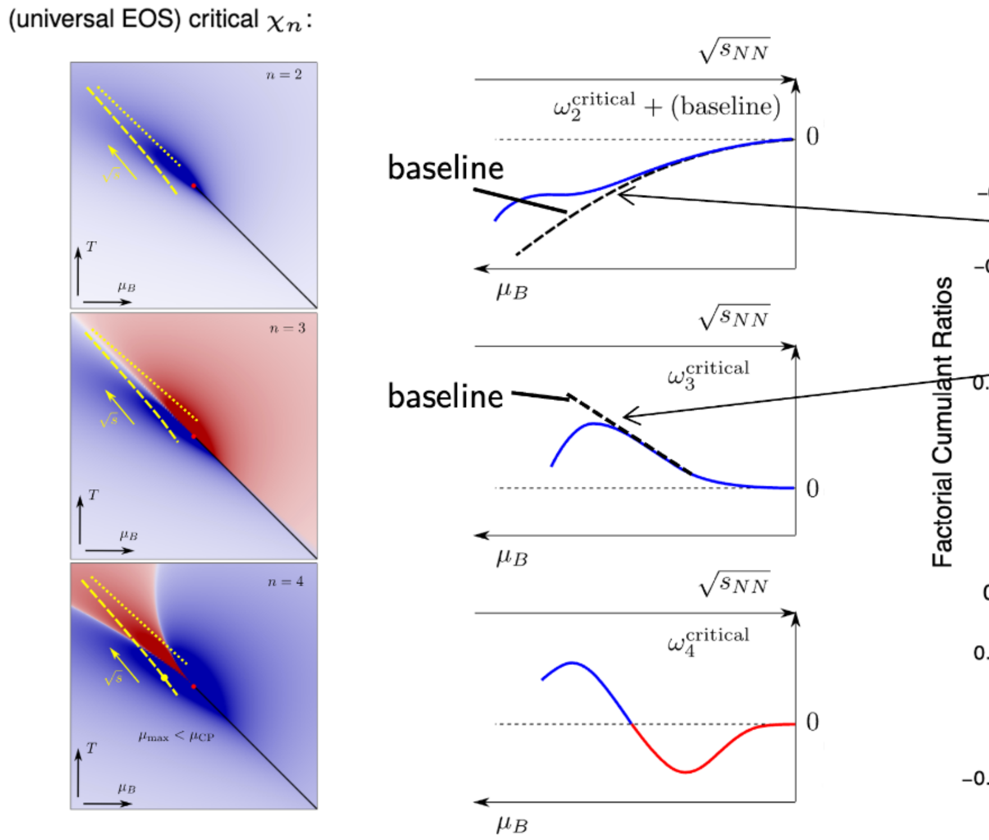


Summary slides

Understanding proton cumulants from RHIC-BES-II data

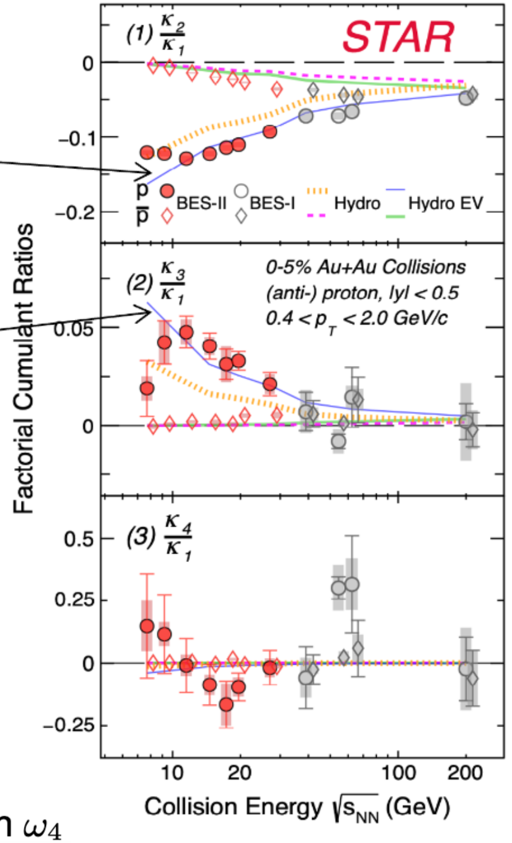
Vovchenko, Koch, arXiv:2504.01368, plot adapted from M. Stephanov, arXiv:2410.02861

$$\omega_n = \hat{C}_n / \hat{C}_1$$



BES-II data:

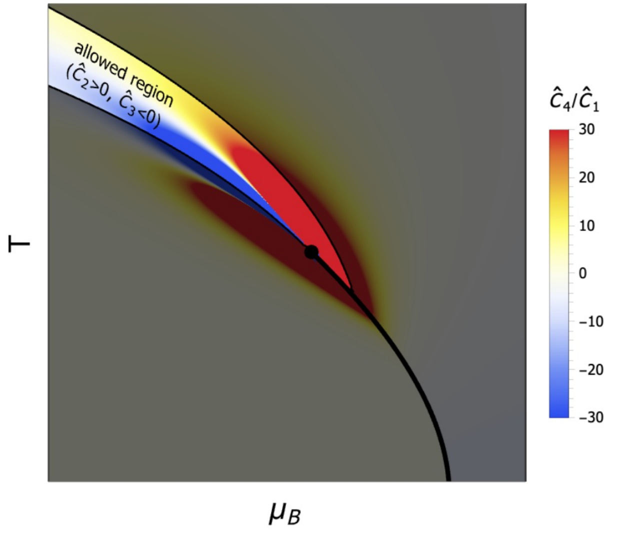
plot from A. Pandav, CPOD2024



Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

- **Factorial cumulants** may be more instructive than ordinary cumulants for CP search
- **signal relative to baseline:**
  - *positive* $\hat{C}_2 - \hat{C}_2^{baseline} > 0$
  - *negative* $\hat{C}_3 - \hat{C}_3^{baseline} < 0$

allowed region near the CP is on QGP side of crossover



Controlling the non-critical baseline is essential

Scaled factorial cumulants and the antiproton puzzle

Bzdak et al. introduced reduced correlation functions – "couplings" [Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)]

$$\boxed{\hat{c}_k = \frac{\hat{C}_k}{\langle N \rangle^k}}$$

$$c_k = \frac{\int \rho_1(y_1)\cdots\rho_1(y_k)c_k(y_1,\ldots,y_k)dy_1\cdots dy_k}{\int \rho_1(y_1)\cdots\rho_1(y_k)dy_1\cdots dy_k}$$

integrated correlation function in rapidity

Long-range correlations lead to acceptance-independent $\hat{c}_k$, including any combination of global baryon conservation and volume fluctuations (no need for CBWC)!

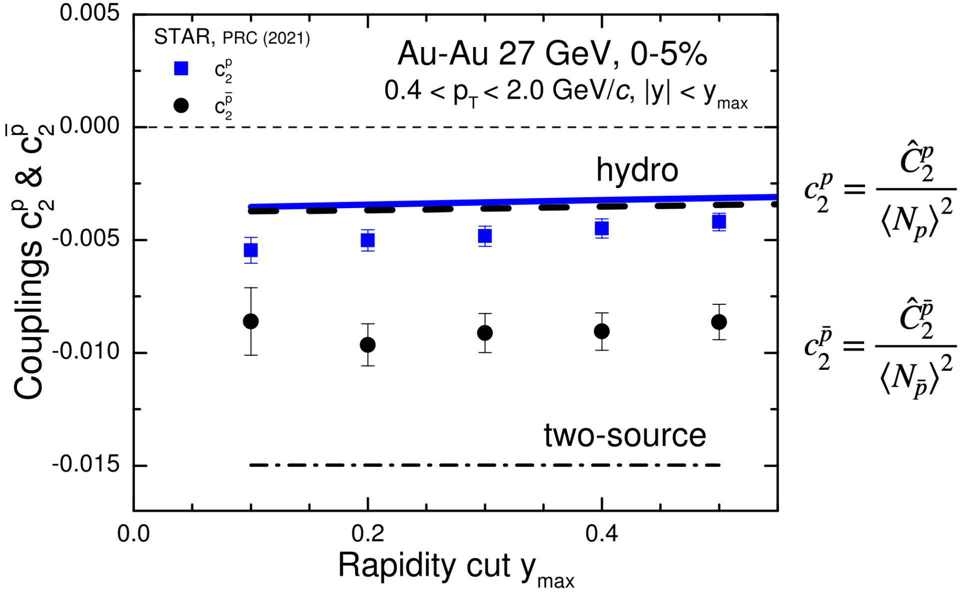
Standard hydro predicts:

$\hat{c}_2^p \approx \hat{c}_2^{\bar{p}} = const.$ at a given $\sqrt{s_{NN}}$

Experiment:

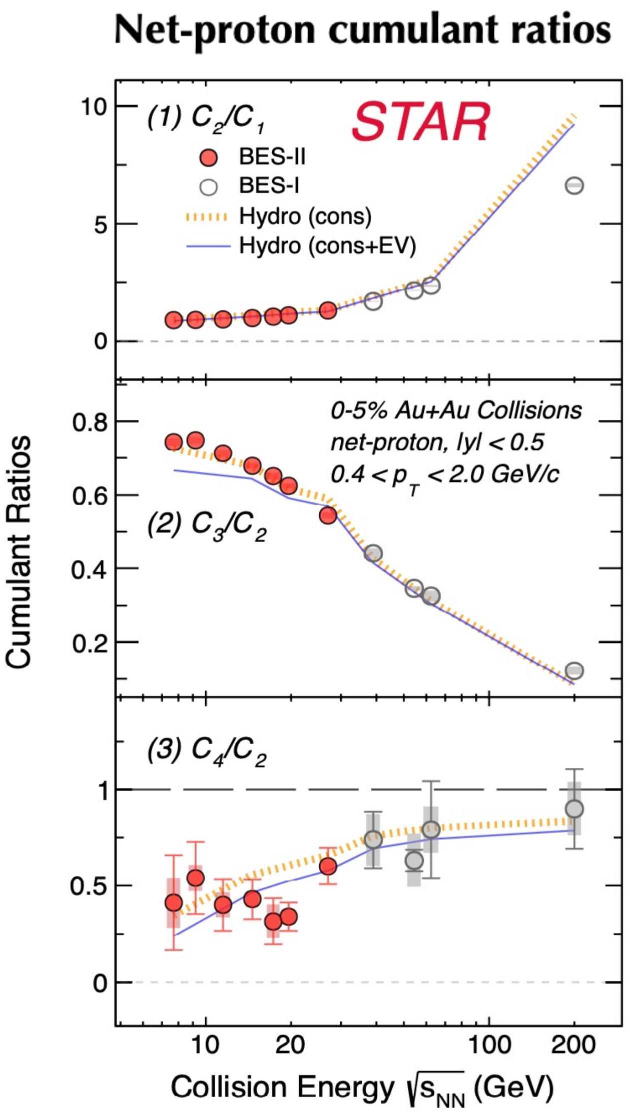
$|\hat{c}_2^p| < |\hat{c}_2^{\bar{p}}|$ **the antiproton puzzle**

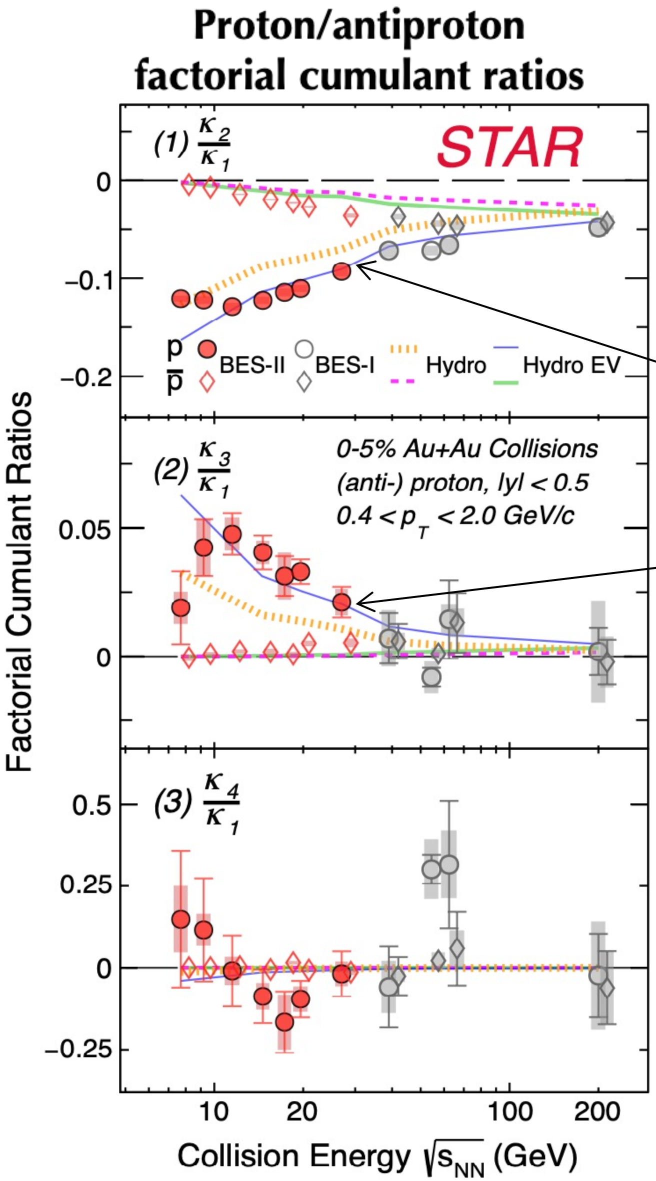
Possible explanation: stopped and produced matter do not thermalize (no single fluid)



A. Bzdak, V. Koch, V. Vovchenko, arXiv:2503.16405

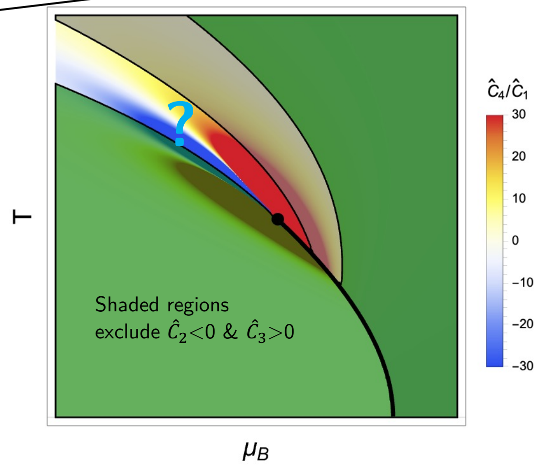
Understanding proton cumulants from RHIC-BES-II data





- Factorial cumulants may be more instructive than ordinary cumulants
- Deviations from available non-critical baseline at $\sqrt{s_{NN}} < 10$ GeV

$\hat{C}_2$ −baseline > 0 & $\hat{C}_3$ −baseline < 0



- Need precise handle on non-critical contributions

Non-critical cumulants: Analytic vs Monte Carlo

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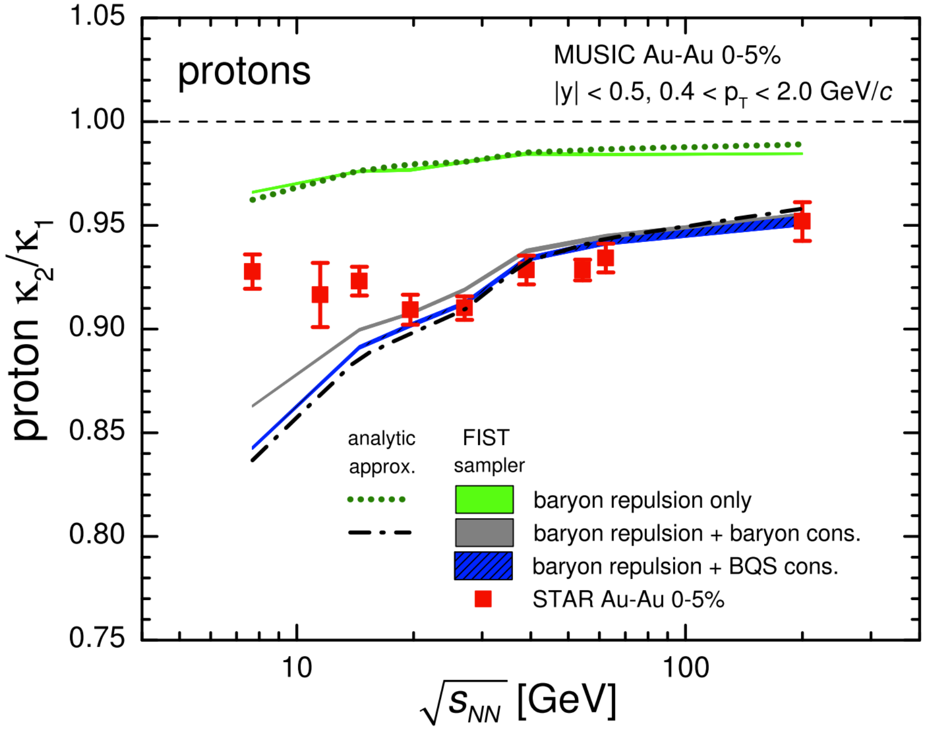
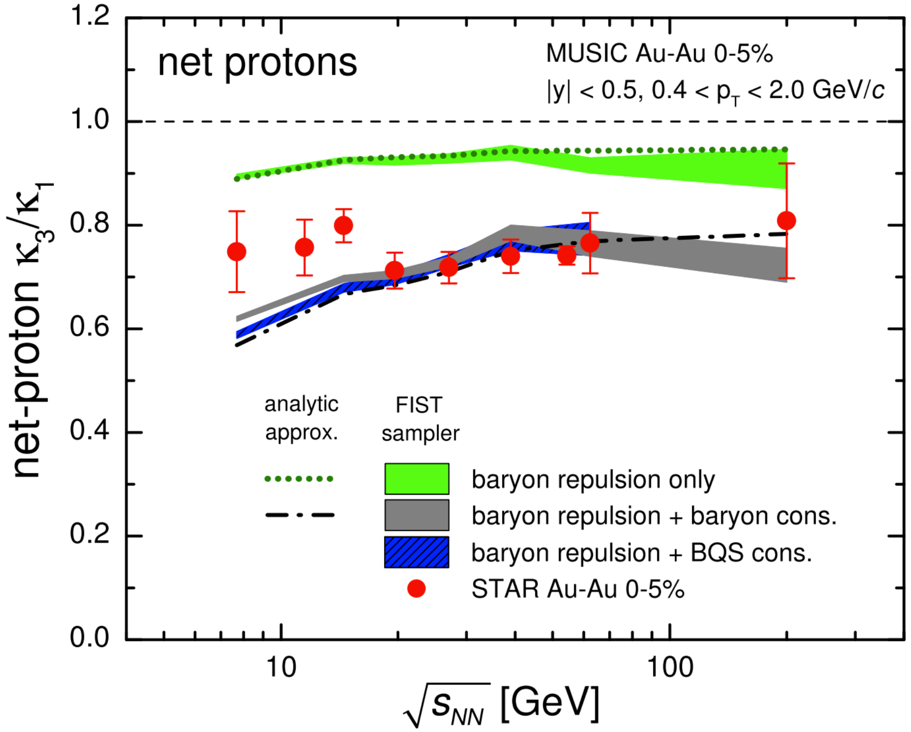
Proton κ_2/κ_1



Net-proton κ_3/κ_1



Non-critical cumulants

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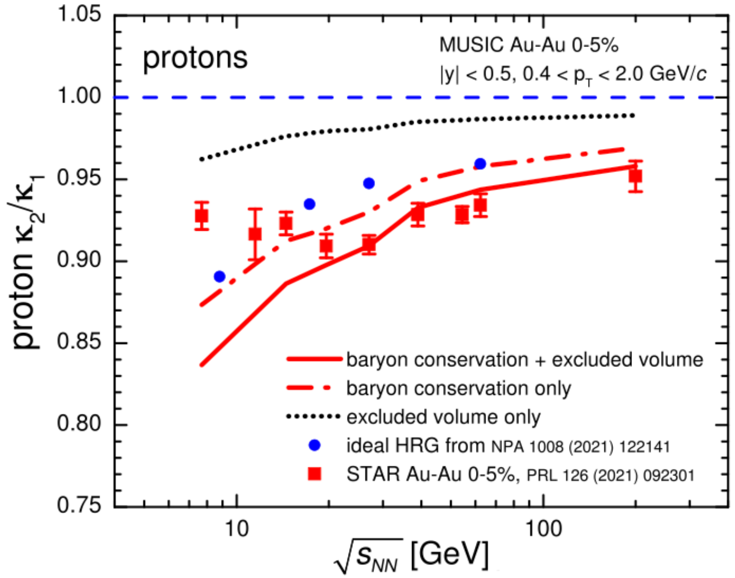
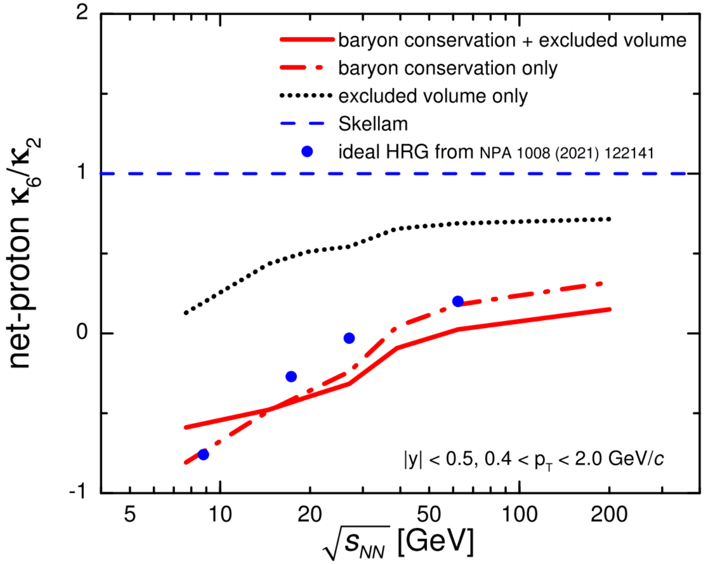
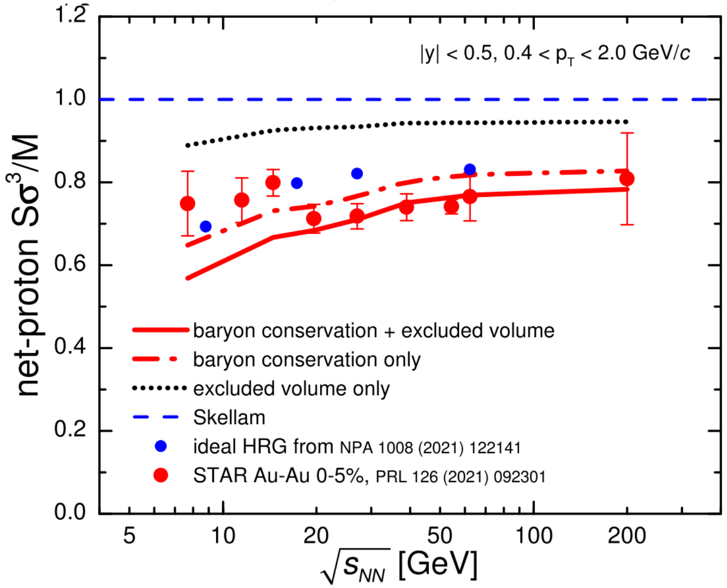
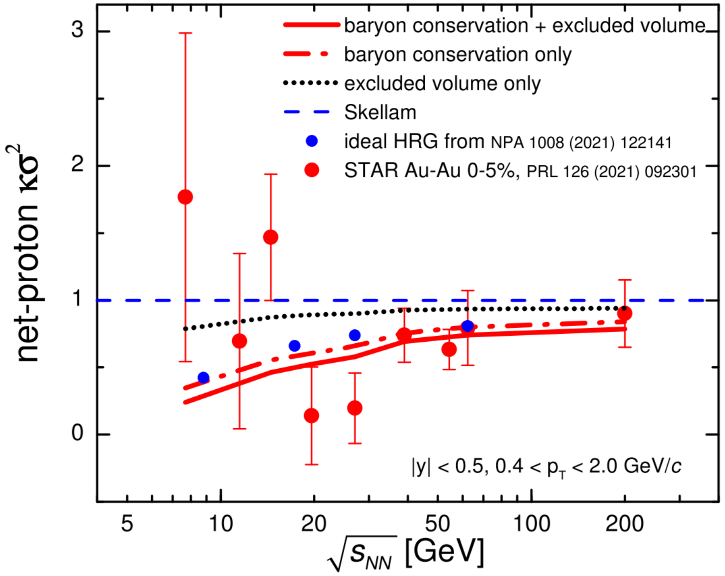


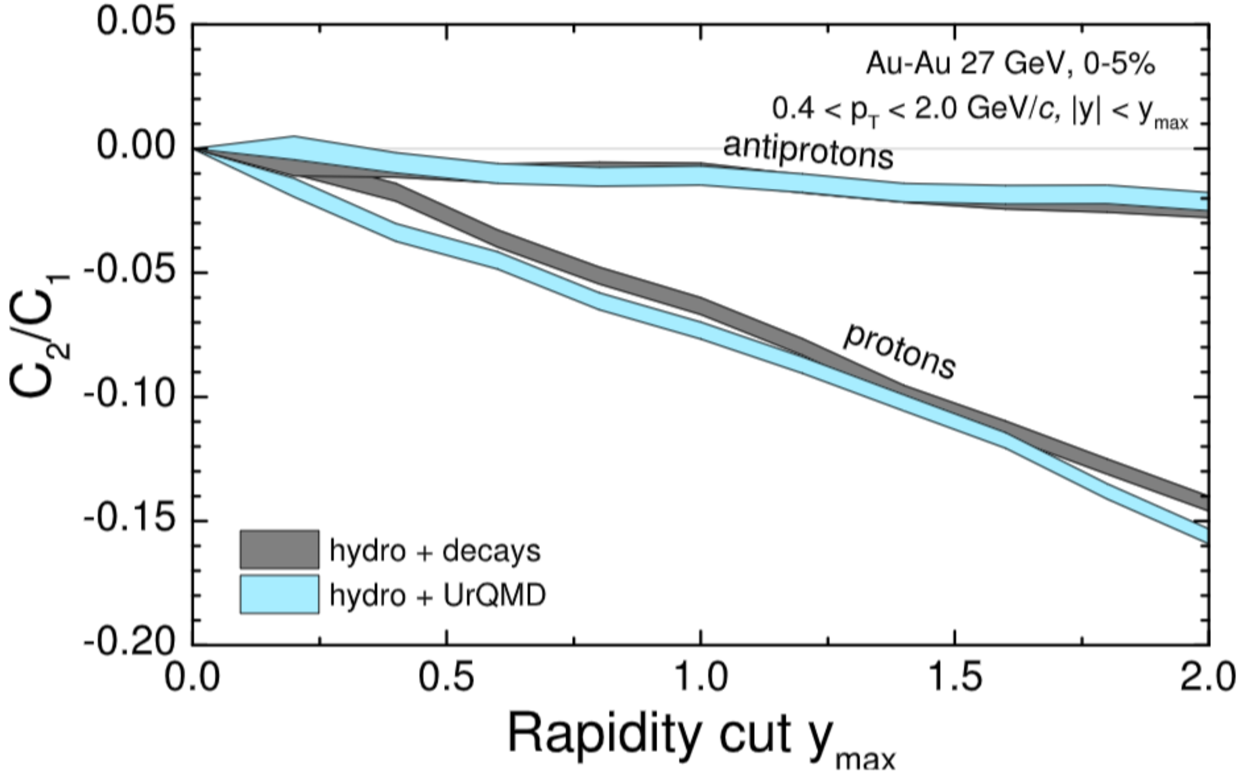






Effect of the hadronic phase

Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD



Dependence on the switching energy density