Fluctuations and correlations in heavy-ion collisions as a probe of the QCD phase structure

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QCD under extreme conditions

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What we know

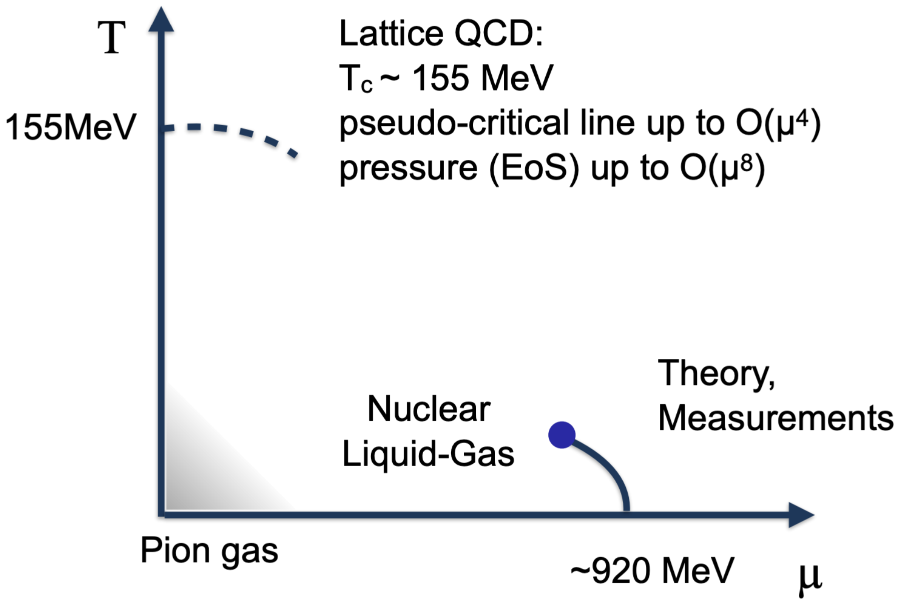

Figure courtesy of V. Koch

- Dilute hadron gas at low T & μ_{B} due to confinement, quark-gluon plasma high T & μ_{B}
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured
- Chiral crossover at $\mu_B = 0$

QCD under extreme conditions

What we know

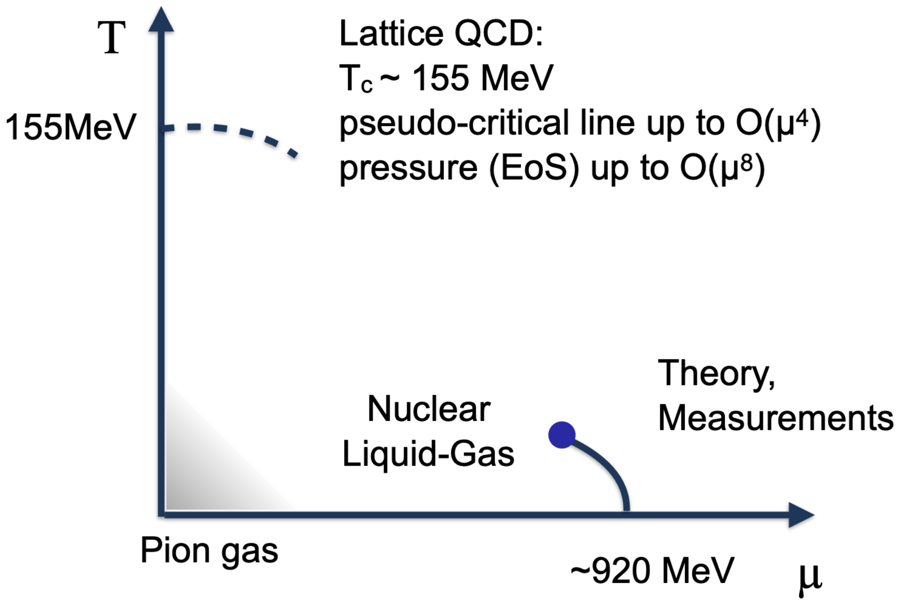

Figure courtesy of V. Koch

What we hope to know

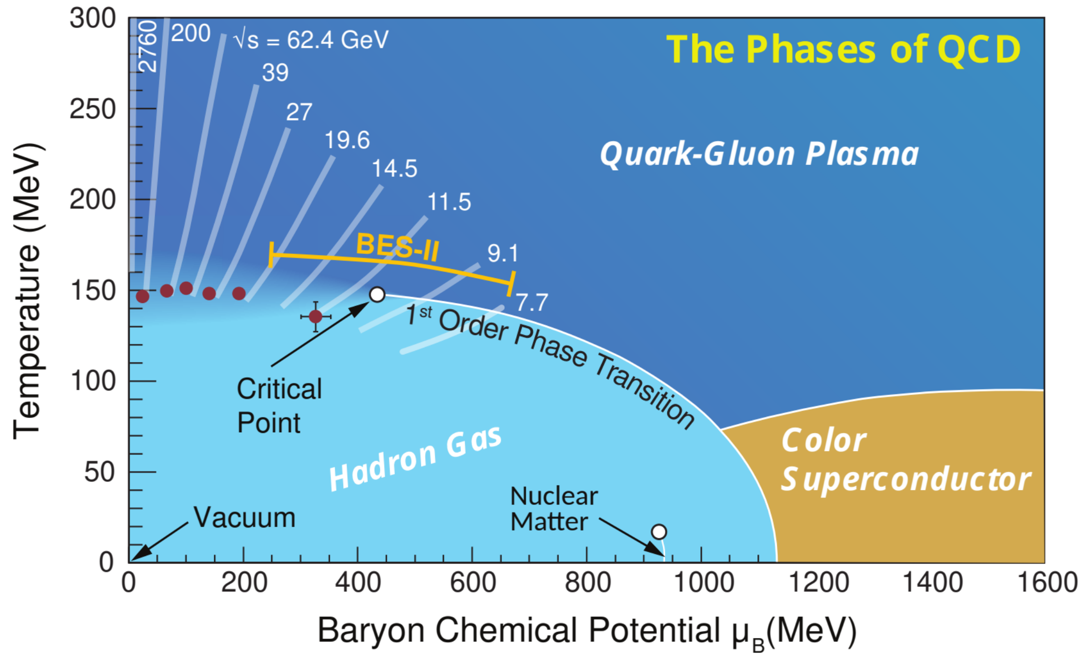

Figure from Bzdak et al., Phys. Rept. '20 & 2015 Long Range Plan

- Dilute hadron gas at low T & μ_B due to confinement, quark-gluon plasma high T & μ_B
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QCD under extreme conditions

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What we know

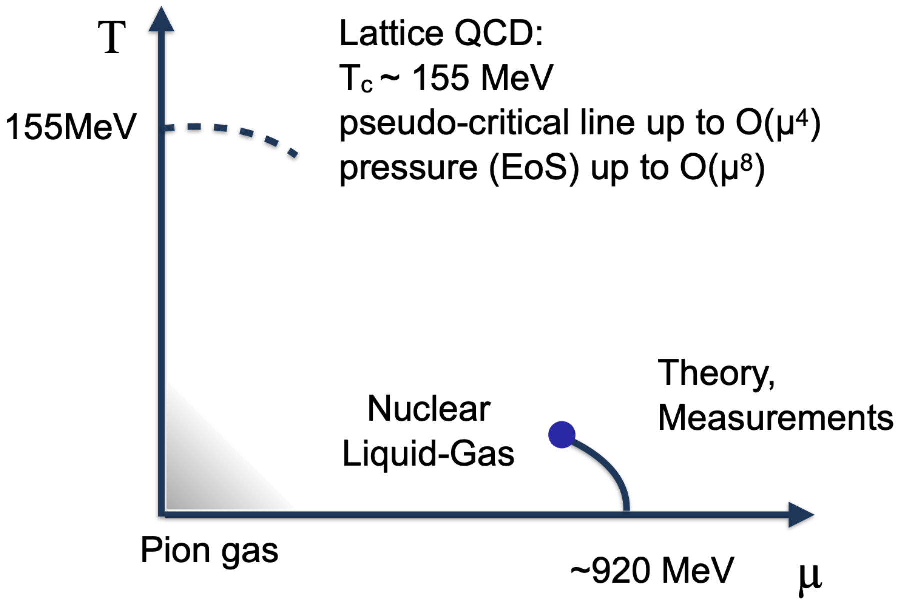
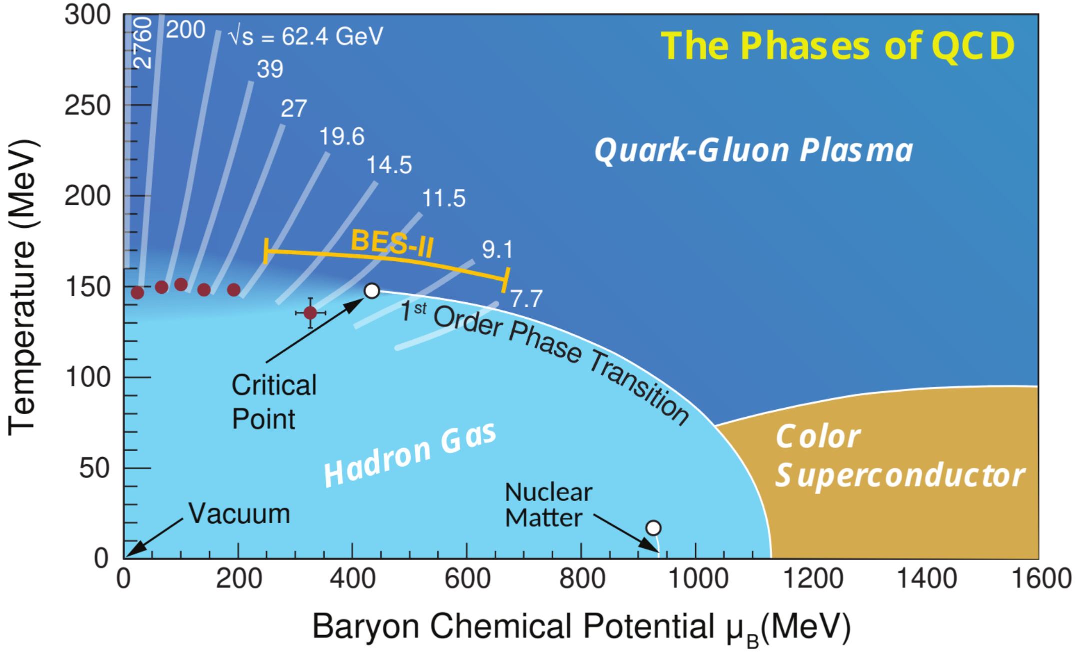


Figure courtesy of V. Koch

What we hope to know



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Key question: *Is there a QCD critical point and how to find it?*

Extrapolations from lattice QCD at $\mu_B = 0$

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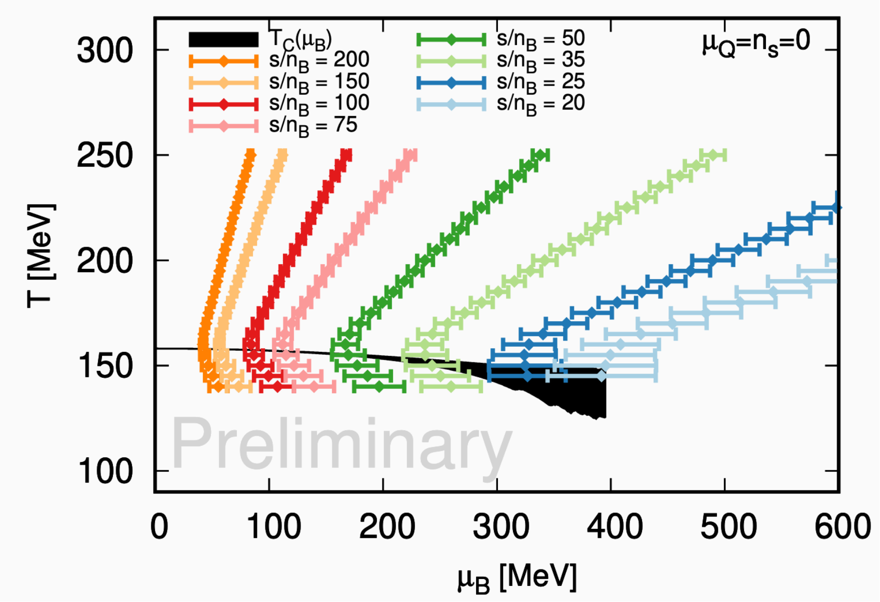
Ideally, find the critical point through first-principle **lattice QCD** simulations at finite μ_B

- Challenging (sign problem), but perhaps not impossible? [Borsanyi et al., Phys. Rev. D 107, 091503L (2023)]

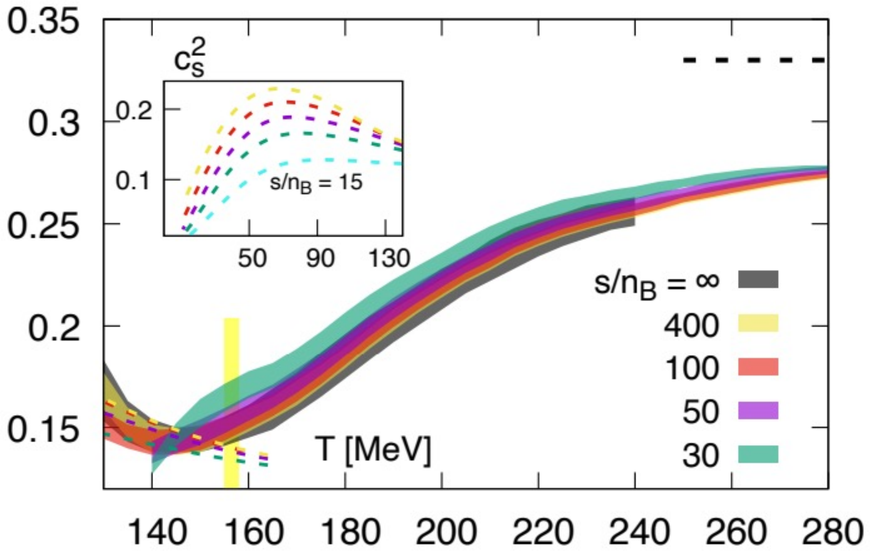
Taylor expansion + various resummations and extrapolation schemes from $\mu_B = 0$

alternative expansion scheme



[Borsanyi et al. (WB), Phys. Rev. D 105, 114504 (2022)]

Padé approximants



[Bollweg et al. (HotQCD), Phys. Rev. D 108, 014510 (2023)]

No indications for the strengthening of the chiral crossover or critical point signals

Disfavors QCD critical point at $\frac{\mu_B}{T} < 3$

Searching for singularities in the complex plane

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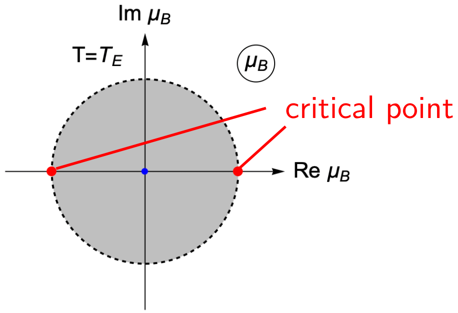
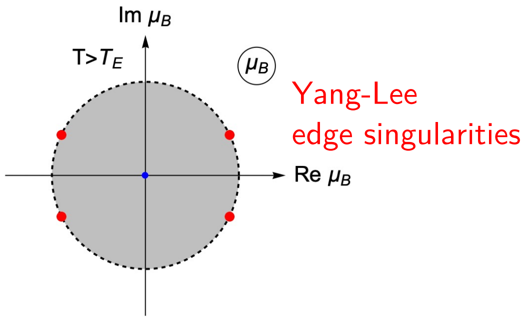
Critical point:

- *singularity in the partition function*
- *$T = T_E$: real μ_B axis*
- *$T > T_E$: Yang-Lee edge singularities*

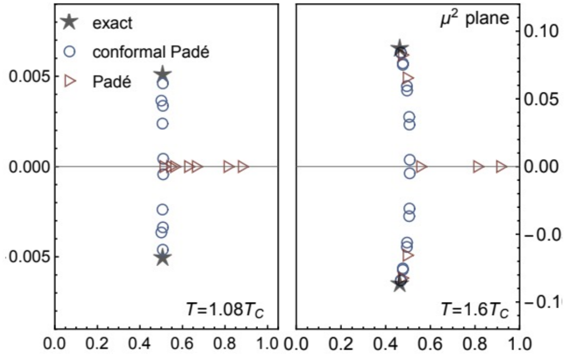
in the complex μ_B plane

[M. Stephanov, Phys. Rev. D 73, 094508 (2006)]



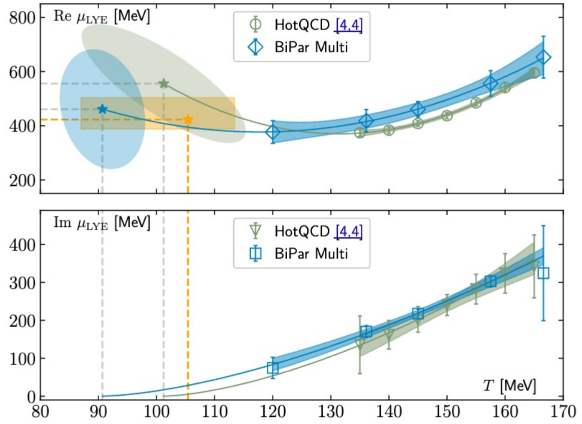


Lattice QCD is at $T > TE$:



[G. Basar, arXiv:2312.06952]

- Extract YL edge singularity through (multi-point*)/(conformal**) Pade fits
- See if it approaches the real axis as temperatures decreases



Critical Point: 3D-Ising scaling inspired fit:

$$\mathrm{Im}\,\mu_{LY} = c(T - T_{CEP})^{\Delta}$$

$$\mathrm{Re}\,\mu_{LY} = \mu_{CEP} + a(T - T_{CEP}) + b(T - T_{CEP})^2$$

Extrapolated CP estimate:
T~90-110 MeV, μ_B~400-600 MeV

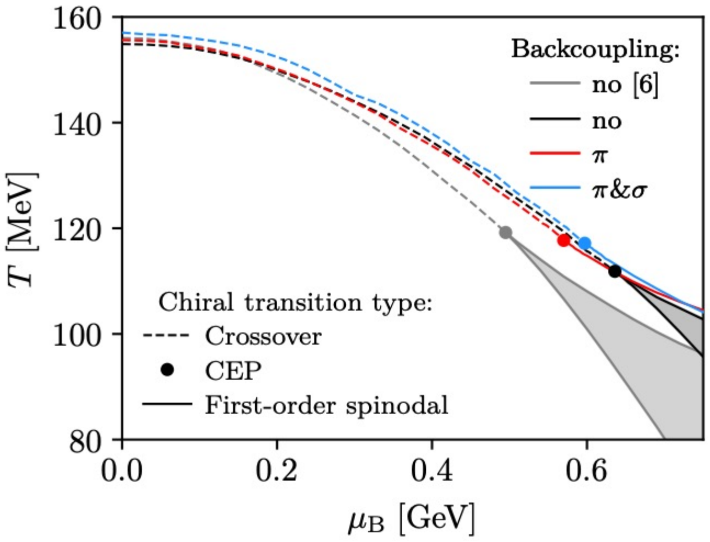
NB: many things have to go right, systematic error still very large (up to 100%), no continuum limit (likely large cut-off effects)

*D.A. Clarke et al. (Bielefeld-Parma), arXiv:2405.10196; **G. Basar, arXiv:2312.06952

Effective QCD theories predictions

Dyson-Schwinger equations

truncation errors

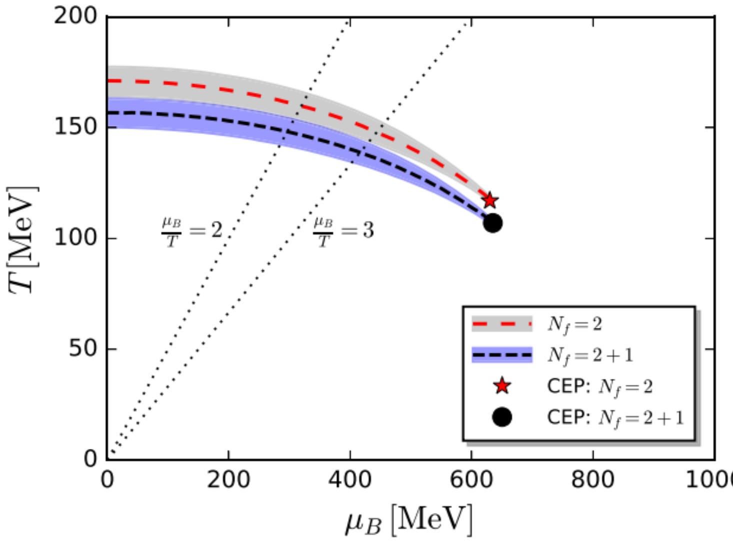


Gunkel, Fischer, PRD 104, 054202 (2021)

T~120 MeV μ_B~600 MeV

Functional renormalization group

truncation errors

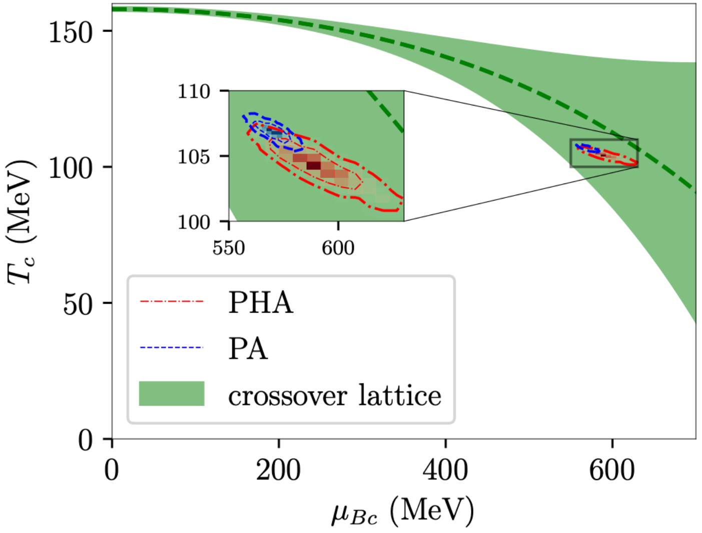


Fu, Pawlowski, Rennecke, PRD 101, 053032 (2020)

T~100 MeV μ_B~600 – 650 MeV

Black-hole engineering

strongly-coupled only ($\eta/s=1/4\pi$)



Hippert et al., arXiv:2309.00579

T~105 MeV μ_B~580 MeV

All in excellent agreement with lattice QCD at $\mu_B = 0$
and predict QCD critical point in a similar ballpark of μ_B/T ~ 5-6

If true, reachable in heavy-ion collisions at $\sqrt{s_{NN}}$~3 – 5 GeV

Search for critical point with heavy-ion collisions

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Control parameters

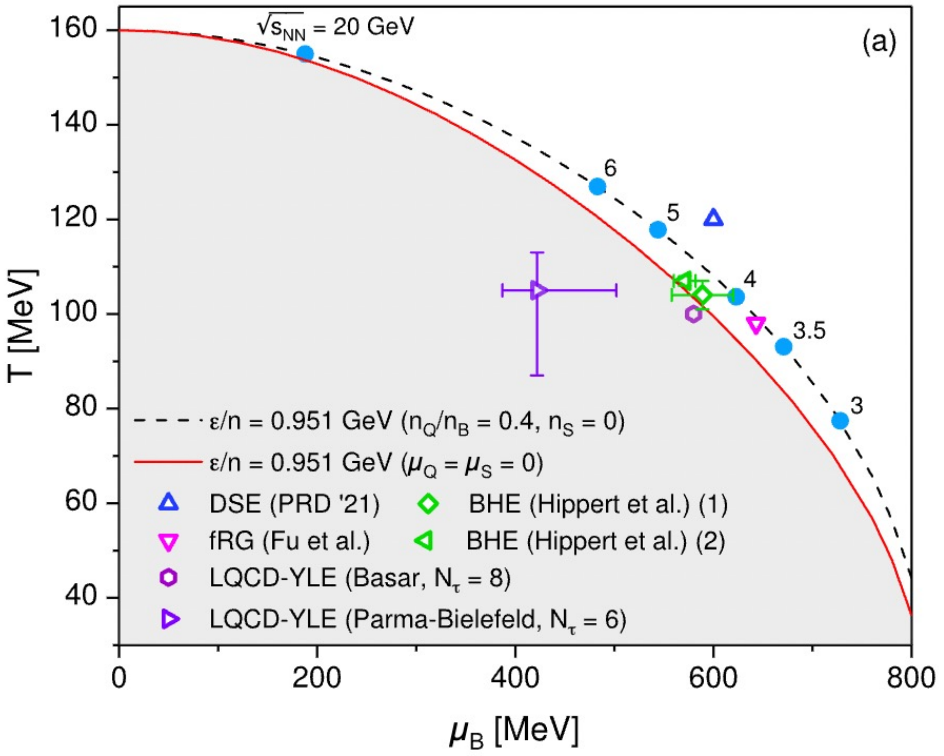
- Collision energy $\sqrt{s_{NN}} = 2.4 - 5020$ GeV
  - Scan the QCD phase diagram
- Size of the collision region
  - Expect stronger signal in larger systems

Measurements

- Final hadron abundances and momentum distributions **event-by-event**

Chemical freeze-out curve and CP

- Sets lower bound on the temperature of the CP
- **Caveats:** strangeness neutrality ($\mu_S \neq 0$), uncertainty in the freeze-out curve



A. Lysenko, Poberezhnyuk, Gorenstein, VV, arXiv:2408.06473

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

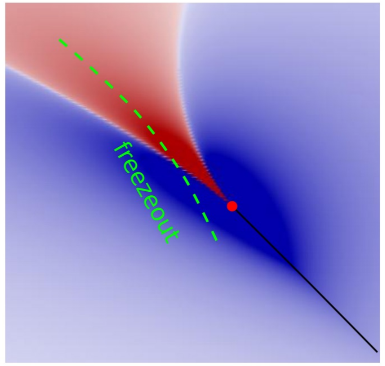
Grand partition function

$$\ln Z^{\rm gce}(T,V,\mu) = \ln\left[\sum_N e^{\mu N/T}\, Z^{\rm ce}(T,V,N)\right]$$

$$\kappa_n \propto \frac{\partial^n(\ln Z^{\rm gce})}{\partial \mu^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- **(QCD) critical point:** large correlation length and fluctuations



M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR) and CERN-SPS (NA61/SHINE)

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

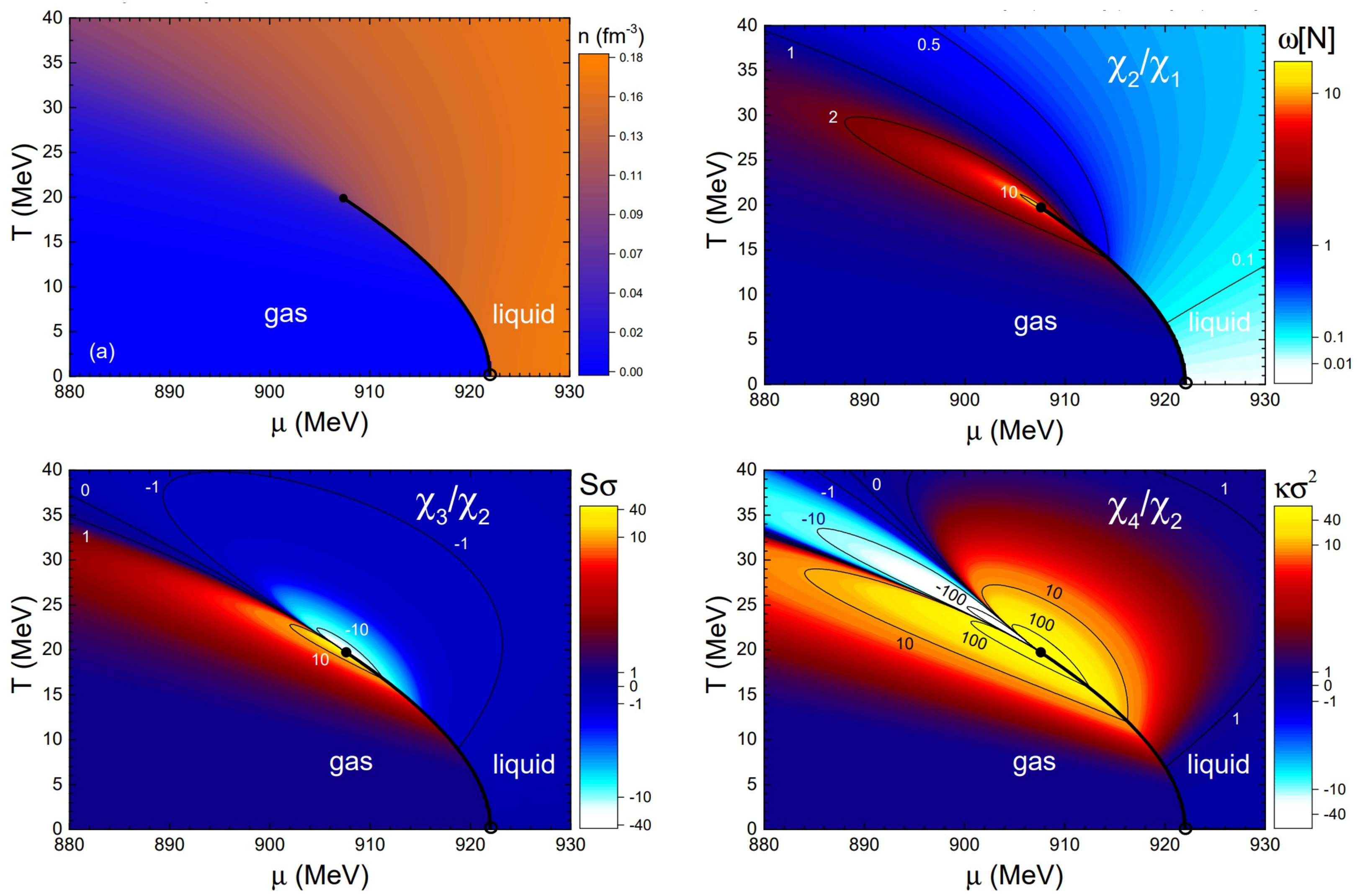
$$\xi \to \infty$$

Looking for enhanced fluctuations and non-monotonicities

Other uses of cumulants:

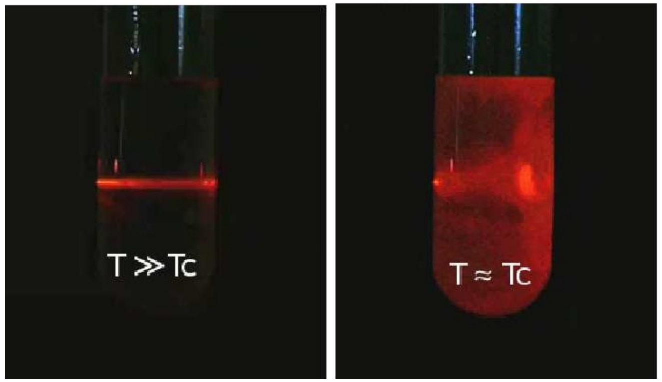
- QCD degrees of freedom
  Jeon, Koch, PRL 85, 2076 (2000)
  Asakawa, Heinz, Muller, PRL 85, 2072 (2000)
- Extracting the speed of sound
  A. Sorensen et al., PRL 127, 042303 (2021)
- Conservation volume V_C
  VV, Donigus, Stoecker, PRC 100, 054906 (2019)

Example: (Nuclear) Liquid-gas transition



VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Critical opalescence



$$\langle N^2\rangle - \langle N\rangle^2 \sim \langle N\rangle \sim 10^{23}$$

in equilibrium

Example: Critical fluctuations in a microscopic simulation

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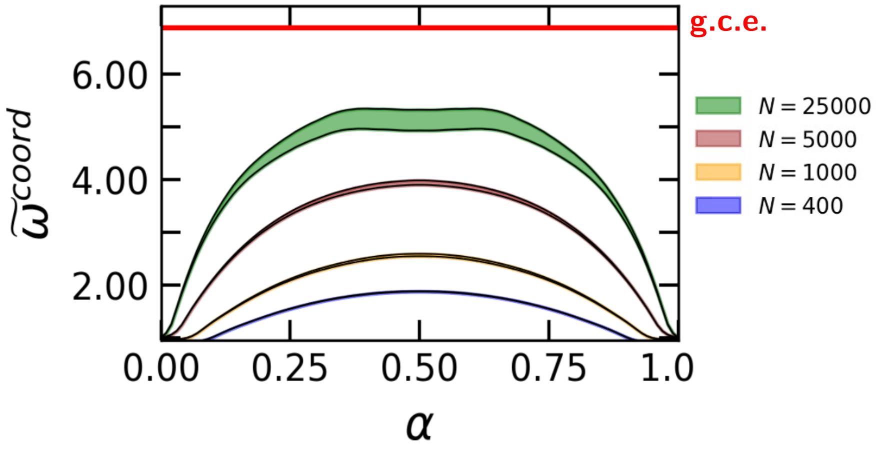
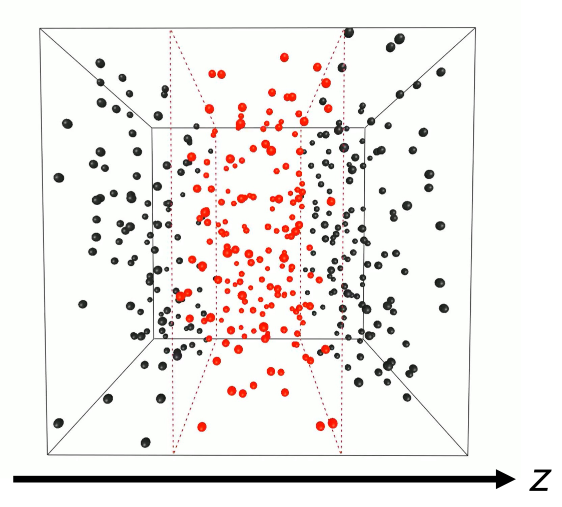
V. Kuznietsov et al., Phys. Rev. C 105, 044903 (2022)

Classical molecular dynamics simulations of the **Lennard-Jones fluid** near Z(2) critical point ($T \approx 1.06 T_c$, $n \approx n_c$) of the liquid-gas transition

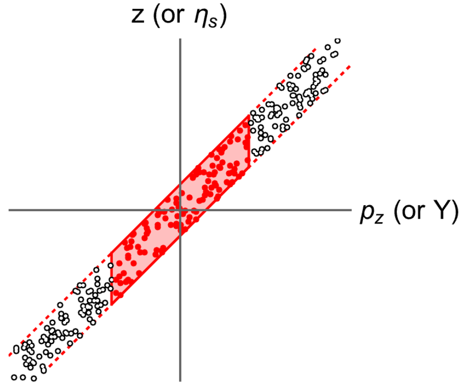
Scaled variance in coordinate space acceptance $|z| < z^{max}$

$$\tilde{\omega}^{\mathrm{coord}} = \frac{1}{1-\alpha} \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$





Heavy-ion collisions:
flow correlates p_z and z cuts



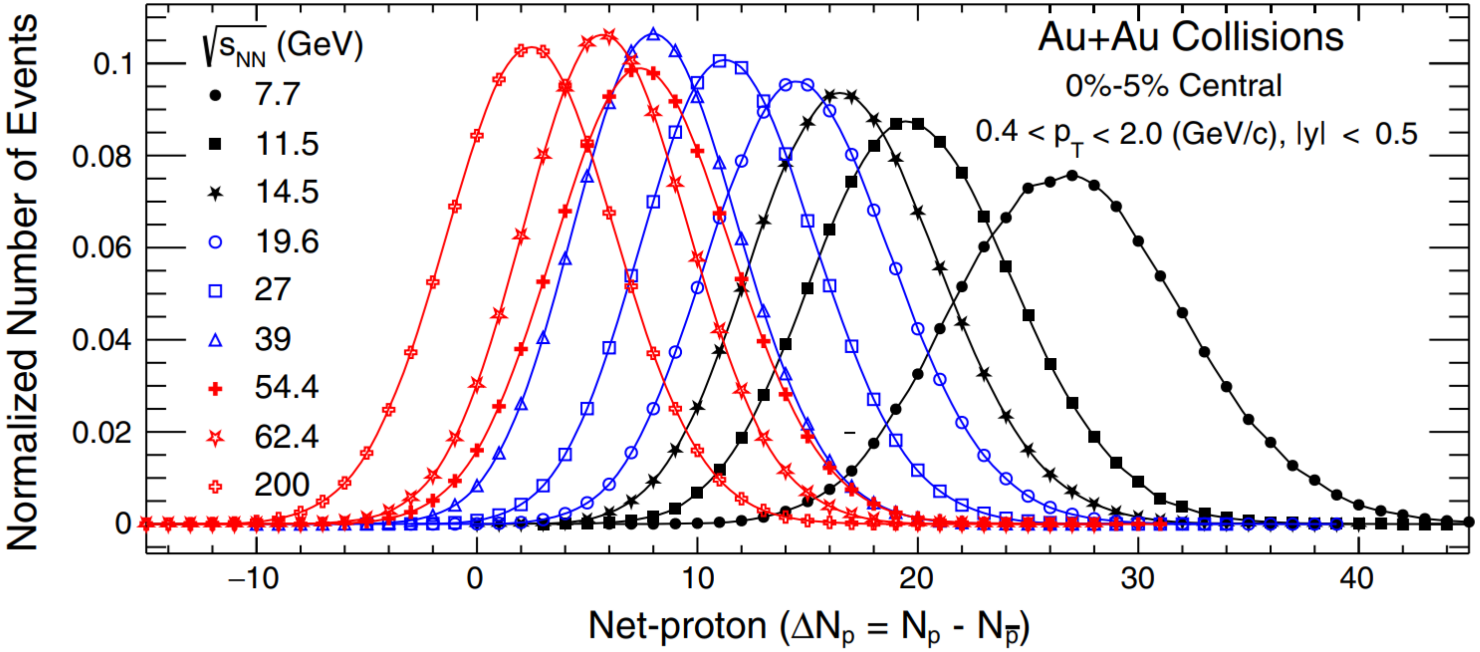
- Large fluctuations survive despite strong finite-size effects
- Need coordinate space cuts (collective flow helps)
- Here no finite-time effects

Collective flow and finite-time effects explored in V. Kuznietsov et al., Phys. Rev. C 110, 015206 (2024)

Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons $P(\Delta N_p) \sim \frac{N_{\text{events}}(\Delta N_p)}{N_{events}^{total}}$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)



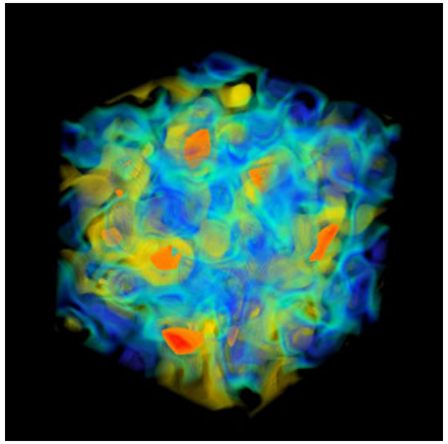
Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

$$\frac{\kappa_2}{\langle N \rangle}, \qquad \frac{\kappa_3}{\kappa_2}, \qquad \frac{\kappa_4}{\kappa_2}$$

Look for subtle critical point signals

Theory vs experiment: Challenges for fluctuations

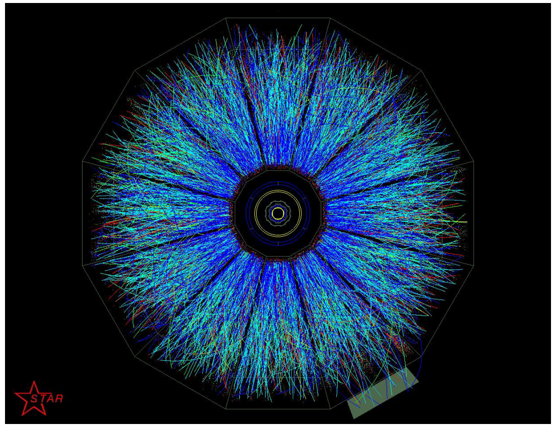
Theory



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

Need dynamical description

Coordinate vs Momentum space

V. Kuznietsov et al., Phys. Rev. C 110 (2024) 015206

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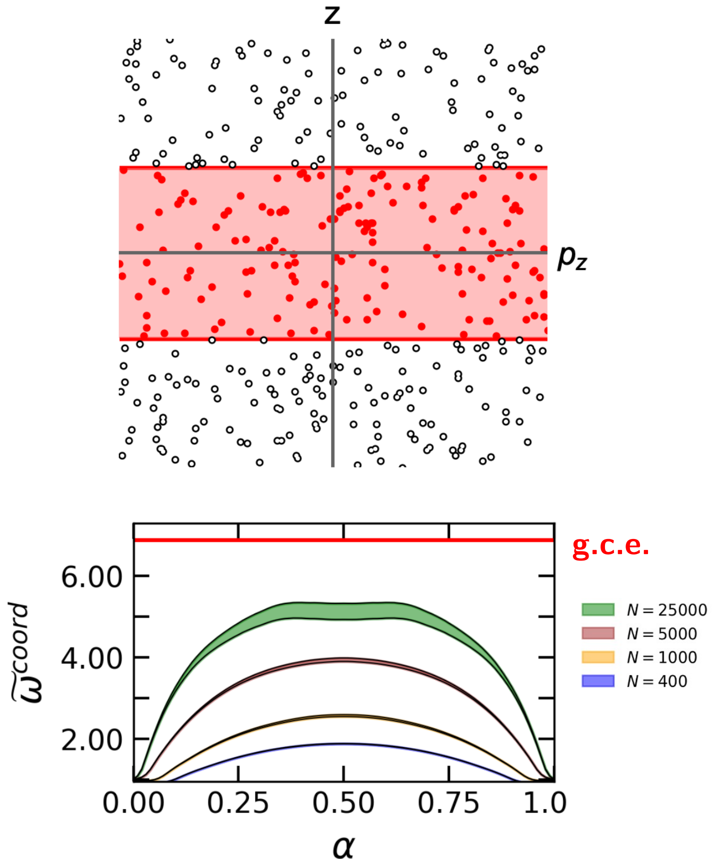
Box setup: Coordinates and momenta are uncorrelated

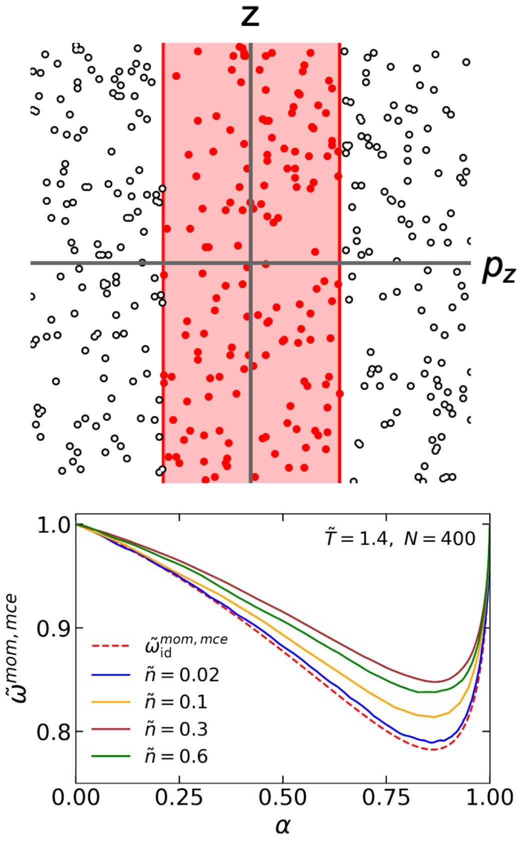
HICs: Flow (e.g. Bjorken)

Coordinate space cut

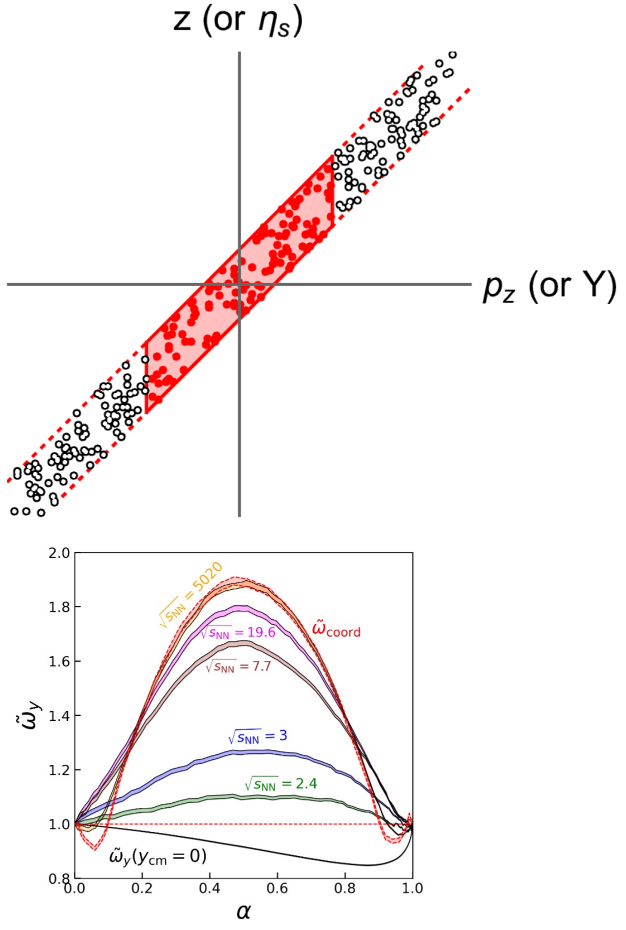


Large correlations

Momentum space cut



Nothing left

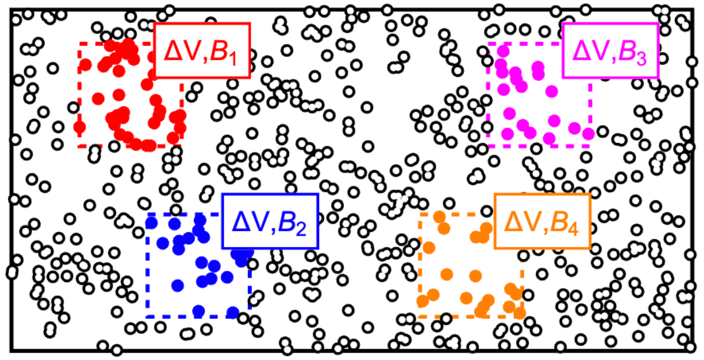


momentum cut ~ coordinate cut + smearing

Exact charge conservation

VV, arXiv:2409.01397

Utilizing the partition function in thermodynamic limit one can compute n-point density correlators



$$\mathcal{C}_1(\mathbf{r}_1) = \rho(\mathbf{r}_1)$$

$$\mathcal{C}_2(\mathbf{r}_1,\mathbf{r}_2) = \chi_2\delta(\mathbf{r}_1-\mathbf{r}_2) - \frac{\chi_2}{V}$$

local correlation — balancing contribution (e.g. baryon conservation)

$$\mathcal{C}_3(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = \chi_3\delta_{1,2,3} - \frac{\chi_3}{V}[\delta_{1,2}+\delta_{1,3}+\delta_{2,3}] + 2\frac{\chi_3}{V^2}$$

$$\delta_{1,\dots,n} = \prod_{i=2}^n \delta(\mathbf{r}_1-\mathbf{r}_i)$$

local correlation — balancing contributions

$$\mathcal{C}_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = \chi_4\delta_{1,2,3,4} - \frac{\chi_4}{V}[\delta_{1,2,3}+\delta_{1,2,4}+\delta_{1,3,4}+\delta_{2,3,4}] - \frac{(\chi_3)^2}{\chi_2 V}[\delta_{1,2}\delta_{3,4}+\delta_{1,3}\delta_{2,4}+\delta_{1,4}\delta_{2,3}]$$

$$+\frac{1}{V^2}\left[\chi_4+\frac{(\chi_3)^2}{\chi_2}\right][\delta_{1,2}+\delta_{1,3}+\delta_{1,4}+\delta_{2,3}+\delta_{2,4}+\delta_{3,4}] - \frac{3}{V^3}\left[\chi_4+\frac{(\chi_3)^2}{\chi_2}\right].$$

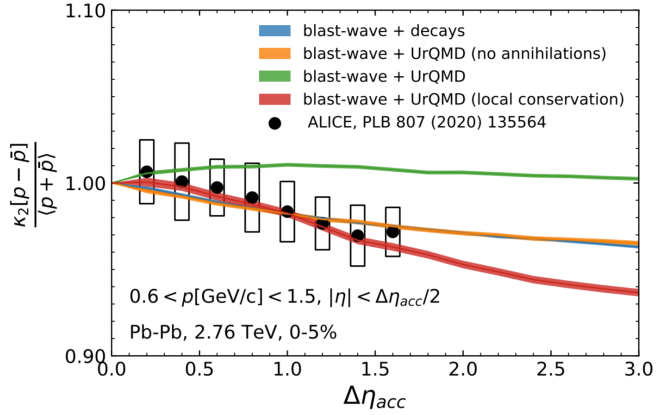
local correlation — balancing contributions

Integrating the correlator reproduces known cumulant inside a subsystem

$$\kappa_n[B_{V_s}] = \int_{\mathbf{r}_1\in V_s} d\mathbf{r}_1 \dots \int_{\mathbf{r}_n\in V_s} d\mathbf{r}_n\, \mathcal{C}_n(\{\mathbf{r}_i\})$$

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, Phys. Lett. B 811, 135868 (2020)

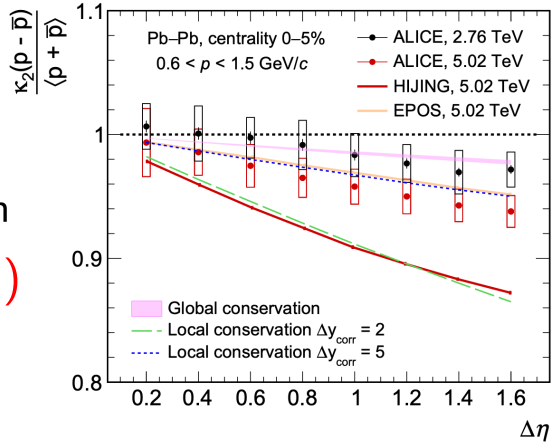
Proton cumulants at high energy



$\kappa_2[p - \bar{p}]/\langle p + \bar{p}\rangle$:

- Largely understood as (global) baryon conservation
  - Larger suppression at 5 TeV contrary to naïve expectation
- Interplay: baryon annihilation(↗) vs local conservation(↘)
  - Additional measurement of $\kappa_2[p + \bar{p}]$ can resolve it

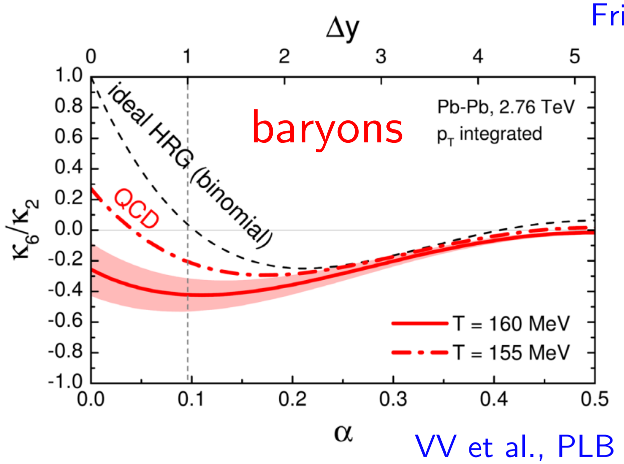
O. Savchuk et al., PLB 827, 136983 (2022)



ALICE Collaboration, PLB 844, 137545 (2023)

High-order cumulants: probe remnants of chiral criticality

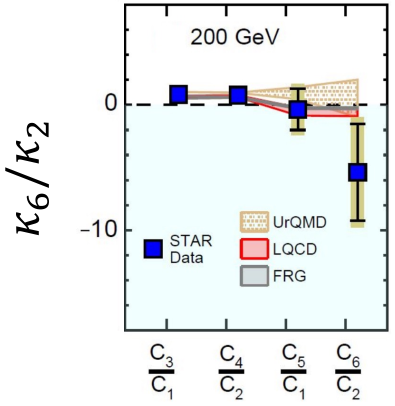
Friman et al., EPJC 71, 1694 (2011)



- negative κ_6 of baryons

VV et al., PLB 811, 135868 (2020)

RHIC 200 GeV: hints of negative $\kappa_6 < 0$ (protons)



- are baryons even more negative?

STAR Collaboration, PRL 130, 082301 (2023)

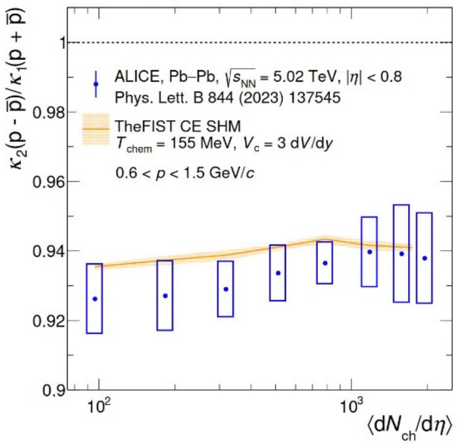
Hadron cumulants at LHC and local conservation

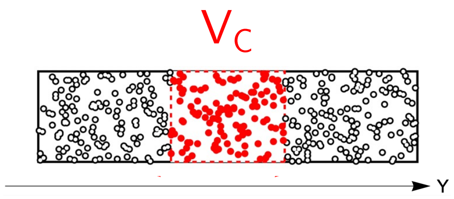
Slide from M. Ciacco, SQM2024

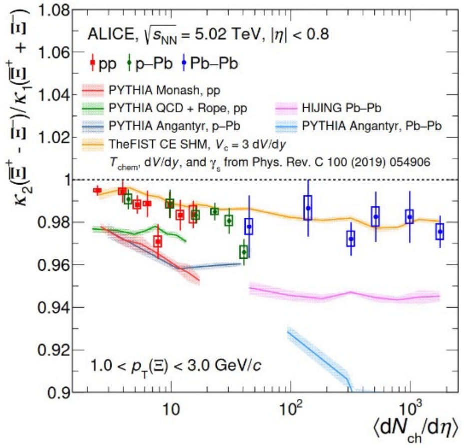
Net-proton	$V_C \sim 3$ dV/dy
Net-Ξ and net-Ξ–net-K correlation	$V_C \sim 3$ dV/dy
Net-Λ	$V_C \sim 3$ dV/dy
Antideuteron–net-Λ correlation	$V_C \sim 3$ dV/dy
Antideuteron–antiproton correlation	$V_C \sim 1.6$ dV/dy

Smaller effective volume
→ stronger correlation between antideuteron and antiproton
→ the correlation strength is enhanced by the (anti)nucleosynthesis mechanism

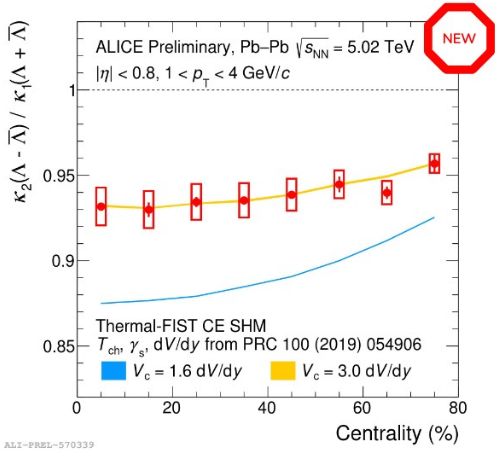
ALICE Coll., Phys. Lett. B 844 (2023) 137545

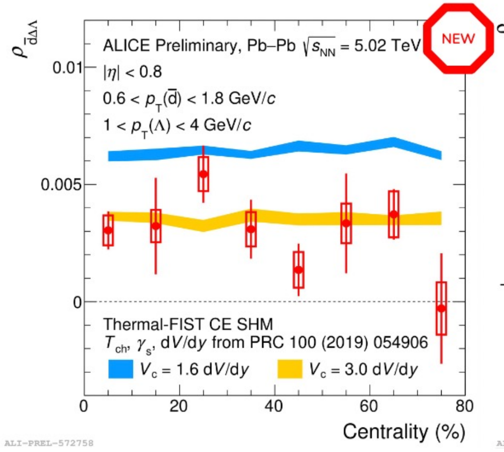


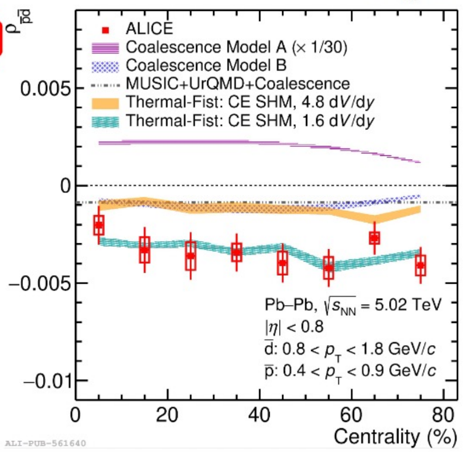




ALICE Coll., arXiv:2405.19890 [nucl-ex]







ALICE Coll., Phys. Rev. Lett. 131, 041901 (2023)

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Correlation volume V_C: truncate the fireball around few units of midrapidity and treat it canonically

VV, Donigus, Stoecker, PRC 100, 054906 (2019)

Local baryon conservation from density correlator

VV, arXiv:2409.01397

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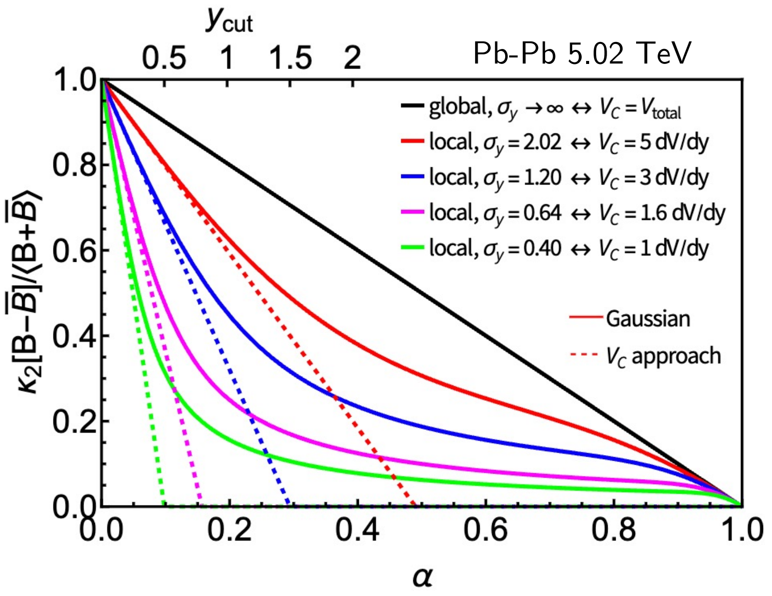
Introduce Gaussian (space-time) rapidity correlation into baryon-conservation balancing term

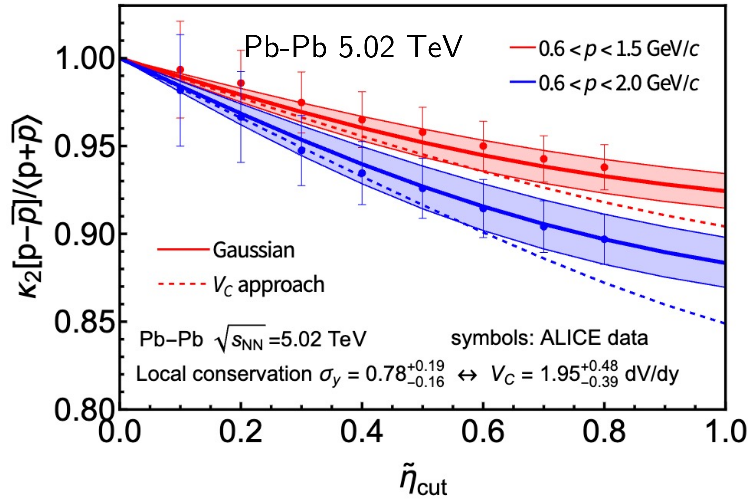
global conservation

$$C_2^B(\eta_1,\eta_2) = \langle n_B + n_{\bar{B}} \rangle \left[\delta(\eta_1-\eta_2) - \frac{1}{2\eta_{\max}}\right]$$

+ local conservation

$$C_2^B(\eta_1,\eta_2) = \langle n_B + n_{\bar{B}} \rangle \left[\delta(\eta_1-\eta_2) - \frac{\tilde{A}\, e^{-\frac{(\eta_1-\eta_2)^2}{2\sigma_\eta^2}}}{2\eta_{\max}}\right]$$





- Linear regime at small a establishes connection to the V_C approach ($V_C = k dV/dy$, $k \approx \sqrt{2\pi}\sigma_\eta$)
- V_C approach has limitations, likely provides upper bound on the conservation volume
- Evidence for local (not just global) baryon conservation for 5 TeV data (in contrast to 2.76 TeV data)

Local baryon conservation and charge balance functions

Density correlators from canonical ensemble allow one to explore relation between susceptibilities and balance functions

$$B(\mathbf{r}_1|\mathbf{r}_2) = \frac{1}{2}\left[\rho_2^{+-}(\mathbf{r}_1|\mathbf{r}_2) - \rho_2^{--}(\mathbf{r}_1|\mathbf{r}_2) + \rho_2^{-+}(\mathbf{r}_1|\mathbf{r}_2) - \rho_2^{++}(\mathbf{r}_1|\mathbf{r}_2)\right]$$

Bass, Danielewicz, Pratt, PRL 85, 2689 (2000)

or

$$B^{+-}(\mathbf{r}_1,\mathbf{r}_2) = \frac{1}{\langle N^- \rangle}\left[C_{11}^{+-}(\mathbf{r}_1,\mathbf{r}_2) - C_2^{--}(\mathbf{r}_1,\mathbf{r}_2)\right],$$
$$B^{-+}(\mathbf{r}_1,\mathbf{r}_2) = \frac{1}{\langle N^+ \rangle}\left[C_{11}^{-+}(\mathbf{r}_1,\mathbf{r}_2) - C_2^{++}(\mathbf{r}_1,\mathbf{r}_2)\right],$$

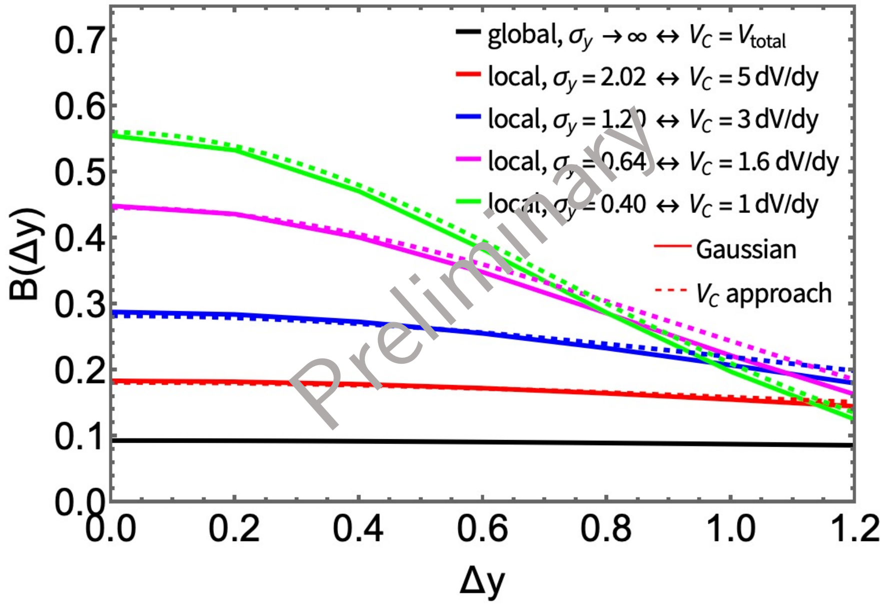
Pruneau et al., PRC 107, 014902 (2023)

Baryons and antibaryons :

$$B^{+-}(\mathbf{r}_1,\mathbf{r}_2) = \frac{1}{\langle B^- \rangle}\left[(\tilde{\chi}_{11}^{B^+B^-} - \tilde{\chi}_2^{B^-})\left(\delta(\mathbf{r}_1-\mathbf{r}_2) - \frac{\varkappa(\mathbf{r}_1,\mathbf{r}_2)}{V}\right) + \langle n_{B^-}\rangle \frac{\varkappa(\mathbf{r}_1,\mathbf{r}_2)}{V}\right]$$
$$B^{-+}(\mathbf{r}_1,\mathbf{r}_2) = \frac{1}{\langle B^+ \rangle}\left[(\tilde{\chi}_{11}^{B^+B^-} - \tilde{\chi}_2^{B^+})\left(\delta(\mathbf{r}_1-\mathbf{r}_2) - \frac{\varkappa(\mathbf{r}_1,\mathbf{r}_2)}{V}\right) + \langle n_{B^+}\rangle \frac{\varkappa(\mathbf{r}_1,\mathbf{r}_2)}{V}\right]$$

Balance functions probe both the (factorial) susceptibilities $\tilde{\chi}_2^B$ and local baryon conservation $\varkappa(r_1, r_2)$

In rapidity:



Dynamical approaches to the QCD critical point search

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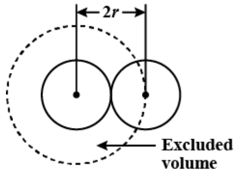
1. Dynamical model calculations of critical fluctuations — BEST COLLABORATION [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]
   - Fluctuating hydrodynamics (hydro+) and (non-equilibrium) evolution of fluctuations
   - Equation of state with a tunable critical point [P. Parotto et al, PRC 101, 034901 (2020); J. Karthein et al., EPJ Plus 136, 621 (2021)]
   - Generalized Cooper-Frye particlization [M. Pradeep, et al., PRD 106, 036017 (2022); PRL 130, 162301 (2023)]

Alternatives at high μ_B: hadronic transport/molecular dynamics with a critical point

[A. Sorensen, V. Koch, PRC 104, 034904 (2021); V. Kuznietsov et al., PRC 105, 044903 (2022)]



2. **Deviations from precision calculations of non-critical fluctuations**
   - Non-critical baseline is not flat [Braun-Munzinger et al., NPA 1008, 122141 (2021)]
   - Include essential non-critical contributions to (net-)proton number cumulants
   - Exact baryon conservation + hadronic interactions (hard core repulsion)
   - Based on realistic hydrodynamic simulations tuned to bulk data

[VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

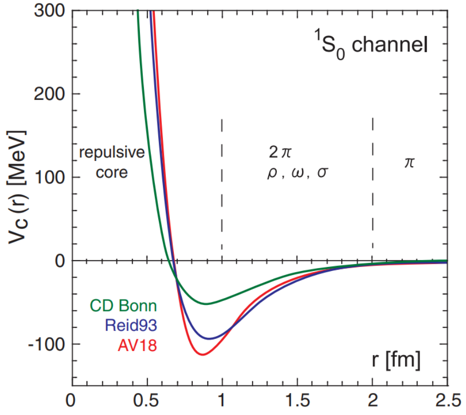
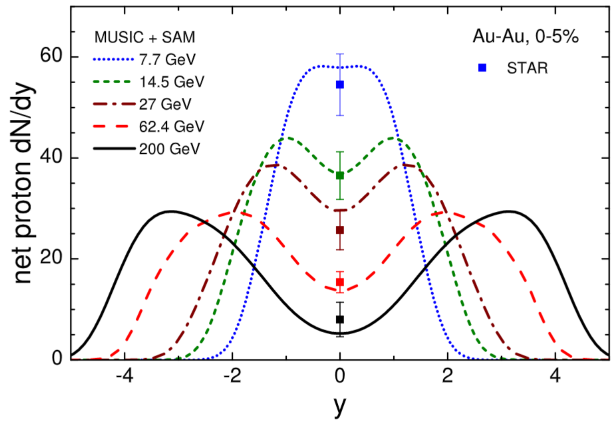


Figure from Ishii et al., PRL ‘07

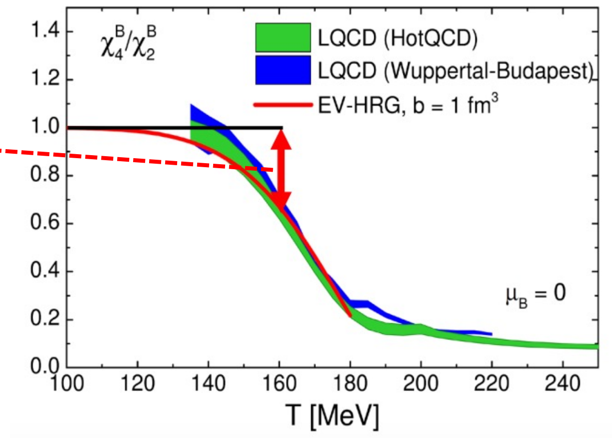
Calculation of non-critical contributions at RHIC-BES

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
  - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
  - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]
  - Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3



- Non-critical contributions are computed at particlization
  - QCD-like baryon number distribution (χ_n^B) via excluded volume b = 1 fm^3 [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
  - Exact global baryon conservation* (and other charges)
    - Subensemble acceptance method 2.0 (analytic) [VV, Phys. Rev. C 105, 014903 (2022)]
    - or FIST sampler (Monte Carlo) [VV, Phys. Rev. C 106, 064906 (2022)] https://github.com/vlvovch/fist-sampler



- Absent: critical point, local conservation, initial-state/volume fluctuations, hadronic phase

*If baryon conservation is the only effect (no other correlations), non-critical baseline can be computed without hydro
Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, NPA 1008, 122141 (2021)

Calculating cumulants from MUSIC

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Cooper-Frye formula:

$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x)\, p^\mu\, f_j[u^\mu(x)p_\mu; T(x), \mu_j(x)]$$

Calculation of the cumulants incorporates **balancing contributions from baryon conservation***

$$C_1^B(x_1) = \chi_1^B(x_1),$$

$$C_2^B(x_1, x_2) = \chi_2^B(x_1)\, \delta(x_1 - x_2) - \frac{\chi_2^B(x_1)\chi_2^B(x_2)}{\int_{\sigma(x)} d\sigma_\mu(x) u^\mu(x)\, \chi_2^B(x)},$$

local correlation

balancing contribution (baryon conservation)

...

Global baryon conservation:

$$\int d\sigma_\mu(x_i) u^\mu(x_i) C_n^B(x_1, \dots, x_n) = 0 \quad \text{for} \quad n > 1$$

Generalized Cooper-Frye:

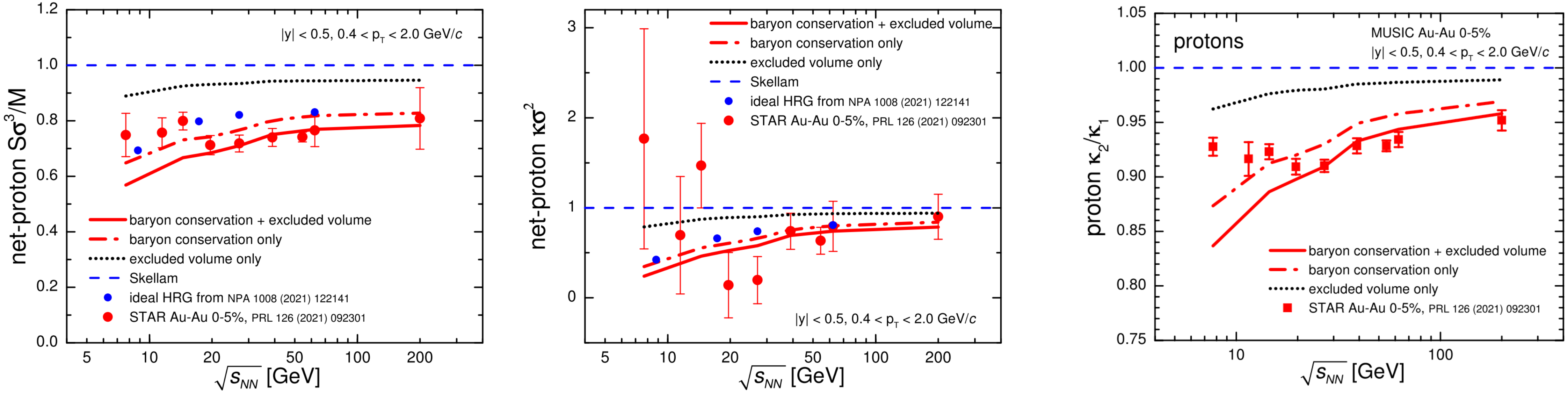
$$\kappa_n^B = \prod_{i=1}^{n} \int_{x_i \in \sigma(x)} d\sigma_\mu(x_i) \int_{|y_i|<0.5,\, 0.4<p_T<2} \frac{d^3 p_i}{\omega_{p_i}} p_i^\mu \exp\left[-\frac{p_i^\mu u_\mu(x_i)}{T(x_i)}\right] C_n^B(x_1, \dots, x_n)$$

*based on SAM-2.0 method, VV, Phys. Rev. C 105, 014903 (2022)

RHIC-BES-I: Net proton cumulant ratios (MUSIC)

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

net proton C_3/C_1 net proton C_4/C_2 proton C_2/C_1

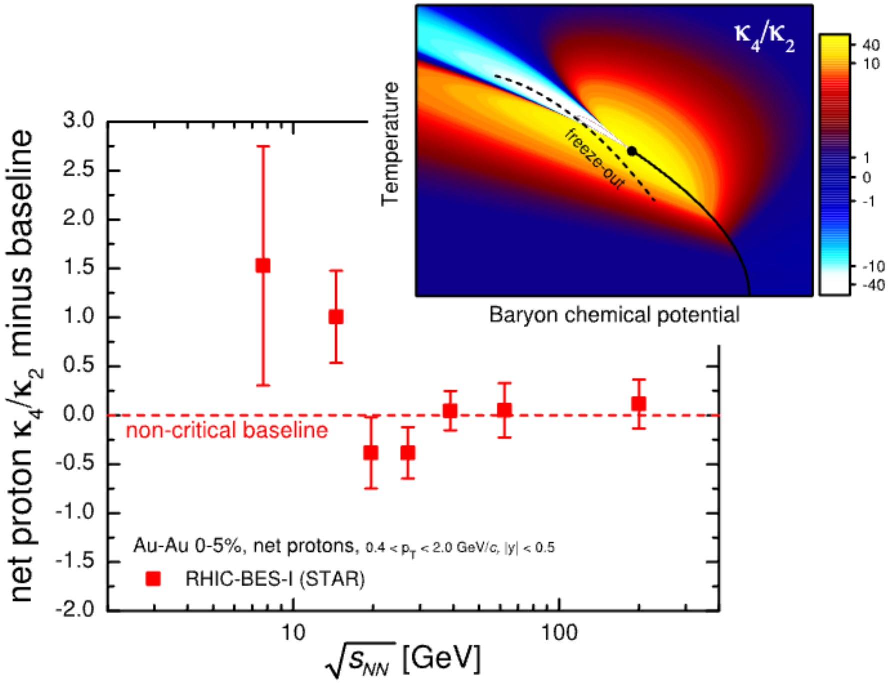


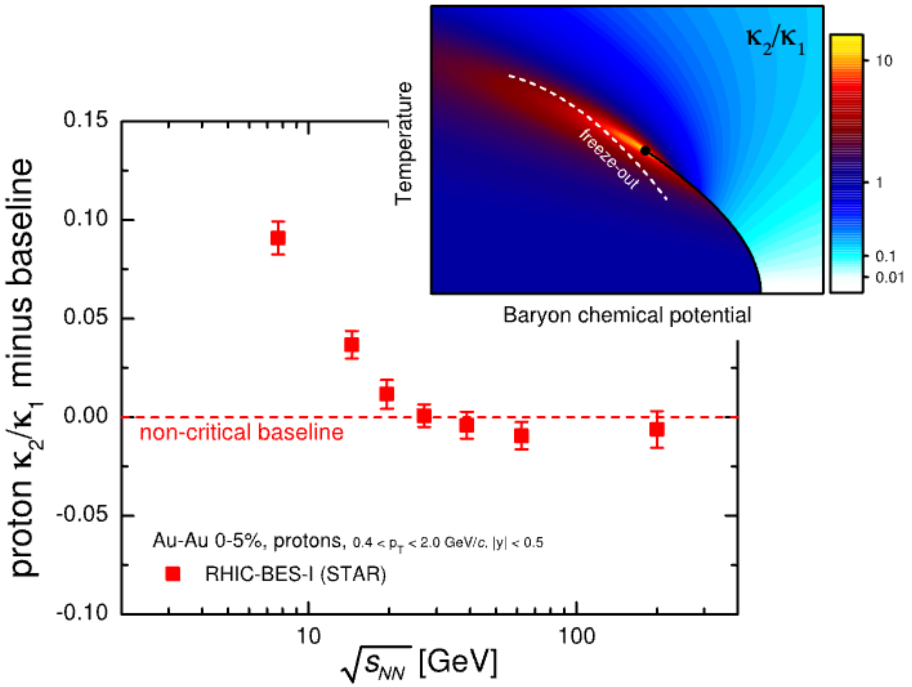
- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (BQS conservation and repulsion)
- Effect from baryon conservation is stronger than repulsion but both are required at $\sqrt{s_{NN}} \geq 20$ GeV
- Deviations from baseline at lower energies?

Hints from RHIC-BES-I

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Subtracting the hydro baseline

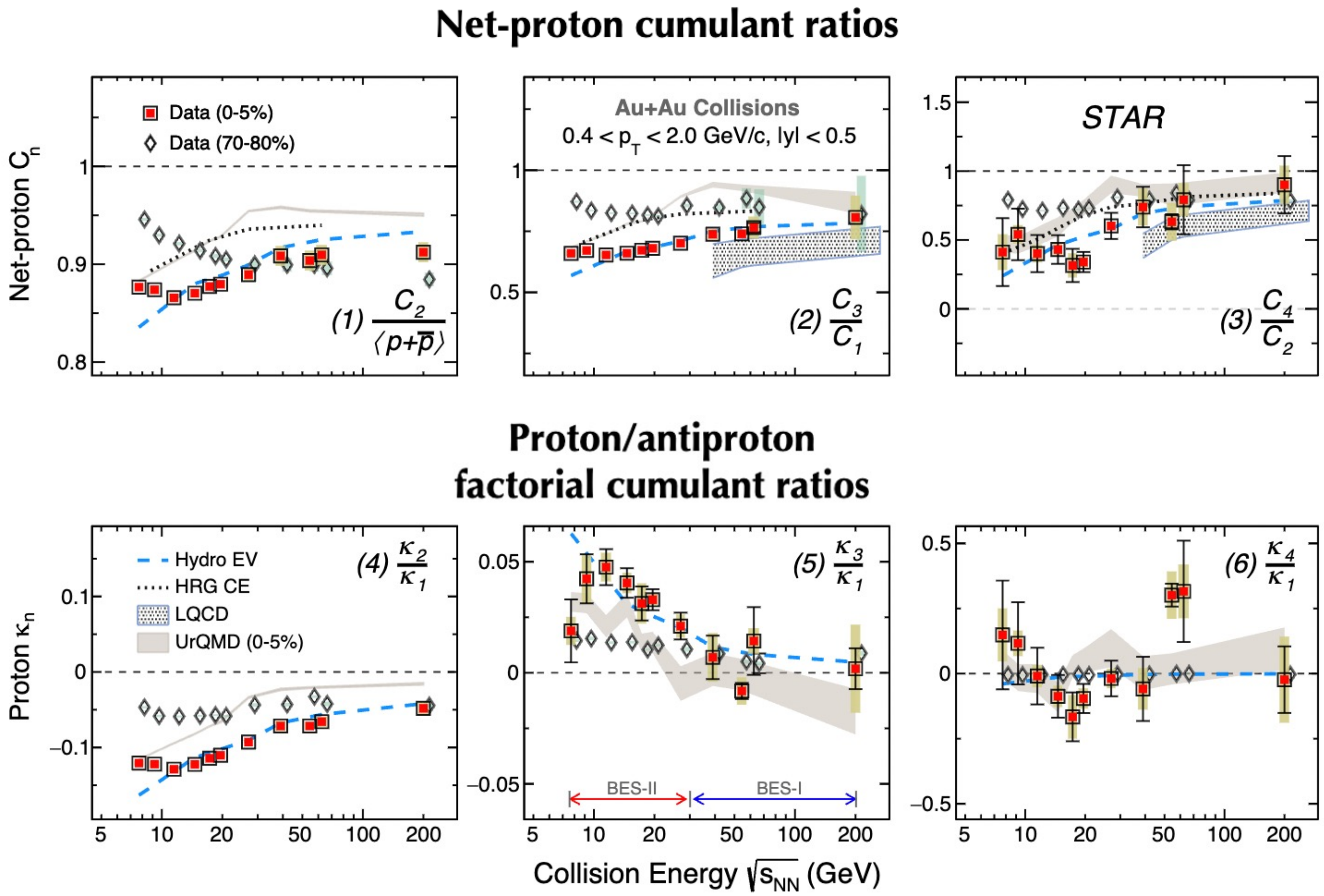




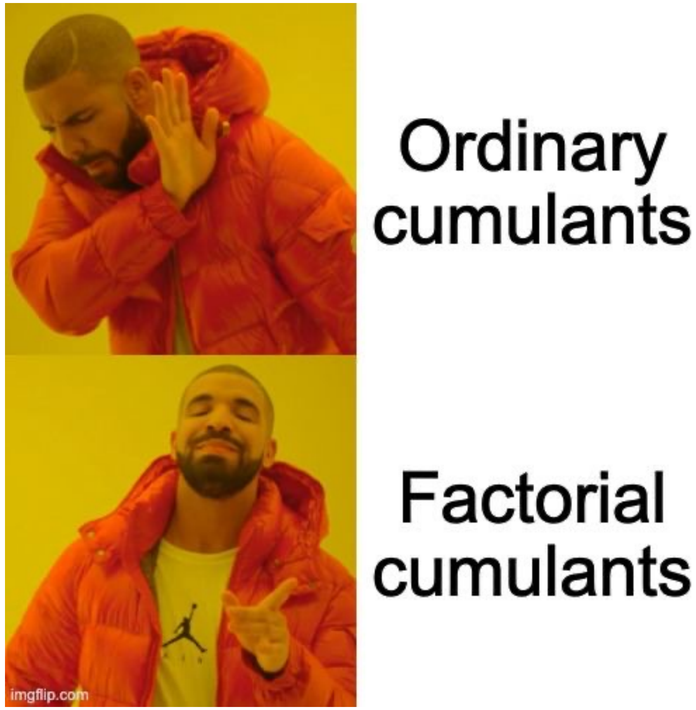
⚠ **Notation:** Here we use κ_n for cumulants and $\hat{C}_n$ for factorial cumulants, STAR uses the opposite

RHIC-BES-II data

A. Pandav, CPOD2024, N. Xu, SQM2024



- No smoking gun signature for CP in ordinary cumulants
- More structure seen in factorial cumulants



Hydro EV: VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

What are factorial cumulants?

Factorial cumulants $\hat{C}_n$ vs ordinary cumulants C_n

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Factorial cumulants: ~irreducible n-particle correlations

$$\hat{C}_n \sim \langle N(N-1)(N-2)\dots\rangle_c$$

$$\hat{C}_1 = C_1$$
$$\hat{C}_2 = C_2 - C_1$$
$$\hat{C}_3 = C_3 - 3C_2 + 2C_1$$
$$\hat{C}_4 = C_4 - 6C_3 + 11C_2 - 6C_1$$

[Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017); Kitazawa, Luo, PRC 96, 024910 (2017); C. Pruneau, PRC 100, 034905 (2019)]

Ordinary cumulants: mix corrs. of different orders

$$C_n \sim \langle \delta N^n \rangle_c$$

$$C_1 = \hat{C}_1$$
$$C_2 = \hat{C}_2 + \hat{C}_1$$
$$C_3 = \hat{C}_3 + 3\hat{C}_2 + \hat{C}_1$$
$$C_4 = \hat{C}_4 + 6\hat{C}_3 + 7\hat{C}_2 + \hat{C}_1$$

Factorial cumulants and different physics mechanisms

- Baryon conservation $\hat{C}_n^{\text{cons}} \propto (\hat{C}_1)^n / \langle N_{\text{tot}} \rangle^{n-1}$ *small*
  [Bzdak, Koch, Skokov, EPJC '17]
- Excluded volume $\hat{C}_n^{\text{EV}} \propto b^n$ *small*
  [VV et al, PLB '17]
- Volume fluctuations $\hat{C}_n^{CF} \sim (\hat{C}_1)^n \kappa_n[V]$ *depends on volume cumulants*
  [Holzman et al., arXiv:2403.03598]
- Critical point $\hat{C}_2^{CP} \sim \xi^2, \quad \hat{C}_3^{CP} \sim \xi^{4.5}, \quad \hat{C}_4^{CP} \sim \xi^7$ ***large***
  [Ling, Stephanov, PRC '16]

- proton vs baryon $\hat{C}_n^B \sim 2^n \times \hat{C}_n^p$ **same sign!**
  [Kitazawa, Asakawa, PRC '12]

Factorial cumulants and long-range correlations

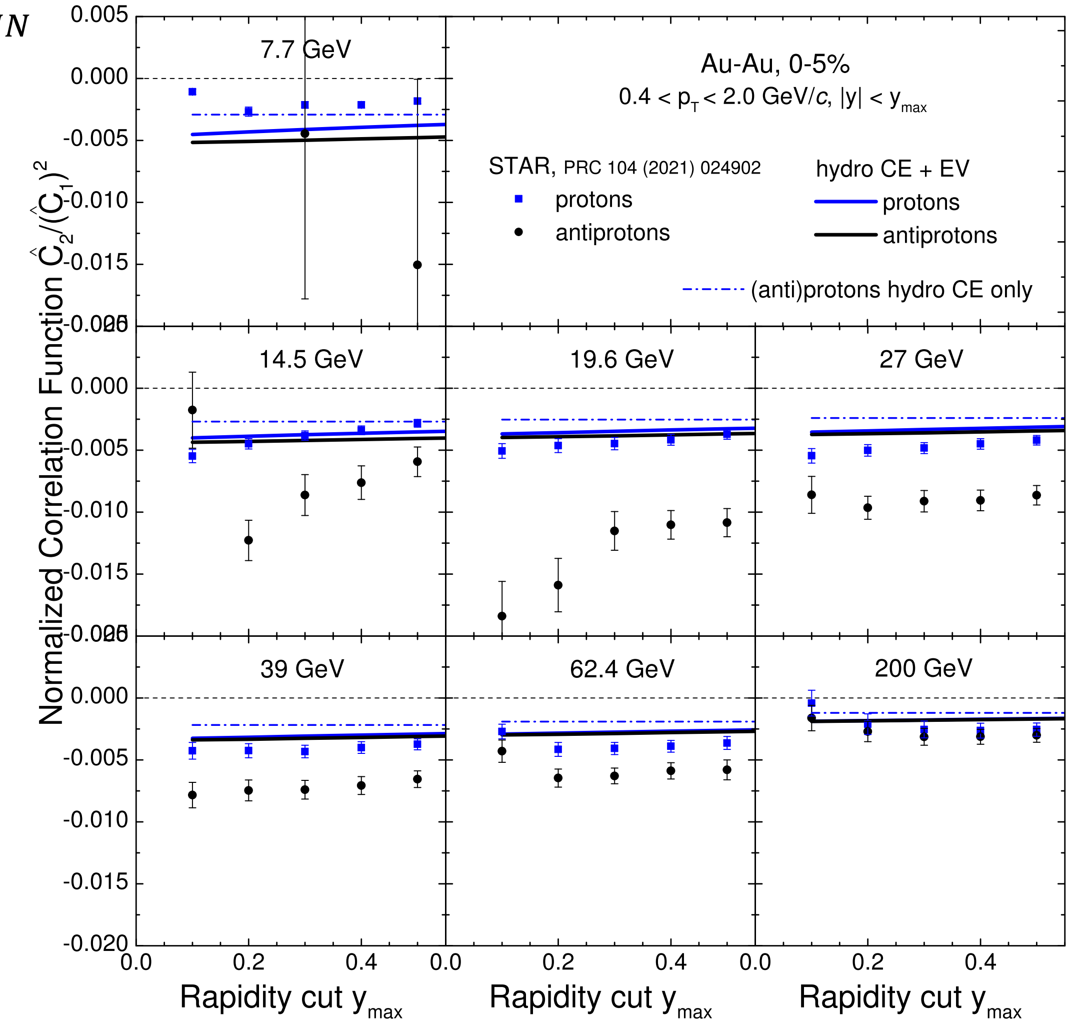
Long-range correlations: $\dfrac{\hat{C}_n}{(\hat{C}_1)^n} = \text{const.}$ at given $\sqrt{s_{NN}}$

- Global (not local) baryon conservation
  [Bzdak, Koch, Skokov, EPJC 77, 288 (2017)]
- + volume fluctuations
  [Holzmann, Koch, Rustamov, Stroth, arXiv:2403.03598]
- + (uniform) efficiency
  [Pruneau, Gavin, Voloshin, PRC 66, 044904 (2002)]

In particular $\dfrac{\hat{C}_2^p}{(\hat{C}_1^p)^2} \approx \dfrac{\hat{C}_2^{\bar{p}}}{(\hat{C}_1^{\bar{p}})^2} = \text{const.}$

- Significant difference between p and $\bar{p}$ in BES-I
  - Missing baryon annihilation?
- With BES-II one can test the scaling with greater precision and extended coverage in rapidity
  - No need for CBWC

BES-I data



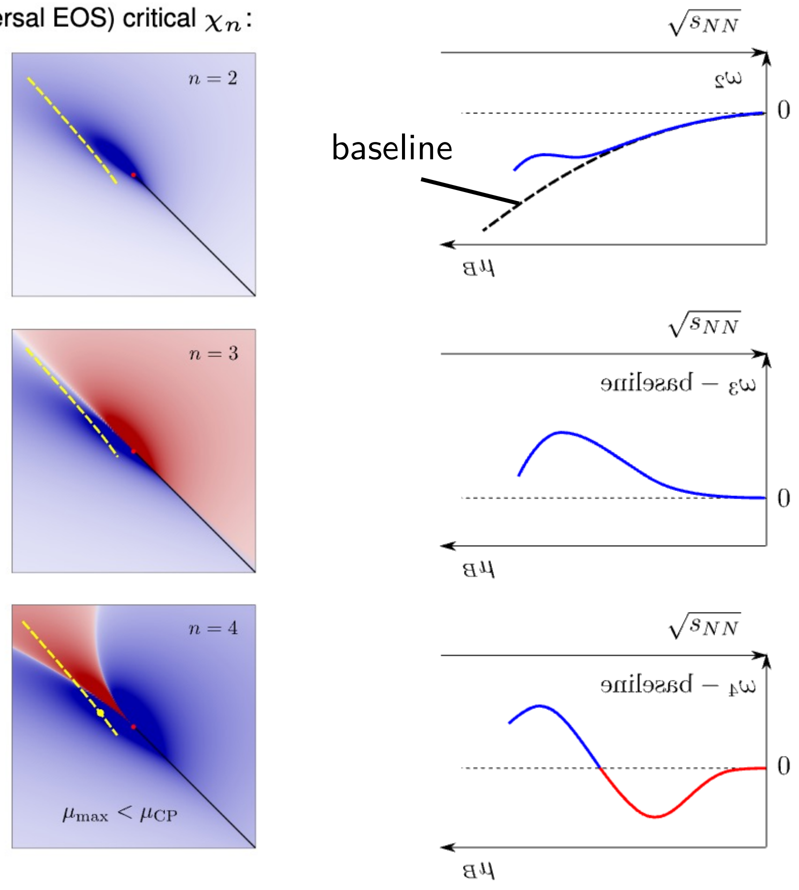
A. Bzdak, V. Koch, VV, in preparation

Factorial cumulants from RHIC-BES-II

From M. Stephanov (SQM2024):

$$\omega_n = \hat{C}_n / \hat{C}_1$$

(universal EOS) critical χ_n:



Bzdak et al review 1906.00936

Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

Factorial cumulants from RHIC-BES-II

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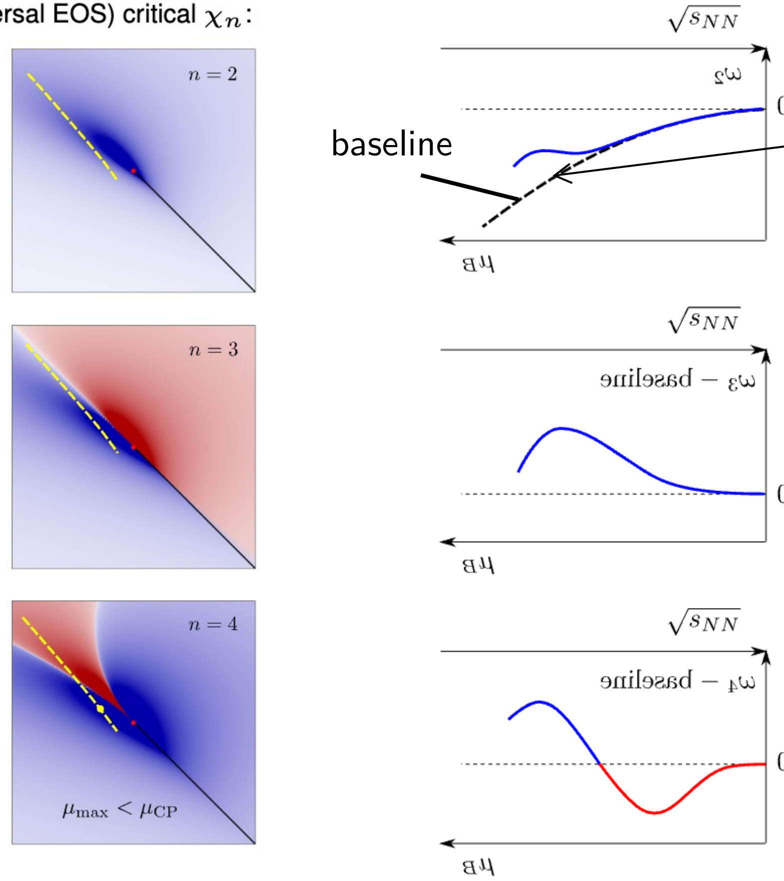
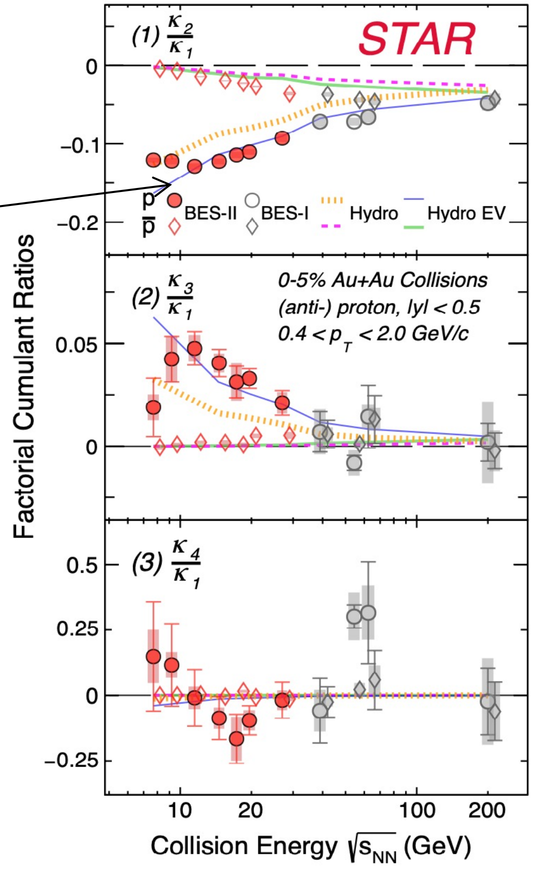


Bzdak et al review 1906.00936

STAR data:



A. Pandav, CPOD2024

baseline (hydro):

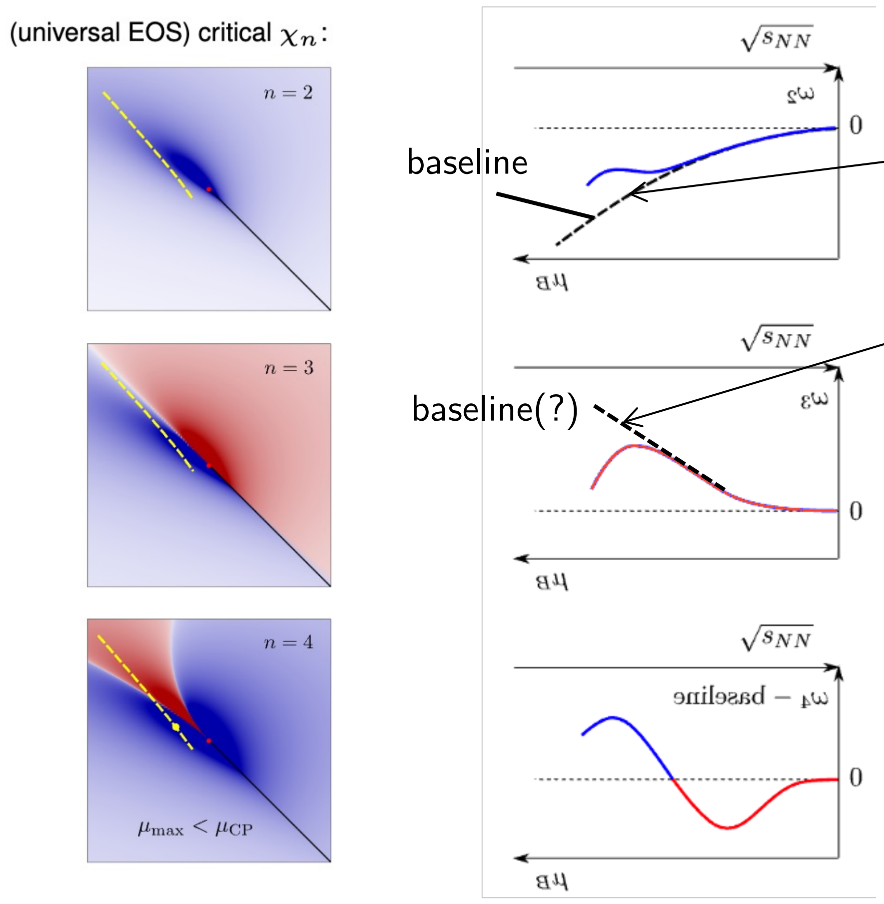
VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

Factorial cumulants from RHIC-BES-II

From M. Stephanov (SQM2024):

$\omega_n = \hat{C}_n/\hat{C}_1$



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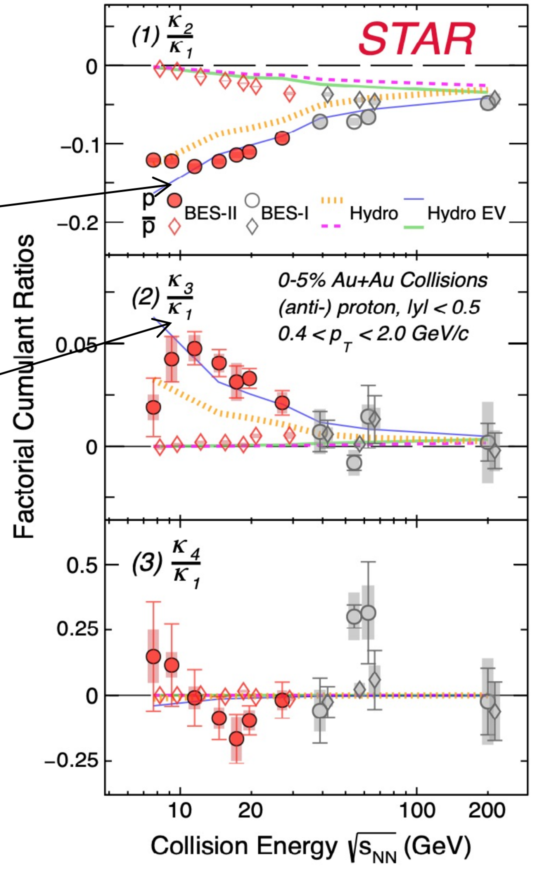


A. Pandav, CPOD2024

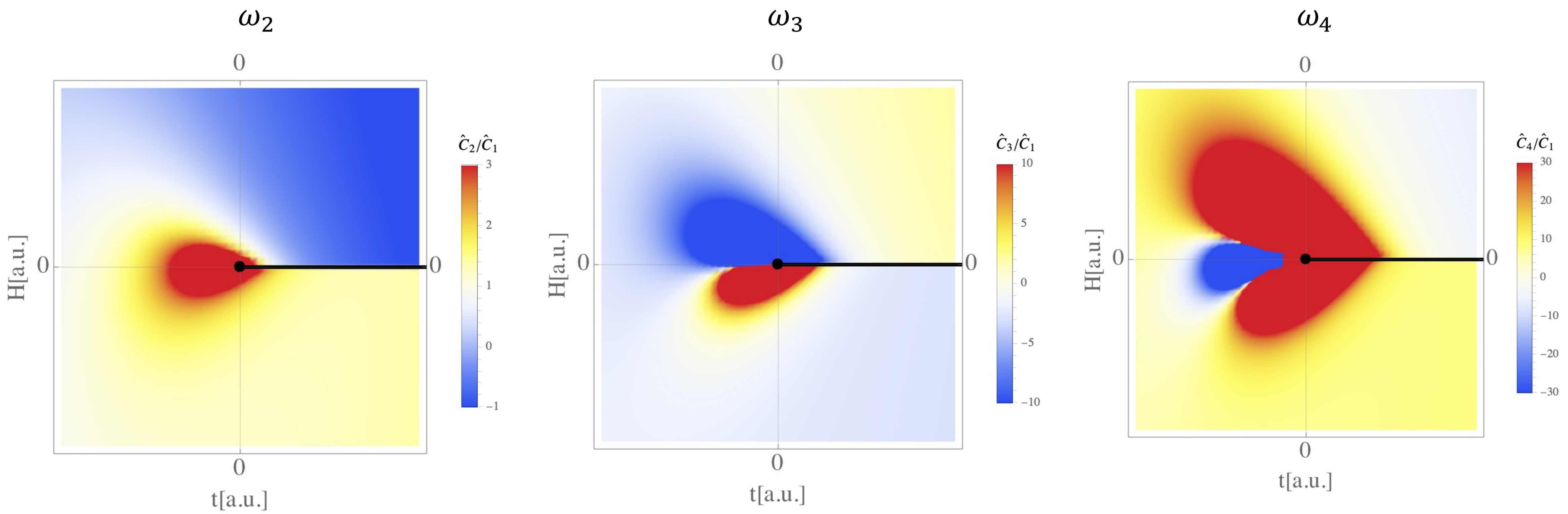
baseline (hydro):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

- describes right side of the peak in $\hat{C}_3$
- implies
  - *positive* $\hat{C}_2$ − baseline > 0
  - *negative* $\hat{C}_3$ − baseline < 0

Factorial cumulants from RHIC-BES-II and CP

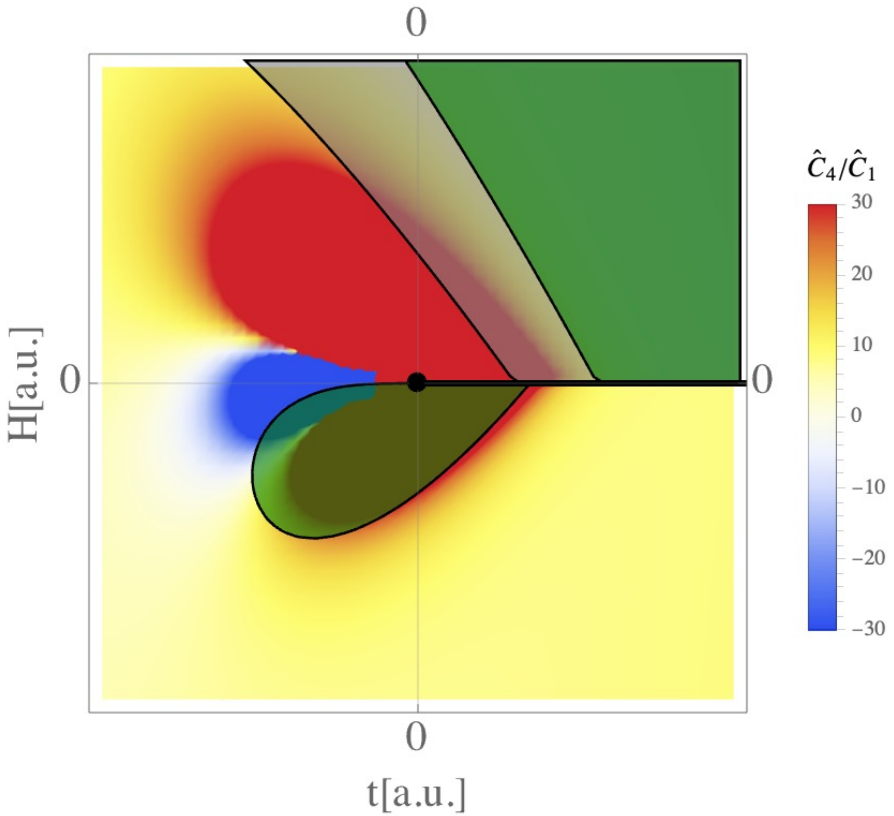
Factorial cumulants in Ising model



Adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)

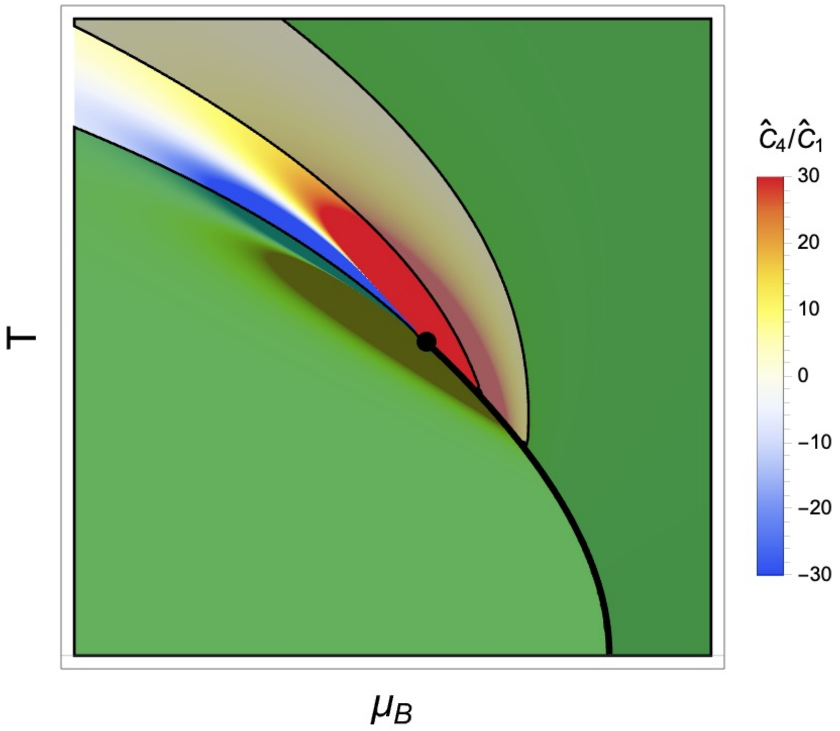
Factorial cumulants from RHIC-BES-II and CP

Exclusion plots



Shaded regions exclude $\hat{C}_2<0$ & $\hat{C}_3>0$

How it may look like in $T-\mu_B$ plane

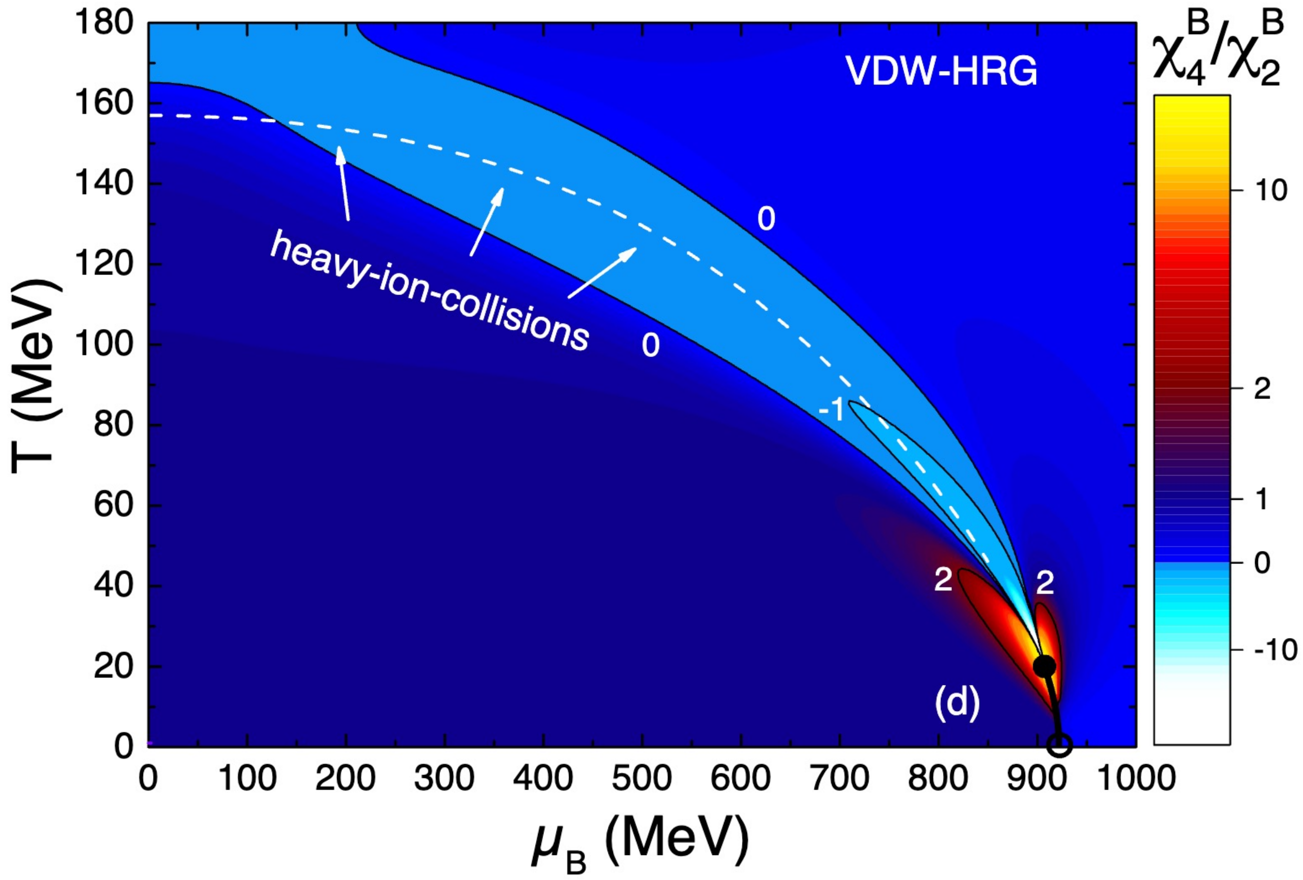


Based on QvdW model of nuclear matter
VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Freeze-out of fluctuations of the QGP side of the crossover?

Nuclear liquid-gas transition

HRG with attractive and repulsive interactions among baryons



VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

Nuclear liquid-gas transition

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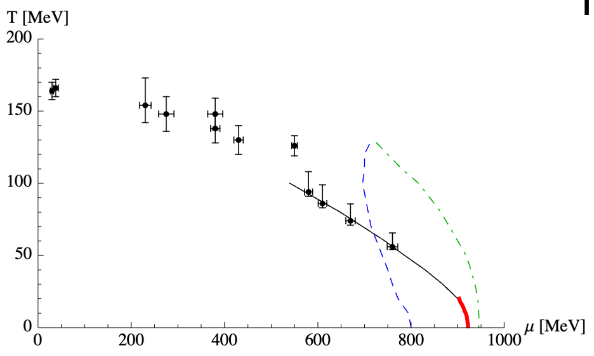
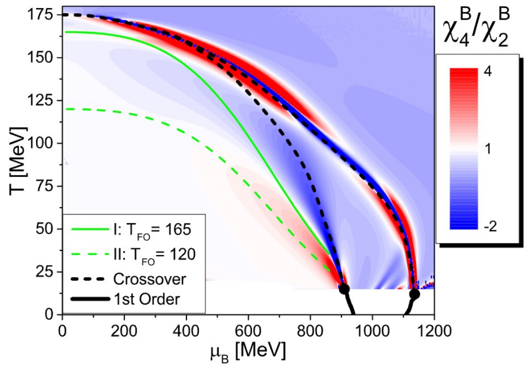
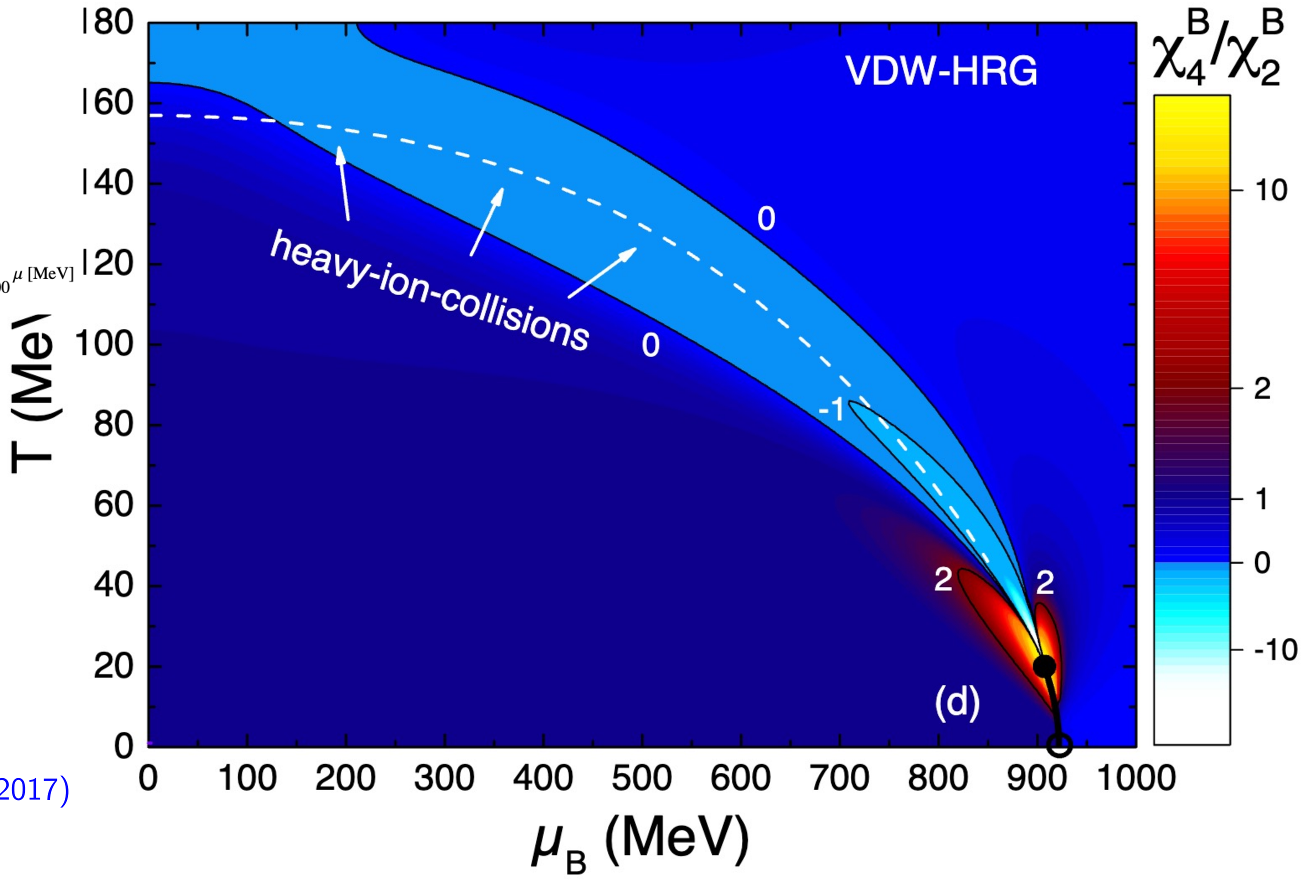


Floerchinger, Wetterich, NPA (2012)

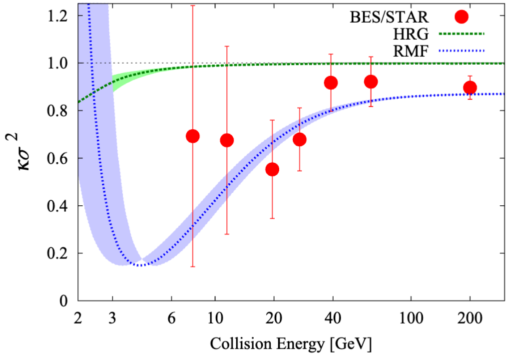


Mukherjee, Steinheimer, Schramm, PRC (2017)

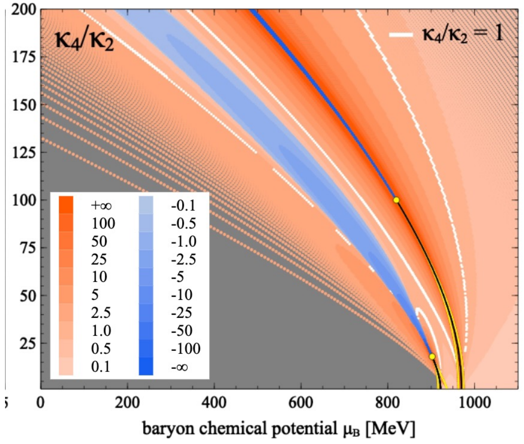
HRG with attractive and repulsive interactions among baryons



VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)



Fukushima, PRC (2014)



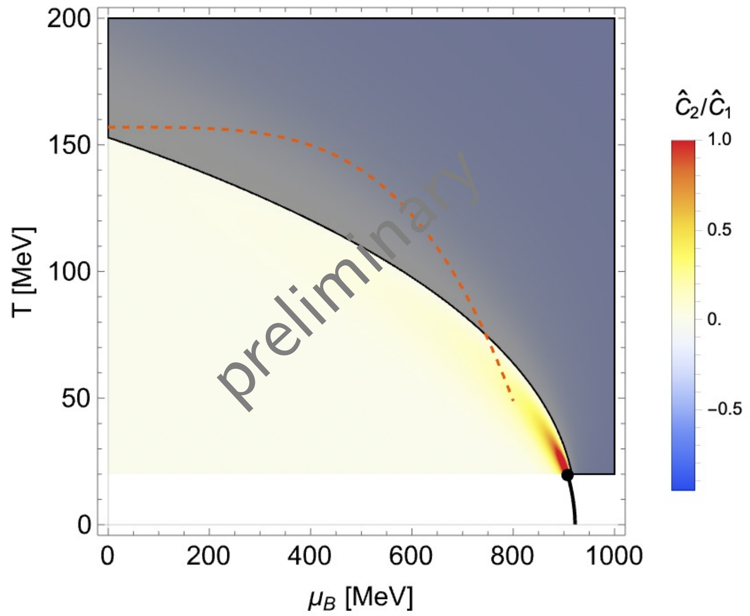
Sorensen, Koch, PRC (2020)

Factorial cumulants and nuclear liquid-gas transition

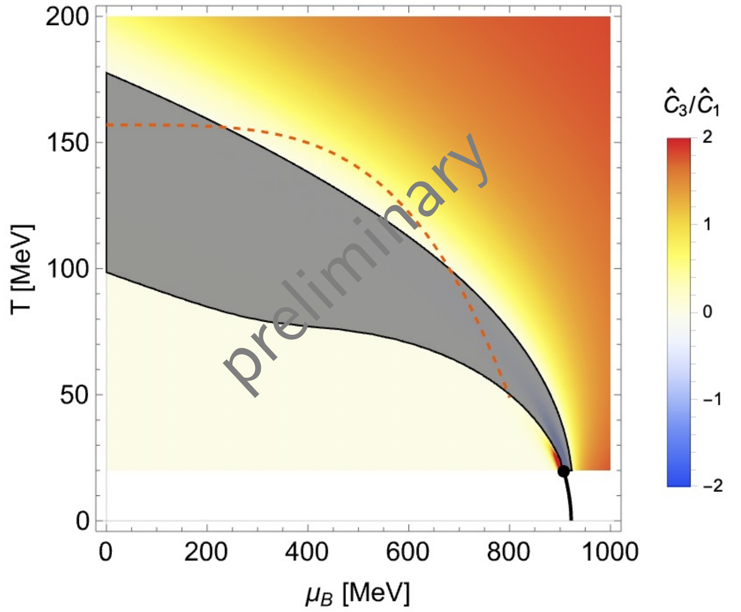
Calculation in a van der Waals-like HRG model

VV, Gorenstein, Stoecker, EPJA 54, 16 (2018)

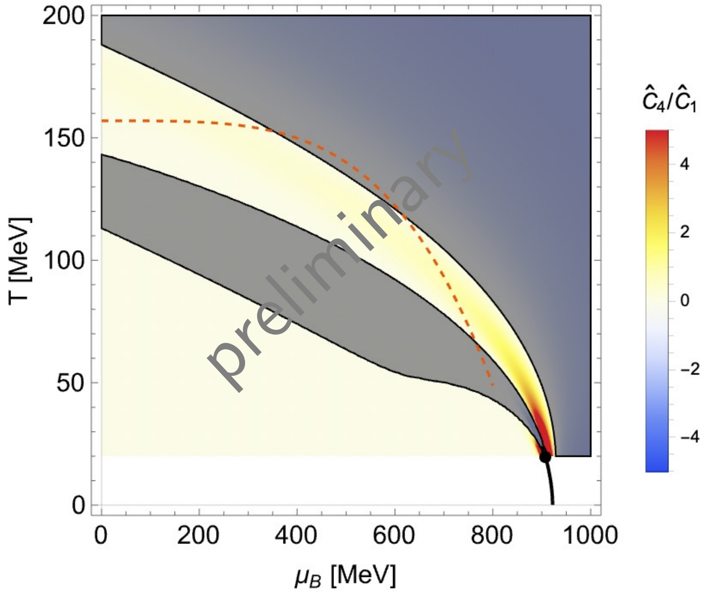
ω_2



ω_3



ω_4

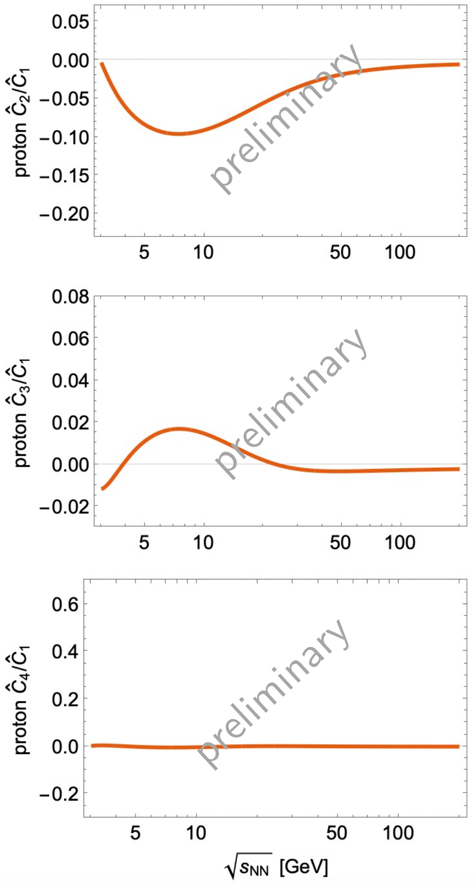


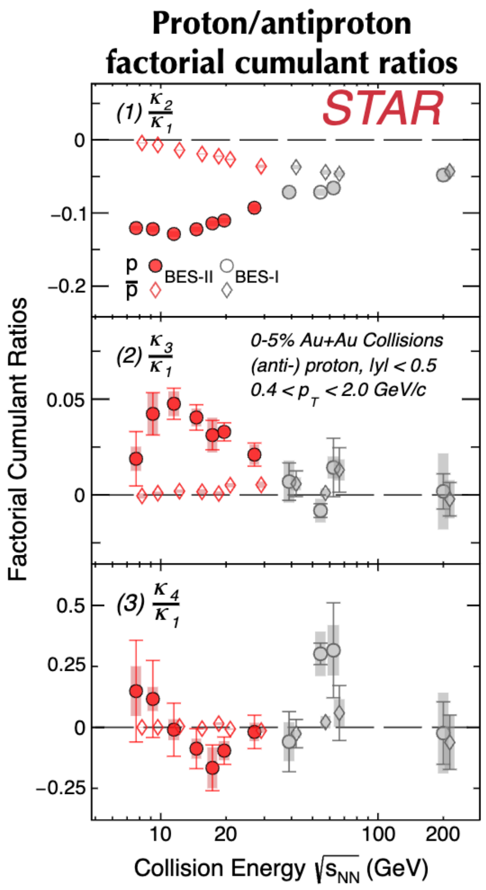
Shaded regions: negative values

Factorial cumulants and nuclear liquid-gas transition

Calculation in a van der Waals-like HRG model along the freeze-out curve*

VV, Gorenstein, Stoecker, EPJA 54, 16 (2018)





NB: The calculation is grand-canonical

*Poberezhnyuk et al., PRC 100, 054904 (2019)

Summary

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- Proton cumulants are uniquely sensitive to the the CP but challenging to model dynamically
  - local charge conservation
  - factorial cumulants are especially advantageous

- BES-II data
  - Protons are consistent with the *prediction* from non-critical hydro at $\sqrt{s_{NN}} \geq 20$ GeV
  - Non-monotonic structure in factorial cumulants
  - Positive $\hat{C}_2$ and negative $\hat{C}_3$ after subtracting non-critical baseline at $\sqrt{s_{NN}} < 10$ GeV
    - QGP side of the crossover using naïve equilibrium interpretation
    - Nuclear liquid-gas contribution?

Outlook:

- Improved description of local conservation, volume fluctuations, and nuclear interactions
- Test global conservation + volume fluctuations baseline through $\hat{C}_n/(\hat{C}_1)^n$ scaling
- Understanding factorial cumulants of antiprotons

Thanks for your attention!