

Fluctuations as a probe of QCD critical point in light of RHIC BES-II

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On-line seminar series on RHIC Beam Energy Scan

October 22, 2024

Thanks to:

M.I. Gorenstein, M. Hippert, M. Kahangirwe, V. Koch, V.A. Kuznetsov, A. Lysenko, J. Noronha,
R. Poberezhniuk, C. Ratti, O. Savchuk, H. Shah, J. Steinheimer, H. Stoecker



QCD under extreme conditions

What we know

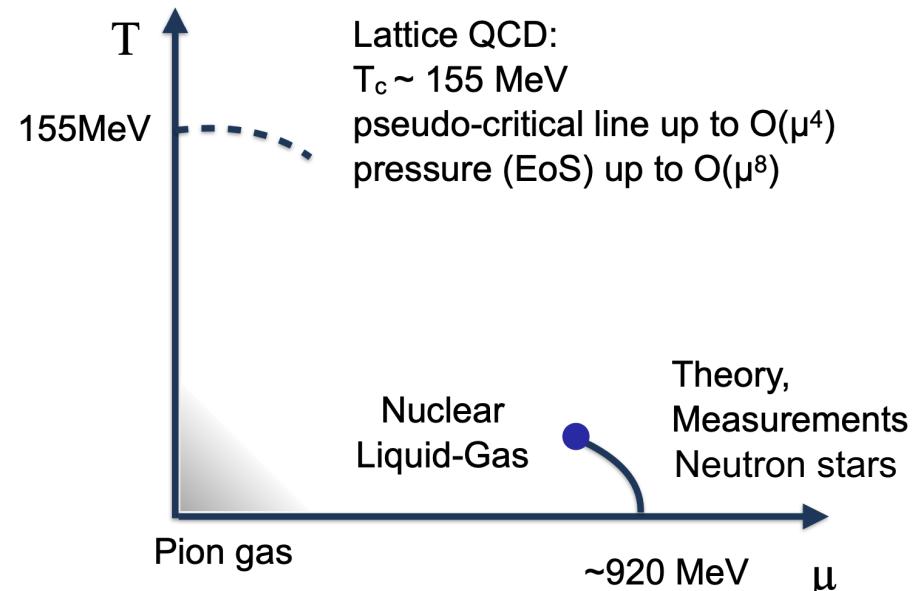


Figure courtesy of V. Koch

What we hope to know

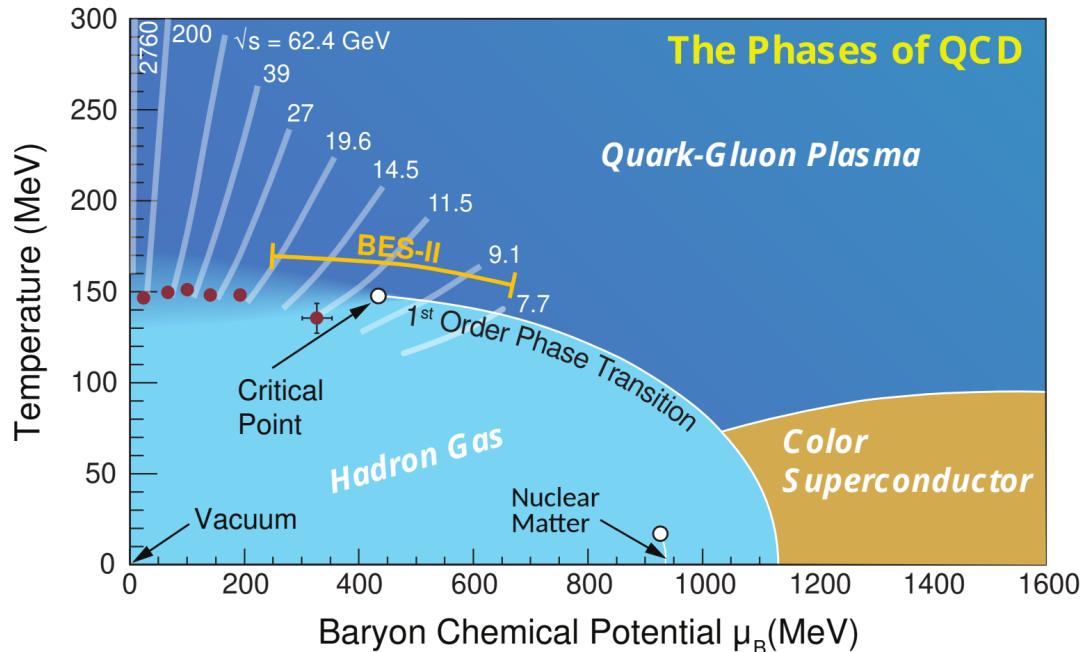


Figure from Bzdak et al., Phys. Rept. '20 & 2015 US Nuclear Long Range Plan

- Dilute hadron gas at low T & μ_B due to confinement, quark-gluon plasma high T & μ_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured
- Chiral crossover at $\mu_B = 0$ which may turn into a *first-order phase transition* at finite μ_B

Key question: *Is there a QCD critical point and how to find it?*

QCD critical point theory estimates: New developments

Critical point predictions as of a few years ago

All over the place...

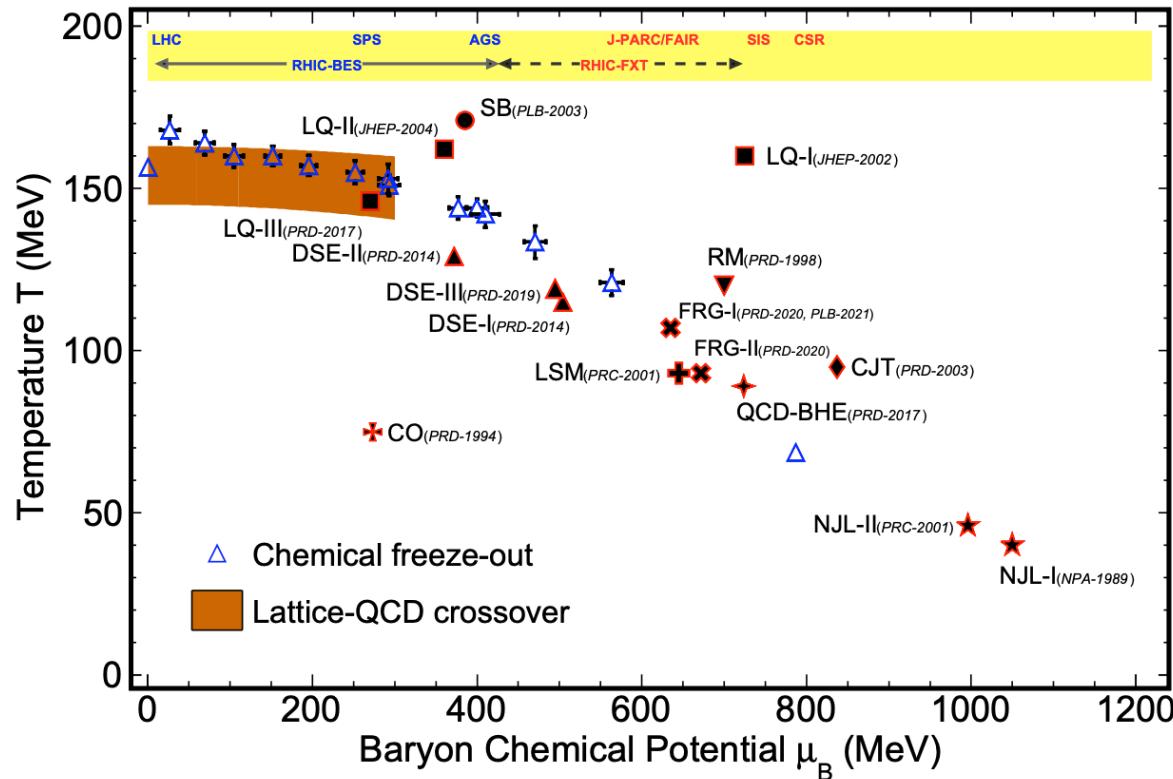


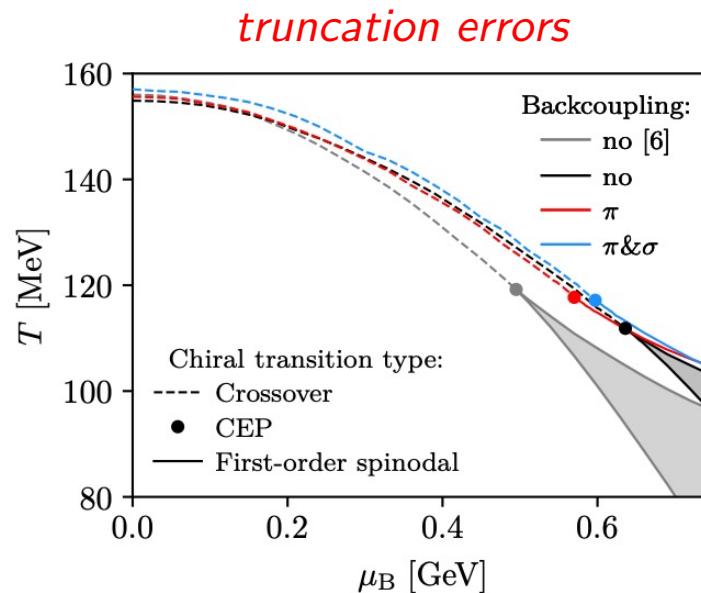
Figure adapted from A. Pandav, D. Mallick, B. Mohanty, Prog. Part. Nucl. Phys. 125 (2022)

Including the possibility that the QCD critical point does not exist at all

de Forcrand, Philipsen, JHEP 01, 077 (2007); VV, Steinheimer, Philipsen, Stoecker, PRD 97, 114030 (2018)

Effective QCD theories anchored with lattice QCD

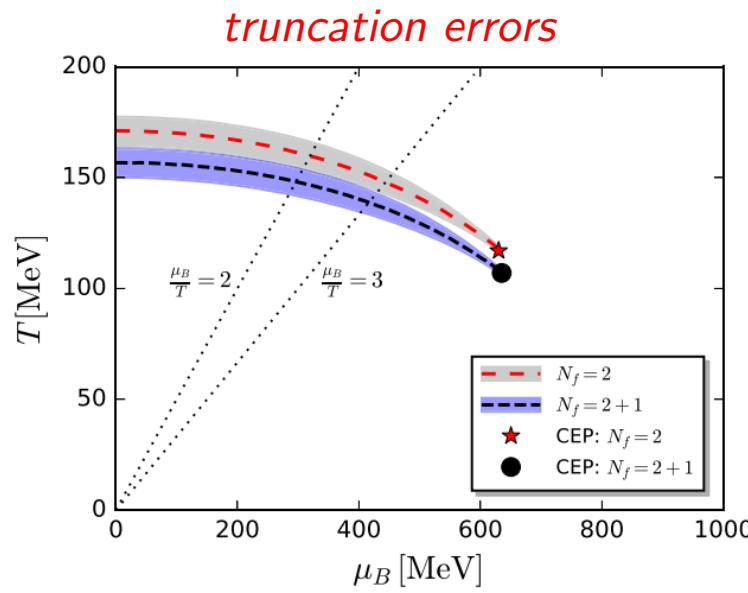
Dyson-Schwinger equations



Gunkel, Fischer, PRD 104, 054202 (2021)

$T \sim 120$ MeV $\mu_B \sim 600$ MeV

Functional renormalization group

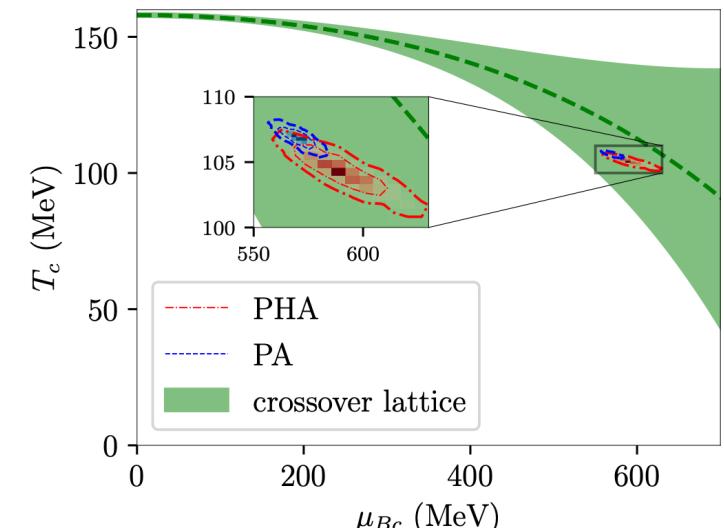


Fu, Pawłowski, Rennecke, PRD 101, 053032 (2020)

$T \sim 100$ MeV $\mu_B \sim 600 - 650$ MeV

Black-hole engineering

strongly-coupled only ($\eta/s = 1/4\pi$)



Hippert et al., arXiv:2309.00579

$T \sim 105$ MeV $\mu_B \sim 580$ MeV

- All in excellent agreement with lattice QCD at $\mu_B = 0$ and predict QCD critical point in a similar ballpark of $\mu_B/T \sim 5-6$

Other estimates:

- Finite-size scaling of heavy-ion observables
- Extrapolation of Yang-Lee edge singularities

[R. Lacey, PRL 114, 142301 (2015); A. Sørensen, P. Sørensen, arXiv:2405.10278]

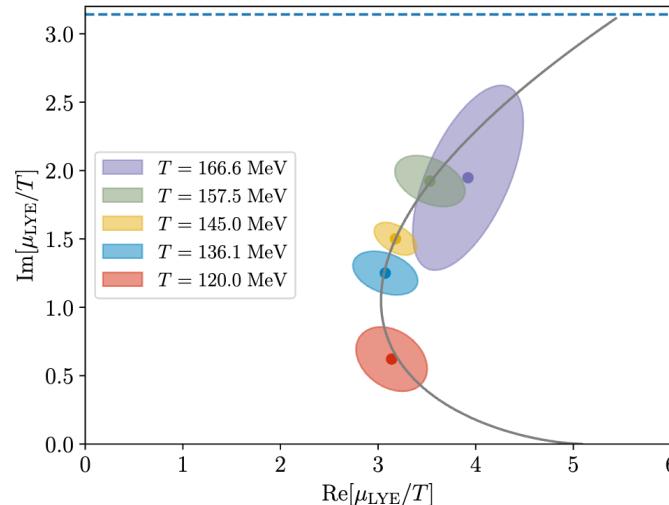
[D.A. Clarke et al. (Bielefeld-Parma), arXiv:2405.10196; G. Basar, PRC 110, 015203 (2024)]

Searching for singularities in the complex plane

Critical point is a singularity on the real μ_B axis,
 which turns into **Yang-Lee edge singularities** above T_c in the complex plane

M. Stephanov, PRD 73, 094508 (2006)

Strategy: Extract YL edge singularity through (multi-point) Pade fits and
 see if it approaches the real axis as temperatures decreases



CP Z(2) scaling inspired fit:

$$\text{Im } \mu_{LY} = c(T - T_{CEP})^\Delta$$



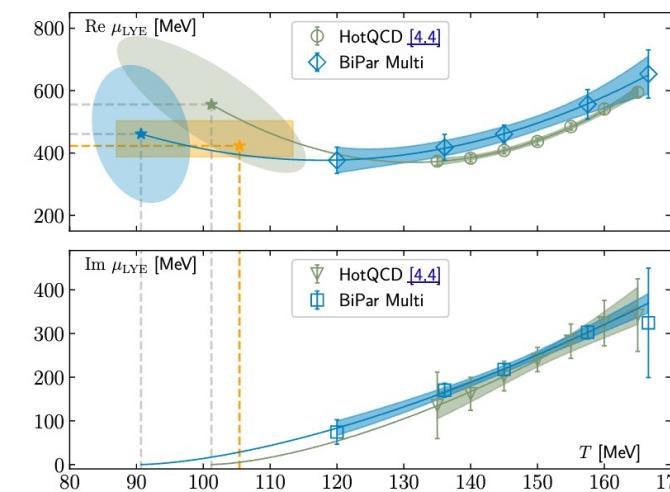
$$\text{Re } \mu_{LY} = \mu_{CEP} + a(T - T_{CEP}) + b(T - T_{CEP})^2$$

Extrapolated CP estimate:

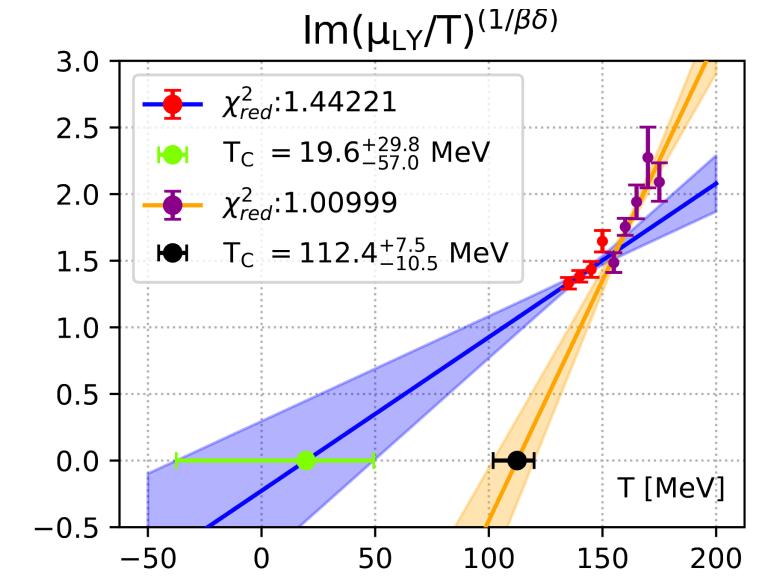
$$T \sim 90-110 \text{ MeV}, \mu_B \sim 400-600 \text{ MeV}$$

D.A. Clarke et al. (Bielefeld-Parma), arXiv:2405.10196; G. Basar, PRC 110, 015203 (2024)

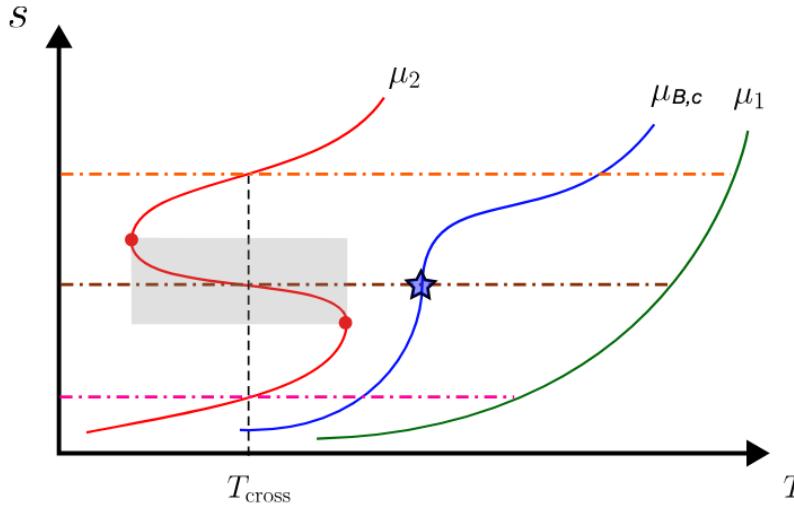
many things have to go right, systematic error still very large (up to 100%)



Variations in fit range etc.



A. Adam et al. (Wuppertal-Budapest), LATTICE2024



Locating the QCD critical point from first principles through contours of constant entropy density

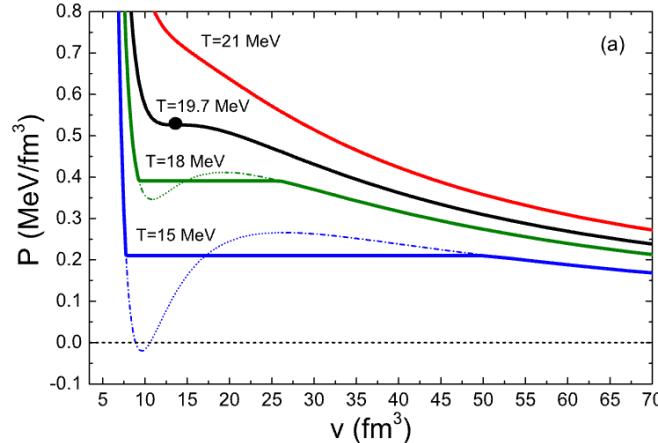


Hitansh Shah, Mauricio Hippert, Jorge Noronha, Claudia Ratti, VV, [arXiv:2410.16206](https://arxiv.org/abs/2410.16206)

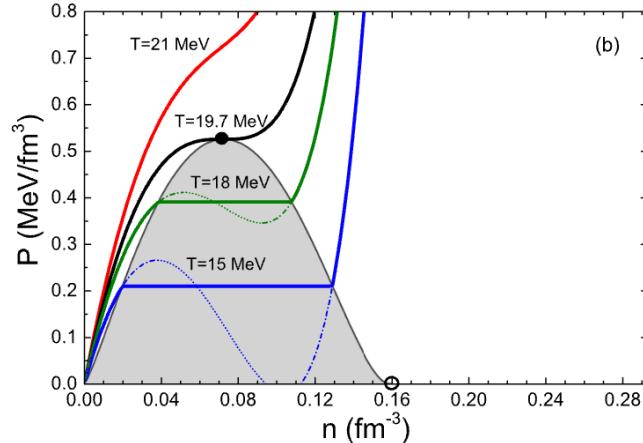
(Dated: October 22, 2024)

Critical point and crossings of entropy density

Usual case: Fix T , scan P over ρ_B



FOPT exists if non-monotonic, apply Maxwell construction



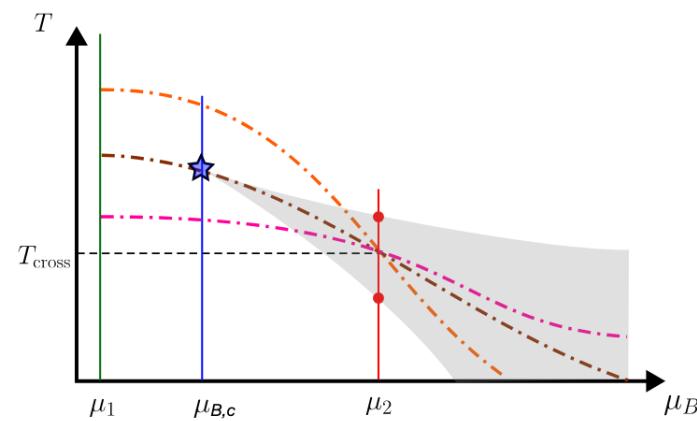
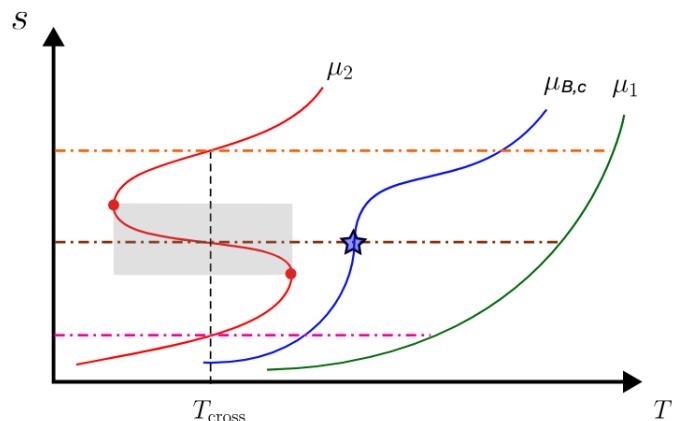
Critical Point:

$$\left(\frac{\partial P}{\partial \rho_B} \right)_T = 0, \quad \left(\frac{\partial^2 P}{\partial \rho_B^2} \right)_T = 0.$$

Figure from VV, Anchishkin, Gorenstein, PRC 91, 064314 (2015)

Alternative: Fix μ_B , scan s over T

FOPT exists if lines of constant entropy cross



Critical Point:

$$\left(\frac{\partial T}{\partial s} \right)_{\mu_B} = 0, \quad \left(\frac{\partial^2 T}{\partial s^2} \right)_{\mu_B} = 0.$$

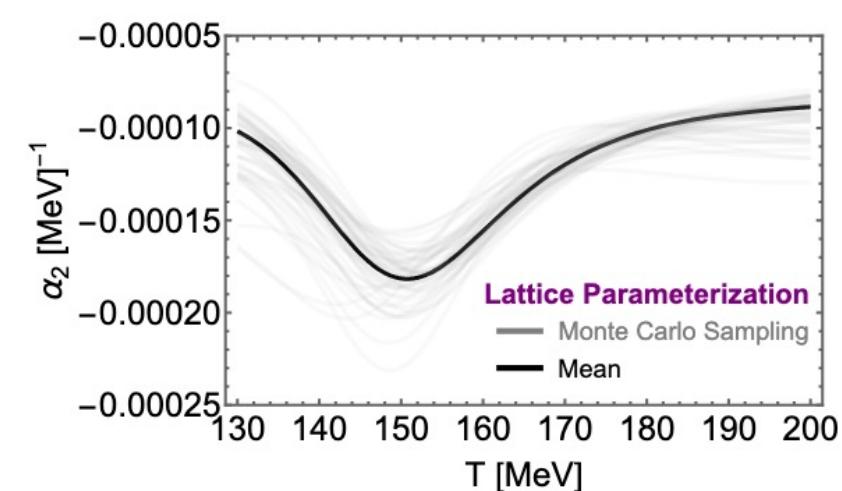
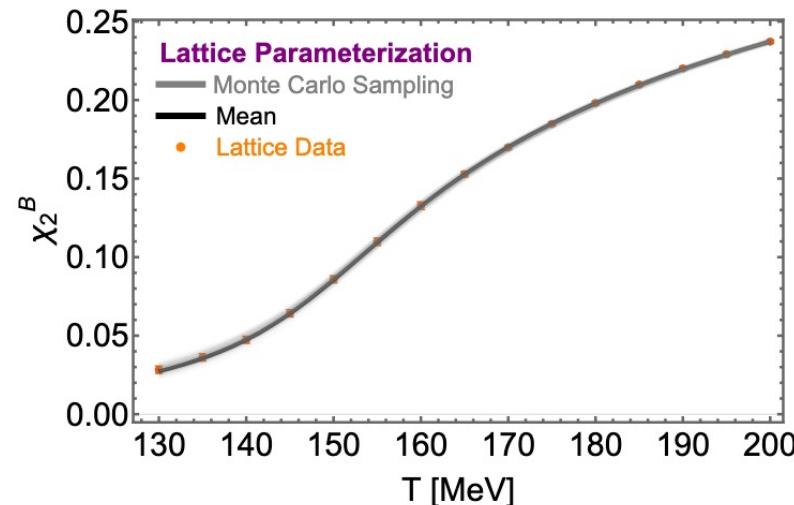
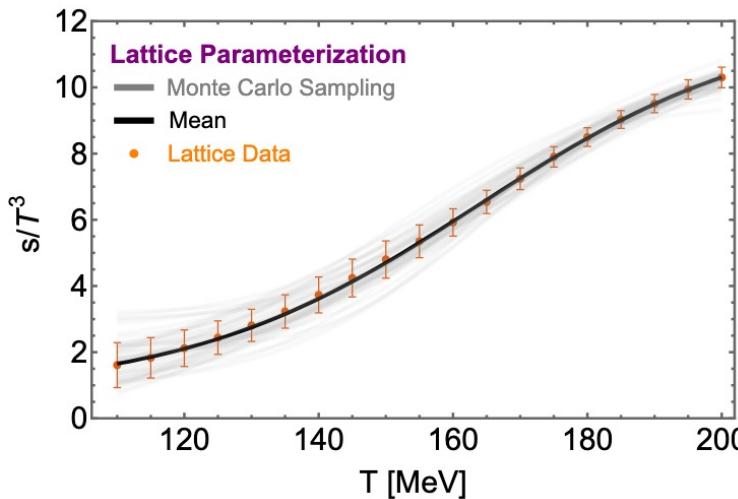
New expansion

Define the $s = \text{const.}$ line in $T-\mu_B$ plane as an expansion around $\mu_B = 0$

$$T_s(\mu_B; T_0) \approx T_0 + \sum_{n=1}^N \alpha_{2n}(T_0) \frac{\mu_B^{2n}}{(2n)!} + \mathcal{O}(\mu_B^{2(N+1)}) \quad \alpha_{2n}(T_0) = \left(\frac{\partial^{2n} T}{\partial \mu_B^{2n}} \right)_s \Big|_{T=T_0, \mu_B=0}$$

$$\alpha_2(T_0) = -\frac{2T_0 \chi_2^B(T_0) + T_0^2 \chi_2^{B'}(T_0)}{s'(T_0)}$$

Parametrized continuum-extrapolated lattice QCD input $[s(T), \chi_2^B(T)]$ from Bayesian analysis

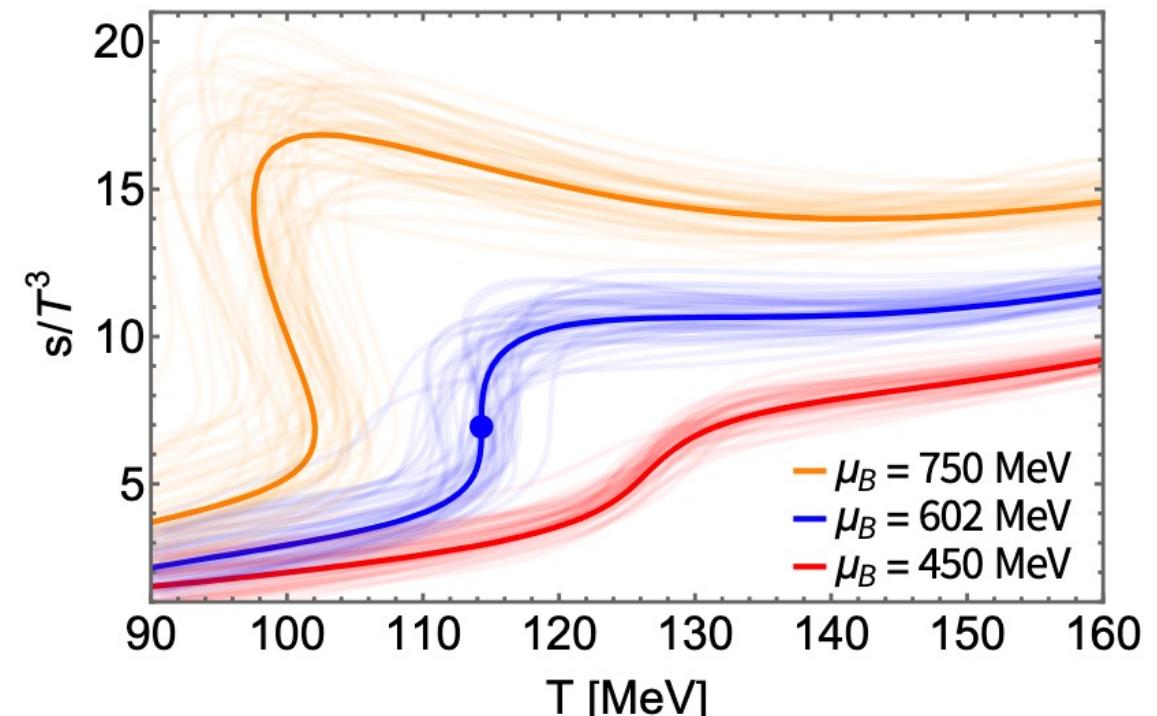
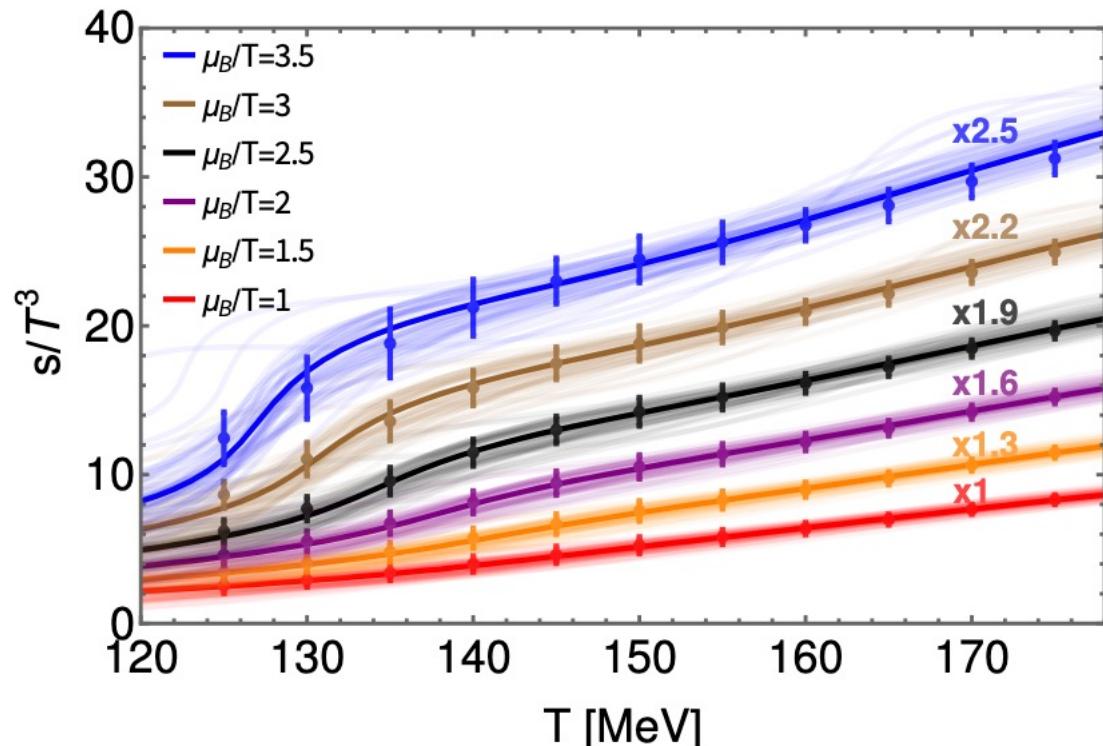


Lattice data for $s(T)$: Borsanyi et al., PLB 730, 99 (2014) Lattice data for $\chi_2^B(T)$: Borsanyi et al., PRL 126, 232001 (2021)

Propagation of the correlated lattice QCD uncertainties through Monte Carlo sampling of the parameter posterior distribution

Entropy density at finite μ_B

$$T_s(\mu_B; T_0) = T_0 + \alpha_2(T_0) \frac{\mu_B^2}{2}$$

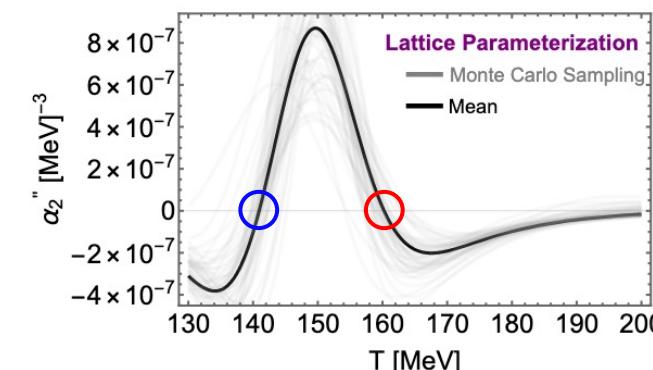
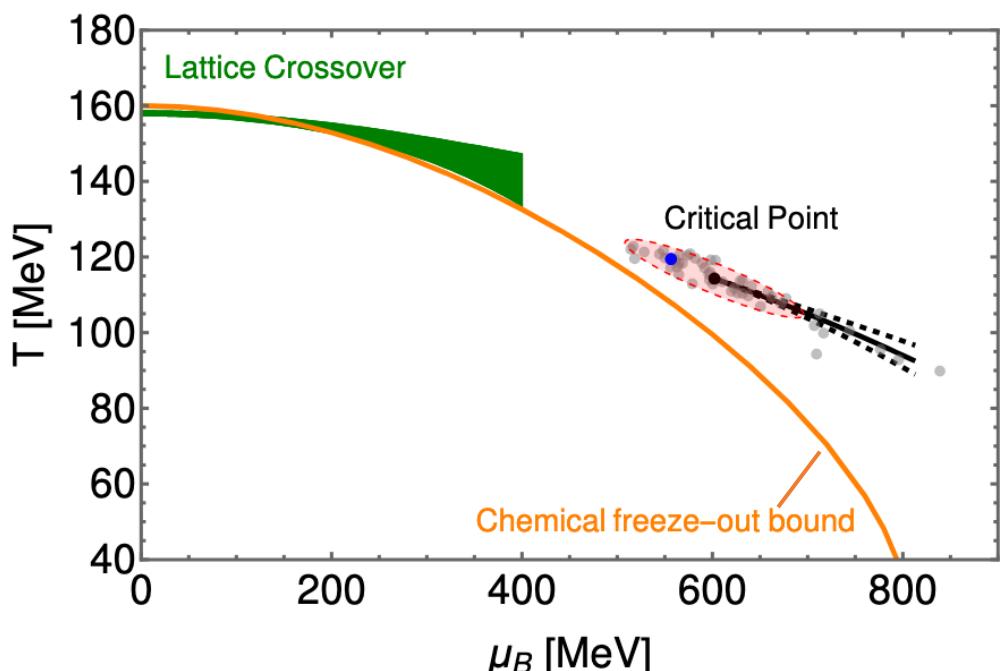


- Excellent agreement at low μ_B/T with available lattice QCD constraints
[Borsanyi et al., PRL 126, 232001 (2021)]
- First-order phase transition emerges at $\mu_B > 600$ MeV

Locating the critical point

$$T_s(\mu_B; T_0) = T_0 + \alpha_2(T_0) \frac{\mu_B^2}{2} \quad \left(\frac{\partial T}{\partial s} \right)_{\mu_B} = 0 \quad \xrightarrow{\text{spinodals}} \quad \mu_{B,c} = \sqrt{-\frac{2}{\alpha'_2(T_{0,c})}} \quad \left(\frac{\partial^2 T}{\partial s^2} \right)_{\mu_B} = 0 \quad \xrightarrow{\text{critical point}} \quad \alpha''_2(T_{0,c}) = 0$$

expansion ***spinodals*** ***critical point***



Critical point location:

$$T_C = 114.3 \pm 6.9 \text{ MeV}, \quad \mu_B = 602.1 \pm 62.1 \text{ MeV}$$

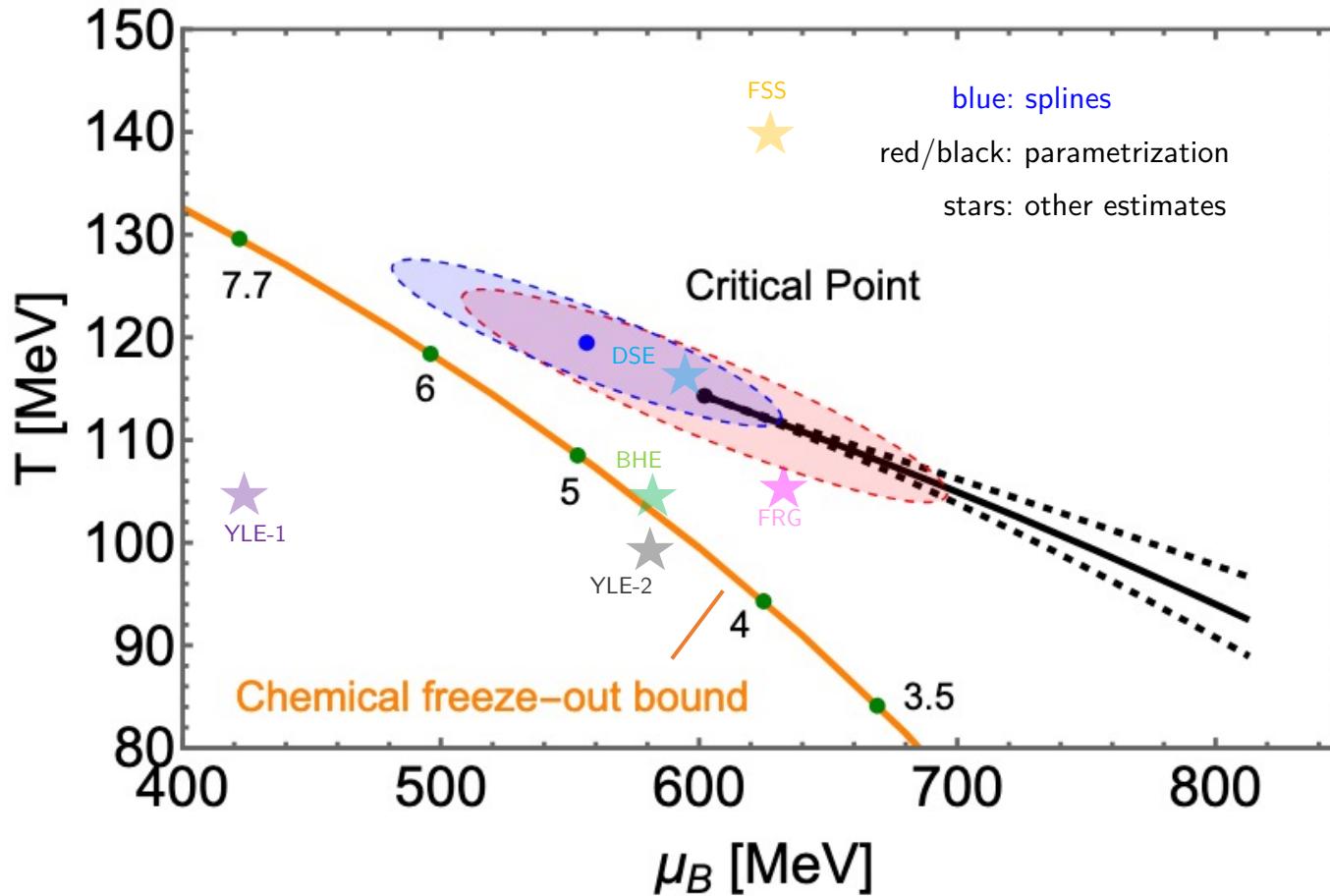
Second solution: CP at *imaginary* μ_B

$$\tilde{T}_c = 197.1 \pm 7.1 \text{ MeV}, \quad \frac{\tilde{\mu}_{B,c}}{\tilde{T}_c} = i(3.50 \pm 0.30)$$

cf. *Roberge-Weiss endpoint* at

$$T_{RW} = 208 \pm 5 \text{ MeV}, \quad \mu_{B,RW}/T_{RW} = i\pi$$

Comparison to other estimates in the literature



Cross-check using **splines** in place of **parametrization** yields consistent results

Other CP estimates from recent literature:

YLE-1: D.A. Clarke et al. (Bielefeld-Parma), arXiv:2405.10196

YLE-2: G. Basar, PRC 110, 015203 (2024)

BHE: M. Hippert et al., arXiv:2309.00579

FRG: W-J. Fu et al., PRD 101, 054032 (2020)

DSE: P.J. Gunkel et al., PRD 104, 052022 (2021)

FSS: A. Sorensen et al., arXiv:2405.10278

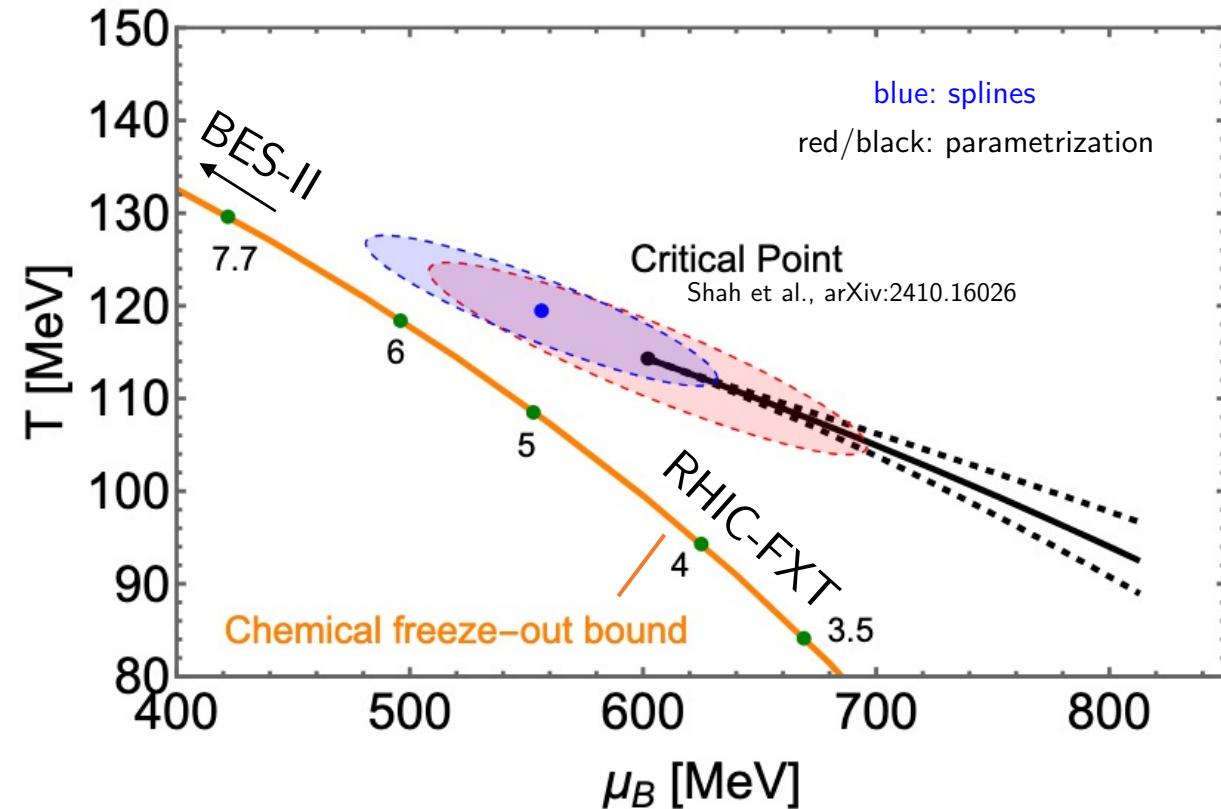
Search for critical point with heavy-ion collisions

Control parameters

- Collision energy $\sqrt{s_{NN}} = 2.4 - 5020$ GeV
 - Scan the QCD phase diagram
- Size of the collision region
 - Expect stronger signal in larger systems

Measurements

- Final hadron abundances and momentum distributions **event-by-event**



Chemical freeze-out curve and CP

[Artemiy Lysenko, Poberezhnyuk, Gorenstein, VV, arXiv:2408.06473]

- Sets the **lower bound** on the temperature of the CP
- **Caveats:** strangeness neutrality ($\mu_S \neq 0$), uncertainty in the freeze-out curve
- Estimate through constant energy-per-hadron criterion [Cleymans, Redlich, arXiv:2408.06473]

Critical point, cumulants, and heavy-ion collisions

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln \langle e^{tN} \rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

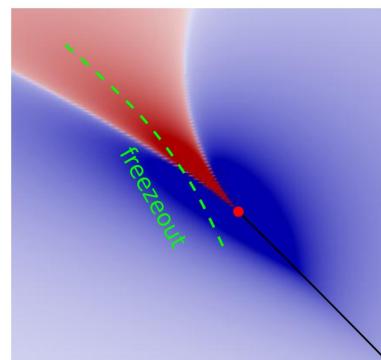
$$\kappa_n \propto \frac{\partial^n (\ln Z^{\text{gce}})}{\partial \mu^n}$$

Grand partition function

$$\ln Z^{\text{gce}}(T, V, \mu) = \ln \left[\sum_N e^{\mu N / T} Z^{\text{ce}}(T, V, N) \right]$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- **(QCD) critical point:** large correlation length and fluctuations



$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

$$\xi \rightarrow \infty$$

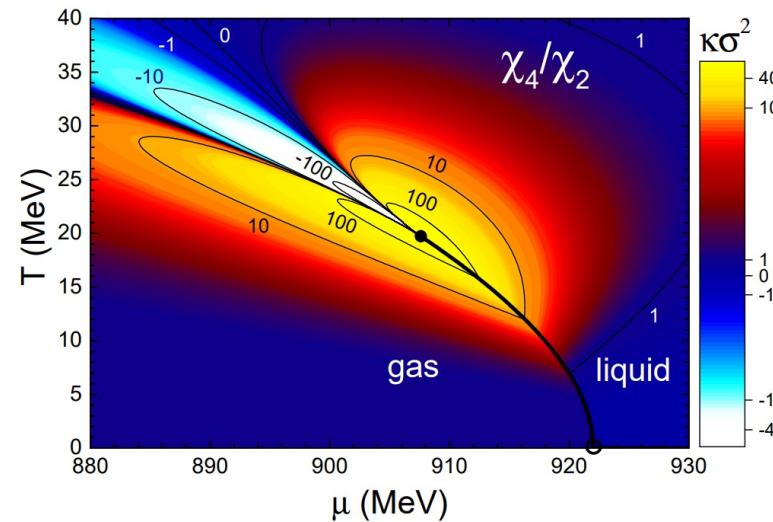
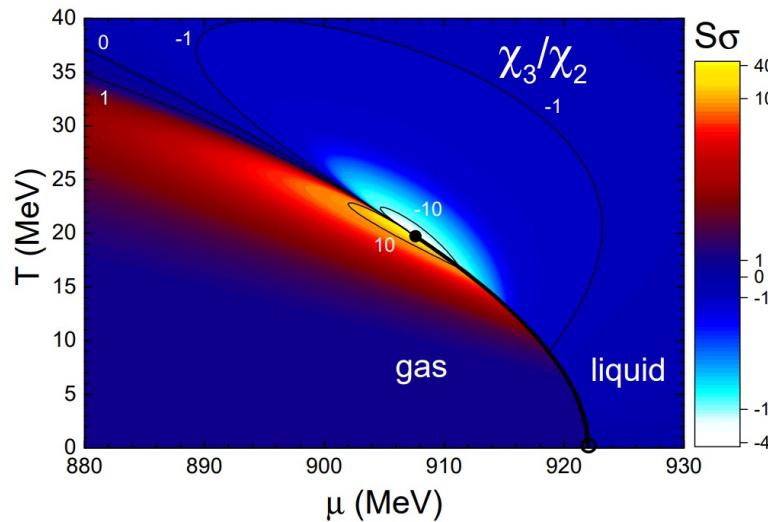
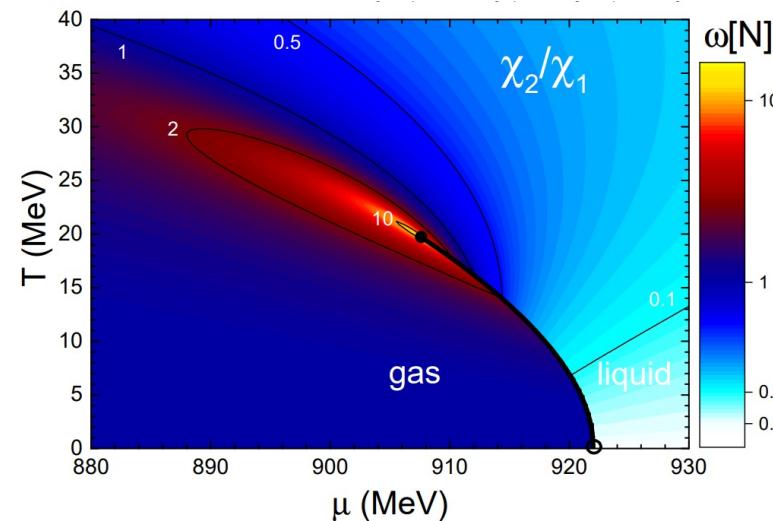
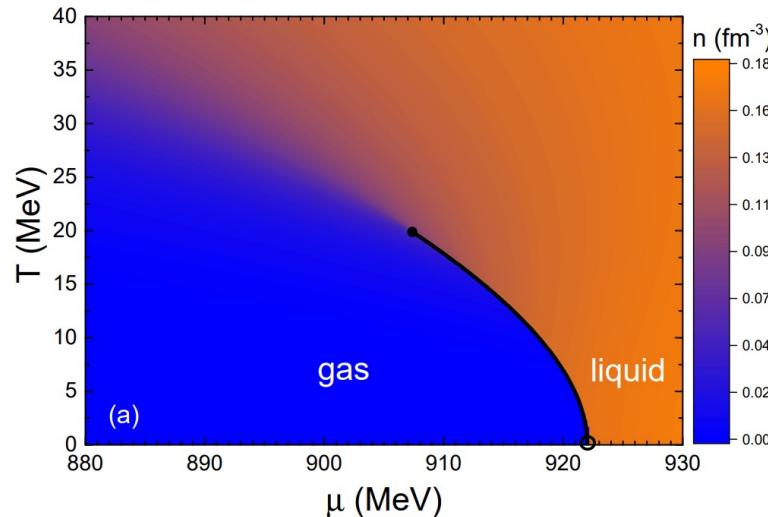
Looking for enhanced fluctuations
and non-monotonocities

M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

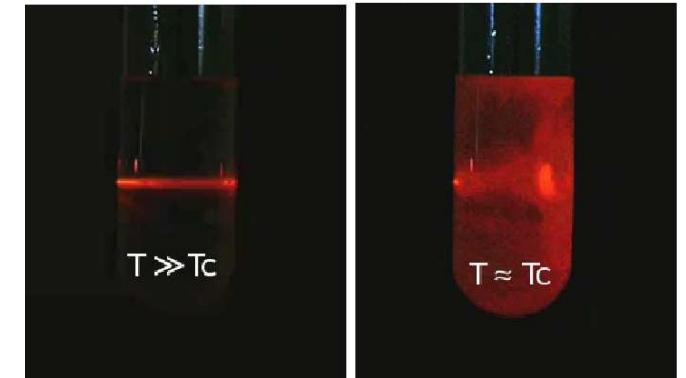
Other uses of cumulants:

- QCD degrees of freedom
Jeon, Koch, PRL 85, 2076 (2000)
Asakawa, Heinz, Muller, PRL 85, 2072 (2000)
- Extracting the speed of sound
A. Sorensen et al., PRL 127, 042303 (2021)
- Conservation volume V_C
VV, Donigus, Stoecker, PRC 100, 054906 (2019)

Example: (Nuclear) Liquid-gas transition



Critical opalescence



$$\langle N^2 \rangle - \langle N \rangle^2 \sim \langle N \rangle \sim 10^{23}$$

in equilibrium

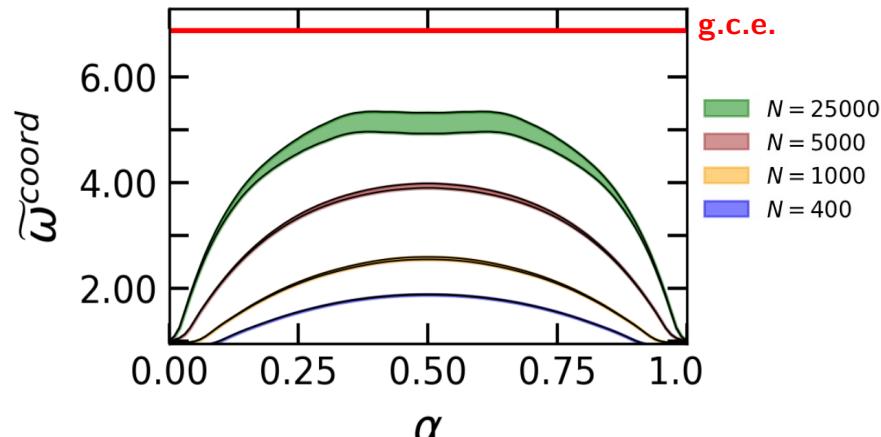
Example: Critical fluctuations in a microscopic simulation

V. Kuznetsov et al., Phys. Rev. C 105, 044903 (2022)

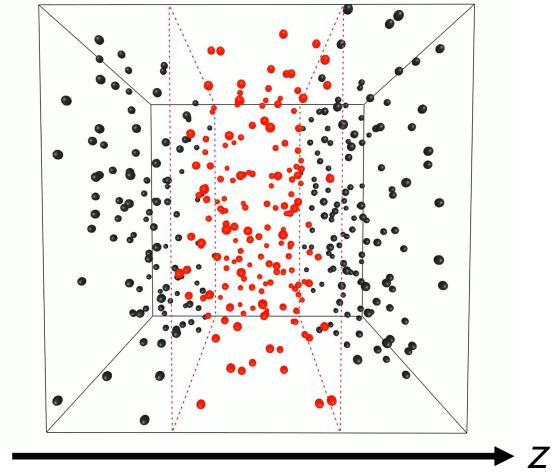
Classical molecular dynamics simulations of the **Lennard-Jones fluid**
 near Z(2) critical point ($T \approx 1.06T_c$, $n \approx n_c$) of the liquid-gas transition

Scaled variance in coordinate space acceptance $|z| < z^{max}$

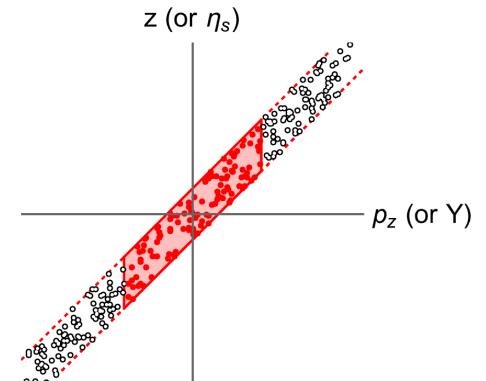
$$\tilde{\omega}^{\text{coord}} = \frac{1}{1-\alpha} \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$



- Large fluctuations survive despite strong finite-size effects
- Need coordinate space cuts (collective flow helps)
- Here no finite-time effects



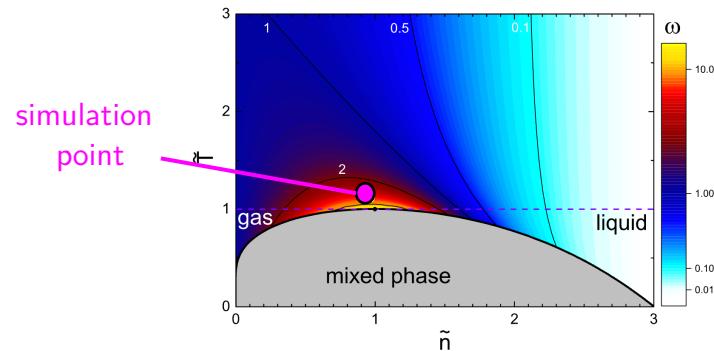
Heavy-ion collisions:
 flow correlates p_z and z cuts



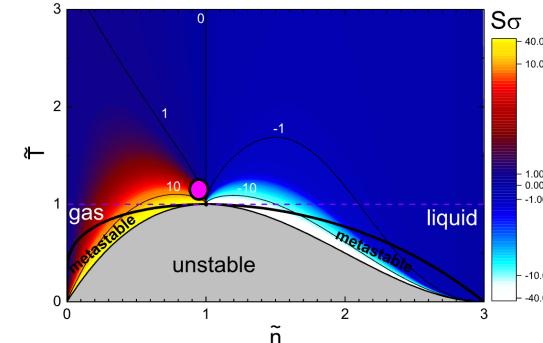
Non-Gaussian fluctuations from molecular dynamics

V. Kuznetsov et al., to appear

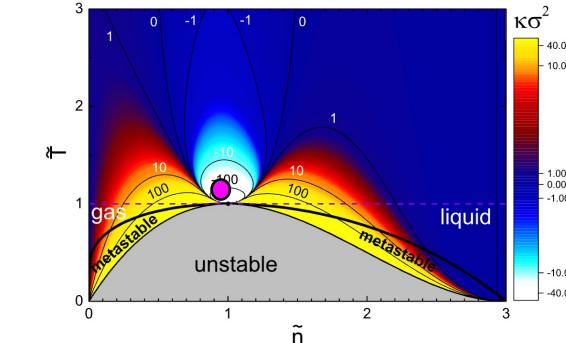
Scaled variance κ_2/κ_1



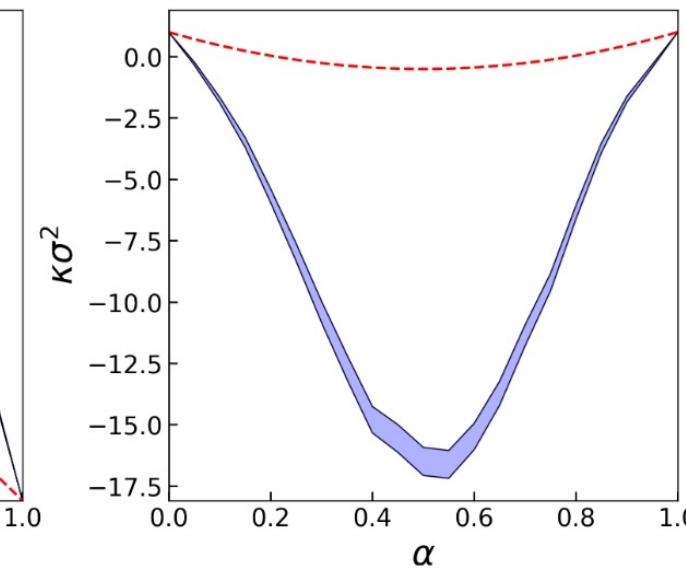
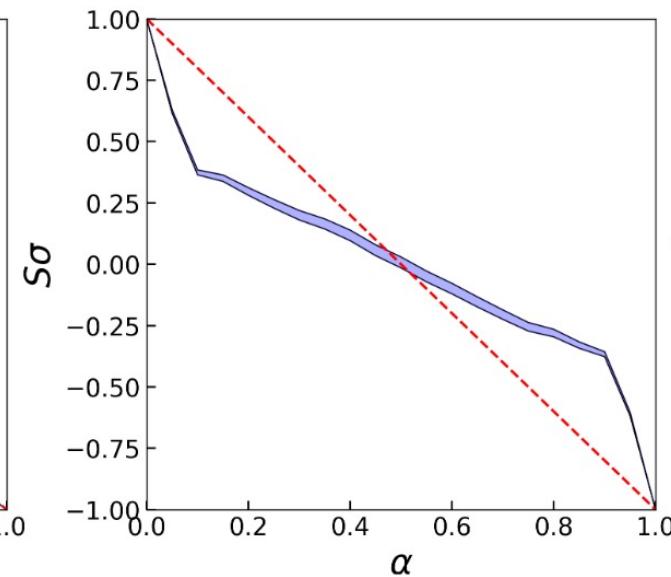
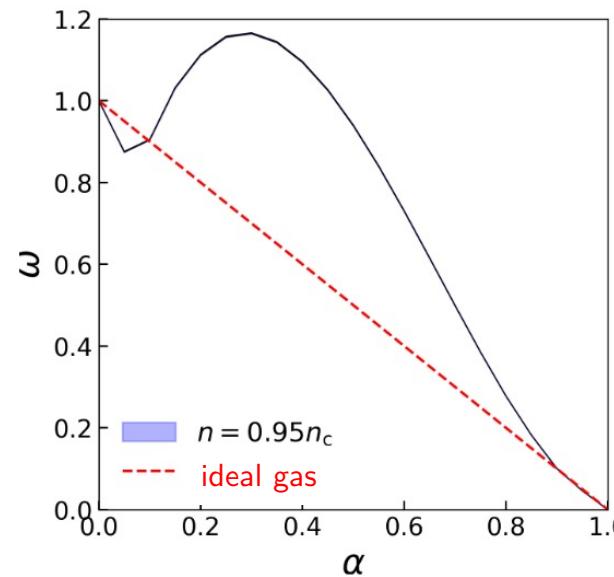
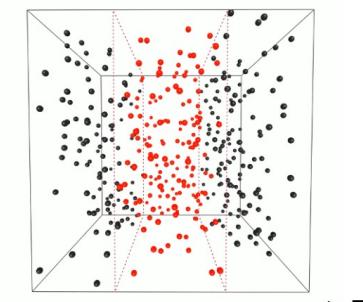
Skewness κ_3/κ_2



Kurtosis κ_4/κ_2



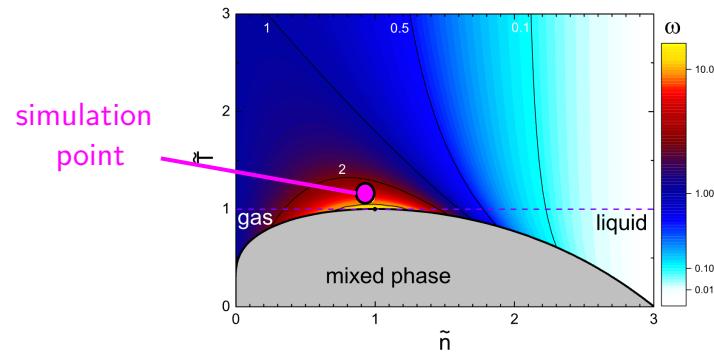
400 nucleons
in a box



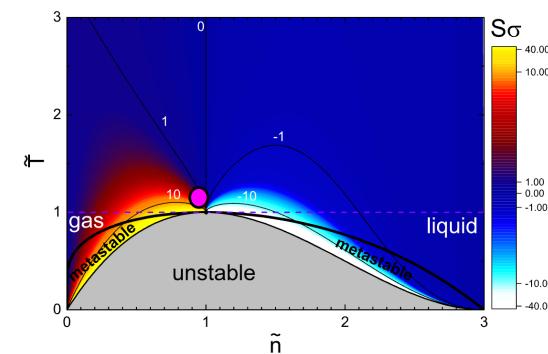
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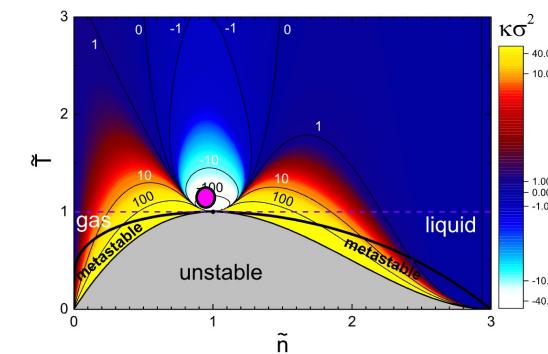
Scaled variance κ_2/κ_1



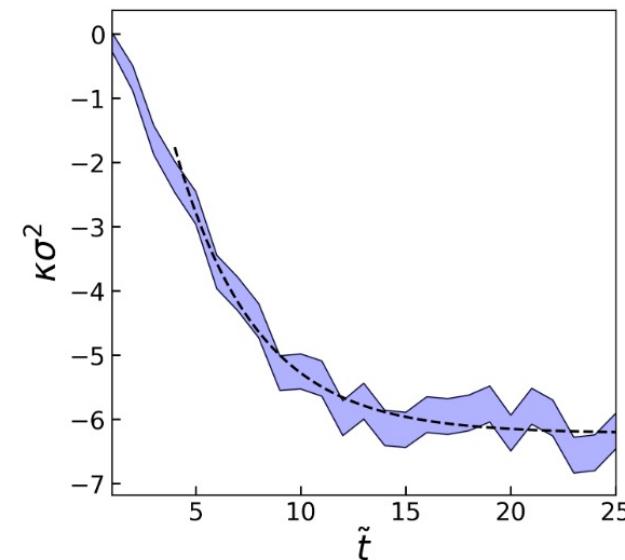
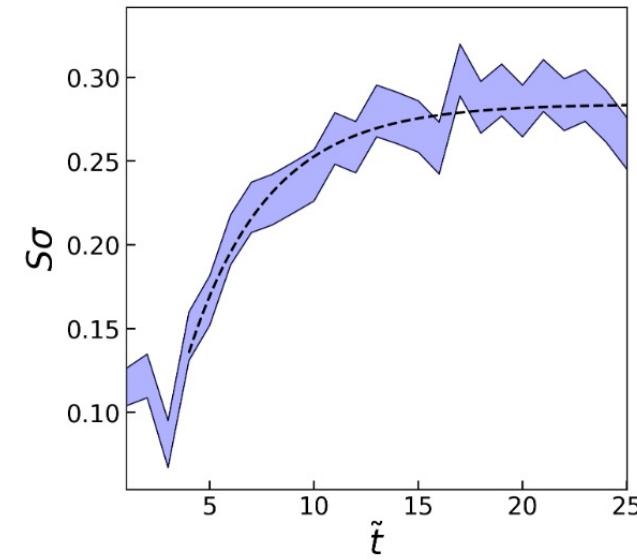
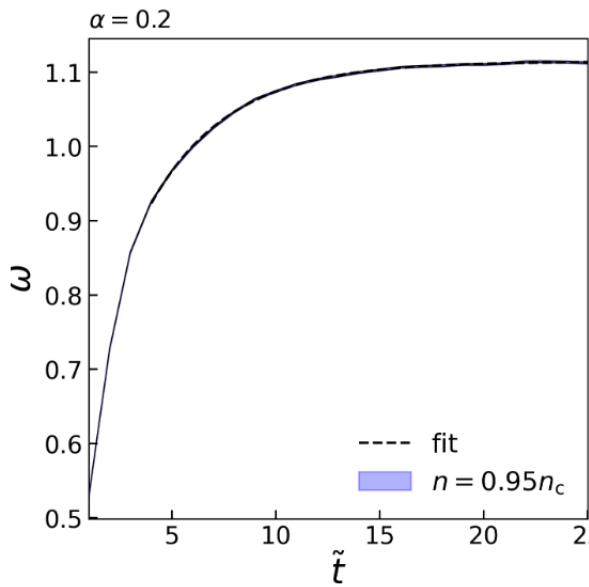
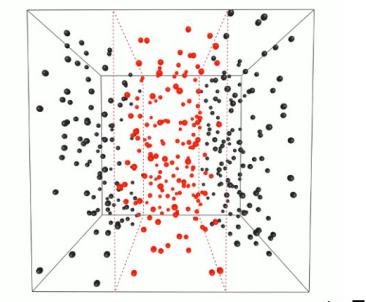
Skewness κ_3/κ_2



Kurtosis κ_4/κ_2



400 nucleons
in a box



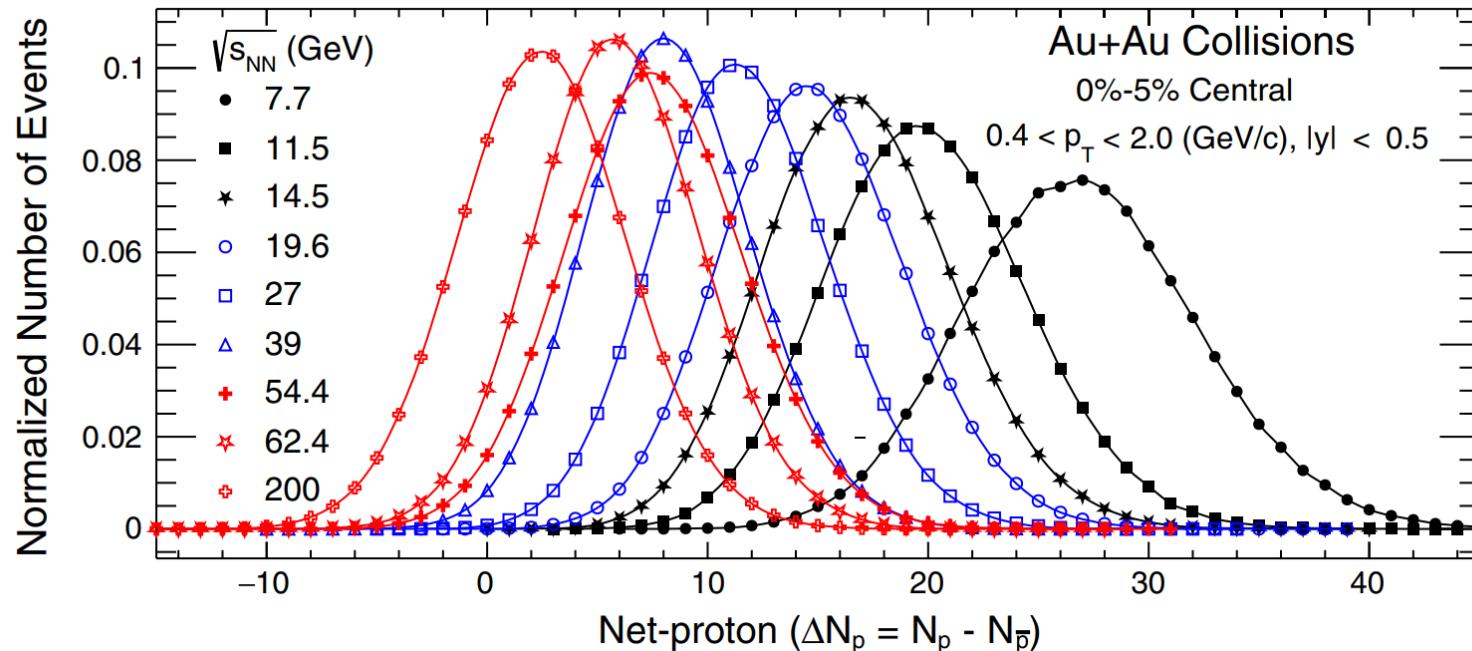
- (Non-)Gaussian cumulants equilibrate on comparable time scales

Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons

$$P(\Delta N_p) \sim \frac{N_{\text{events}}(\Delta N_p)}{N_{\text{events}}^{\text{total}}}$$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)



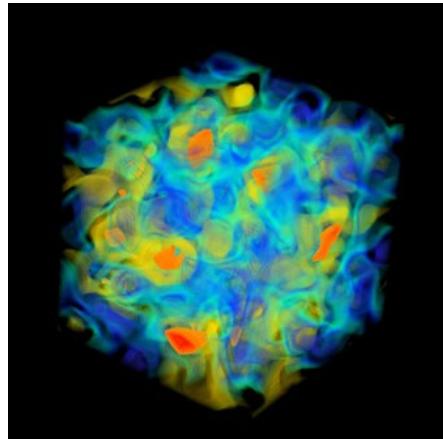
Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

$$\frac{\kappa_2}{\langle N \rangle}, \quad \frac{\kappa_3}{\kappa_2}, \quad \frac{\kappa_4}{\kappa_2}$$

Look for subtle critical point signals

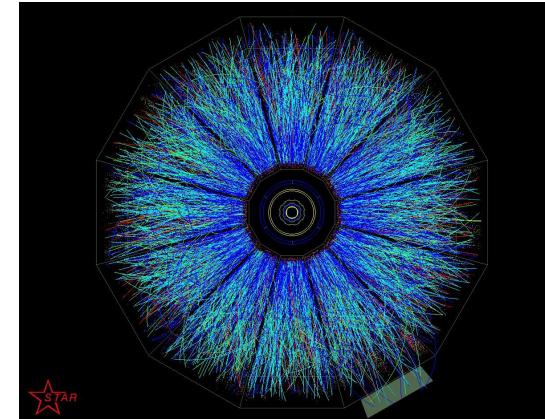
Theory vs experiment: Challenges for fluctuations

Theory



© Lattice QCD@BNL

Experiment



STAR event display

- Coordinate space
 - In contact with the heat bath
 - Conserved charges
 - Uniform
 - Fixed volume
- Momentum space
 - Expanding in vacuum
 - Non-conserved particle numbers
 - Inhomogenous
 - Fluctuating volume

Need dynamical description

Exact charge conservation

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020); VV, arXiv:2409.01397

Utilizing the canonical partition function in thermodynamic limit
 compute **n-point density correlators**

$$\mathcal{C}_1(\mathbf{r}_1) = \rho(\mathbf{r}_1)$$

$$\mathcal{C}_2(\mathbf{r}_1, \mathbf{r}_2) = \chi_2 \delta(\mathbf{r}_1 - \mathbf{r}_2) - \frac{\chi_2}{V}$$

local correlation

balancing contribution
 (e.g. baryon conservation)

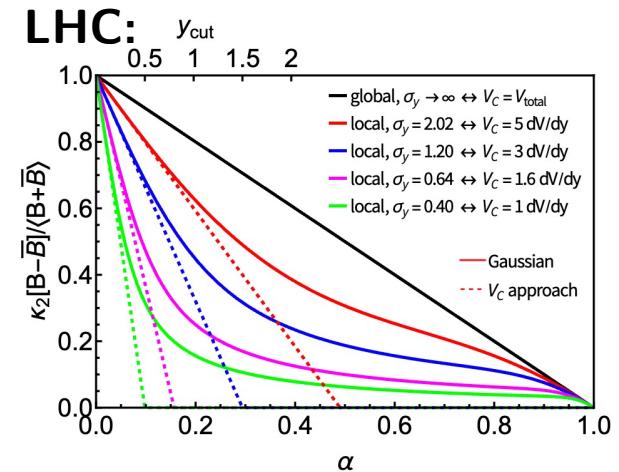
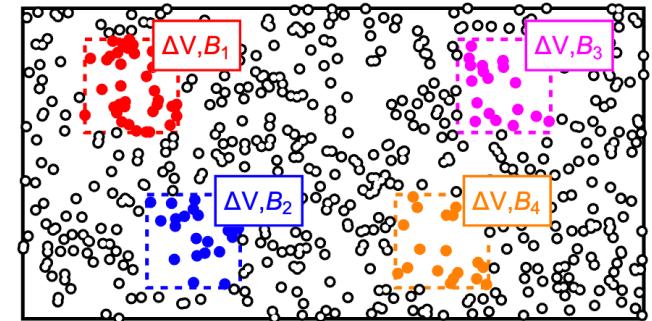
$$\mathcal{C}_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \chi_3 \delta_{1,2,3} - \frac{\chi_3}{V} [\delta_{1,2} + \delta_{1,3} + \delta_{2,3}] + 2 \frac{\chi_3}{V^2} \quad \delta_{1,\dots,n} = \prod_{i=2}^n \delta(\mathbf{r}_1 - \mathbf{r}_i)$$

local correlation

balancing contributions

$$\begin{aligned} \mathcal{C}_4(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = & \chi_4 \delta_{1,2,3,4} - \frac{\chi_4}{V} [\delta_{1,2,3} + \delta_{1,2,4} + \delta_{1,3,4} + \delta_{2,3,4}] - \frac{(\chi_3)^2}{\chi_2 V} [\delta_{1,2} \delta_{3,4} + \delta_{1,3} \delta_{2,4} + \delta_{1,4} \delta_{2,3}] \\ & + \frac{1}{V^2} \left[\chi_4 + \frac{(\chi_3)^2}{\chi_2} \right] [\delta_{1,2} + \delta_{1,3} + \delta_{1,4} + \delta_{2,3} + \delta_{2,4} + \delta_{3,4}] - \frac{3}{V^3} \left[\chi_4 + \frac{(\chi_3)^2}{\chi_2} \right]. \end{aligned}$$

balancing contributions



Integrating the correlator yields cumulant inside a subsystem of the canonical ensemble

$$\kappa_n[B_{V_s}] = \int_{\mathbf{r}_1 \in V_s} d\mathbf{r}_1 \dots \int_{\mathbf{r}_n \in V_s} d\mathbf{r}_n \mathcal{C}_n(\{\mathbf{r}_i\})$$

Momentum space: Fold with Maxwell-Boltzmann in LR frame and integrate out the coordinates

Fluctuations and beam energy scan

Dynamical approaches to the QCD critical point search

1. Dynamical model calculations of critical fluctuations

- Fluctuating hydrodynamics (hydro+) and (non-equilibrium) evolution of fluctuations
- Equation of state with a tunable critical point [P. Parotto et al, PRC 101, 034901 (2020); J. Karthein et al., EPJ Plus 136, 621 (2021)]
- Generalized Cooper-Frye particlization [M. Pradeep, et al., PRD 106, 036017 (2022); PRL 130, 162301 (2023)]



[X. An et al., Nucl. Phys. A 1017, 122343 (2022)]

Alternatives at high μ_B : hadronic transport/molecular dynamics with a critical point

[A. Sorensen, V. Koch, PRC 104, 034904 (2021); V. Kuznetsov et al., PRC 105, 044903 (2022)]

2. Deviations from precision calculations of non-critical fluctuations

- Non-critical baseline is not flat [Braun-Munzinger et al., NPA 1008, 122141 (2021)]
- Include essential non-critical contributions to (net-)proton number cumulants
- Exact baryon conservation + hadronic interactions (hard core repulsion)
- Based on realistic hydrodynamic simulations tuned to bulk data

[VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

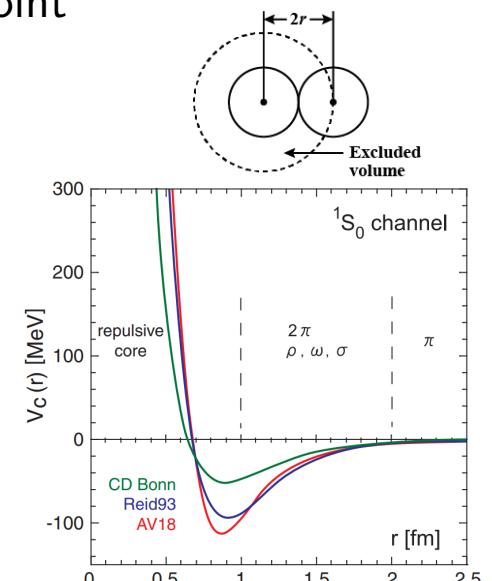
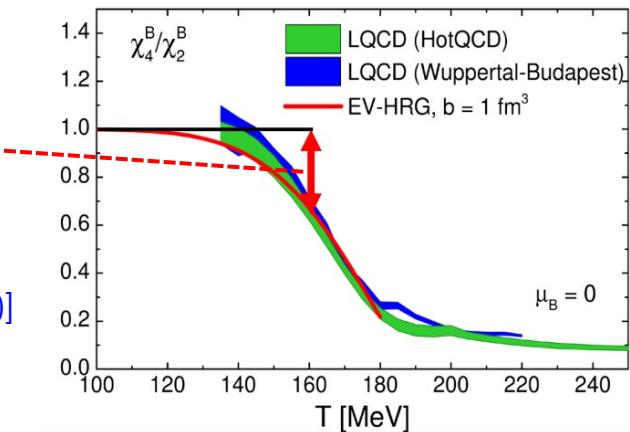
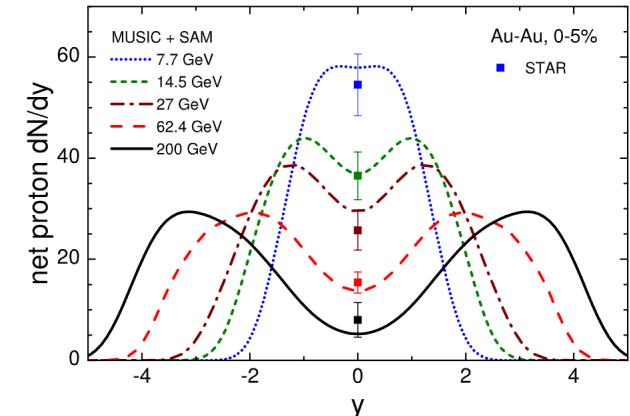


Figure from Ishii et al., PRL '07

Calculation of non-critical contributions at RHIC-BES

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

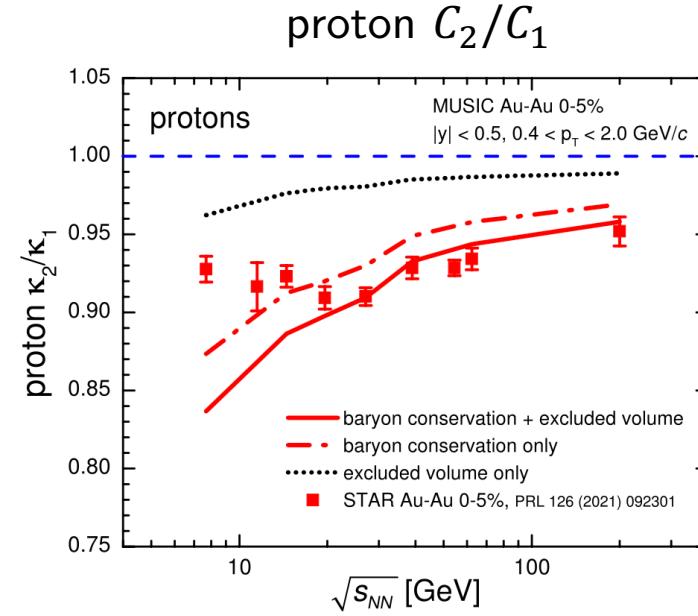
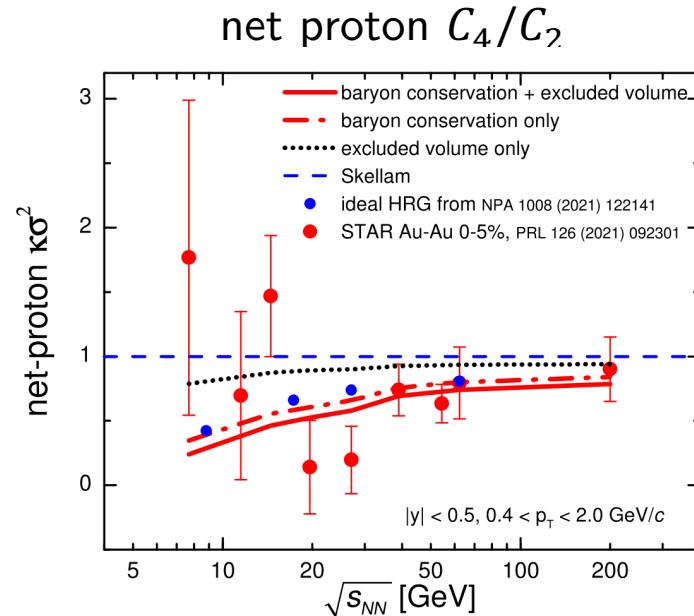
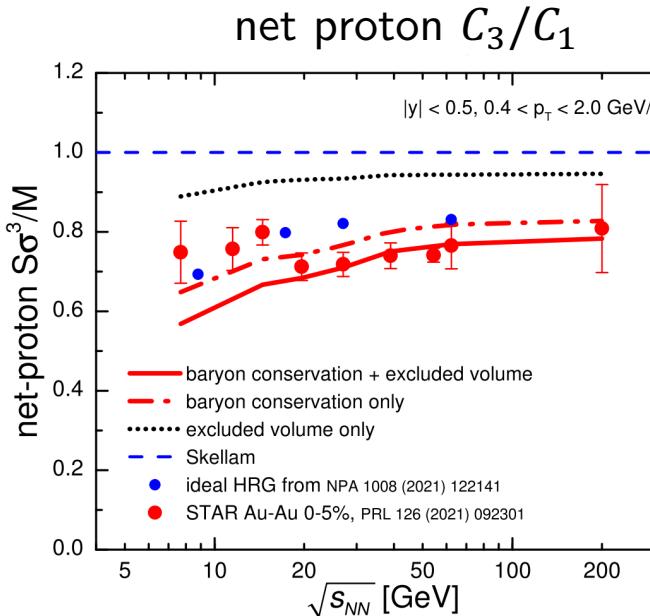
- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
 - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
 - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]
 - Cooper-Frye particlization at $\epsilon_{sw} = 0.26 \text{ GeV/fm}^3$
- Non-critical contributions are computed at particlization
 - QCD-like baryon number distribution (χ_n^B) via **excluded volume** $b = 1 \text{ fm}^3$ [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
 - **Exact global baryon conservation*** (and other charges)
 - Subensemble acceptance method 2.0 (analytic) [VV, Phys. Rev. C 105, 014903 (2022)]
 - or FIST sampler (Monte Carlo) [VV, Phys. Rev. C 106, 064906 (2022)]
<https://github.com/vlvovch/fist-sampler>
- **Absent:** critical point, local conservation, initial-state/volume fluctuations, hadronic phase



*If baryon conservation is the only effect (no other correlations), non-critical baseline can be computed without hydro

RHIC-BES-I: Net proton cumulant ratios (MUSIC)

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

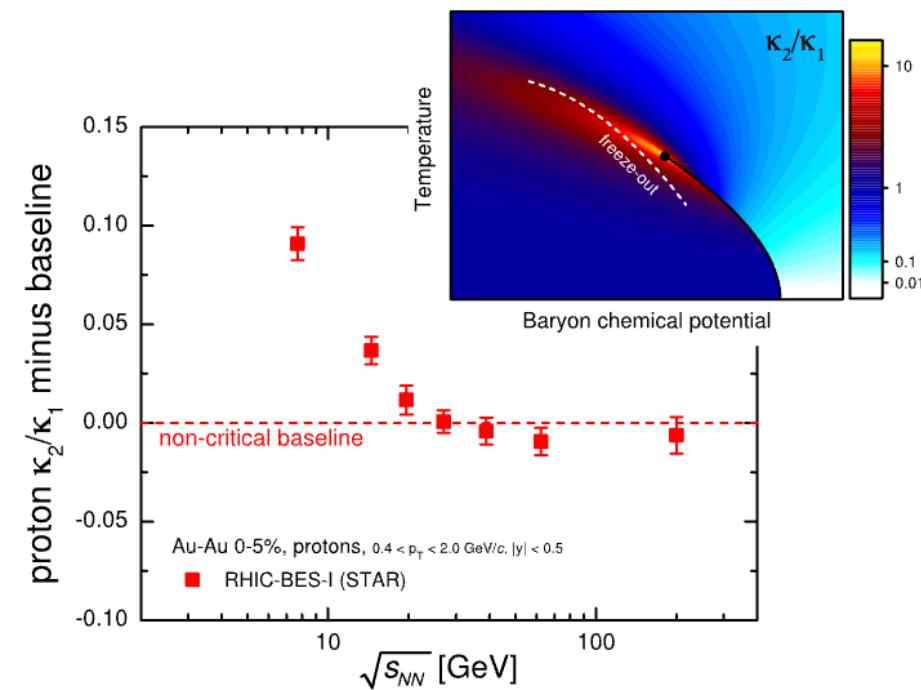
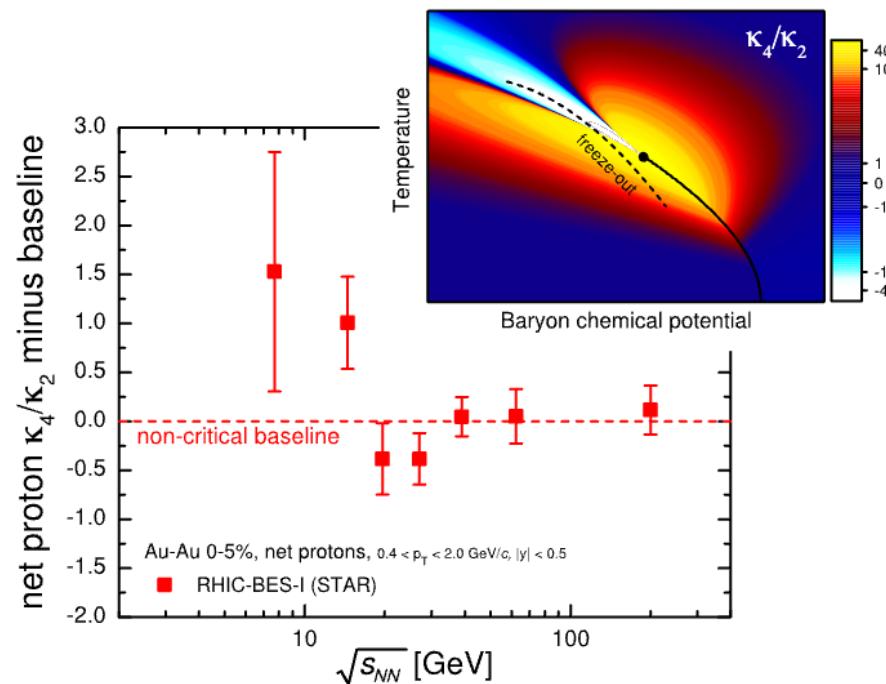


- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (BQS conservation and repulsion)
- Effect from baryon conservation is stronger than repulsion but both are required at $\sqrt{s_{NN}} \geq 20$ GeV
- Deviations from baseline at lower energies?

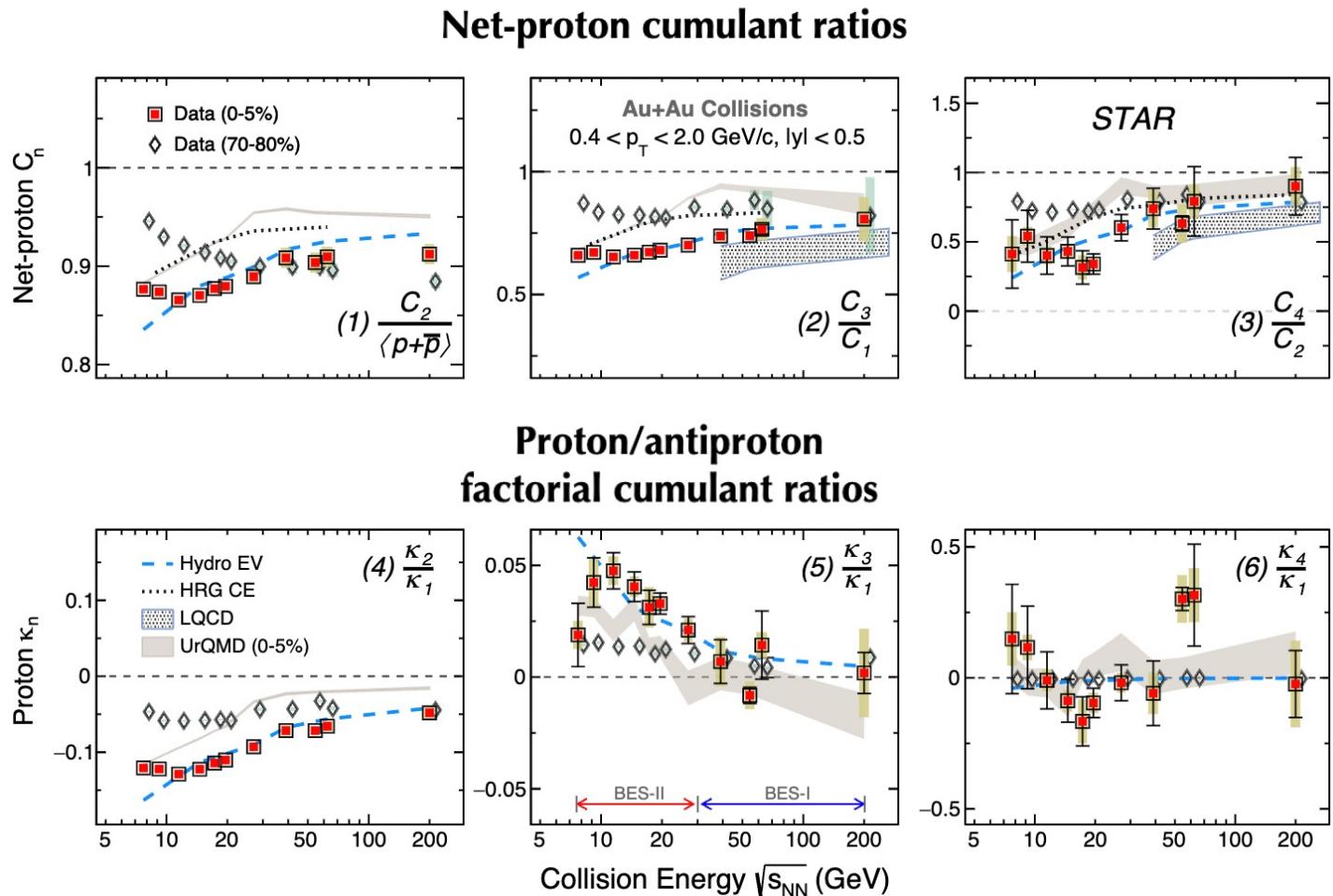
Hints from RHIC-BES-I

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

Subtracting the hydrodynamic non-critical baseline



Proton cumulants from RHIC-BES-II



- No smoking gun signature for CP in ordinary cumulants
- More structure seen in factorial cumulants

Conclusion 1:



Hydro EV: [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 \(2022\)](#)

**Ordinary
cumulants**

**Factorial
cumulants**

Factorial cumulants \hat{C}_n vs ordinary cumulants C_n

Factorial cumulants: \sim irreducible n-particle correlations

$$\hat{C}_n \sim \langle N(N-1)(N-2)\dots \rangle_c$$

Ordinary cumulants: mix corrs. of different orders

$$C_n \sim \langle \delta N^n \rangle_c$$

$$\hat{C}_1 = C_1$$

$$C_1 = \hat{C}_1$$

$$\hat{C}_2 = C_2 - C_1$$

$$C_2 = \hat{C}_2 + \hat{C}_1$$

$$\hat{C}_3 = C_3 - 3C_2 + 2C_1$$

$$C_3 = \hat{C}_3 + 3\hat{C}_2 + \hat{C}_1$$

$$\hat{C}_4 = C_4 - 6C_3 + 11C_2 - 6C_1$$

$$C_4 = \hat{C}_4 + 6\hat{C}_3 + 7\hat{C}_2 + \hat{C}_1$$

[Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017); Kitazawa, Luo, PRC 96, 024910 (2017); C. Pruneau, PRC 100, 034905 (2019)]

Factorial cumulants and different effects

- Baryon conservation
[Bzdak, Koch, Skokov, EPJC '17]

$$\hat{C}_n^{\text{cons}} \propto (\hat{C}_1)^n / \langle N_{\text{tot}} \rangle^{n-1} \quad \text{small}$$

- proton vs baryon $\hat{C}_n^B \sim 2^n \times \hat{C}_n^p$ **same sign!**
[Kitazawa, Asakawa, PRC '12]

- Excluded volume
[VV et al, PLB '17]

$$\hat{C}_n^{\text{EV}} \propto b^n \quad \text{small}$$

- Volume fluctuations
[Holzman et al., arXiv:2403.03598]

$$\hat{C}_n^{\text{CF}} \sim (\hat{C}_1)^n \kappa_n[V] \quad \text{depends on volume cumulants}$$

- Critical point**
[Ling, Stephanov, PRC '16]

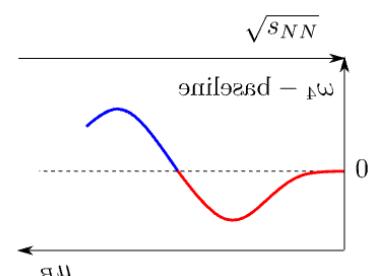
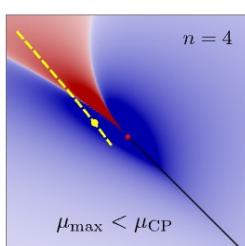
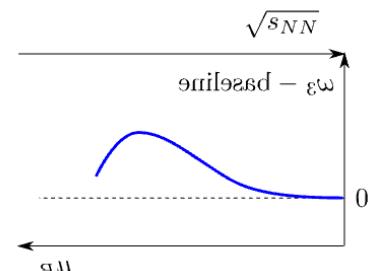
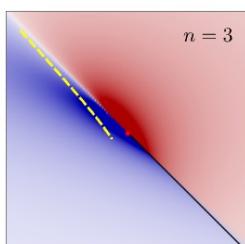
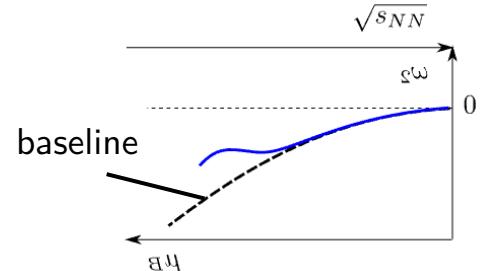
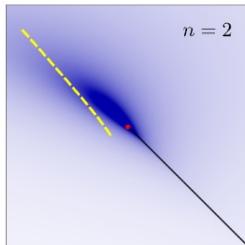
$$\hat{C}_2^{\text{CP}} \sim \xi^2, \quad \hat{C}_3^{\text{CP}} \sim \xi^{4.5}, \quad \hat{C}_4^{\text{CP}} \sim \xi^7 \quad \text{large}$$

Factorial cumulants from RHIC-BES-II

From M. Stephanov, SQM2024 & arXiv:2410.02861

$$\omega_n = \hat{C}_n / \hat{C}_1$$

(universal EOS) critical χ_n :



Bzdak et al review 1906.00936

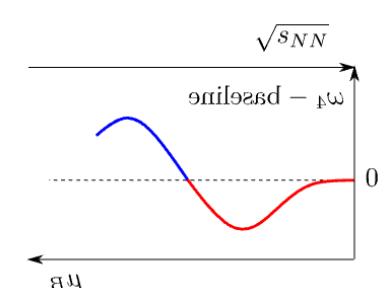
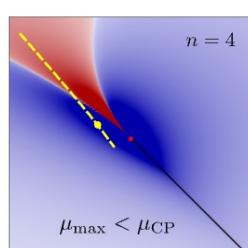
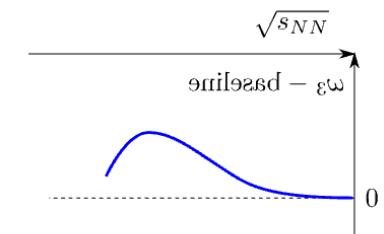
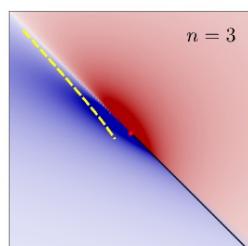
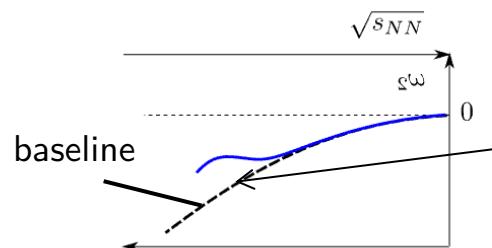
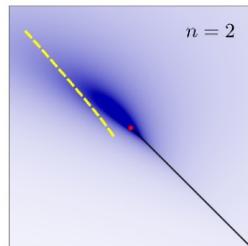
Expected signatures: **bump** in ω_2 and ω_3 , **dip** then **bump** in ω_4
 for CP at $\mu_B > 420$ MeV

Factorial cumulants from RHIC-BES-II

From M. Stephanov, SQM2024 & arXiv:2410.02861

$$\omega_n = \hat{C}_n / \hat{C}_1$$

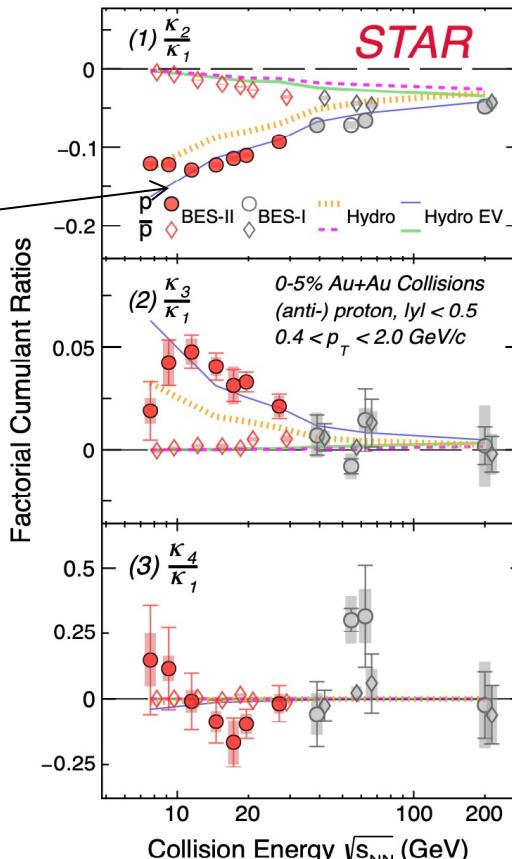
(universal EOS) critical χ_n :



Bzdak et al review 1906.00936

Expected signatures: **bump** in ω_2 and ω_3 , **dip** then **bump** in ω_4
 for CP at $\mu_B > 420$ MeV

STAR data:



baseline (hydro EV):

V.V. Koch, C. Shen, PRC 105, 014904 (2022)

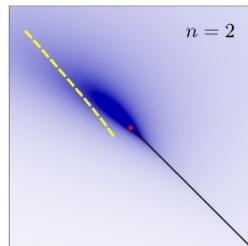
A. Pandav, CPOD2024

Factorial cumulants from RHIC-BES-II

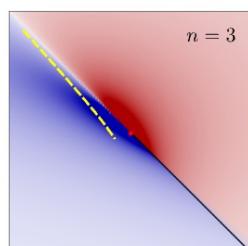
From M. Stephanov, SQM2024 & arXiv:2410.02861

$$\omega_n = \hat{C}_n / \hat{C}_1$$

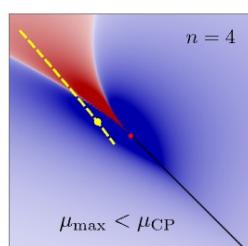
(universal EOS) critical χ_n :



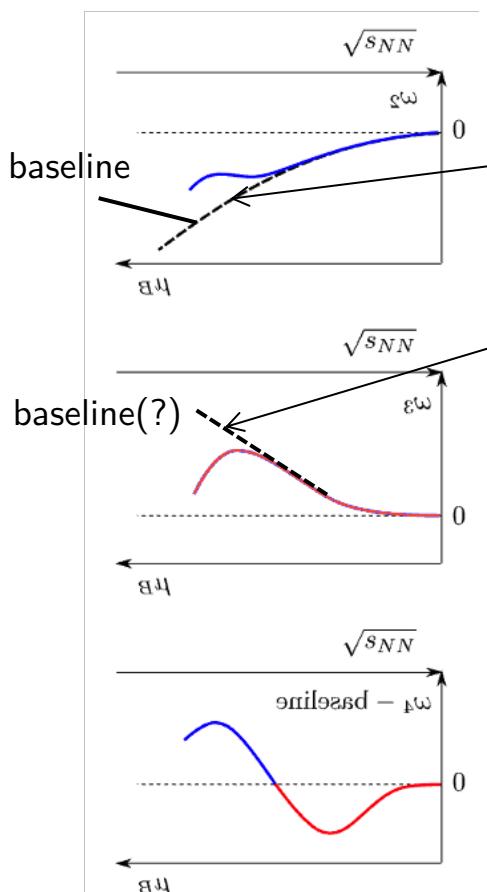
$n = 2$



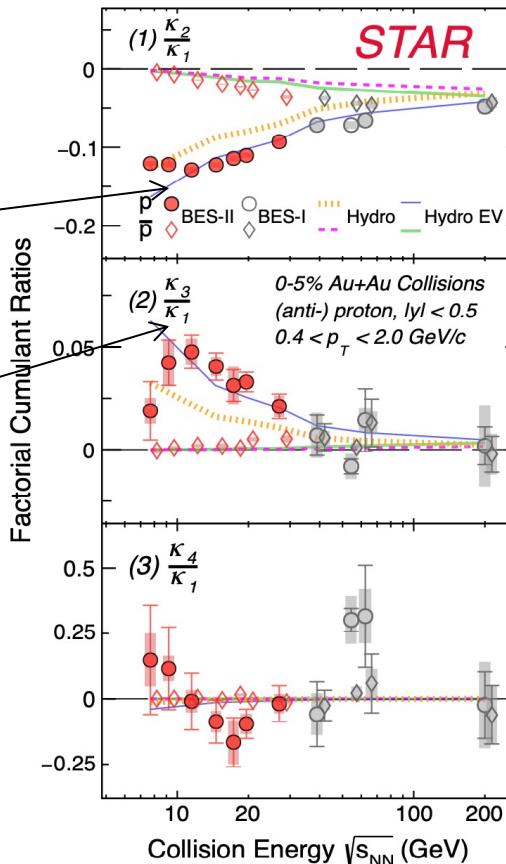
$n = 3$



$\mu_{\max} < \mu_{CP}$



STAR data:



A. Pandav, CPOD2024

baseline (hydro EV):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

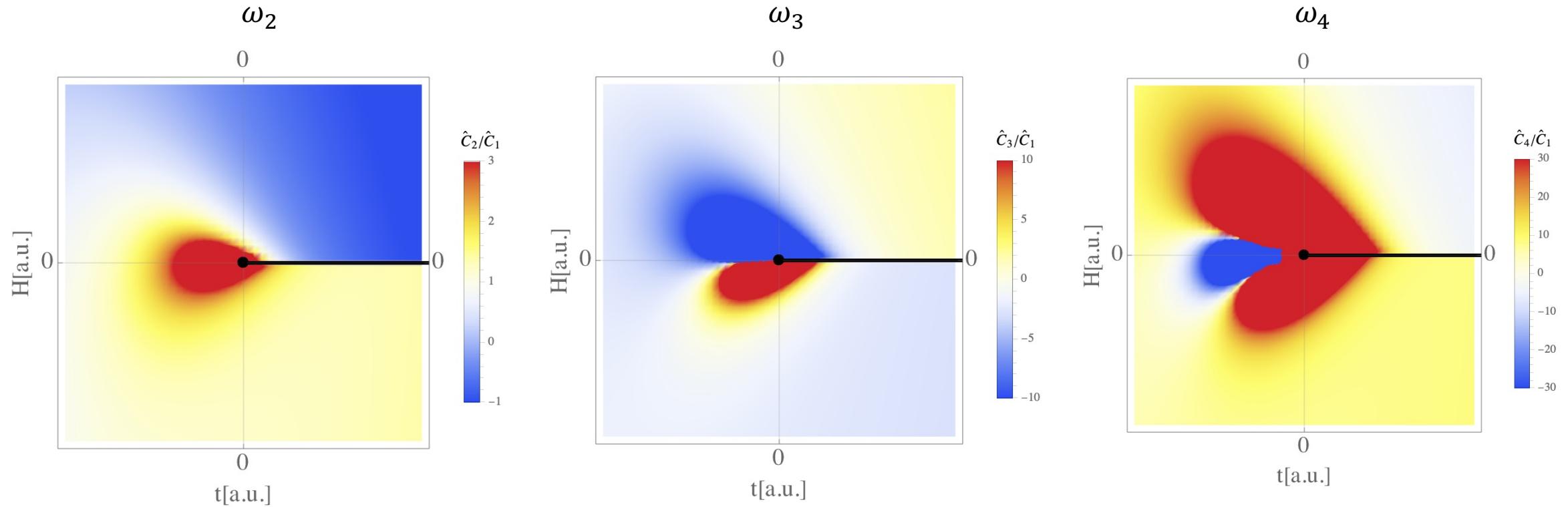
- describes right side of the peak in \hat{C}_3
- signal relative to baseline:**
 - positive* $\hat{C}_2 > 0$
 - negative* $\hat{C}_3 < 0$

Conclusion 2:

Controlling the non-critical baseline
is essential

Factorial cumulants from RHIC-BES-II and CP

Factorial cumulants in 3D-Ising model

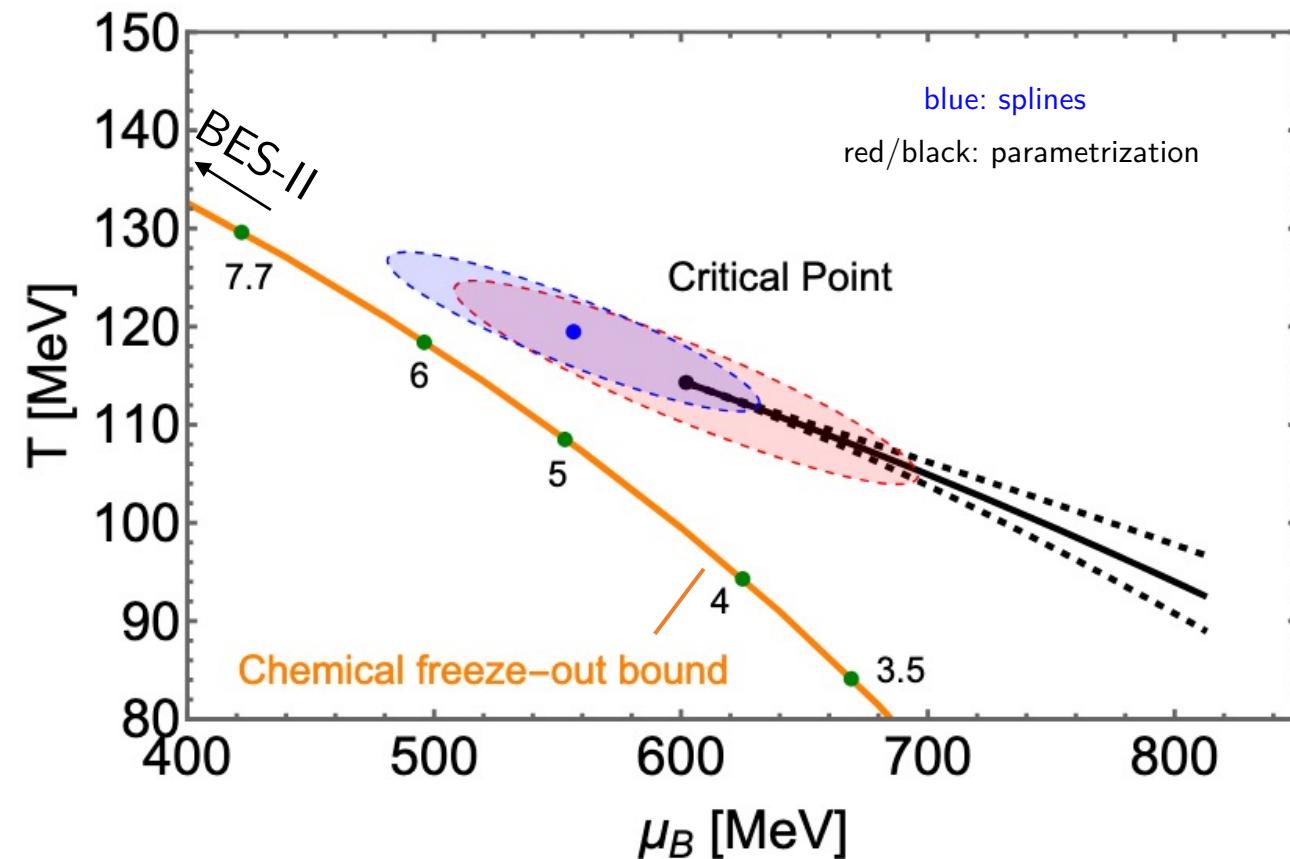


Adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)

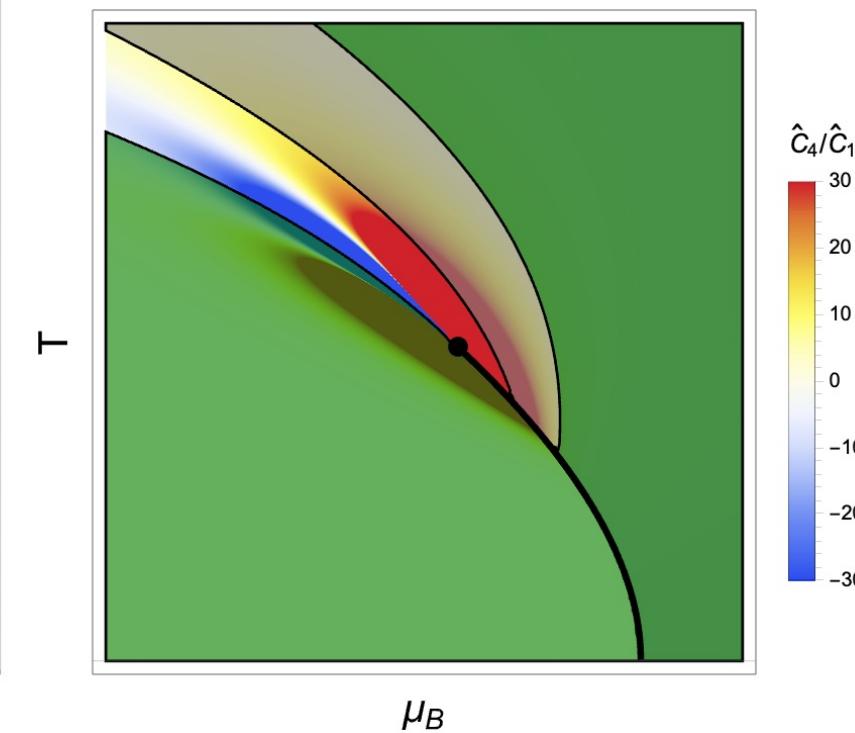
$$\omega_n = \hat{C}_n / \hat{C}_1$$

Factorial cumulants from RHIC-BES-II and CP

Exclusion plots



How it may look like in $T - \mu_B$ plane

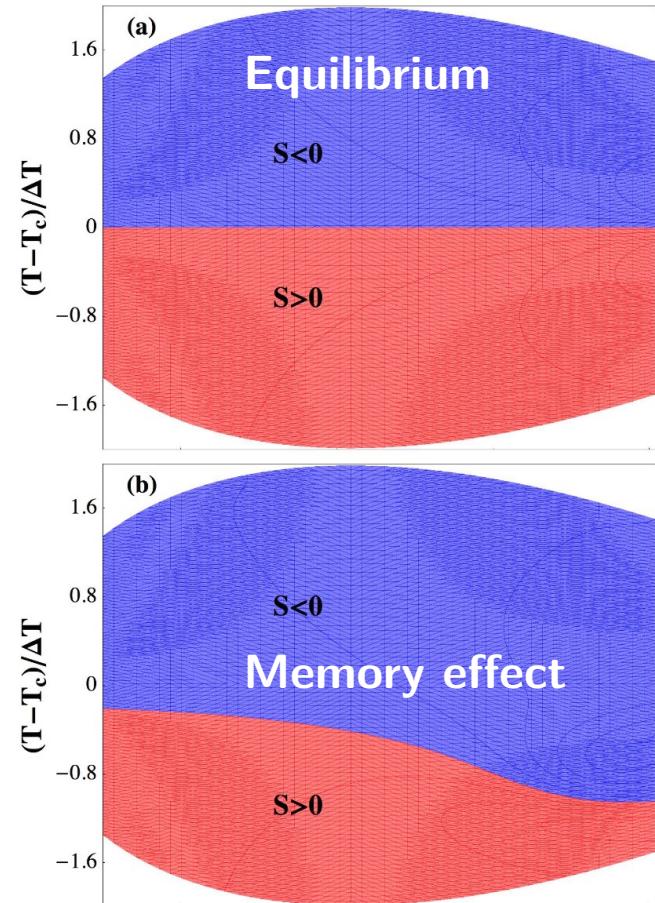


Based on QvdW model of nuclear matter

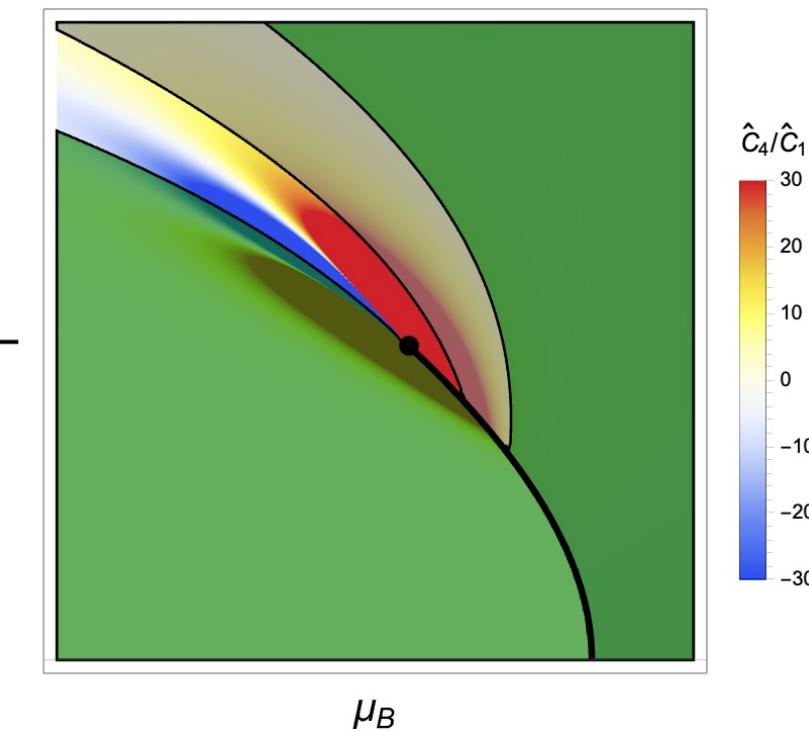
VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Freeze-out of fluctuations on the QGP side of the crossover?

Factorial cumulants from RHIC-BES-II and CP



How it may look like in $T - \mu_B$ plane

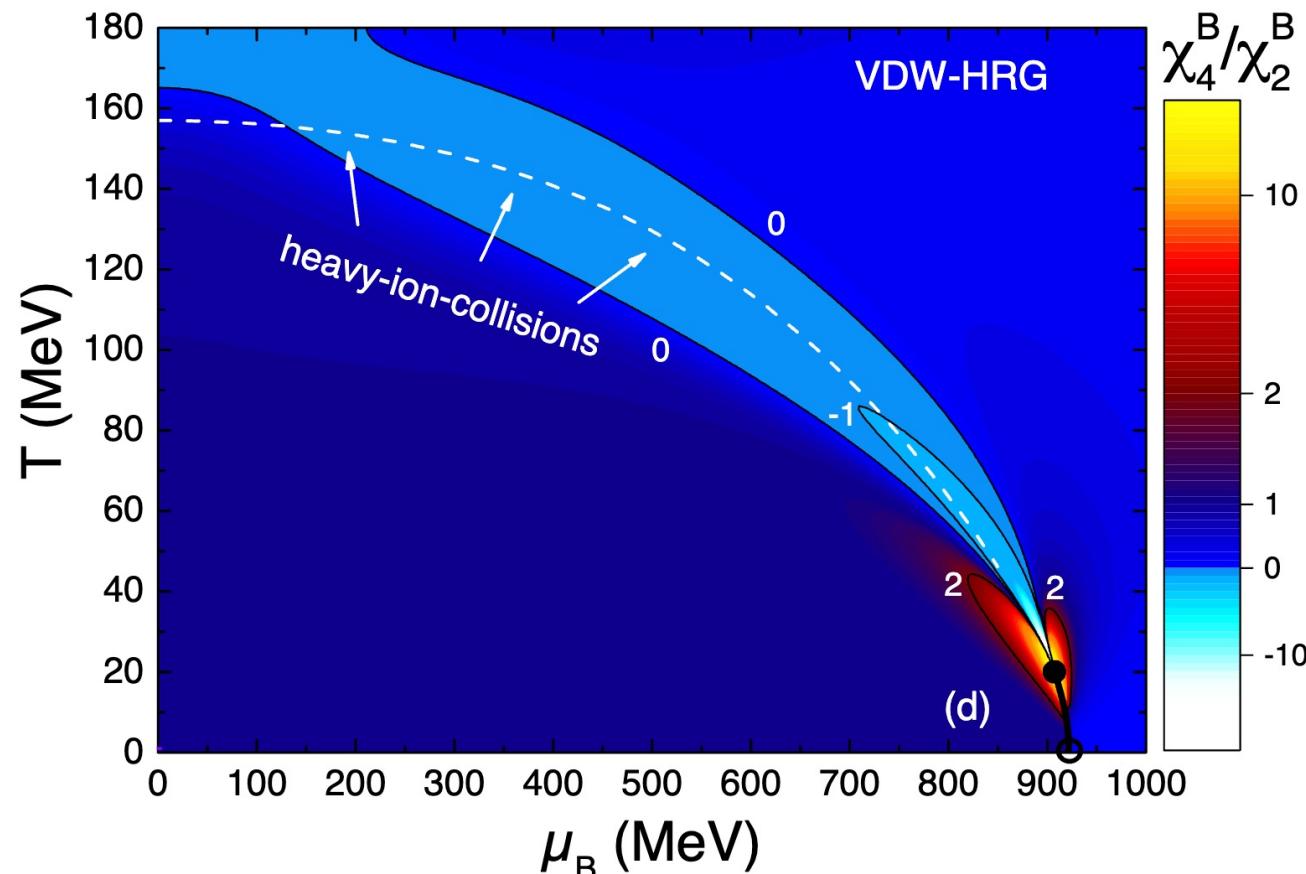


Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)

Freeze-out of fluctuations on the QGP side of the crossover?

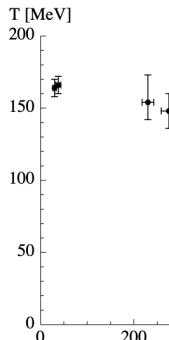
Interplay with nuclear liquid-gas transition

HRG with attractive and repulsive interactions among baryons

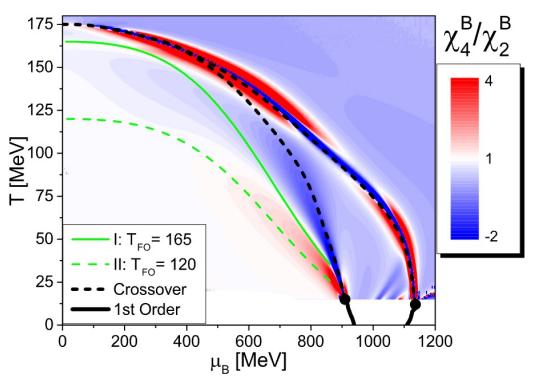


VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

Interplay with nuclear liquid-gas transition

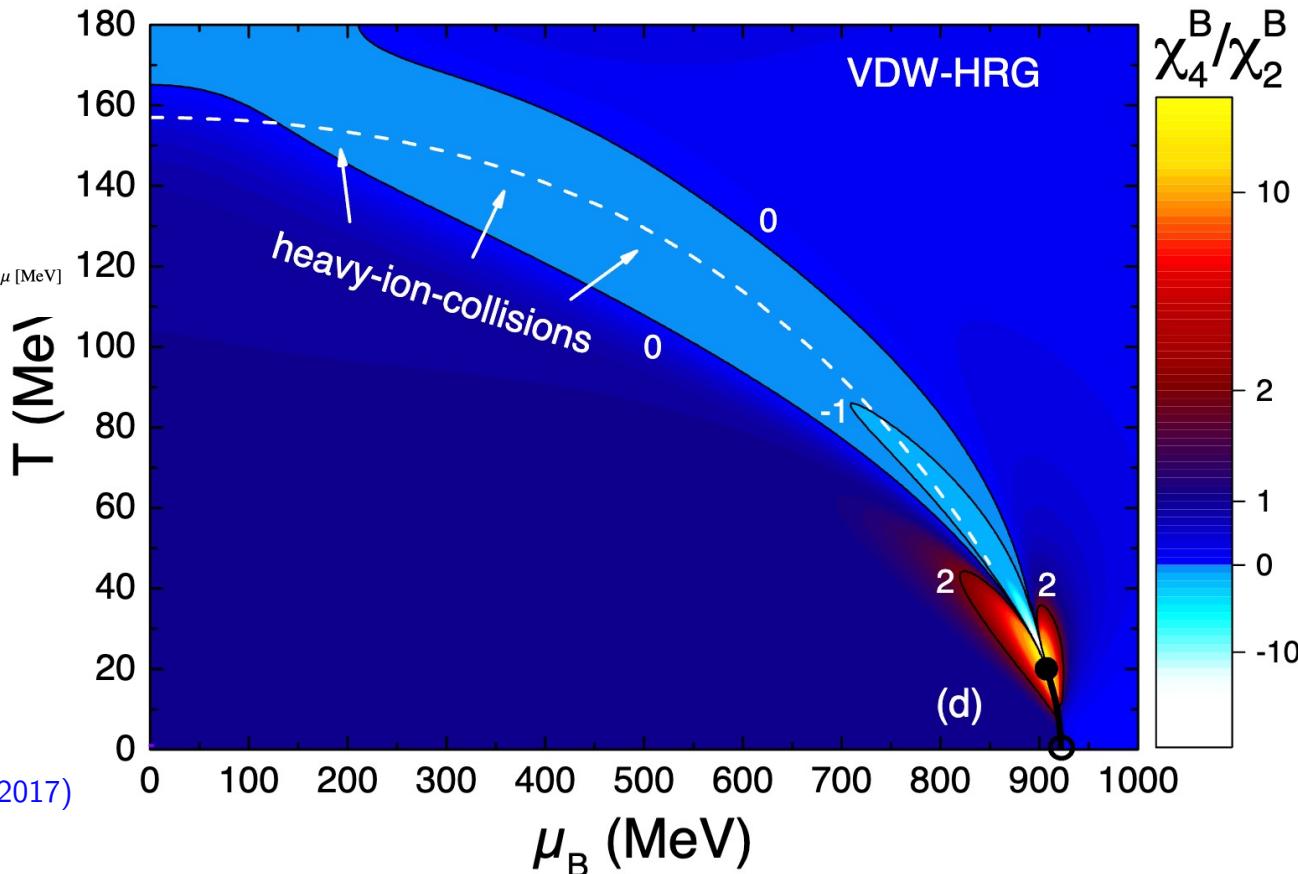


Floerchinger, Wetterich, NPA (2012)

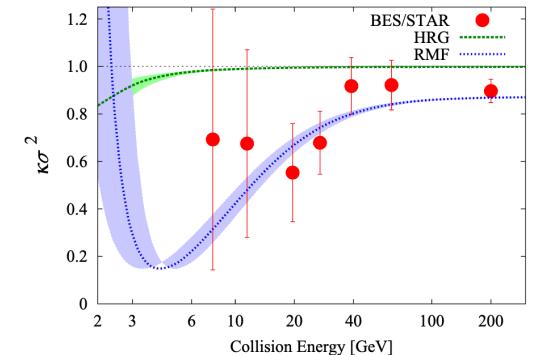


Mukherjee, Steinheimer, Schramm, PRC (2017)

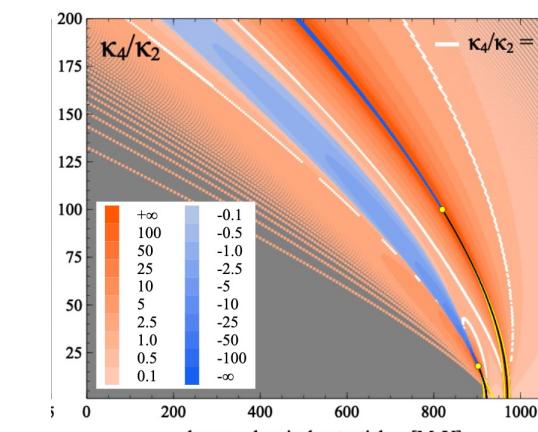
HRG with attractive and repulsive interactions among baryons



VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)



Fukushima, PRC (2014)



A phase diagram of κ_4 / κ_2 versus baryon chemical potential μ_B [MeV]. The x-axis ranges from 0 to 1000 MeV, and the y-axis ranges from 25 to 200. The plot shows regions with different values of κ_4 / κ_2 , indicated by a color bar on the left. A white dashed line represents the condition $\kappa_4 / \kappa_2 = 1$. The plot is color-coded by the ratio κ_4 / κ_2 , with values ranging from -100 to +100. A color bar on the right indicates the scale for κ_4 / κ_2 .

Sorensen, Koch, PRC (2020)

Scaled factorial cumulants, long-range correlations, and the antiproton puzzle

Acceptance dependence and long-range correlations

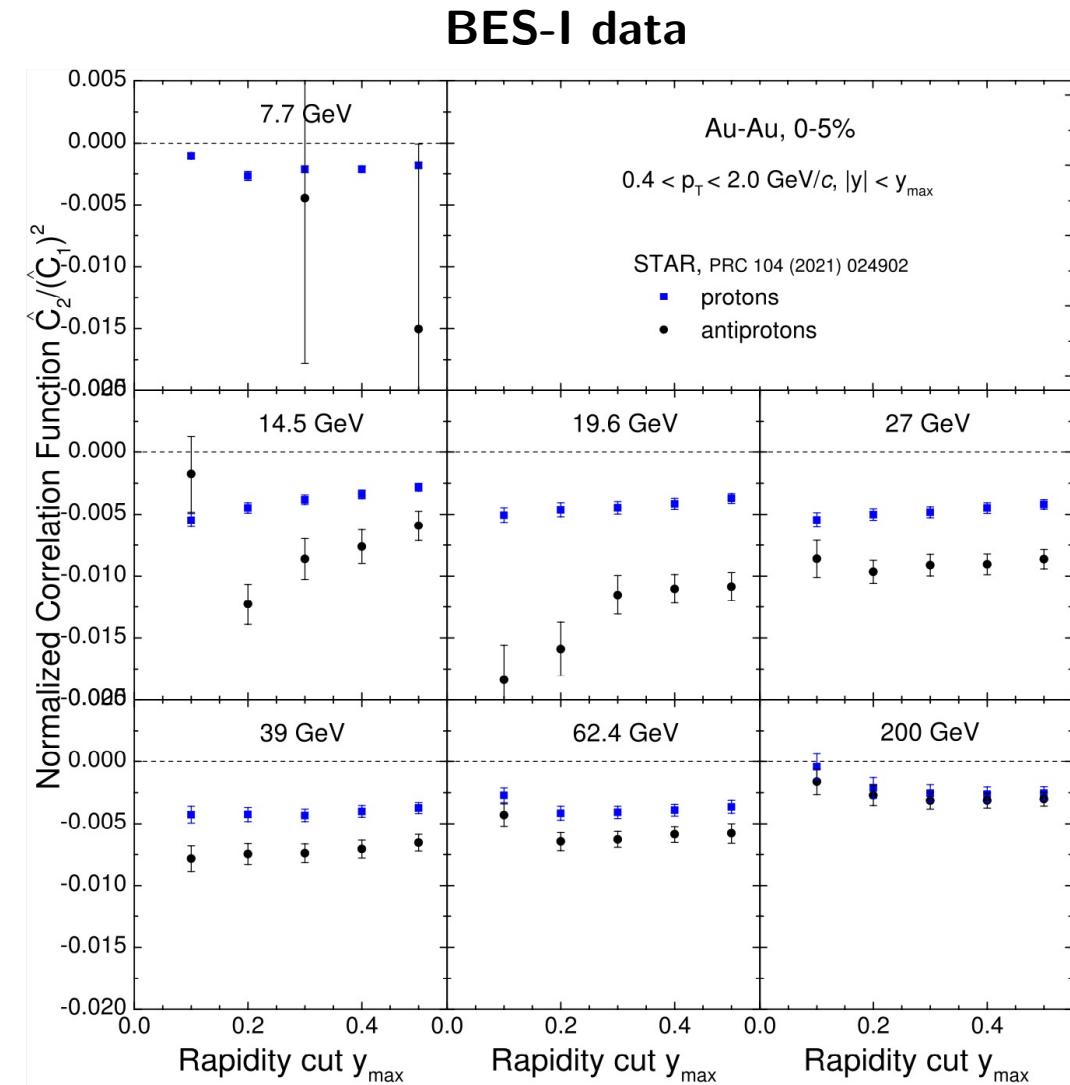
Long-range correlations: $\frac{\hat{C}_n}{(\hat{C}_1)^n} = \text{const.}$ at given $\sqrt{s_{NN}}$

- Global (not local) baryon conservation
[Bzdak, Koch, Skokov, EPJC 77, 288 (2017)]
- + volume fluctuations
[Holzmann, Koch, Rustamov, Stroth, arXiv:2403.03598]
- + (uniform) efficiency
[Pruneau, Gavin, Voloshin, PRC 66, 044904 (2002)]

In particular

$$\frac{\hat{C}_2^p}{(\hat{C}_1^p)^2} \approx \frac{\hat{C}_2^{\bar{p}}}{(\hat{C}_1^{\bar{p}})^2} = \text{const.}$$

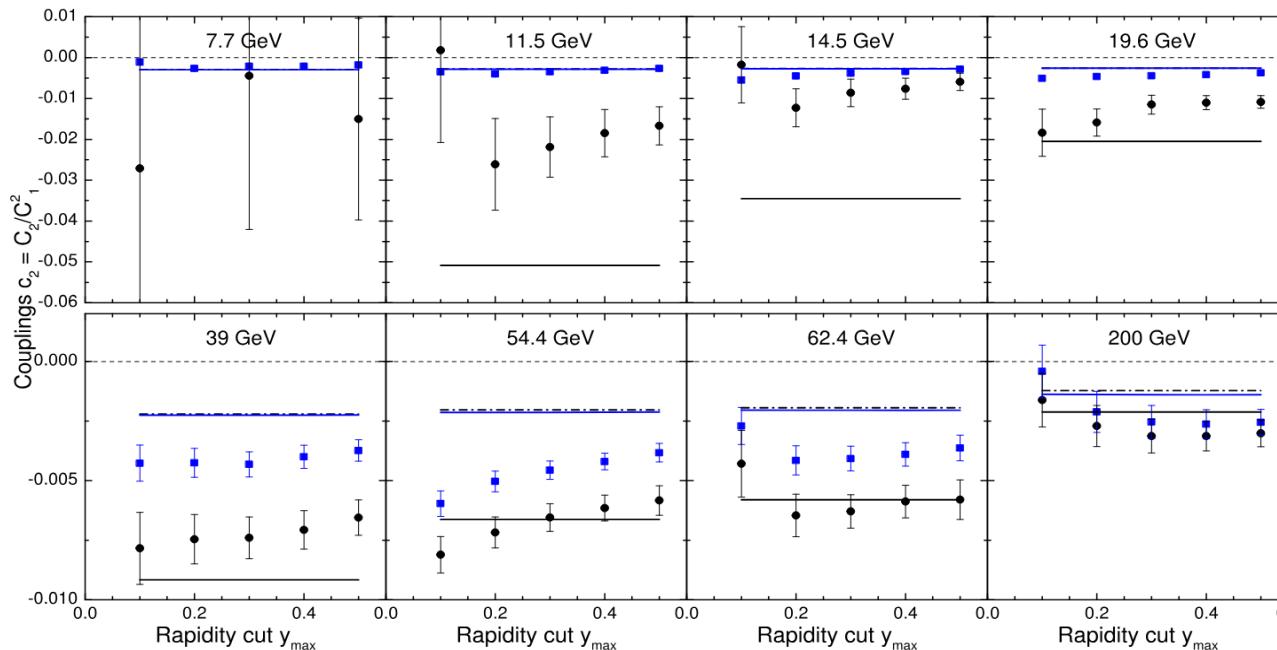
- Significant difference between p and \bar{p} in BES-I
 - **The antiproton puzzle?**
- With BES-II one can test the scaling with greater precision and extended coverage in rapidity
 - No need for volume fluctuations corrections (CBWC)



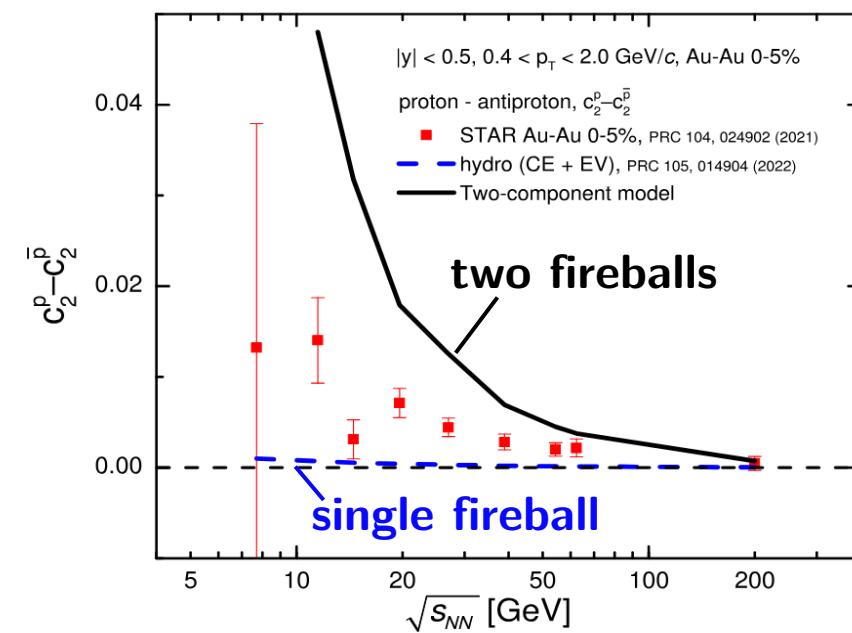
Factorial cumulants and two-component model

Two-component model: produced ($p\bar{p}$ pairs) and stopped protons come from two independent fireballs

Data lie in-between single and two-fireball models



Difference between p and \bar{p}



Opportunities for BES-II:

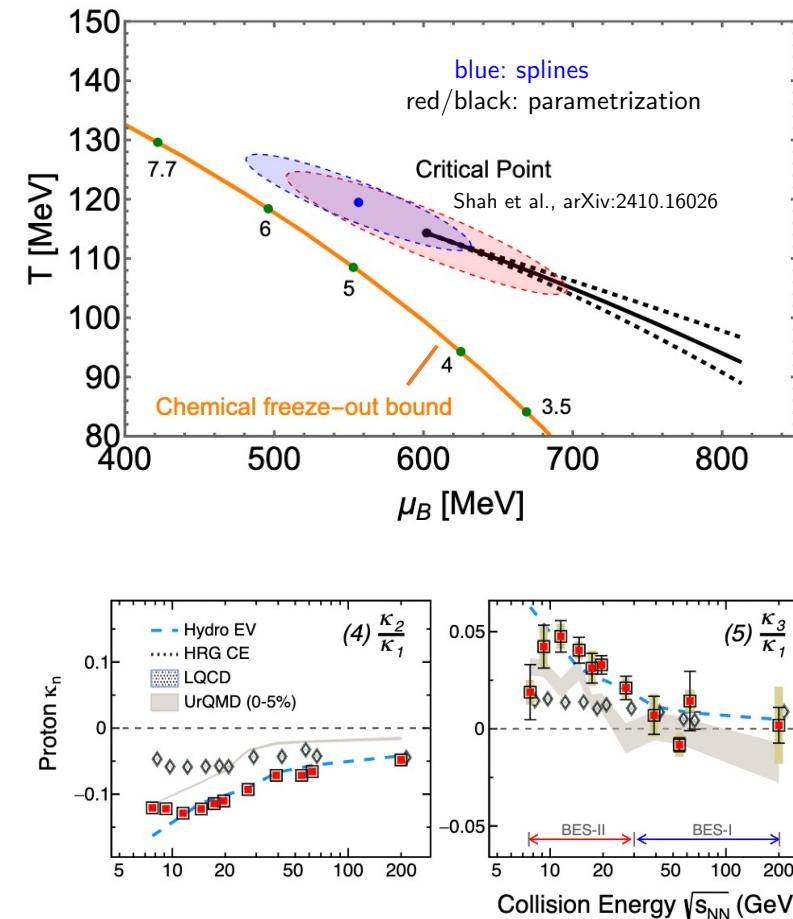
- Further tests of the splitting between protons and antiprotons in 2nd order cumulants with extended y coverage
- Critical point signal expected to break the scaling $\frac{\hat{C}_n}{(\hat{C}_1)^n} = \text{const.}$

Summary

- Locating the QCD critical point from first-principles
 - New method based on contours of constant entropy places QCD CP at $T_c = 114.3 \pm 6.9$ MeV, $\mu_B = 602.1 \pm 62.1$ MeV
- Proton cumulants are uniquely sensitive to the CP but challenging to model dynamically, factorial cumulants are advantageous
- BES-II data is in
 - Consistent with predictions from non-critical physics @ $\sqrt{s_{NN}} \geq 20$ GeV
 - Shows (non-monotonic) structure in factorial cumulants
 - Positive \hat{C}_2 and negative \hat{C}_3 after subtracting non-critical baseline at $\sqrt{s_{NN}} < 10$ GeV

Outlook:

- Improving first-principles constraints on CP location
- Improved description of non-critical baselines and quantitative predictions of critical fluctuations
- Acceptance dependence of factorial cumulants, understanding antiprotons



Thanks for your attention!