

QCD at finite temperature and density: Criticality

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Houston, TX, USA

September 7, 2023



What we know

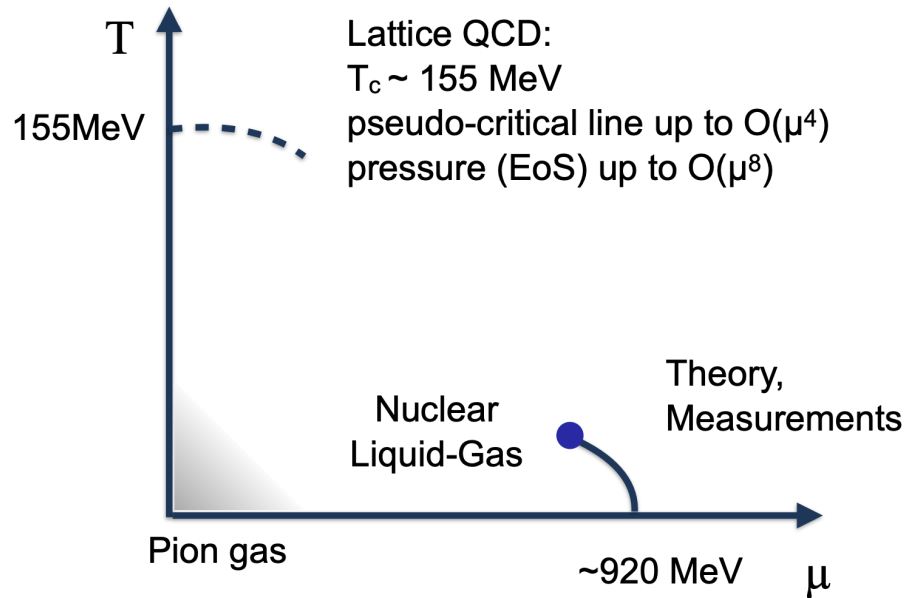


Figure courtesy of V. Koch

- Dilute hadron gas at low T & μ_B due to confinement, quark-gluon plasma high T & μ_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured
- Chiral crossover at $\mu_B = 0$

What we know

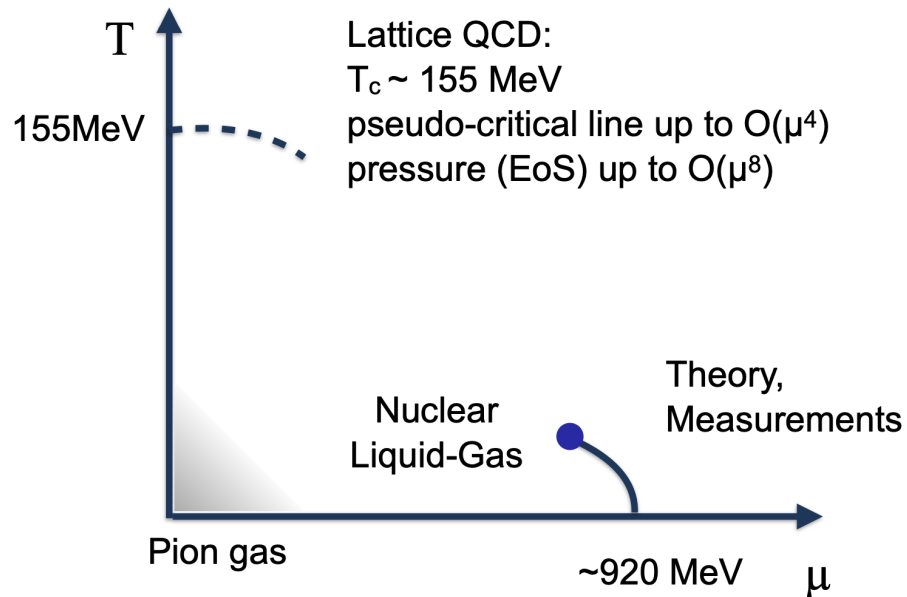


Figure courtesy of V. Koch

What we hope to know

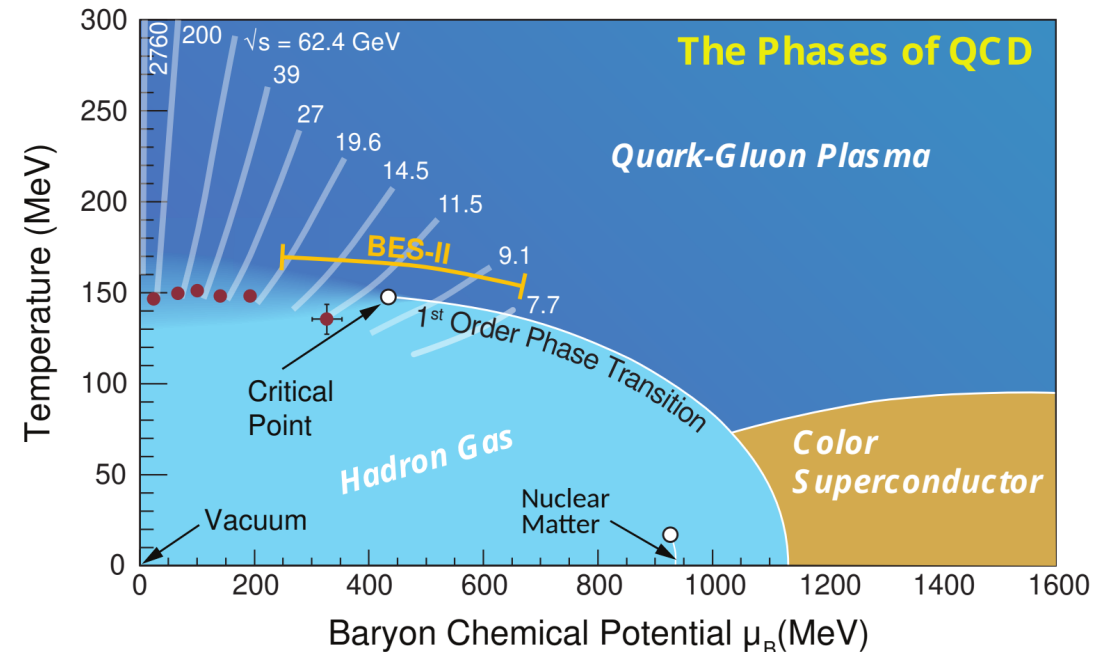


Figure from Bzdak et al., Phys. Rept. '20 & 2015 Long Range Plan

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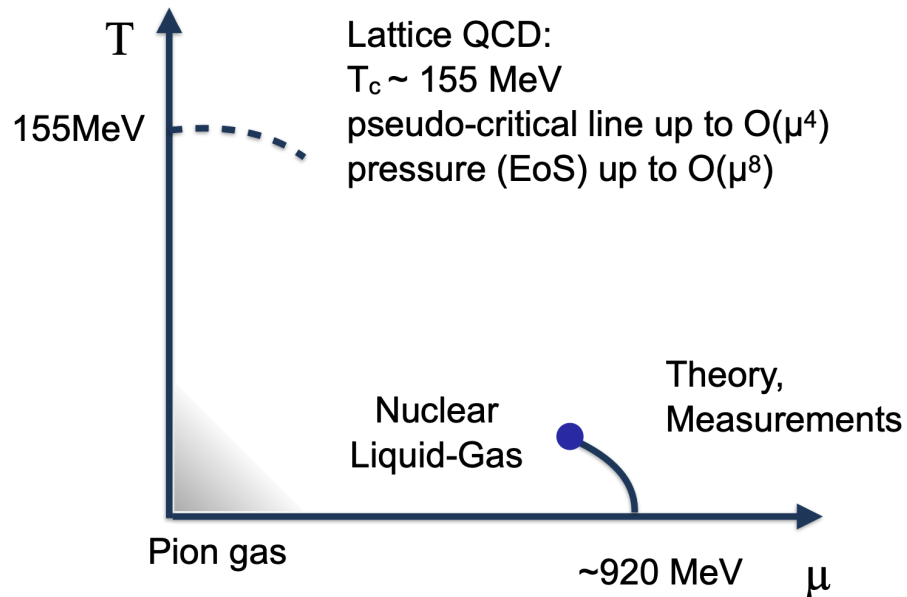


Figure courtesy of V. Koch

What we hope to know

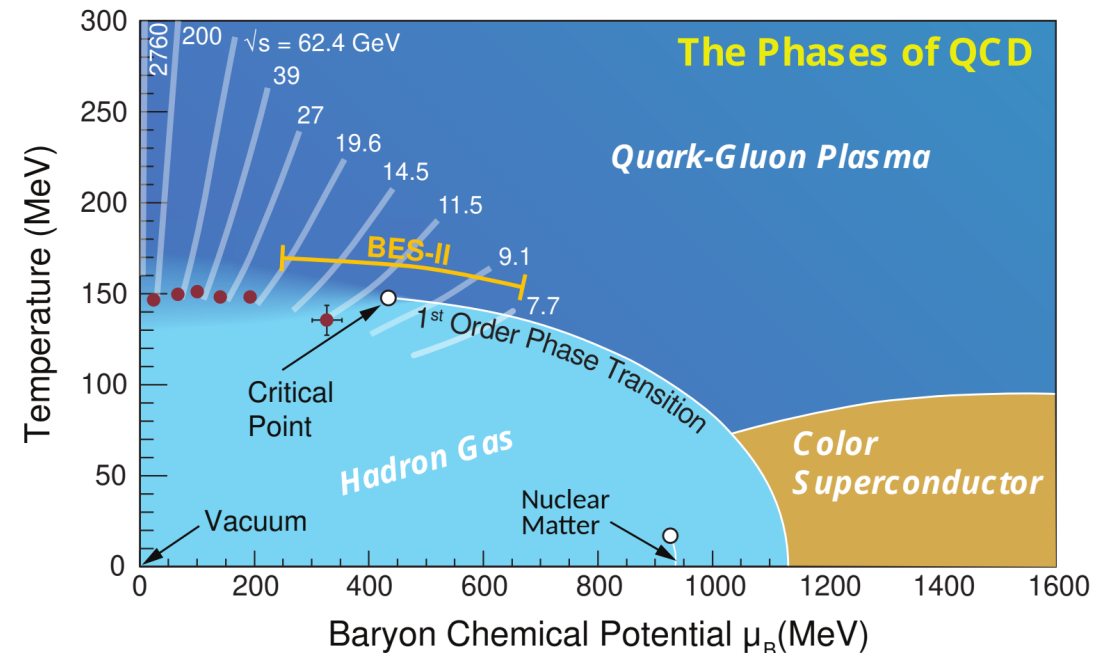


Figure from Bzdak et al., Phys. Rept. '20 & 2015 Long Range Plan

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Key question: *Is there a QCD critical point and how to find it?*

Where is the critical point? Ask AI

ChatGPT struggles



Is there a QCD critical point and where is it?



As of my knowledge cutoff in September 2021, the existence and precise location of a critical point in the phase diagram of Quantum Chromodynamics (QCD), the theory of the strong interaction between quarks and gluons, was not definitively established.

...

However, finding the QCD critical point using lattice QCD is a particularly challenging problem due to the so-called "sign problem." The sign problem arises when trying to extend lattice QCD calculations from zero to non-zero baryon chemical potential, which is necessary to locate the critical point on the phase diagram. At non-zero chemical potential, the action of the QCD path integral becomes complex, which makes the standard Monte Carlo techniques used in lattice QCD simulations ineffective.

...



Experimental searches for the QCD critical point in heavy-ion collisions generally rely on identifying fluctuations and correlations in the properties of the particles produced in the collisions, which are expected to exhibit unique features near the critical point.

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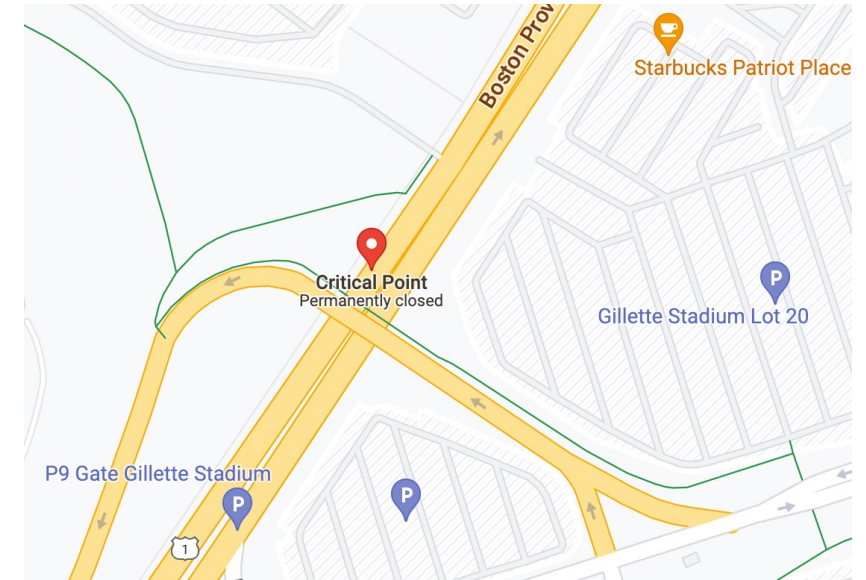
However, finding the QCD critical point using lattice QCD is a particularly challenging problem due to the so-called "sign problem." The sign problem arises when trying to extend lattice QCD calculations from zero to non-zero baryon chemical potential, which is necessary to locate the critical point on the phase diagram. At non-zero chemical potential, the action of the QCD path integral becomes complex, which makes the standard Monte Carlo techniques used in lattice QCD simulations ineffective.

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No issues for Google



Critical point predictions from theory as of previous QM

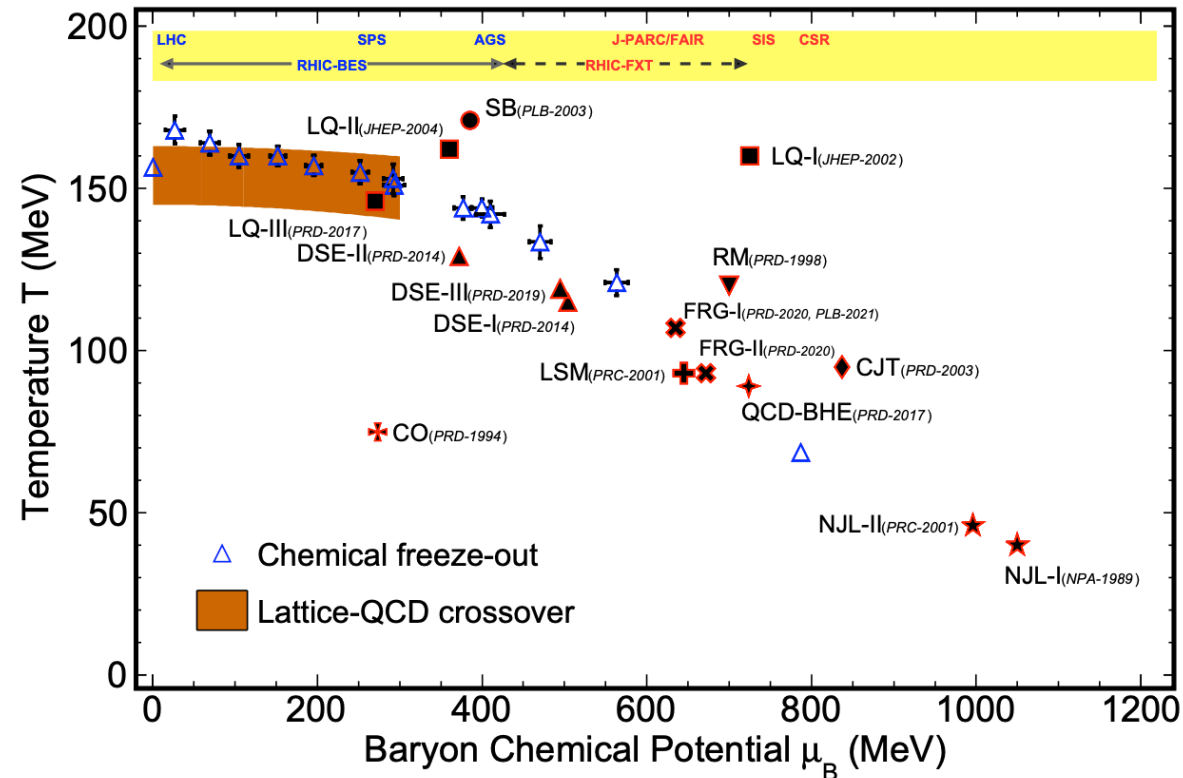


Figure adapted from A. Pandav, D. Mallick, B. Mohanty, Prog. Part. Nucl. Phys. 125 (2022)

Including the possibility that the QCD critical point does not exist at all

de Forcrand, Philipsen, JHEP 01, 077 (2007); VV, Steinheimer, Philipsen, Stoecker, PRD 97, 114030 (2018)

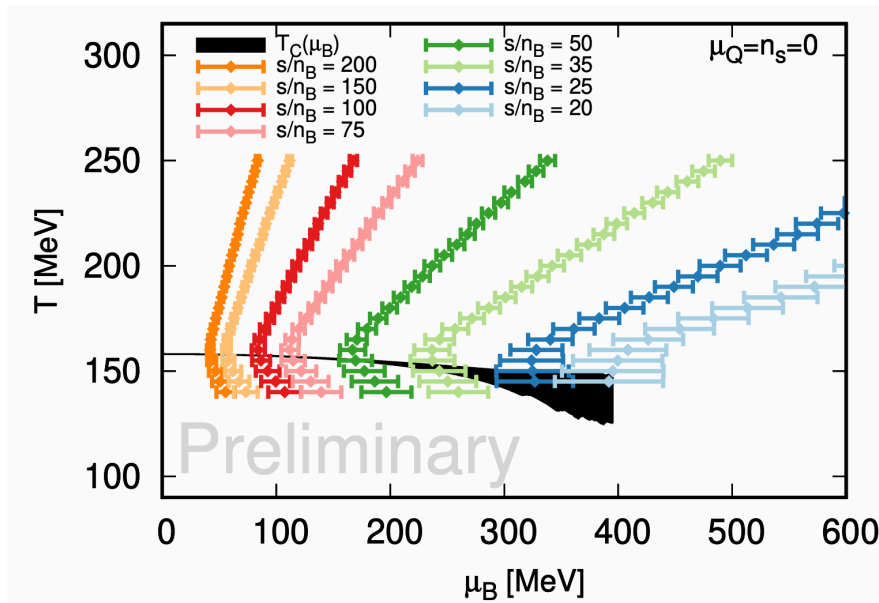
Extrapolations from $\mu_B = 0$

Ideally, find the critical point through first-principle lattice QCD simulations at finite μ_B

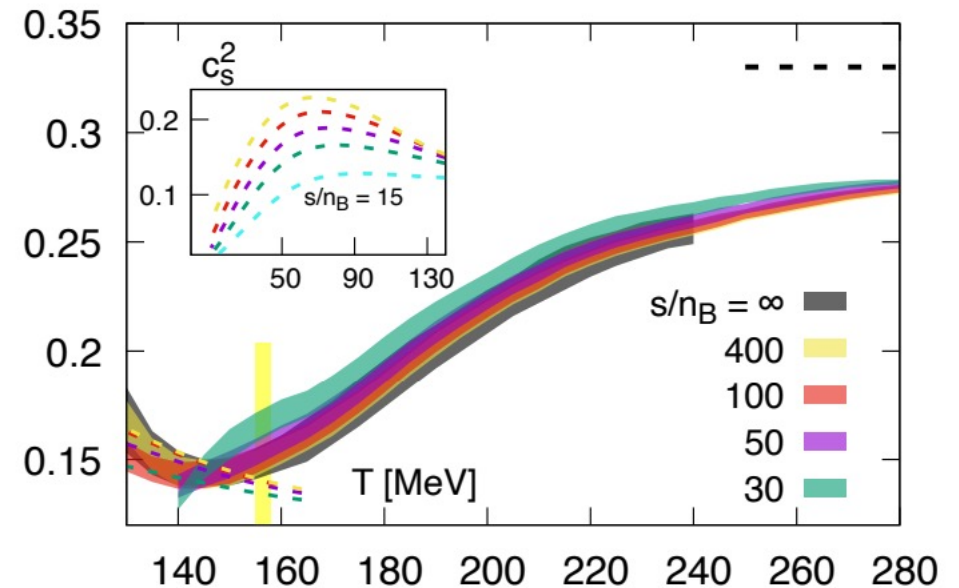
- Challenging (sign problem), but perhaps not impossible?

Talk by C.H. Wong, Tue 4:10 PM

Various resummations and extrapolation schemes from $\mu_B = 0$



Talk by P. Parotto, Tue 4:30 PM



Talk by D. Clarke, Wed 2:40 PM

No indications for the strengthening of the chiral crossover or critical point signals

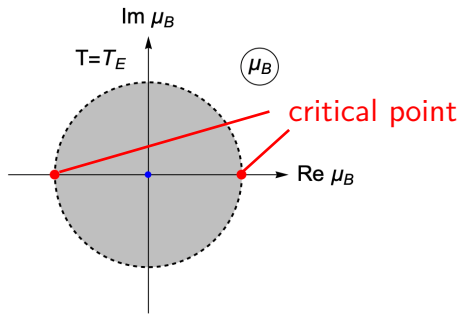
Disfavors QCD critical point at $\frac{\mu_B}{T} < 3$

Searching for singularities in the complex plane

Talks by G. Basar, Tue 4:30 PM & J. Goswami, Wed 3:20 PM

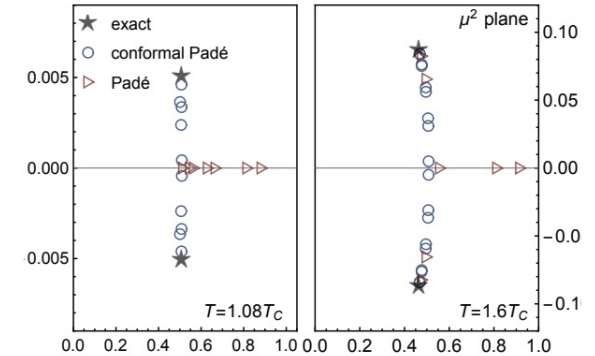
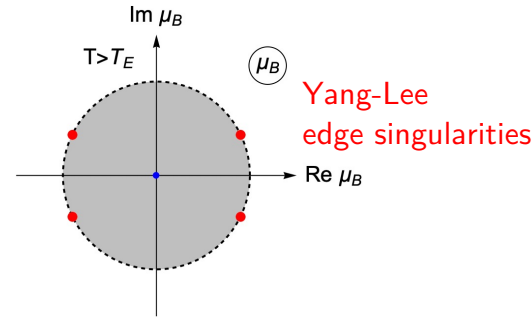
Critical point:

- singularity in the partition function
- real μ_B axis

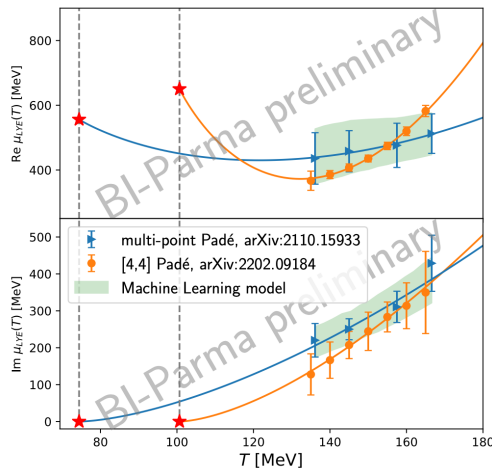


Above the critical temperature:

Yang-Lee edge singularities in the complex plane



- Extract YL edge singularity through (multi-point)/(conformal) Padé fits
- See if it approaches the real axis as temperatures decreases



Critical Point: Z(2) scaling inspired fit:

$$\text{Im } \mu_{LY} = c(T - T_{CEP})^\Delta$$

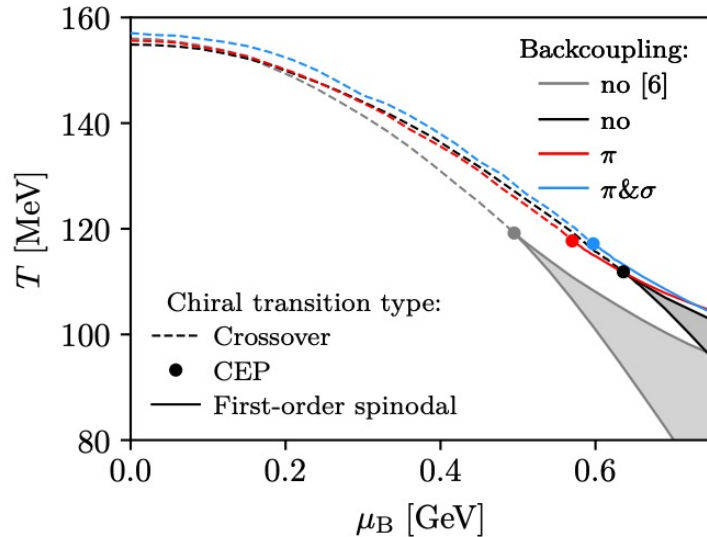
$$\text{Re } \mu_{LY} = \mu_{CEP} + a(T - T_{CEP}) + b(T - T_{CEP})^2$$



Extrapolated CP estimate:
 $T \sim 90-100$ MeV, $\mu_B \sim 500-600$ MeV

NB: many things have to go right, systematic error still very large (up to 100%)

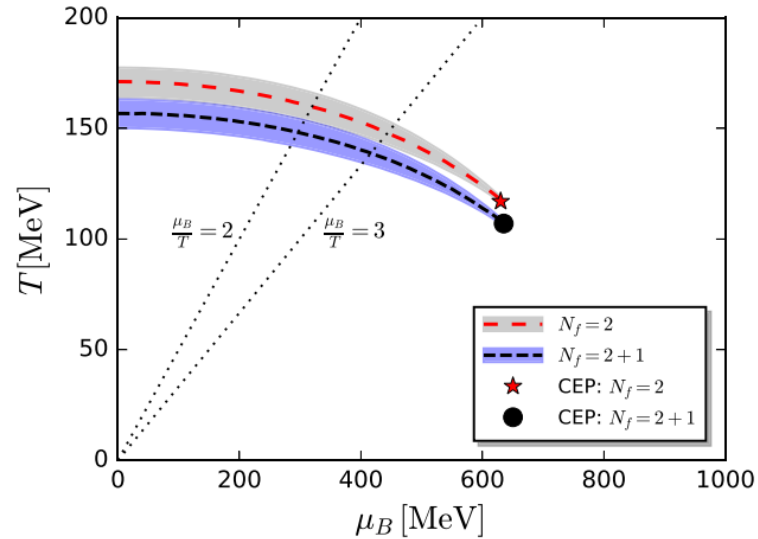
Dyson-Schwinger equations



Gunkel, Fischer, PRD 104, 054202 (2021)

$T \sim 120 \text{ MeV}$ $\mu_B \sim 600 \text{ MeV}$

Functional renormalization group

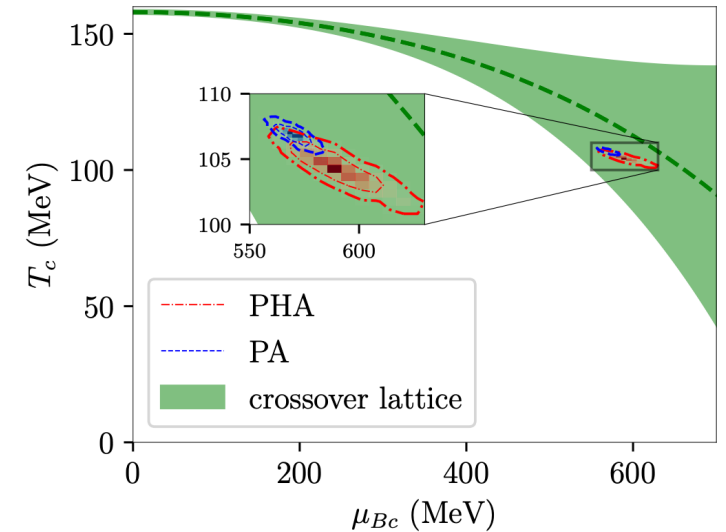


Fu, Pawłowski, Rennecke, PRD 101, 053032 (2020)

Talk by Wei-Jie Fu, Tue 9:30 AM

$T \sim 100 \text{ MeV}$ $\mu_B \sim 600 - 650 \text{ MeV}$

Black-hole engineering



Hippert et al., arXiv:2309.00579

Talk by M. Hippert, Tue 9:50 AM

$T \sim 105 \text{ MeV}$ $\mu_B \sim 580 \text{ MeV}$

All in excellent agreement with lattice QCD at $\mu_B = 0$
and predict QCD critical point in a similar ballpark of $\mu_B/T \sim 5-6$

If true, reachable in heavy-ion collisions at $\sqrt{s_{NN}} \sim 3 - 5 \text{ GeV}$

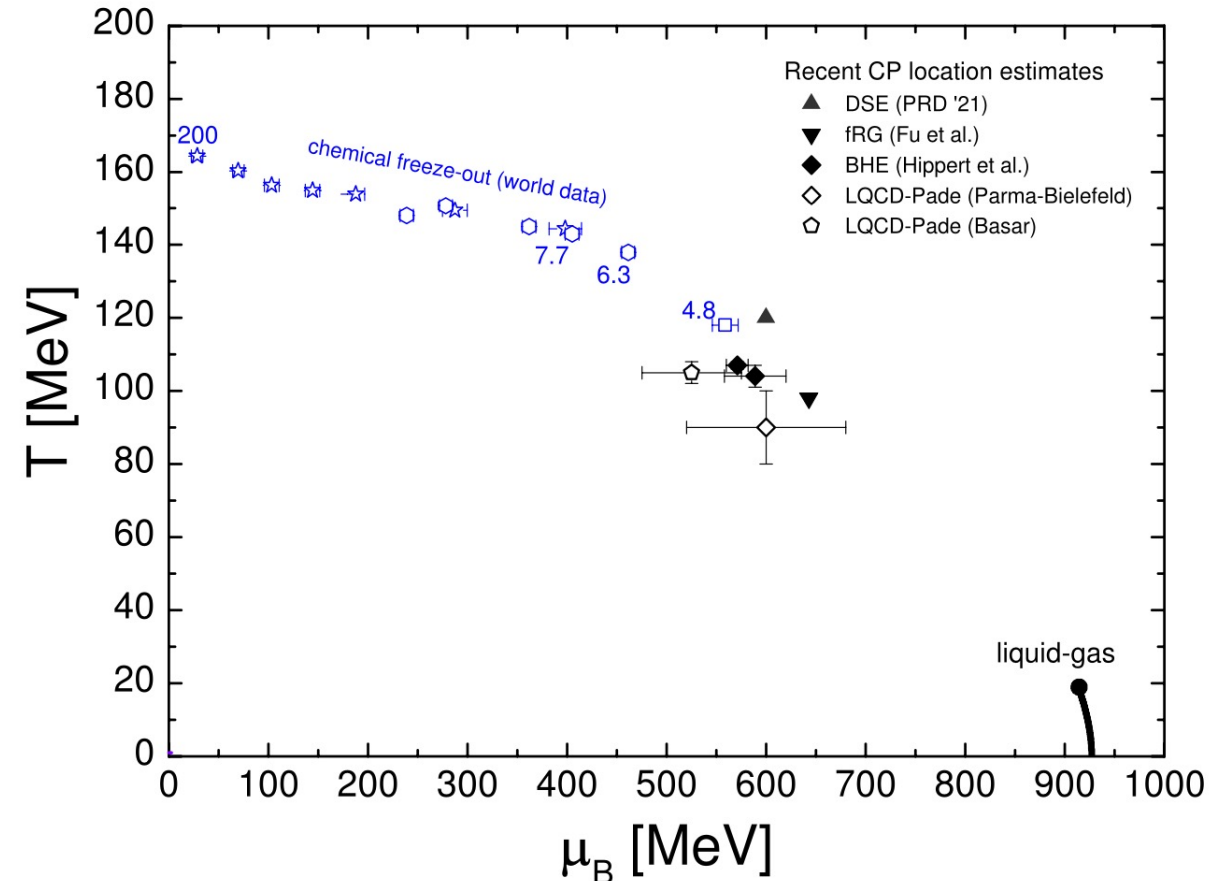
Search for critical point with heavy-ion collisions

Control parameters

- Collision energy $\sqrt{s_{NN}} = 2.4 - 5020$ GeV
 - Scan the QCD phase diagram
- Size of the collision region
 - Expect stronger signal in larger systems

Measurements

- Final hadron abundances and momentum distributions **event-by-event**



Cumulant generating function

$$K_N(t) = \ln \langle e^{tN} \rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

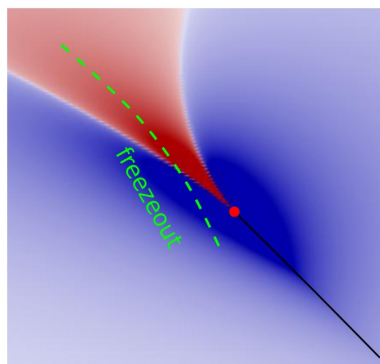
$$\kappa_n \propto \frac{\partial^n (\ln Z^{\text{gce}})}{\partial \mu^n}$$

Grand partition function

$$\ln Z^{\text{gce}}(T, V, \mu) = \ln \left[\sum_N e^{\mu N/T} Z^{\text{ce}}(T, V, N) \right]$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- **(QCD) critical point:** large correlation length and fluctuations



M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

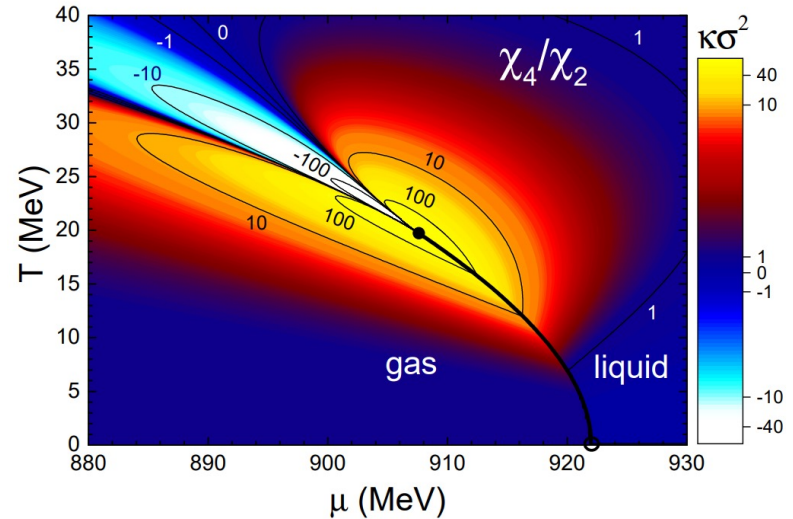
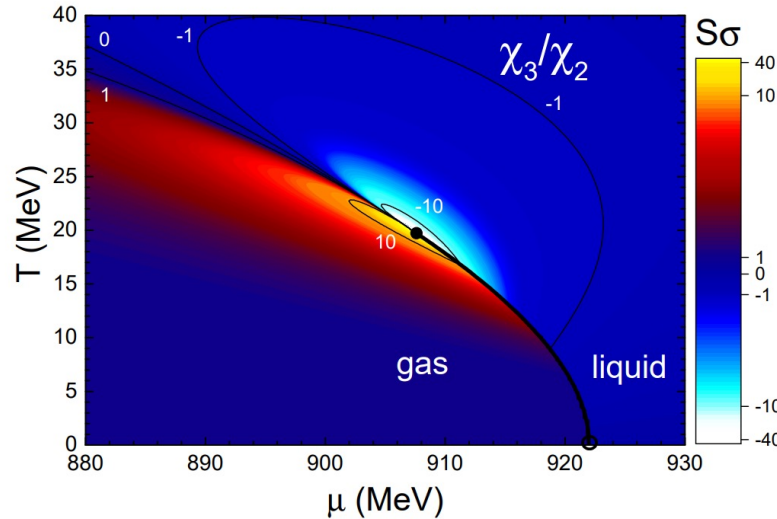
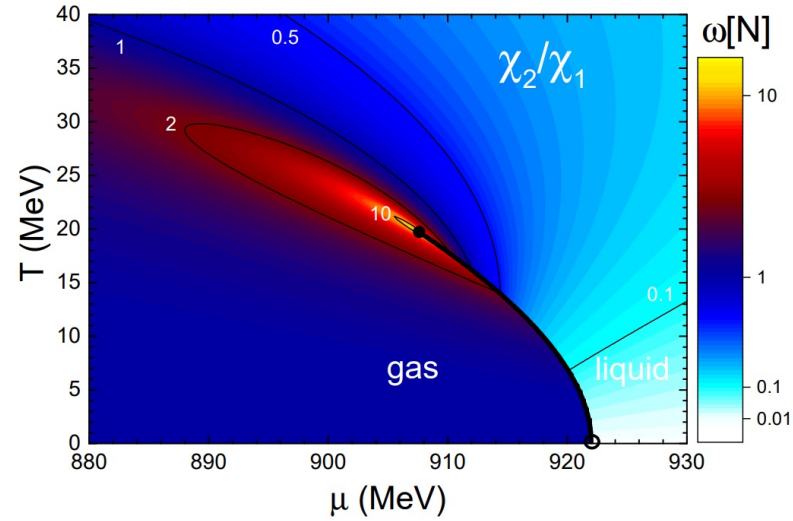
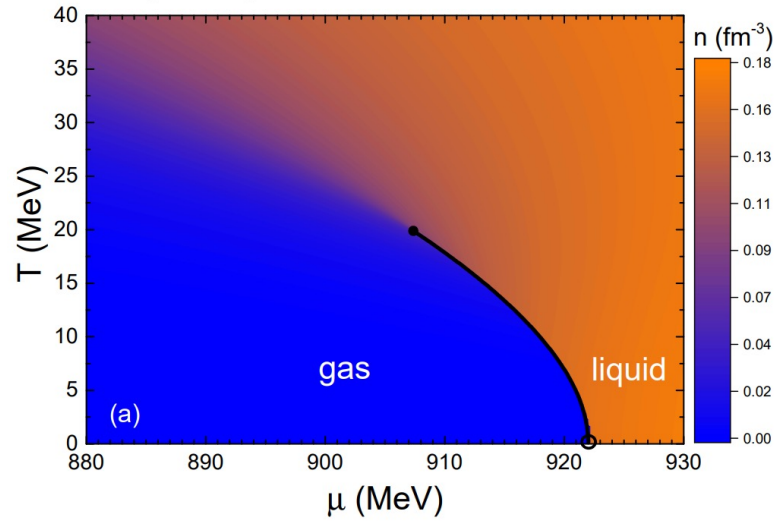
$$\xi \rightarrow \infty$$

Looking for enhanced fluctuations
and non-monotonocities

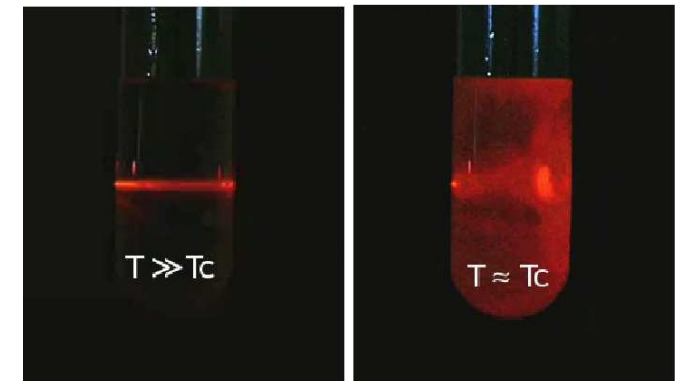
Other uses of cumulants:

- QCD degrees of freedom
[Jeon, Koch, PRL 85, 2076 \(2000\)](#)
[Asakawa, Heinz, Muller, PRL 85, 2072 \(2000\)](#)
- Extracting the speed of sound
[A. Sorensen et al., PRL 127, 042303 \(2021\)](#)
- Probing the magnetic field
Posters by [J. Jahan](#) and [I. Fokin](#)

Example: (Nuclear) Liquid-gas transition



Critical opalescence



$$\langle N^2 \rangle - \langle N \rangle^2 \sim \langle N \rangle \sim 10^{23}$$

in equilibrium

Example: Critical fluctuations in a microscopic simulation

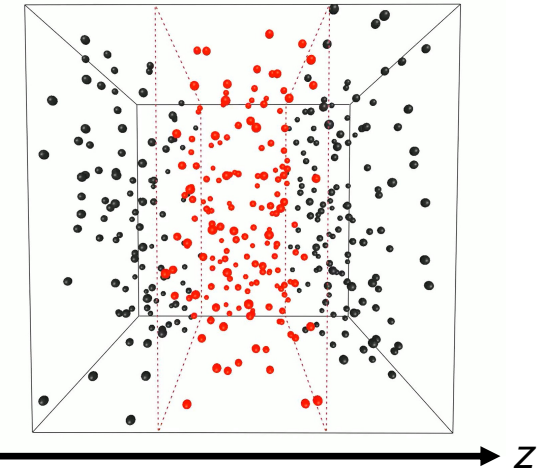
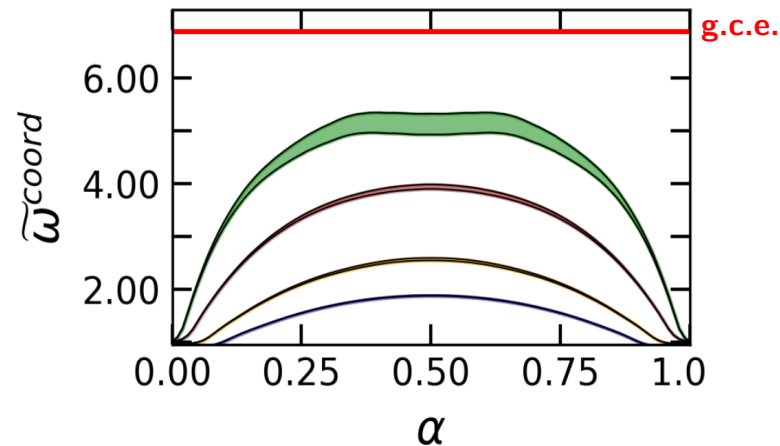


V. Kuznetsov et al., Phys. Rev. C 105, 044903 (2022); Poster by V. Kuznetsov

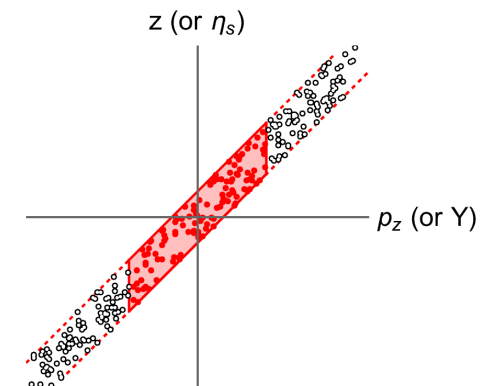
Classical molecular dynamics simulations of the **Lennard-Jones fluid** near Z(2) critical point ($T \approx 1.06T_c$, $n \approx n_c$) of the liquid-gas transition

Scaled variance in coordinate space acceptance $|z| < z^{max}$

$$\tilde{\omega}^{coord} = \frac{1}{1 - \alpha} \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$



Heavy-ion collisions:
flow correlates p_z and z cuts

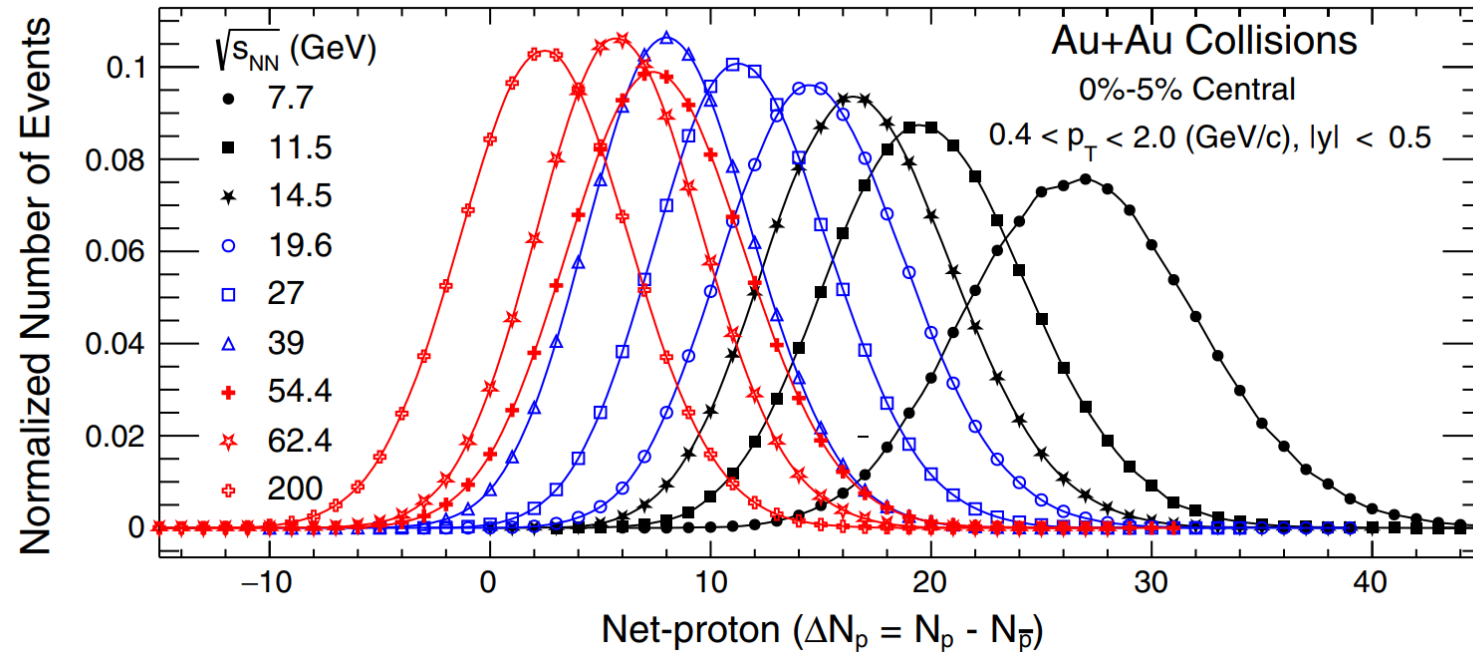


- Large fluctuations survive despite strong finite-size effects
- Need coordinate space cuts (collective flow helps)
- Here no finite-time effects

Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons $P(\Delta N_p) \sim \frac{N_{\text{events}}(\Delta N_p)}{N_{\text{events}}^{\text{total}}}$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)



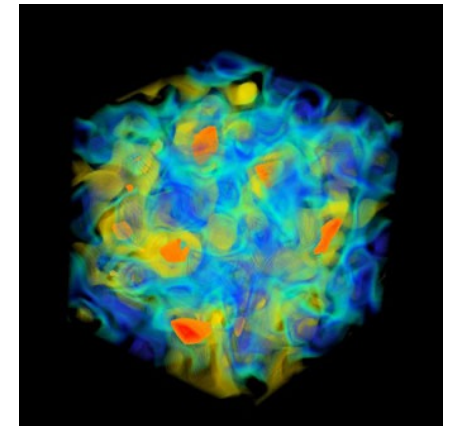
Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

$$\frac{\kappa_2}{\langle N \rangle}, \quad \frac{\kappa_3}{\kappa_2}, \quad \frac{\kappa_4}{\kappa_2}$$

Look for subtle critical point signals (tails of the distribution)

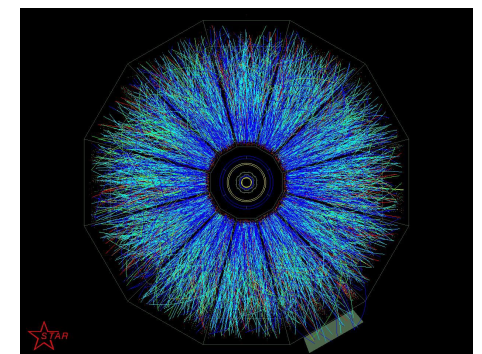
- grand-canonical heat bath vs expansion in vacuum
 - subensemble acceptance method (SAM) Talks by R. Poberezhnyuk, Tue 10:10 & W.J. Fu, Tue 9:30
VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020); JHEP 089(2020); PRC 105, 014903 (2022)
 - ideal gas limit (works also in small systems)
Bzdak, Koch, Skokov, PRC 87, 014901 (2013); Braun-Munzinger et al., NPA 1008, 122141 (2021)
- coordinate vs momentum space
Ling, Stephanov, PRC 93, 034915 (2016); Ohnishi, Kitazawa, Asakawa, PRC 94, 044905 (2016)
- QCD conserved charges (net-baryon) vs proxies (net-proton)
Kitazawa, Asakawa, PRC 85, 021901 (2012); VV, Koch, PRC 103, 044903 (2021)
- volume, initial state, and baryon stopping: fixed vs fluctuating
Gorenstein, Gazdzicki, PRC 84, 014904 (2011); Skokov, et al., PRC 88, 034911 (2013)
Talk by A. Bzdak, Wed 14:20; Poster by A. Rustamov
- hadronic phase
Steinheimer, VV, Aichelin, Bleicher, Stoecker, PLB 776, 32 (2018)
Savchuk, VV, Koch, Steinheimer, Stoecker, PLB 827, 136983 (2022)
- non-equilibrium (memory) effects
Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015); Asakawa, Kitazawa, Müller, PRC 101, 034913 (2020)
Talks by X. An, Tue 12:40 & M. Pradeep, Tue 8:50

Theory



© Lattice QCD@BNL

Experiment



STAR event display

1. Dynamical model calculations of critical fluctuations [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]

- Fluctuating hydrodynamics (hydro+) and (non-equilibrium) evolution of fluctuations
- Equation of state with a tunable critical point [P. Parotto et al, PRC 101, 034901 (2020); J. Karthein et al., EPJ Plus 136, 621 (2021)]
- Generalized Cooper-Frye particlization [M. Pradeep, et al., PRD 106, 036017 (2022); PRL 130, 162301 (2023)]

Alternatives at high μ_B : hadronic transport/molecular dynamics with a critical point

[A. Sorensen, V. Koch, PRC 104, 034904 (2021); V. Kuznetsov et al., PRC 105, 044903 (2022)]

2. Deviations from precision calculations of non-critical fluctuations

- Non-critical baseline is not flat [Braun-Munzinger, Rustamov, Stachel, NPA 1008, 122141 (2021)]
- Include essential non-critical contributions to (net-)proton number cumulants
- Exact **baryon conservation** + **hadronic interactions** (hard core repulsion)
- Based on realistic hydrodynamic simulations tuned to bulk data

[VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

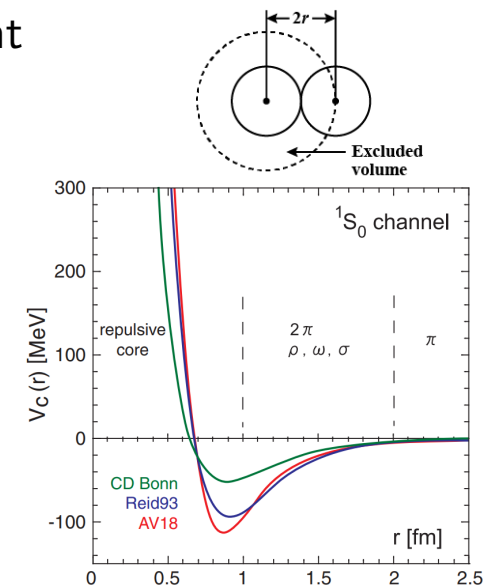
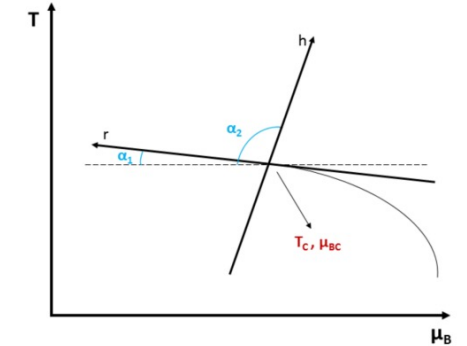
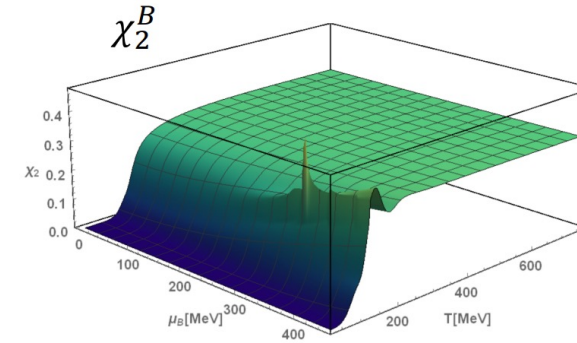


Figure from Ishii et al., PRL '07

Equation of state with a tunable critical point

BEST equation of state: [P. Parotto et al, PRC 101, 034901 \(2020\)](#)

- 3D-Ising CP mapped onto the QCD
- Tunable CP location along the pseudocritical line
- Matched to lattice data at $\mu_B = 0$

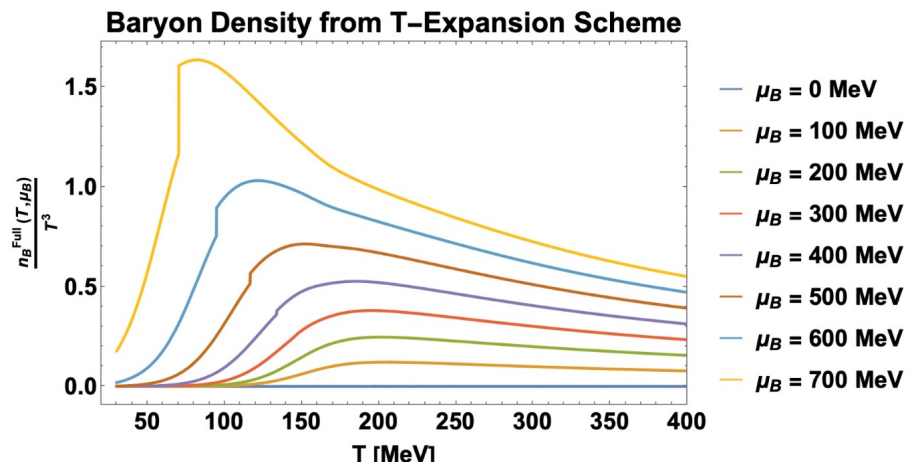


New development: [Talk by M. Kahangirwe](#)

Match to **alternative expansion scheme** from lattice QCD instead of Taylor expansion, extending the range to whole BES range

$$p(T, \mu_B) = p^{\text{non-Ising}}(T, \mu_B) + p^{\text{Ising}}(T, \mu_B)$$

regular
critical



Alternative ways to embed the critical point:

[\[J. Kapusta, T. Welle, C. Plumberg, PRC 106, 014909 \(2022\); PRC 106, 044901 \(2022\)\]](#)

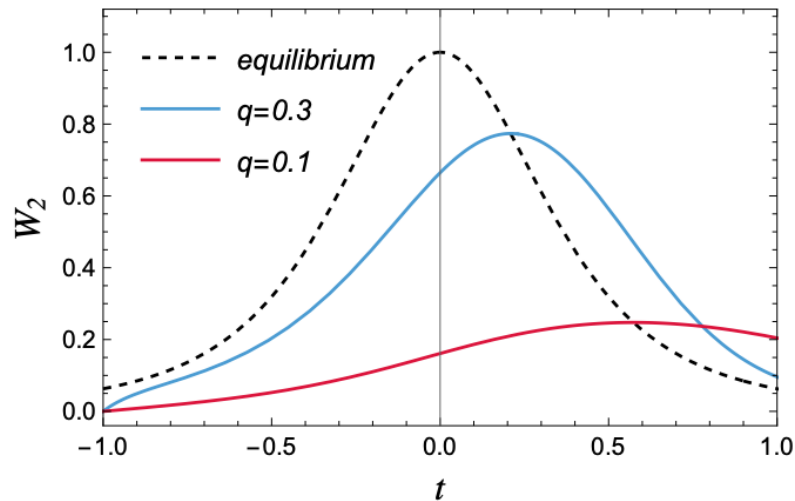
Equilibrium expectations for fluctuations:

[Talks by M. Pradeep, Tue 8:50 & J.M. Karthein, Thu 12:00](#)

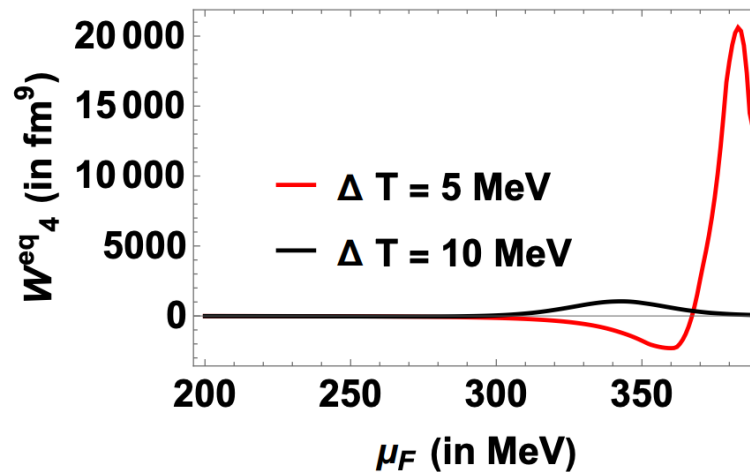
Non-equilibrium evolution and critical slowing down



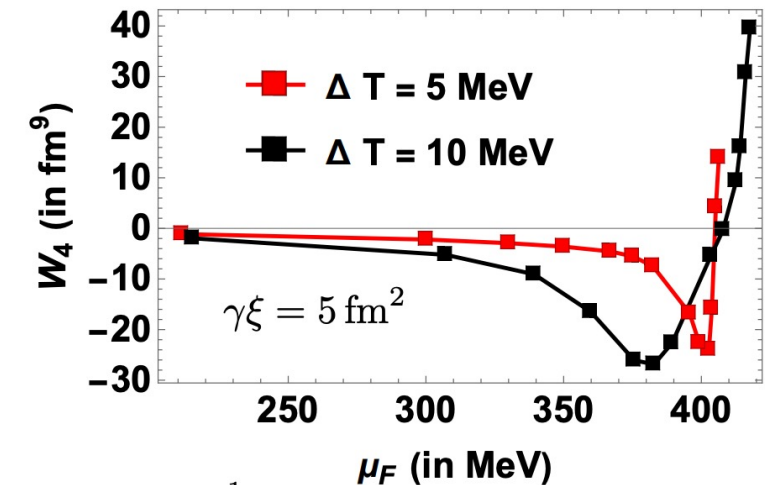
- Non-equilibrium evolution of (non-)Gaussian fluctuations
 - Strong suppression of critical point signals due to critical slowing down and (local) conservation



Talk by X. An, Tue 12:40



Talk by M. Pradeep, Tue 8:50



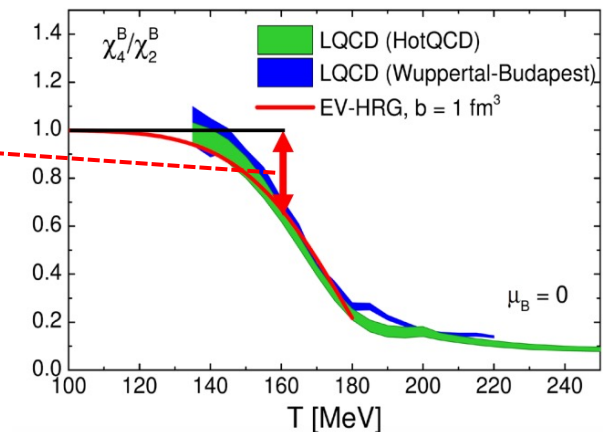
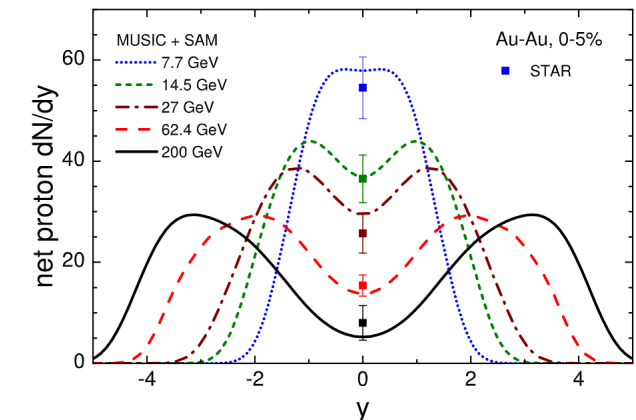
- Generalized Cooper-Frye particlization: maximum entropy freeze-out of fluctuations
[M. Pradeep, M. Stephanov, PRL 130, 162301 (2023)]
- Diffusion and cross-correlations of multiple conserved charges and energy-momentum

Poster by O. Savchuk

Calculation of non-critical contributions

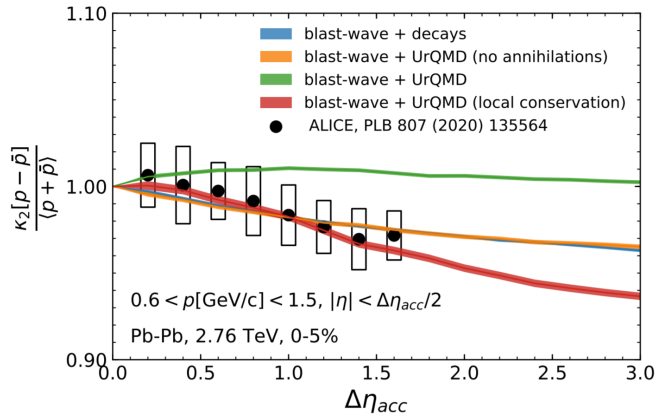
VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
 - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
 - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]
 - Cooper-Frye particlization at $\epsilon_{sw} = 0.26 \text{ GeV}/\text{fm}^3$
- Non-critical contributions are computed at particlization
 - QCD-like baryon number distribution via excluded volume $b = 1 \text{ fm}^3$ [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
 - Exact global baryon conservation* (and other charges)
 - Subensemble acceptance method 2.0 (analytic) [VV, Phys. Rev. C 105, 014903 (2022)]
 - or FIST sampler (Monte Carlo) [VV, Phys. Rev. C 106, 064906 (2022)]
<https://github.com/vlvovch/fist-sampler>
- **Absent:** critical point, local conservation, initial-state/volume fluctuations



*If baryon conservation is the only effect (no other correlations), non-critical baseline can be computed without hydro

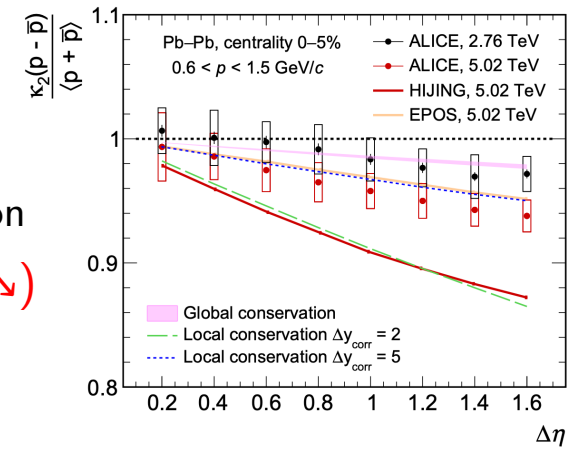
Proton cumulants at high energy



$$\kappa_2[p - \bar{p}] / \langle p + \bar{p} \rangle:$$

- Largely understood as (global) baryon conservation
 - Larger suppression at 5 TeV contrary to naïve expectation
- Interplay: **baryon annihilation**(↗) vs **local conservation**(↘)
 - Additional measurement of $\kappa_2[p + \bar{p}]$ can resolve it

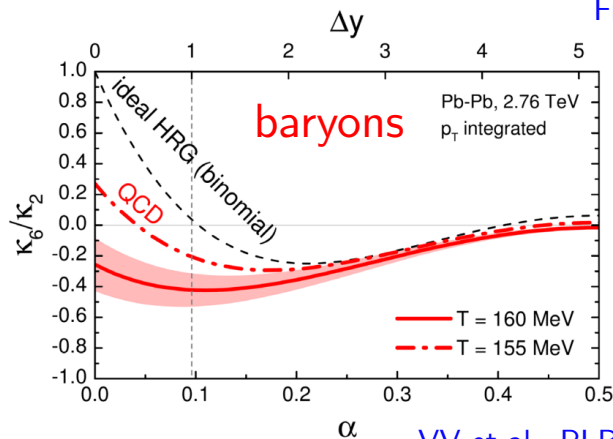
O. Savchuk et al., PLB 827, 136983 (2022)



ALICE Collaboration, PLB 844, 137545 (2023)

High-order cumulants: probe remnants of **chiral criticality**

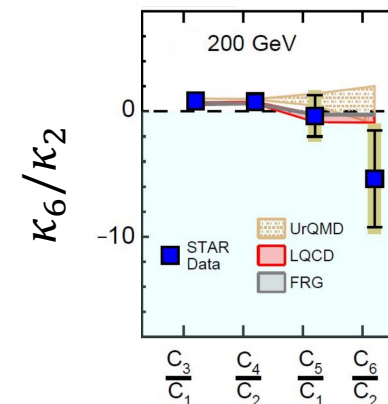
Friman et al., EPJC 71, 1694 (2011)



VV et al., PLB 811, 135868 (2020)

- negative κ_6 of **baryons**

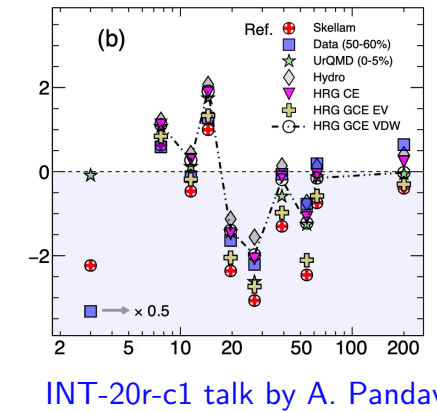
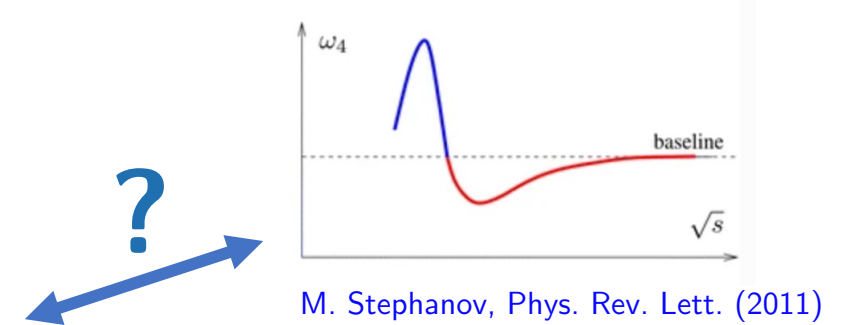
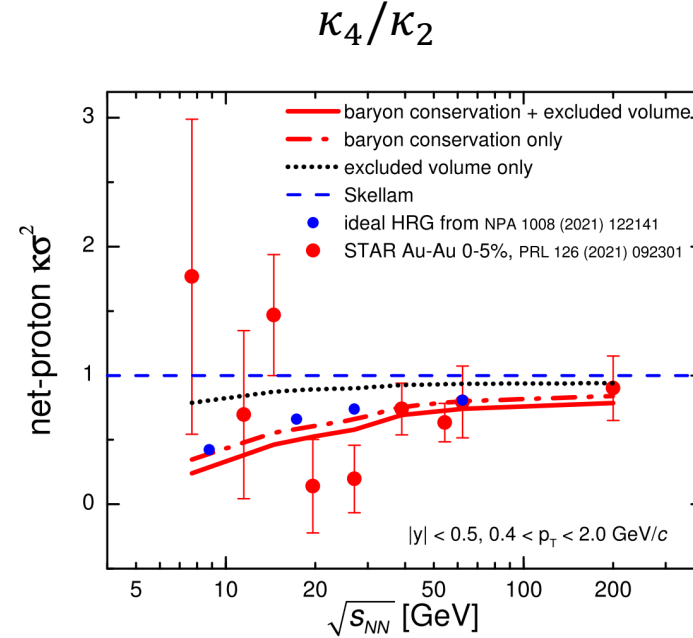
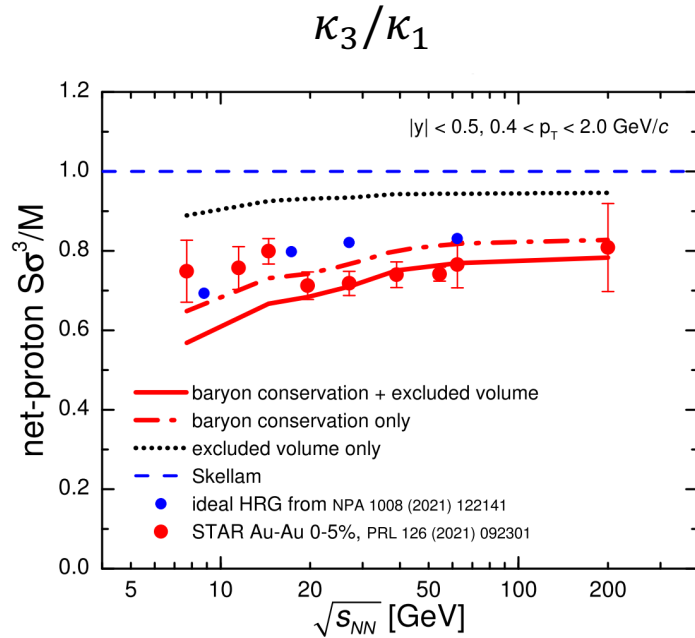
RHIC 200 GeV: hints of negative $\kappa_6 < 0$ (**protons**)



- are **baryons** even more negative?

STAR Collaboration, PRL 130, 082301 (2023)

RHIC-BES: Net proton cumulant ratios (MUSIC)



- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (BQS conservation and repulsion)
- Effect from baryon conservation is stronger than repulsion but both are required at $\sqrt{s_{NN}} \geq 20$ GeV
- Reduced errors to come from BES-II

Can we learn more from the more accurate data available for κ_2 and κ_3 ?

Removing the “net” part: Proton variance



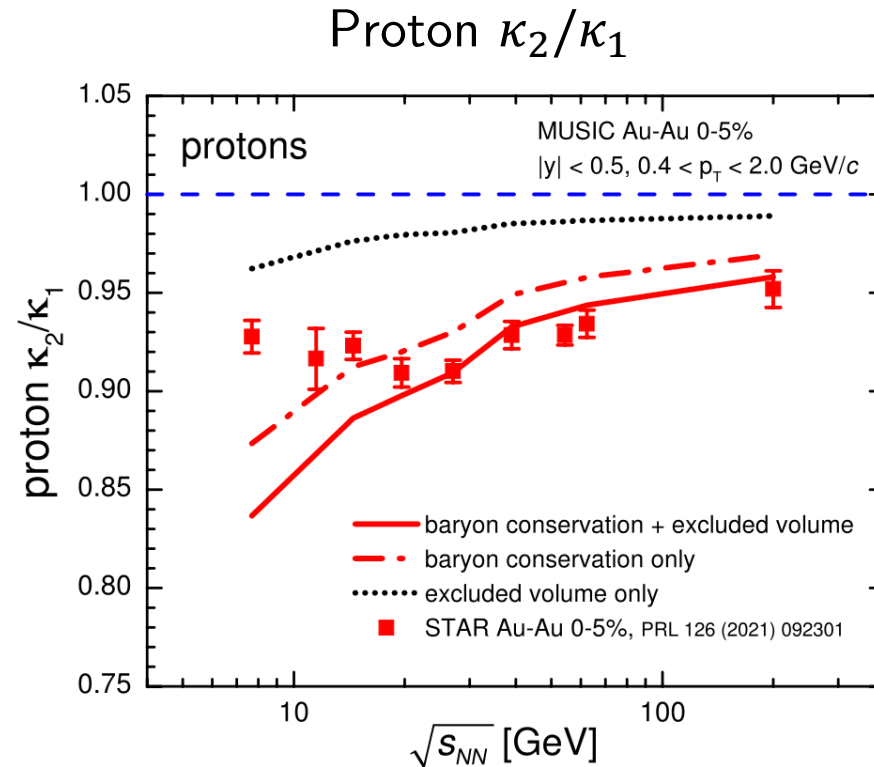
Net-proton $\kappa_2/\kappa_1 \sim \frac{\langle p+\bar{p} \rangle}{\langle p-\bar{p} \rangle} \sim \coth\left(\frac{\mu_B}{T}\right)$ in free gas

Proton $\kappa_2/\kappa_1 \sim \frac{\langle p \rangle}{\langle p \rangle} = 1$ in free gas

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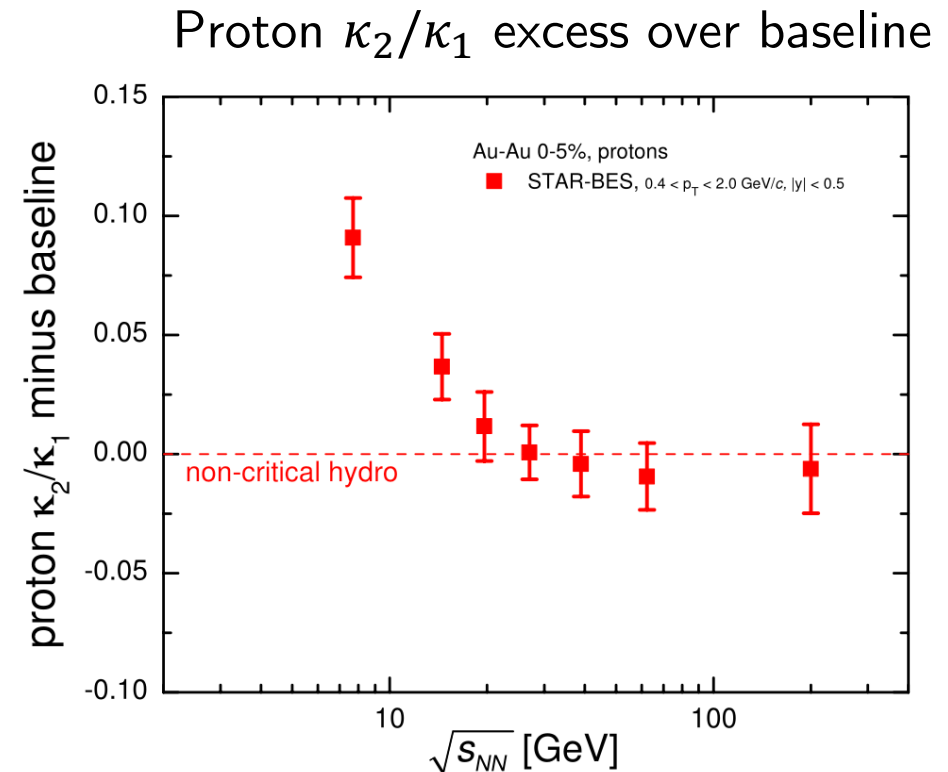
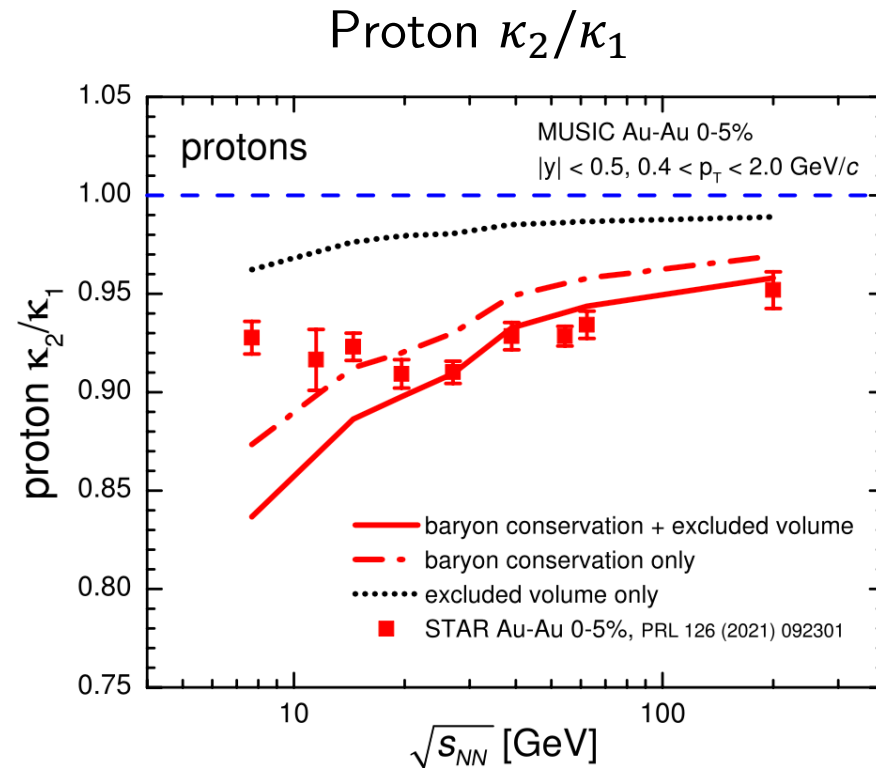


- Data at $\sqrt{s_{NN}} \geq 20 \text{ GeV}$ consistent with non-critical physics (BQS conservation and repulsion)
- Clear excess of proton variance at $\sqrt{s_{NN}} < 20 \text{ GeV}$ – *hint of attractive interactions?*

Removing the “net” part: Proton variance

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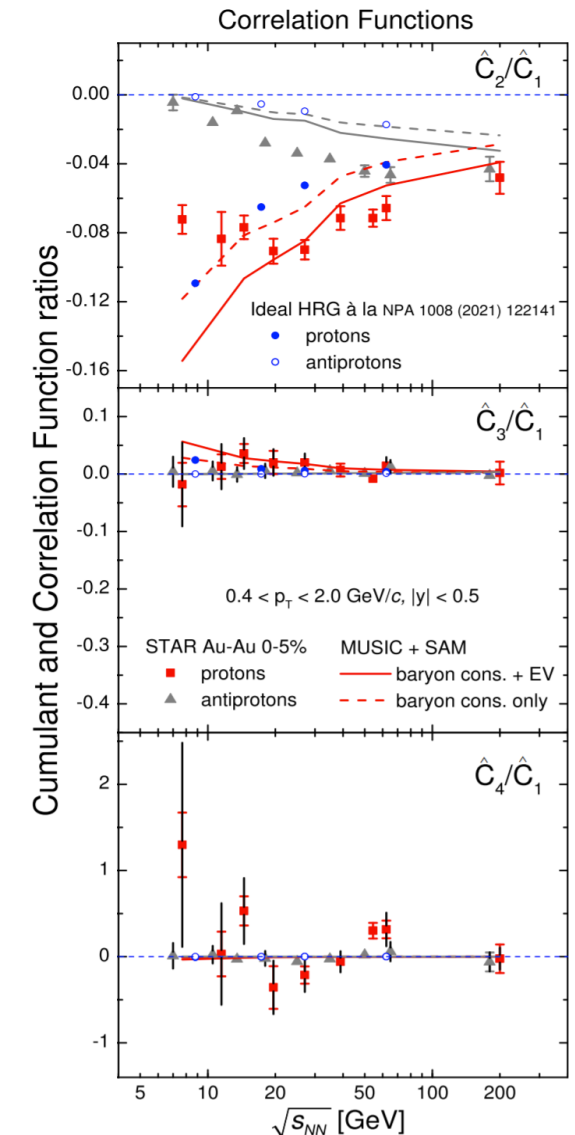
- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (BQS conservation and repulsion)
- Clear excess of proton variance at $\sqrt{s_{NN}} < 20$ GeV – *hint of attractive interactions?*

Correlation Functions (factorial cumulants)

- **Factorial cumulants \hat{C}_n** [Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)]
 - Remove the Poisson contribution and probe genuine correlations

$$\hat{C}_1 = \kappa_1, \quad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$

$$\hat{C}_2 = -\kappa_1 + \kappa_2, \quad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$
- **Expectation:** High-order ($n > 3$) factorial cumulants
 - have small contributions from non-critical effects [Bzdak, Koch, Skokov, EPJC '17; VV et al, PLB '17]
 - are as singular as ordinary cumulants near the critical point [Ling, Stephanov, PRC '16]
- **Observations from STAR data:**
 - \hat{C}_3 & \hat{C}_4 are largely consistent with zero within errors
 - Reanalyze (non-)monotonic energy dependence for \hat{C}_4/\hat{C}_1 instead of κ_4/κ_2 ?
 - Statistically significant effects appear to be driven by two-proton correlations in \hat{C}_2



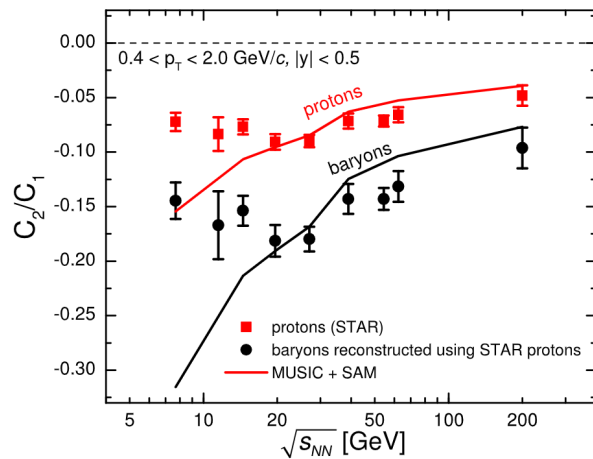
Baryon cumulants from protons

- **net baryon \neq net proton**
 - protons are a *subset* of all baryons
 - effectively amounts to additional efficiency correction
 - “Poissonizer” of proton cumulants relative to baryons
 - **error:** $\sim 50\%$ in \hat{C}_2 , $\sim 75\%$ in \hat{C}_3 , $\sim 87.5\%$ in \hat{C}_4

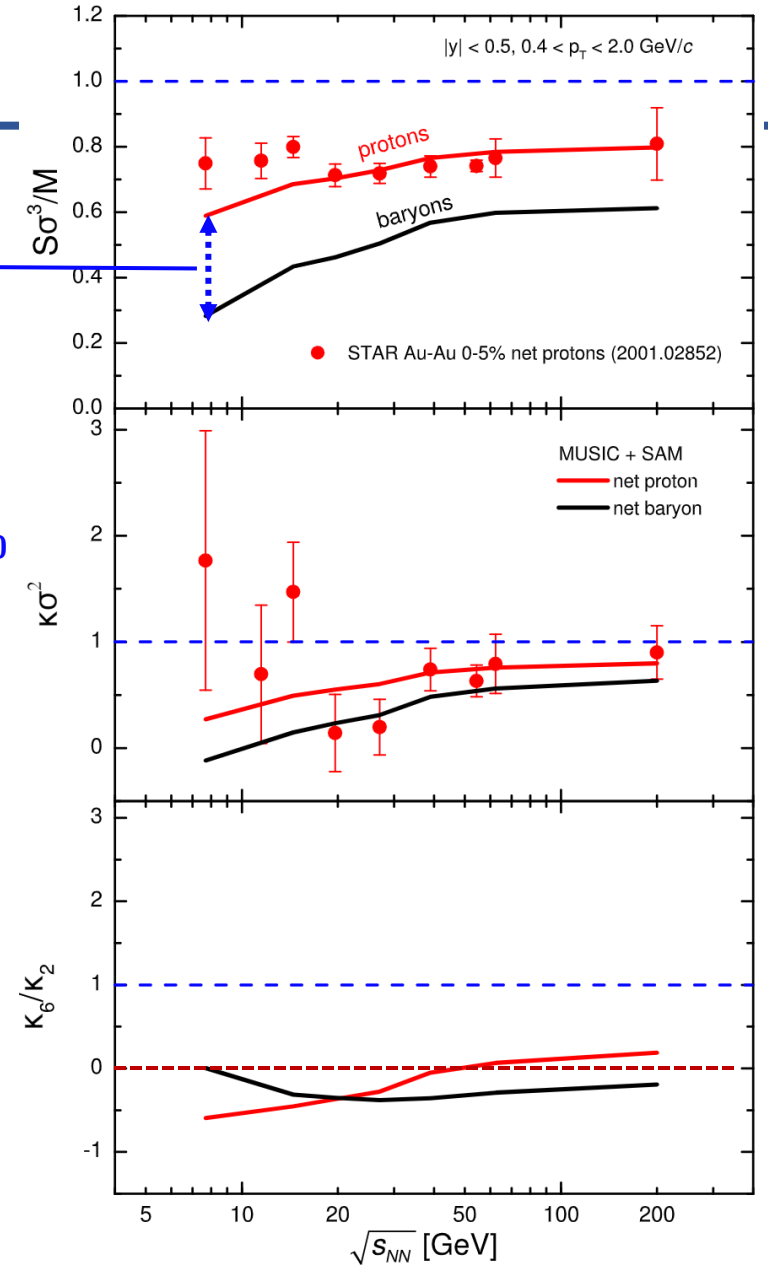
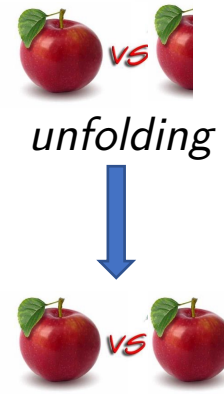
see also talk by M. Marczenko, Wed 14:20

- Baryon cumulants can be reconstructed from proton cumulants based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]

- Requires the use of joint factorial moments, experiment can do it



$$\frac{\hat{C}_2^B}{\hat{C}_1^B} \approx 2 \frac{\hat{C}_2^P}{\hat{C}_1^P}$$



Time to stop equating proton and baryon cumulants

Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV

- Intriguing hints from HADES@2.4 GeV and STAR-FXT@3GeV: huge excess of two-proton correlations!

[HADES Collaboration, Phys. Rev. C 102, 024914 (2020)]

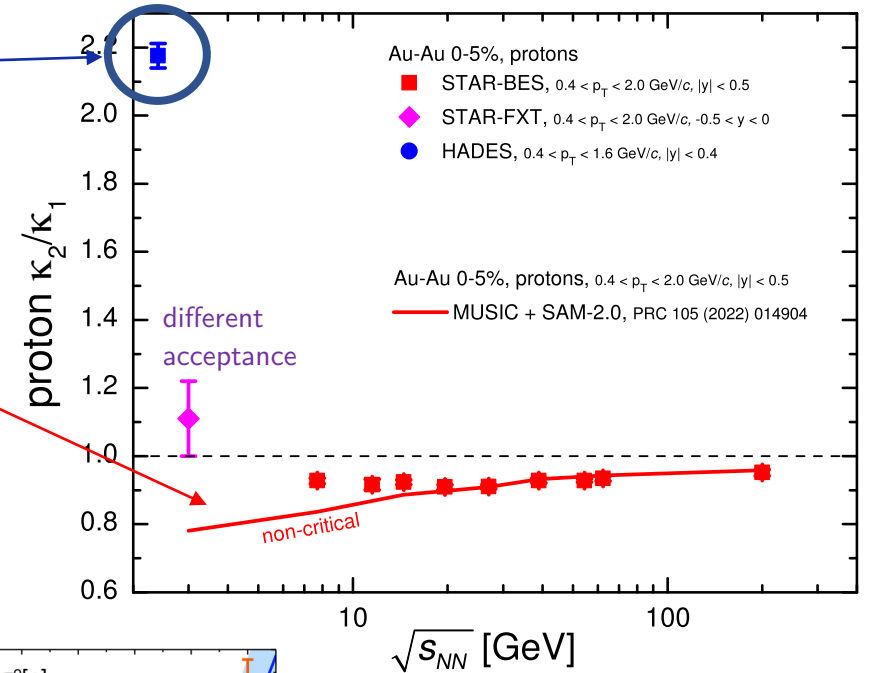
[STAR Collaboration, Phys. Rev. Lett. 128, 202303 (2022)]

- No change of trend in the non-critical reference
- Additional mechanisms:
 - Nuclear liquid-gas transition (the other QCD critical point)
 - Light nuclei formation/fragmentation
 - Stronger initial state, volume, and baryon stopping fluctuations

Talk by A. Bzdak, Wed 14:20; Poster by A. Rustamov

- Difference in acceptance ($-0.5 < y < 0$ vs $|y| < 0.5$)

- Improved modeling of lower energies required



VV, Phys. Rev. C 106, 064906 (2022)

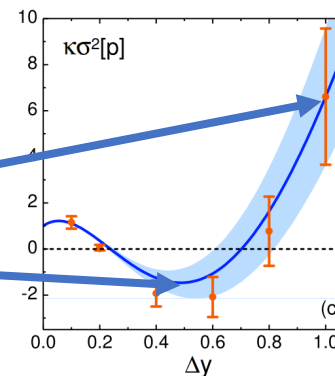
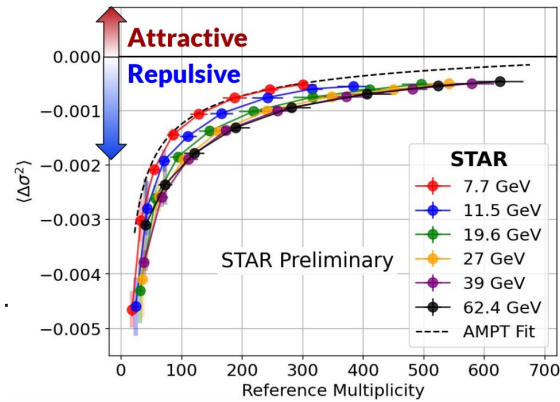


Figure from O. Savchuk et al., PLB 835, 137540 (2022)

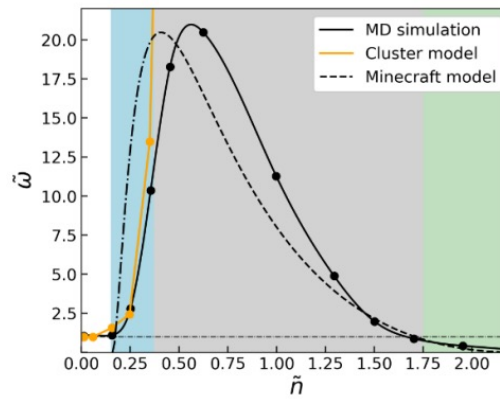
We may want to understand κ_2 first

- Azimuthal correlations of protons
 - points to repulsion at RHIC-BES

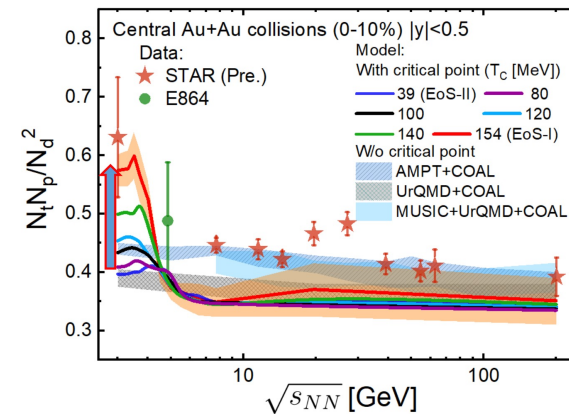


Talk by D. Neff, Tue 8:30

- Light nuclei
 - Spinodal/critical point enhancement of density fluctuations and light nuclei production



Talk by R. Poberezhnyuk, Tue 10:10
Poster by V. Kuznietsov



Talk by K.-J. Sun, Wed 15:00
Posters by K. Murase & S. Wu

- Proton intermittency
 - No structure indicating power-law seen (NA61/SHINE)

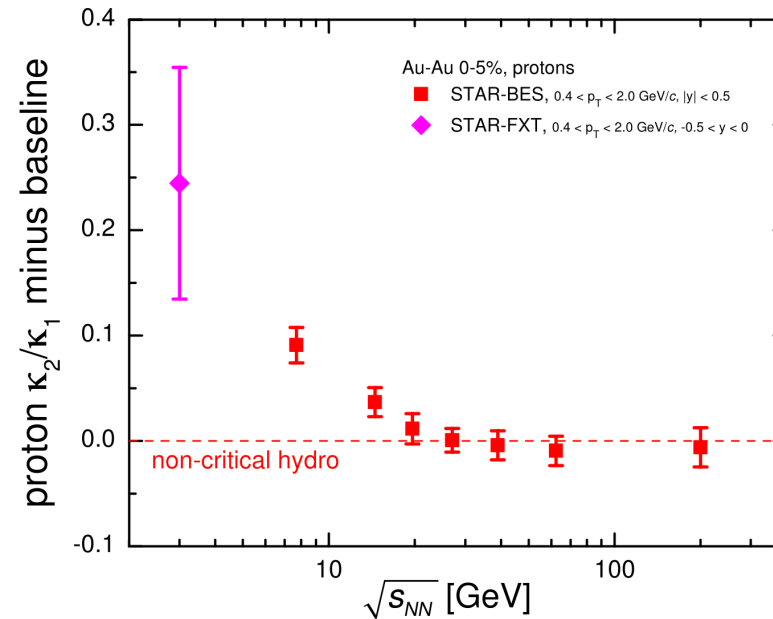
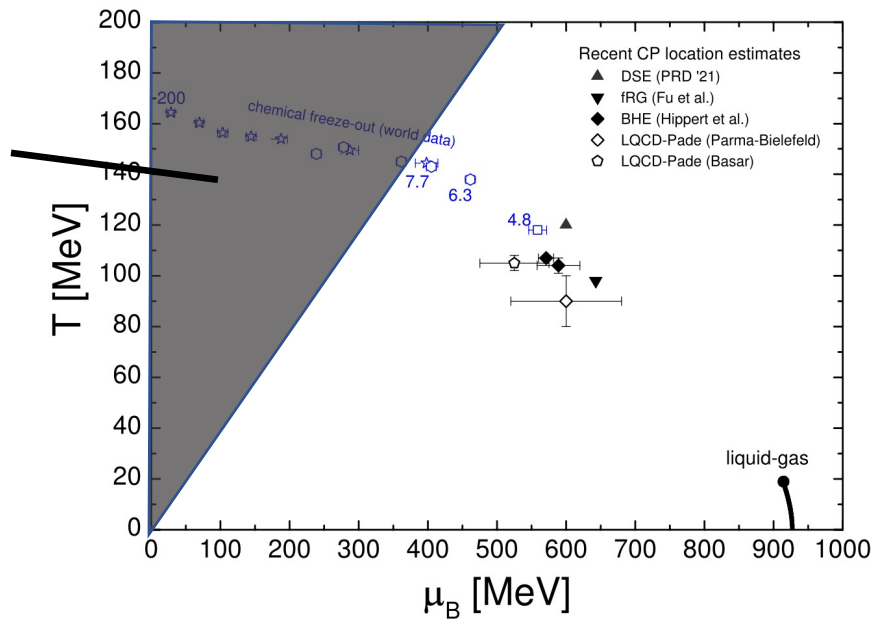
Talk by B. Porfy, Tue 9:10

- Directed flow, speed of sound

Talk by C. Bernardes, Wed 3:40

Eventually, consistency in understanding all the observables will be required

Critical point disfavored



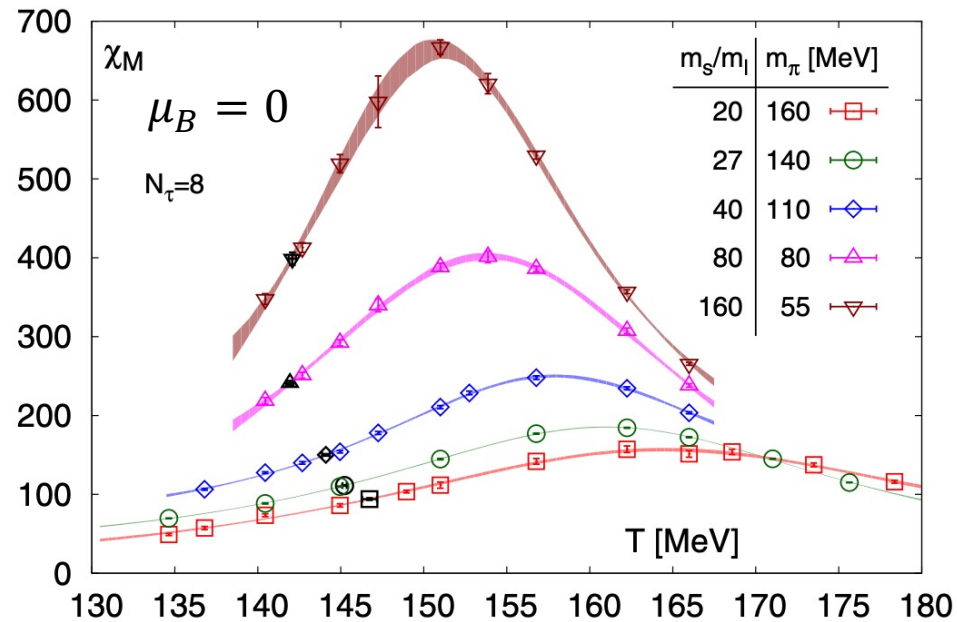
- Theory estimates
 - No indications of critical behavior from lattice QCD at small $\mu_B/T < 2-3$
 - Several effective theories and lattice-Pade predictions point to $T \sim 90-120$ MeV and $\mu_B \sim 500 - 650$ MeV
- Heavy-ion collisions
 - Proton cumulants are consistent with non-critical physics at $\sqrt{s_{NN}} \geq 20$ GeV
 - Significant excess of two-proton correlations at $\sqrt{s_{NN}} \leq 20$ GeV that has no clear explanation yet

Thanks for your attention!

Backup slides

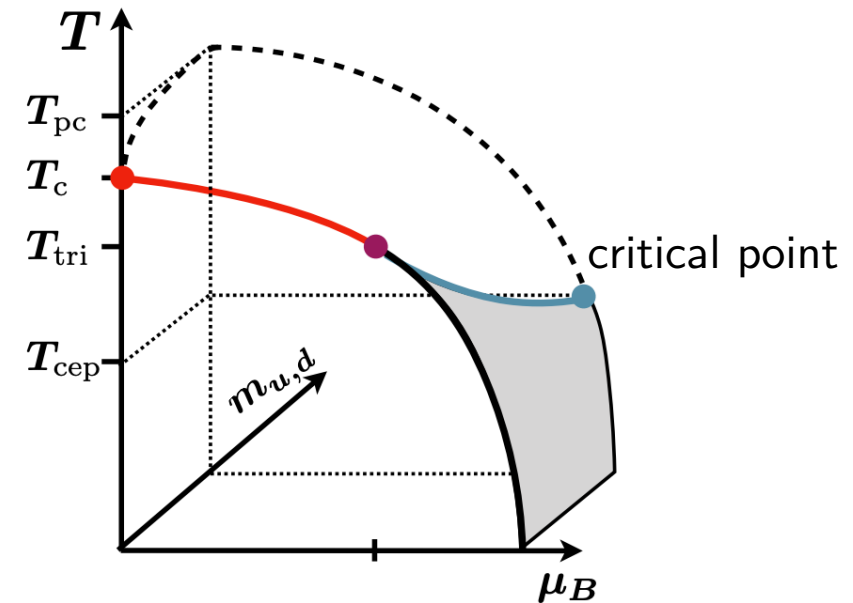
Hunting for the QCD critical point with lattice QCD

Remnants of $O(4)$ chiral criticality at $\mu_B = 0$ quite well established with lattice QCD



HotQCD Collaboration, PRL 123, 062002 (2019)

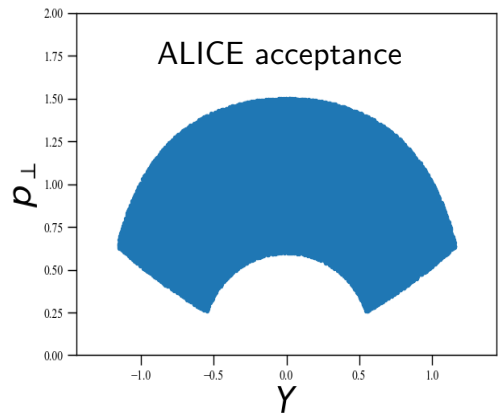
Physical quark masses away from the chiral limit: Expect a $Z(2)$ critical point at finite μ_B



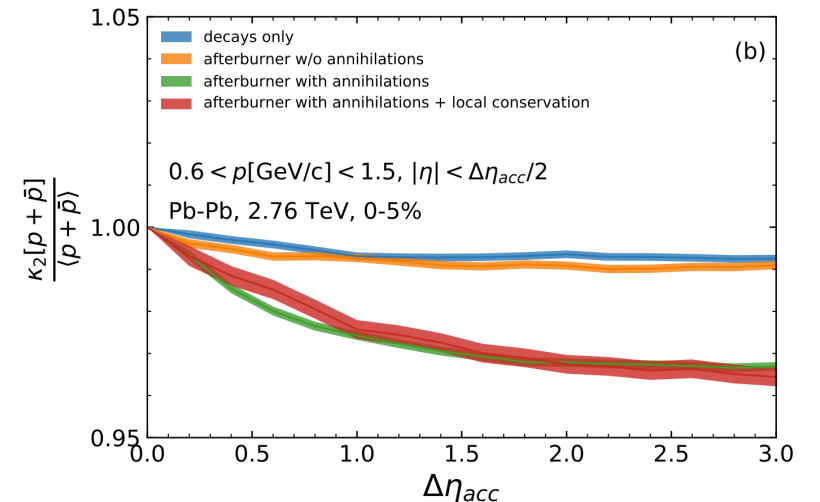
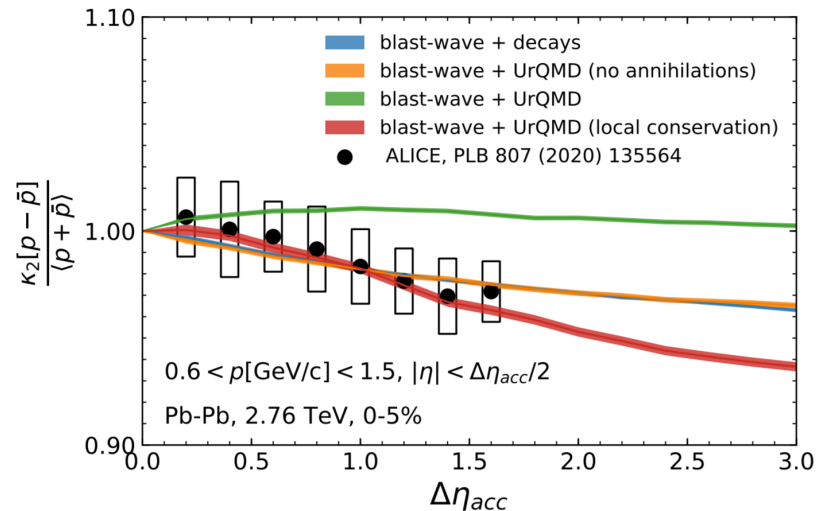
C. Schmidt

Net-particle fluctuations at the LHC (blast-wave model)

- Net protons described within errors and consistent with either VV, Koch, Phys. Rev. C 103, 044903 (2021)
 - **global** baryon conservation without $B\bar{B}$ annihilations in the hadronic phase
see e.g. ALICE Coll. arXiv:2206.03343
 - or **local** baryon conservation with $B\bar{B}$ annihilations in the hadronic phase
O. Savchuk et al., Phys. Lett. B 827, 136983 (2022)

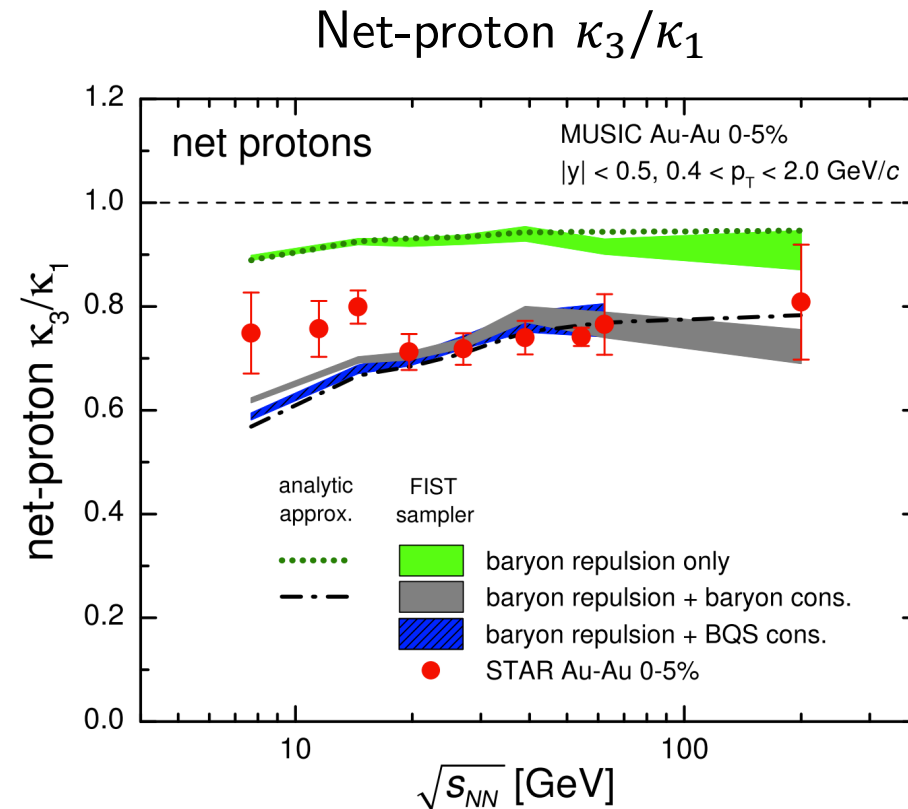
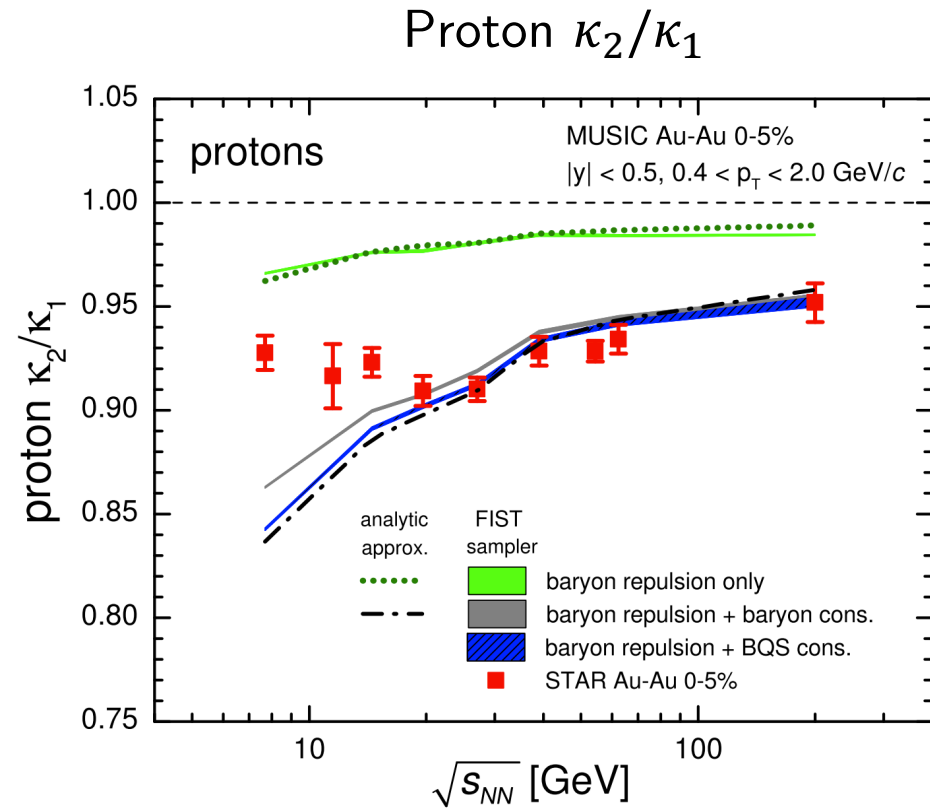


$0.6 < p < 1.5 \text{ GeV}/c, \Delta\eta_{acc} = 1.6$

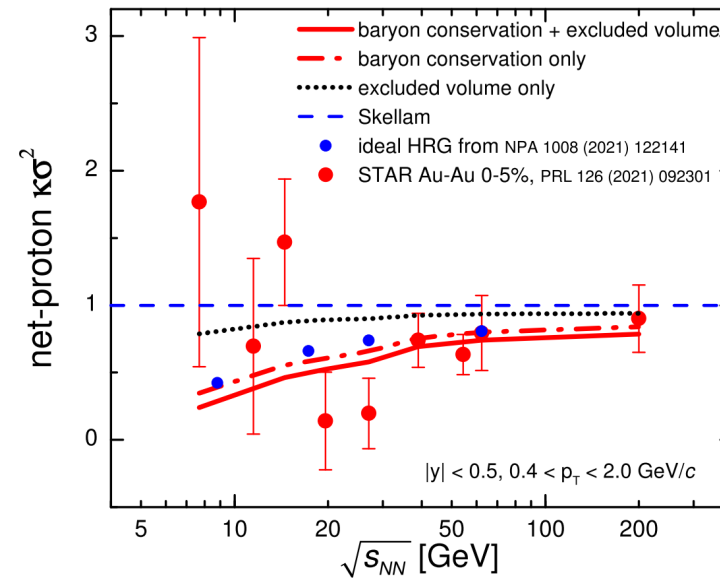
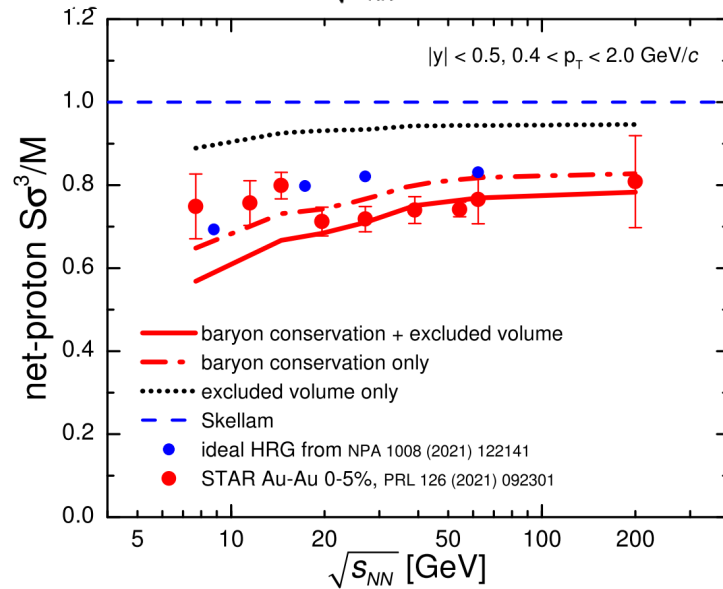
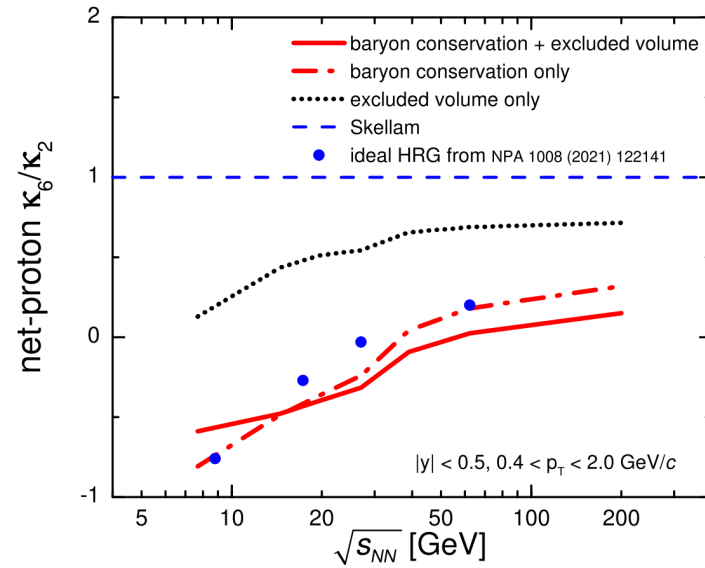
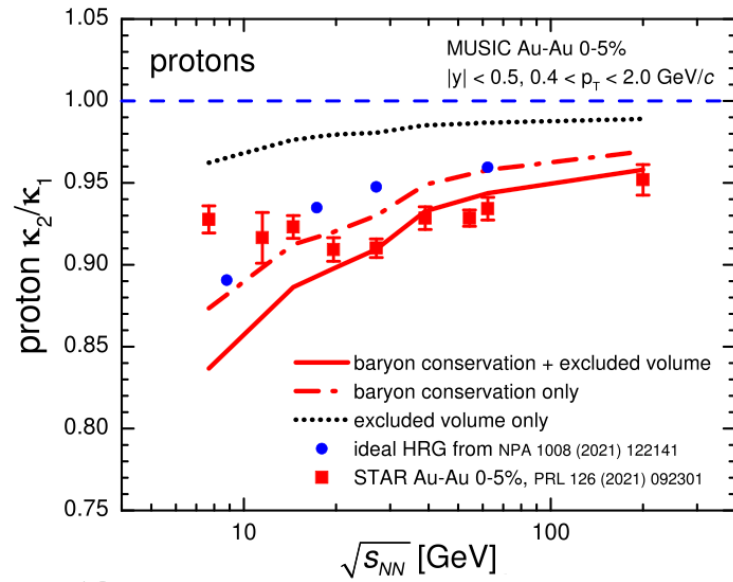


Data on (net-)proton fluctuations can constrain the effect of annihilations in the hadronic phase

Non-critical cumulants: Analytic vs Monte Carlo



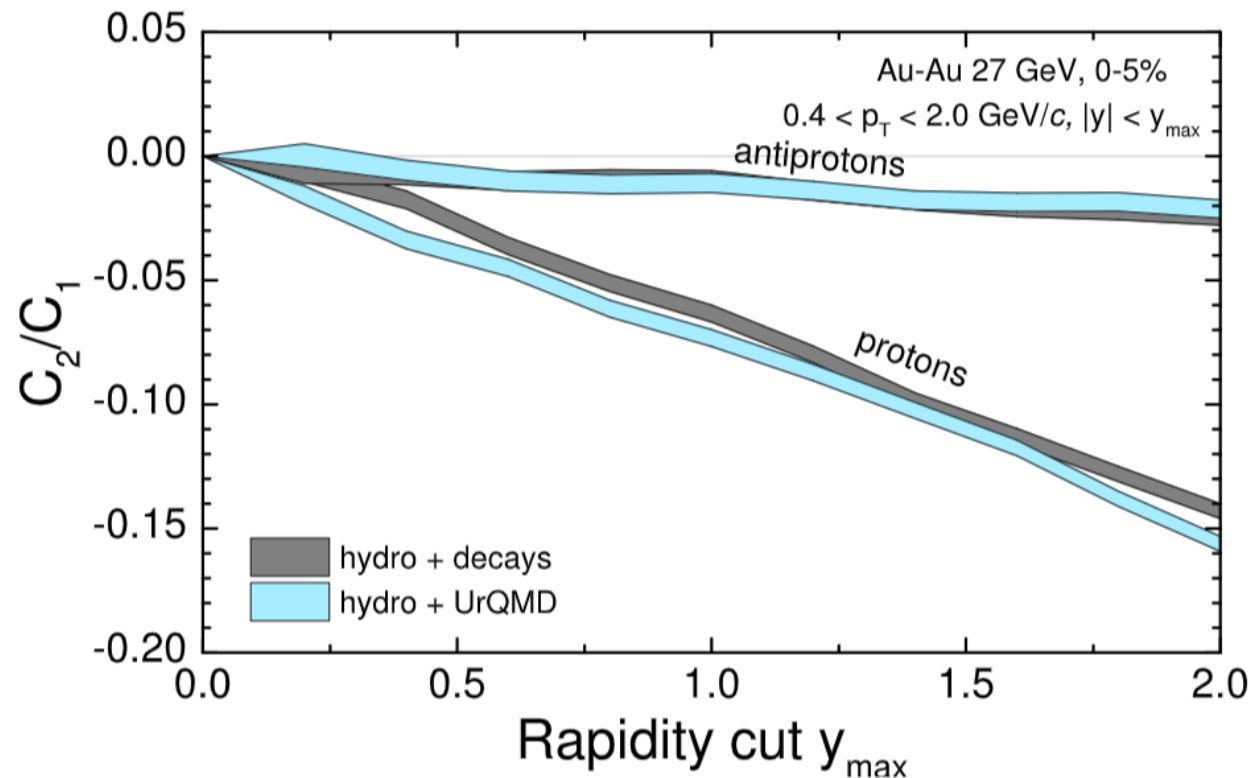
Non-critical cumulants



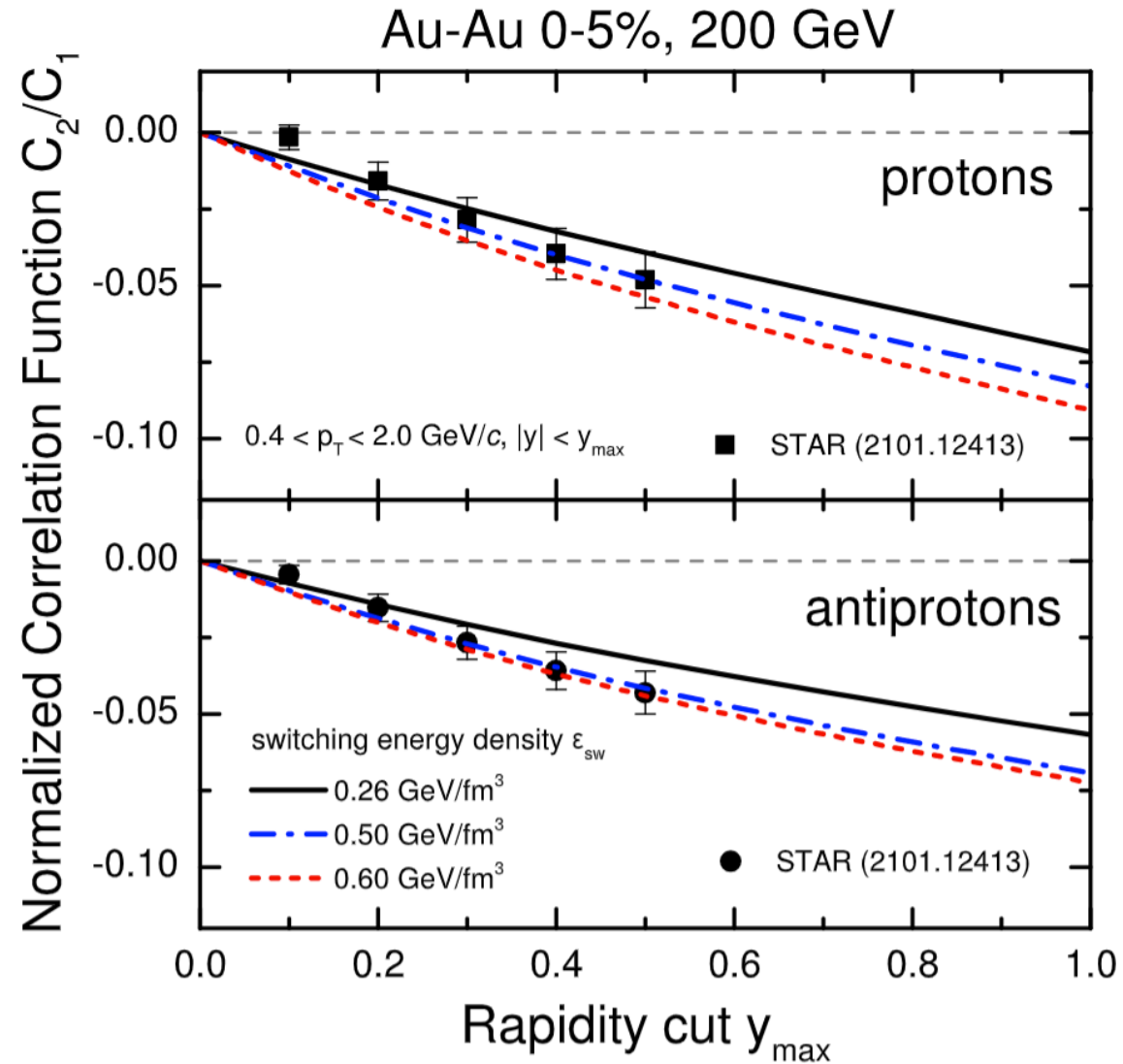
Non-critical cumulants

Effect of the hadronic phase

Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD



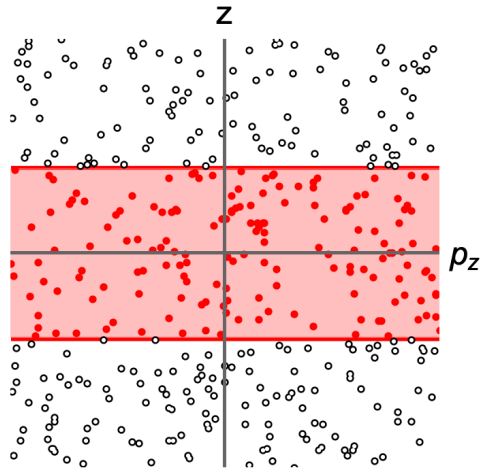
Dependence on the switching energy density



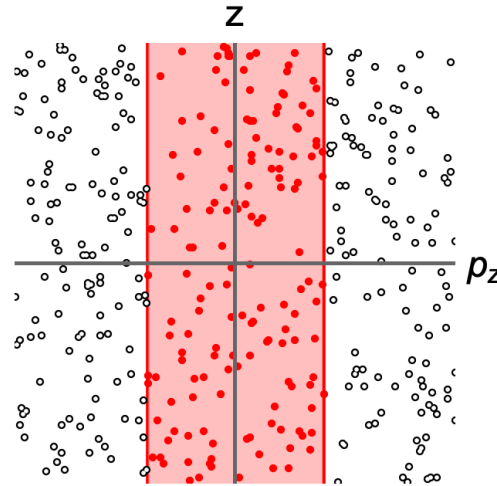
Coordinate vs Momentum space

Box setup: Coordinates and momenta are uncorrelated

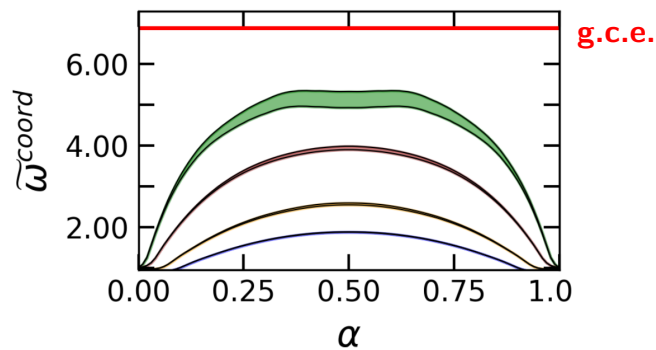
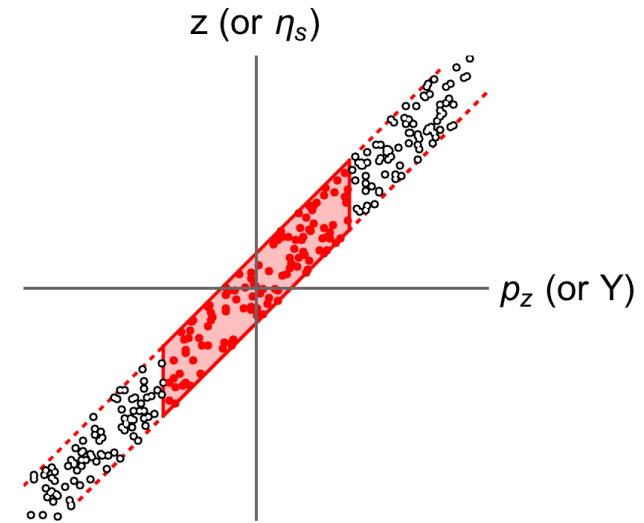
Coordinate space cut



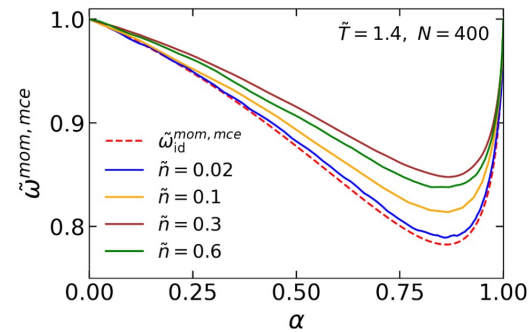
Momentum space cut



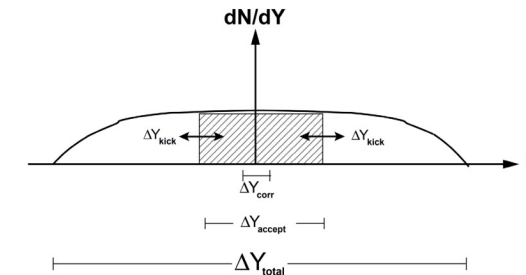
HICs: Flow (e.g. Bjorken)



Large correlations



Nothing left

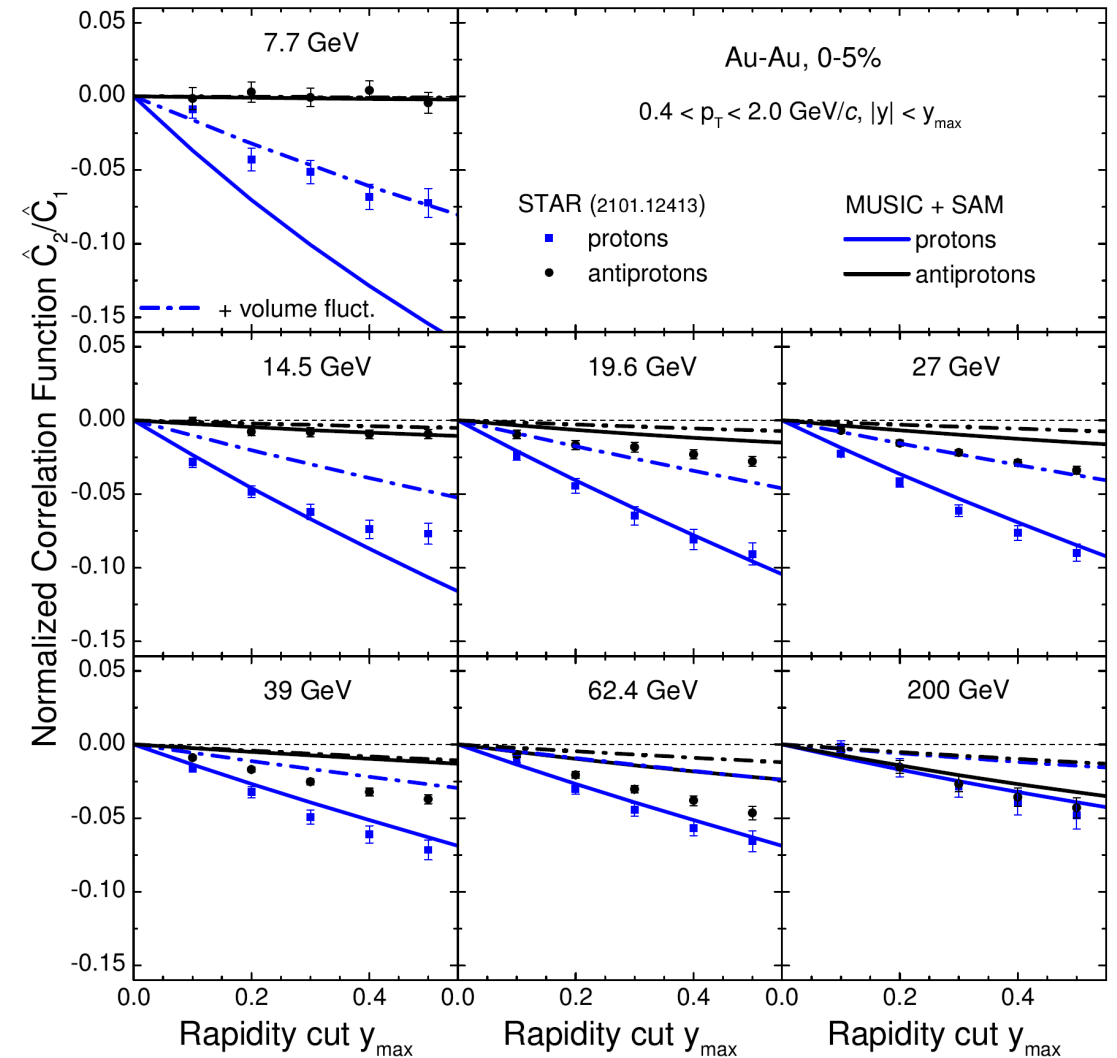


V. Koch, arXiv:0810.2520

momentum cut \sim coordinate cut + smearing

Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?
- Volume fluctuations? [Skokov, Friman, Redlich, PRC '13]
 - $C_2/C_1 \neq C_1 * \Delta v^2$
 - Can improve low energies but spoil high energies?
- **Attractive interactions?**
 - Could work if baryon repulsion turns into attraction in the high- μ_B regime
 - **Critical point?**



Net baryon fluctuations at LHC

- Global baryon conservation distorts the cumulant ratios already for one unit of rapidity acceptance

e.g. $\left. \frac{\chi_4^B}{\chi_2^B} \right|_{T=160\text{MeV}}^{\text{GCE}} \stackrel{\text{"lattice QCD"}}{\simeq 0.67} \neq \left. \frac{\chi_4^B}{\chi_2^B} \right|_{\Delta Y_{\text{acc}}=1}^{\text{HIC}} \stackrel{\text{experiment}}{\simeq 0.56}$

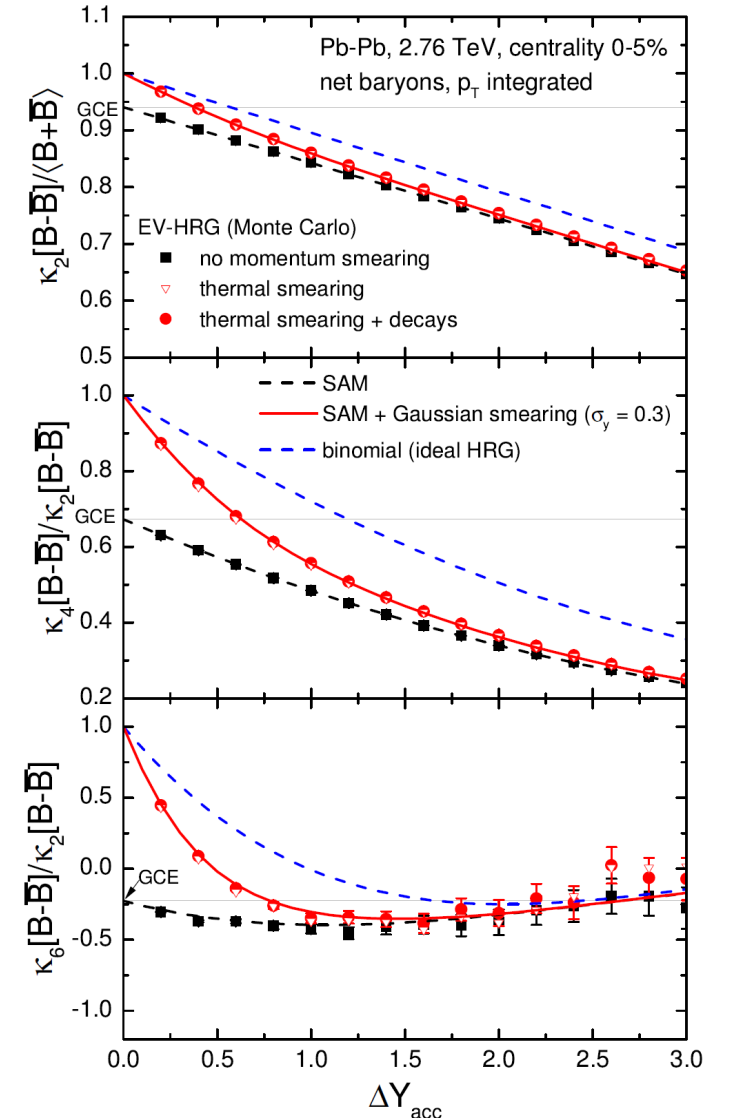
- Neglecting thermal smearing, effects of global conservation can be described analytically via SAM

$$\frac{\kappa_2}{\langle B + \bar{B} \rangle} = (1 - \alpha) \frac{\kappa_2^{\text{gce}}}{\langle B + \bar{B} \rangle}, \quad \alpha = \frac{\Delta Y_{\text{acc}}}{9.6}, \quad \beta \equiv 1 - \alpha$$

$$\frac{\kappa_4}{\kappa_2} = (1 - 3\alpha\beta) \frac{\chi_4^B}{\chi_2^B},$$

$$\frac{\kappa_6}{\kappa_2} = [1 - 5\alpha\beta(1 - \alpha\beta)] \frac{\chi_6^B}{\chi_2^B} - 10\alpha(1 - 2\alpha)^2\beta \left(\frac{\chi_4^B}{\chi_2^B} \right)^2$$

- Effect of resonance decays is negligible



Net baryon vs net proton

- Thermal smearing distorts the signal at $\Delta Y_{accept} \leq 1$. Net baryons converge to model-independent SAM result at larger ΔY_{accept}
- net baryon \neq net proton, e.g.

$$\left. \frac{\chi_4^B}{\chi_2^B} \right|_{\Delta Y_{acc}=1}^{\text{HIC}} \simeq 0.56 \neq \left. \frac{\chi_4^p}{\chi_2^p} \right|_{\Delta Y_{acc}=1}^{\text{HIC}} \simeq 0.83$$

- Baryon cumulants can be reconstructed from proton cumulants via binomial (un)folding based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]
 - Requires the use of joint factorial moments, only experiment can do it model-independently

