Probing the QCD phase structure with fluctuations in heavy-ion collisions

Volodymyr Vovchenko (University of Houston)

MIT Nuclear and Particle Physics Seminar

May 15, 2023

QCD phase structure

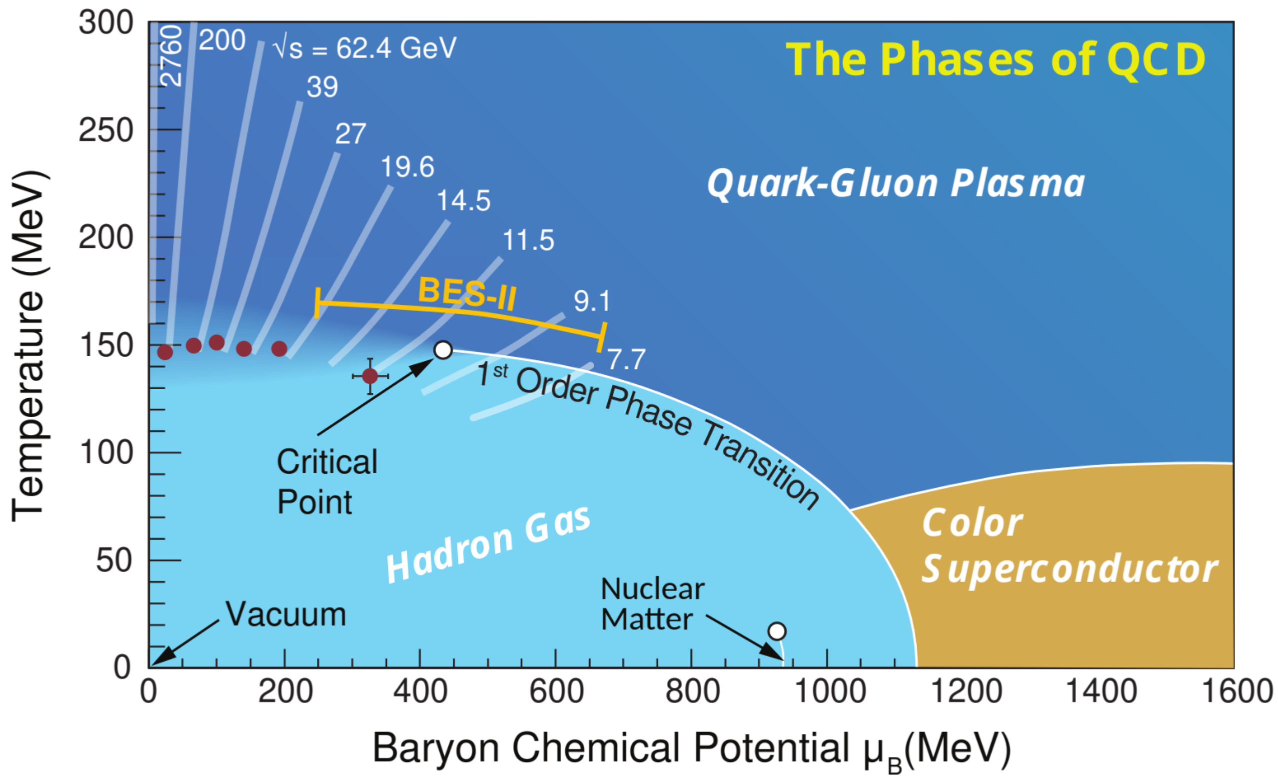


Figure from Bzdak et al., Phys. Rept. '20

- Dilute hadron gas at low T & μ_B due to confinement, quark-gluon plasma high T & μ_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured

Is there a critical point and how to find it?

Where is the critical point? Ask AI

UNIVERSITY OF HOUSTON

ChatGPT struggles

Is there a QCD critical point and where is it?

As of my knowledge cutoff in September 2021, the existence and precise location of a critical point in the phase diagram of Quantum Chromodynamics (QCD), the theory of the strong interaction between quarks and gluons, was not definitively established.

...

However, finding the QCD critical point using lattice QCD is a particularly challenging problem due to the so-called "sign problem." The sign problem arises when trying to extend lattice QCD calculations from zero to non-zero baryon chemical potential, which is necessary to locate the critical point on the phase diagram. At non-zero chemical potential, the action of the QCD path integral becomes complex, which makes the standard Monte Carlo techniques used in lattice QCD simulations ineffective.

...

Experimental searches for the QCD critical point in heavy-ion collisions generally rely on identifying fluctuations and correlations in the properties of the particles produced in the collisions, which are expected to exhibit unique features near the critical point.

Where is the critical point? Ask AI

ChatGPT struggles

Is there a QCD critical point and where is it?

As of my knowledge cutoff in September 2021, the existence and precise location of a critical point in the phase diagram of Quantum Chromodynamics (QCD), the theory of the strong interaction between quarks and gluons, was not definitively established.

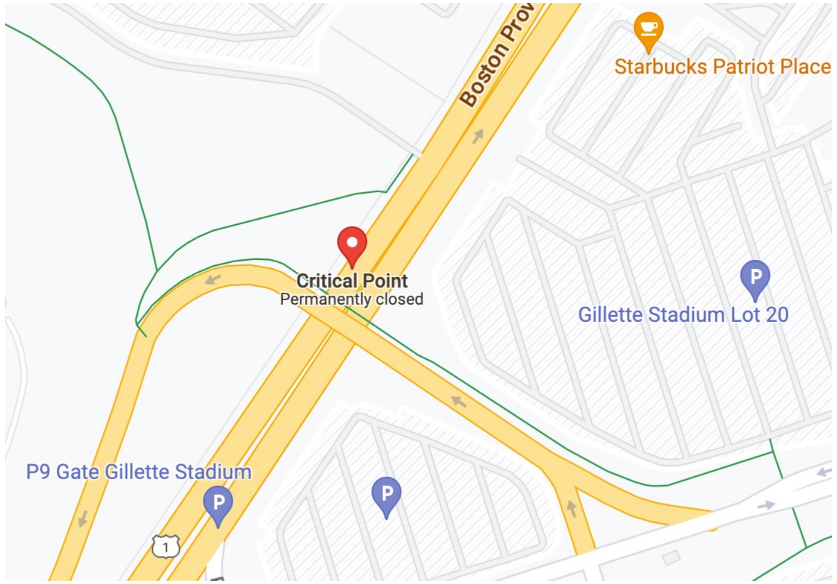
...

However, finding the QCD critical point using lattice QCD is a particularly challenging problem due to the so-called "sign problem." The sign problem arises when trying to extend lattice QCD calculations from zero to non-zero baryon chemical potential, which is necessary to locate the critical point on the phase diagram. At non-zero chemical potential, the action of the QCD path integral becomes complex, which makes the standard Monte Carlo techniques used in lattice QCD simulations ineffective.

...

Experimental searches for the QCD critical point in heavy-ion collisions generally rely on identifying fluctuations and correlations in the properties of the particles produced in the collisions, which are expected to exhibit unique features near the critical point.

No issues for Google

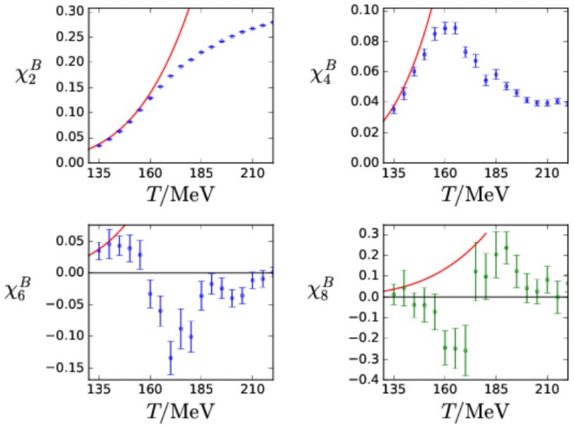


Lattice QCD constraints on the QCD critical point

Indirect lattice QCD methods offer glimpse into small μ_B/T

- Taylor expansion around $\mu_B/T=0$

$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \frac{\chi_2^B(T,0)}{2!}(\mu_B/T)^2 + \frac{\chi_4^B(T,0)}{4!}(\mu_B/T)^4 + \dots$$

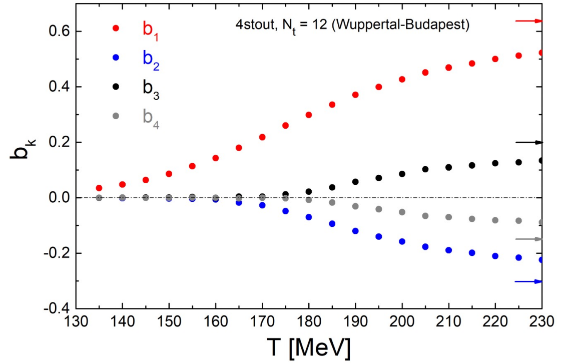


[WB Collaboration, JHEP 10, 205 (2018)]

- Relativistic virial expansion in fugacities via analytic continuation from imaginary μ_B/T (Laurent series in $\lambda_B = e^{\mu_B/T}$)

$$p(T,\mu_B) = \sum_{k=-\infty}^{\infty} \tilde{p}_k(T) e^{\frac{k\mu_B}{T}} = \sum_{k=0}^{\infty} p_k(T) \cosh\left(\frac{k\mu_B}{T}\right)$$

$$\rho_B(T,\mu_B) = \sum_{k=0}^{\infty} b_k(T) \sinh\left(\frac{k\mu_B}{T}\right) = i\sum_{k=0}^{\infty} b_k(T) \sin(\theta_B) \qquad b_k = k p_k, \quad \frac{\mu_B}{T} = i\theta_B$$



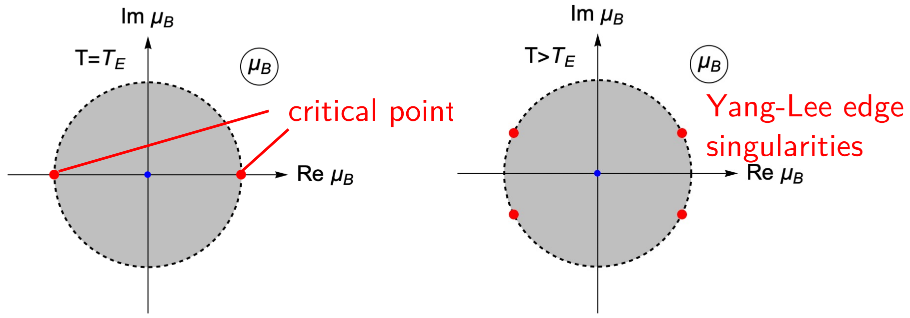
[VV, Pasztor, Fodor, Katz, Stoecker, PLB 71, 775 (2017)]

Critical point: singularity of the expansion(s) located on real μ_B axis

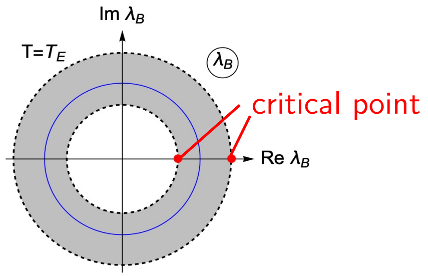
[Lee, Yang, PR 87, 410 (1952); M. Stephanov, PRD 73, 094508 (2006))]

Lattice QCD constraints on the QCD critical point

Taylor expansion



Virial expansion



Above T_E: Yang-Lee edge singularities in the *complex plane*

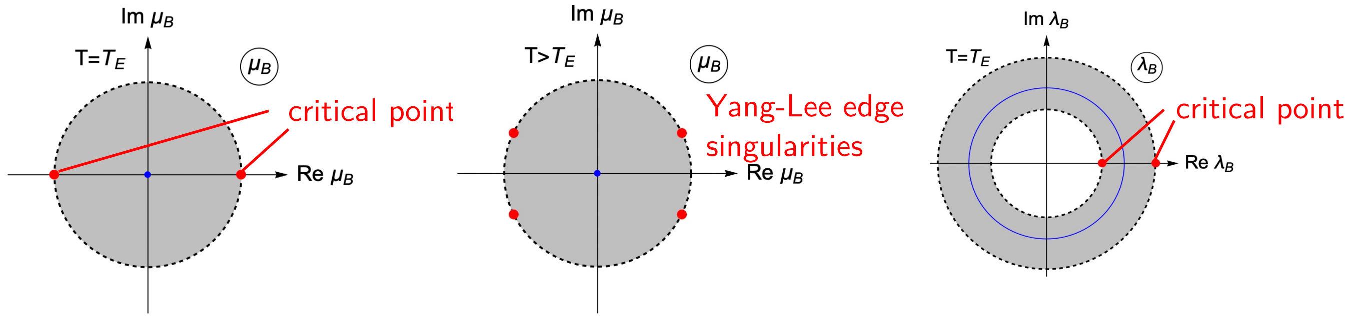
Challenges:

- Need many expansion coefficients
  - Use Pade & resummation schemes
- Interference from other singularities
  - Roberge-Weiss transition
  - Remnants of nuclear liquid-gas

    Savchuk et al., PRC 101, 035205 (2020)
  - Thermal singularities (Fermi-Dirac)

Lattice QCD constraints on the QCD critical point

Taylor expansion   Virial expansion

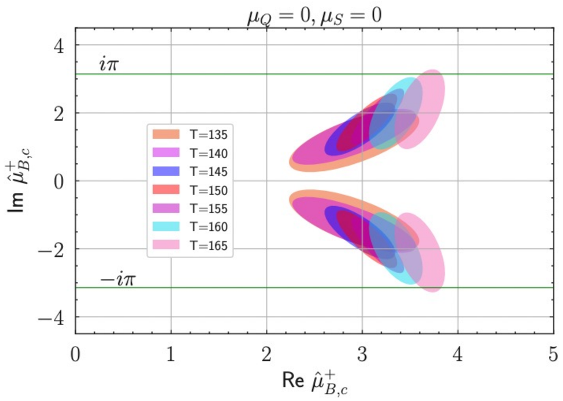


Above T_E: Yang-Lee edge singularities in the *complex plane*

Challenges:

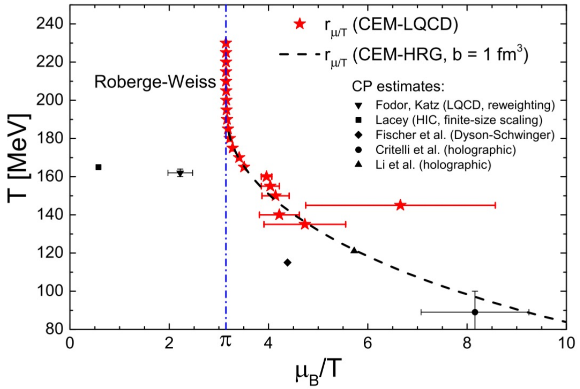
- Need many expansion coefficients
  - Use Pade & resummation schemes
- Interference from other singularities
  - Roberge-Weiss transition
  - Remnants of nuclear liquid-gas
    Savchuk et al., PRC 101, 035205 (2020)
  - Thermal singularities (Fermi-Dirac)

Poles from [4,4]-Pade approximant



HotQCD Collab., PRD 105, 074511 (2022)

Resummation of the virial expansion (cluster expansion model) sees Roberge-Weiss-like singularity

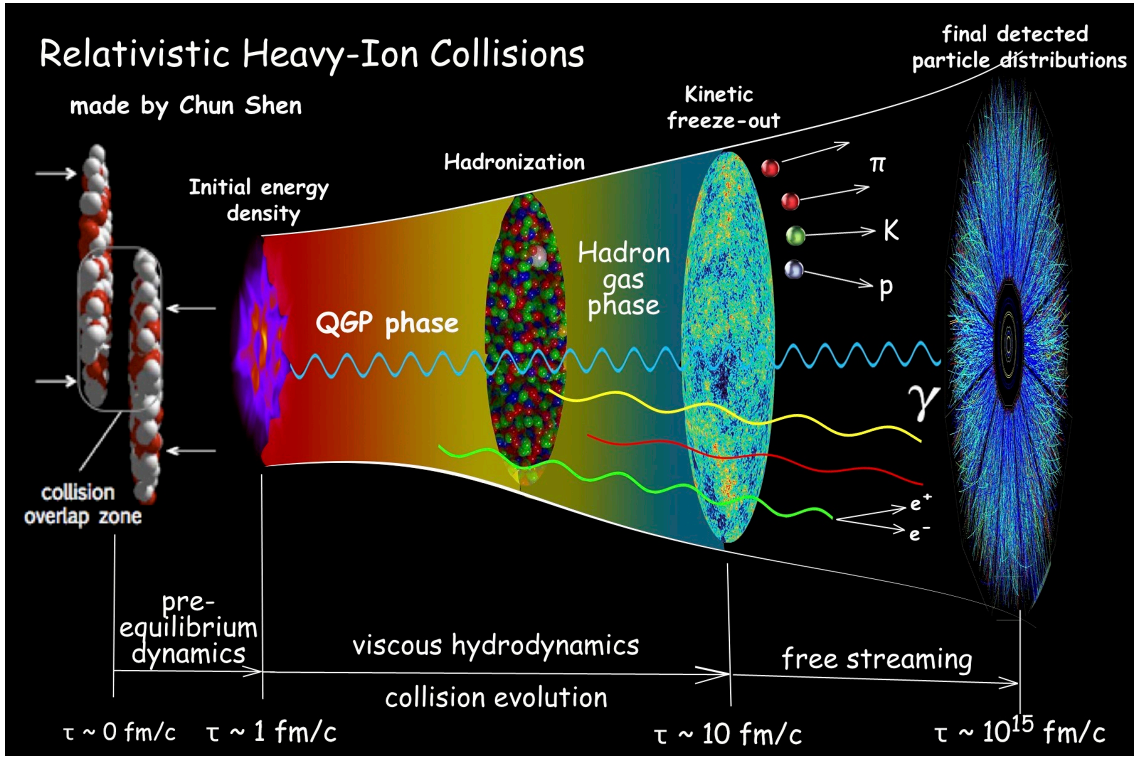


VV, Steinheimer, Philipsen, Stoecker, PRD 97, 114030 (2018)

Critical point disfavored at $\frac{\mu_B}{T} < 2 - 3$ & T>135 MeV

Critical point, if it exists, is likely located beyond the reach of current lattice methods

Relativistic heavy-ion collisions – "Little Bangs"



Control parameters

- Collision energy $\sqrt{s_{NN}} = 2.4 - 5020$ GeV
- Size of the collision region

Measurements

- Final hadron abundances and momentum distributions **event-by-event**

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

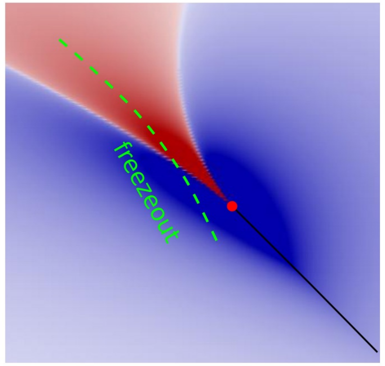
Grand partition function

$$\ln Z^{\text{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\text{ce}}(T,V,N)\right]$$

$$\kappa_n \propto \frac{\partial^n(\ln Z^{\text{gce}})}{\partial \mu^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- **(QCD) critical point** – large correlation length, critical fluctuations of baryon number



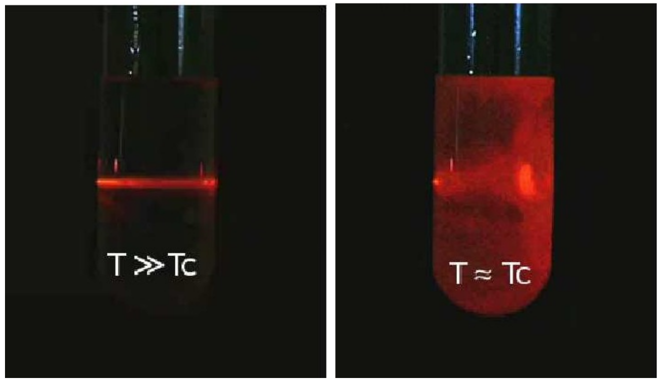
M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

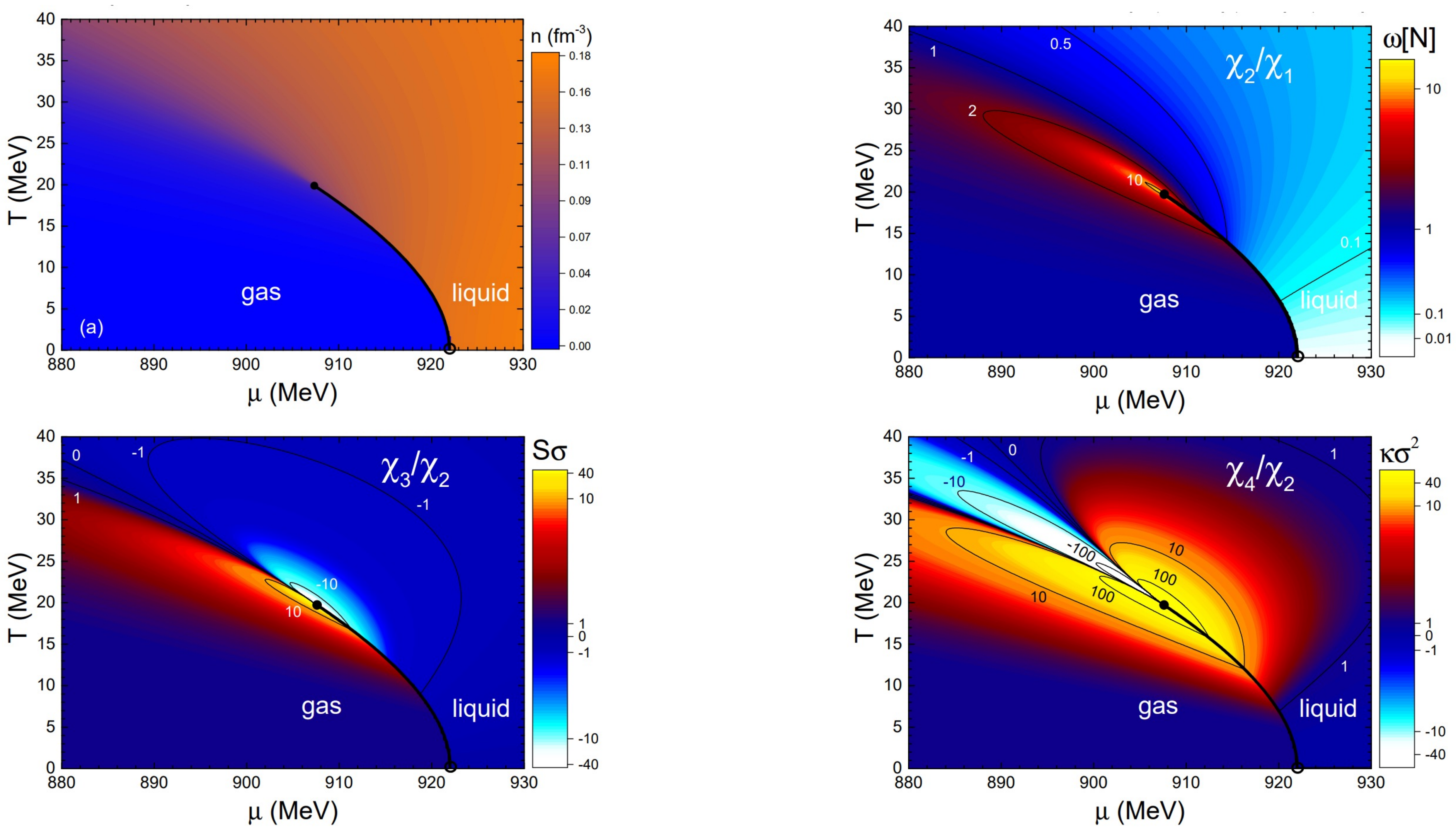
$$\xi \to \infty$$

Looking for enhanced fluctuations
and non-monotonicities

Critical opalescence



Example: Nuclear liquid-gas transition

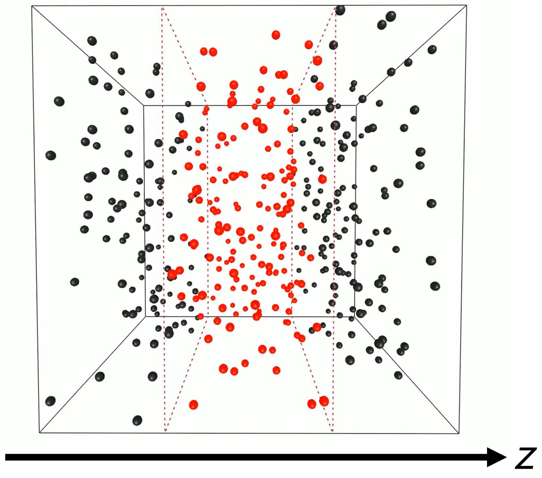


VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Example: Lennard-Jones fluid

Kuznietsov, Savchuk, Gorenstein, Koch, VV, Phys. Rev. C 105, 044903 (2022)

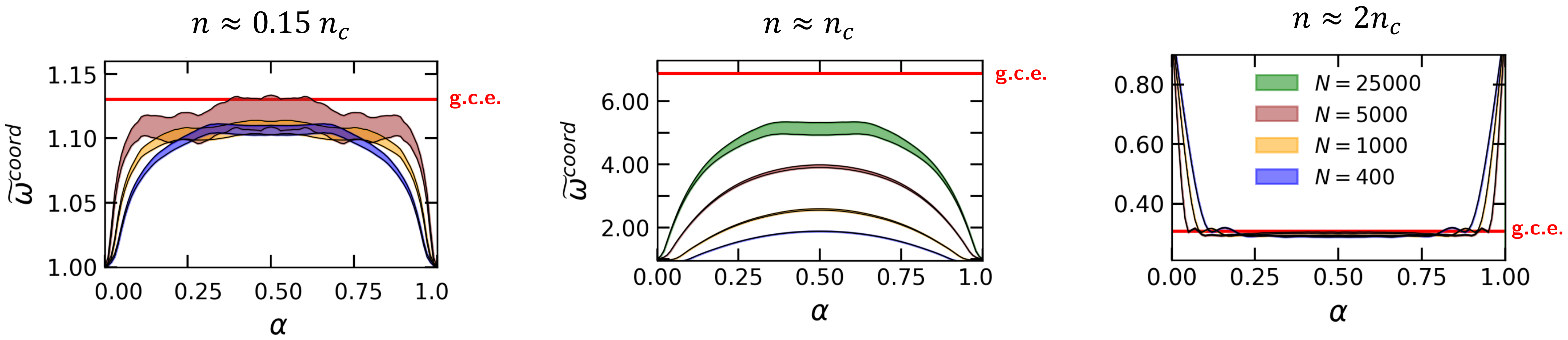
Classical molecular dynamics simulations* of a **Lennard-Jones fluid** along the (super)critical isotherm of the liquid-gas transition



Microcanonical (const. *EVN*) ensemble with periodic boundary conditions

Variance of conserved particle number distribution inside coordinate space subvolume $|z| < z^{max}$ as time average

$$\tilde{\omega}^{\rm coord} = \frac{1}{1-\alpha} \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$

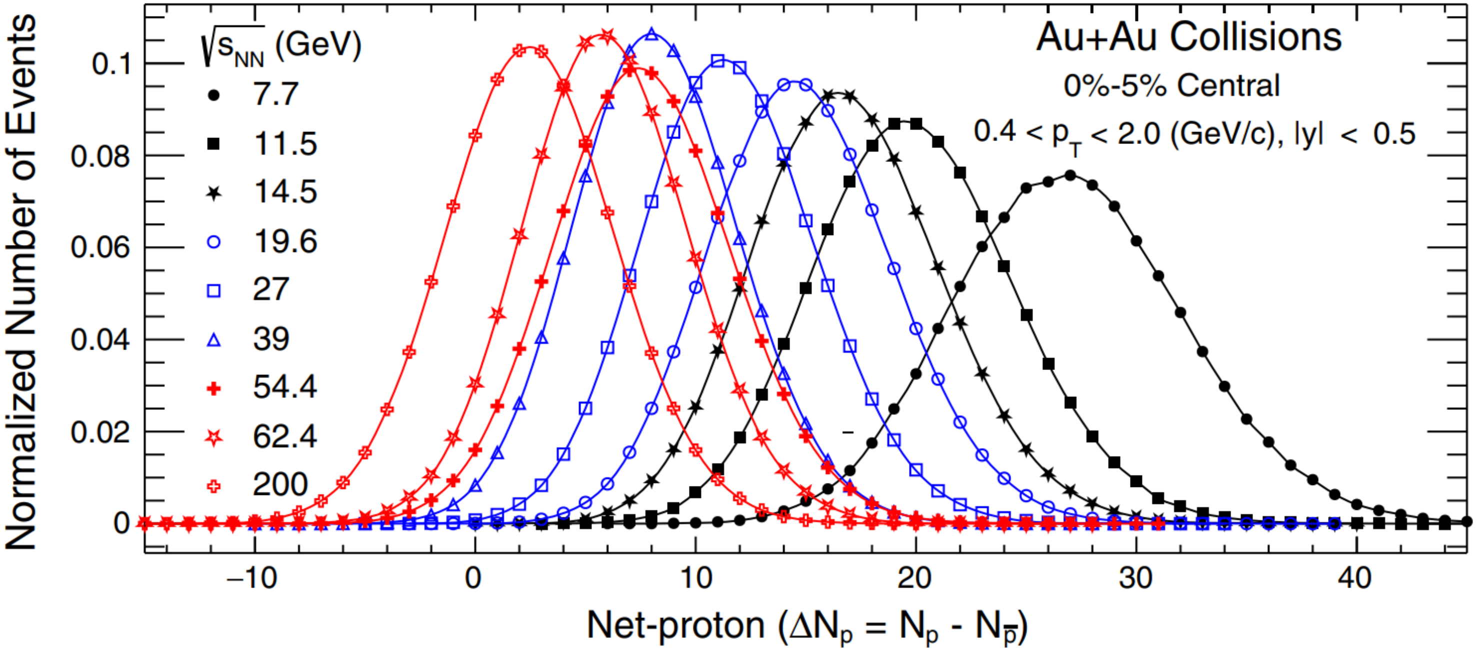


*Molecular dynamics code from https://github.com/vlvovch/lennard-jones-cuda

Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons $P(\Delta N_p) \sim \frac{N_{events}(\Delta N_p)}{N_{events}^{total}}$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)



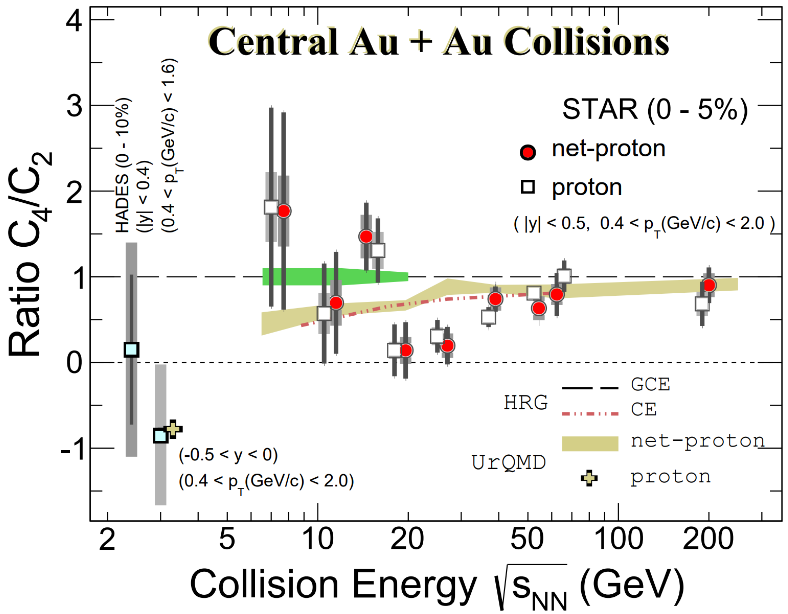
Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

$$\frac{\kappa_2}{\langle N \rangle}, \qquad \frac{\kappa_3}{\kappa_2}, \qquad \frac{\kappa_4}{\kappa_2}$$

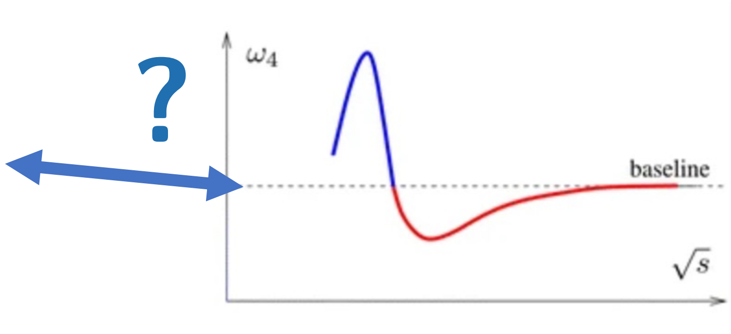
Experimental measurements

UNIVERSITY OF HOUSTON

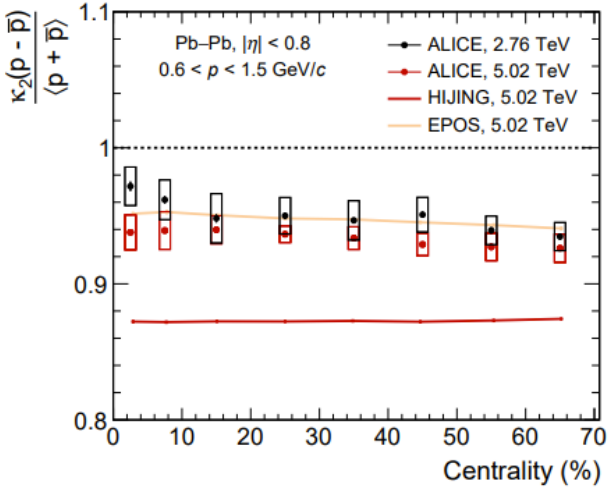
Beam energy scan in search for the critical point (STAR Coll.)



STAR Coll., Phys. Rev. Lett. 126, 092301 (2021); arXiv:2112.00240



M. Stephanov, Phys. Rev. Lett. (2011)



ALICE Coll., arXiv:2206.03343

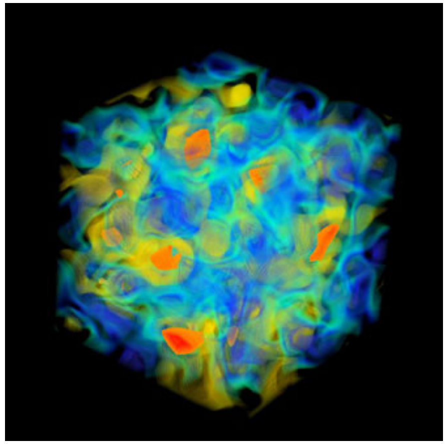
Reduced errors (better statistics), more energies, to come soon from RHIC-BES-II program, STAR-FXT etc.

Can we learn more from the more accurate data available for κ_2 and κ_3?

Other measurements: LHC-ALICE, GSI-HADES & CERN-NA61/SHINE Collaborations

Theory vs experiment: Challenges for fluctuations

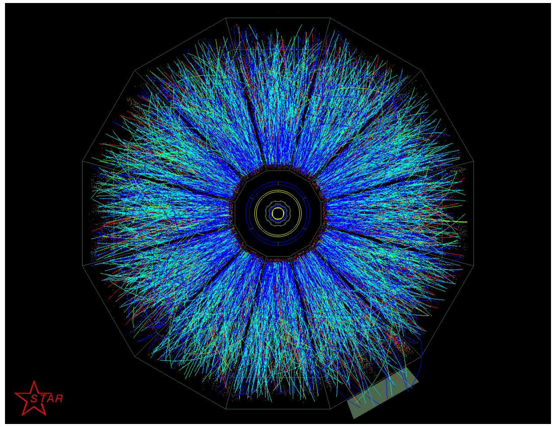
Theory



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

Need dynamical description

Dynamical approaches to the QCD critical point search

1. Dynamical model calculations of critical fluctuations BEST COLLABORATION [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]
   - Fluctuating hydrodynamics (hydro+) or molecular dynamics
   - Equation of state with a tunable critical point [P. Parotto et al, PRC 101, 034901 (2020); J. Karthein et al., EPJ Plus 136, 621 (2021)]
   - Generalized Cooper-Frye particlization [M. Pradeep, et al., PRD 106, 036017 (2022); PRL 130, 162301 (2023)]

Alternatives at high μ_B: hadronic transport/molecular dynamics with a critical point
[A. Sorensen, V. Koch, PRC 104, 034904 (2021); V. Kuznietsov et al., PRC 105, 044903 (2022)]

2. **Deviations from precision calculations of non-critical fluctuations**
   - Include essential non-critical contributions to (net-)proton number cumulants
   - Exact baryon conservation + hadronic interactions (hard core repulsion)
   - Based on realistic hydrodynamic simulations tuned to bulk data

[VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

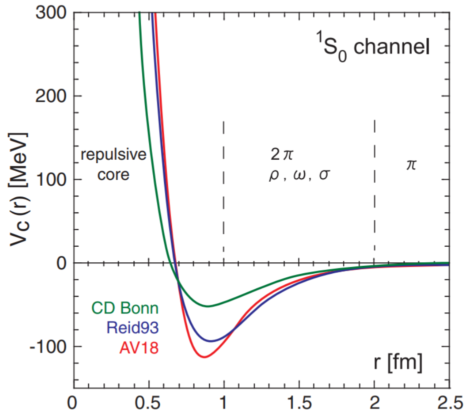


Figure from Ishii et al., PRL '07

Excluded volume effect

Incorporate repulsive baryon (nucleon) hard core via excluded volume

VV, M.I. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

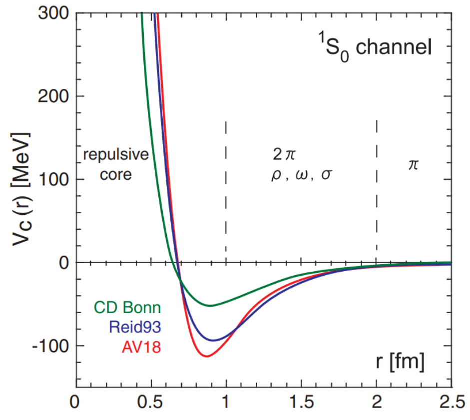
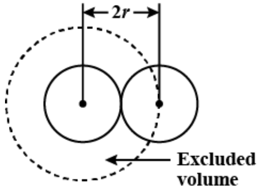


Figure from Ishii et al., PRL '07

Amounts to a van der Waals correction for baryons in the HRG model

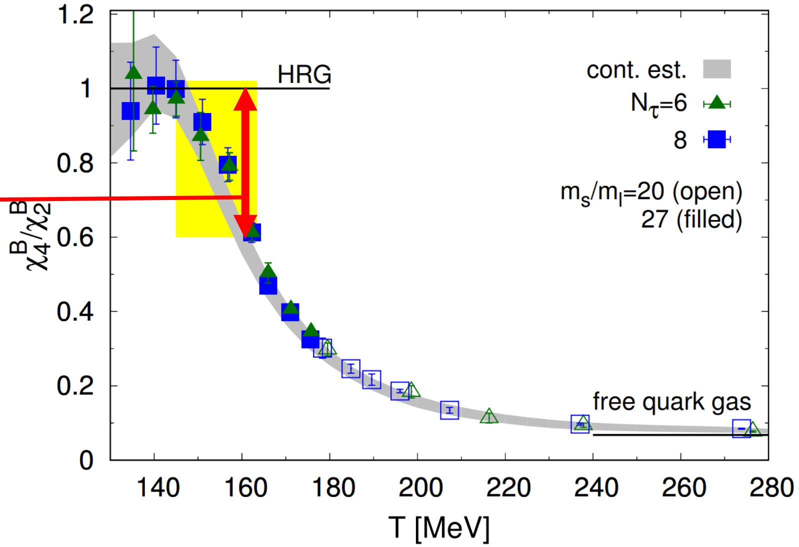
$$V \to V - bN$$



$$p^{\text{ev}}_{B(\bar{B})} = p^{\text{id}}_{B(\bar{B})}\, e^{-b p^{\text{ev}}_{B(\bar{B})}/T}$$

- Net baryon kurtosis suppressed as in lattice QCD*

$$\frac{\chi_4^B}{\chi_2^B} \simeq 1 - 12 b \phi_B(T) + O(b^2)$$



- Reproduces virial coefficients of baryon interaction from lattice QCD

VV, A. Pasztor, S. Katz, Z. Fodor, H. Stoecker, Phys. Lett. B 755, 71 (2017)

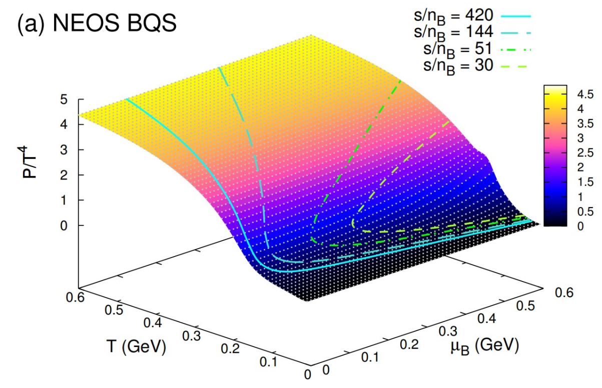
Excluded volume from lattice QCD: $b \approx 1$ fm^3

*J.M. Karthein, V. Koch, C. Ratti, VV, PRD 104, 094009 (2021)

Hydrodynamic description within non-critical physics

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- Collision geometry based 3D initial state
  - Constrained to net proton distributions [Shen, Alzhrani, Phys. Rev. C 102, 014909 (2020)]
- Viscous hydrodynamics evolution – MUSIC-3.0
  - Energy-momentum and baryon number conservation
  - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]



- Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3

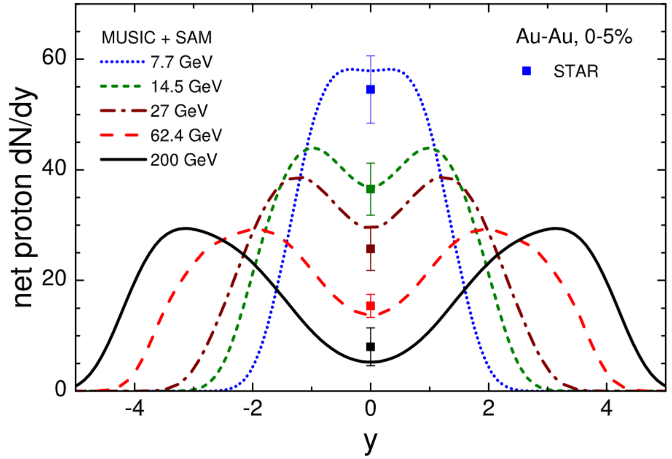
$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x)\, p^\mu \, \frac{d_j\, \lambda_j^{\text{ev}}(x)}{(2\pi)^3} \exp\left[\frac{\mu_j(x) - u^\mu(x) p_\mu}{T(x)}\right].$$

- Particlization respects QCD-based baryon number distribution
  - Incorporated via baryon excluded volume b = 1 fm^3

[VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]

- Incorporates exact global baryon conservation (and other charges)

[VV, Phys. Rev. C 105, 014903 (2022)]



- Absent: critical point, local conservation, initial-state/volume fluctuations, hadronic phase

Calculating cumulants from hydrodynamics

UNIVERSITY OF HOUSTON

- **Analytic approach** VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)
  - Calculate proton cumulants in the experimental acceptance in the grand-canonical limit using the Cooper-Frye formula to model acceptance effect
  - Apply correction for the exact global baryon number conservation (SAM-2.0)
    VV, Phys. Rev. C 105, 014903 (2022)

Pros: Calculate high-order cumulants (up to 8th order) without the need for large statistics

Cons: The method is approximate and not easily extendable to other observables

- **Monte Carlo approach (FIST sampler)** VV, Phys. Rev. C 106, 064906 (2022) https://github.com/vlvovch/fist-sampler
  - Event generator (Cooper-Frye particlization)
  - Conservation laws (baryon number but also charge and strangeness) via rejection sampling
  - Excluded volume effect by rejecting coordinate space overlap of baryons

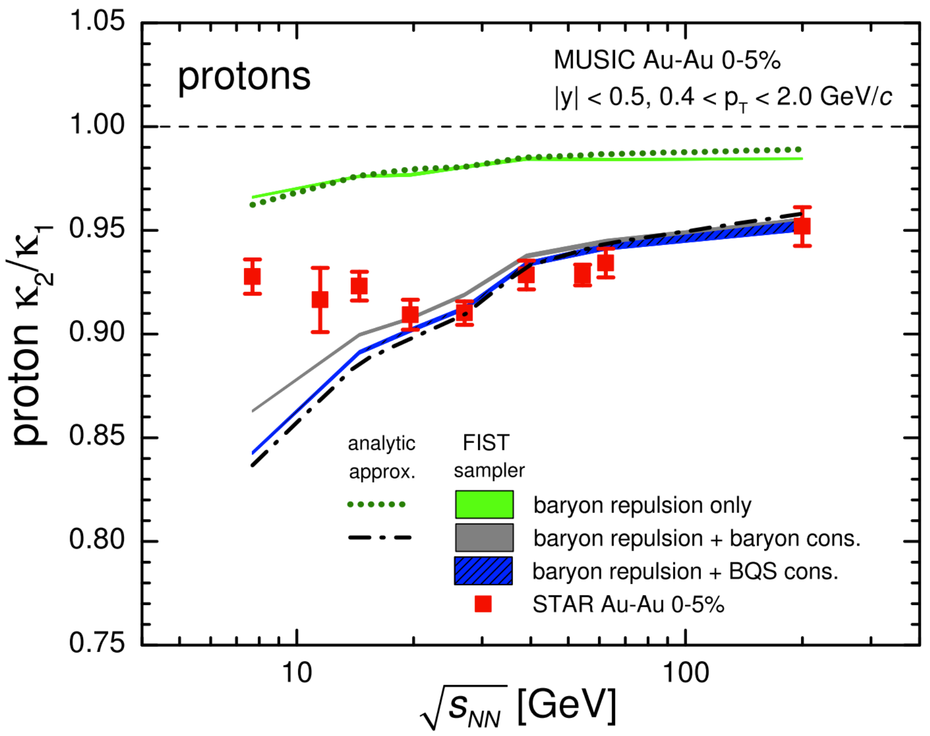
Pros: Flexibility of an event generator, more accurate

Cons: Need large statistics for high-order cumulants

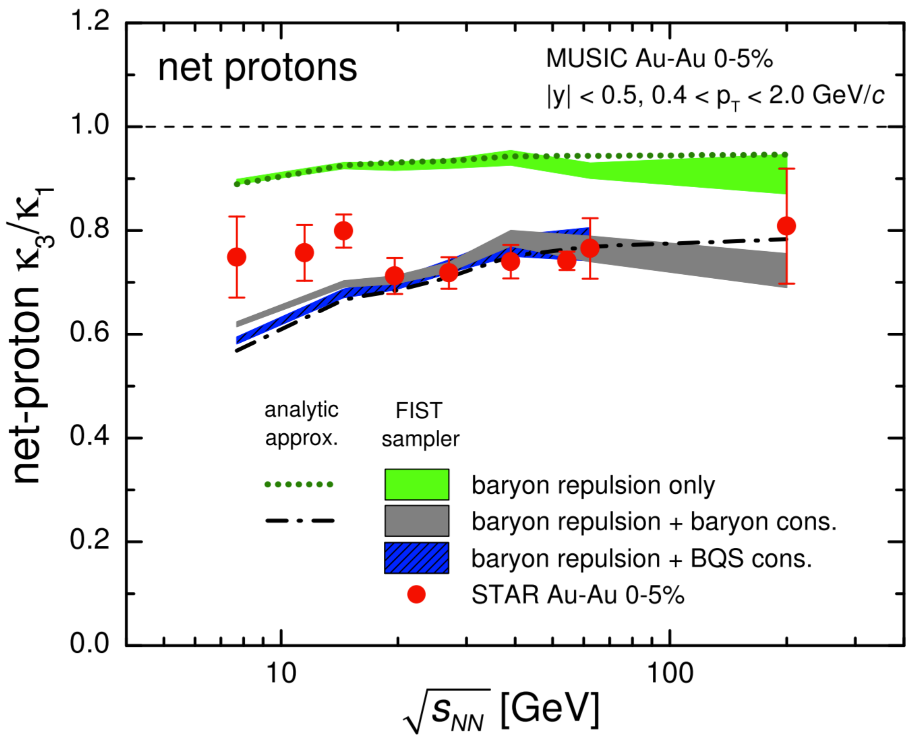
RHIC-BES: Net proton cumulant ratios (MUSIC)

UNIVERSITY OF HOUSTON

Proton κ_2/κ_1



Net-proton κ_3/κ_1



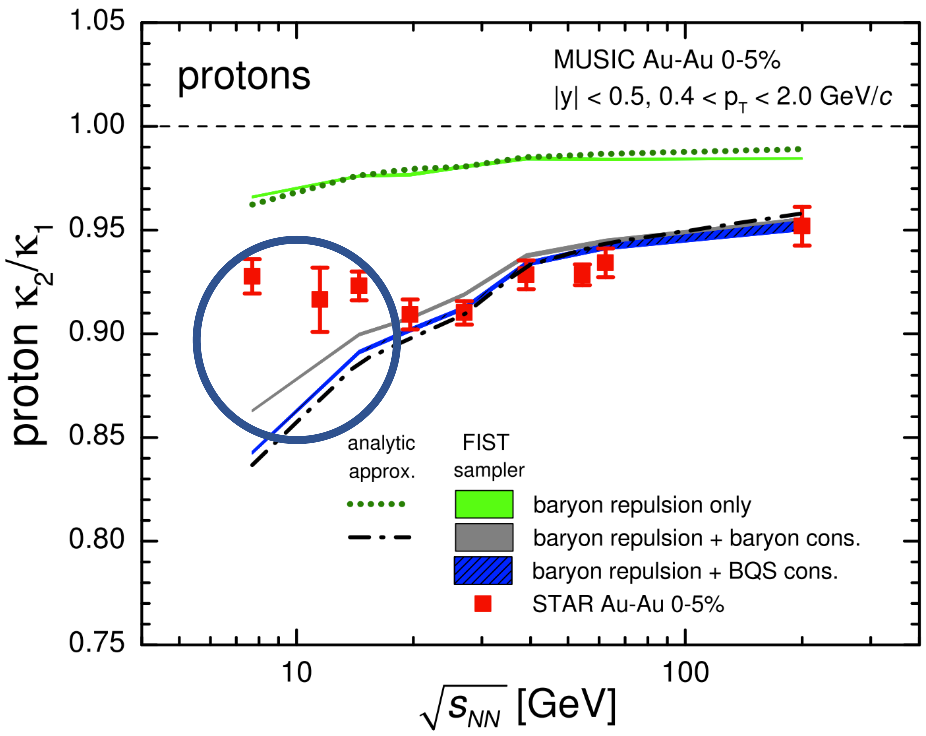
- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (BQS conservation and repulsion)
- Effect from baryon conservation is larger than from repulsion
- Excess of fluctuations in data at $\sqrt{s_{NN}} < 20$ GeV – *hint of attractive interactions?*

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022); VV, Phys. Rev. C 106, 064906 (2022)

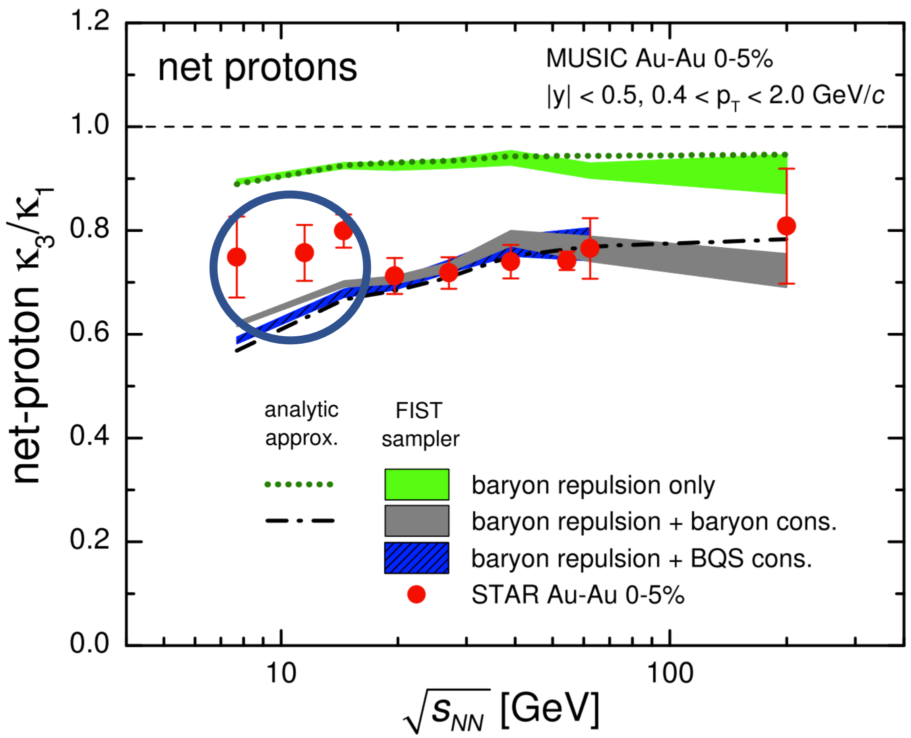
RHIC-BES: Net proton cumulant ratios (MUSIC)

UNIVERSITY OF HOUSTON

Proton κ_2/κ_1



Net-proton κ_3/κ_1

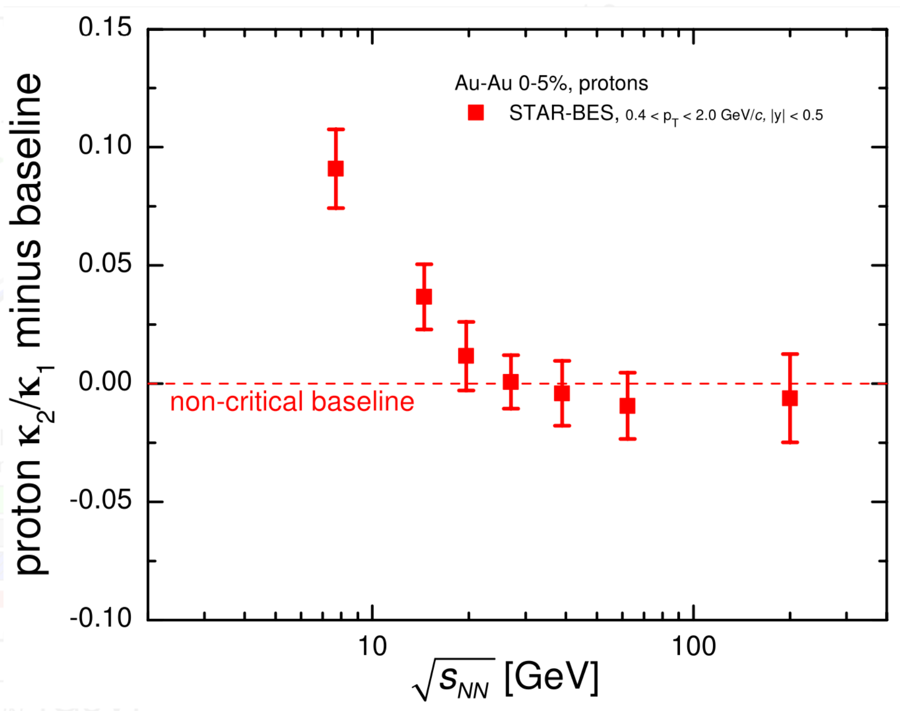


- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (BQS conservation and repulsion)
- Effect from baryon conservation is larger than from repulsion
- Excess of fluctuations in data at $\sqrt{s_{NN}} < 20$ GeV – *hint of attractive interactions?*

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022); VV, Phys. Rev. C 106, 064906 (2022)

RHIC-BES: Net proton cumulant ratios (MUSIC)

Proton κ_2/κ_1 excess over baseline



- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (BQS conservation and repulsion)
- Effect from baryon conservation is larger than from repulsion
- Excess of fluctuations in data at $\sqrt{s_{NN}} < 20$ GeV – *hint of attractive interactions?*

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022); VV, Phys. Rev. C 106, 064906 (2022)

Correlation Functions (factorial cumulants)

- Analyze genuine multi-particle correlations via **factorial cumulants** $\hat{C}_n$
  [Bzdak, Koch, Strodthoff, Phys. Rev. C '17]

$$\hat{C}_1 = \kappa_1, \qquad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$
$$\hat{C}_2 = -\kappa_1 + \kappa_2, \quad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$

- Three- and four-particle correlations are small without a CP

$\hat{C}_n^{\text{cons}} \propto \alpha^n,$ [Bzdak, Koch, Skokov, EPJC '17]

$\hat{C}_n^{\text{EV}} \propto b^n$ [VV et al, PLB '17]

  - Multi-particle correlations expected near the critical point [Ling, Stephanov, PRC '15]

Correlation Functions (factorial cumulants)

- Analyze genuine multi-particle correlations via **factorial cumulants** $\hat{C}_n$ [Bzdak, Koch, Strodthoff, Phys. Rev. C '17]

$$\hat{C}_1 = \kappa_1, \qquad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$
$$\hat{C}_2 = -\kappa_1 + \kappa_2, \quad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$

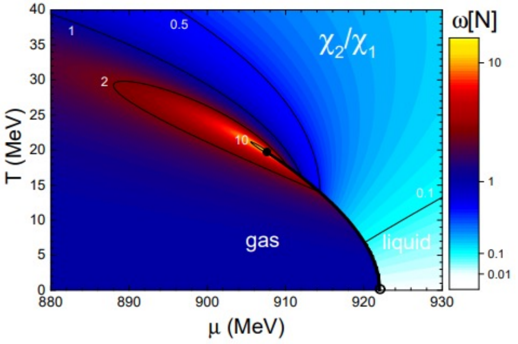
- Three- and four-particle correlations are small without a CP

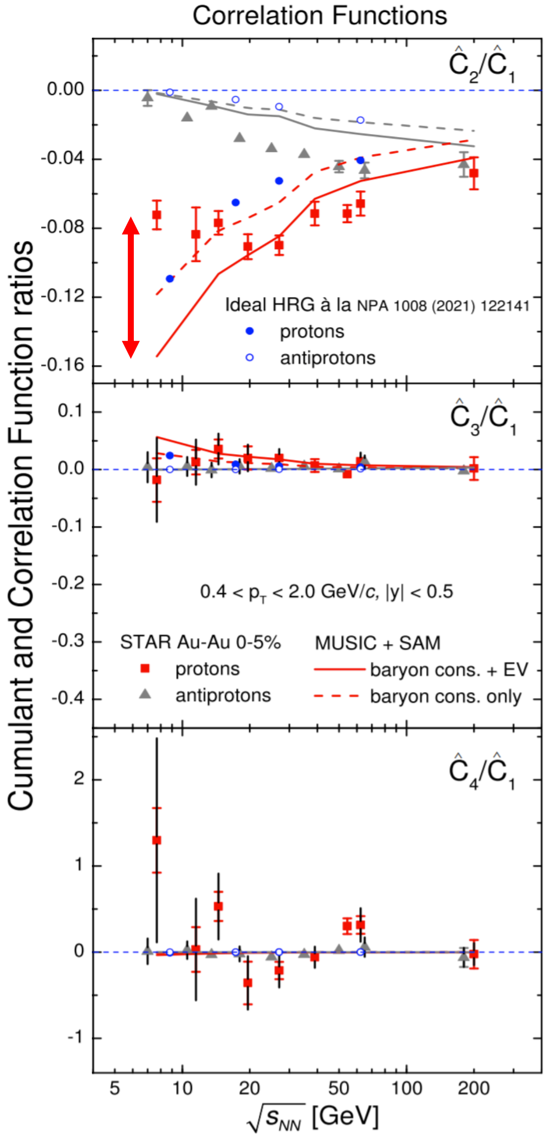
$$\hat{C}_n^{\text{cons}} \propto \alpha^n, \qquad \hat{C}_n^{\text{EV}} \propto b^n$$

[Bzdak, Koch, Skokov, EPJC '17] [VV et al, PLB '17]

  - Multi-particle correlations expected near the critical point [Ling, Stephanov, PRC '15]

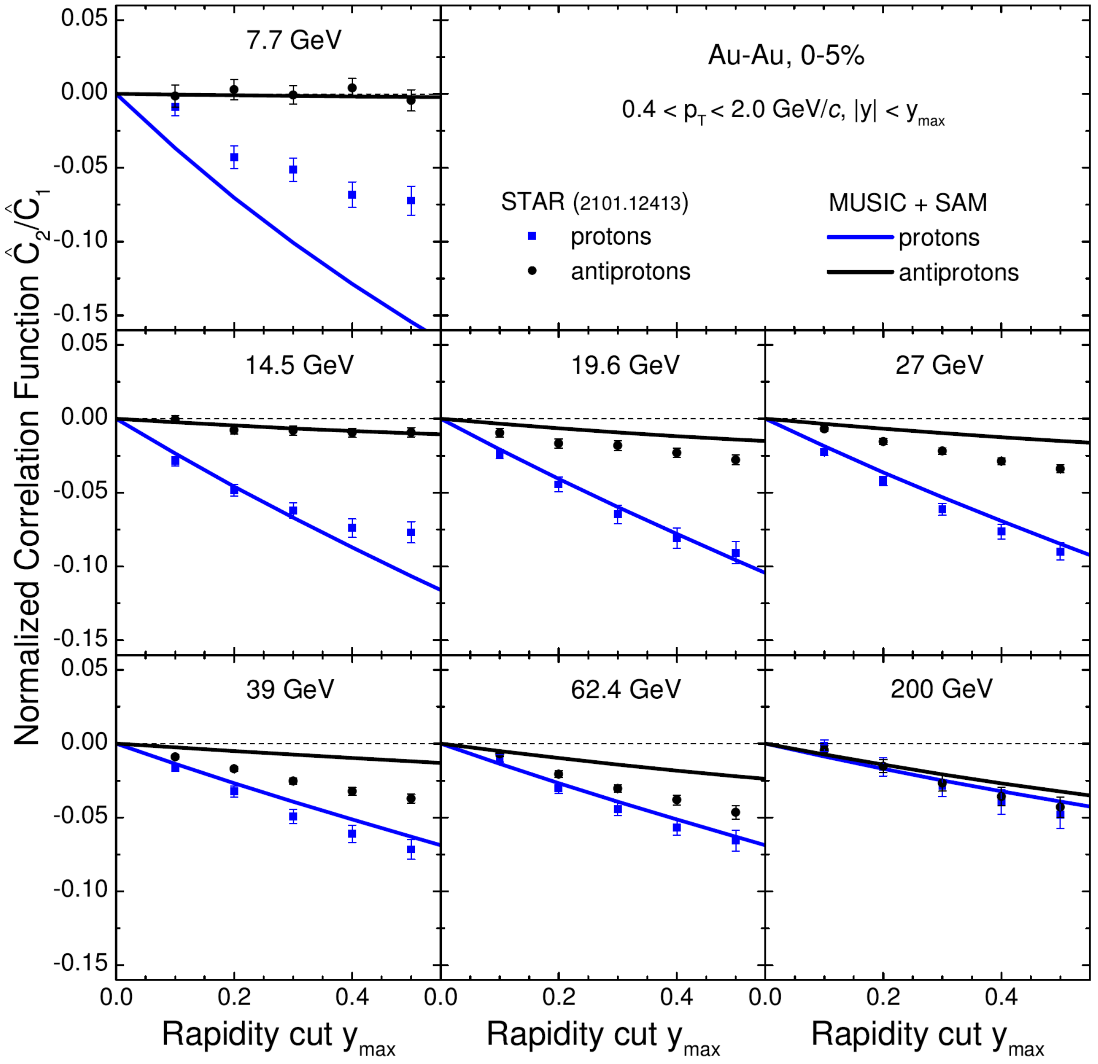
- Signals from the data at $\sqrt{s_{NN}} \leq 20$ GeV
  - Excess of two-proton correlations
  - Possibility of significant 4-proton correlations
  - ***Critical point?***





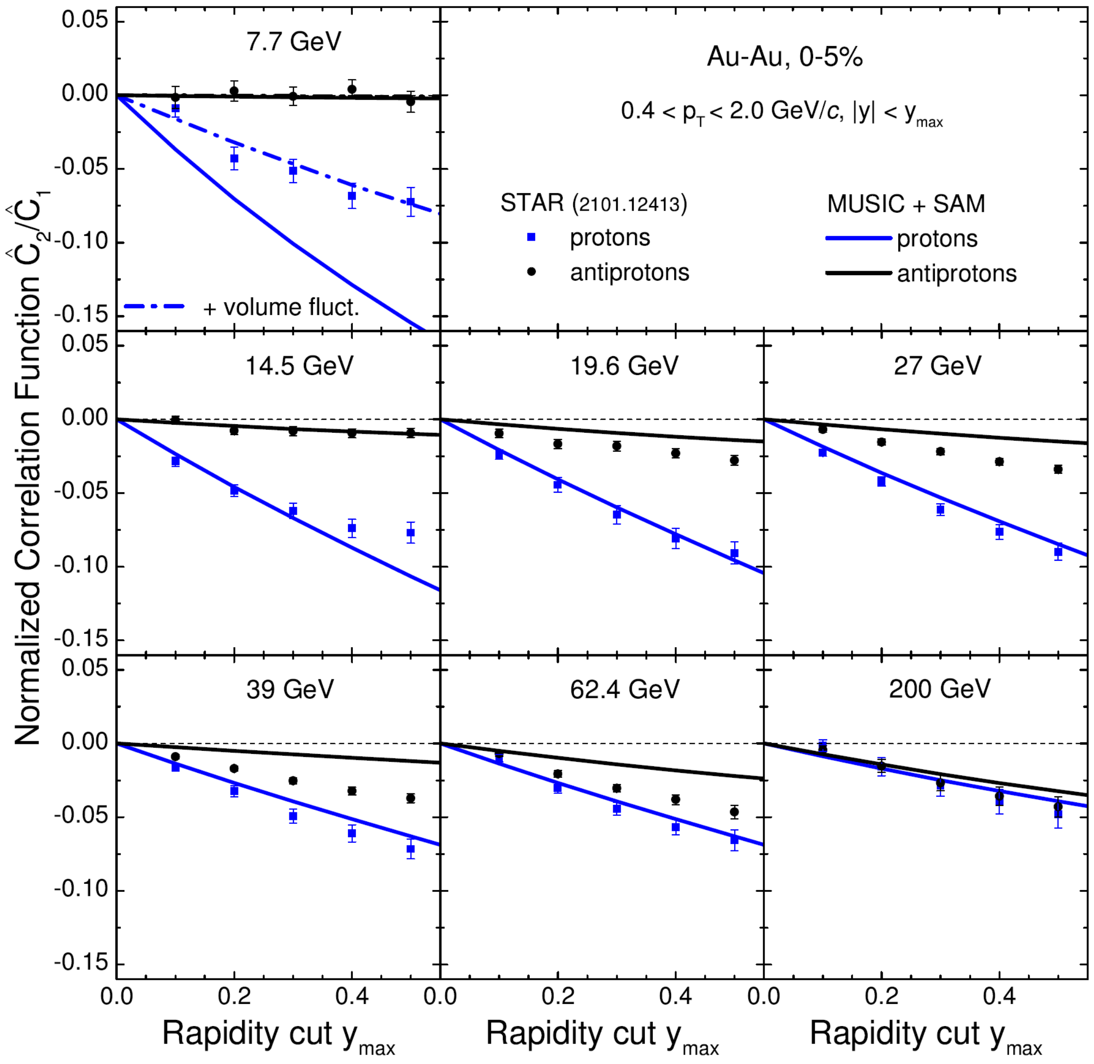
Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?



Acceptance dependence of two-particle correlations

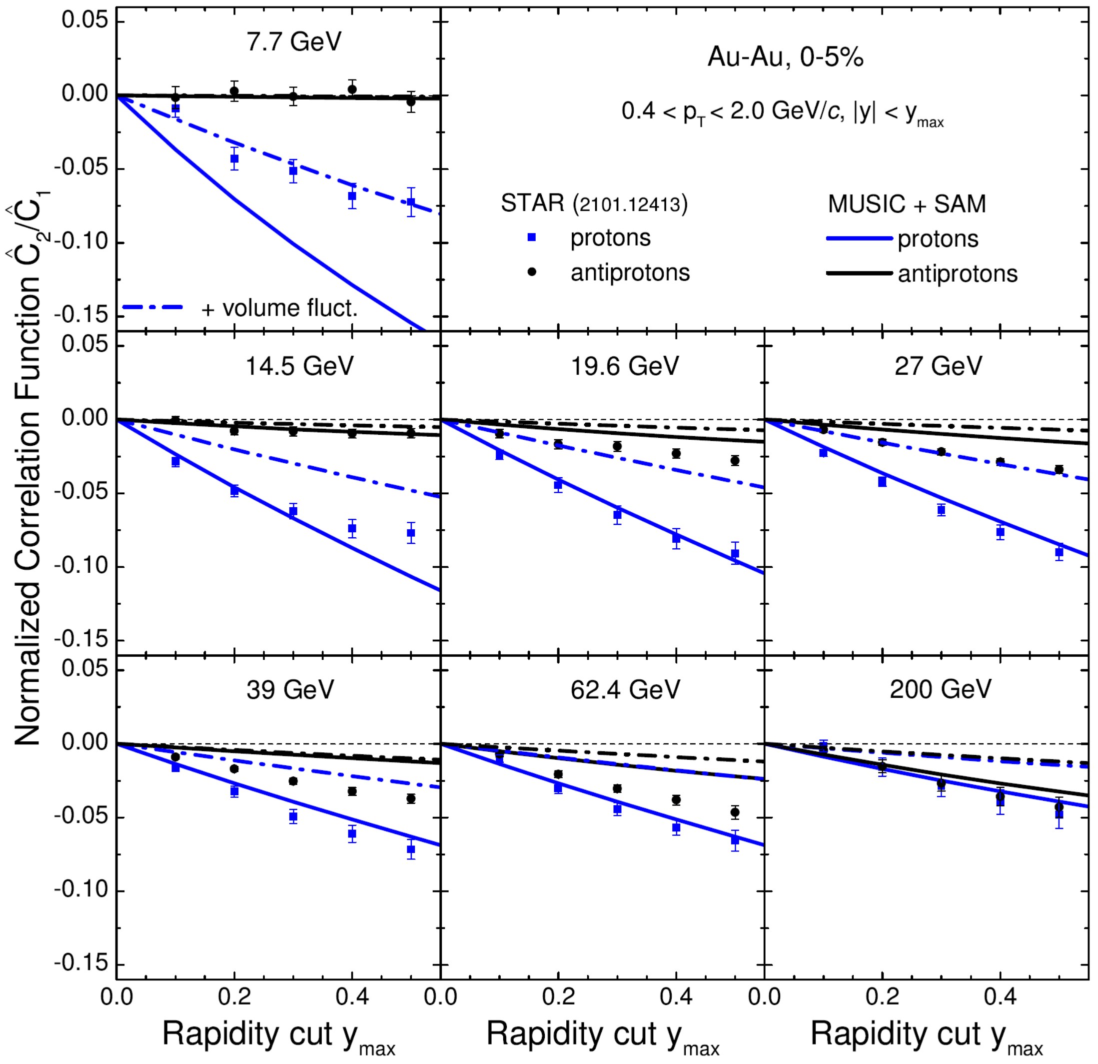
- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?
- **Volume fluctuations?** [Skokov, Friman, Redlich, PRC '13]
  - $C_2/C_1 \mathrel{+}= C_1 * \Delta v^2$



Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?

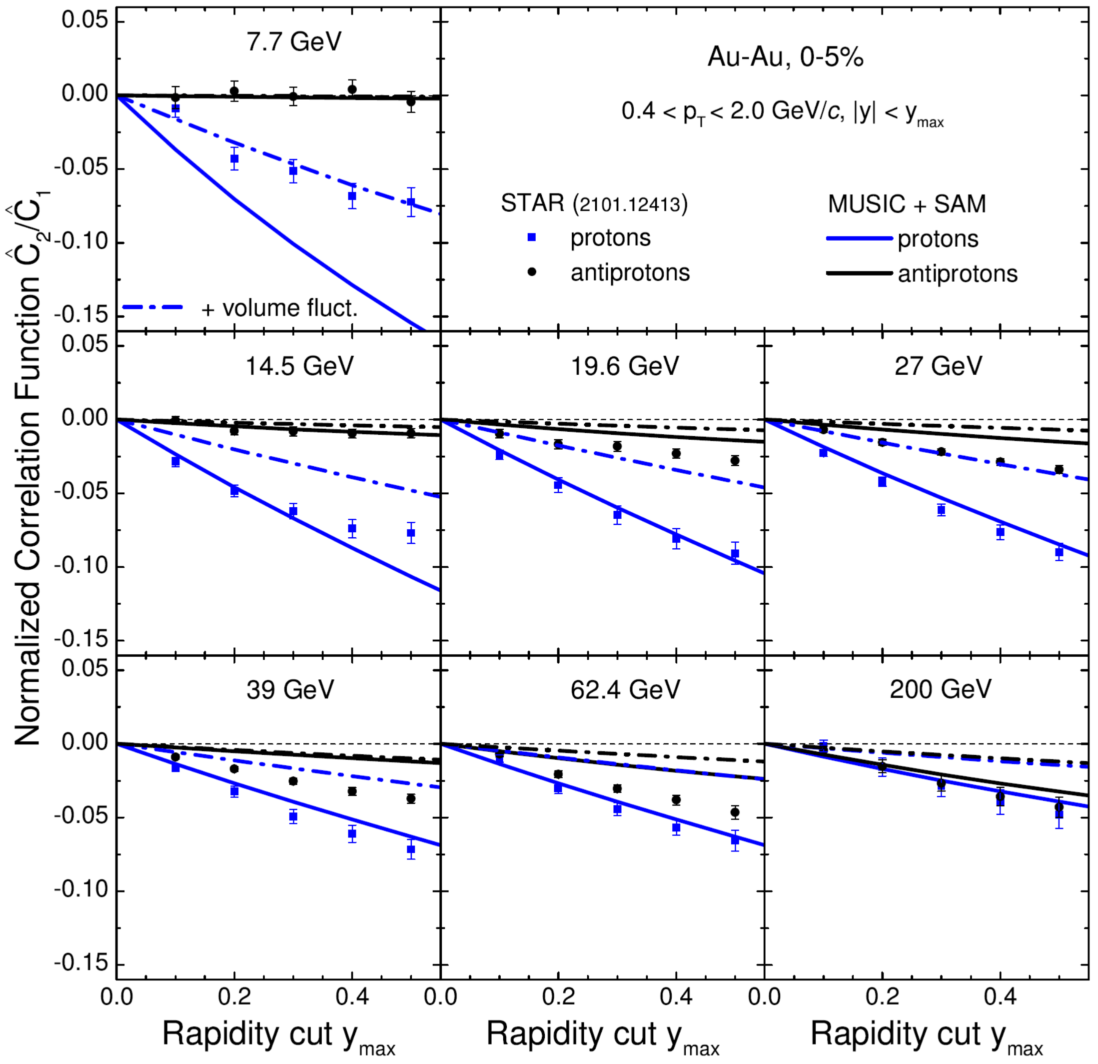
- **Volume fluctuations?** [Skokov, Friman, Redlich, PRC '13]
  - $C_2/C_1 \mathrel{+}= C_1 * \Delta v^2$
  - Can improve low energies but spoil high energies?



Acceptance dependence of two-particle correlations

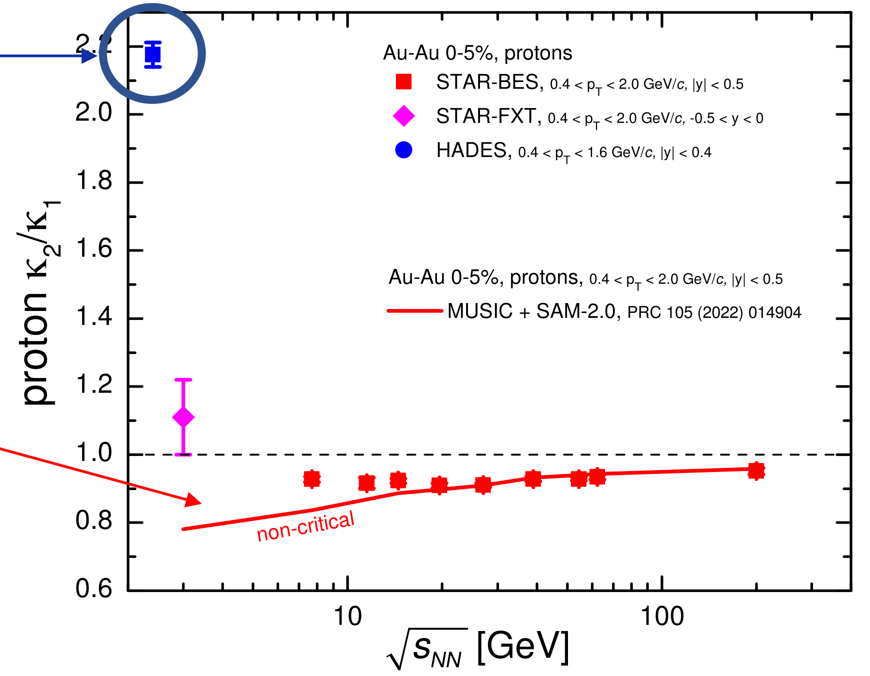
UNIVERSITY OF HOUSTON

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?
- Volume fluctuations? [Skokov, Friman, Redlich, PRC '13]
  - $C_2/C_1 \mathrel{+}= C_1 * \Delta v^2$
  - Can improve low energies but spoil high energies?
- **Attractive interactions?**
  - Could work if baryon repulsion turns into attraction in the high-μ_B regime
  - **Critical point?**

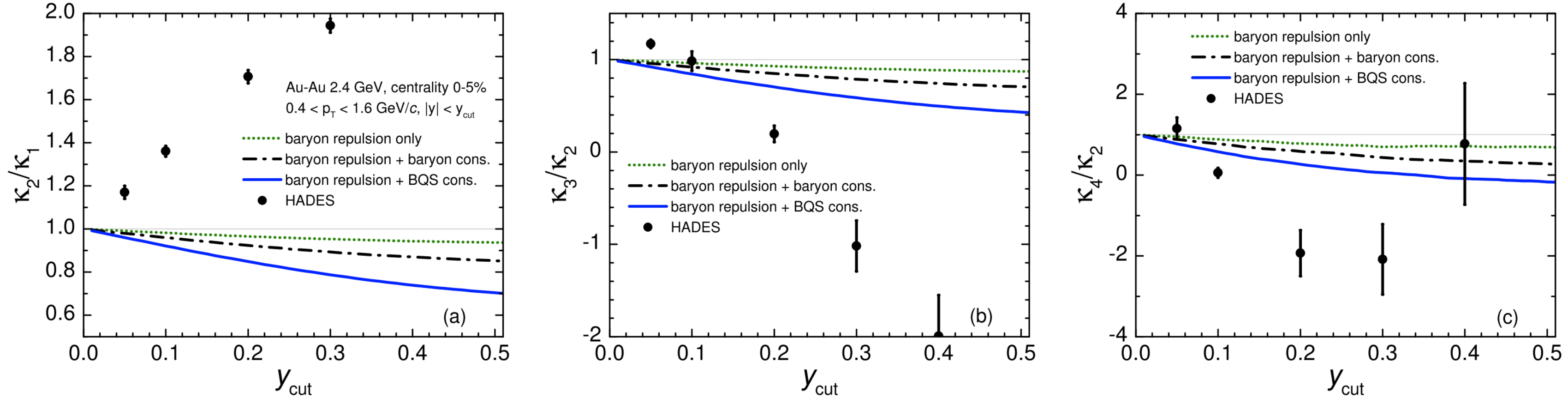


Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV

- Intriguing hints from HADES@ 2.4 GeV and STAR-FXT@3GeV: huge excess of two-proton correlations! [HADES Collaboration, Phys. Rev. C 102, 024914 (2020)] [STAR Collaboration, Phys. Rev. Lett. 128, 202303 (2022)]

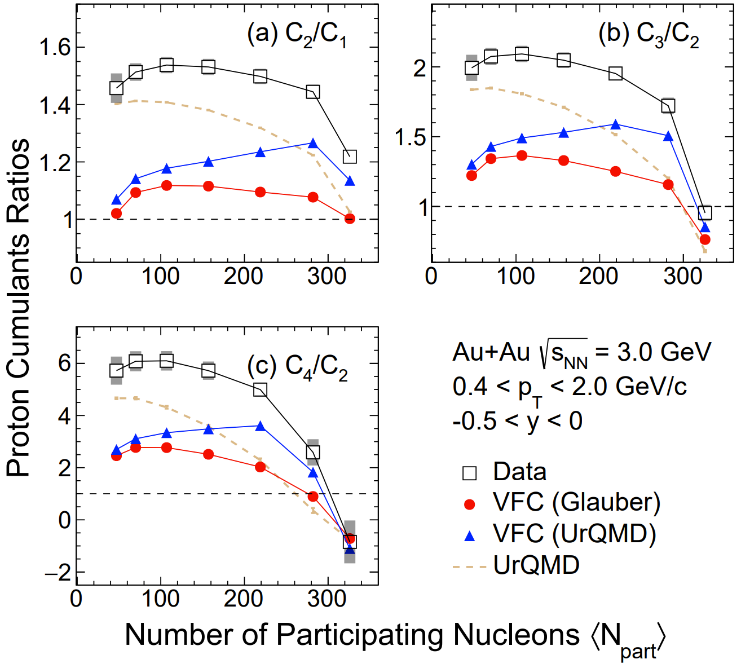


- No change of trend in the non-critical reference
- Additional mechanisms:
  - Nuclear liquid-gas transition
  - Light nuclei formation/fragmentation



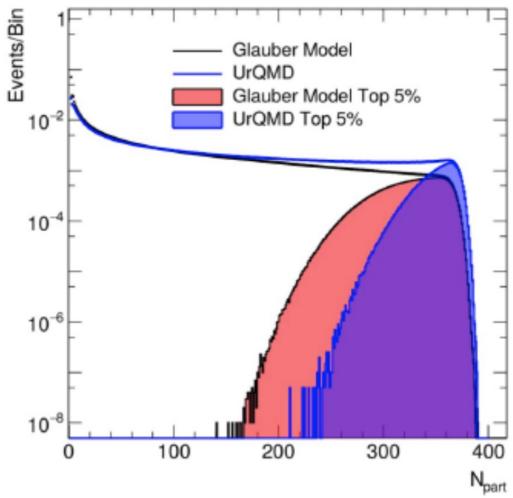
VV, Phys. Rev. C 106, 064906 (2022)

Closer look at data at lower energies



STAR Collaboration, Phys. Rev. Lett. 128 (2022) 202303

STAR-FXT



HADES

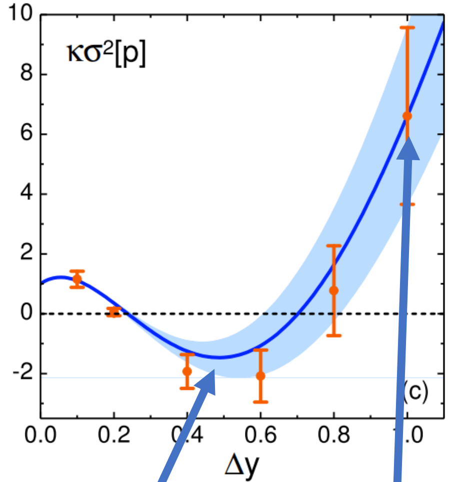


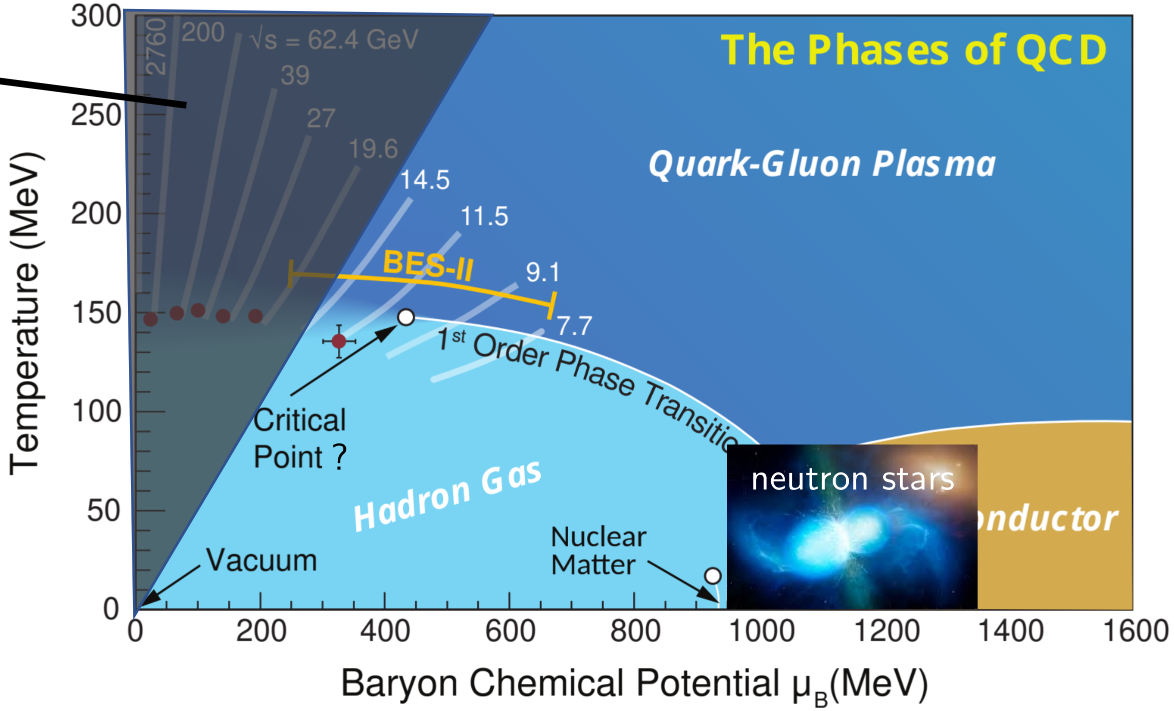
Figure from O. Savchuk et al., PLB 835, 137540 (2022)

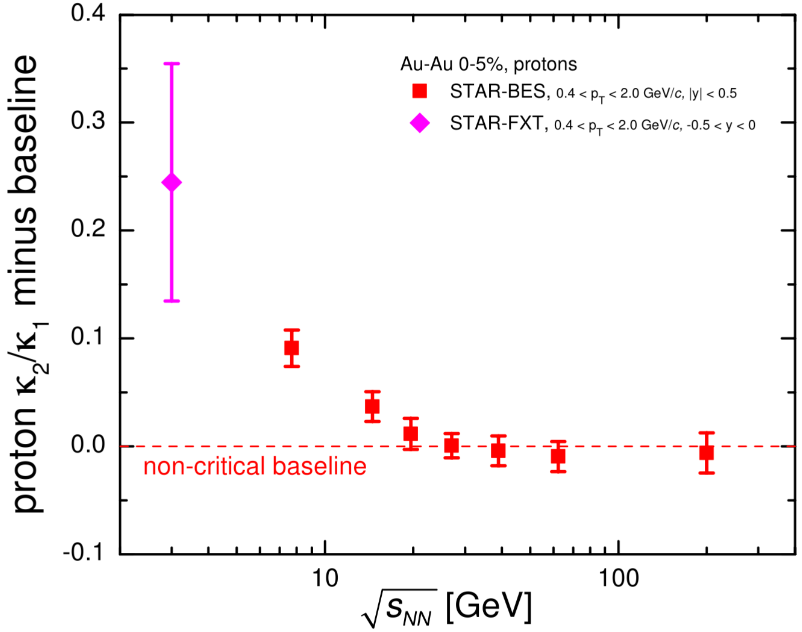
- Volume fluctuations/centrality selection appear to play an important role
  - UrQMD is useful for understanding basic systematics associated with it
- Indications for enhanced scaled variance, $\kappa_2/\kappa_1 > 1$
- κ_4/κ_2 negative and described by UrQMD (purely hadronic?), note -0.5<y<0 instead of |y|<0.5

Proper understanding of $\kappa_2/\kappa_1 > 1$ in both HADES and STAR-FXT is missing

Summary: What we learned so far from fluctuations

Critical point disfavored





- Data at high energies ($\sqrt{s_{NN}} \geq 20$ GeV) consistent with "non-critical" physics
  - Disfavors QCD critical point at $\mu_B/T < 2$-3, consistent with what we know from lattice QCD
- Interesting indications for (multi)-proton correlations at $\sqrt{s_{NN}} \leq 7.7$ GeV, better modeling required

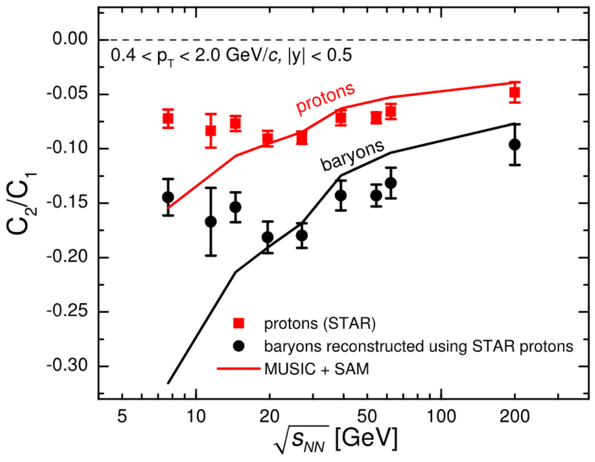
Thanks for your attention!

Backup slides

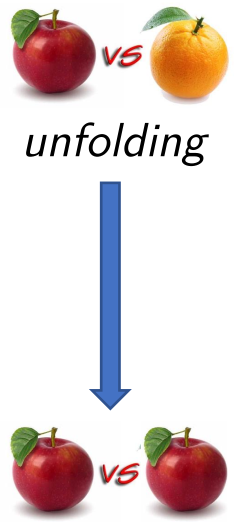
Baryon cumulants from protons

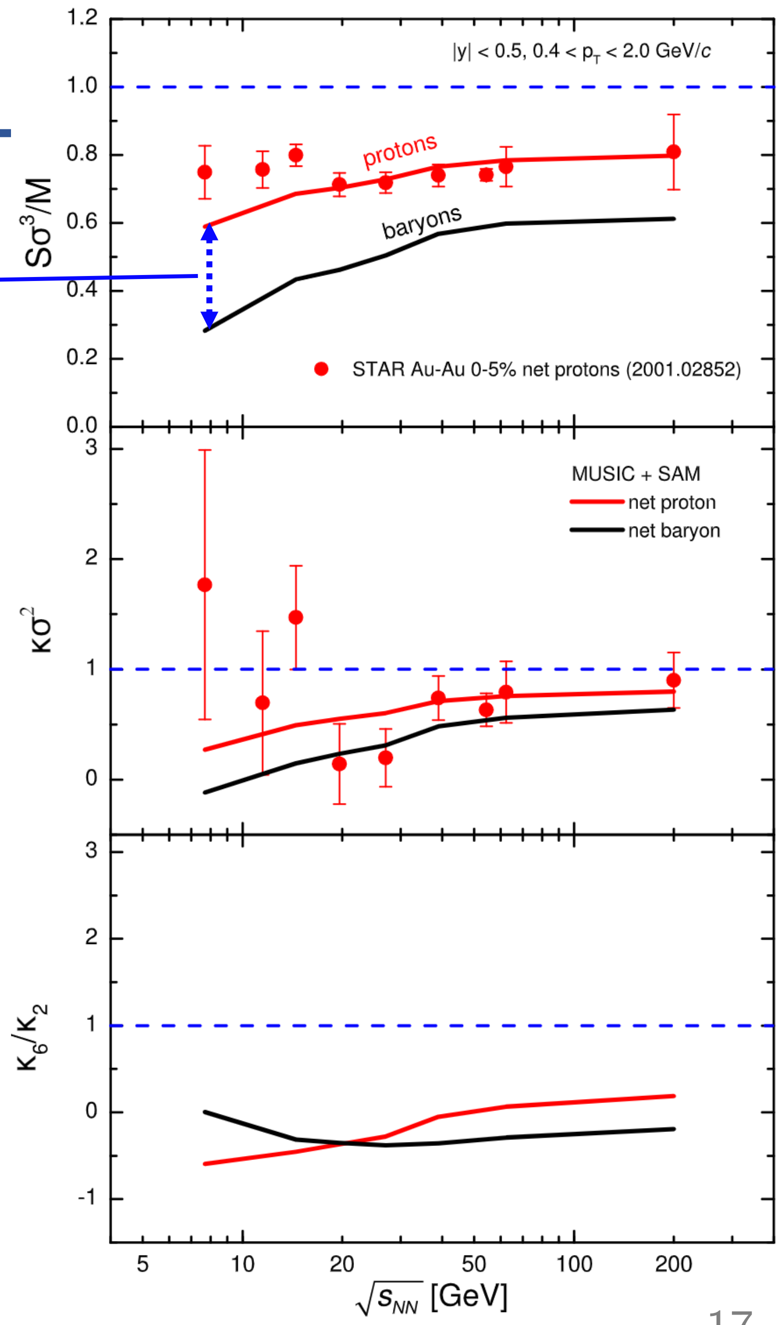
- **net baryon** $\neq$ **net proton** (protons are ***subset*** of all baryons)

- Baryon cumulants can be reconstructed from proton cumulants via binomial (un)folding based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]
  - Amounts to an additional "efficiency correction" and requires the use of joint factorial moments, only experiment can do it model-independently



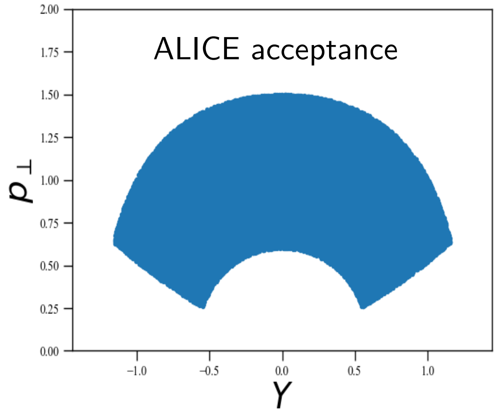
$$\frac{\hat{C}_2^B}{\hat{C}_1^B} \approx 2\,\frac{\hat{C}_2^p}{\hat{C}_1^p}$$



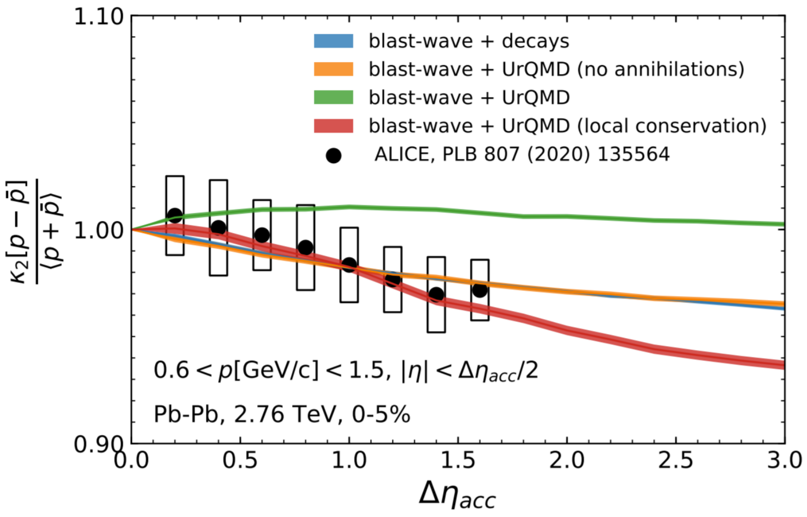


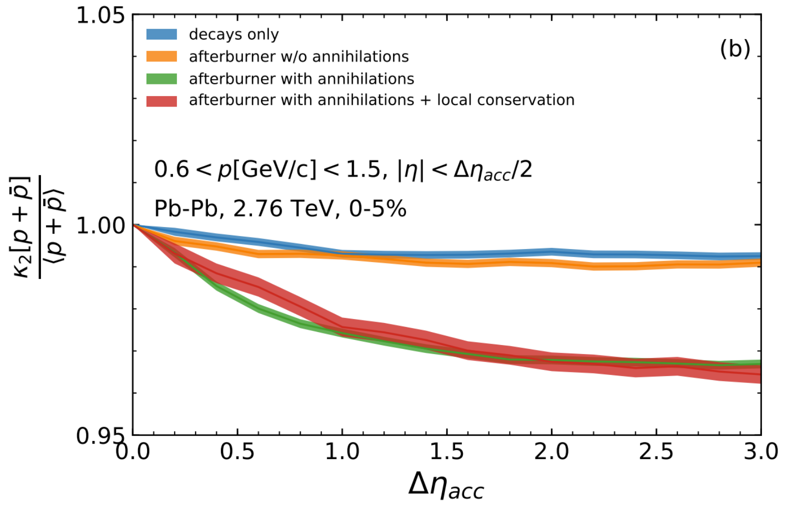
Net-particle fluctuations at the LHC (blast-wave model)

- Net protons described within errors and consistent with either VV, Koch, Phys. Rev. C 103, 044903 (2021)
  - global baryon conservation without $B\bar{B}$ annihilations in the hadronic phase
    see e.g. ALICE Coll. arXiv:2206.03343
  - or local baryon conservation with $B\bar{B}$ annihilations in the hadronic phase
    O. Savchuk et al., Phys. Lett. B 827, 136983 (2022)



$0.6 < p < 1.5$ GeV/c, $\Delta\eta_{acc} = 1.6$





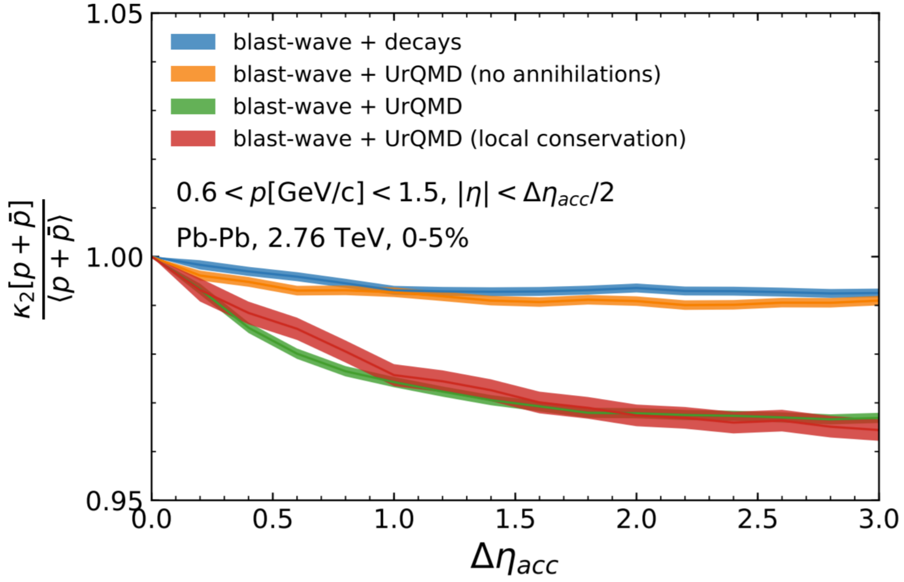
Data on (net-)proton fluctuations can constrain the effect of annihilations in the hadronic phase

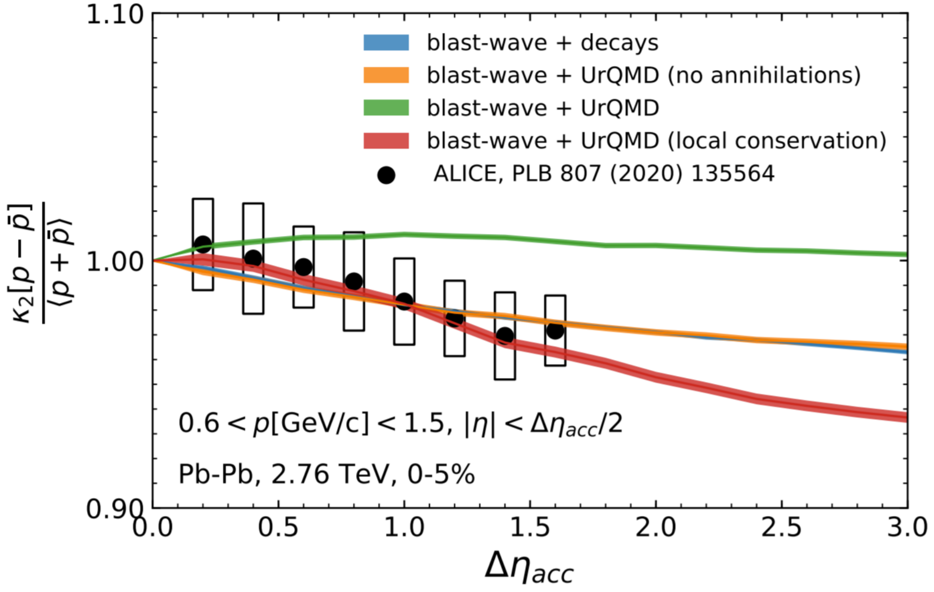
Effects of baryon annihilation and local conservation

O. Savchuk, V.V., V. Koch, J. Steinheimer, H. Stoecker, arXiv:2106.08239

Baryon annihilation $B\bar{B} \to n\pi$ in afterburners (UrQMD, SMASH) suppresses baryon yields

$\langle p + \bar{p} \rangle \searrow$ $\rightarrow$ $\frac{\kappa_2[p-\bar{p}]}{\langle p + \bar{p} \rangle} \nearrow$





- ALICE data requires local baryon conservation across $\Delta y \sim \pm 1.5$ with UrQMD annihilations (no regenerations) or global conservation ($\Delta y \sim \Delta y_{tot}$) without annihilations
- Local conservation and $B\bar{B}$ annihilation can be constrained from data through the combined analysis of $\kappa_2[p-\bar{p}]$ and $\kappa_2[p+\bar{p}]$

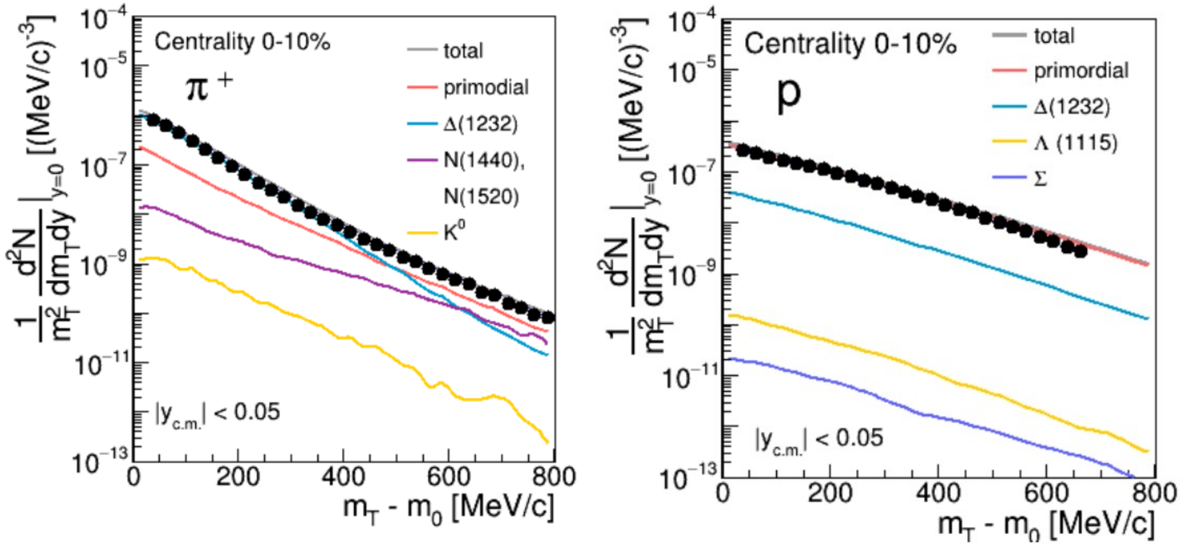
Thermodynamic analysis of HADES data

VV, Koch, Phys. Lett. B 833, 137368 (2022)

- Single freeze-out scenario: Emission from Siemens-Rasmussen hypersurface with Hubble-like flow
  - → Pion and proton spectra o.k.

  [S. Harabasz et al., PRC 102, 054903 (2020)]

- Uniform $T \approx 70$ MeV, $\mu_B \approx 875$ MeV across the fireball [A. Motornenko et al., PLB 822, 136703 (2021)]



- Fluctuations:
  - Same as before but incorporate additional binomial filtering to account for protons bound in light nuclei
  - Uniform fireball → Final proton cumulants are linear combinations of baryon susceptibilities χ_n^B at freeze-out

$$\kappa_n^p = \sum_{m=1}^{n} \alpha_{n,m}\, \chi_m^B$$

Extract χ_n^B directly from experimental data

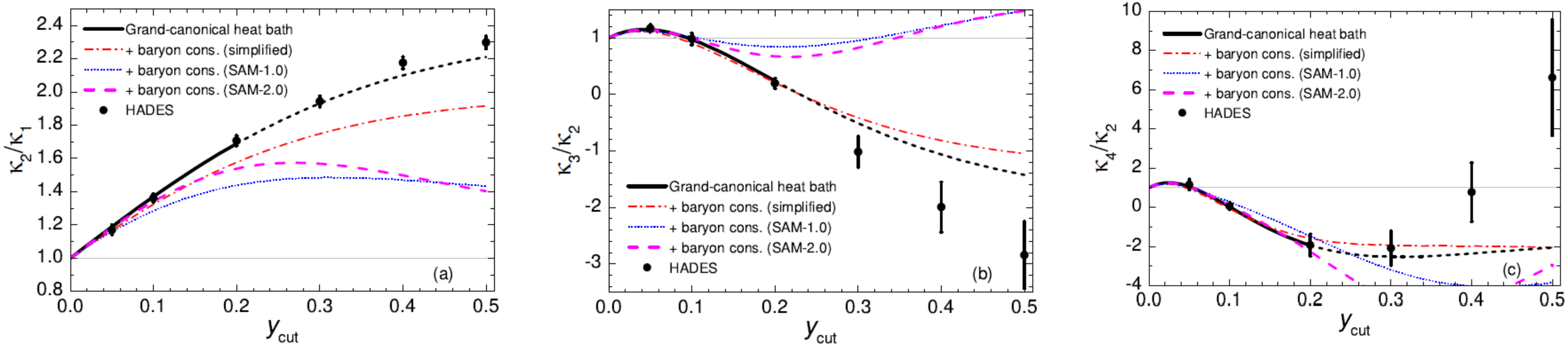
A closer look at the HADES data

VV, Koch, Phys. Lett. B 833, 137368 (2022)

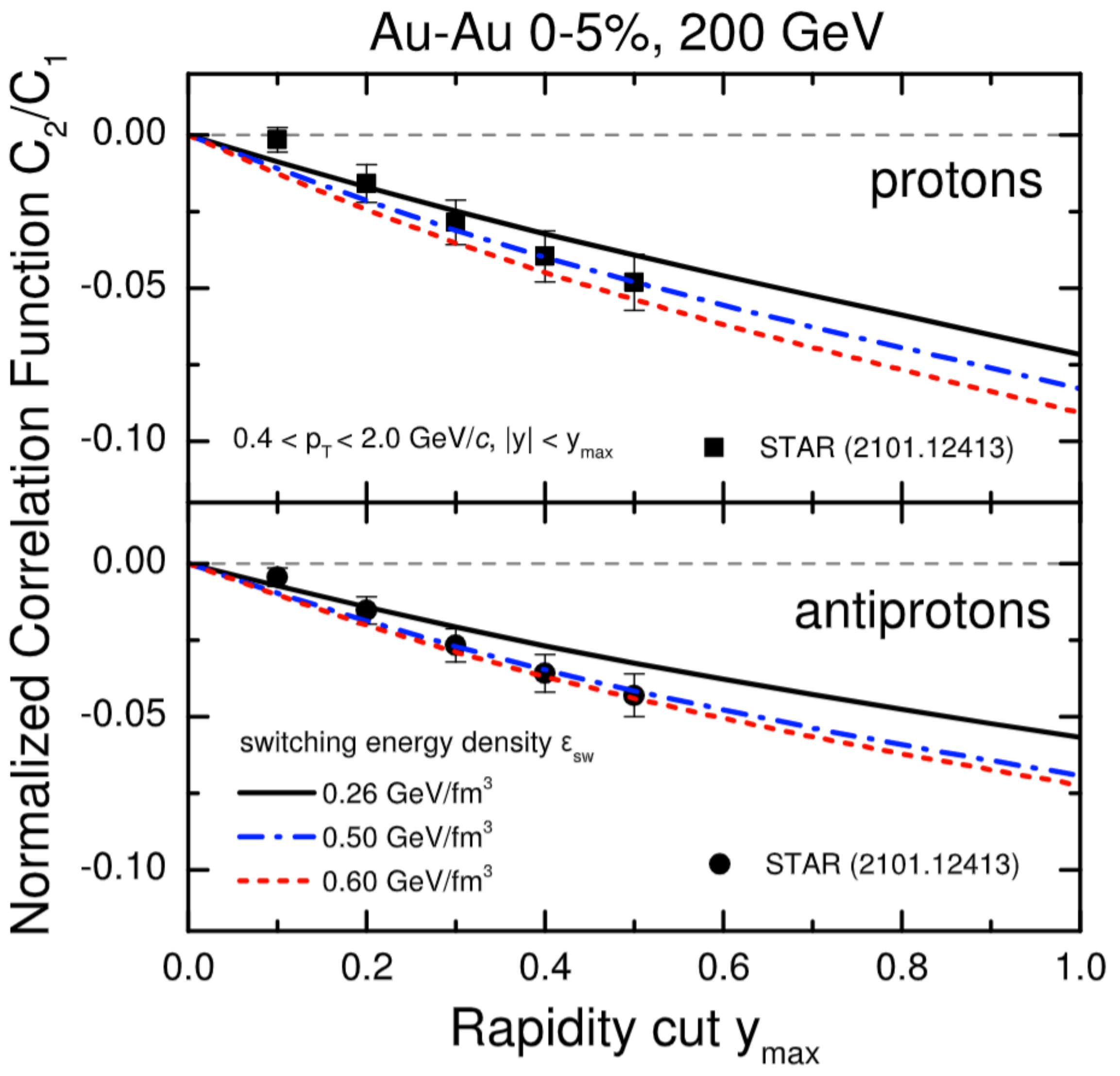
- Fit baryon susceptibilities to data within a fireball model (Siemens-Rasmussen*)
- In the grand-canonical limit (no baryon conservation, small y_{cut}) the data are described well with

$$\frac{\chi_2^B}{\chi_1^B} \sim 9.17 \pm 0.21, \qquad \frac{\chi_3^B}{\chi_2^B} \sim -33.1 \pm 0.8, \qquad \frac{\chi_4^B}{\chi_2^B} \sim 691 \pm 50, \quad \text{i.e.} \quad \chi_4^B \gg -\chi_3^B \gg \chi_2^B \gg \chi_1^B$$

- Could be indicative of a *critical point* near the HADES freeze-out at T~70 MeV, μ_B~875 MeV
- However, the results for $y_{cut} > 0.2$ are challenging to describe with baryon conservation included

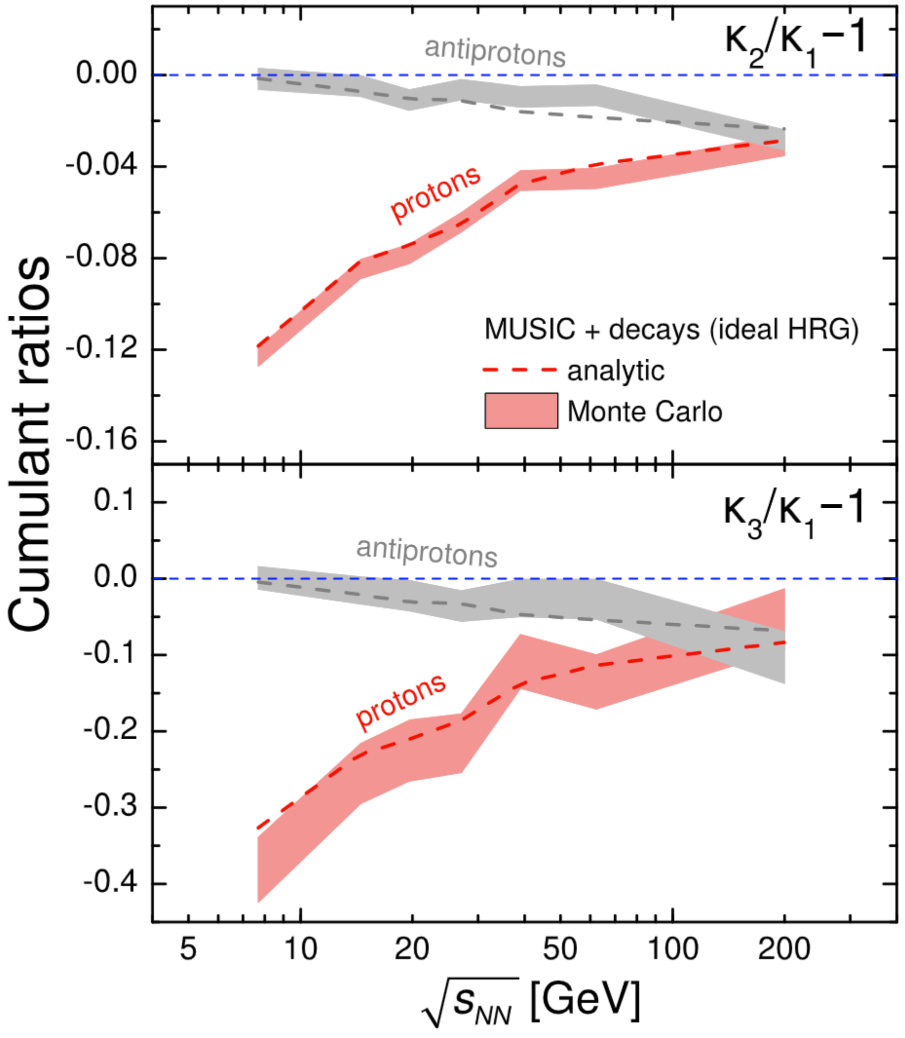


Dependence on the switching energy density



Cross-checking the cumulants with Monte Carlo

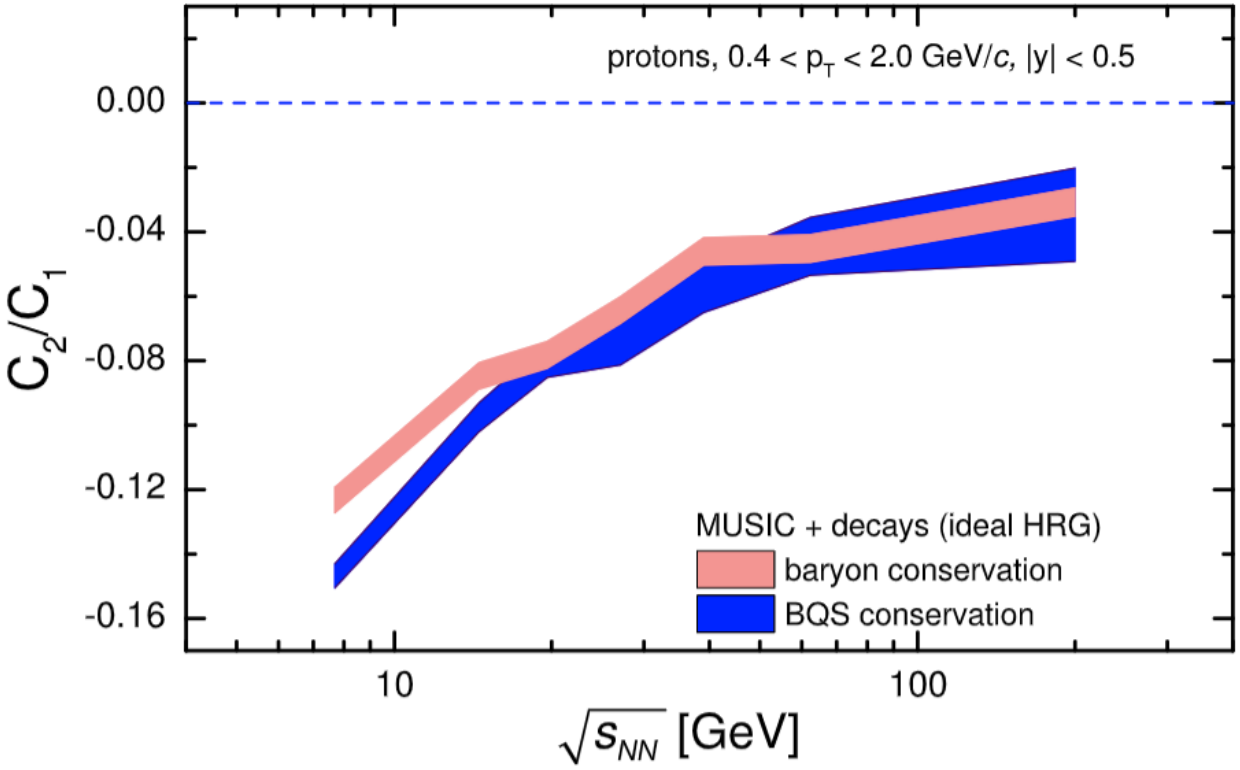
- Sample canonical ideal HRG model at particlization with Thermal-FIST
- Analytic results agree with Monte Carlo within errors



Exact conservation of electric charge

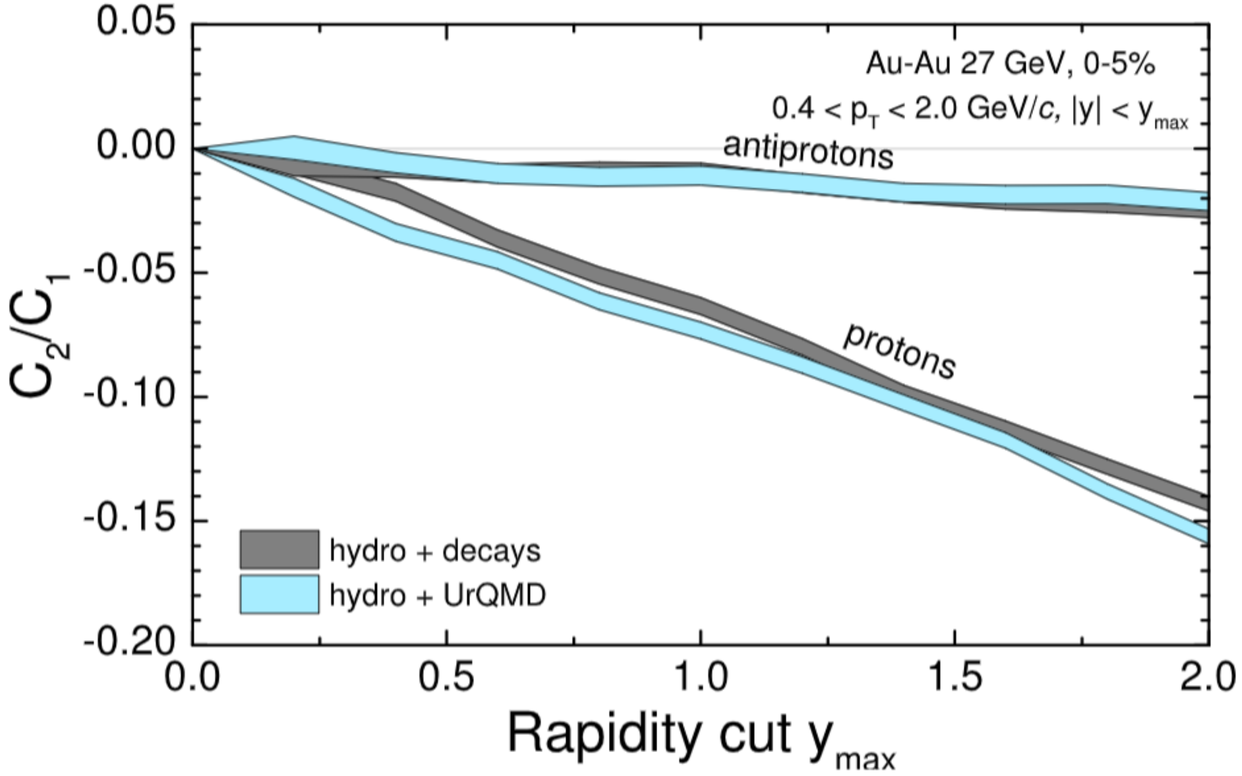
- Sample ideal HRG model at particlization with exact conservation of baryon number, electric charge, and strangeness using Thermal-FIST

- Protons are affected by electric charge conservation at $\sqrt{s_{NN}} \leq 14.5$ GeV



Effect of the hadronic phase

Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD



Net baryon fluctuations at LHC

- Global baryon conservation distorts the cumulant ratios already for one unit of rapidity acceptance

e.g. $\left.\frac{\chi_4^B}{\chi_2^B}\right|_{T=160MeV}^{GCE} \simeq 0.67$ *"lattice QCD"* $\neq$ $\left.\frac{\chi_4^B}{\chi_2^B}\right|_{\Delta Y_{acc}=1}^{HIC} \simeq 0.56$ *experiment*

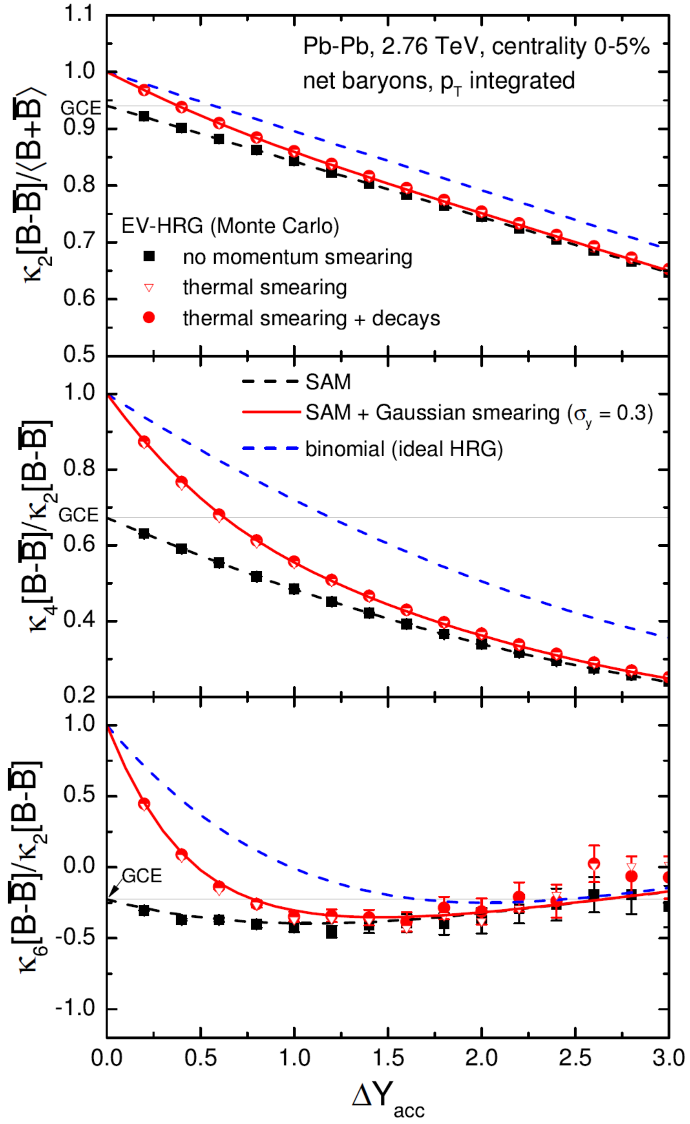
- Neglecting thermal smearing, effects of global conservation can be described analytically via SAM

$$\frac{\kappa_2}{\langle B+\bar{B}\rangle} = (1-\alpha)\frac{\kappa_2^{gce}}{\langle B+\bar{B}\rangle}, \qquad \alpha = \frac{\Delta Y_{acc}}{9.6}, \quad \beta \equiv 1-\alpha$$

$$\frac{\kappa_4}{\kappa_2} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B},$$

$$\frac{\kappa_6}{\kappa_2} = [1-5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$

- Effect of resonance decays is negligible



VV, Koch, arXiv:2012.09954

Effect of nuclear-liquid gas transition

Savchuk et al., 1909.04461

- Take 4 different models describing nuclear liquid-gas transition:

  van der Waals classical/quantum, Skyrme, Walecka

- Fit each model to liquid-gas transition. What happens at small μ?