



Quarkyonic or Baryquark matter?

On the dynamical generation of momentum space shell structure

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Nuclear Theory Seminar at Iowa State University

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Based on

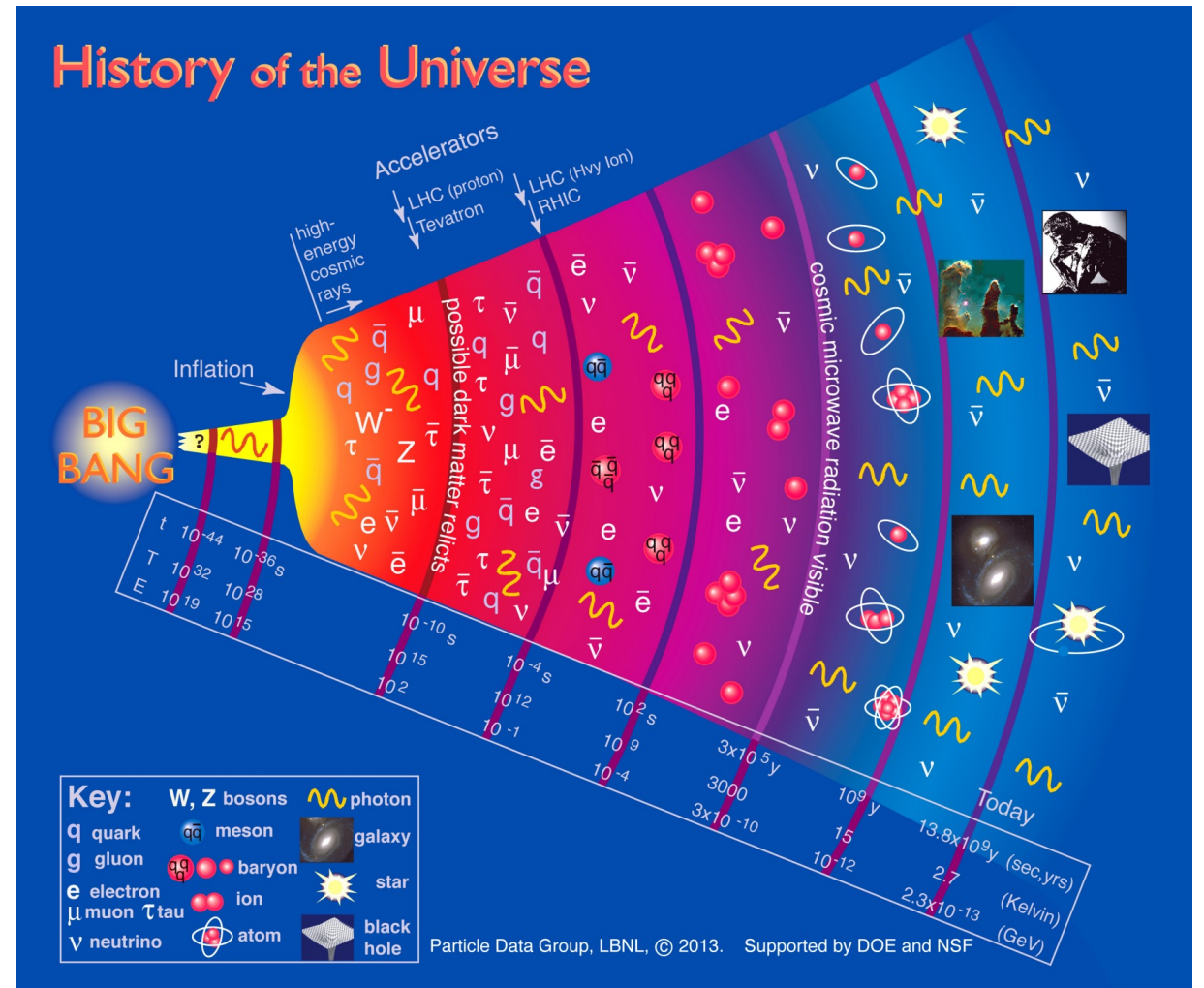
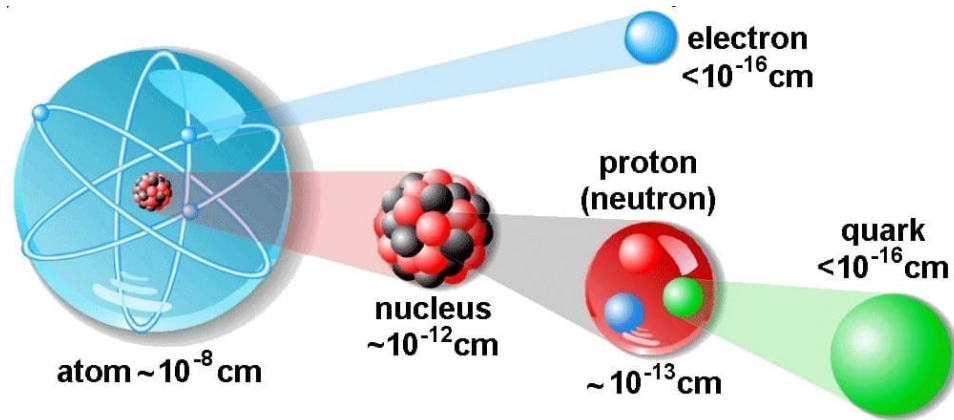
V. Koch, VV, [arXiv:2211.14674](https://arxiv.org/abs/2211.14674), Phys. Lett. B 841, 137942 (2023)

R. Poberezhnyuk, VV, in preparation



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Structure of matter

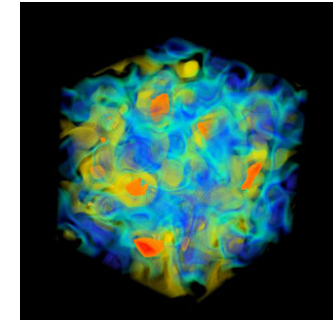


Strongly interacting matter

- Theory of strong interactions: *Quantum Chromodynamics* (QCD)

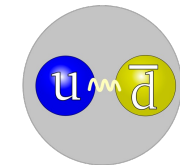
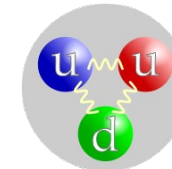
$$\mathcal{L} = \sum_{q=u,d,s,\dots} \bar{q} [i\gamma^\mu (\partial_\mu - igA_\mu^a \lambda_a) - m_q] q - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

- Basic degrees of freedom: quarks and gluons that carry color charge
- At smaller energies confined into baryons (qqq) and mesons ($q\bar{q}$)



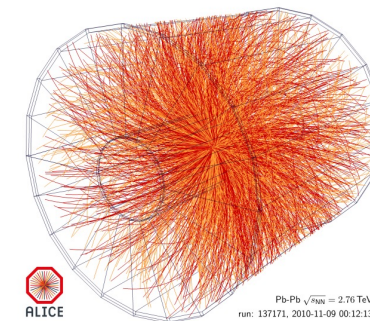
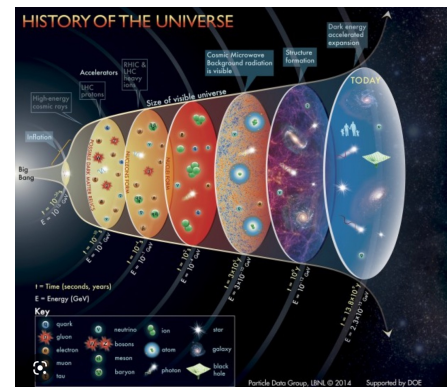
Scales

- Length:** 1 femtometer = 10^{-15} m
- Temperature:** $100 \text{ MeV}/k_B = 10^{12}$ K



Where is it relevant?

- Early Universe
- Laboratory: heavy-ion collisions
- Astrophysics: Neutron star (mergers)



(c) NASA

QCD matter under extreme conditions

Regulate temperature (heating) and/or baryon density (compression) → QCD phase diagram

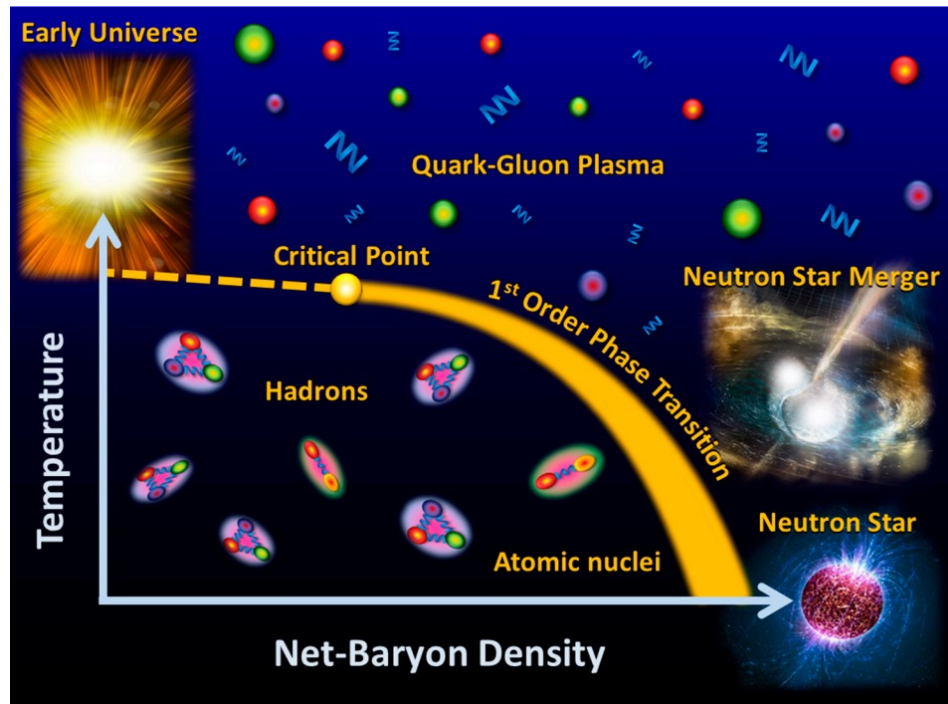


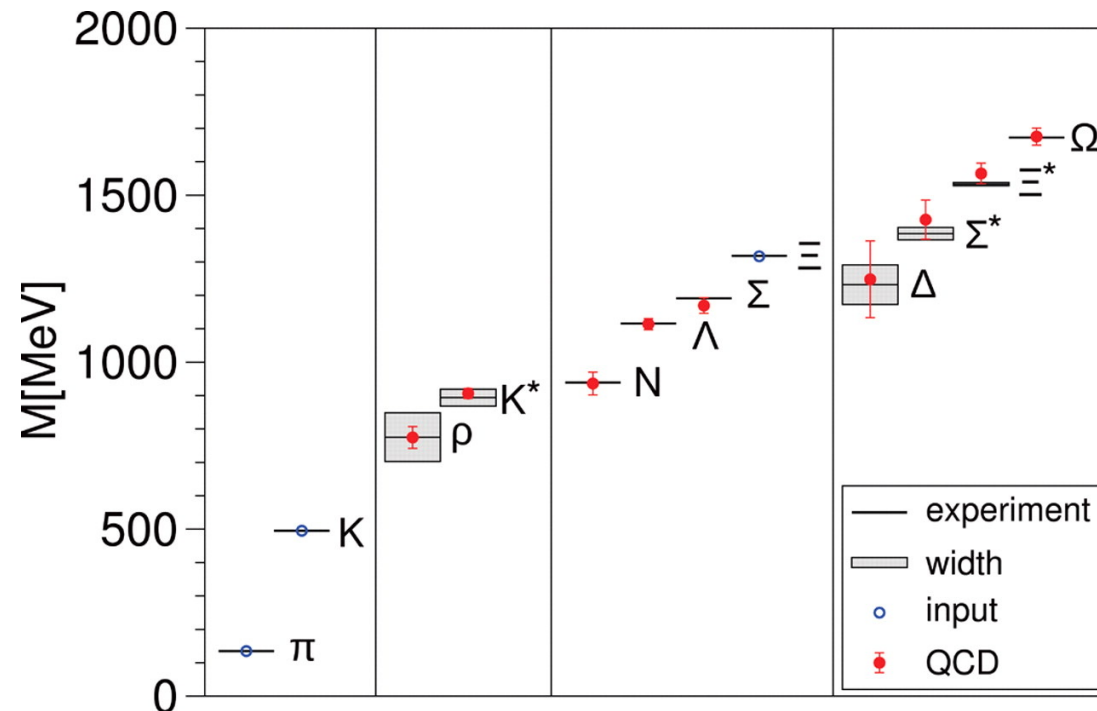
Figure from P. Senger, N. Herrman, Nucl. Phys. News

$$Z = \text{Tr}(e^{-(\hat{H} - \mu\hat{N})/T})$$

- Hadrons at low densities (confinement), quarks at high densities (asymptotic freedom)
- What is the nature of hadron-quark transition and/or coexistence?
 - The question is inherently non-perturbative

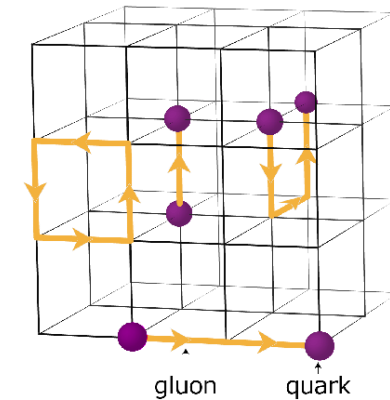
First-principle tool: Lattice QCD

Ab-initio calculation of hadron masses



BMW Collaboration, Science 322, 1224 (2008)

Remarkable agreement of QCD with the experiment



QCD transition at $\mu_B = 0$ from lattice QCD

$$Z = \text{Tr}(e^{-(\hat{H}-\mu\hat{N})/T}) = \int DU \det M[U, \mu] e^{-S_{YM}}$$

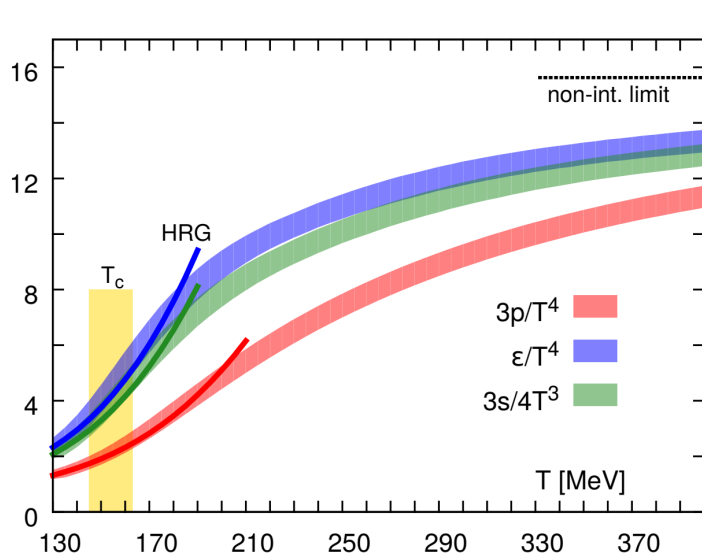


Figure from HotQCD coll., PRD '14

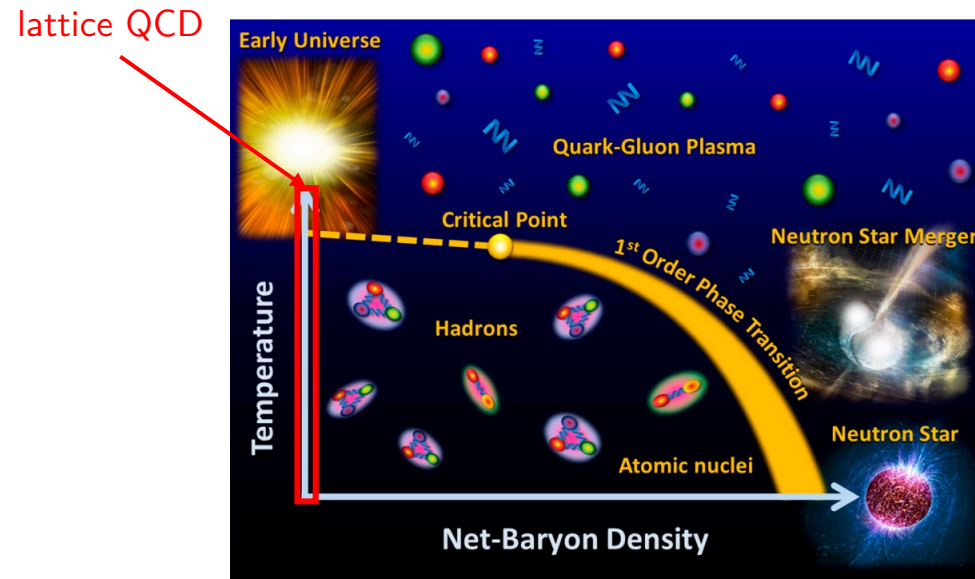


Figure from P. Senger, N. Herrman, Nucl. Phys. News

- Analytic crossover at vanishing net baryon density at $k_B T_{pc} \approx 155$ MeV – a first-principle result
- Smooth connection between hadrons and quarks – **quark-hadron duality?** [Y. Aoki et al., Nature 443, 675 (2006)]
 - On the hadronic side realized by adding resonances (HRG model) and eigenvolumes
 - Quark side – (hard-thermal loop) perturbation theory

[M. Albright, J. Kapusta, C. Young, Phys. Rev. C 90, 024915 (2014)]

QCD transition at finite μ_B

Finite baryon densities inaccessible with lattice QCD due to the sign problem

Extrapolations from $\mu_B = 0$:

- Crossover at $T_{pc} \approx 155$ MeV maintained till $\frac{\mu_B}{T} < 2 - 3$
 - No evidence for critical point
 - Consistent with heavy-ion data on proton number fluctuations

lattice QCD

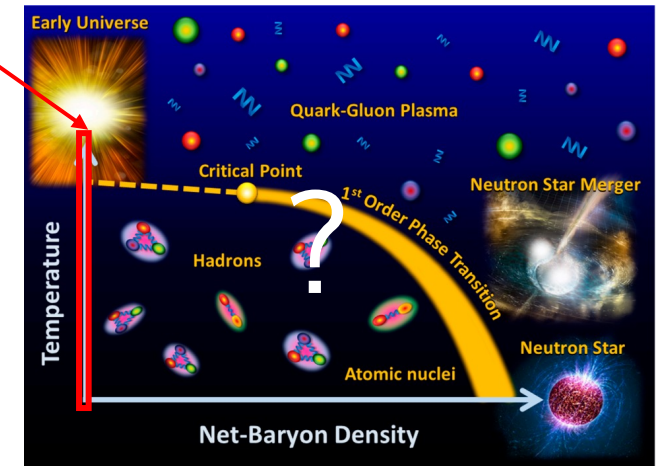


Figure from P. Senger, N. Herrman, Nucl. Phys. News

Ultimate limit: zero temperature, $T = 0$ ($\mu_B/T \rightarrow \infty$)

- No thermal excitations (resonances less relevant)
- Pauli blocking between “deconfined” and confined (baryons) quarks
- Accessible in neutron stars
- Perturbative QCD: high densities only, $n_B > 40n_0$

QCD transition at zero temperature and neutron stars

- QCD matter at zero temperature is found in interior of neutron stars
- Its pressure balances the gravitational pull

QCD EoS

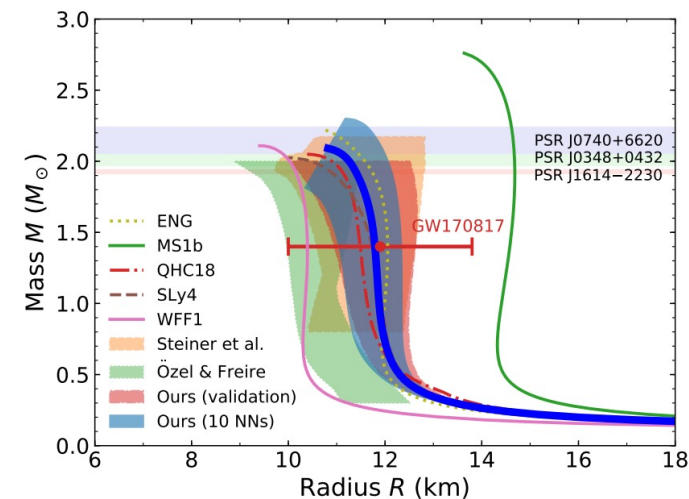
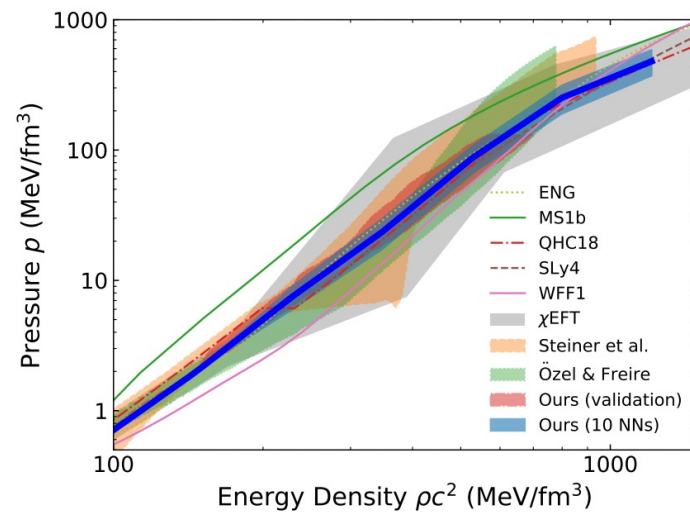
$$P = P(\rho)$$

Tolman-Oppenheimer-Volkoff equation

$$\frac{dP}{dr} = -\frac{Gm}{r^2} \rho \left(1 + \frac{P}{\rho c^2}\right) \left(1 + \frac{4\pi r^3 P}{mc^2}\right) \left(1 - \frac{2Gm}{rc^2}\right)^{-1}$$

NS mass-radius curve

$$M = M(R)$$



Figures from Fujimoto, Fukushima, Phys. Rev. D (2020)

+ multi-messenger constraints (neutron star mergers, nuclear physics, heavy-ion coll.)

QCD EoS in the cold and dense regime from neutron stars

Speed of sound: $c_s^2 = \frac{n_B}{\mu_B} \frac{d\mu_B}{dn_B}$

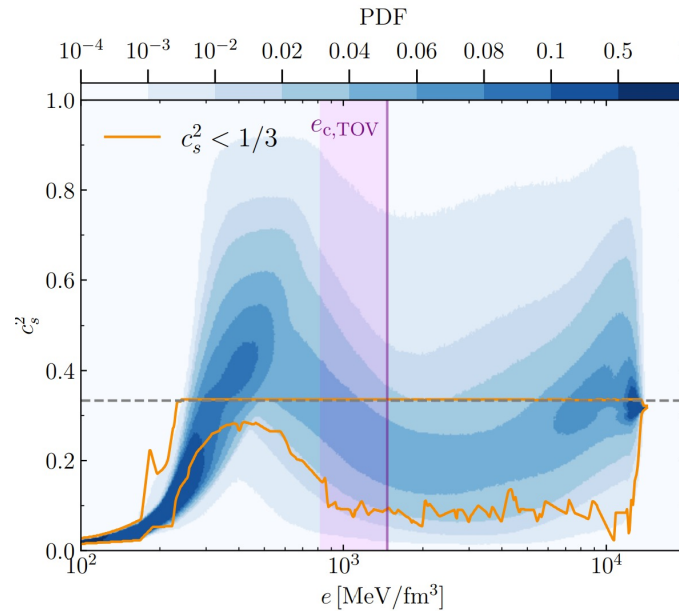
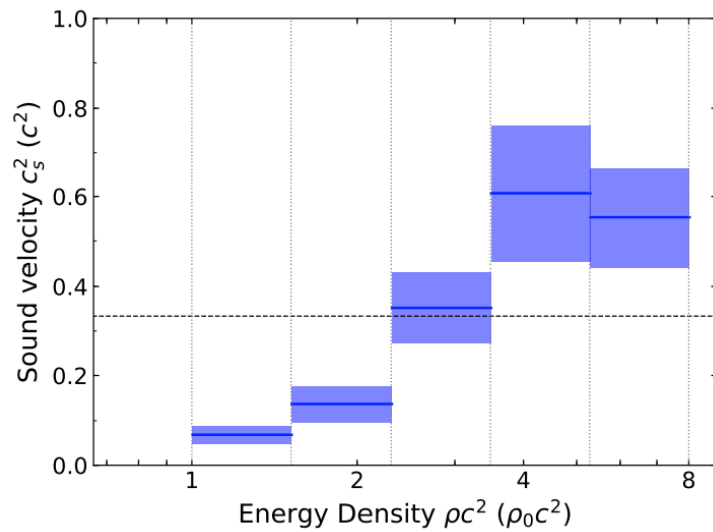
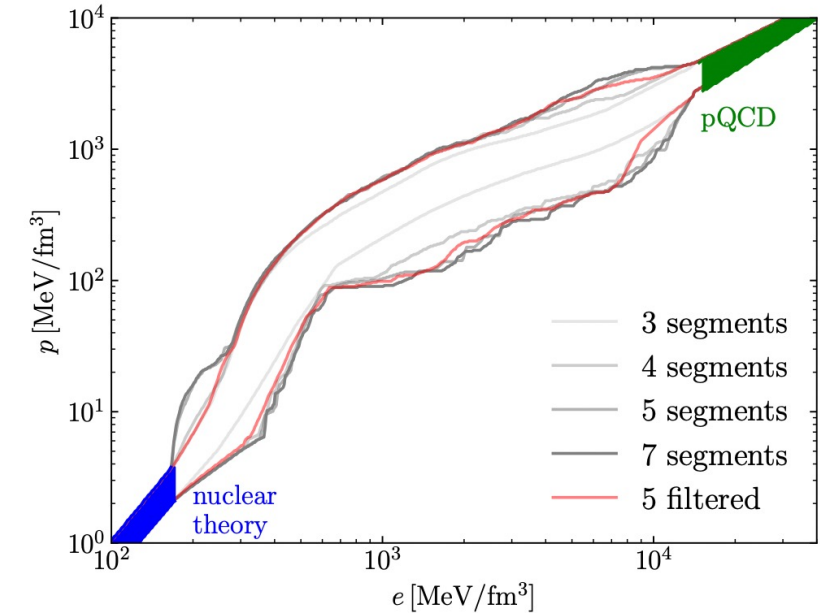


Figure from Fujimoto, Fukushima, Phys. Rev. D (2020)

Pressure



Figures from Altiparmak, Ecker, Rezzolla, ApJL (2022)

Many constraints from neutron star observations indicate a strong rise of c_s^2 beyond the conformal limit

Tews, Carlson, Gandolfi, Reddy, ApJ 860 (2018) 149;
 Fujimoto, Fukushima, PRD 101 (2020) 054016;
 Tang, Noronha-Hostler, Yunes, PRL 125 (2020) 261104;
 Altiparmak, Ecker, Rezzolla, ApJL 939 (2022) L34;

...

QCD the cold and dense regime and quarkyonic matter

We have neutron-star matter EoS based on astrophysical observations

- Consistent with nuclear physics at low densities (neutrons)
- Perturbative QCD at high densities (quarks)
- Does not elucidate the state of matter in-between

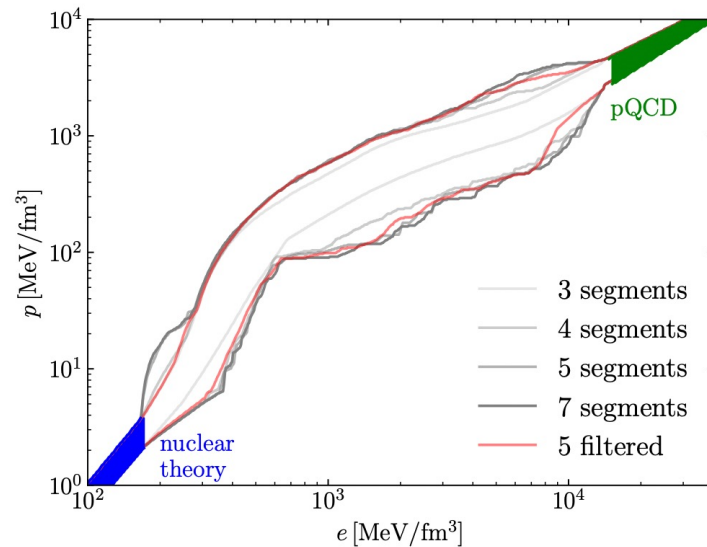


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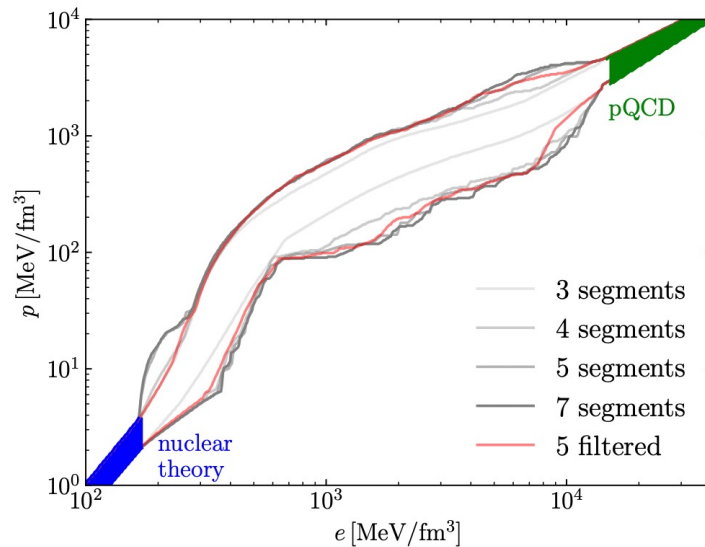


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Quarkyonic matter?

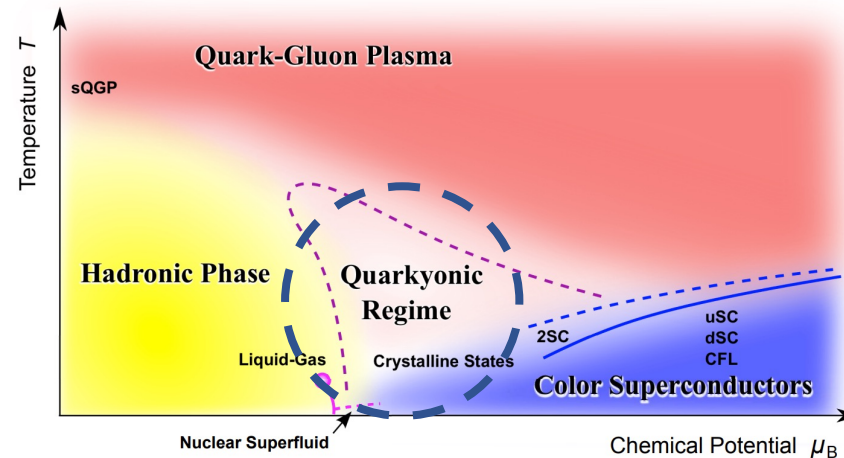
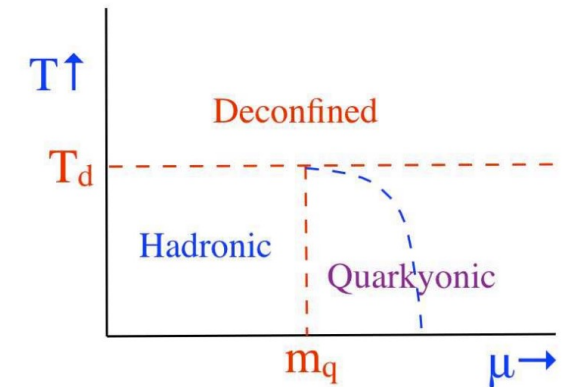


Figure adapted from Fukushima, Sasaki, PPNP '13

Large N_c



McLerran, Pisarski, NPA '07

Quarkyonic matter: baryon-quark coexistence, baryonic excitations around the Fermi surface

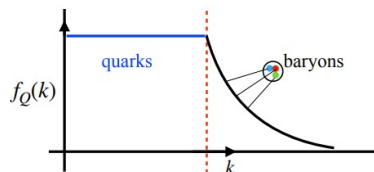
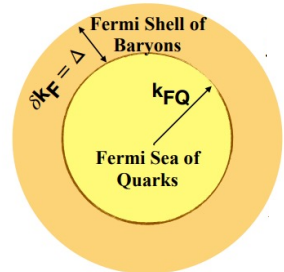
McLerran, Pisarski, NPA 796, 83 (2007)

Quarkyonic matter and neutron stars

First(?) practical realization of quarkyonic matter ($T=0$)
 [McLerran, Reddy, PRL 122, 122701 (2019)]

Mixture of “confined” quarks (baryons) and deconfined quarks with Pauli principle*

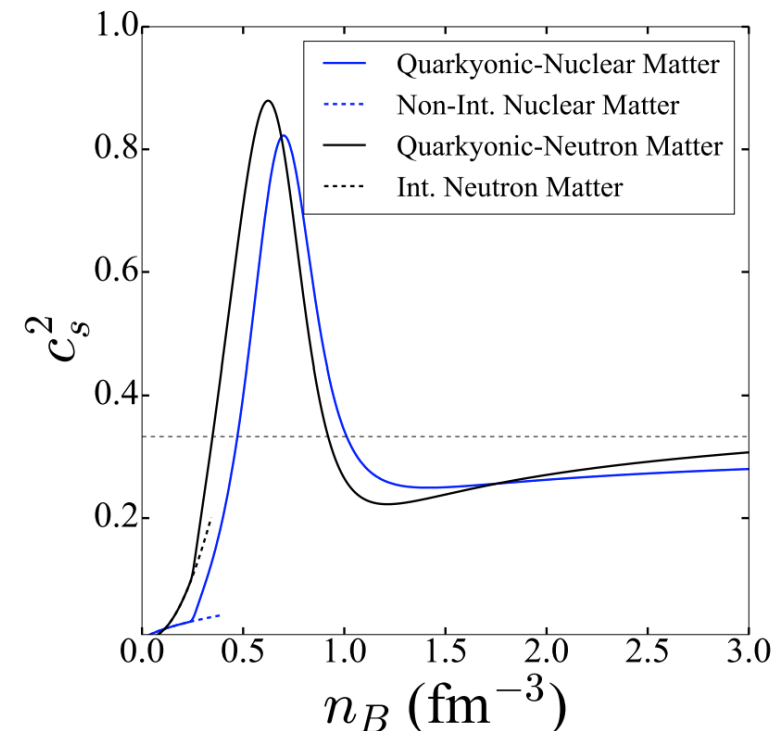
Enforce *momentum space shell structure* (baryonic Fermi surface) and its density evolution



$$\Delta = \frac{\Lambda^3}{k_{FB}^2} + \kappa \frac{\Lambda}{N_c^2} \quad m_N = N_c m_Q$$

$$(n_B) = 4 \int_{N_c k_{FQ}}^{k_{FB}} \frac{d^3 k}{(2\pi)^3} \sqrt{k^2 + M_n^2},$$

$$+ 2 \times N_c \int_0^{k_{FQ}} \frac{d^3 k}{(2\pi)^3} \sqrt{k^2 + M_q^2}$$



*Due to coinciding spin-isospin degeneracies, baryon and quark states cannot overlap

Dynamical generation of momentum space shell structure

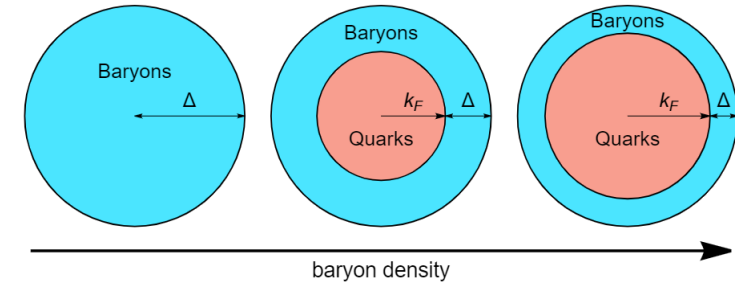
PHYSICAL REVIEW C **101**, 035201 (2020)

Dynamically generated momentum space shell structure of quarkyonic matter via an excluded volume model

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²Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195, USA



AN EXCLUDED VOLUME THEORY OF NUCLEAR INTERACTIONS

$$n_{ex}^N = \frac{n_N^N}{1 - n_N^N/n_0}$$

Minimize energy density at fixed n_B to find k_F and Δ

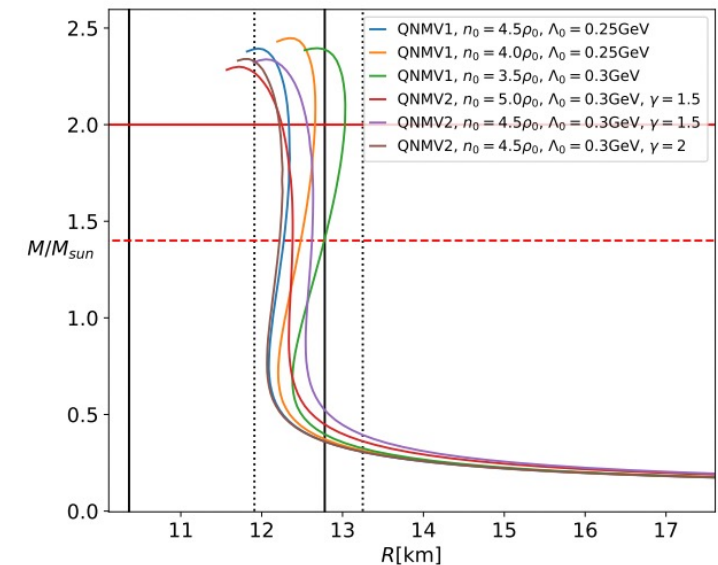
$$\tilde{\epsilon} = 4 \left(1 - \frac{n_N^N}{n_0}\right) \int_{k_F}^{k_F+\Delta} \frac{d^3k}{(2\pi)^3} \left((N_c m_Q)^2 + k^2 \right)^{\frac{1}{2}} + \frac{2N_c}{\pi^2} \int_0^{k_F/N_c} dk k (\Lambda^2 + k^2)^{\frac{1}{2}} (m_Q^2 + k^2)^{\frac{1}{2}}$$

Requires infrared regulator to avoid superluminal speed of sound

Works reasonably well for neutron stars, extendable to strange quarks

D. Duarte, S. Hernandez-Ortiz, K. Jeong, PRC 102 (2020) 025203; PRC 102 (2020) 065202

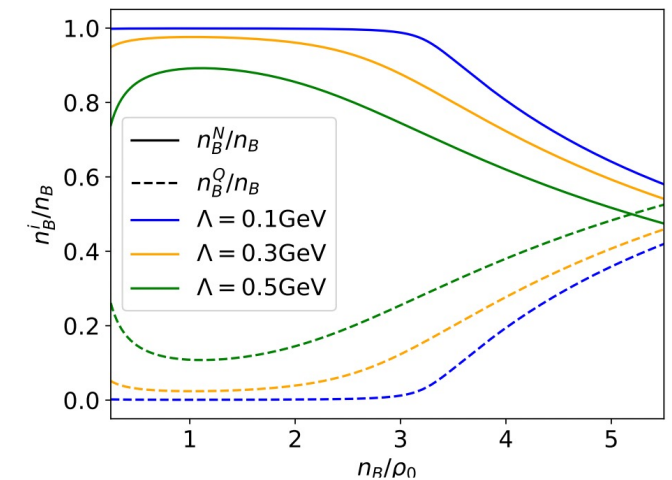
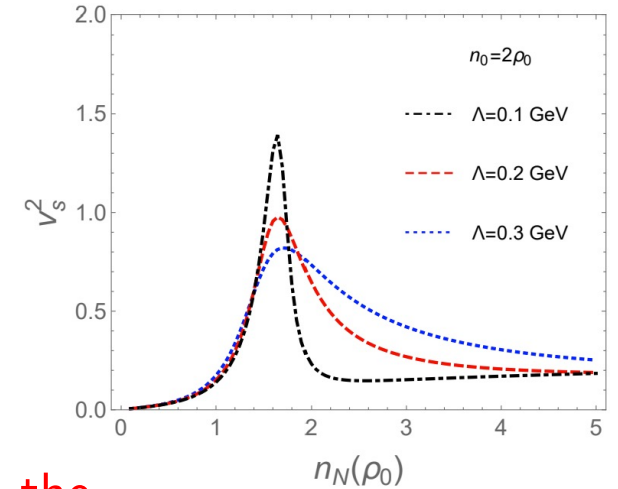
S. Sen, L. Sivertsen, ApJ 915, 109 (2021)



The excluded volume mechanism applied to baryon-quark mixture

- Helps to explain how quarks appear with baryon density
 - When baryon cores start to overlap, it becomes energetically unfavorable to have nucleons only
- Does not explain why quarks are in the Fermi sea and baryons are on the Fermi surface
- Requires infrared regulator
 - Hard to constrain its value independently
 - Quarks appear at low densities

addressed in [S. Sen, L. Sivertsen, ApJ 915, 109 \(2021\)](#)

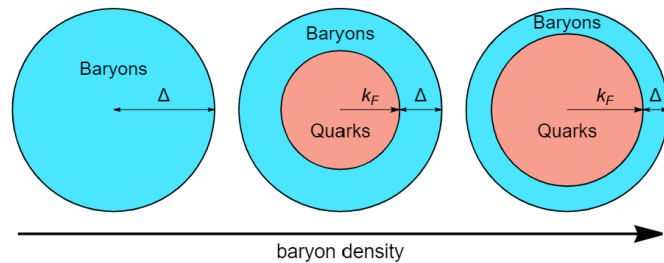


Key question: Is quarkyonic matter momentum shell structure the energetically preferred state of dense QCD matter? Will it emerge in a true dynamical mechanism (e.g. transport simulations)?

Quarkyonic vs baryquark matter

Two opposite scenarios for the realization of Pauli exclusion principle in baryon-quark mixture

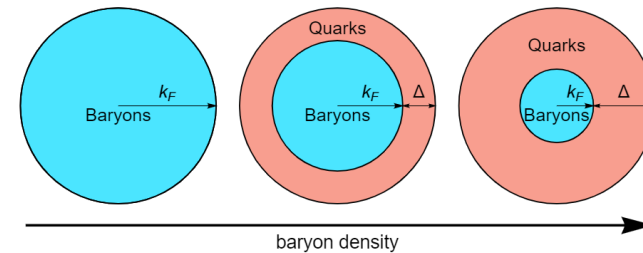
Quarkyonic



$$n_B = n_N + n_Q$$

$$\varepsilon = \varepsilon_N + \varepsilon_Q$$

Baryquark



$$n_Q = \frac{2}{\pi^2} \int_0^{k_F/N_c} dk k^2 = \frac{2 k_F^3}{3\pi N_c^3}$$

$$n_N = f_{ev} \int_{k_F}^{k_F+\Delta} dk k^2 = f_{ev} \frac{2[(k_F + \Delta)^3 - k_F^3]}{3\pi^2}$$

$$\varepsilon_Q = \frac{2N_c}{\pi^2} \int_0^{k_F/N_c} dk k^2 \sqrt{m_Q^2 + k^2},$$

$$\varepsilon_N = f_{ev} \frac{2}{\pi^2} \int_{k_F}^{k_F+\Delta} dk k^2 \sqrt{m_N^2 + k^2}.$$

$$n_Q = \frac{2}{\pi^2} \int_{k_F/N_c}^{(k_F+\Delta)/N_c} dk k^2 = \frac{2[(k_F + \Delta)^3 - k_F^3]}{3\pi N_c^3}$$

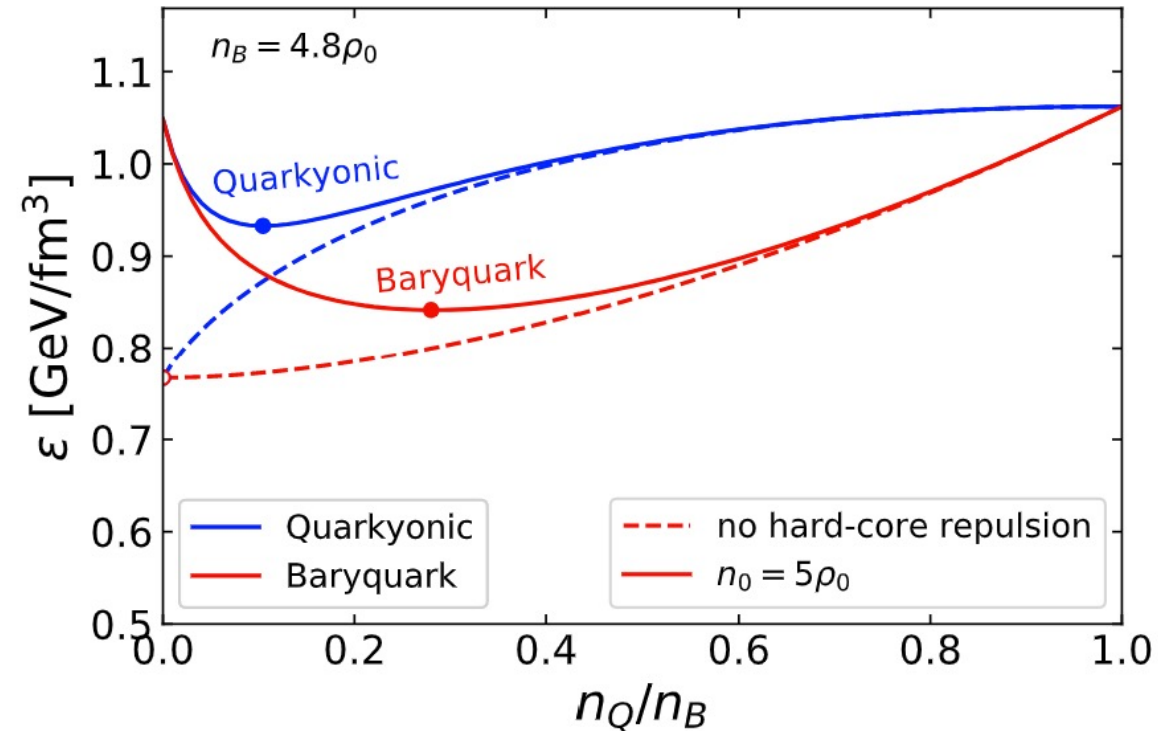
$$n_N = f_{ev} \frac{2}{\pi^2} \int_0^{k_F} dk k^2 = f_{ev} \frac{2 k_F^3}{3\pi}$$

$$\varepsilon_Q = \frac{2N_c}{\pi^2} \int_{k_F/N_c}^{(k_F+\Delta)/N_c} dk k^2 \sqrt{m_Q^2 + k^2}$$

$$\varepsilon_N = f_{ev} \frac{2}{\pi^2} \int_0^{k_F} dk k^2 \sqrt{m_N^2 + k^2}.$$

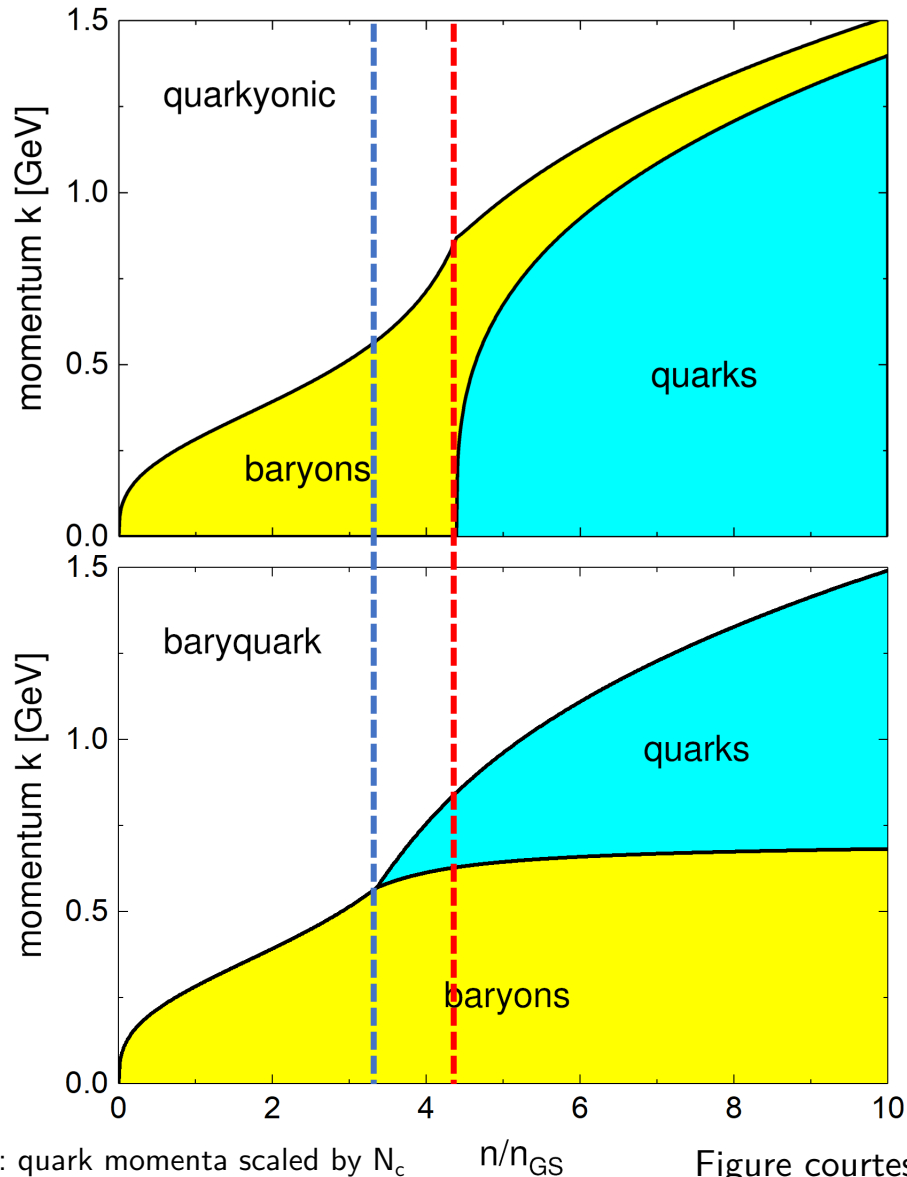
Quarkyonic vs baryquark matter: energy minimization

At each baryon density n_B minimize energy density wrt to quark fraction n_Q/n_B



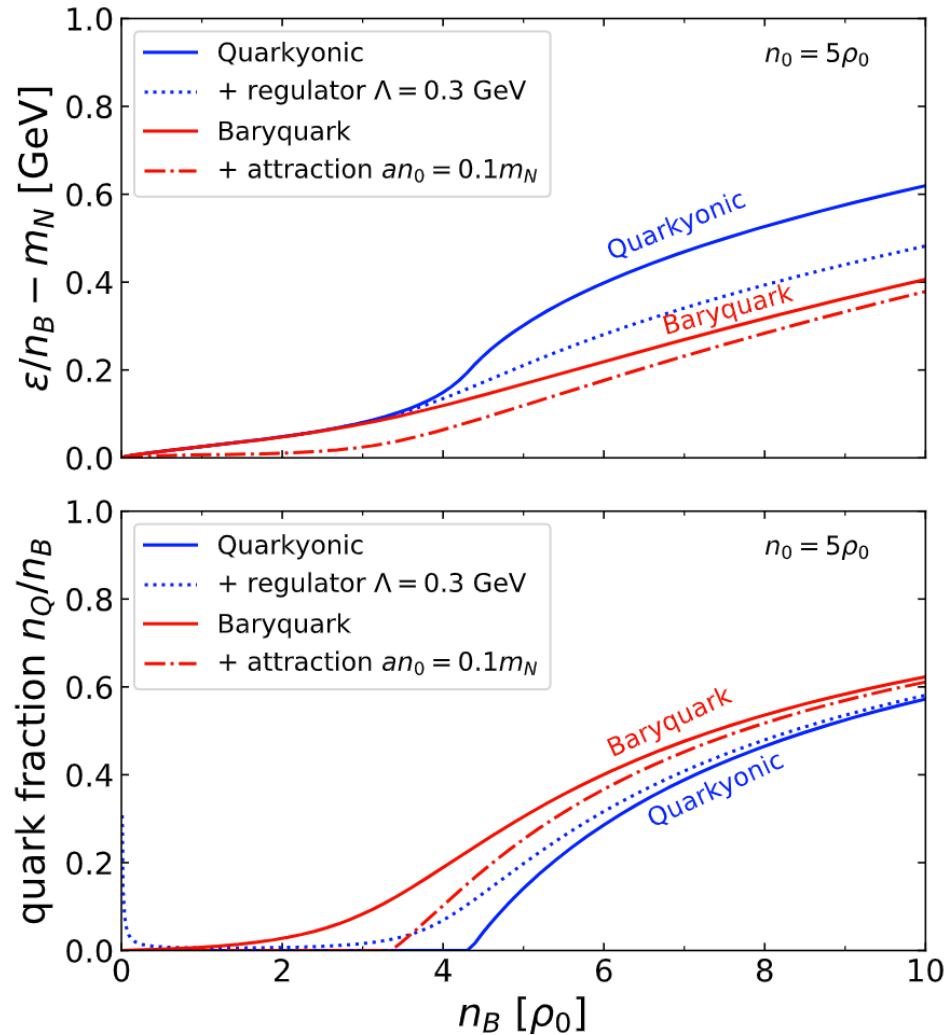
- Without excluded volume, the pure nucleon phase is always preferred
 - Already in this limit evident that adding quarks to the surface is energetically favorable
- With hard-core repulsion minimum may be at finite n_Q/n_B , baryquark has a deeper minimum

Quarkyonic vs baryquark matter: momentum shell structure



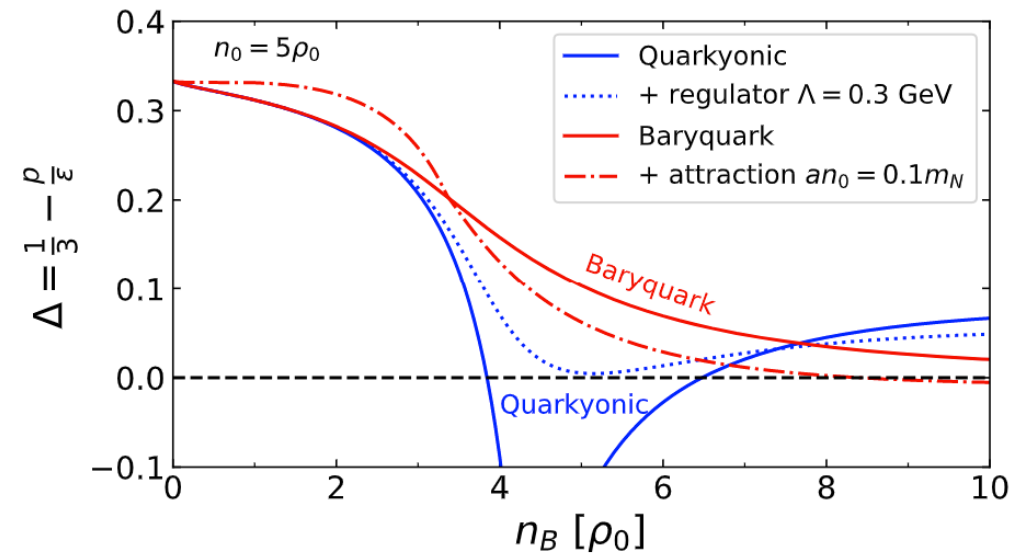
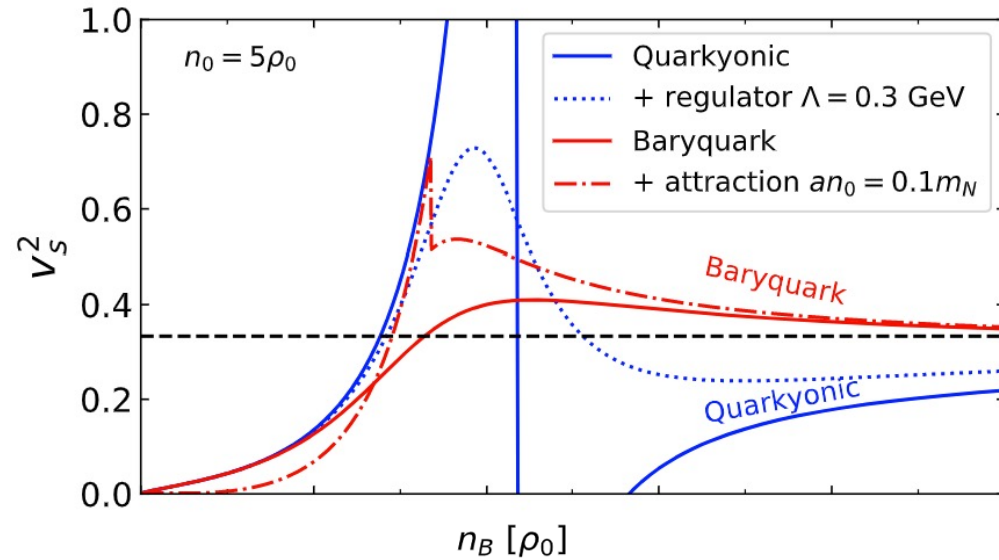
- Non-zero quark fraction emerges at a certain density as a result of energy minimization
- Both scenarios give the same EoS (pure nucleon matter) before the quark onset
- Appearance of quarks in baryquark matter is earlier and smoother

Quarkyonic vs baryquark matter: equation of state



- For most parameter setups the quark onset corresponds to a 2nd-order phase transition
- “Early” appearance of quarks
 - In baryquark matter likely an artifact of missing nucleon attraction, $\epsilon_N \rightarrow \epsilon_N - an_N^2$
 - In quarkyonic matter appears due to infrared regulator
- Infrared regulator does not make quarkyonic energetically favored over baryquark

Quarkyonic vs baryquark: speed of sound, conformality

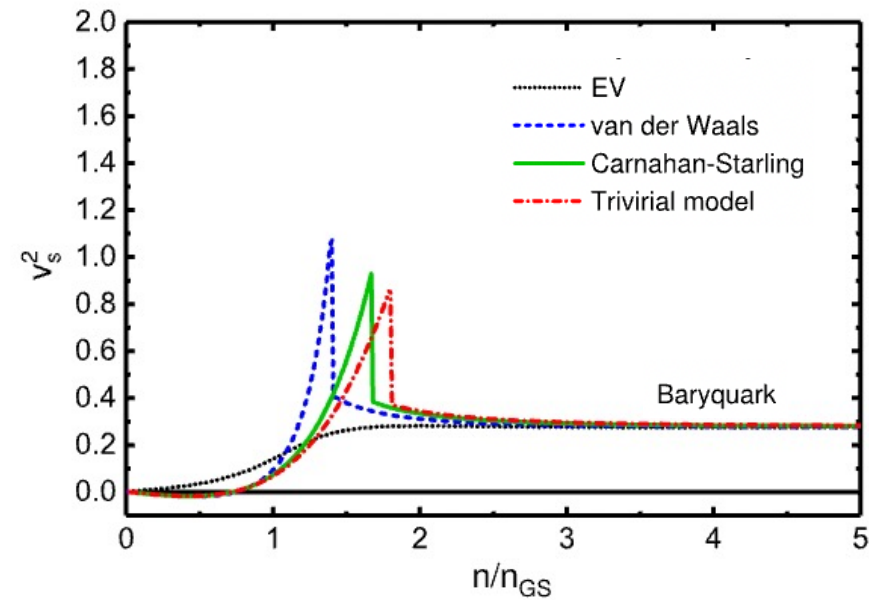
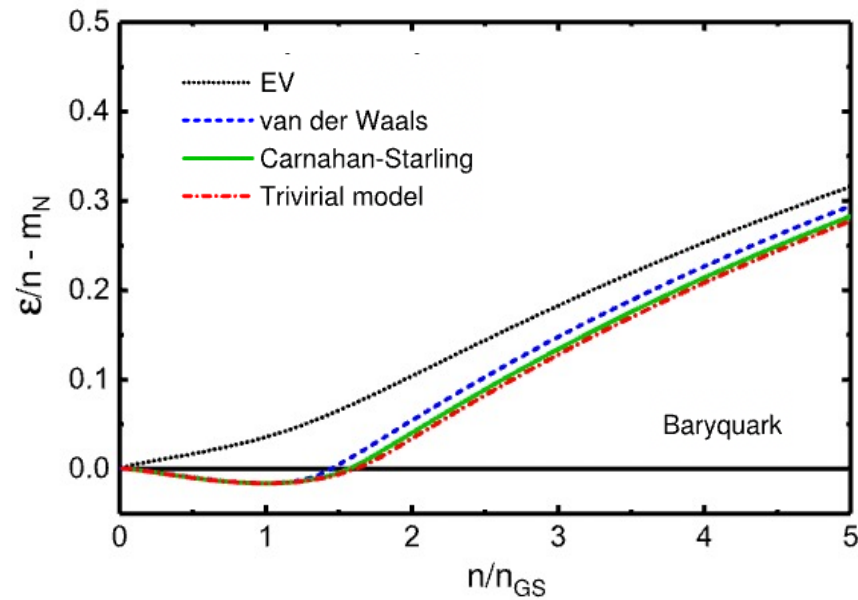


- In quarkyonic matter need to introduce regulator to obtain physically acceptable speed of sound
- In baryquark matter the behavior is acceptable without the need to introduce regulators
- The speed of sound exceeds the conformal limit ($c_s^2 = 1/3$) in all cases
- Trace anomaly: exceeding the conformal limit ($\varepsilon = 3p$) is less obvious

Outlook: Realistic low-density equation of state

Achieved by matching with quantum real gas model

- Generalized excluded-volume model
- Attractive mean-field



R. Poberezhnyuk, VV, to appear

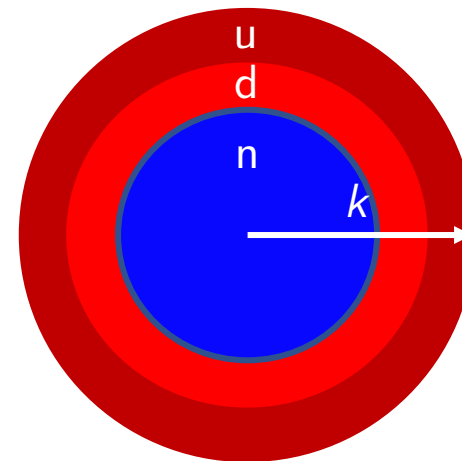
Outlook: Neutron-star baryquark matter

Isospin-symmetric matter:

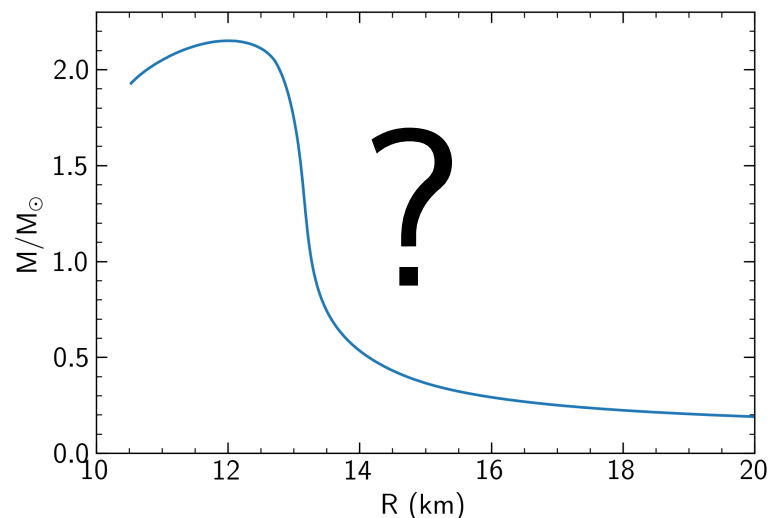
- same # of protons (uud) and neutrons (udd)
- Fermi surfaces of u & d quarks coincide

Pure neutron matter:

- Neutrons (udd) only \rightarrow nud-matter
- charge neutrality ($n_u=2n_d$)
- different Fermi surfaces for u & d quarks



Mass-radius relation



- Quark-hadron coexistence at $T = 0$ implies a mixed phase in the momentum space
- Equation of state of baryon-quark mixture with Pauli principle and baryonic hard-core
 - Disfavors **quarkyonic matter** (**baryonic Fermi surface**) compared to **baryquark** (**quark Fermi surface**)
 - Qualitatively similar resulting EoS but baryquark does not require infrared regulator
- Existing quarkyonic matter descriptions require modifications if this picture is to be preserved
 - Momentum-dependence nuclear interactions?
 - Abandon the quasiparticle picture? (too naïve)
 - A judicious regulator?
- **Outlook:** [R. Poberezhnyuk, VV, to appear]
 - Match to realistic low-density EoS
 - Variations on Pauli exclusion principle and quark-hadron duality implementations
 - Isospin asymmetry and neutron stars

Thanks for your attention!