

# Cooper-Frye sampling with short-range repulsion

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# QCD phase structure and heavy-ion collisions

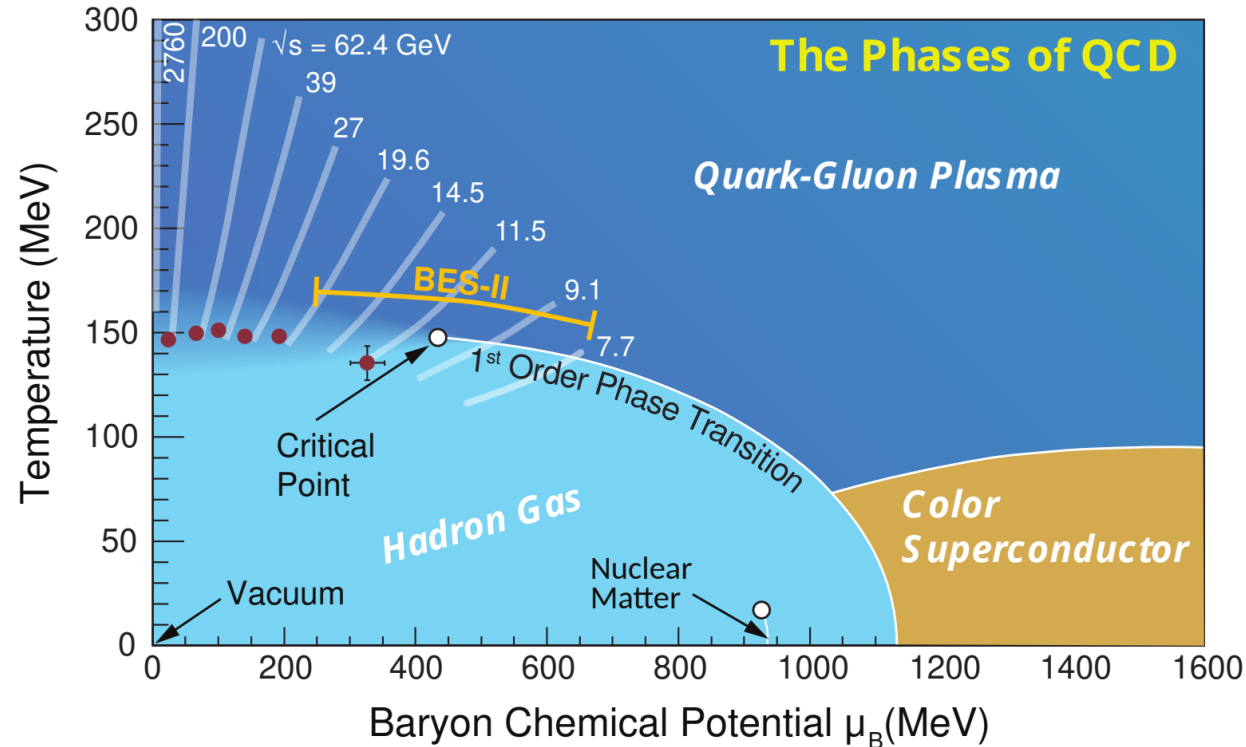
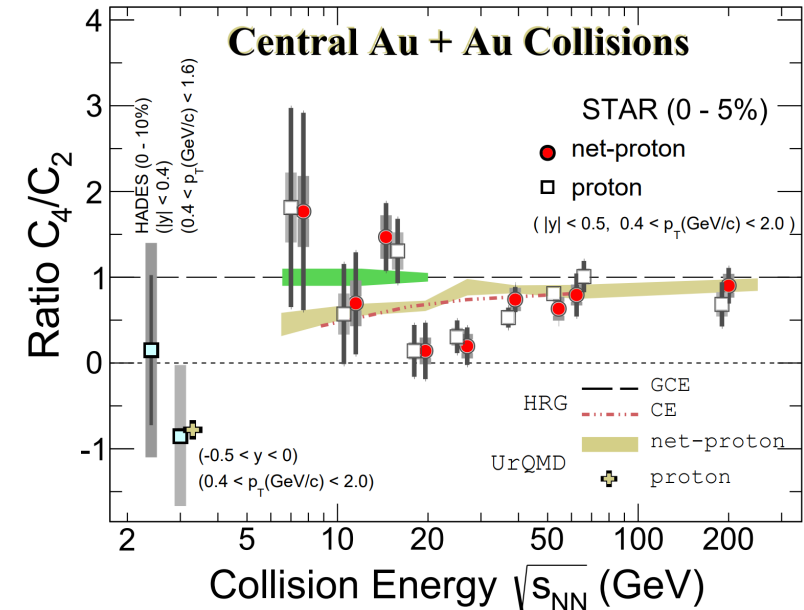


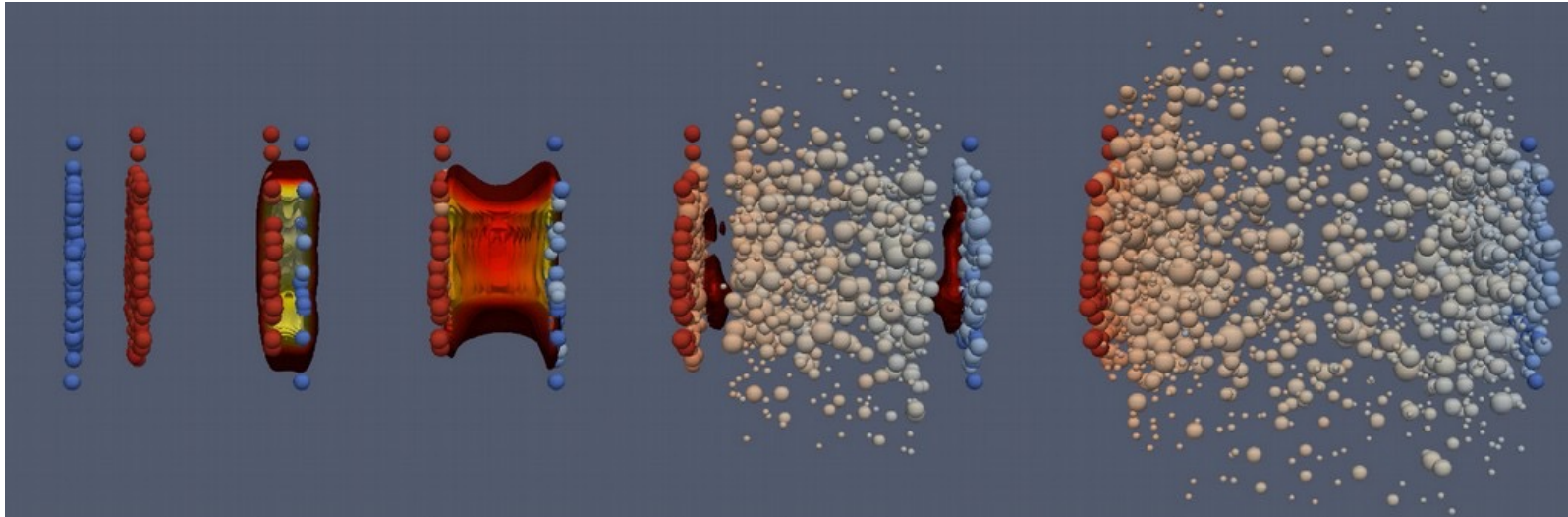
Figure from Bzdak et al., Phys. Rept. '20 & 2015 Long Range Plan



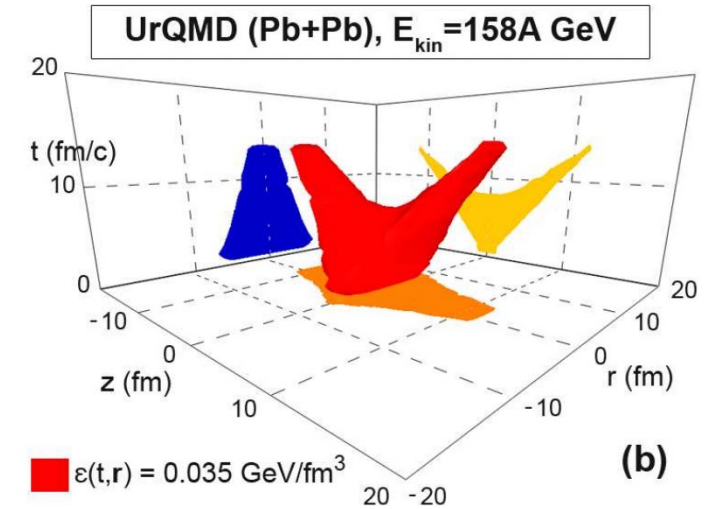
STAR Coll., PRL 126, 092301 (2021), PRL 128, 202303 (2022)

- Scanning the QCD phase diagram with heavy-ion collisions at different energies
- Event-by-event fluctuations probe the QCD phase structure, in particular the critical point
- Effects like baryon conservation are essential and make even non-critical baseline non-trivial

# Heavy-ion collisions and particlization



MADAI Collaboration



Anchishkin, VV, Csernai, PRC '13

**Particlization:** Mapping the expanding hydrodynamic fluid into hadron resonance gas on a Cooper-Frye hypersurface (typically constant energy density surface)

$$\omega_p \frac{dN_j}{d^3p} = \int_{\Sigma(x)} d\Sigma_\mu(x) p^\mu f_j[u^\mu(x)p_\mu; T(x), \mu_j(x)],$$

What is the fate of event-by-event fluctuations?

# Existing particlization techniques

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- **Standard Cooper-Frye (grand-canonical) particlization**
  - Examples: iSS, frzout, BEST sampler, ... [C. Shen et al.; J. Bernhard; S. Pratt,...]
  - Conservation laws are enforced on average + viscous corrections
    - Ok for spectra and flow
  - Each hydrodynamic cell is sampled independently
    - **Hadron number fluctuations follow Poisson statistics**
    - **Not very suitable for event-by-event fluctuations**
- **(Micro)canonical particlization** [Oliinychenko, Koch, PRL 123, 182302 (2019); Schwartz et al., JPG 45, 015001 (2018)]
  - Exact conservation of conserved charges, energy and momentum
  - **No hadronic interactions**
- **FIST sampler** VV, Phys. Rev. C 106, 064906 (2022) <https://github.com/vlvovch/fist-sampler>
  - Exact conservation of baryon number, electric charge, and strangeness
  - Hard-core repulsion

# Hard-core repulsion

- Two hadrons with hard-core repulsion cannot overlap
  - Reject the configuration if hadrons overlap\*  $|\mathbf{r}_i - \mathbf{r}_j| < 2r_c$

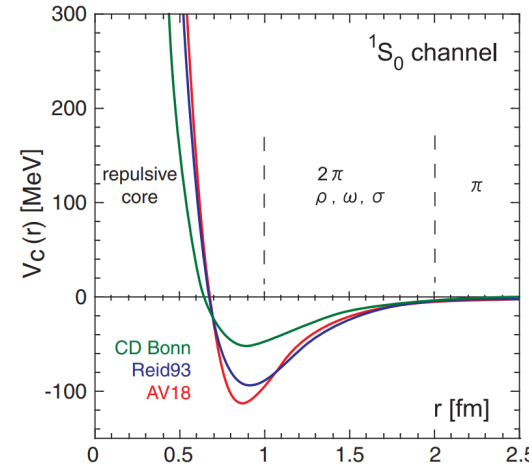
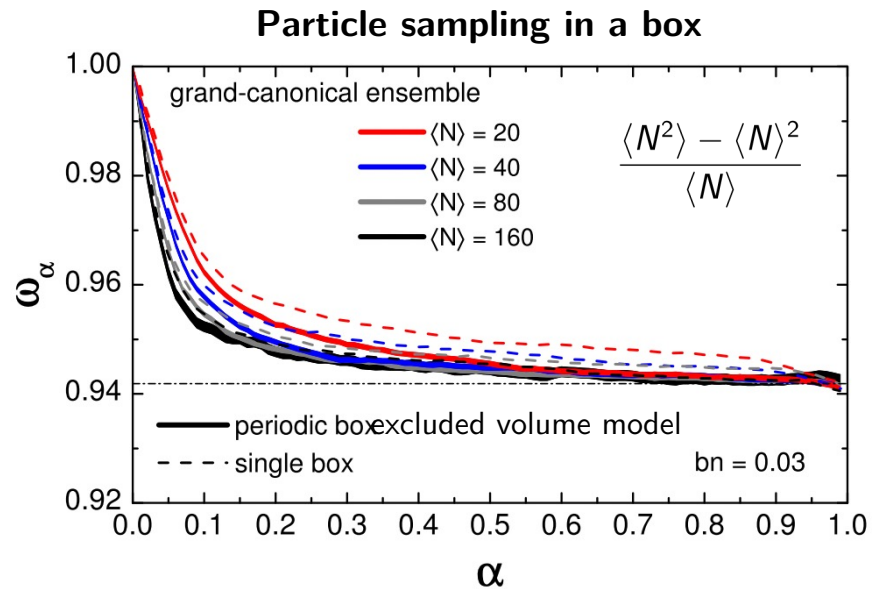
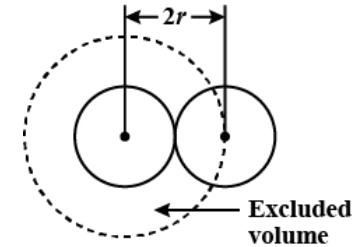
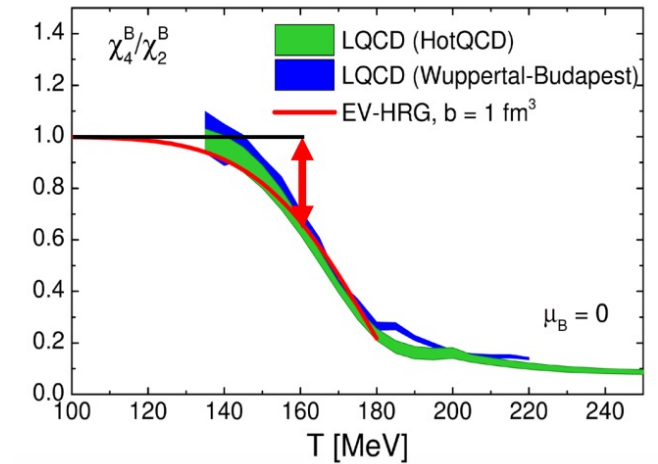


Figure from Ishii et al., PRL '07



VV et al., Phys. Lett. B 755, 71 (2017)

- Box setup reproduces the fluctuations in **excluded volume model** ( $V \rightarrow V - bN$  with  $b = \frac{16\pi r_c^3}{3}$ )

\*For Cooper-Frye hypersurface,  $|\mathbf{r}_i - \mathbf{r}_j|$  is calculated as the equal-time distance in the center-of-mass frame

## Full algorithm

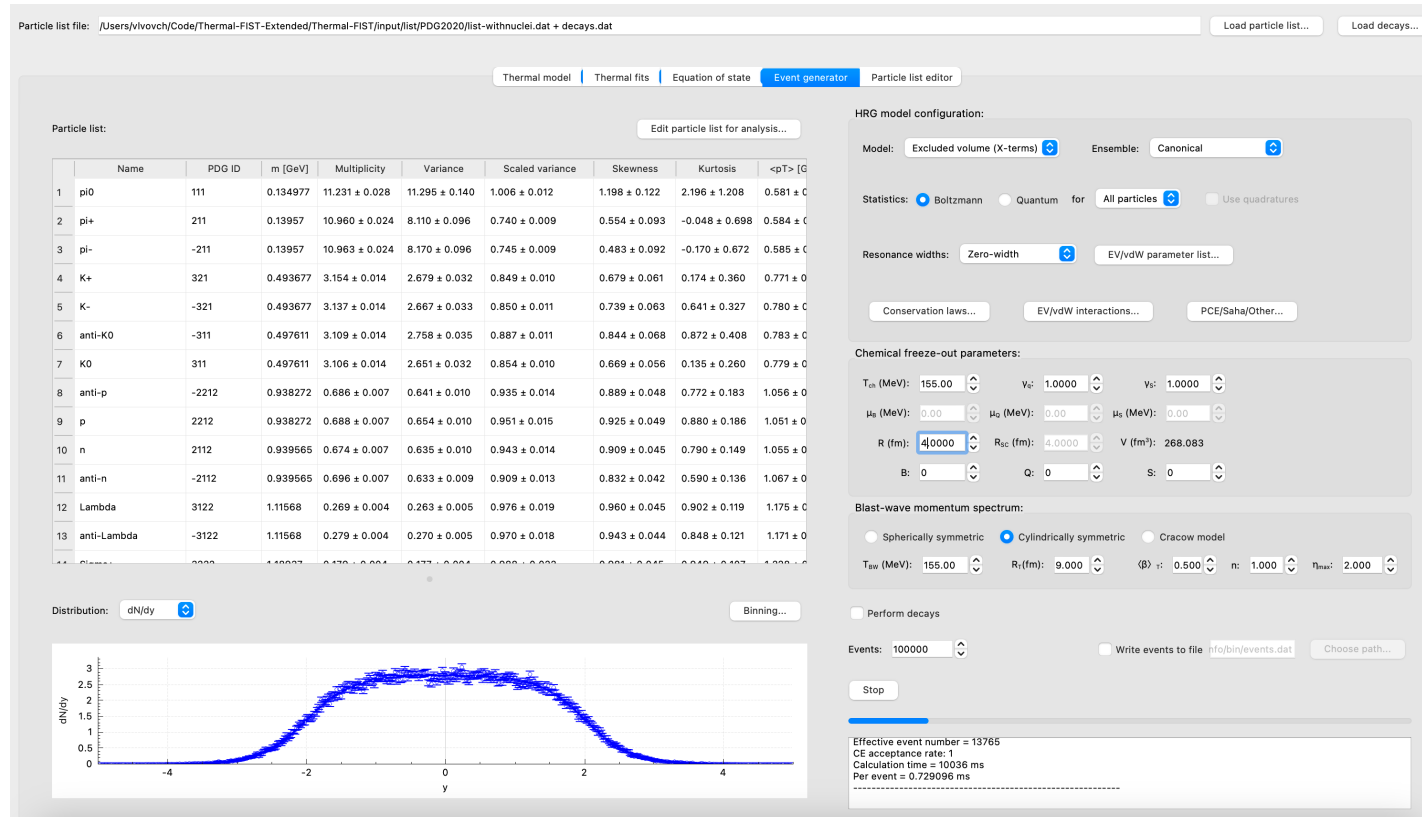
1. Sample the total number of each hadron species from Poisson distribution based on Cooper-Frye integral
2. Optionally enforce canonical treatment of global conservation laws via rejection sampling
3. Sample coordinates and momenta of hadrons one-by-one
  - Choose hydro cell from multinomial distribution
  - Momentum from thermal distribution + Lorenz boost
4. Reject the configuration if any two hadrons with hard-core repulsion overlap

## Inputs

- List of hadrons and hard-core radii  $\sigma_{ij}$  for each pair of hadron species
  - $b \sim 1 \text{ fm}^3$  in baryon-baryon interaction is motivated by lattice QCD [[Karthein, Koch, Ratti, VV, PRD 104, 094009 \(2021\)](#)]
- Cooper-Frye hypersurface: numerical from hydro (e.g. MUSIC) or parameterized blast-wave

**Implementation:** available in **Thermal-FIST** out-of-the box since version 1.4

**open source:** <https://github.com/vlvovch/Thermal-FIST>



The screenshot displays the Thermal-FIST software interface. The top bar shows the particle list file path: `/Users/vlvovch/Code/Thermal-FIST-Extended/Thermal-FIST/input/list/PDG2020/list-withnuclci.dat + decays.dat`. The main window is divided into several sections:

- Particle list:** A table with columns: Name, PDG ID, m [GeV], Multiplicity, Variance, Scaled variance, Skewness, Kurtosis, and <pT> [GeV]. The table lists particles such as pi0, pi+, pi-, K+, K-, anti-K0, K0, anti-p, p, n, anti-n, Lambda, and anti-Lambda.
- HRG model configuration:** Includes Model (Excluded volume (X-terms)), Ensemble (Canonical), Statistics (Boltzmann), Resonance widths (Zero-width), and Conservation laws.
- Chemical freeze-out parameters:** Includes T<sub>ch</sub> (MeV), V<sub>0</sub>, V<sub>1</sub>, μ<sub>B</sub> (MeV), μ<sub>S</sub> (MeV), μ<sub>3</sub> (MeV), R (fm), R<sub>0</sub> (fm), V (fm<sup>3</sup>), B, Q, and S.
- Blast-wave momentum spectrum:** Includes Spherically symmetric, Cylindrically symmetric, and Cracow model options, along with T<sub>sw</sub> (MeV), R<sub>1</sub> (fm), β, n, and η<sub>max</sub>.
- Events:** Includes a field for the number of events (100000) and a checkbox for writing events to file.
- Stop:** A button to stop the simulation.
- Summary:** A box showing simulation statistics: Effective event number = 13765, CE acceptance rate = 1, Calculation time = 10036 ms, and Per event = 0.729096 ms.

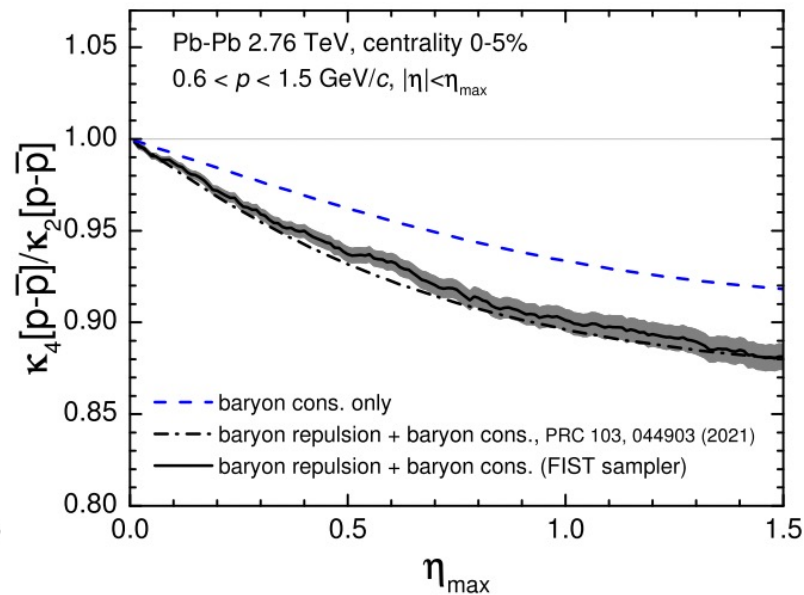
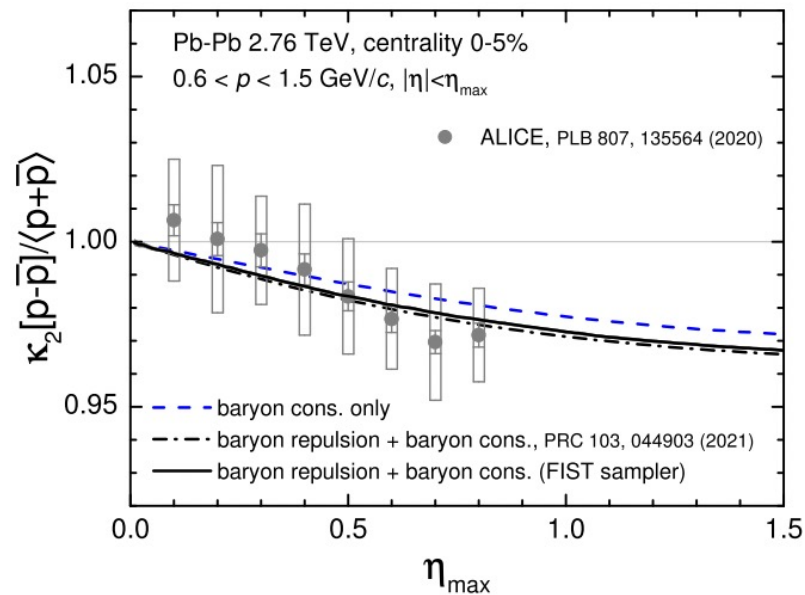
At the bottom left, a plot shows the distribution of dN/dy versus y, with a bell-shaped curve centered at y=0.

- Examples for use at LHC and RHIC: <https://github.com/vlvovch/fist-sampler>



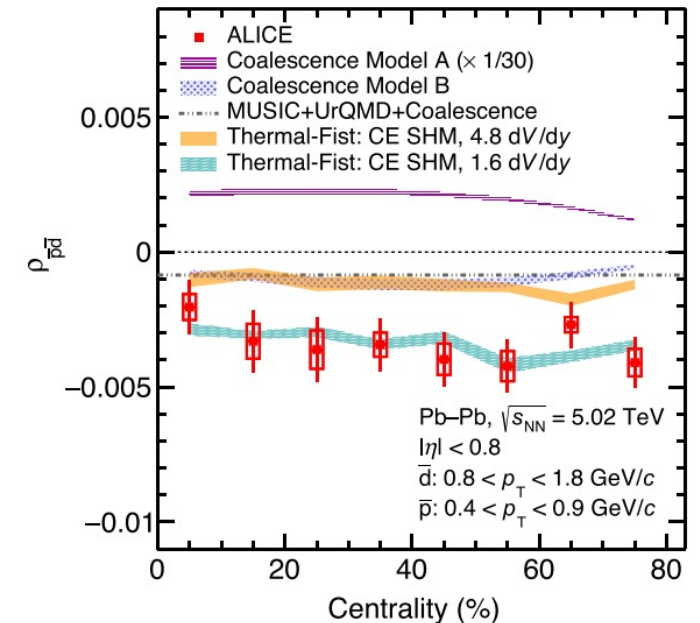
Using the blast-wave model hypersurface ( $\sim 20$  million events)

Proton cumulants



VV, Phys. Rev. C 106, 064906 (2022)

Proton-deuteron correlations



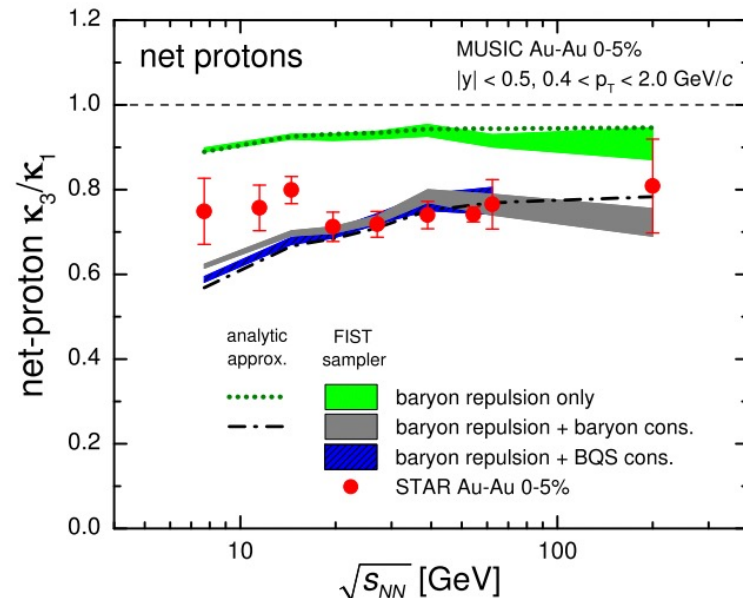
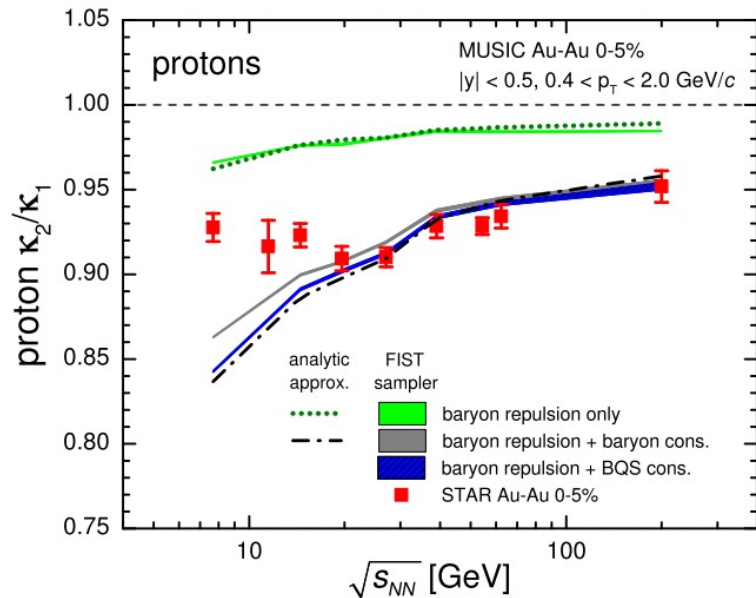
ALICE Coll., PRL 131, 041901 (2023)

Effects of hadronic afterburner UrQMD (mainly baryon annihilation) investigated in [Savchuk et al., PLB 827, 136983 (2023)]

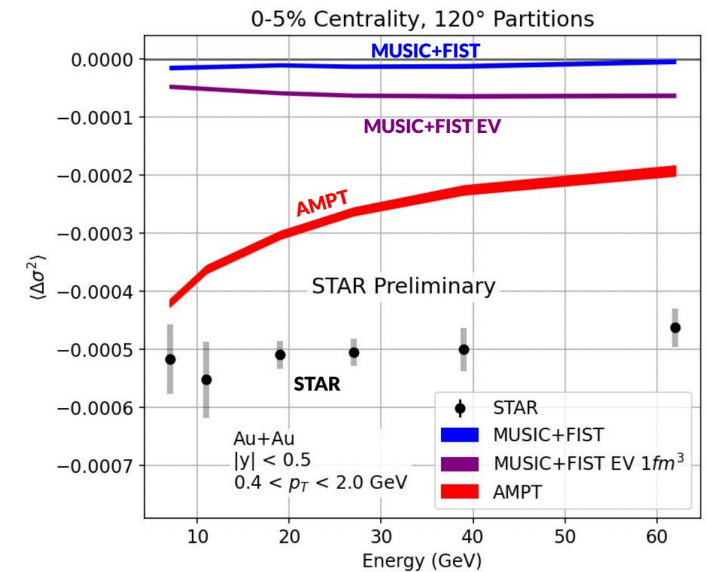


# FIST sampler at RHIC-BES

Using the single-shot hydro hypersurfaces from MUSIC (sample several million events per energy)



VV, Phys. Rev. C 106, 064906 (2022)

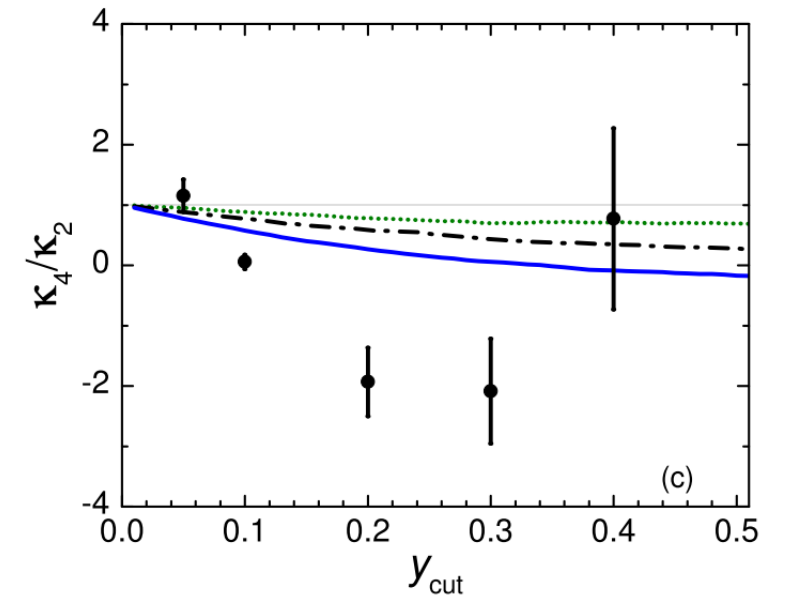
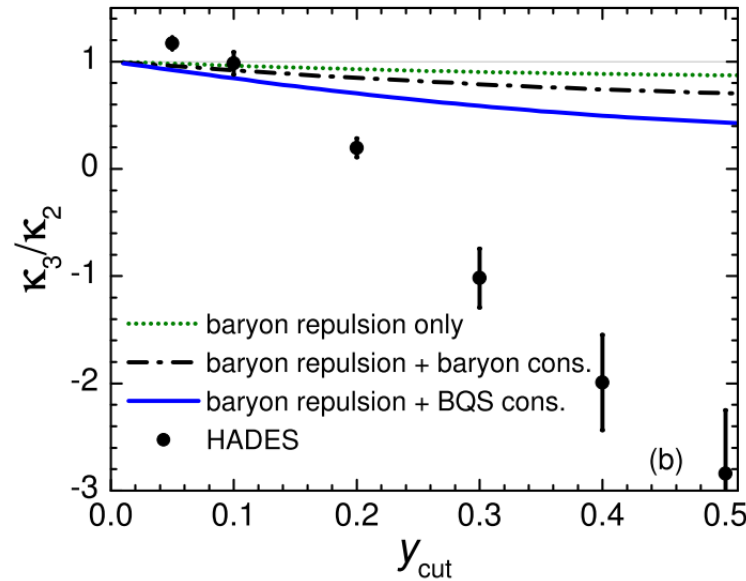
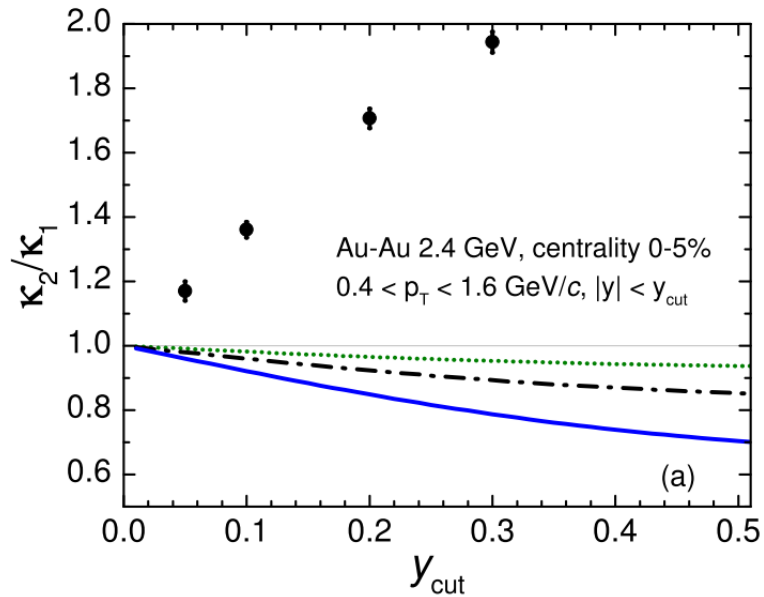


D. Neff, QM2023

- Validating earlier analytic calculations [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)] within Monte Carlo event generator
- Baryon conservation + excluded volume describe the data above 20 GeV
- **New element:** Simultaneous effects of baryon, charge, strangeness, and excluded volume
  - Electric charge conservation becomes essential below 7.7 GeV

# FIST sampler at HADES

Using Siemens-Rasmussen-Hubble fireball parameterization



VV, Phys. Rev. C 106, 064906 (2022)

- Strong effect of simultaneous baryon and electric charge conservation
- No description of HADES data

- FIST sampler is a fast Monte Carlo routine for Cooper-Frye particlization
  - Canonical treatment of conservation laws
  - Short-range hard-core repulsion among hadrons
  - Included in Thermal-FIST since v1.4 and available publicly
- Provides an **event generator** giving non-critical baseline for event-by-event fluctuations observables
- Outlook:
  - Effects of hadronic afterburner
  - Observables other than proton number cumulants
  - Viscous corrections

**Thanks for your attention!**