Proton number cumulants and correlation functions at RHIC-BES from hydrodynamics

Volodymyr Vovchenko (University of Houston)

10th workshop of the APS Topical Group on Hadronic Physics (GHP2023), Minneapolis, Minnesota

April 12, 2022

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QCD phase structure

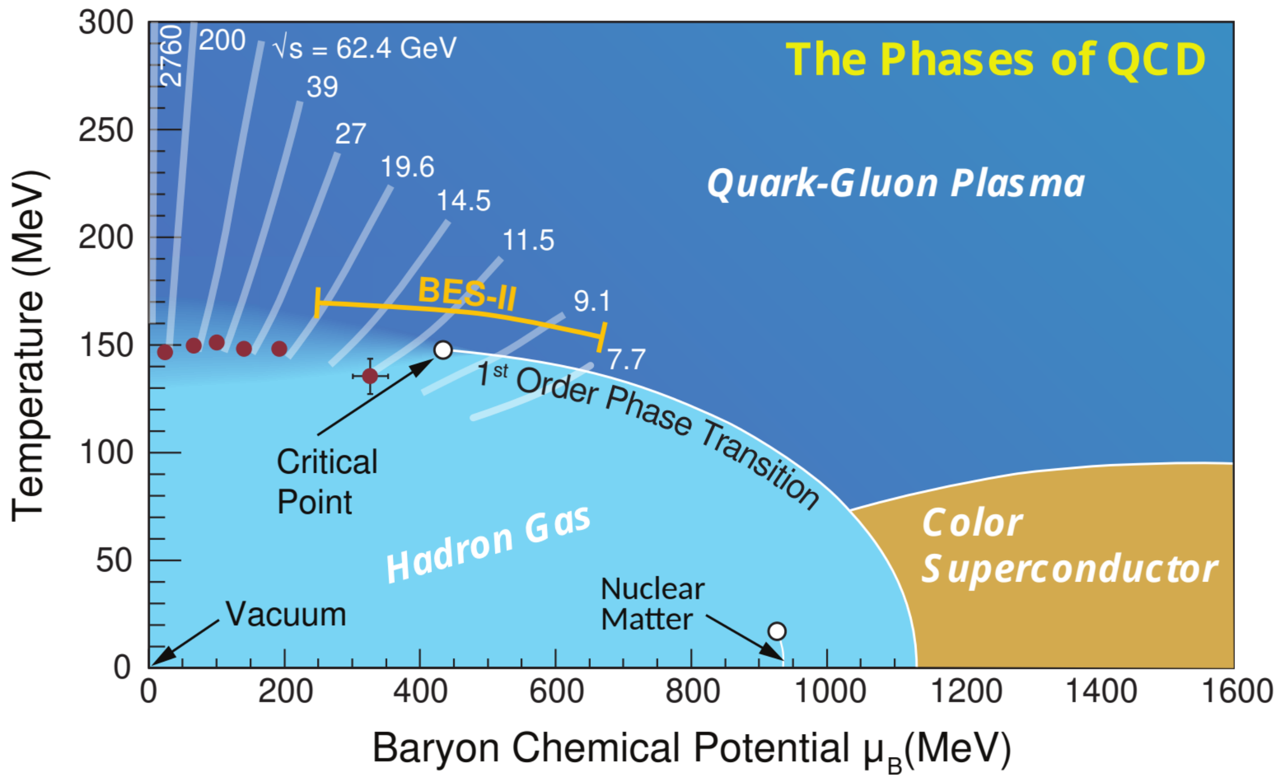


Figure from Bzdak et al., "Mapping the Phases of Quantum Chromodynamics with Beam Energy Scan", Phys. Rept. '20

- Dilute hadron gas at low T/ρ_B due to confinement, quark-gluon plasma high T/ρ_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured

Is there a critical point and how to find it with heavy-ion collisions?

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

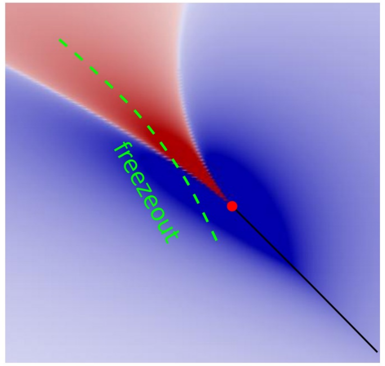
Grand partition function

$$\ln Z^{\rm gce}(T,V,\mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\rm ce}(T,V,N)\right]$$

$$\kappa_n \propto \frac{\partial^n(\ln Z^{\rm gce})}{\partial \mu^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- **(QCD) critical point** – large correlation length, critical fluctuations of baryon number



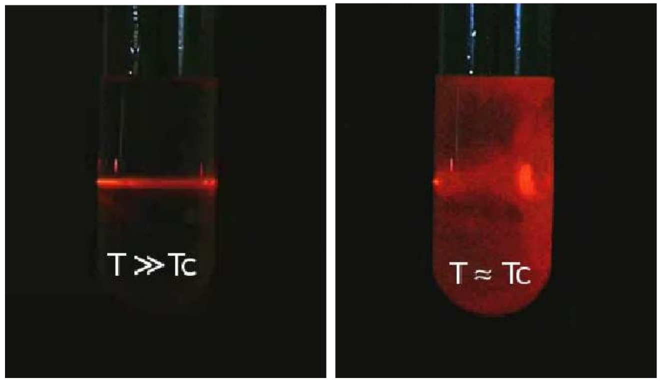
M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

$$\xi \to \infty$$

Looking for enhanced fluctuations
and non-monotonicities

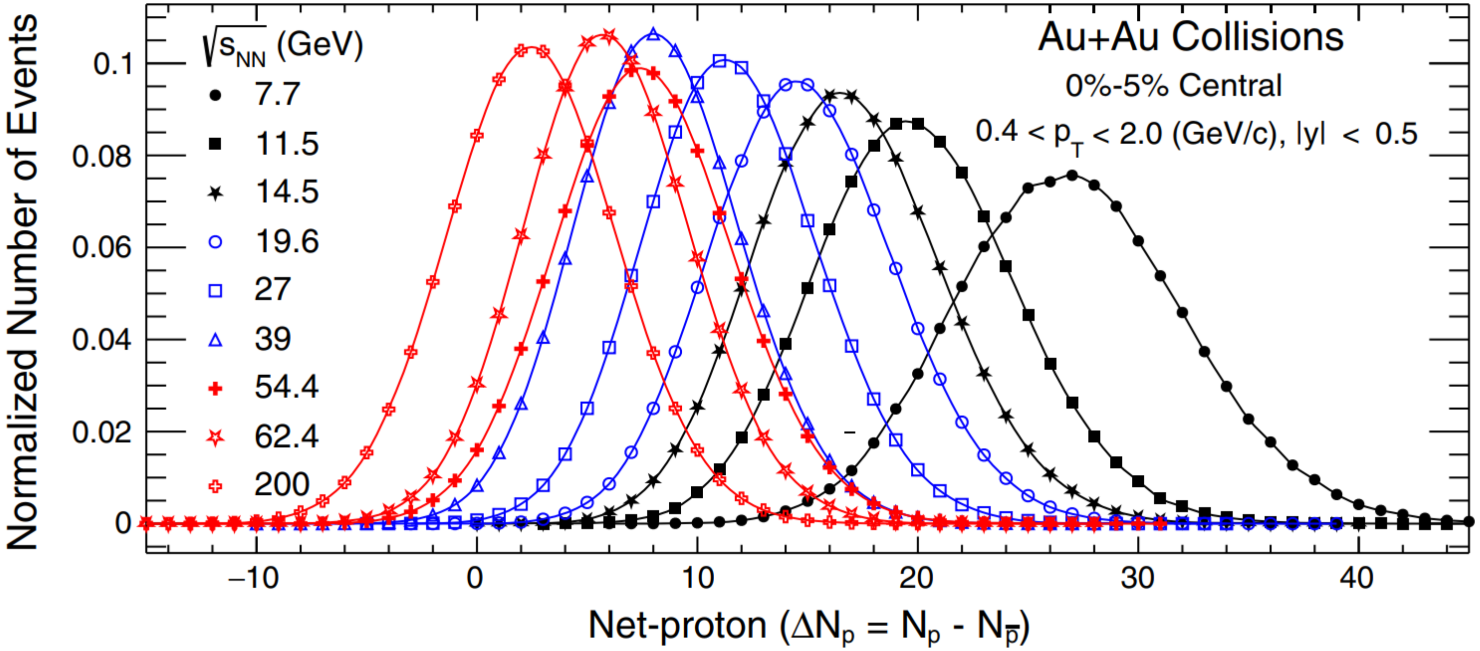
Critical opalescence



Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons $P(\Delta N_p) \sim \frac{N_{events}(\Delta N_p)}{N_{events}^{total}}$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)

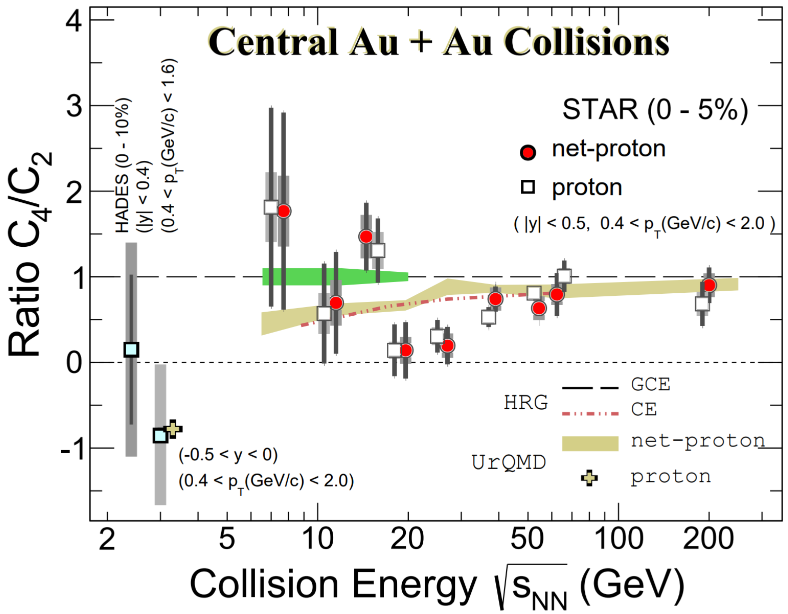


Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

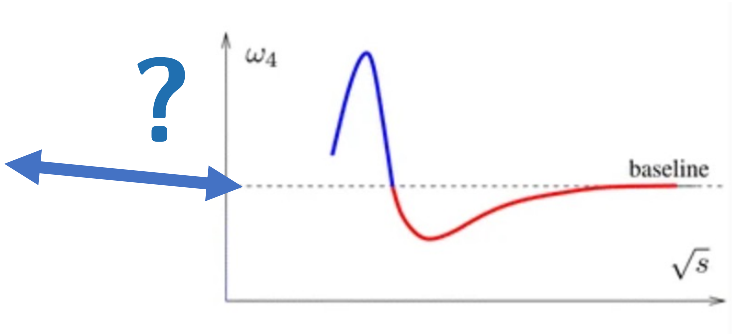
$$\frac{\kappa_2}{\langle N \rangle}, \qquad \frac{\kappa_3}{\kappa_2}, \qquad \frac{\kappa_4}{\kappa_2}$$

Experimental measurements

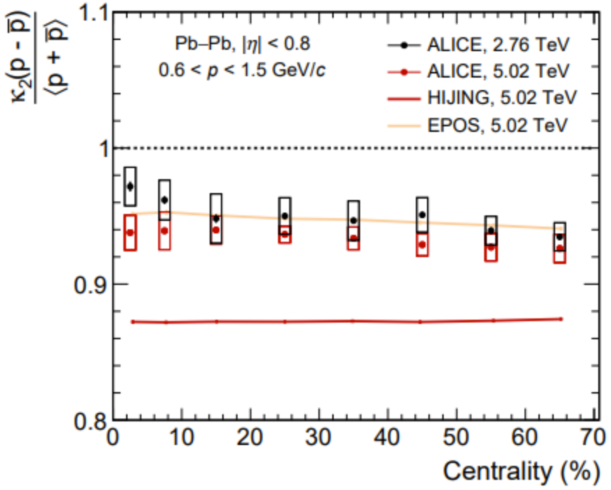
Beam energy scan in search for the critical point (STAR Coll.) or chiral crossover remnants (ALICE Coll.)



STAR Coll., Phys. Rev. Lett. 126, 092301 (2021); arXiv:2112.00240



M. Stephanov, Phys. Rev. Lett. (2011)



ALICE Coll., arXiv:2206.03343

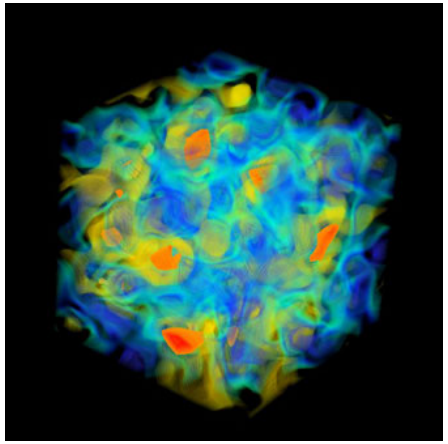
Reduced errors (better statistics) & more energies to come soon from RHIC-BES-II, STAR-FXT

Can we learn more from the more accurate data available for κ_2 and κ_3?

Other measurements: LHC-ALICE, GSI-HADES & CERN-NA61/SHINE Collaborations

Theory vs experiment: Challenges for fluctuations

Theory



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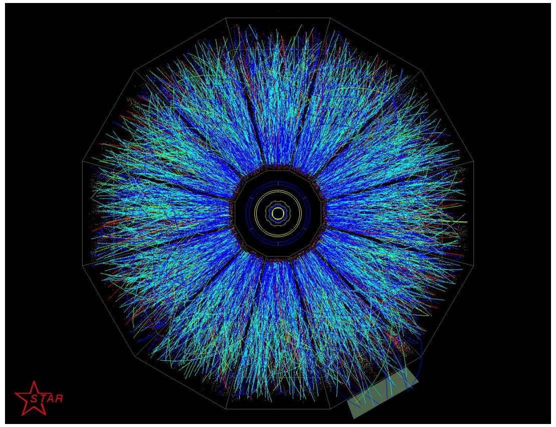
- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

Need dynamical description

Dynamical approaches to the QCD critical point search

1. Dynamical model calculations of critical fluctuations
   - Fluctuating hydrodynamics (hydro+) or molecular dynamics
   - Equation of state with a tunable critical point

   Example: effort from the Beam Energy Scan Theory (BEST) Collaboration

   BEST COLLABORATION [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]

2. **Deviations from precision calculations of non-critical fluctuations**
   - Include essential non-critical contributions to (net-)proton number cumulants
   - Exact baryon conservation + hadronic interactions (hard core repulsion)
   - Based on realistic hydrodynamic simulations tuned to bulk data

   [VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

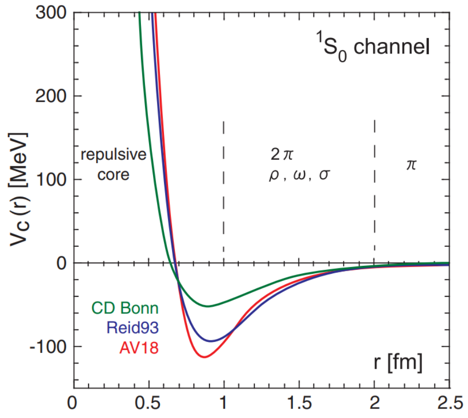
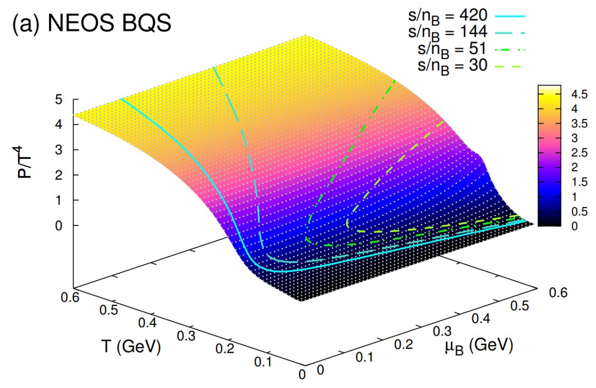


Figure from Ishii et al., PRL ‘07

Hydrodynamic description within non-critical physics

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

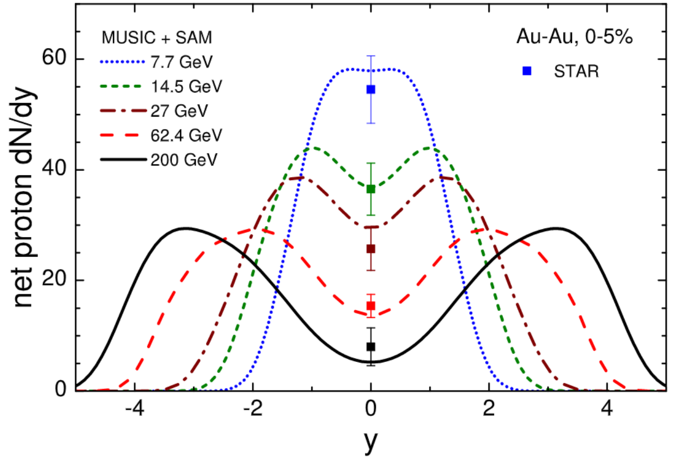
- Collision geometry based 3D initial state
  - Constrained to net proton distributions [Shen, Alzhrani, Phys. Rev. C ‘20]
- Viscous hydrodynamics evolution – MUSIC-3.0
  - Energy-momentum and baryon number conservation
  - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C ’19]



- Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3

$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x)\, p^\mu \, \frac{d_j\, \lambda_j^{\rm ev}(x)}{(2\pi)^3} \exp\left[\frac{\mu_j(x) - u^\mu(x) p_\mu}{T(x)}\right].$$

- Particlization respects QCD-based baryon number distribution
  - Incorporated via baryon excluded volume b = 1 fm^3 [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
- Incorporates exact global baryon conservation (and other charges) [VV, Phys. Rev. C 105, 014903 (2022)]



- Absent: critical point, local conservation, initial-state/volume fluctuations, hadronic phase

Calculating cumulants from hydrodynamics

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- **Analytic approach** VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)
  - Calculate proton cumulants in the experimental acceptance in the grand-canonical limit using the Cooper-Frye formula
  - Apply correction for the exact global baryon number conservation (SAM-2.0) VV, Phys. Rev. C 105, 014903 (2022)

  Pros: Calculate high-order cumulants (up to 8^{th} order) without the need for large statistics

  Cons: The method is approximate and not easily extendable to other observables

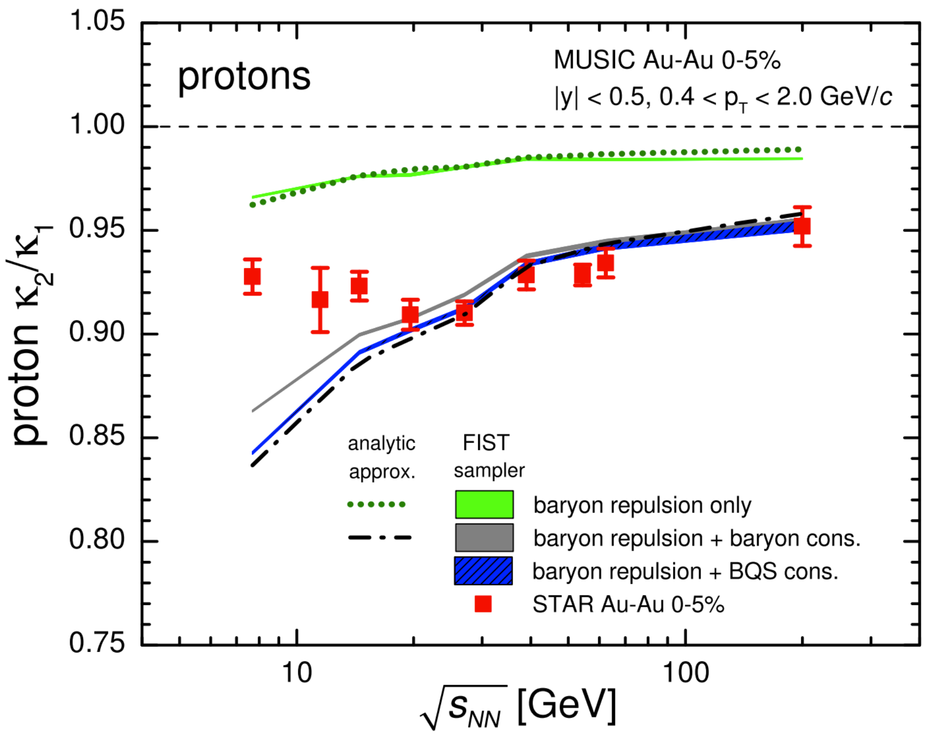
- **Monte Carlo approach (FIST sampler)** VV, Phys. Rev. C 106, 064906 (2022) https://github.com/vlvovch/fist-sampler
  - Event generator (Cooper-Frye particlization)
  - Conservation laws (baryon number but also charge and strangeness) via rejection sampling
  - Excluded volume effect by rejecting coordinate space overlap of baryons

  Pros: Flexibility of an event generator, more accurate

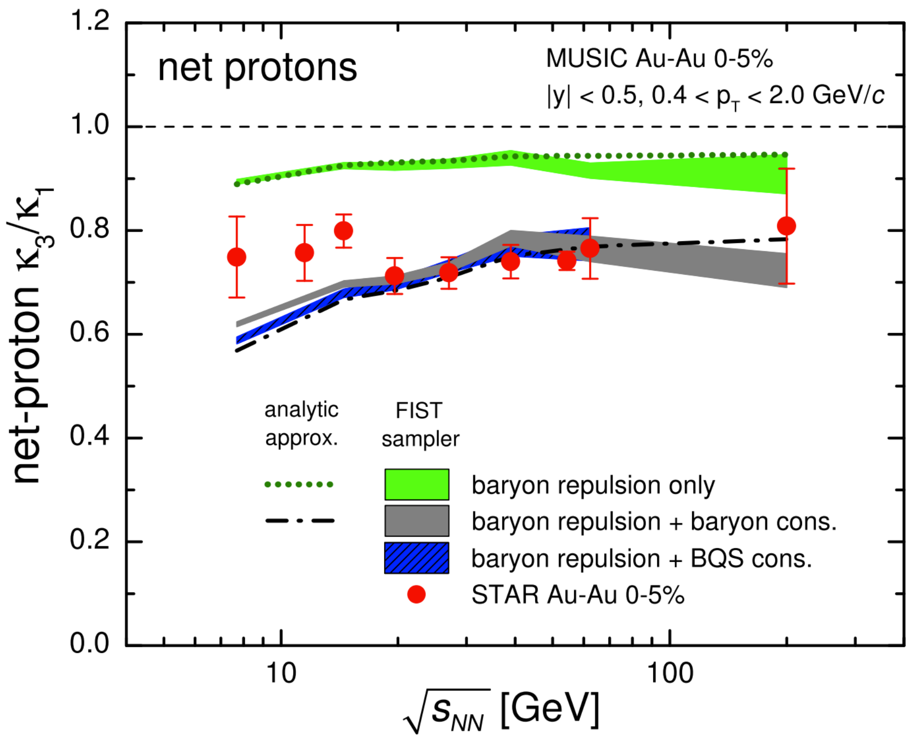
  Cons: Need large statistics for high-order cumulants

RHIC-BES: Net proton cumulant ratios (MUSIC)

Proton κ_2/κ_1



Net-proton κ_3/κ_1



- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (BQS conservation and repulsion)
- Effect from baryon conservation is larger than from repulsion
- Excess of fluctuations in data at $\sqrt{s_{NN}} < 20$ GeV – *hint of attractive interactions?*

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022); VV, arXiv:2208.13693

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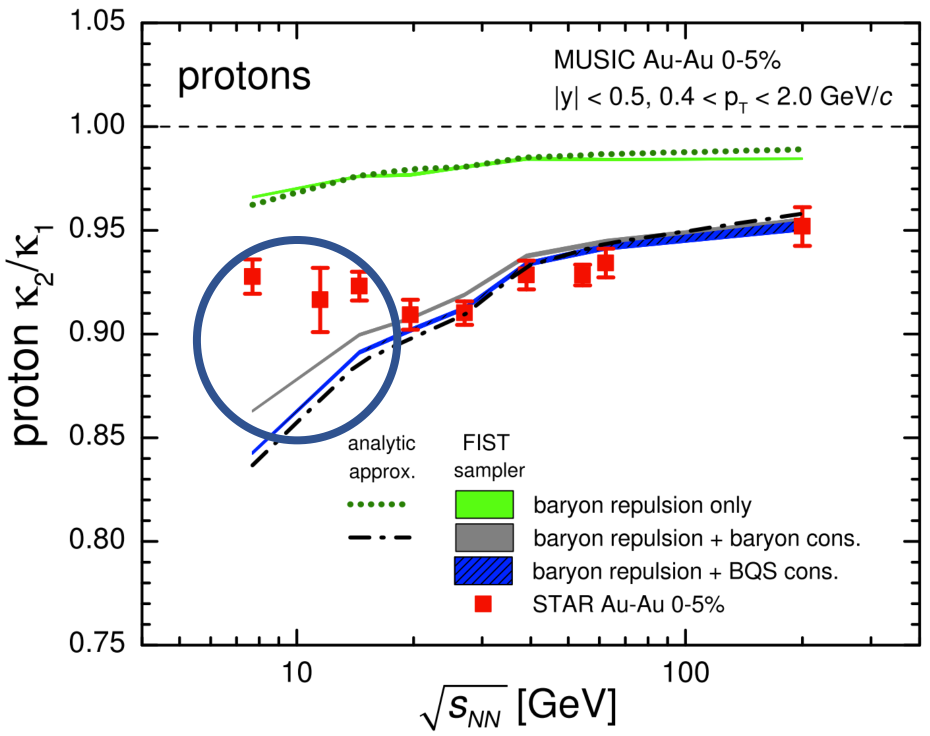

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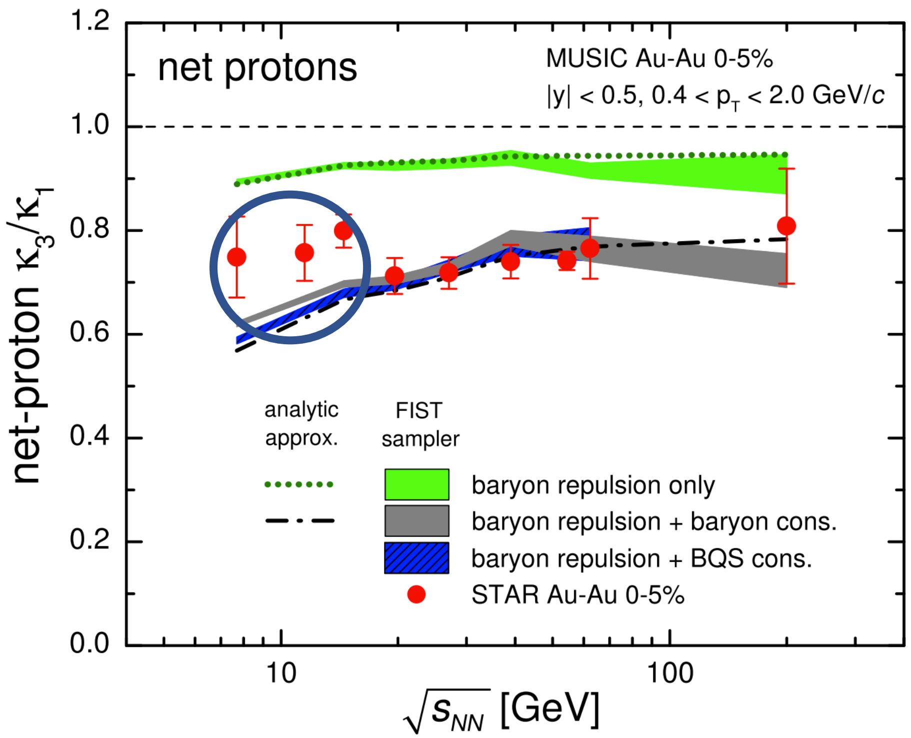

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VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022); VV, arXiv:2208.13693

Correlation Functions (factorial cumulants)

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- Analyze genuine multi-particle correlations via **factorial cumulants** $\hat{C}_n$
  [Bzdak, Koch, Strodthoff, Phys. Rev. C '17]

$$\hat{C}_1 = \kappa_1, \qquad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$
$$\hat{C}_2 = -\kappa_1 + \kappa_2, \quad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$

- Three- and four-particle correlations are small without a CP

$$\hat{C}_n^{\text{cons}} \propto \alpha^n, \qquad \hat{C}_n^{\text{EV}} \propto b^n$$

[Bzdak, Koch, Skokov, EPJC '17] [VV et al, PLB '17]

  - Multi-particle correlations expected near the critical point [Ling, Stephanov, PRC '15]

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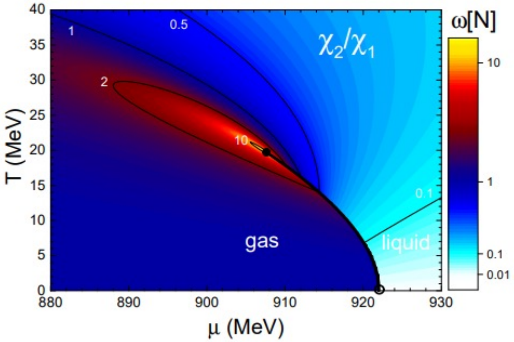
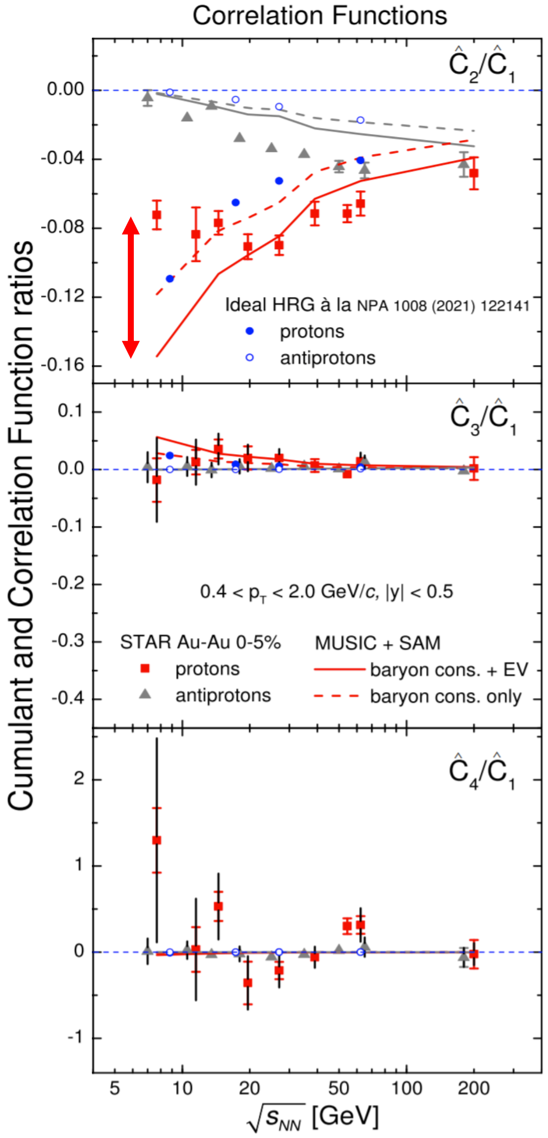
$\hat{C}_n^{\text{EV}} \propto b^n$ [VV et al, PLB '17]

  - Multi-particle correlations expected near the critical point [Ling, Stephanov, PRC '15]

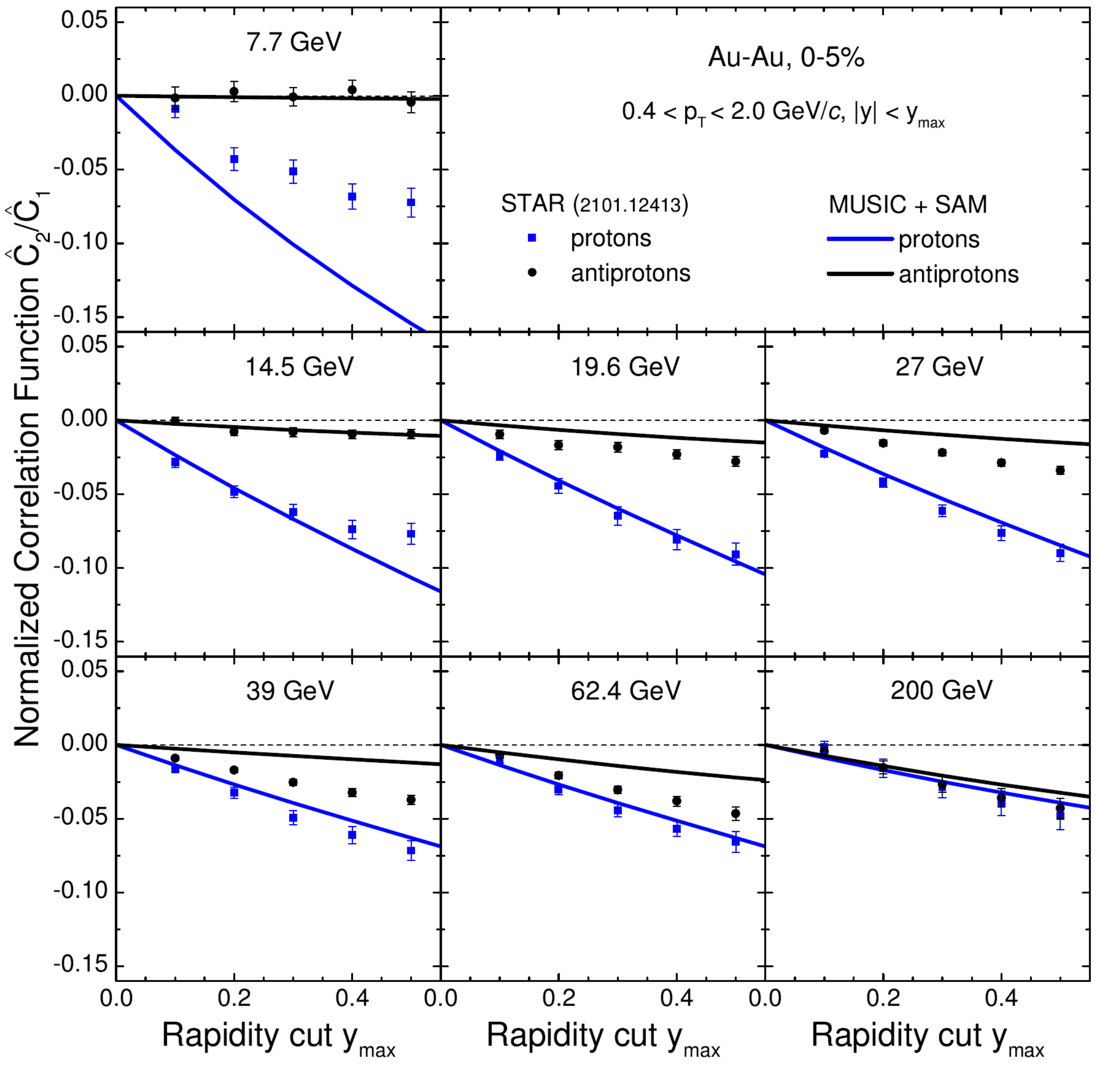
- Signals from the data at $\sqrt{s_{NN}} \leq 20$ GeV
  - Excess of two-proton correlations
  - Possibility of significant 4-proton correlations
  - ***Critical point?***





Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?



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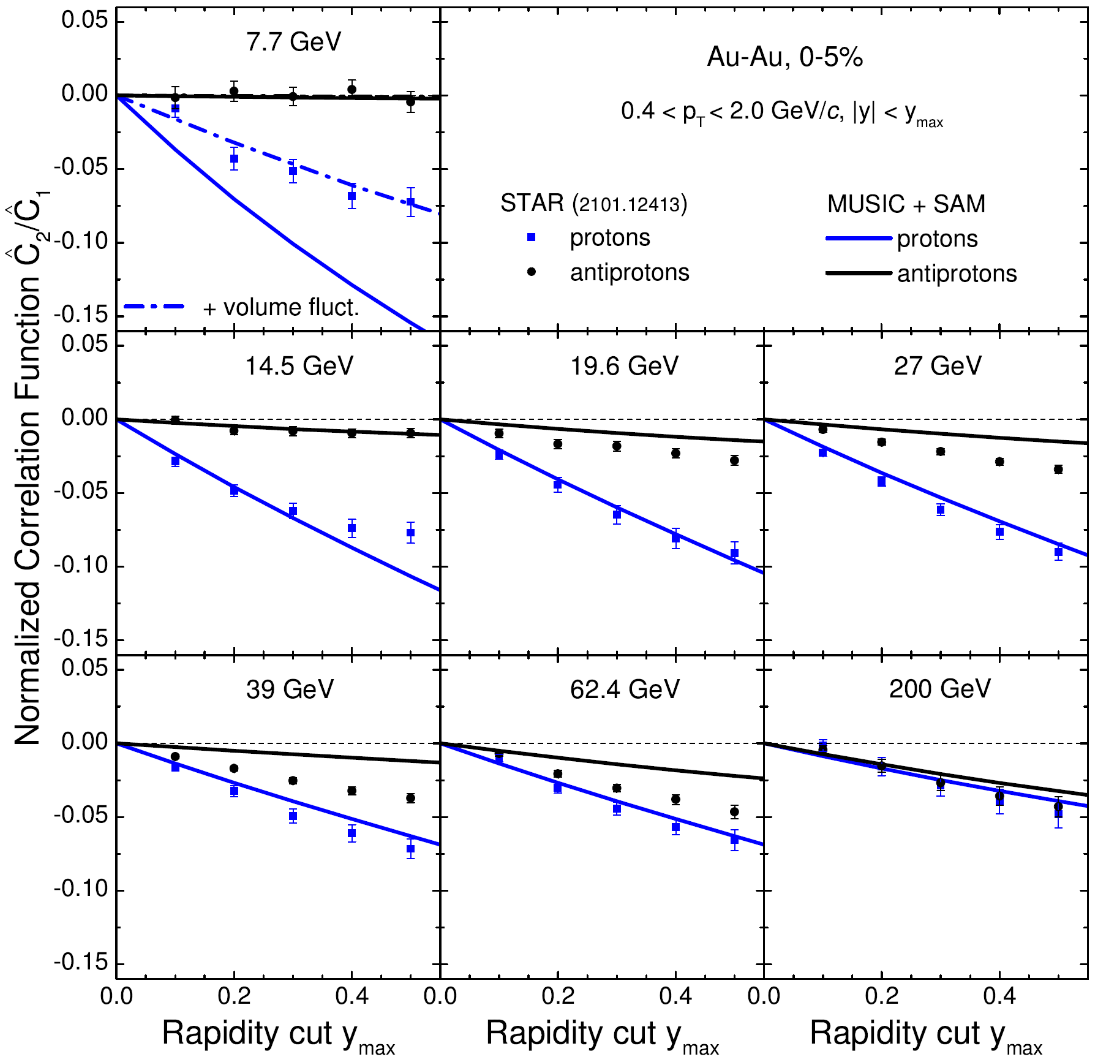
- **Volume fluctuations?** [Skokov, Friman, Redlich, PRC '13]
  - $C_2/C_1 \mathrel{+}= C_1 * \Delta v^2$



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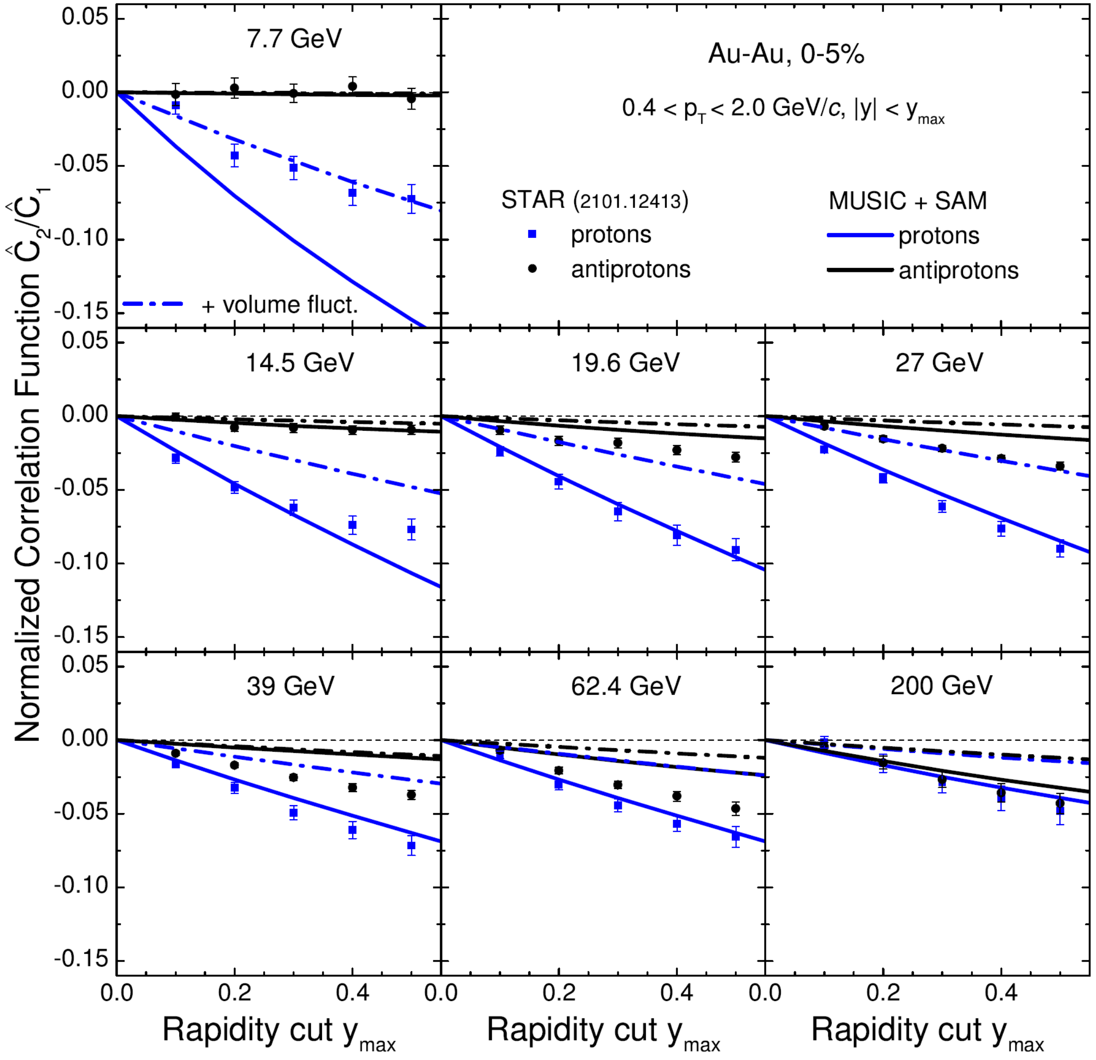

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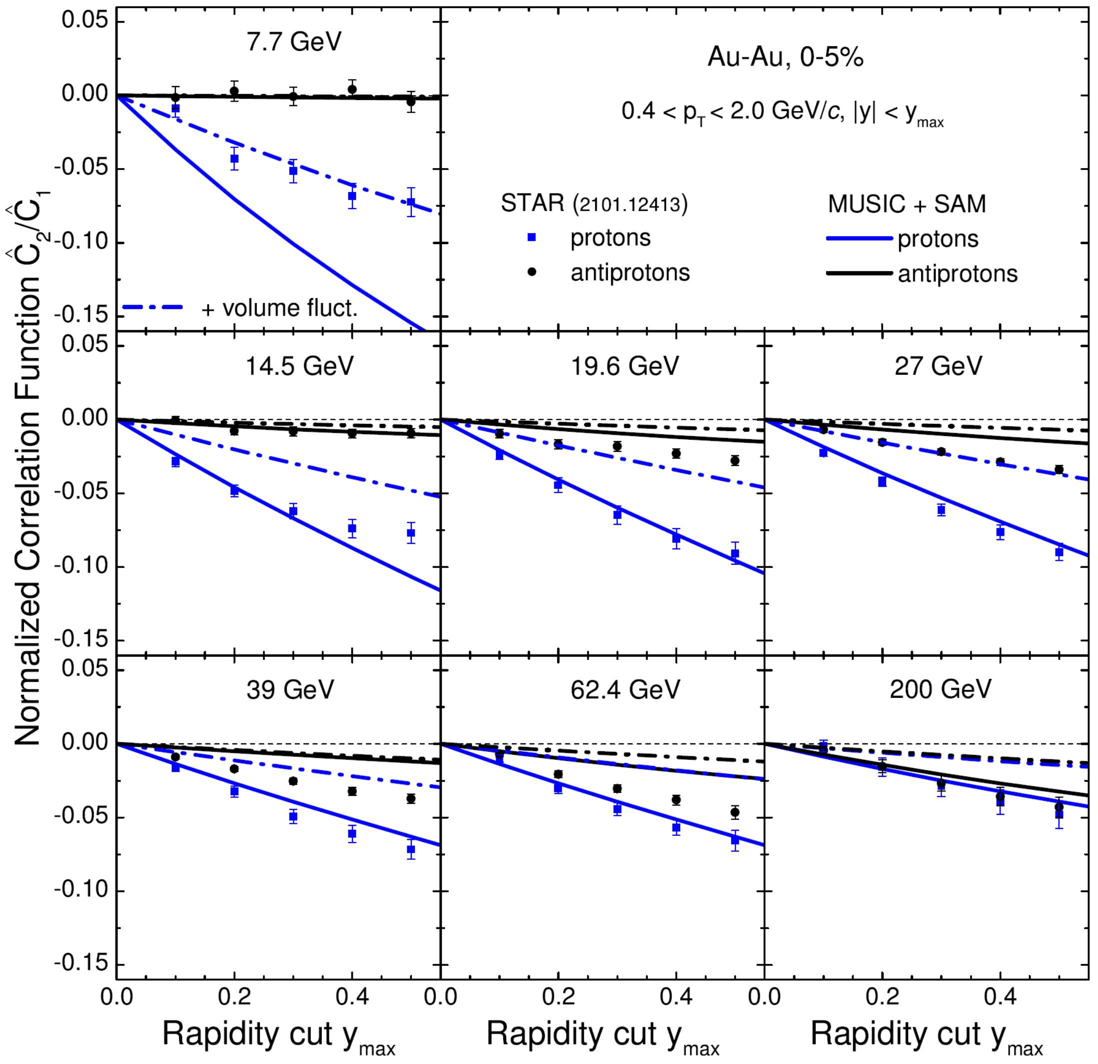
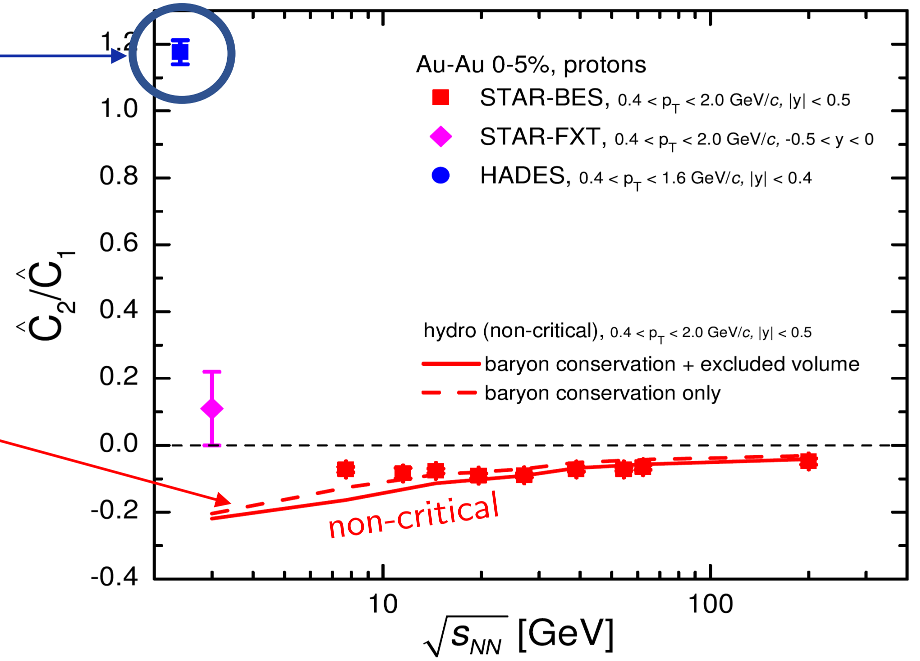
  - $C_2/C_1 \mathrel{+}= C_1 * \Delta v^2$
  - Can improve low energies but spoil high energies?
- **Attractive interactions?**
  - Could work if baryon repulsion turns into attraction in the high-μ_B regime
  - **Critical point?**

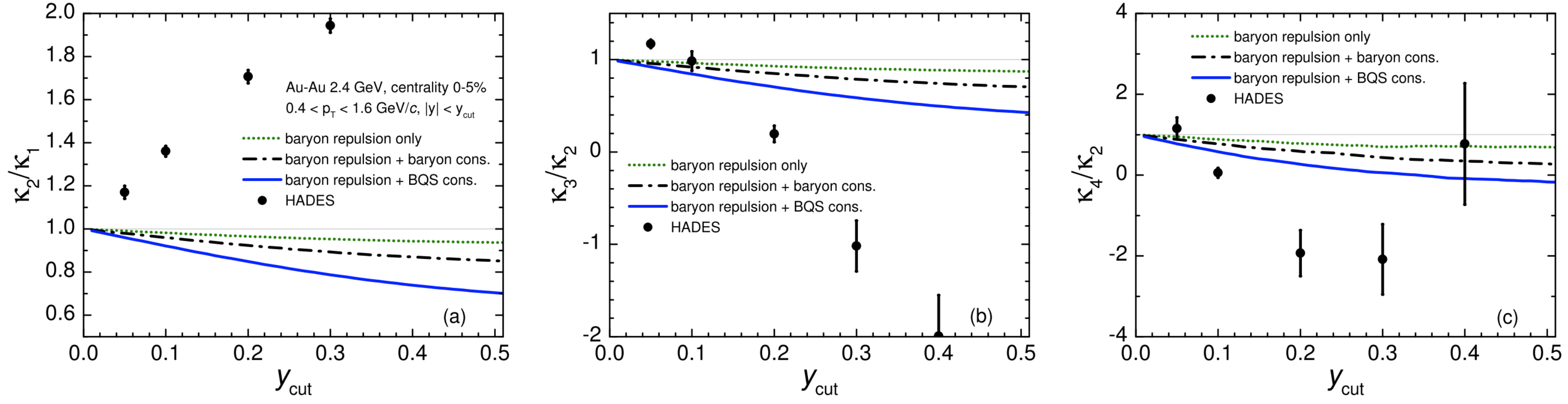


Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV

- Intriguing hints from HADES@ 2.4 GeV and STAR-FXT@3GeV: huge excess of two-proton correlations! [HADES Collaboration, Phys. Rev. C 102, 024914 (2020)] [STAR Collaboration, Phys. Rev. Lett. 128, 202303 (2022)]

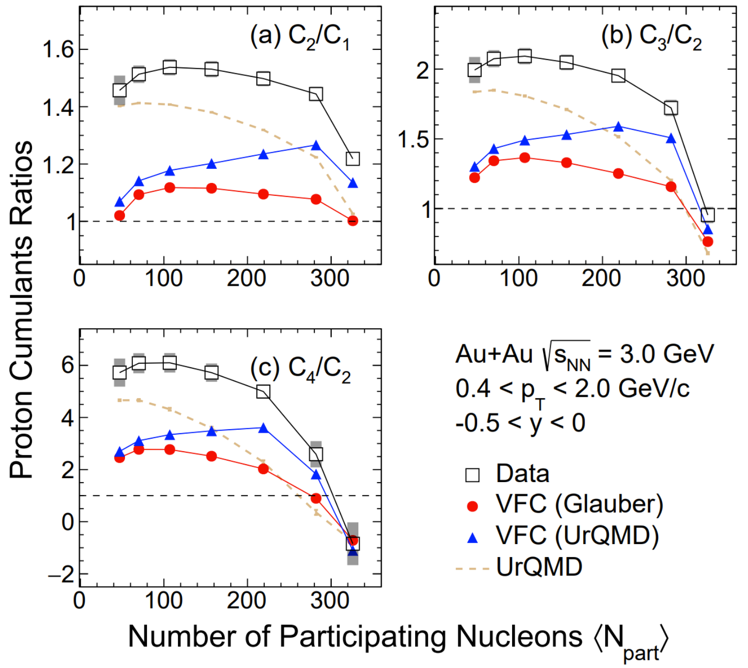


- No change of trend in the non-critical reference
- Additional mechanisms:
  - Nuclear liquid-gas transition
  - Light nuclei formation



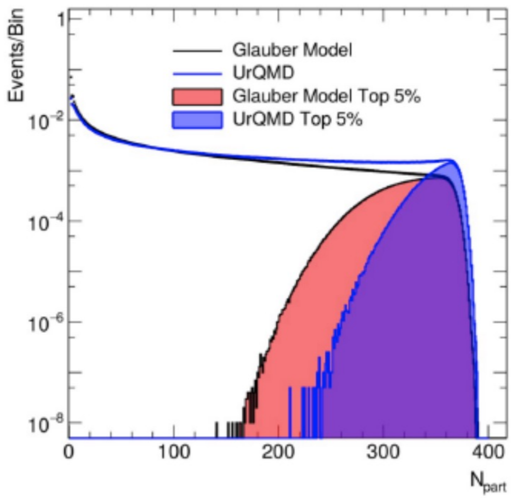
VV, Phys. Rev. C 106, 064906 (2022)

Closer look at data at lower energies



STAR Collaboration, Phys. Rev. Lett. 128 (2022) 202303

STAR-FXT



HADES

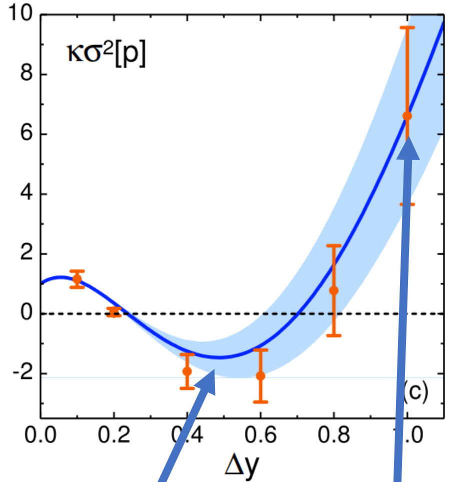


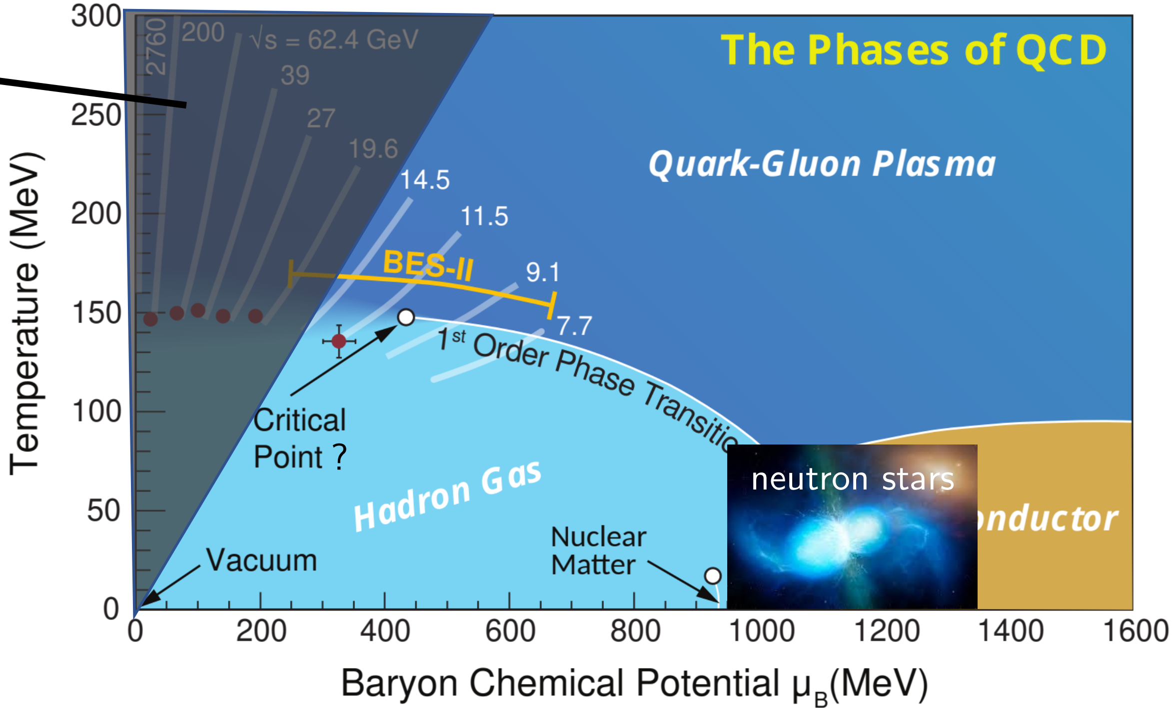
Figure from O. Savchuk et al., PLB 835, 137540 (2022)

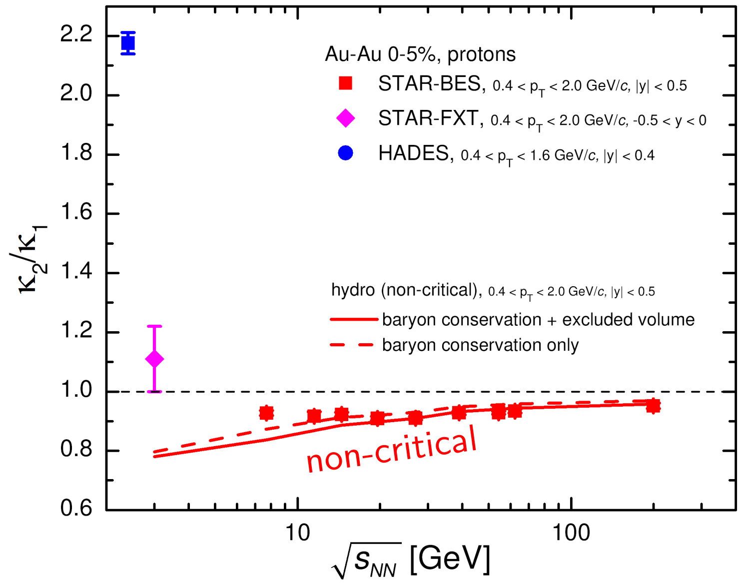
- Volume fluctuations/centrality selection appear to play an important role
  - UrQMD is useful for understanding basic systematics associated with it
- Indications for enhanced scaled variance, $\kappa_2/\kappa_1 > 1$
- κ_4/κ_2 negative and described by UrQMD (purely hadronic?), note -0.5<y<0 instead of |y|<0.5

Proper understanding of $\kappa_2/\kappa_1 > 1$ in both HADES and STAR-FXT is missing

Summary: What we learned from fluctuations at RHIC-BES

Critical point disfavored



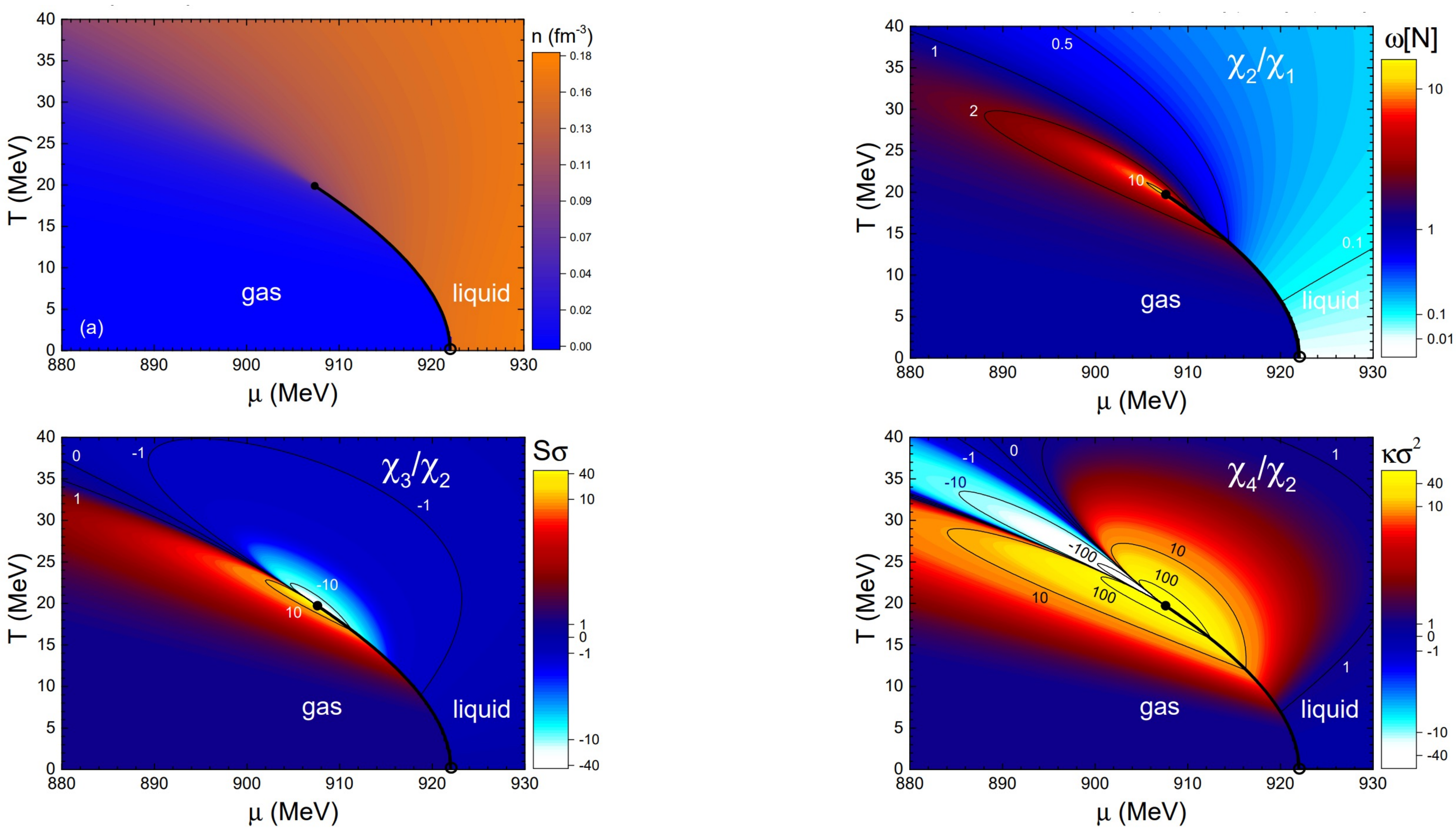


- Data at high energies ($\sqrt{s_{NN}} \geq 20$ GeV) consistent with "non-critical" physics
  - Disfavors QCD critical point at μ_B/T<2-3, consistent with what we know from lattice QCD
- Interesting indications for (multi)-proton correlations at $\sqrt{s_{NN}} \leq 7.7$ GeV, better modeling required

Thanks for your attention!

Backup slides

Example: Cumulants near the nuclear liquid-gas transition



VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Excluded volume effect

Incorporate repulsive baryon (nucleon) hard core via excluded volume

VV, M.I. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

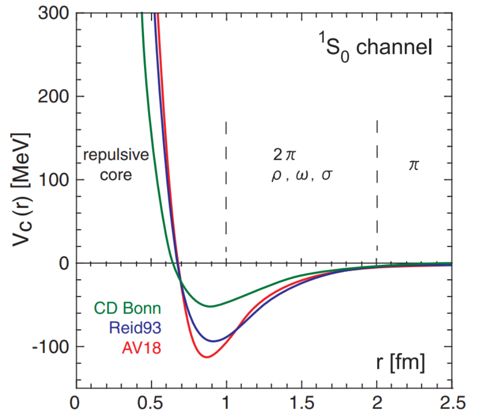
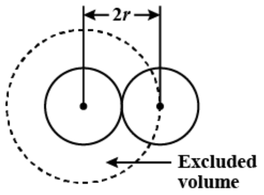


Figure from Ishii et al., PRL '07

Amounts to a van der Waals correction for baryons in the HRG model

$$V \to V - bN$$

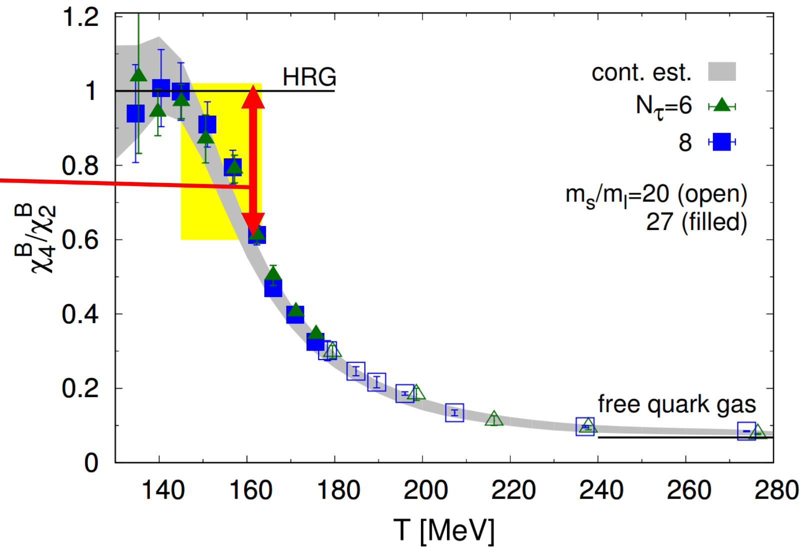


$$p^{\rm ev}_{B(\bar{B})} = p^{\rm id}_{B(\bar{B})}\, e^{-b p^{\rm ev}_{B(\bar{B})}/T}$$

- Net baryon kurtosis suppressed as in lattice QCD

$$\frac{\chi_4^B}{\chi_2^B} \simeq 1 - 12 b \phi_B(T) + O(b^2)$$

- Reproduces virial coefficients of baryon interaction from lattice QCD

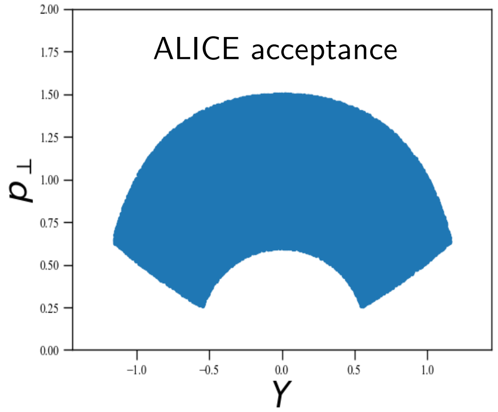


Excluded volume from lattice QCD: **b ≈ 1 fm³**

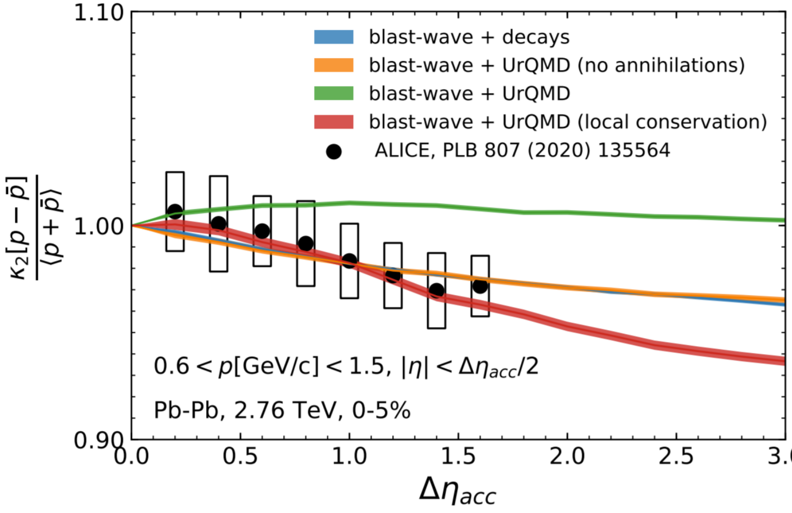
VV, A. Pasztor, S. Katz, Z. Fodor, H. Stoecker, Phys. Lett. B 755, 71 (2017)

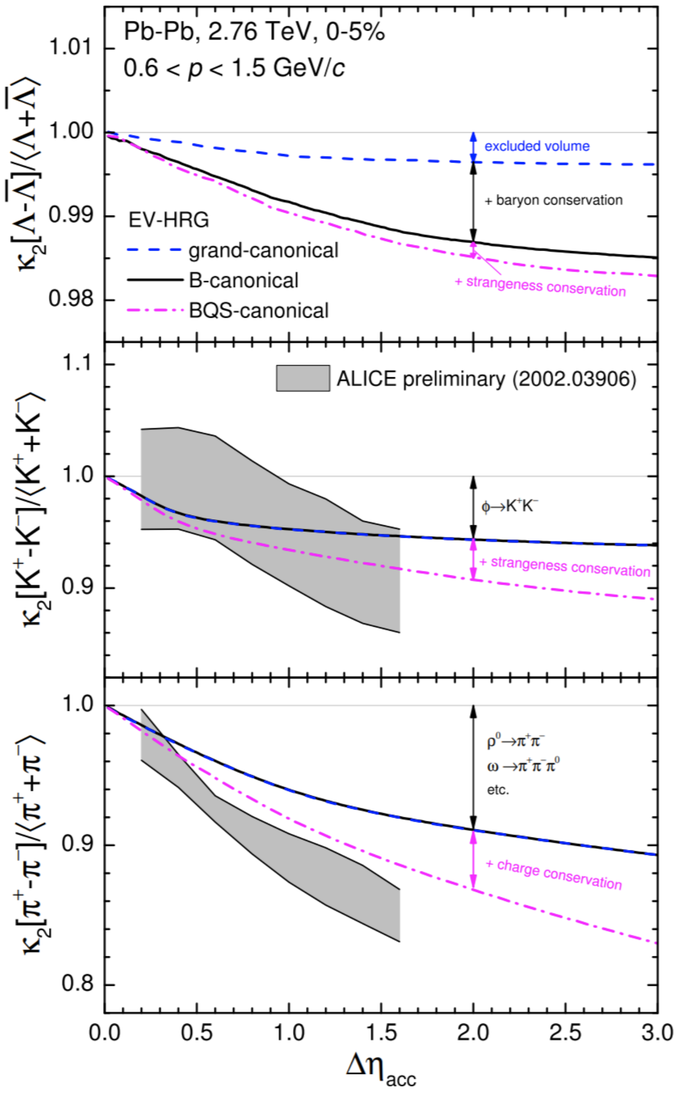
Net-particle fluctuations at the LHC (blast-wave model)

- Net protons described within errors and consistent with either
  - global baryon conservation without $B\bar{B}$ annihilations
    see e.g. ALICE Coll. arXiv:2206.03343
  - or local baryon conservation with $B\bar{B}$ annihilations
    O. Savchuk et al., Phys. Lett. B 827, 136983 (2022)



$0.6 < p < 1.5$ GeV/c, $\Delta\eta_{acc} = 1.6$





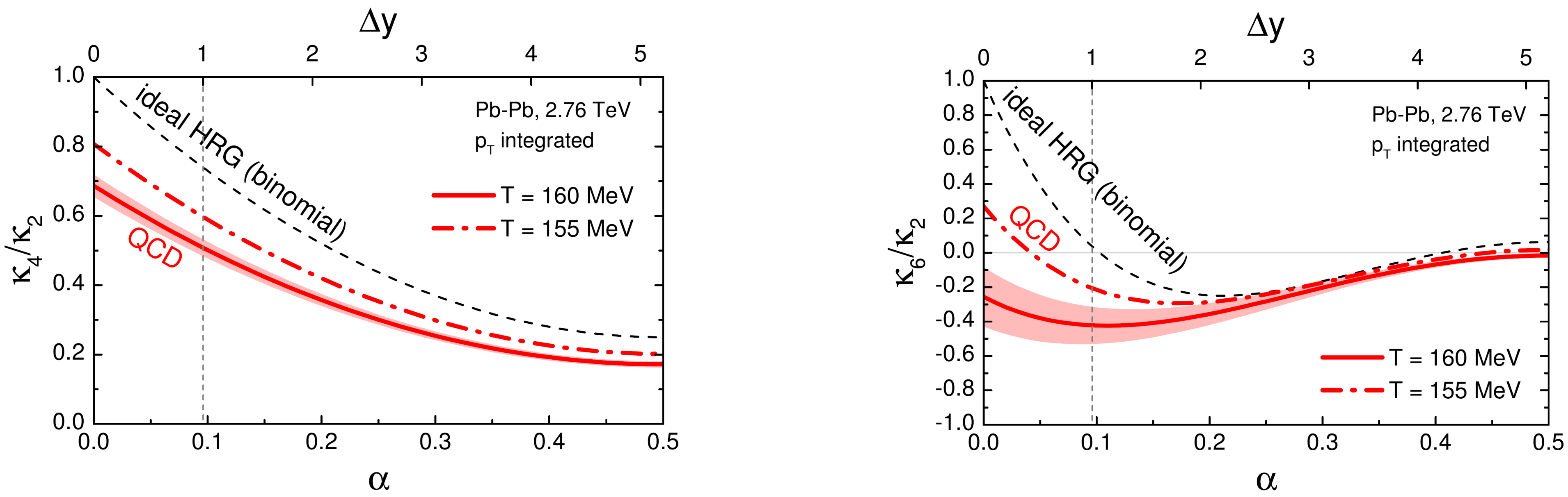
- Large effect from resonance decays for pions and kaons + exact conservation of electric charge/strangeness

VV, Koch, Phys. Rev. C 103, 044903 (2021)

Net baryon fluctuations at LHC ($\mu_B = 0$)

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

$$\left(\frac{\kappa_4}{\kappa_2}\right)_{LHC} = (1 - 3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} \qquad \left(\frac{\kappa_6}{\kappa_2}\right)_{LHC} = [1 - 5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$



Lattice data for χ_4^B/χ_2^B and χ_6^B/χ_2^B from Borsanyi et al., 1805.04445

Theory: negative χ_6^B/χ_2^B is a possible signal of chiral criticality [Friman, Karsch, Redlich, Skokov, EPJC '11]

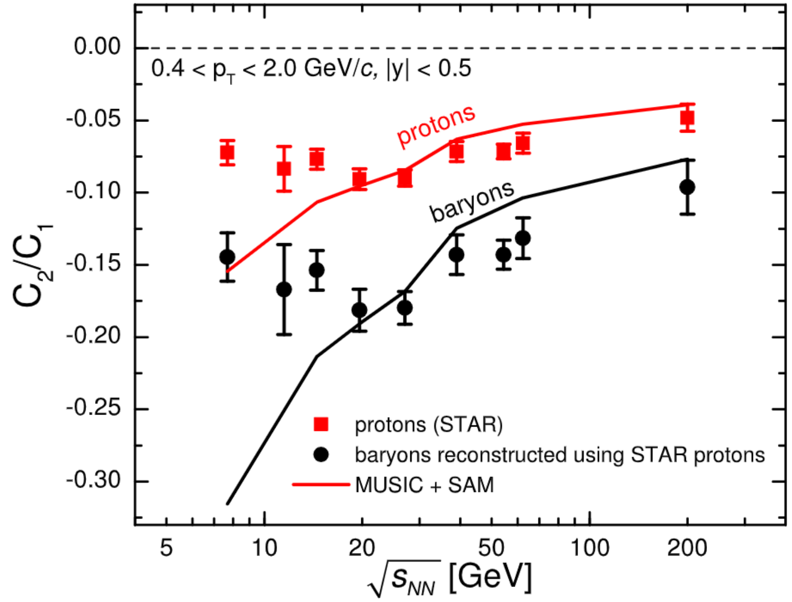
Experiment: $\alpha \approx \frac{N_{ch}(\Delta y)}{N_{ch}(\infty)} \approx \mathrm{erf}\left(\frac{\Delta y}{2\sqrt{2}\sigma_y}\right)$, for $\Delta y \approx 1$ the κ_6/κ_2 is mainly sensitive to the EoS

$N_{ch}(\Delta y)$ measurement: ALICE Collaboration, PLB 726 (2013) 610-622

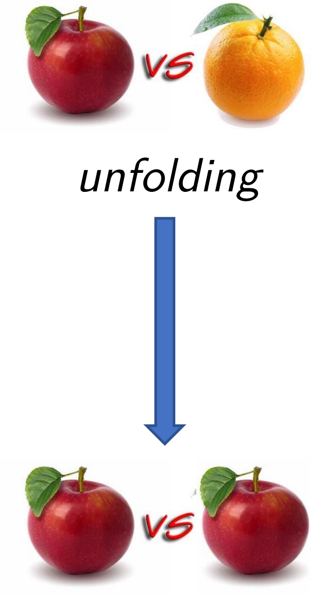
Baryon cumulants from protons

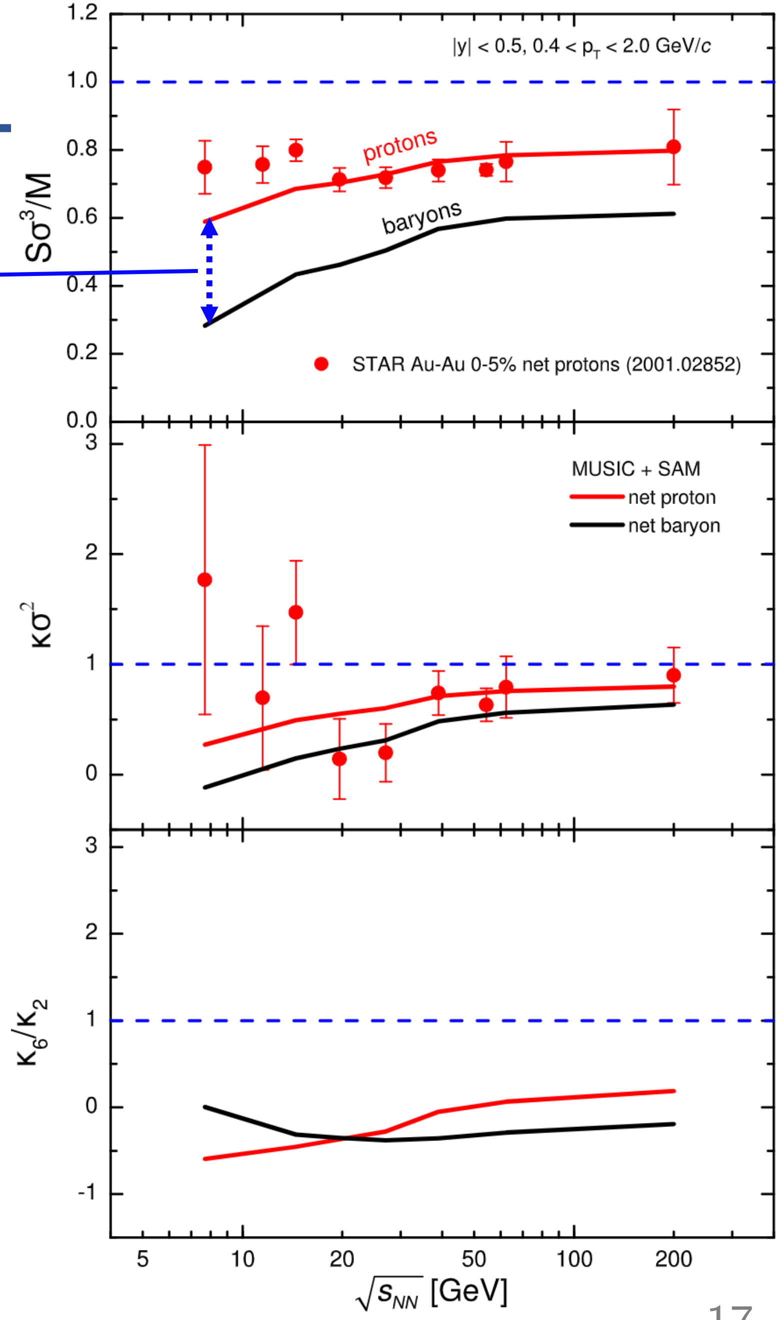
- **net baryon** $\neq$ **net proton** (protons are ***subset*** of all baryons)

- Baryon cumulants can be reconstructed from proton cumulants via binomial (un)folding based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]
  - Amounts to an additional “efficiency correction” and requires the use of
    ...xperiment can do it mod



$$\frac{\hat{C}_2^B}{\hat{C}_1^B} \approx 2\,\frac{\hat{C}_2^p}{\hat{C}_1^p}$$



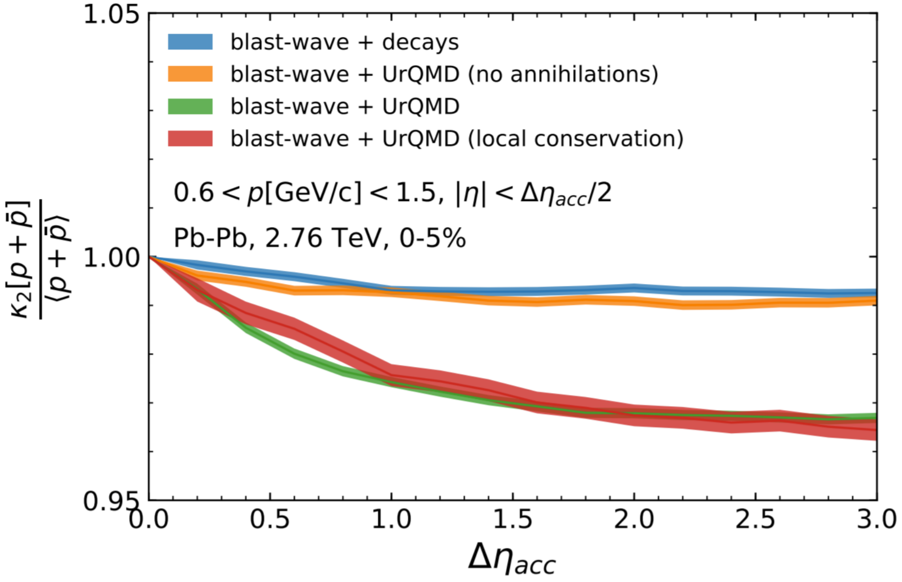


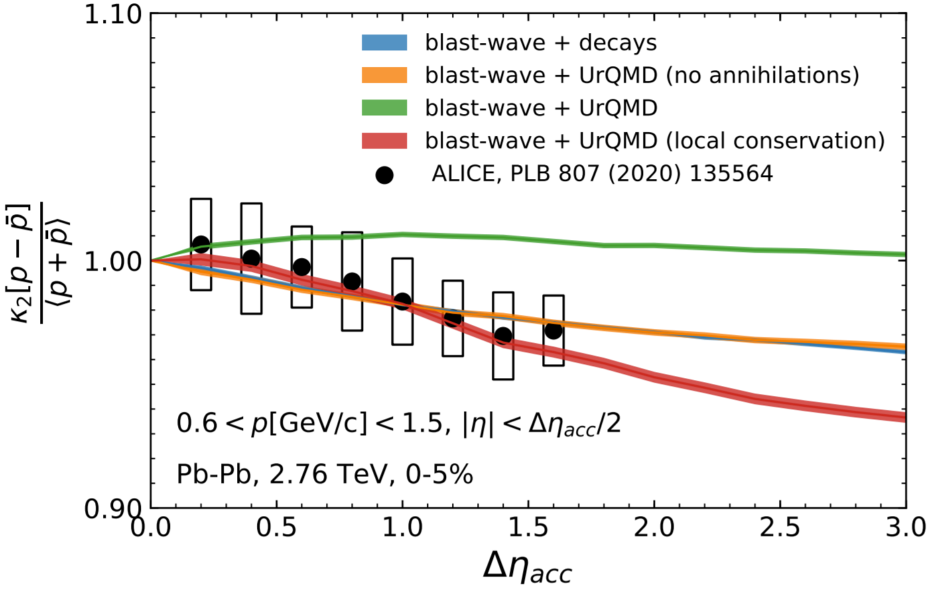
Effects of baryon annihilation and local conservation

O. Savchuk, V.V., V. Koch, J. Steinheimer, H. Stoecker, arXiv:2106.08239

Baryon annihilation $B\bar{B} \to n\pi$ in afterburners (UrQMD, SMASH) suppresses baryon yields

$\langle p + \bar{p} \rangle \searrow$ $\longrightarrow$ $\frac{\kappa_2[p-\bar{p}]}{\langle p+\bar{p} \rangle} \nearrow$





- ALICE data requires local baryon conservation across $\Delta y \sim \pm 1.5$ with UrQMD annihilations (no regenerations) or global conservation ($\Delta y \sim \Delta y_{tot}$) without annihilations
- Local conservation and $B\bar{B}$ annihilation can be constrained from data through the combined analysis of $\kappa_2[p-\bar{p}]$ and $\kappa_2[p+\bar{p}]$

Thermodynamic analysis of HADES data

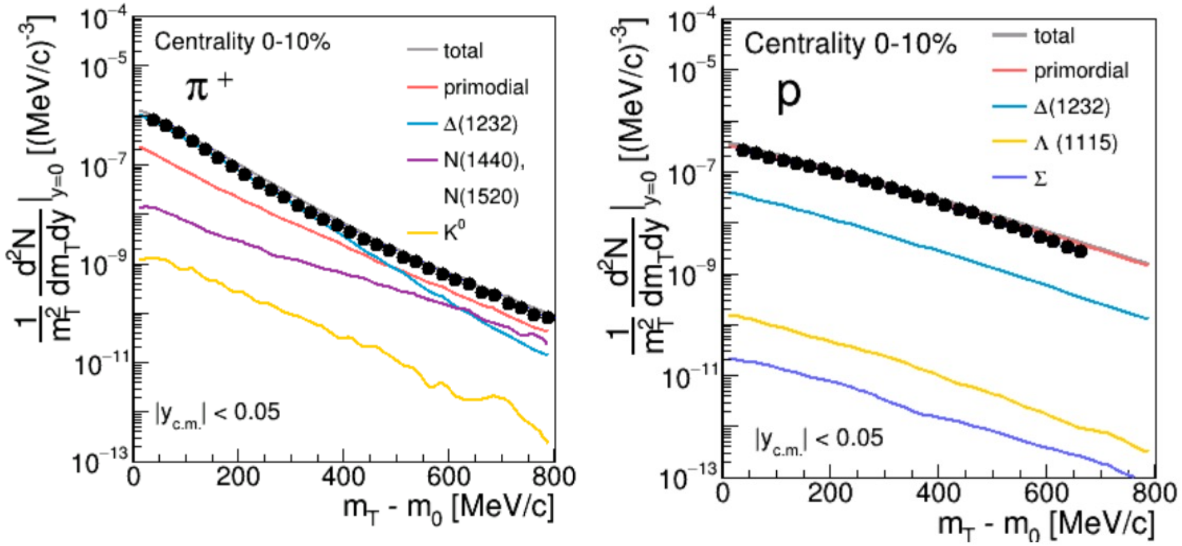
VV, Koch, Phys. Lett. B 833, 137368 (2022)

- Single freeze-out scenario: Emission from Siemens-Rasmussen hypersurface with Hubble-like flow

  → Pion and proton spectra o.k.

  [S. Harabasz et al., PRC 102, 054903 (2020)]

- Uniform $T \approx 70$ MeV, $\mu_B \approx 875$ MeV across the fireball [A. Motornenko et al., PLB 822, 136703 (2021)]



- Fluctuations:
  - Same as before but incorporate additional binomial filtering to account for protons bound in light nuclei
  - Uniform fireball → Final proton cumulants are linear combinations of baryon susceptibilities χ_n^B at freeze-out

$$\kappa_n^p = \sum_{m=1}^{n} \alpha_{n,m} \, \chi_m^B$$

Extract χ_n^B directly from experimental data

A closer look at the HADES data

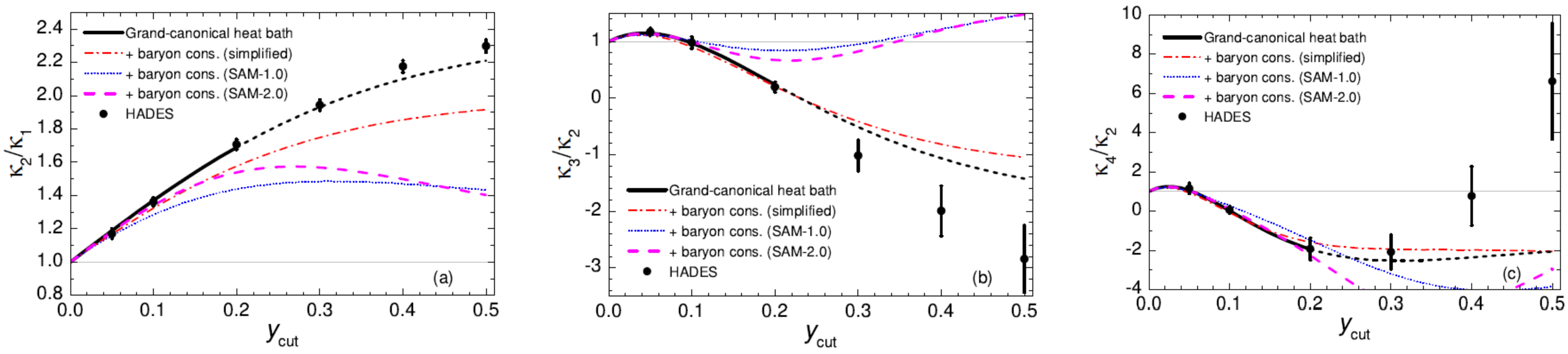
VV, Koch, Phys. Lett. B 833, 137368 (2022)

- Fit baryon susceptibilities to data within a fireball model (Siemens-Rasmussen*)

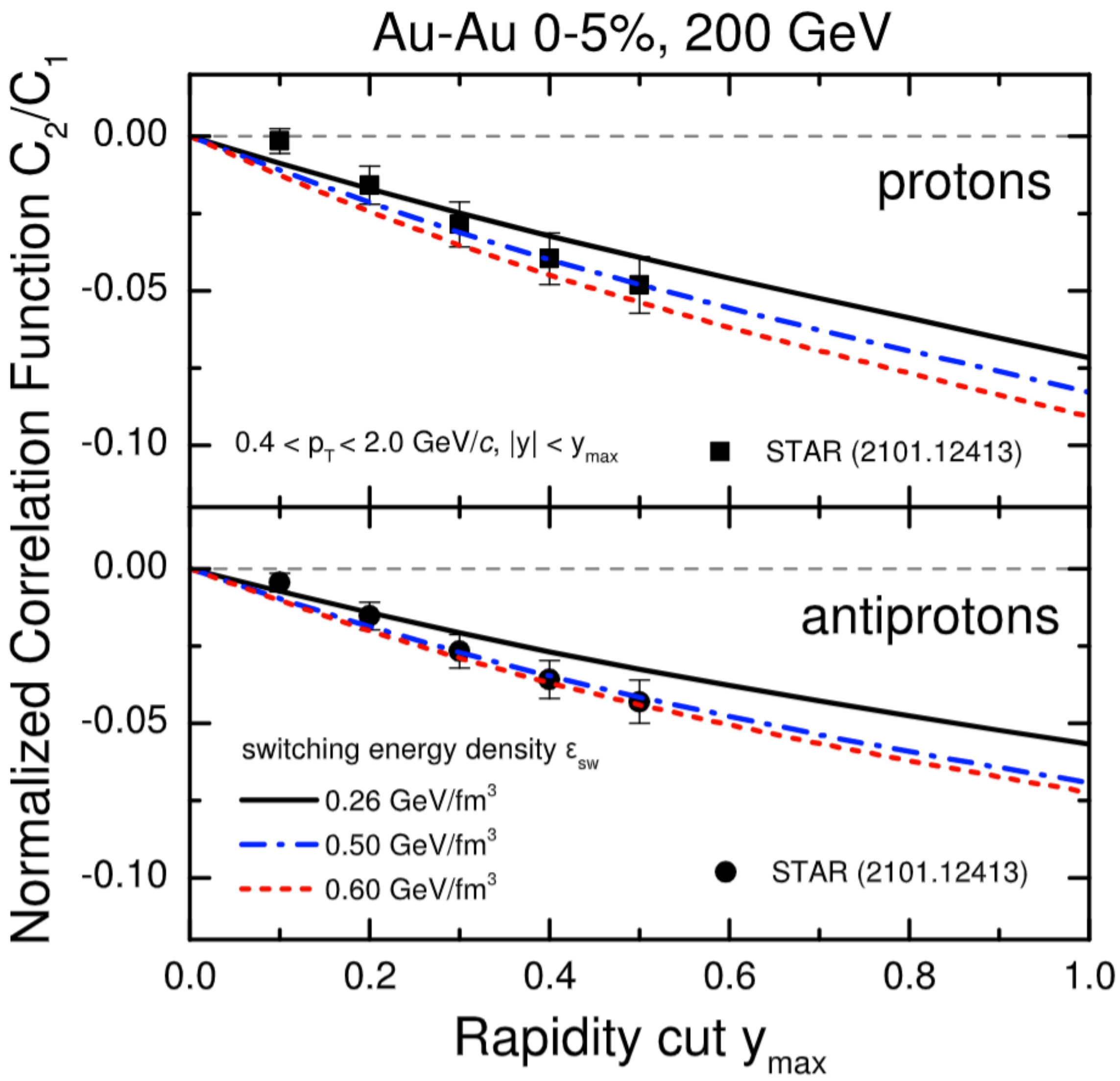
- In the grand-canonical limit (no baryon conservation, small y_{cut}) the data are described well with

$$\frac{\chi_2^B}{\chi_1^B} \sim 9.17 \pm 0.21, \qquad \frac{\chi_3^B}{\chi_2^B} \sim -33.1 \pm 0.8, \qquad \frac{\chi_4^B}{\chi_2^B} \sim 691 \pm 50, \quad \text{i.e.} \quad \chi_4^B \gg -\chi_3^B \gg \chi_2^B \gg \chi_1^B$$

- Could be indicative of a *critical point* near the HADES freeze-out at T~70 MeV, μ_B~875 MeV
- However, the results for $y_{cut} > 0.2$ are challenging to describe with baryon conservation included



Dependence on the switching energy density



Effect of the hadronic phase

Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD