Experimental results vs theory predictions

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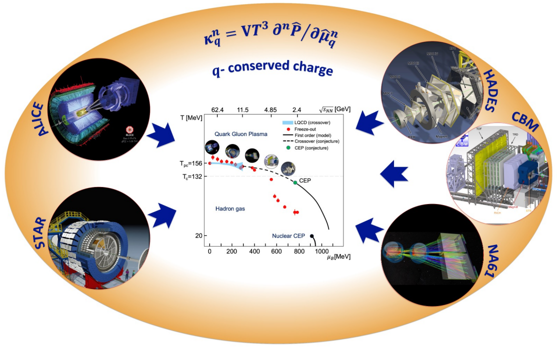


EMMI RRTF "Fluctuations and correlations of conserved charges"

GSI, Darmstadt, Germany

November 9, 2023

Outline

- Introduction
- Some remarks on subensemble acceptance method (SAM)
  - Charge conservation for any equation of state in thermodynamic limit
  - Quantifying finite volume corrections in ideal gas
- Multiple conserved charges
- Non-critical baseline from hydrodynamics
  - Low-order vs high-order cumulants
  - Ordinary vs factorial cumulants
  - Net protons vs protons vs antiprotons
  - Protons vs baryons

Introduction

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

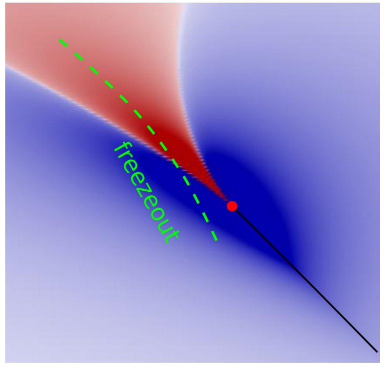
Grand partition function

$$\ln Z^{\mathrm{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\mathrm{ce}}(T,V,N)\right]$$

$$\kappa_n \propto \frac{\partial^n (\ln Z^{\mathrm{gce}})}{\partial \mu^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- **(QCD) critical point** – large correlation length, critical fluctuations of baryon number



M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

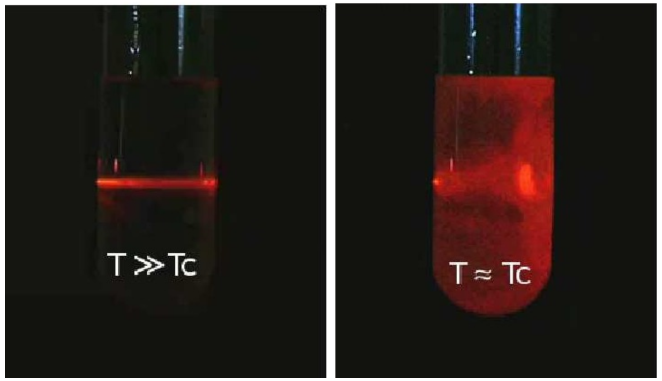
$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

$$\xi \to \infty$$

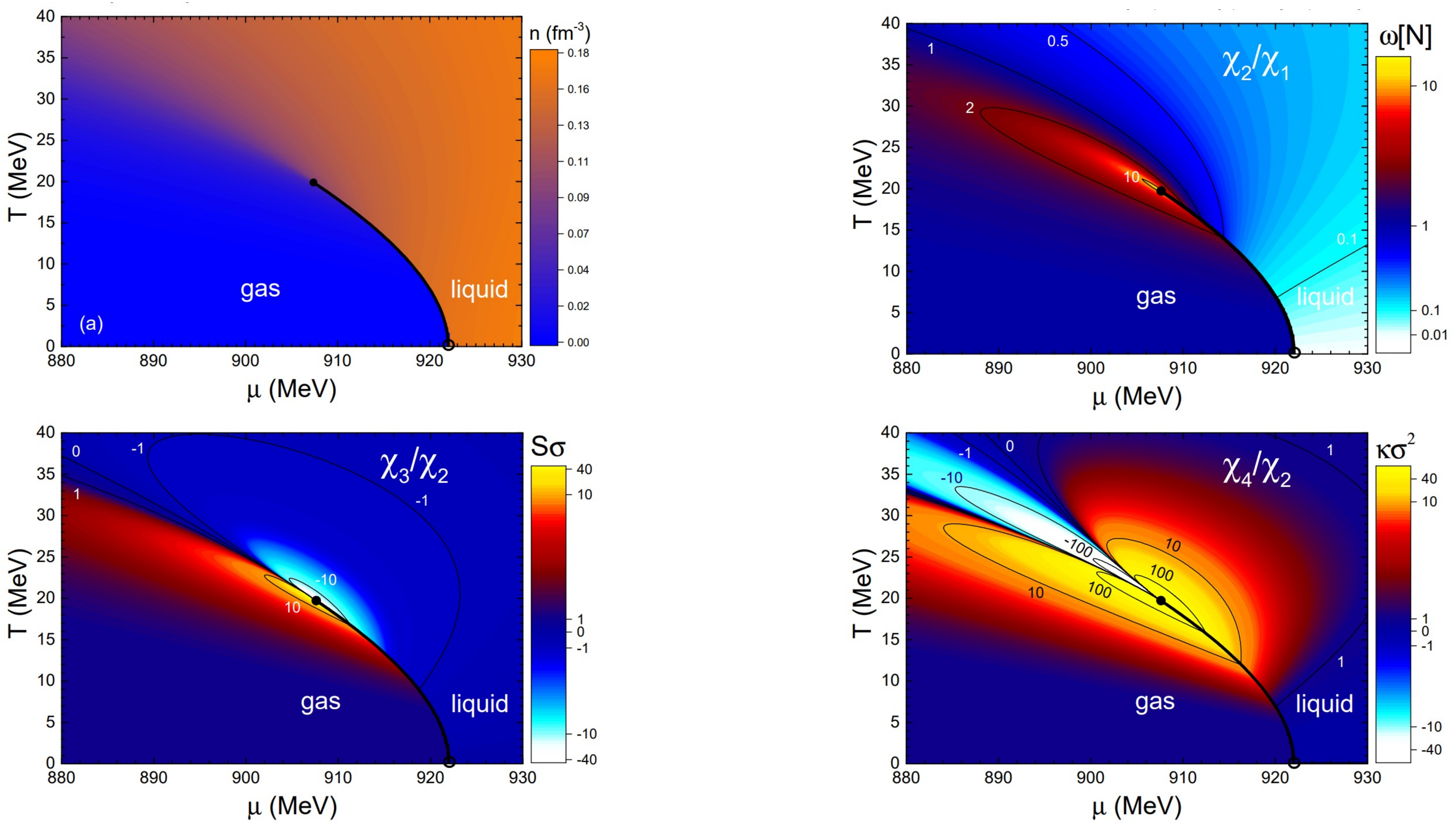
Holds both for ordinary and *factorial* cumulants

Looking for enhanced fluctuations
and non-monotonicities

Critical opalescence



Example: Nuclear liquid-gas transition

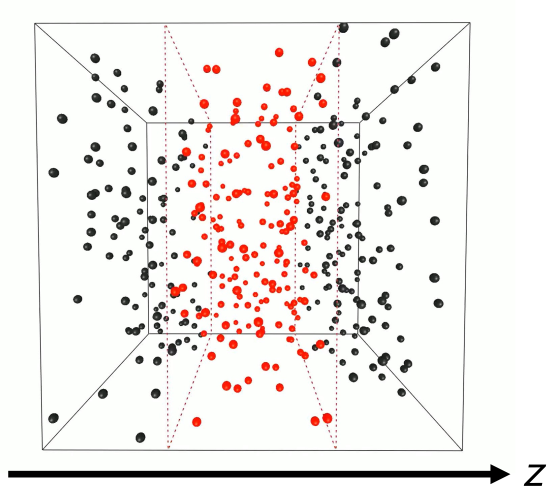


VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Example: Critical fluctuations in a microscopic simulation

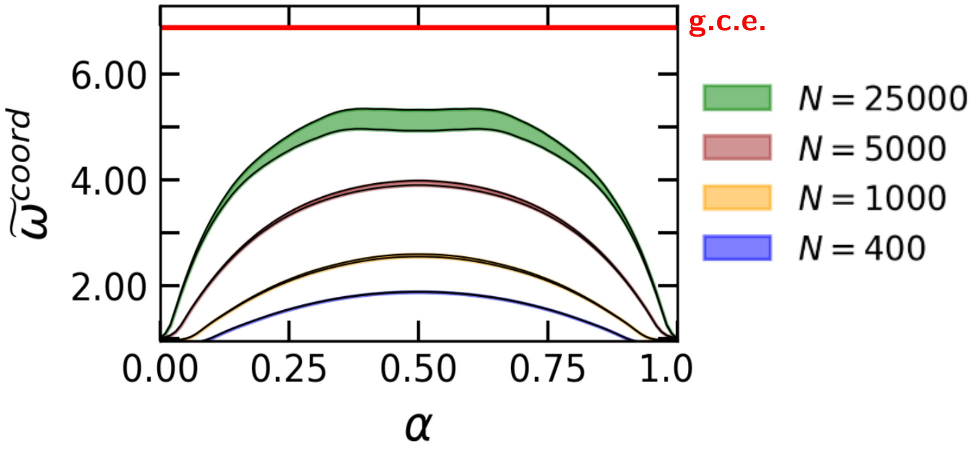
V. Kuznietsov et al., Phys. Rev. C 105, 044903 (2022)

Classical molecular dynamics simulations of the **Lennard-Jones fluid** near Z(2) critical point ($T \approx 1.06 T_c$, $n \approx n_c$) of the liquid-gas transition

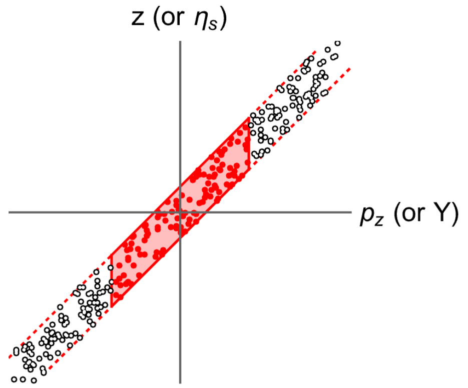


Scaled variance in coordinate space acceptance $|z| < z^{max}$

$$\tilde{\omega}^{\text{coord}} = \frac{1}{1-\alpha} \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$



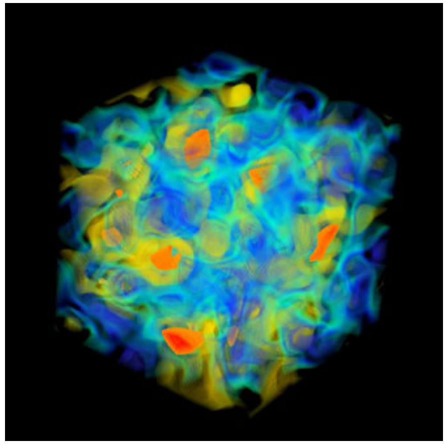
Heavy-ion collisions:
flow correlates p_z and z cuts



- Large fluctuations survive despite strong finite-size effects
- Need coordinate space cuts (collective flow helps)
- Here no finite-time effects

Theory vs experiment: Challenges for fluctuations

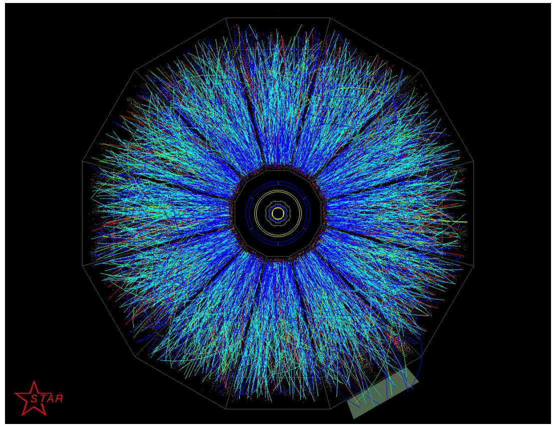
Theory (lattice QCD, fRG/DSE,...)



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

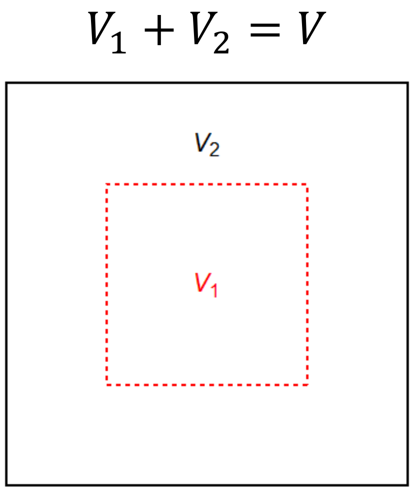
Some remarks on subensemble acceptance method

Baryon number conservation: Subensemble acceptance method

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

Subensemble acceptance method (SAM) – method to correct *any* EoS (e.g. *lattice QCD, fRG, ideal HRG*) for **charge conservation**

Partition a thermal system with a globally conserved charge B *(canonical ensemble)* into two subsystems which can exchange the charge



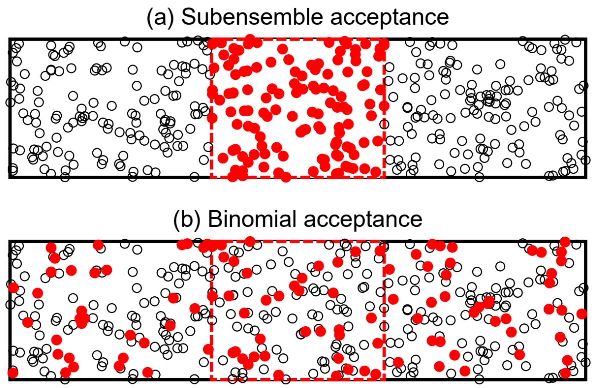
Assume thermodynamic limit:

$$V, V_1, V_2 \to \infty; \quad \frac{V_1}{V} = \alpha = const; \quad \frac{V_2}{V} = (1-\alpha) = const;$$ $\alpha = 0$: **grand-canonical ensemble**

$$V_1, V_2 \gg \xi^3, \qquad \xi = \text{correlation length} \ (= 0 \text{ in ideal gas})$$

The canonical partition function then reads:

$$Z^{ce}(T,V,B) = \mathrm{Tr}\, e^{-\beta \hat{H}} \approx \sum_{B_1} Z^{ce}(T,V_1,B_1)\, Z^{ce}(T,V-V_1,B-B_1)$$



$$Z^{ce}(T,V,B) \overset{V\to\infty}{\simeq} \exp\left[-\frac{V}{T} f(T,\rho_B)\right] \quad \mu_B(T,\rho_B) = \partial f(T,\rho_B)/\partial \rho_B \quad \chi_n^B \equiv \partial^{n-1}(\rho_B/T^3)/\partial(\mu_B/T)^{n-1}$$

SAM: Computing the cumulants

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

Cumulant generating function for B_1:

$$G_{B_1}(t) \equiv \ln\langle e^{t\,B_1}\rangle = \ln\left\{\sum_{B_1} e^{t\,B_1}\,\exp\left[-\frac{\alpha V}{T}\,f(T,\rho_{B_1})\right]\exp\left[-\frac{\beta V}{T}\,f(T,\rho_{B_2})\right]\right\} + \tilde{C} \qquad \beta = 1-\alpha$$

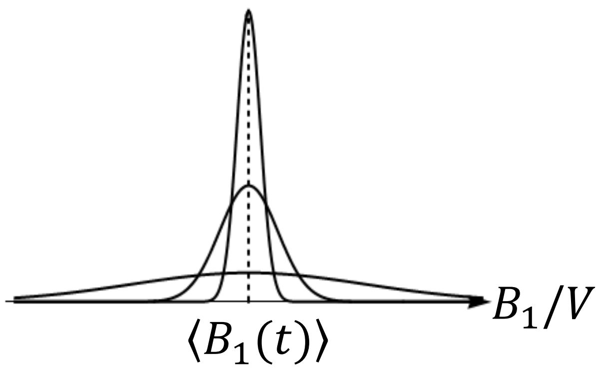
$$\tilde{\kappa}_1[B_1(t)] = \frac{\sum_{B_1} B_1\,\tilde{P}(B_1;t)}{\sum_{B_1}\tilde{P}(B_1;t)} \equiv \langle B_1(t)\rangle \qquad \text{with} \qquad \tilde{P}(B_1;t) = \exp\left\{tB_1 - V\,\frac{\alpha f(T,\rho_{B_1}) + \beta f(T,\rho_{B_2})}{T}\right\}.$$

Thermodynamic limit: $\tilde{P}(B_1;t)$ *highly peaked* at $\langle B_1(t)\rangle$

$\langle B_1(t)\rangle$ is a solution to equation $d\tilde{P}/\mathrm{d}B_1 = 0$:

$$t = \hat{\mu}_B[T,\rho_{B_1}(t)] - \hat{\mu}_B[T,\rho_{B_2}(t)]$$

$$\text{where} \quad \hat{\mu}_B \equiv \mu_B/T, \qquad \mu_B(T,\rho_B) = \partial f(T,\rho_B)/\partial\rho_B$$



$t = 0$: $\rho_{B_1} = \rho_{B_2} = B/V$, $B_1 = \alpha B$, i.e. charge uniformly distributed between the subsystems

SAM: Full result up to κ_4

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

$$\kappa_n[B_1] = \left.\frac{\partial^{n-1}\tilde{\kappa}_1[B_1(t)]}{\partial t^{n-1}}\right|_{t=0} \quad \longleftrightarrow \quad \frac{\partial^n}{\partial t^n} \quad t = \hat{\mu}_B[T,\rho_{B_1}(t)] - \hat{\mu}_B[T,\rho_{B_2}(t)]$$

$$\kappa_1[B_1] = \alpha\, VT^3\, \chi_1^B \qquad \beta = 1 - \alpha$$

$$\kappa_2[B_1] = \alpha\, VT^3\, \beta\, \chi_2^B$$

$$\kappa_3[B_1] = \alpha\, VT^3\, \beta\, (1-2\alpha)\, \chi_3^B$$

$$\kappa_4[B_1] = \alpha\, VT^3\, \beta\, \left[\chi_4^B - 3\alpha\beta \frac{(\chi_3^B)^2 + \chi_2^B\,\chi_4^B}{\chi_2^B}\right]$$

scaled variance $$\frac{\kappa_2[B_1]}{\kappa_1[B_1]} = (1-\alpha)\,\frac{\chi_2^B}{\chi_1^B},$$

skewness $$\frac{\kappa_3[B_1]}{\kappa_2[B_1]} = (1-2\alpha)\,\frac{\chi_3^B}{\chi_2^B},$$

kurtosis $$\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta\left(\frac{\chi_3^B}{\chi_2^B}\right)^2.$$

$$\chi_n^B = \frac{\partial^n (p/T^4)}{\partial(\mu_B/T)^n}$$ – grand-canonical susceptibilities, e.g. from lattice QCD!

SAM vs ideal gas

Ideal gas of baryons and antibaryons:

$$VT^3\chi_n^B = N_B + (-1)^n N_{\bar{B}}$$

Ideal gas: $$g(t) = \ln\left[\left(\frac{q_+}{q_-}\right)^{B/2} \frac{I_B(2z\sqrt{q_+q_-})}{I_B(2z)}\right],$$

where $q_+ = 1 - p_B + p_B e^t$ and $q_- = 1 - p_{\bar{B}} + p_{\bar{B}} e^{-t}$.

Bzdak, Koch, Skokov, PRC 87, 014901 (2013)

Braun-Munzinger, Rustamov, Stachel, NPA 960, 114 (2017)

SAM:

$$\kappa_1 = \alpha B$$
$$\kappa_2 = \alpha(1-\alpha)N$$
$$\kappa_3 = \alpha(1-2\alpha)B$$
$$\kappa_4 = \alpha\beta\left[(1-3\alpha\beta)N - 3\alpha\beta\frac{B^2}{N}\right]$$

$$B = \langle N_B\rangle - \langle N_{\bar{B}}\rangle, \qquad N = \langle N_B\rangle + \langle N_{\bar{B}}\rangle$$

$$c_1 = pB,$$
$$c_2 = p(1-p)\langle N\rangle_C,$$
$$c_3 = c_1(1-p)(1-2p),$$
$$c_4 = c_2 + 3(p^2q^2B^2 - c_2^2) + 6pq(2z^2pq - c_2),$$

$$z = \sqrt{\langle N_B\rangle\langle N_{\bar{B}}\rangle}. \qquad \langle N_{B,\bar{B}}\rangle_C = z\frac{I_{B\mp1}(2z)}{I_B(2z)},$$

$$c \equiv \kappa, \qquad p \equiv \alpha, \qquad q = 1-\alpha$$

Why are the expressions different, especially for κ_4?

Ideal gas: LO and NLO in system size

$$g(t) = \ln\left[\left(\frac{q_+}{q_-}\right)^{B/2} \frac{I_B(2z\sqrt{q_+q_-})}{I_B(2z)}\right],$$

Large volume limit: $N = \langle N_B \rangle + \langle N_{\bar{B}} \rangle \to \infty$ at fixed $r_B = \dfrac{B}{N}$

Use asymptotic expansions: $I_\nu(\nu\tilde{z}) \overset{\nu\to\infty}{\sim} \dfrac{1}{\sqrt{2\pi\nu}} \dfrac{e^{\nu\eta}}{(1+\tilde{z}^2)^{1/4}} \{1 + O(\nu^{-1})\},$

$I_0(z) \overset{z\to\infty}{\sim} \dfrac{e^z}{\sqrt{2\pi z}}$

for B = 0

$$g(t) \sim \ln\{\exp[f(t)N]\, a_1(t)\}$$

LO NLO

$$f(t) = \frac{r_B}{2}\ln\frac{q_+}{q_-} + \sqrt{r_B^2 + (1-r_B^2)q_+q_-} - 1 + |r_B| \ln\left[\frac{(1+|r_B|)\sqrt{q_+q_-}}{|r_B| + \sqrt{r_B^2 + (1-r_B^2)q_+q_-}}\right],$$

$$a_1(t) = \frac{1}{[r_B^2 + (1-r_B^2)q_+q_-]^{1/4}}$$

Ideal gas: LO and NLO in system size

Ideal gas:

LO

$$\kappa_1^{\mathrm{LO}} = N\,\alpha r_B,$$
$$\kappa_2^{\mathrm{LO}} = N\,(1-\alpha)\alpha$$
$$\kappa_3^{\mathrm{LO}} = N\,\alpha(1-\alpha)(1-2\alpha)r_B$$
$$\kappa_4^{\mathrm{LO}} = N\,(1-\alpha)\alpha\left[1-3\alpha(1-\alpha)(1+r_B^2)\right]$$

NLO

$$\kappa_1^{\mathrm{NLO}} = 0,$$
$$\kappa_2^{\mathrm{NLO}} = -\frac{1}{2}(1-\alpha)\alpha\left(1-r_B^2\right),$$
$$\kappa_3^{\mathrm{NLO}} = 0,$$
$$\kappa_4^{\mathrm{NLO}} = -\frac{1}{2}(1-\alpha)\alpha(1-r_B^2)\left[1-6\alpha(1-\alpha)\left(1-r_B^2\right)\right].$$

SAM:

$$\kappa_1^{\mathrm{SAM}} = \alpha N r_B,$$
$$\kappa_2^{\mathrm{SAM}} = \alpha(1-\alpha)N,$$
$$\kappa_3^{\mathrm{SAM}} = \alpha(1-\alpha)(1-2\alpha)N r_B,$$
$$\kappa_4^{\mathrm{SAM}} = \alpha(1-\alpha)N[1-3\alpha(1-\alpha)(1+r_B^2)].$$

SAM at NLO: Barej, Bzdak, PRC 107, 034914 (2023)

$$\kappa_1^{\mathrm{SAM,NLO}} = 0,$$
$$\kappa_2^{\mathrm{SAM,NLO}} = -\frac{1}{2}(1-\alpha)\alpha\left(1-r_B^2\right),$$
$$\kappa_3^{\mathrm{SAM,NLO}} = 0,$$
$$\kappa_4^{\mathrm{SAM,NLO}} = -\frac{1}{2}(1-\alpha)\alpha(1-r_B^2)\left[1-6\alpha(1-\alpha)\left(1-r_B^2\right)\right]$$

Use NLO to estimate relative errors

$$\frac{\kappa_2^{LO}}{\kappa_2} = 1 + \frac{1 - r_B^2}{N} + O(N^{-2})$$

Total number of baryons is at least 400, therefore $\delta\kappa_2$~0.25% or less

For comparison, LO error due to Fermi statistics is $\delta\kappa_{2,FD}^{\mathrm{gce}} \propto \frac{K_2(2m/T)}{K_2(m/T)} \frac{\cosh(2\mu_B/T)}{\cosh(\mu_B/T)}$ $\sim$ 0.2% at $\mu_B = 0$

$$\frac{\kappa_4}{\kappa_2} = [1 - 3\alpha(1-\alpha)(1+r_B^2)] + \frac{3\alpha(1-\alpha)(1-4r_B^2+3r_B^4)}{2N} + O(N^{-2}),$$

$$\frac{\kappa_6}{\kappa_2} = 1 - 15\alpha(1-\alpha)[1 + r_B^2 - \alpha(1-\alpha)](3 + 6r_B^2 - r_B^4) + \frac{15\alpha(1-\alpha)(1-r_B^2)[1 - 3r_B^2 - \alpha(1-\alpha)(5 - 22r_B^2 + 9r_B^4)]}{2N} + O(N^{-2})$$

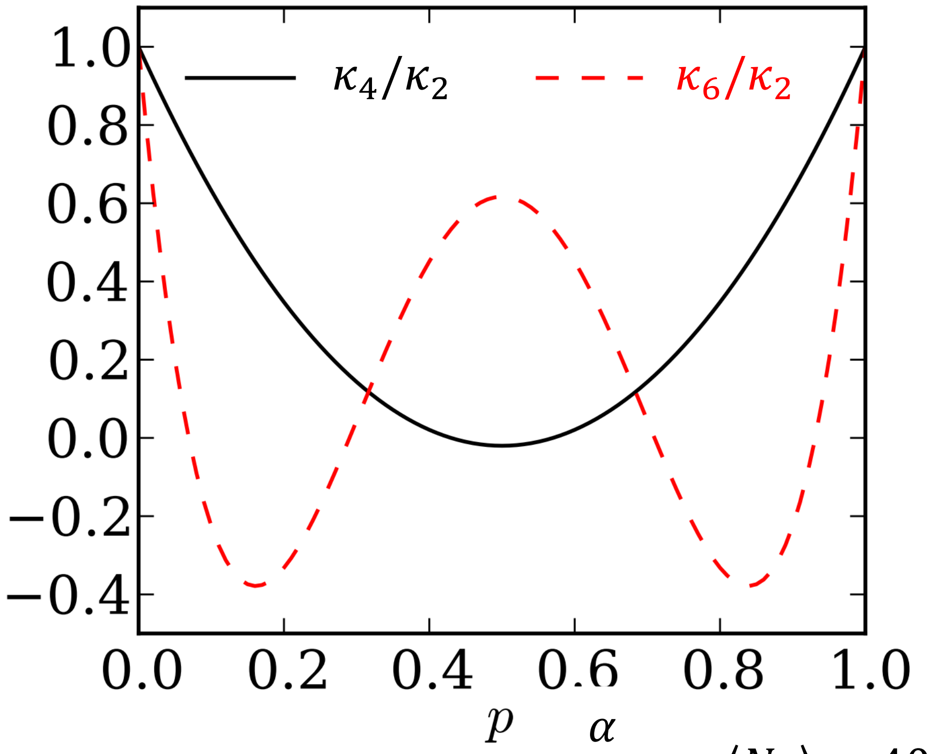
When LO corrections might be sizable?

- Strangeness conservation at low energies
- pp collisions

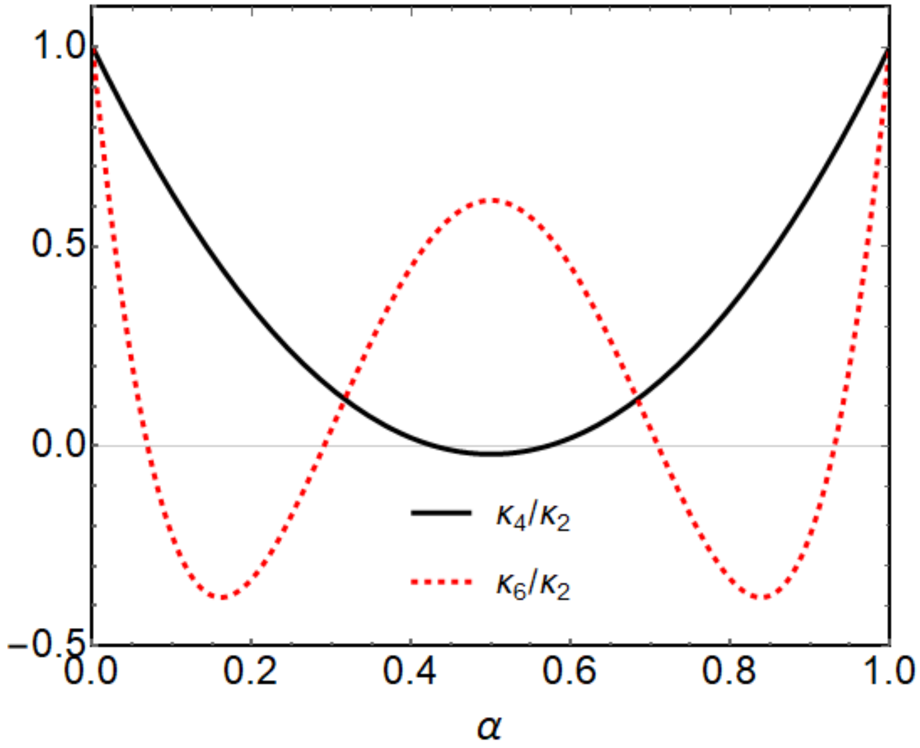
Comparison

Ideal gas of baryons and antibaryons: $\chi_{2n}^B \propto \langle N_B \rangle + \langle N_{\bar{B}} \rangle$, $\chi_{2n-1}^B \propto \langle N_B \rangle - \langle N_{\bar{B}} \rangle$

Ideal gas [Bzdak, Koch, Skokov, PRC 87, 014901 (2013)]



SAM/LO



$\langle N_B \rangle = 400, \qquad \langle N_{\bar{B}} \rangle = 100$

SAM-2.0: Unequal acceptances

VV, PRC 105, 014903 (2022)

SAM-2.0: apply the correction for *arbitrary* distributions inside and outside the acceptance that are peaked at the mean

- Inhomogeneous systems (e.g. RHIC and below)
- Momentum space and unequal acceptances
- Map "grand-canonical" cumulants inside and outside the acceptance to the "canonical" cumulants inside the acceptance

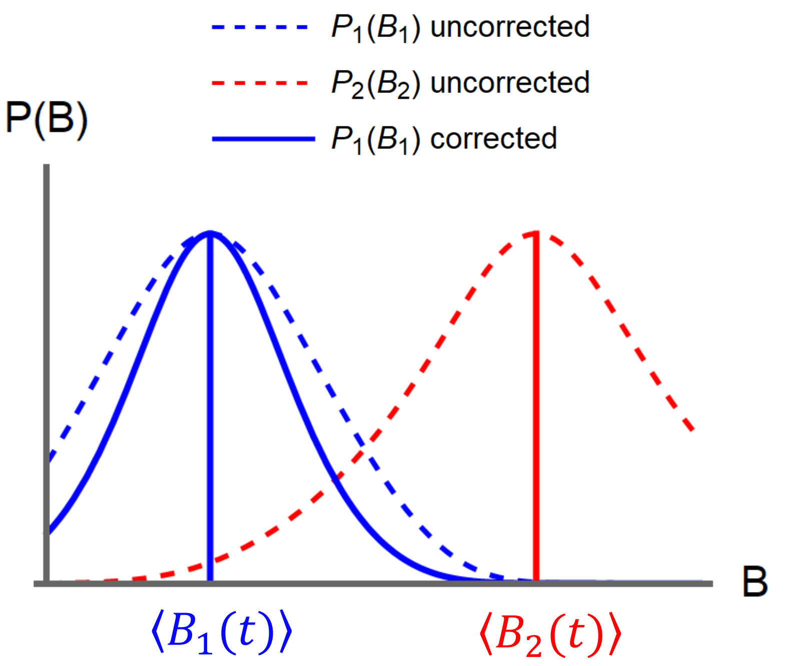
$$\kappa_{\text{in}}^{\text{ce}} = \text{SAM}[\kappa_{\text{in}}^{\text{gce}}, \kappa_{\text{out}}^{\text{gce}}]$$

$$\kappa_1^{\text{SAM2.0}} = \kappa_{1,in}^{\text{gce}},$$

$$\kappa_2^{\text{SAM2.0}} = \frac{\kappa_{2,\text{in}}^{\text{gce}}\,\kappa_{2,\text{out}}^{\text{gce}}}{\kappa_{2,\text{in}}^{\text{gce}} + \kappa_{2,\text{out}}^{\text{gce}}},$$

$$\kappa_3^{\text{SAM2.0}} = \frac{\kappa_{3,\text{in}}^{\text{gce}}(\kappa_{2,\text{out}}^{\text{gce}})^3 - \kappa_{3,\text{out}}^{\text{gce}}(\kappa_{2,\text{in}}^{\text{gce}})^3}{\left(\kappa_{2,\text{in}}^{\text{gce}} + \kappa_{2,\text{out}}^{\text{gce}}\right)^3},$$

$$\kappa_4^{\text{SAM2.0}} = \frac{1}{\left(\kappa_{2,\text{in}}^{\text{gce}} + \kappa_{2,\text{out}}^{\text{gce}}\right)^5} \times \left\{\kappa_{4,\text{in}}^{\text{gce}}(\kappa_{2,\text{out}}^{\text{gce}})^5 + \kappa_{4,\text{out}}^{\text{gce}}(\kappa_{2,\text{in}}^{\text{gce}})^5 + (\kappa_{2,\text{out}}^{\text{gce}})^4\left[\kappa_{2,\text{in}}^{\text{gce}}\kappa_{4,\text{in}}^{\text{gce}} - 3(\kappa_{3,\text{in}}^{\text{gce}})^2\right]\right.$$
$$\left. + (\kappa_{2,\text{in}}^{\text{gce}})^4\left[\kappa_{2,\text{out}}^{\text{gce}}\kappa_{4,\text{out}}^{\text{gce}} - 3(\kappa_{3,\text{out}}^{\text{gce}})^2\right] - 6(\kappa_{2,\text{in}}^{\text{gce}})^2(\kappa_{2,\text{out}}^{\text{gce}})^2\,\kappa_{3,\text{in}}^{\text{gce}}\,\kappa_{3,\text{out}}^{\text{gce}}\right\}.$$



$$P_1^{\text{ce}}(B_1) \propto \sum_{B_1,B_2} P_1^{\text{gce}}(B_1)P_2^{\text{gce}}(B_2) \times \delta_{B,B_1+B_2}$$

*Explicit expressions for any cumulant available via a Mathematica notebook at https://github.com/vlvovch/SAM

SAM-2.0: Ideal gas

VV, PRC 105, 014903 (2022)

Grand-canonical cumulants read:

$$\kappa_{n,in}^{\rm gce} = \alpha_B \langle N_B \rangle_{GC} + (-1)^n \alpha_{\bar{B}} \langle N_{\bar{B}} \rangle_{GC}$$

$$\kappa_{n,out}^{\rm gce} = (1-\alpha_B) \langle N_B \rangle_{GC} + (-1)^n (1-\alpha_{\bar{B}}) \langle N_{\bar{B}} \rangle_{GC}$$

$$\kappa_1^{\rm SAM2.0} = N \, \frac{1}{2} \left[\alpha_B (r_B + 1) + \alpha_{\bar{B}} (r_B - 1) \right],$$

$$\kappa_2^{\rm SAM2.0} = -N \, \frac{1}{4} \left(\alpha_B^2 (r_B + 1)^2 - 2\alpha_B \left(\alpha_{\bar{B}} \left(r_B^2 - 1 \right) + r_B + 1 \right) + \alpha_{\bar{B}} (r_B - 1)(\alpha_{\bar{B}} (r_B - 1) + 2) \right)$$

$$\kappa_3^{\rm SAM2.0} = -N \, \frac{1}{8} \left(\alpha_B^3 (r_B - 3)(r_B + 1)^3 - 3\alpha_B^2 (r_B + 1)^2 \left(\alpha_{\bar{B}} (r_B - 1)^2 - 2 \right) \right.$$
$$\left. + \, \alpha_B \left(3\alpha_{\bar{B}}^2 \left(r_B^2 - 1 \right)^2 - 4(r_B + 1) \right) - \alpha_{\bar{B}} (r_B - 1) \left(\alpha_{\bar{B}}^2 (r_B + 3)(r_B - 1)^2 + 6\alpha_{\bar{B}} (r_B - 1) + 4 \right) \right)$$

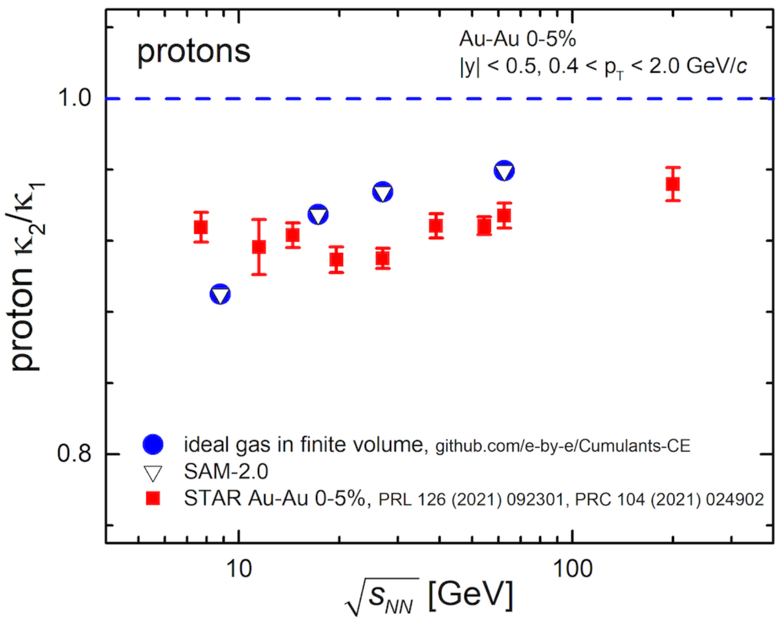
$$\kappa_4^{\rm SAM2.0} = -N \, \frac{1}{16} \left(3\alpha_B^4 \left(r_B^2 - 4r_B + 5 \right) (r_B + 1)^4 - 12\alpha_B^3 (r_B + 1)^3 \left(\alpha_{\bar{B}} (r_B - 1)^3 - r_B + 3 \right) \right.$$
$$+ 2\alpha_B^2 (r_B + 1)^2 \left(3\alpha_{\bar{B}}^2 \left(3r_B^2 - 1 \right) (r_B - 1)^2 - 6\alpha_{\bar{B}} (r_B - 1)^2 + 14 \right)$$
$$- 4\alpha_B \left(3\alpha_{\bar{B}}^3 \left(r_B^2 - 1 \right)^3 + 3\alpha_{\bar{B}}^2 \left(r_B^2 - 1 \right)^2 + 2\alpha_{\bar{B}} \left(r_B^2 - 1 \right) + 2(r_B + 1) \right)$$
$$\left. + \alpha_{\bar{B}} (r_B - 1) \left(3\alpha_{\bar{B}}^3 \left(r_B^2 + 4r_B + 5 \right) (r_B - 1)^3 + 12\alpha_{\bar{B}}^2 (r_B + 3)(r_B - 1)^2 + 28\alpha_{\bar{B}} (r_B - 1) + 8 \right) \right),$$

in agreement with LO results from finite volume calculation

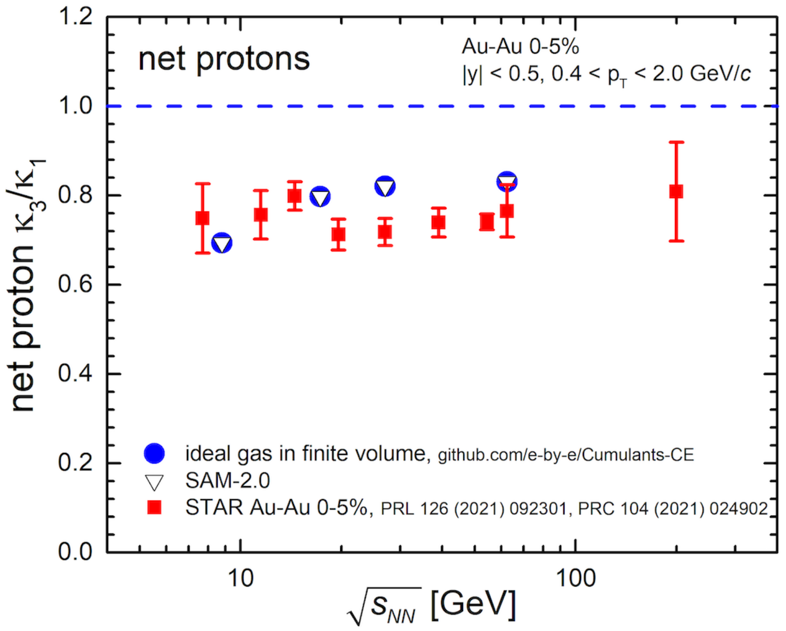
Comparison at RHIC-BES

Using α_B and $\alpha_{\bar{B}}$ at different energies from Braun-Munzinger et al. NPA 1008, 122141 (2021)

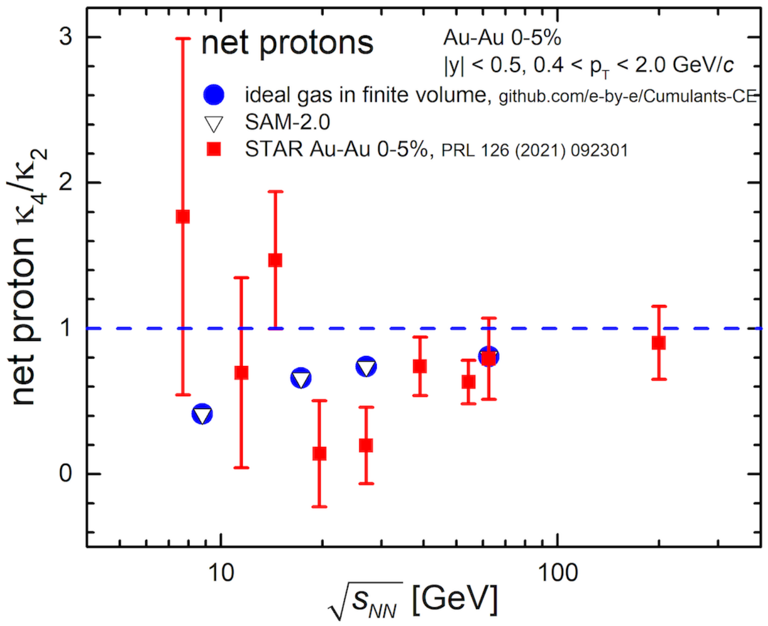
Proton κ_2/κ_1



Net Proton κ_3/κ_1



Net Proton κ_4/κ_2



Ideal gas: using https://github.com/e-by-e/Cumulants-CE

No visible difference to SAM-2.0

Multiple conserved charges

SAM for multiple conserved charges (B,Q,S)

VV, Poberezhnyuk, Koch, JHEP 10, 089 (2020)

Key findings:

- Cumulants up to 3rd order factorize into product of binomial and grand-canonical cumulants

$$\kappa_2^{B,Q,S} = (1-\alpha)\, VT^3\, \chi_2^{B,Q,S} \qquad \kappa_3^{B,Q,S} = (1-\alpha)(1-2\alpha)\, VT^3\, \chi_3^{B,Q,S}$$

  - Ratios of second and third order cumulants of **conserved charges** are NOT sensitive to charge conservation (requires same acceptance)

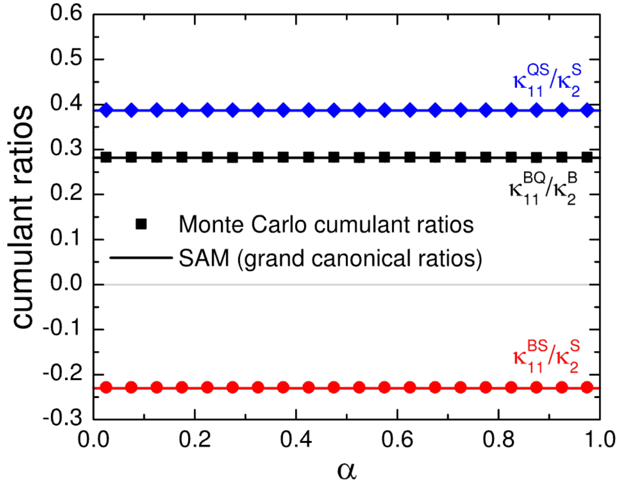
e.g. $\kappa_2^B/\kappa_2^Q = \chi_2^B/\chi_2^Q$

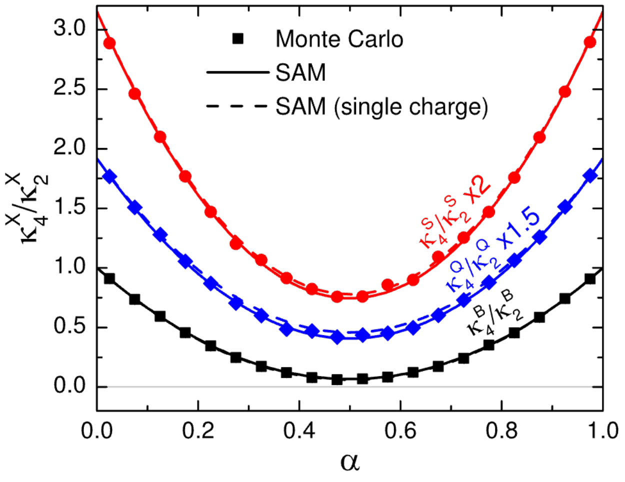
  - Also true for the measurable ratios of covariances involving one non-conserved charge, such as κ_{pQ}/κ_{kQ}

- For order $n > 3$ charge cumulants "mix". Effect in HRG is tiny

$$\kappa_4^B = \kappa_4^{B,\mathrm{gce}}\, \beta \left[(1-3\alpha\beta)\, \chi_4^B - 3\,\alpha\,\beta\, \frac{(\chi_3^B)^2\chi_2^Q - 2\chi_{21}^{BQ}\chi_{11}^{BQ}\chi_3^B + (\chi_{21}^{BQ})^2\chi_2^B}{\chi_2^B\chi_2^Q - (\chi_{11}^{BQ})^2} \right]$$

Monte Carlo HRG





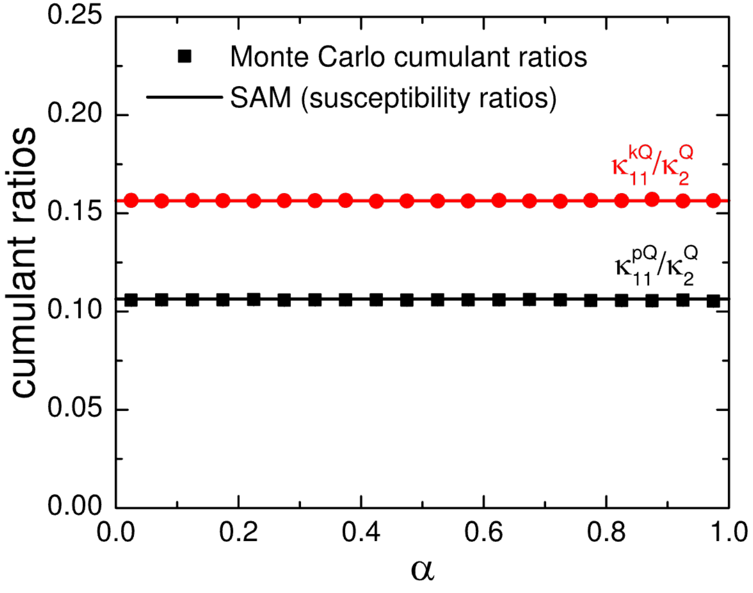
Mathematica notebook to express any B,Q,S-cumulant of order $n \leq 6$ in terms of grand-canonical susceptibilities available at https://github.com/vlvovch/SAM

SAM and non-conserved quantities

VV, Poberezhnyuk, Koch, JHEP 10, 089 (2020)

$$\kappa_{XY} = (1-\alpha)\,\kappa_{XY}^{\mathrm{gce}} + \alpha\,\kappa_{XY}^{\mathrm{ce}}$$

- Mixed cumulants involving one conserved charge e.g. pQ have $\kappa_{pQ}^{ce} = 0$ thus they scale like second order charge cumulants
  - p and Q, again, must have the same a
- Cancellation does NOT occur for two non-conserved quantities, such as κ_{pK}

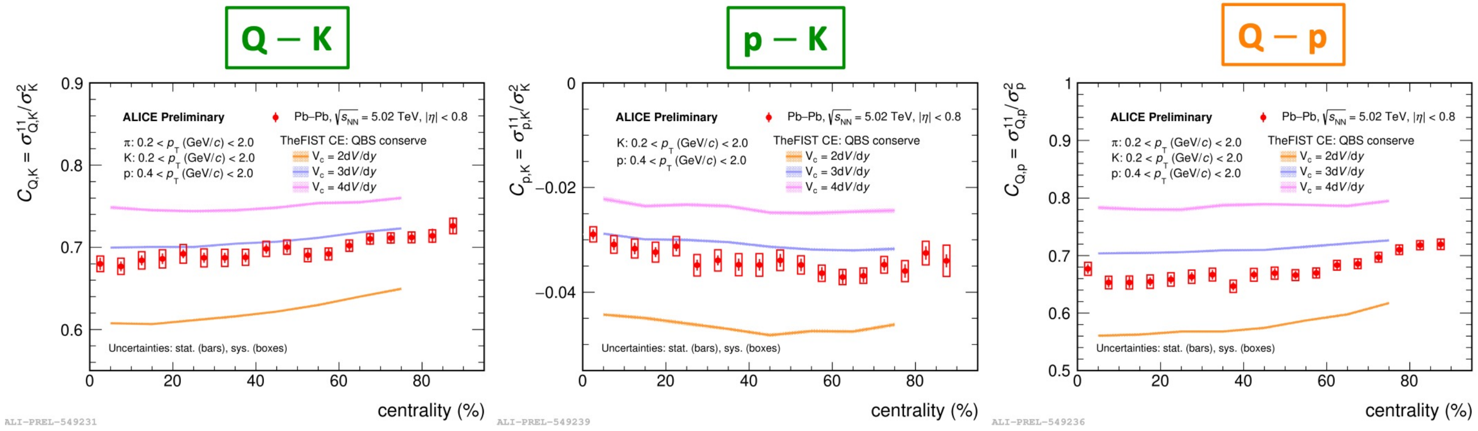


Replace $\sigma_{QK}^{11}/\sigma_K^2$ and $\sigma_{Qp}^{11}/\sigma_p^2$ by

$\sigma_{QK}^{11}/\sigma_Q^2$ and $\sigma_{Qp}^{11}/\sigma_Q^2$

Eliminate V_c dependence?

ALICE, M. Arslandok (Monday)



Net-proton and net-Λ fluctuations

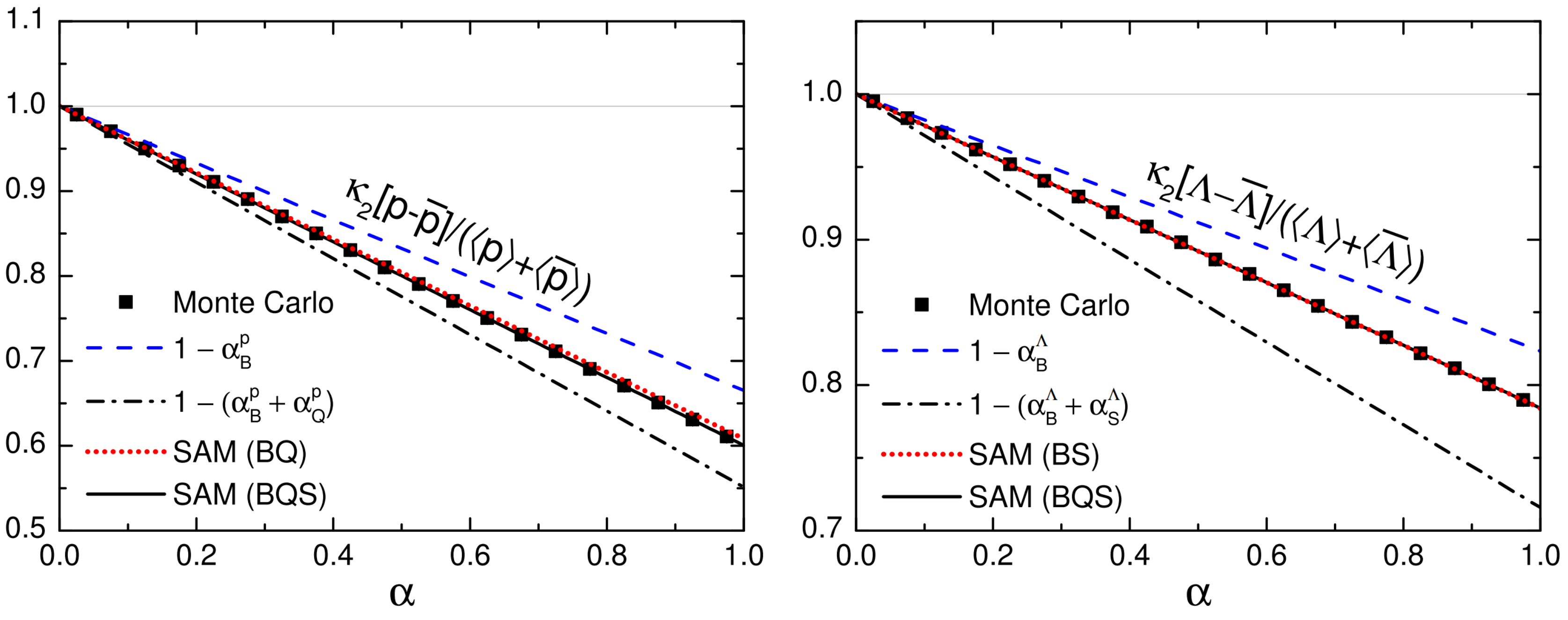
$$\kappa_{pp} = (1-\alpha)\,\kappa_{pp}^{\text{gce}} + \alpha\,\kappa_{pp}^{\text{ce}}$$

- Allows for corrections due to electric charge (protons) or strangeness (Λ) conservation in addition to baryon number conservation.

$$\alpha = \frac{\langle p \rangle_{\text{acc}}}{\langle p \rangle_{4\pi}}$$

$$\alpha_B^p = \frac{\langle p \rangle_{\text{acc}}}{\langle B \rangle_{4\pi}}$$

$$\alpha_Q^p = \frac{\langle p \rangle_{\text{acc}}}{\langle Q \rangle_{4\pi}}$$

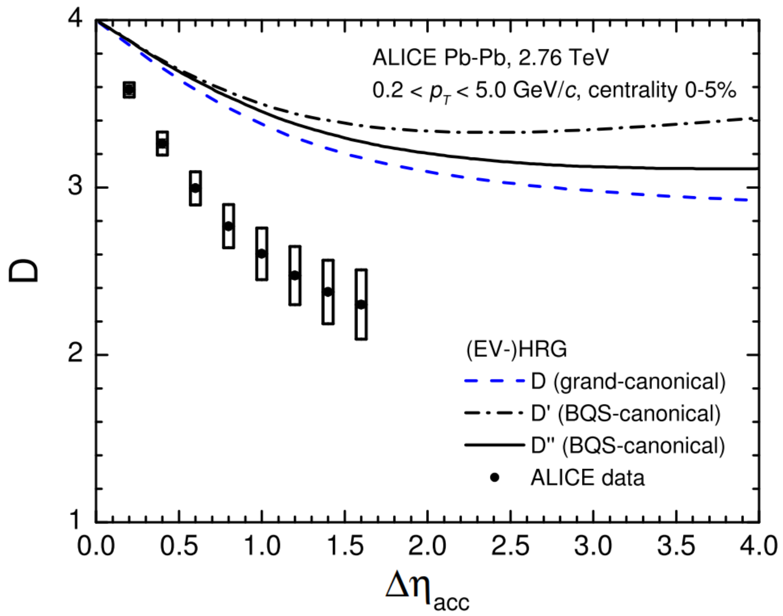


Truth lies in between the "naïve" corrections

Likely bigger effect for higher orders

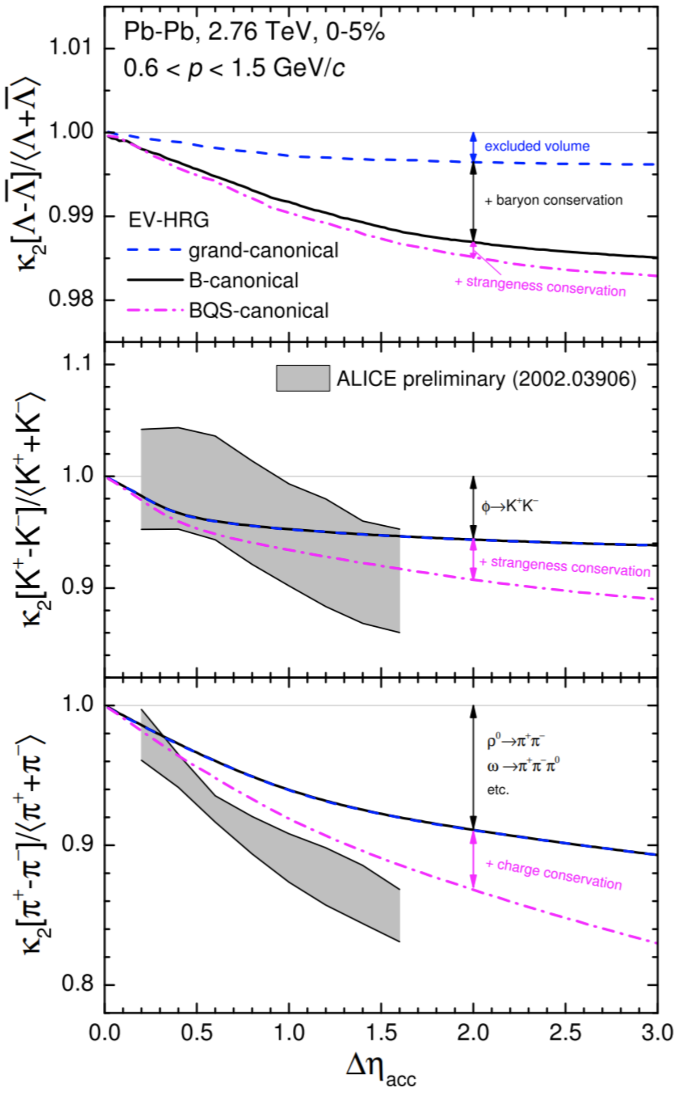
Quantitative calculation with blast-wave model

- Large effect from resonance decays for pions and kaons + exact conservation of electric charge/strangeness
- D-measure $D = \frac{\langle \delta Q^2 \rangle}{\langle N_{\rm ch} \rangle}$



HRG: $D \sim 3 - 4$

QGP?: $D \sim 1 - 1.5$

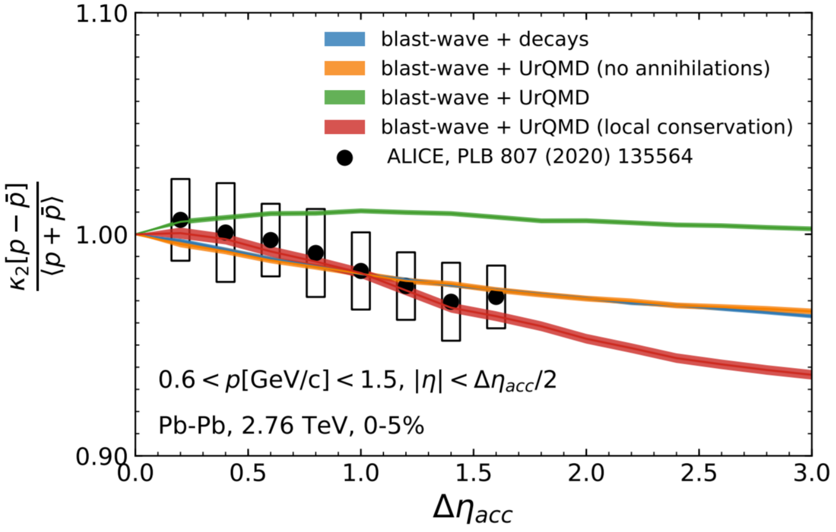


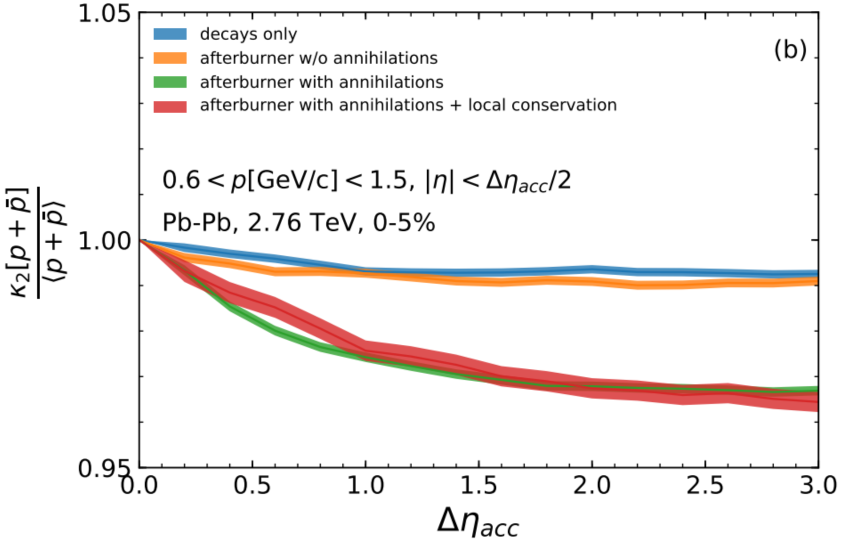
Hadronic description with global conservation challenging

VV, Koch, Phys. Rev. C 103, 044903 (2021)

Baryon annihilation

- Net protons described within errors and consistent with either
  - global baryon conservation without $B\bar{B}$ annihilations
  - or local baryon conservation with $B\bar{B}$ annihilations



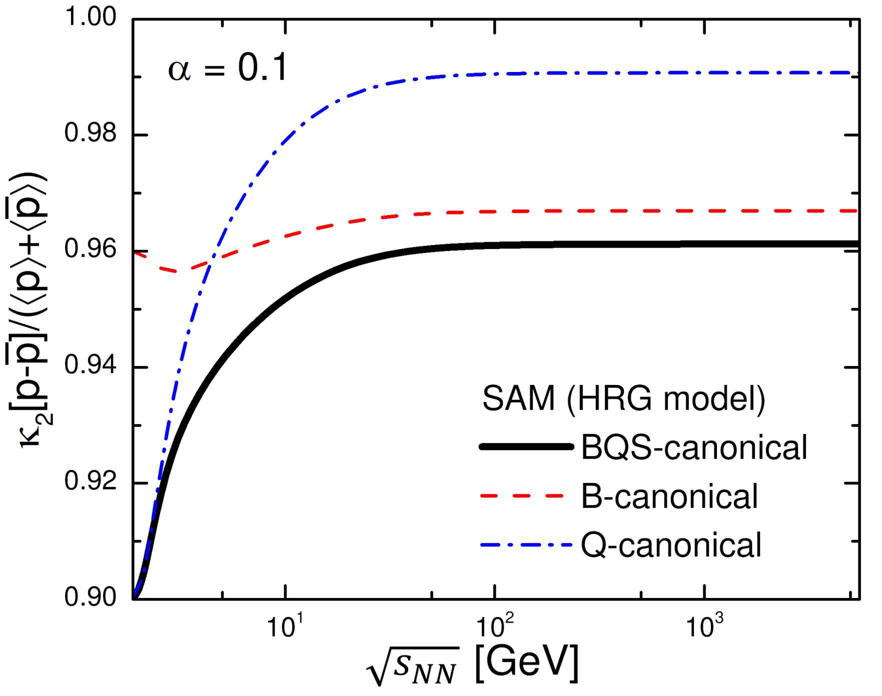


O. Savchuk et al., Phys. Lett. B 827, 136983 (2022)

- $\kappa_2[p-\bar{p}]/\langle p+\bar{p}\rangle$: Interplay of baryon annihilation(↗) and local conservation(↘)
  - Additional measurement of $\kappa_2[p+\bar{p}]$ can resolve it
- $\kappa_2[p+\bar{p}]/\langle p+\bar{p}\rangle$: Insensitive to baryon conservation at LHC, $cov(p+\bar{p}, B-\bar{B})=0$
  - Good measure for volume fluctuations?

Multiple conserved charges as fct. of collision energy

Schematic calculation in HRG model along the chemical freeze-out line, $\alpha = 0.1$

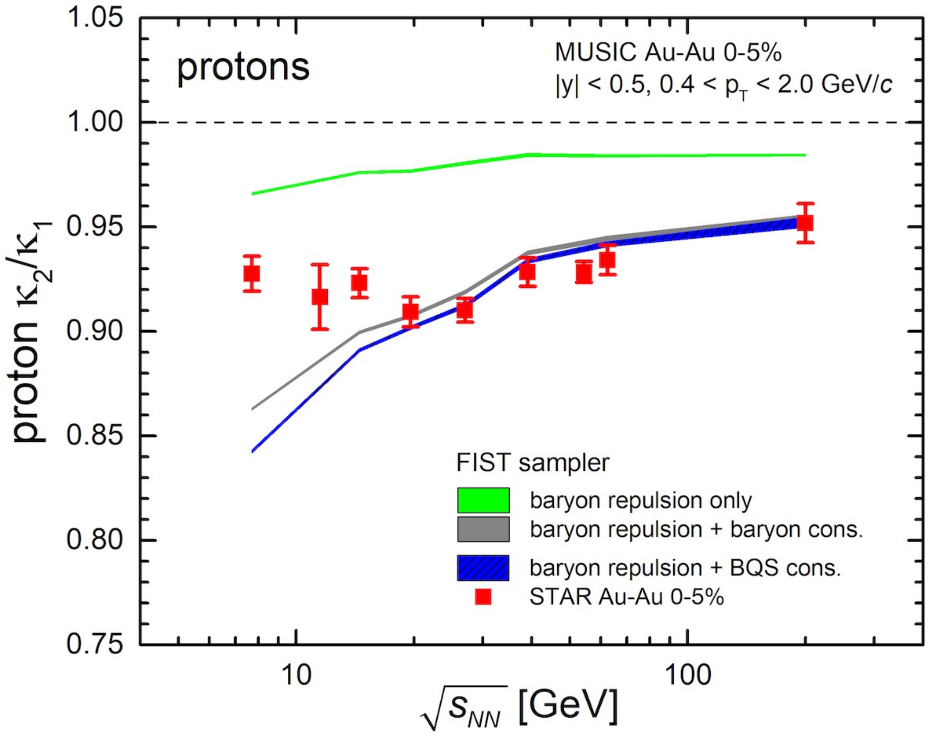


- LHC: Dominant effect from baryon conservation
- Very low energies: net-p ≈ net-Q $\Longrightarrow$ electric charge conservation dominates
- Simultaneous treatment of B and Q conservation is important

Quantitative calculation for BES energies

Canonical sampling of HRG over MUSIC Cooper-Frye hypersurfaces – FIST sampler

Proton κ_2/κ_1 https://github.com/vlvovch/fist-sampler



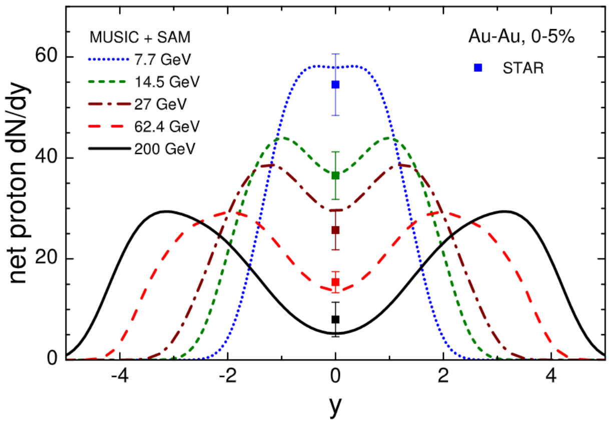
- Effect of charge conservation in addition to baryon conservation becomes visible as energy is decreased
- Becomes dominant effect at 2.4 & 3 GeV?

Non-critical baseline from hydrodynamics

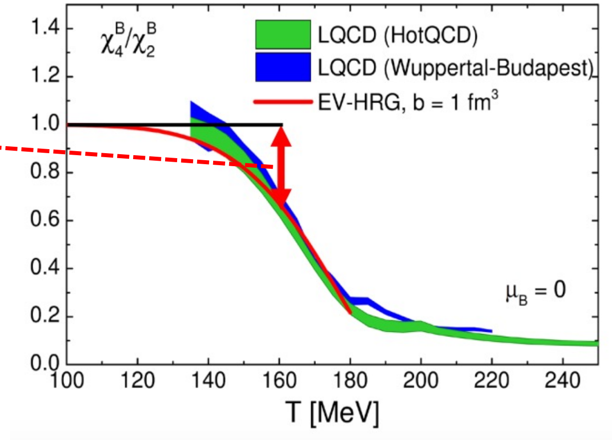
Calculation of non-critical contributions

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
  - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
  - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]
  - Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3



- Non-critical contributions are computed at particlization
  - Cumulants matched to QCD at $\mu_B = 0$ via excluded volume b = 1 fm^3 [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
  - Exact global baryon conservation* (and other charges)
    - SAM-2.0 [VV, Phys. Rev. C 105, 014903 (2022)]
    - or FIST sampler (Monte Carlo) [VV, Phys. Rev. C 106, 064906 (2022)] https://github.com/vlvovch/fist-sampler

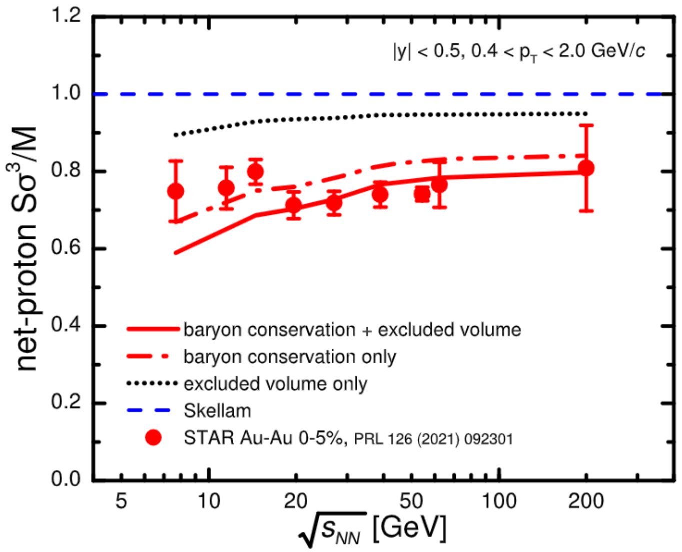


- Absent: critical point, local conservation, initial-state/volume fluctuations

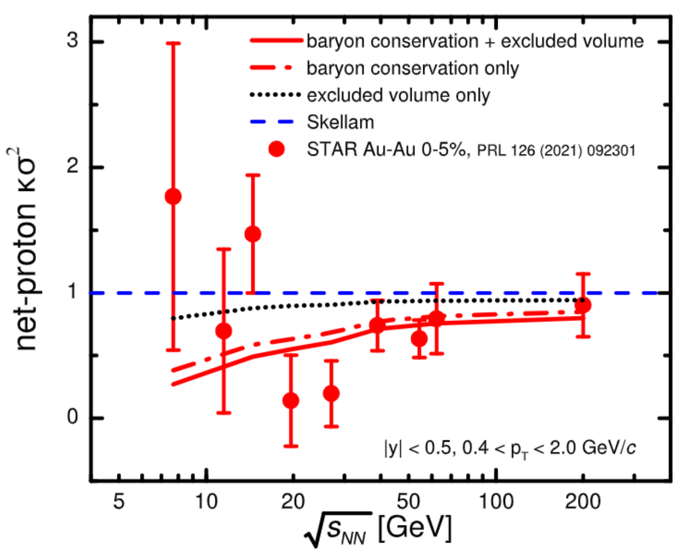
*If baryon conservation is the only effect (no other correlations), non-critical baseline can be computed without hydro
Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, NPA 1008, 122141 (2021)

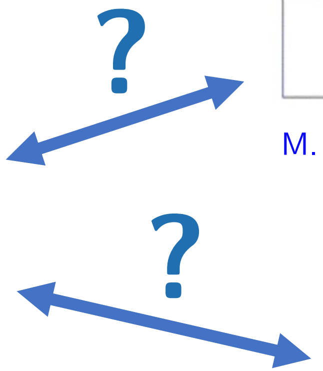
RHIC-BES: Net proton cumulant ratios (MUSIC)

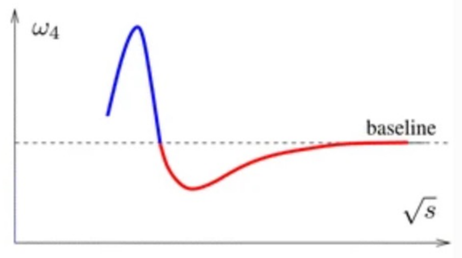
κ_3/κ_1



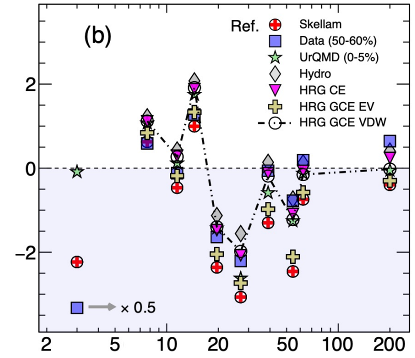
κ_4/κ_2







M. Stephanov, Phys. Rev. Lett. (2011)



INT-20r-c1 talk by A. Pandav

- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (BQS conservation and repulsion)
- Effect from baryon conservation is stronger than repulsion but both are required at $\sqrt{s_{NN}} \geq 20$ GeV
- Reduced errors to come from BES-II

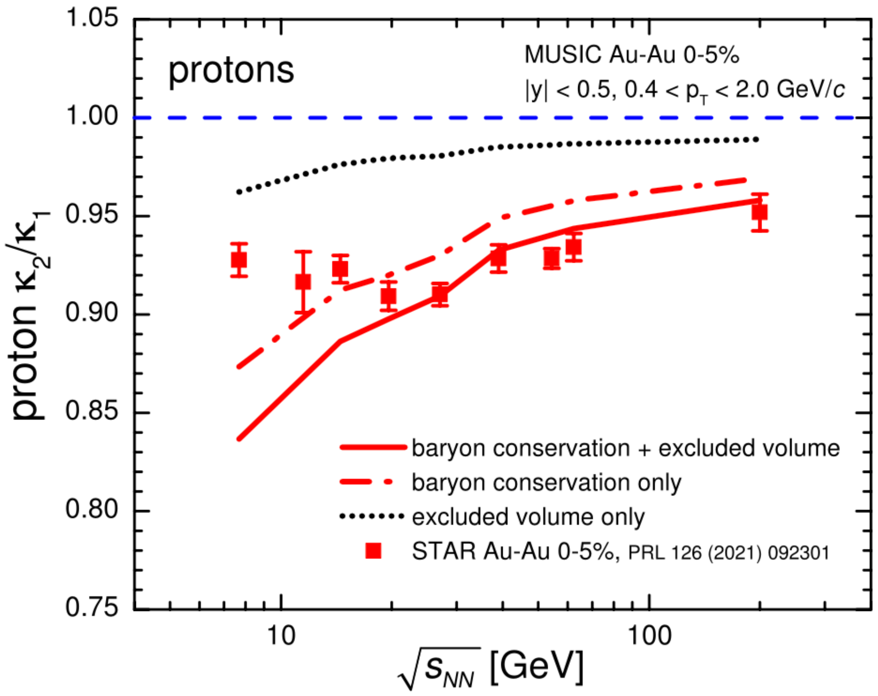
Can we learn more from the more accurate data available for κ_2 and κ_3?

Removing the "net" part: Proton variance

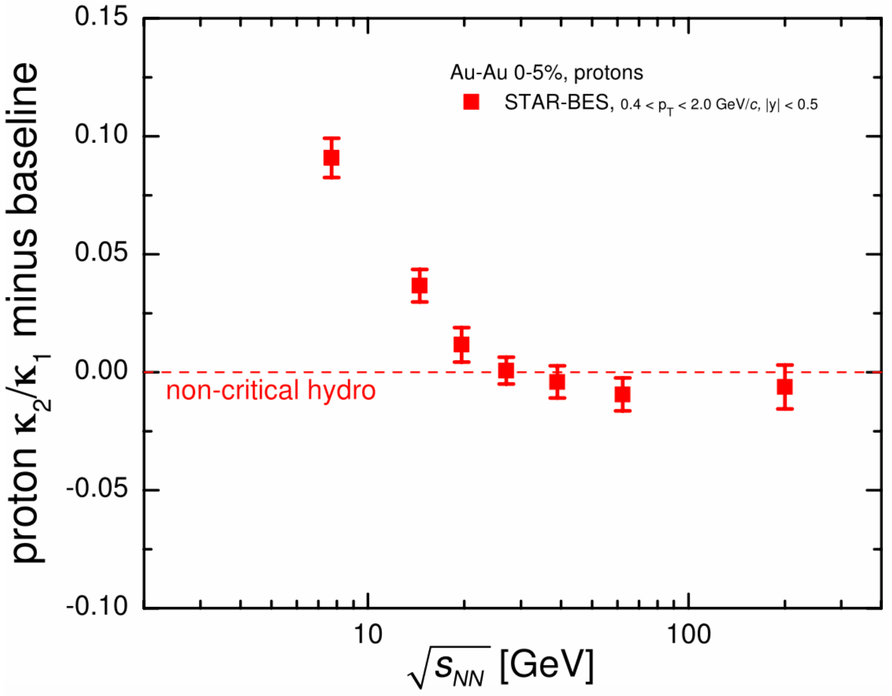
Net-proton $\kappa_2/\kappa_1 \sim \frac{\langle p+\bar{p}\rangle}{\langle p-\bar{p}\rangle} \sim \coth(\frac{\mu_B}{T})$ in free gas

Proton $\kappa_2/\kappa_1 \sim \frac{\langle p\rangle}{\langle p\rangle} = 1$ in free gas

Proton κ_2/κ_1



Proton κ_2/κ_1 excess over baseline



- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (BQS conservation and repulsion)
- Clear excess of proton variance at $\sqrt{s_{NN}} < 20$ GeV – *hint of attractive interactions?*

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022); VV, Phys. Rev. C 106, 064906 (2022)

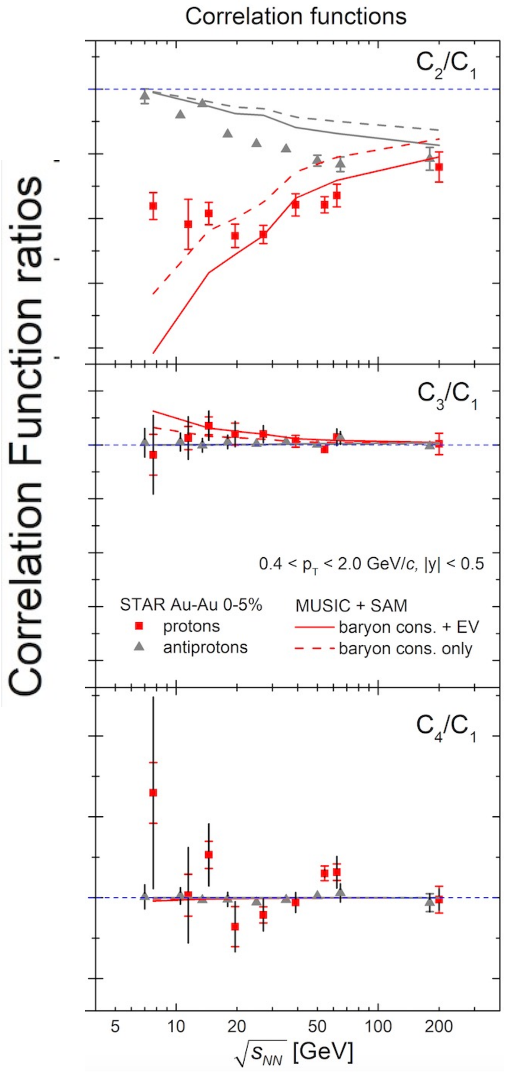
Correlation Functions (factorial cumulants)

- **Factorial cumulants $\hat{C}_n$** [Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)]
  - Remove the Poisson contribution and probe genuine correlations

$$\hat{C}_1 = \kappa_1, \qquad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$
$$\hat{C}_2 = -\kappa_1 + \kappa_2, \quad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$

- **Expectation:** High-order ($n > 3$) factorial cumulants
  - have small contributions from non-critical effects (baryon cons. or excluded volume) [Bzdak, Koch, Skokov, EPJC '17; VV et al, PLB '17]
  - are as singular as ordinary cumulants near the critical point [Ling, Stephanov, PRC '16]

- **Observations from STAR data:**
  - $\hat{C}_3$ & $\hat{C}_4$ are largely consistent with zero within (large) errors
    - Reanalyze (non-)monotonic energy dependence for $\hat{C}_4/\hat{C}_1$ instead of κ_4/κ_2?
  - Statistically significant effects appear to be driven by two-proton correlations in $\hat{C}_2$



⚠️ **Notation:** We use κ_n for cumulants and $\hat{C}_n$ for factorial cumulants, STAR uses the opposite

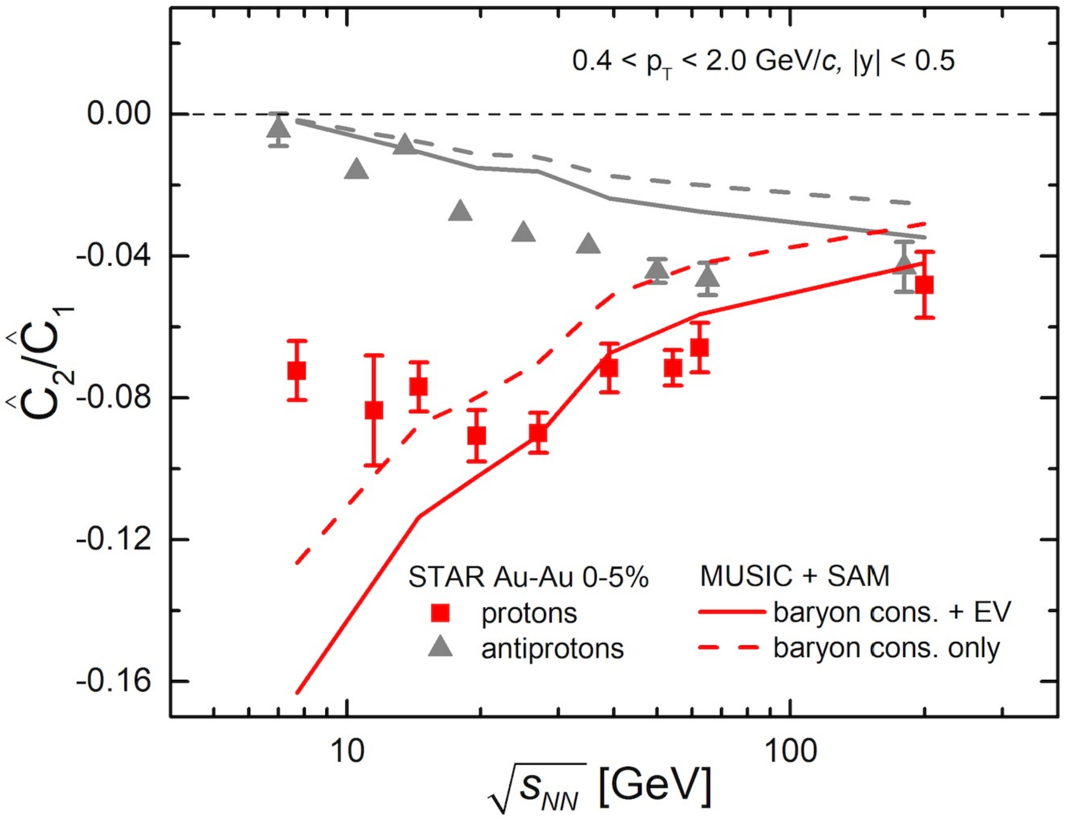
Two-proton correlations

- Protons
  - Consistent with non-critical physics above 20 GeV
  - Enhancement at lower energies

- Antiproton description has issues
  - Correlations in data underestimated by ~factor 2

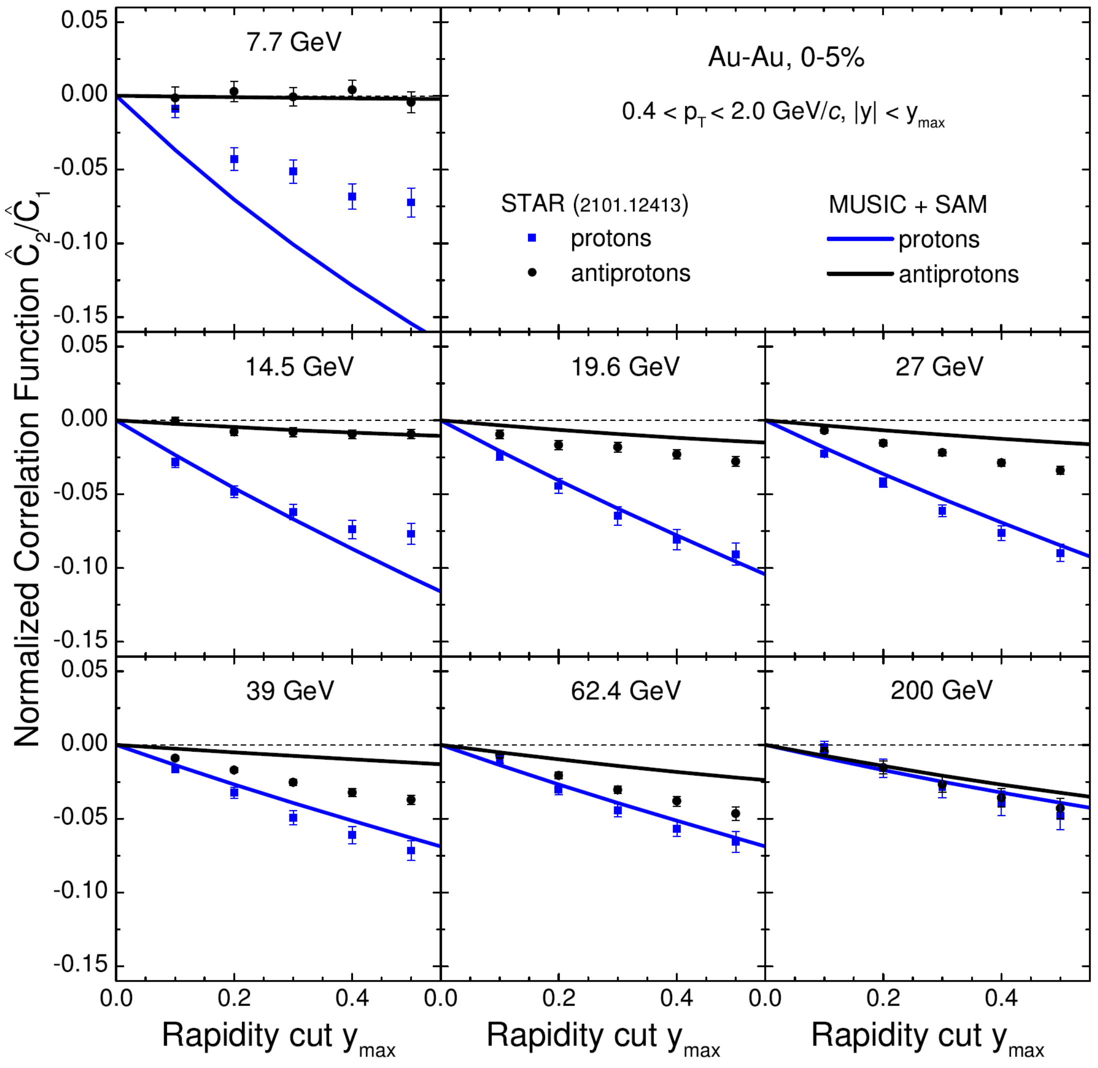
Differences:

- Produced vs stopped?
- Annihilation?



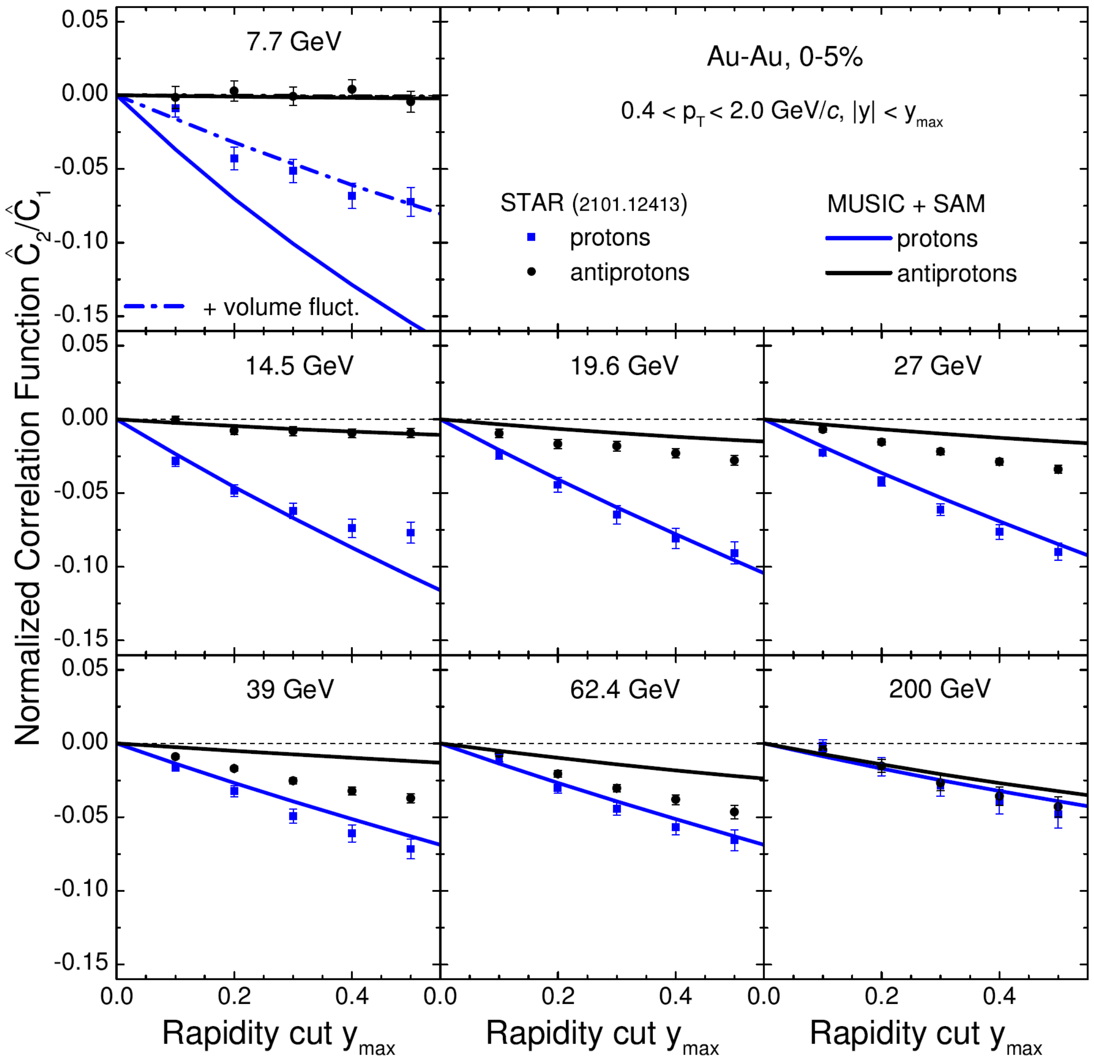
Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?



Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?
- **Volume fluctuations?** [Skokov, Friman, Redlich, PRC '13]
  - $C_2/C_1 += C_1 * \Delta v^2$



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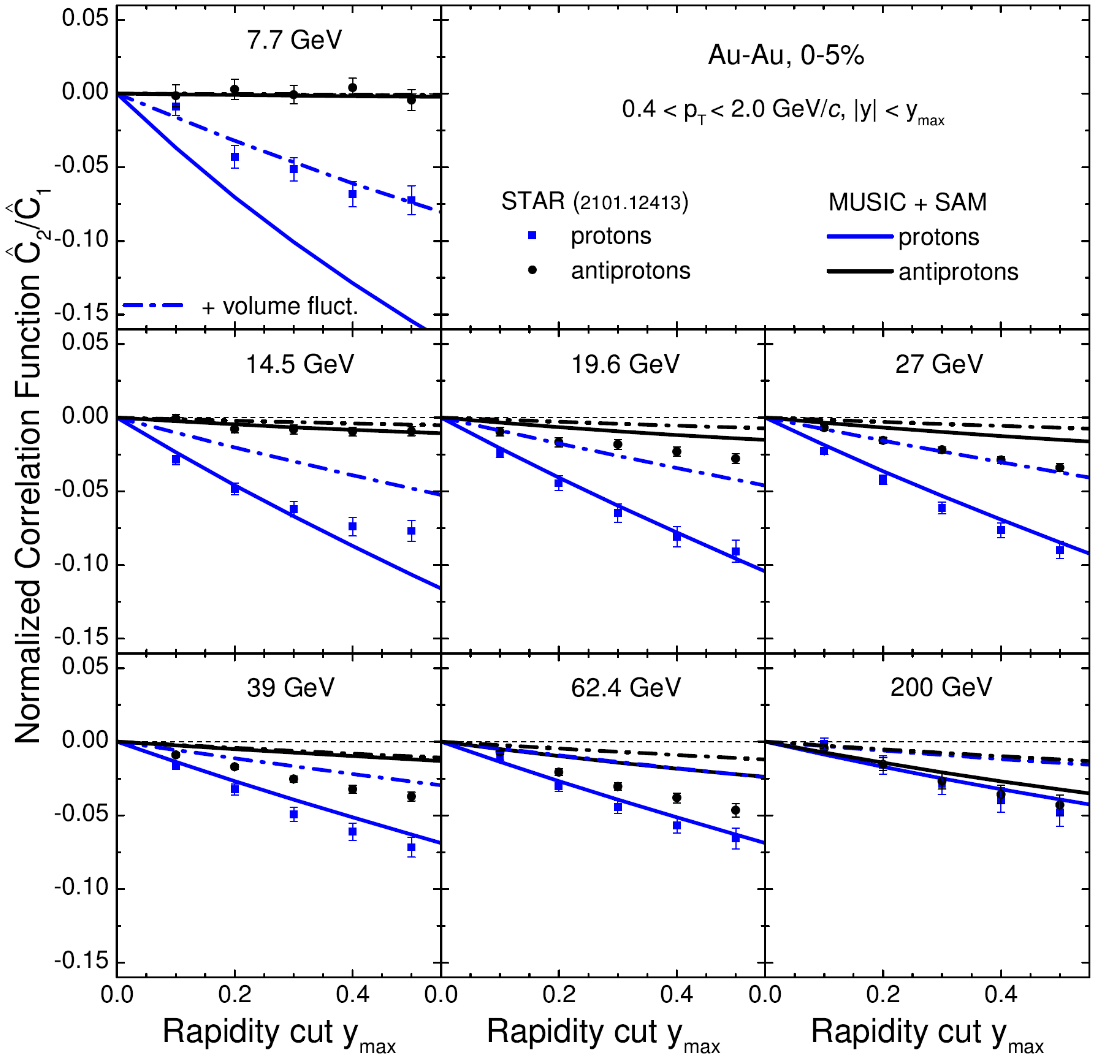
  - Can improve low energies but spoil high energies?



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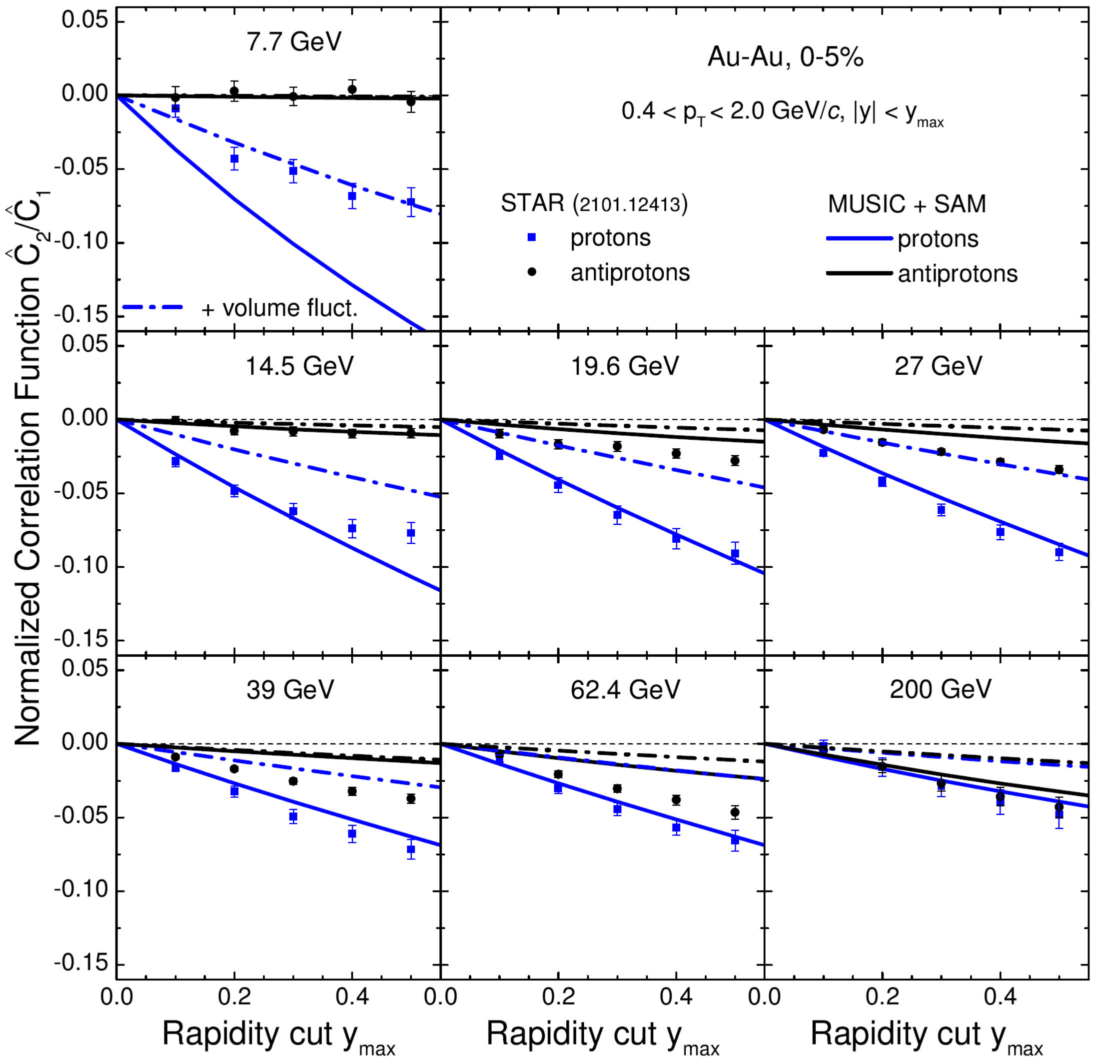
- Volume fluctuations? [Skokov, Friman, Redlich, PRC '13]
  - $C_2/C_1 \mathrel{+}= C_1 * \Delta v^2$
  - Can improve low energies but spoil high energies?
- **Attractive interactions?**
  - Could work if baryon repulsion turns into attraction in the high-μ_B regime
  - **Critical point?**

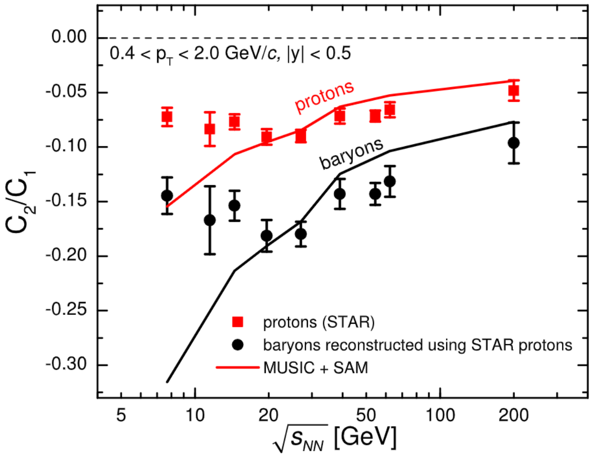


Baryon cumulants from protons

- **net baryon $\neq$ net proton**
  - protons are a ***subset*** of all baryons
  - effectively amounts to additional efficiency correction

    → "Poissonizer" of proton cumulants relative to baryons
  - **loss:** ~50% in $\hat{C}_2$, ~75% in $\hat{C}_3$, ~87.5% in $\hat{C}_4$

- Baryon cumulants can be reconstructed from proton cumulants based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]
  - Requires the use of joint factorial moments



$$\frac{\hat{C}_2^B}{\hat{C}_1^B} \approx 2\,\frac{\hat{C}_2^p}{\hat{C}_1^p}$$