QCD phase structure at finite baryon density from heavy-ion collisions

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Juniors Day at STAR

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BEST COLLABORATION

BERKELEY LAB

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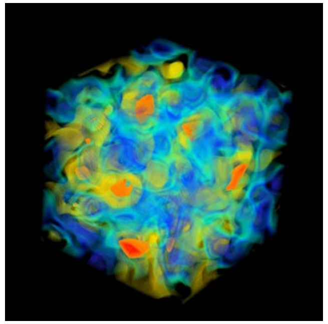
INSTITUTE for NUCLEAR THEORY

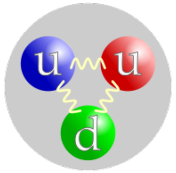
Strongly interacting matter

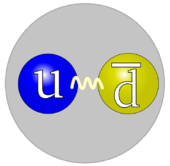
- Theory of strong interactions: *Quantum Chromodynamics* (QCD)

$$\mathcal{L} = \sum_{q=u,d,s,\dots} \bar{q}\left[i\gamma^{\mu}(\partial_{\mu} - igA^{a}_{\mu}\lambda_{a}) - m_{q}\right] q - \frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a}$$



- Basic degrees of freedom: quarks and gluons that carry color charge
- At smaller energies confined into baryons (qqq) and mesons $(q\bar{q})$



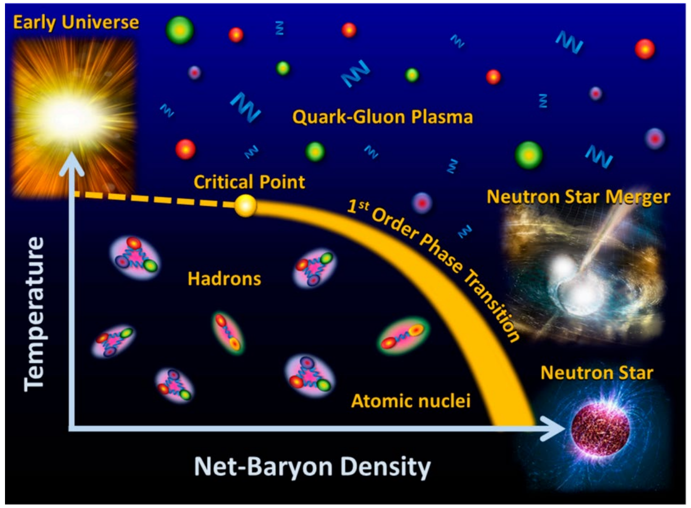


Scales

- Length: 1 femtometer $= 10^{-15}$ m
- Temperature: 100 MeV$/k_B = 10^{12}$ K

Where is it relevant?

- Early Universe
- Astrophysics: Neutron star (mergers)

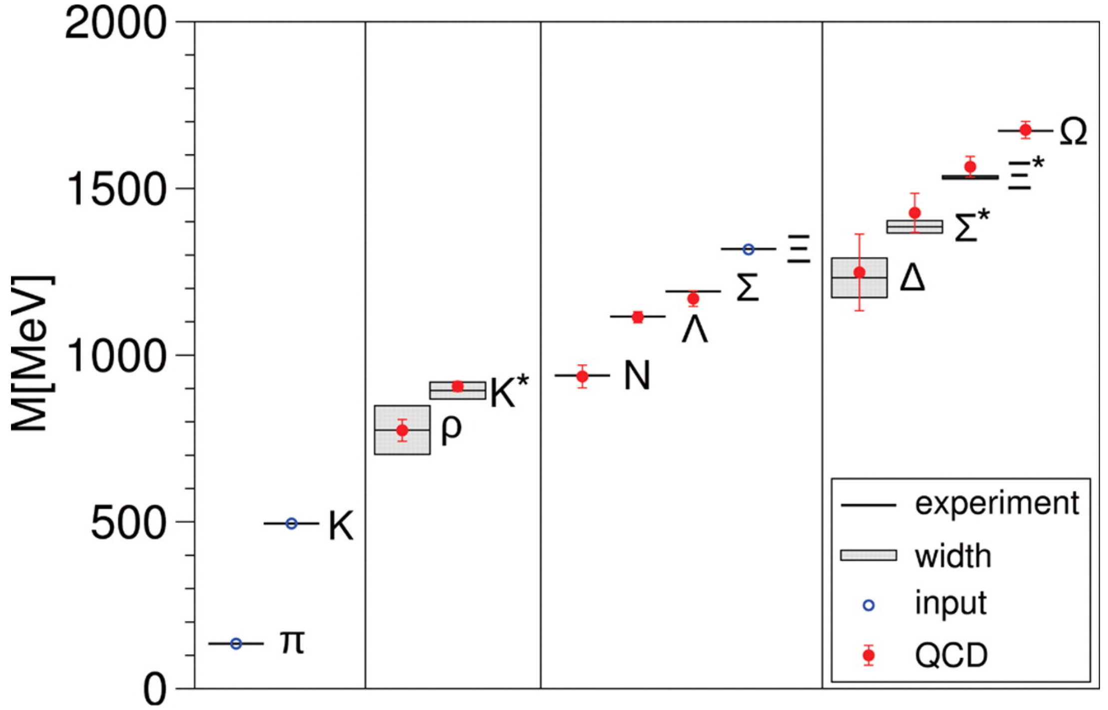


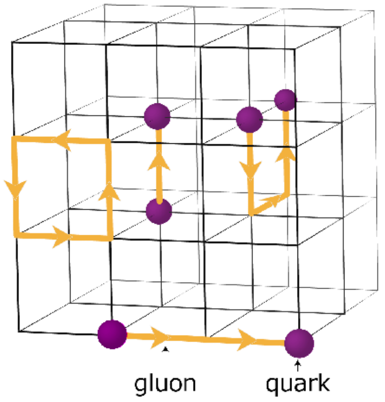
Studied in **laboratory** with heavy-ion collisions

Non-perturbative methods

First-principle tool: **Lattice QCD**

Ab-initio calculation of hadron masses





BMW Collaboration, Science 322, 1224 (2008)

Remarkable agreement of QCD with the experiment

QCD transition from lattice QCD

$$Z = \mathrm{Tr}\left(e^{-(\hat{H}-\mu\hat{N})/T}\right) = \int DU \,\det M[U,\mu]\, e^{-S_{YM}}$$

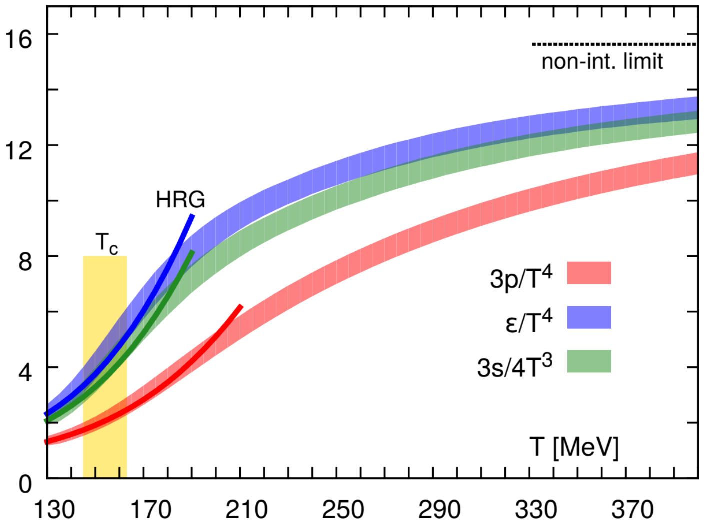


Figure from HotQCD coll., PRD '14

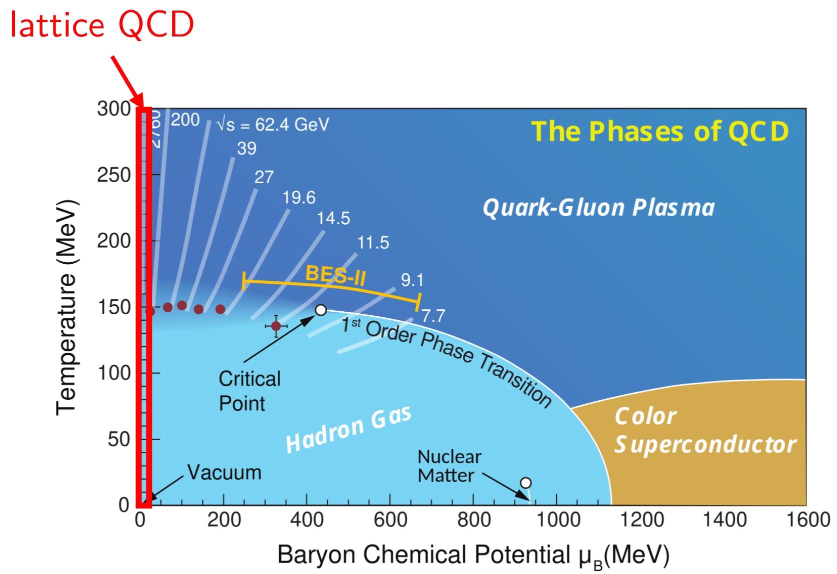


Figure from Bzdak et al., Phys. Rept. '20

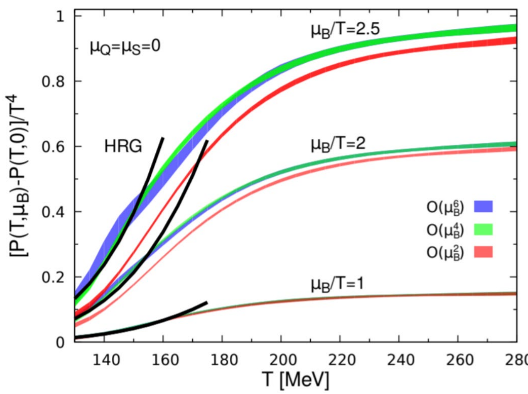
- Analytic crossover at vanishing net baryon density at $k_B T_{pc} \approx 155$ MeV – a first-principle result
  [Y. Aoki et al., Nature 443, 675 (2006)]
- Finite densities inaccessible due to **sign problem**, but many effective theories predict first-order phase transition and the **QCD critical point**

First-principle constraints on the QCD critical point

Indirect lattice QCD methods offer glimpse into small μ_B/T

- Taylor expansion around $\mu_B/T = 0$

$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \frac{\chi_2^B(T,0)}{2!}(\mu_B/T)^2 + \frac{\chi_4^B(T,0)}{4!}(\mu_B/T)^4 + \ldots$$



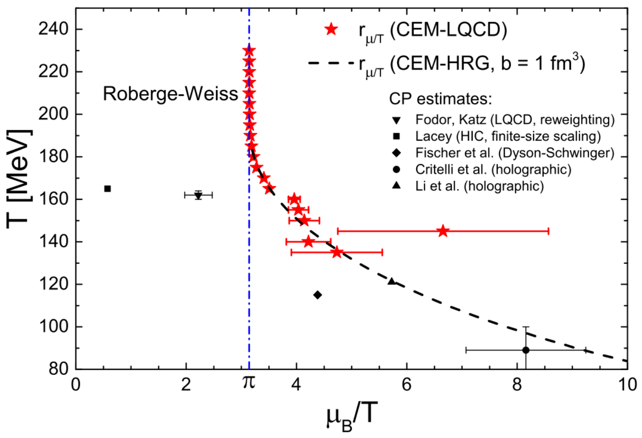
No hints for the critical point at T $>$ 135 MeV

Critical point $\mu_B/T < 3$ disfavored

[HotQCD Collaboration, PRD 95, 054504 (2017)]

- Relativistic virial expansion in fugacities via analytic continuation from imaginary μ_B/T

$$\frac{p(T,\mu_B)}{T^4} = \sum_{k=0}^{\infty} p_k(T) \cosh\left(\frac{k\,\mu_B}{T}\right)$$



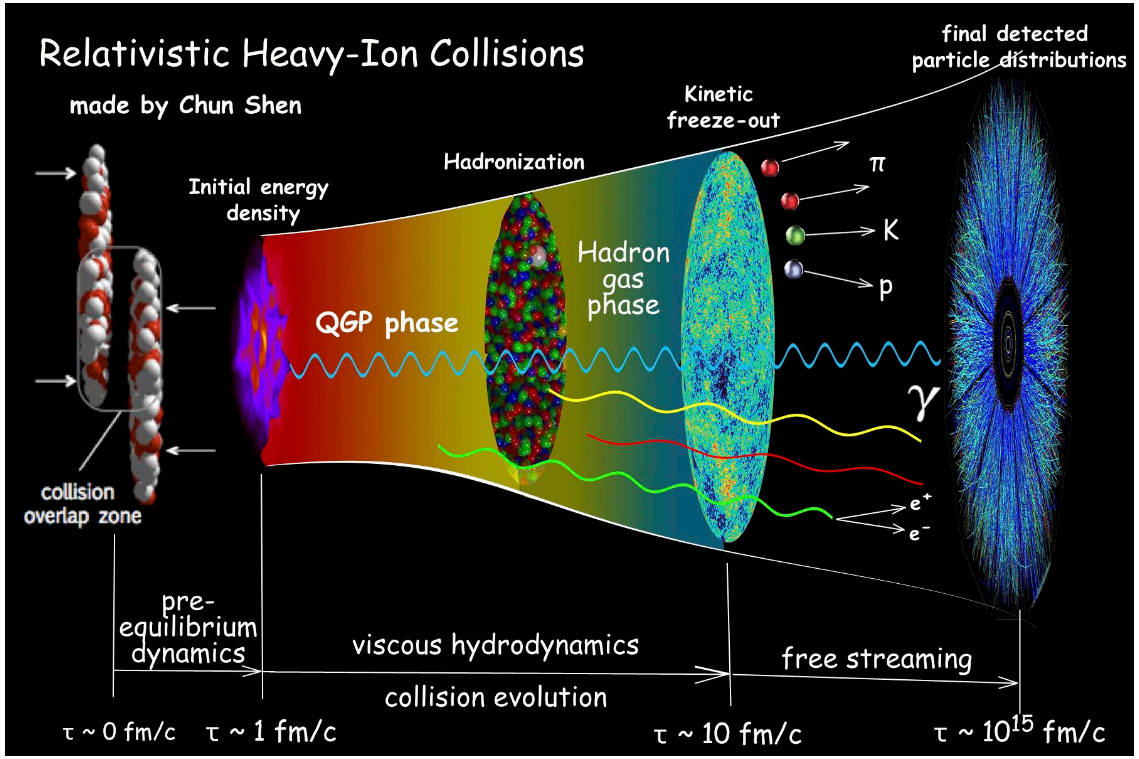
Expansion sees singularity in the complex plane, Im $[\mu_B/T] = \pi$

Critical point at $\mu_B/T < \pi$ disfavored

[V.V., Steinheimer, Philipsen, Stoecker, PRD 97, 114030 (2018)]

Critical point, if it exists, likely located beyond the reach of lattice methods

Relativistic heavy-ion collisions – “Little Bangs”



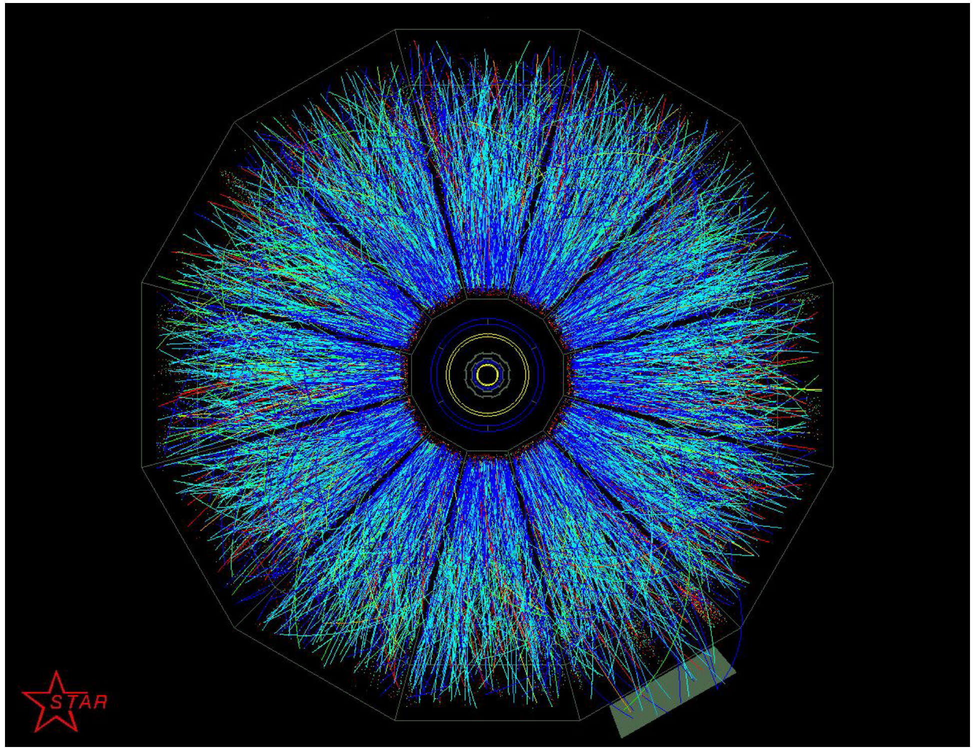
Control parameters

- Collision energy $\sqrt{s_{NN}} = 2.4 – 5020$ GeV
- Size of the collision region

Measurements

- Final hadron abundances and momentum distributions

QCD phase diagram with heavy-ion collisions



STAR event display

Thousands of particles created in relativistic heavy-ion collisions

Apply concepts of statistical mechanics

Particle production in heavy-ion collisions

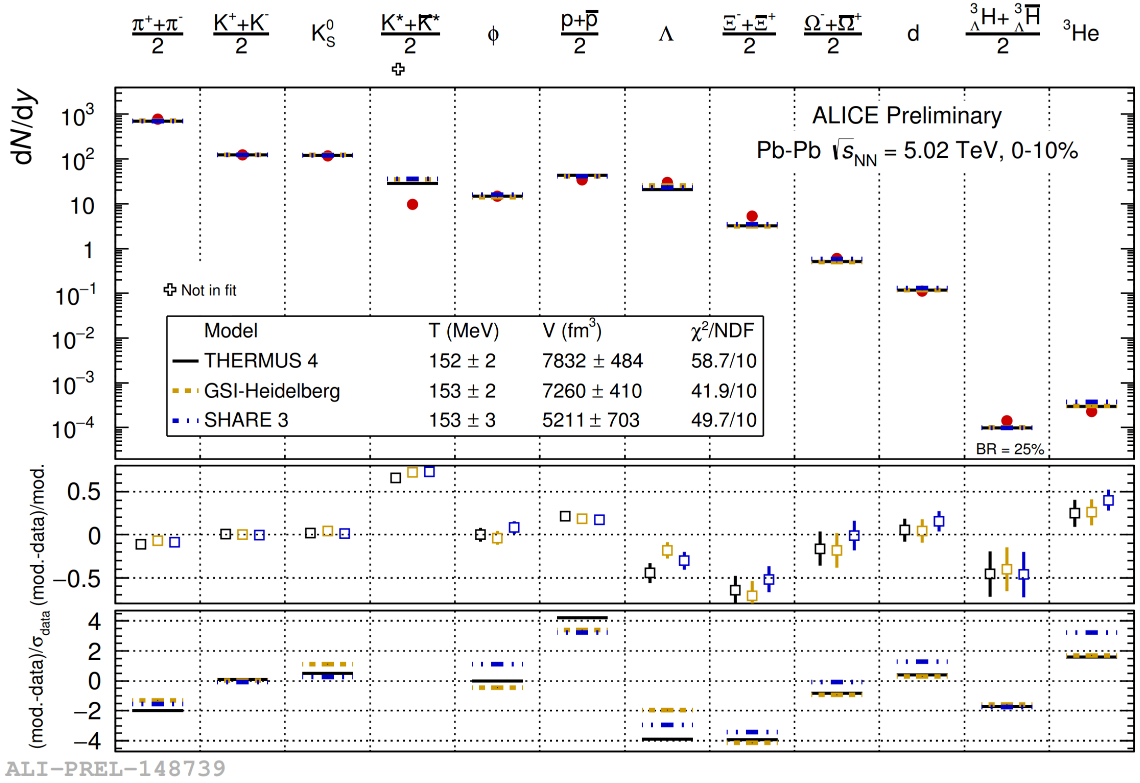
Ideal gas law (E. Clapeyron, 1834)

$$P_i V = N_i k_B T$$

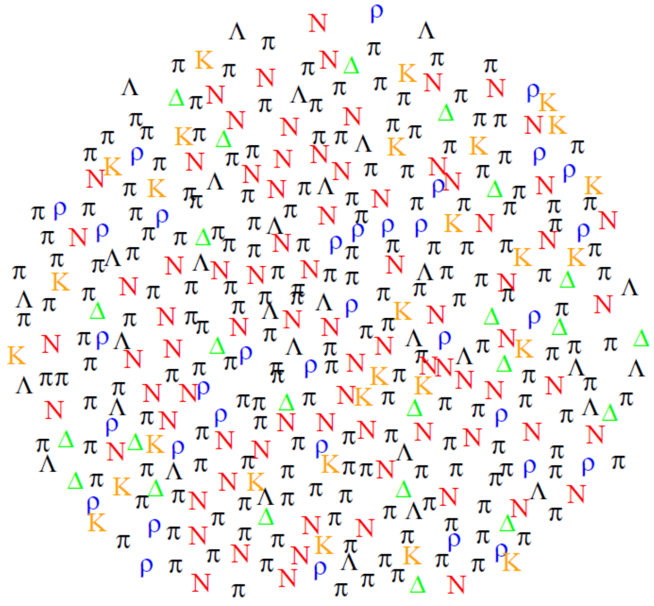
is the simplest model of particle production

$$N_i = \frac{d_i V}{2\pi^2} \int dk\, k^2 \left[1 \pm \exp\left(\frac{\sqrt{k^2 + m_i^2} - \mu_i}{T}\right)\right]^{-1}$$

Bose-Einstein & Fermi-Dirac, 1924-1926



ALICE collaboration (Quark Matter 2018)



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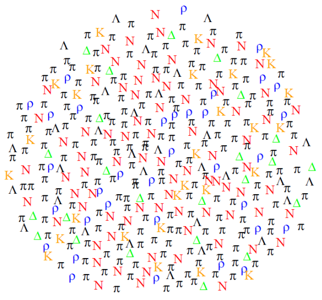
Hadron resonance gas (HRG) model

- **HRG model:** free gas of known hadrons and resonances



$$p(T,\mu_B) = \underbrace{T\,\phi_M(T)}_{\textit{mesons}} + \underbrace{2\,T\,\phi_B(T)}_{\textit{baryons}}\cosh(\mu_B/T)$$

$$\phi_{M(B)}(T) = \sum_{i\in M(B)} \frac{d_i}{2\pi^2}\int dk\, k^2 \exp\left(-\frac{\sqrt{m_i^2+k^2}}{T}\right)$$

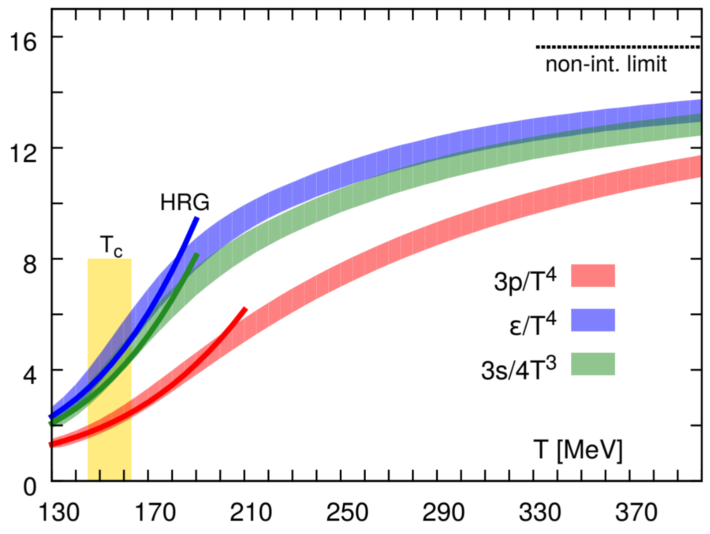


Figure from HotQCD coll., PRD '14

  - Hadronic interactions dominated by resonance formation*
  - Leading order in relativistic virial expansion
  - Matches well with lattice QCD below T_{pc}
  - Non-resonant interactions incorporated in extended descriptions

HRG model and heavy-ion collisions:

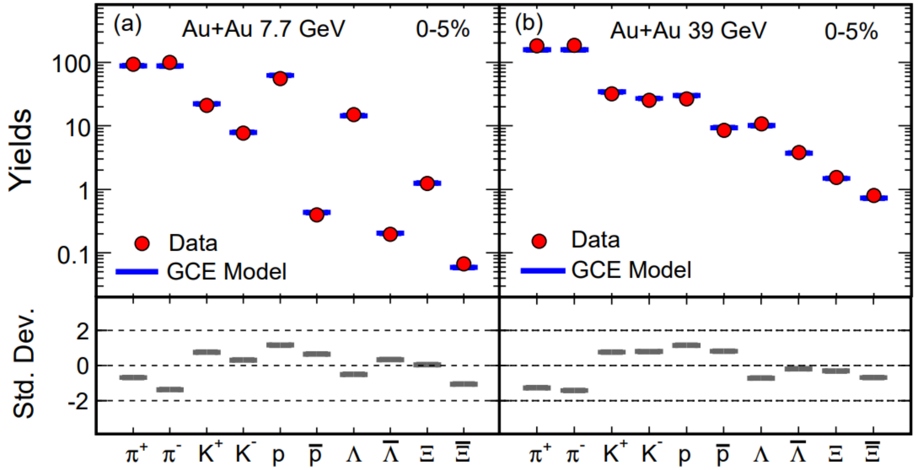
- Basis for the thermal model of particle production

All bells and whistles implemented in open source codes, e.g. Thermal-FIST [VV, Stoecker, Comput. Phys. Commun. 244, 295 (2019)]

** Dashen, Ma, Bernstein, "S-matrix formulation of statistical mechanics", Phys. Rev. (1969); Prakash, Venugopalan, Nucl. Phys. A (1992)*

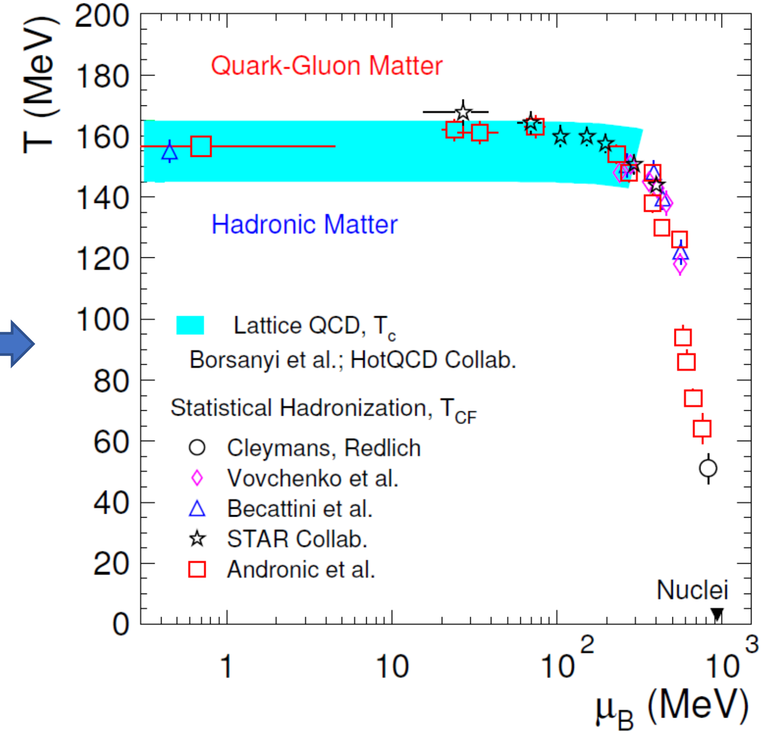
Mapping heavy-ion collisions onto the QCD phase diagram

Fit hadron yields with the HRG model



STAR Collaboration, Phys. Rev. C 96, 044904 (2017)

$$\mathrm{T}(\sqrt{s_{NN}})$$
$$\mu_B(\sqrt{s_{NN}})$$



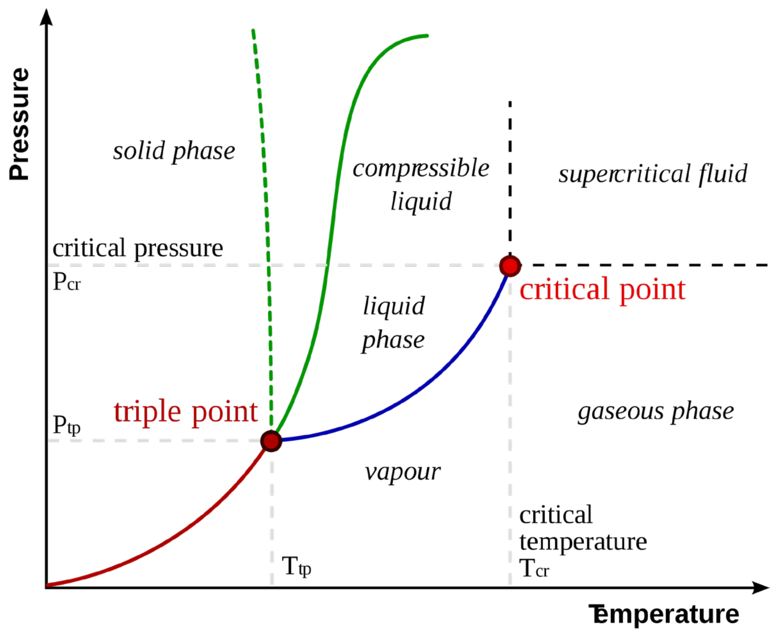
A. Andronic et al., Nature 561, 321 (2018)

$\sqrt{s_{NN}} \searrow$ → $\mu_B \nearrow$

For p_T differential observables (spectra, flow, ...) use relativistic hydrodynamics

QCD critical point

Ordinary fluid (e.g. water)



QCD

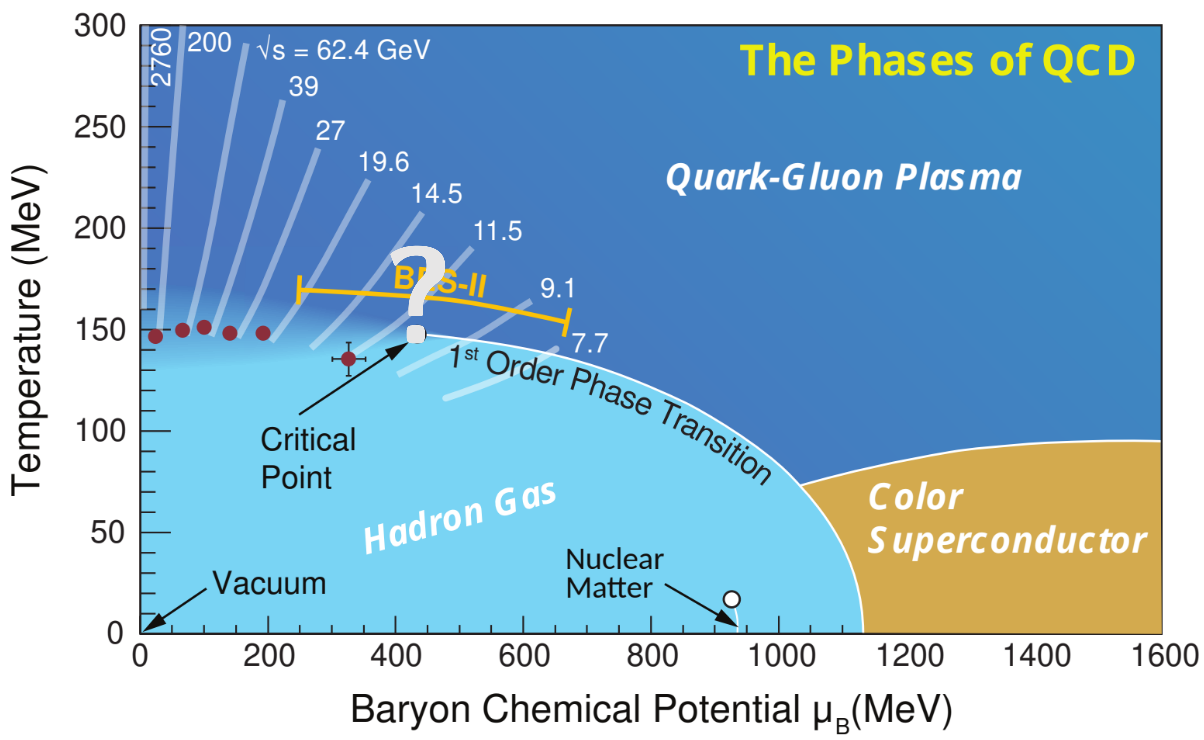


Figure from Bzdak et al., Phys. Rept. '20

What is the nature of the quark-hadron transition at finite baryon density?

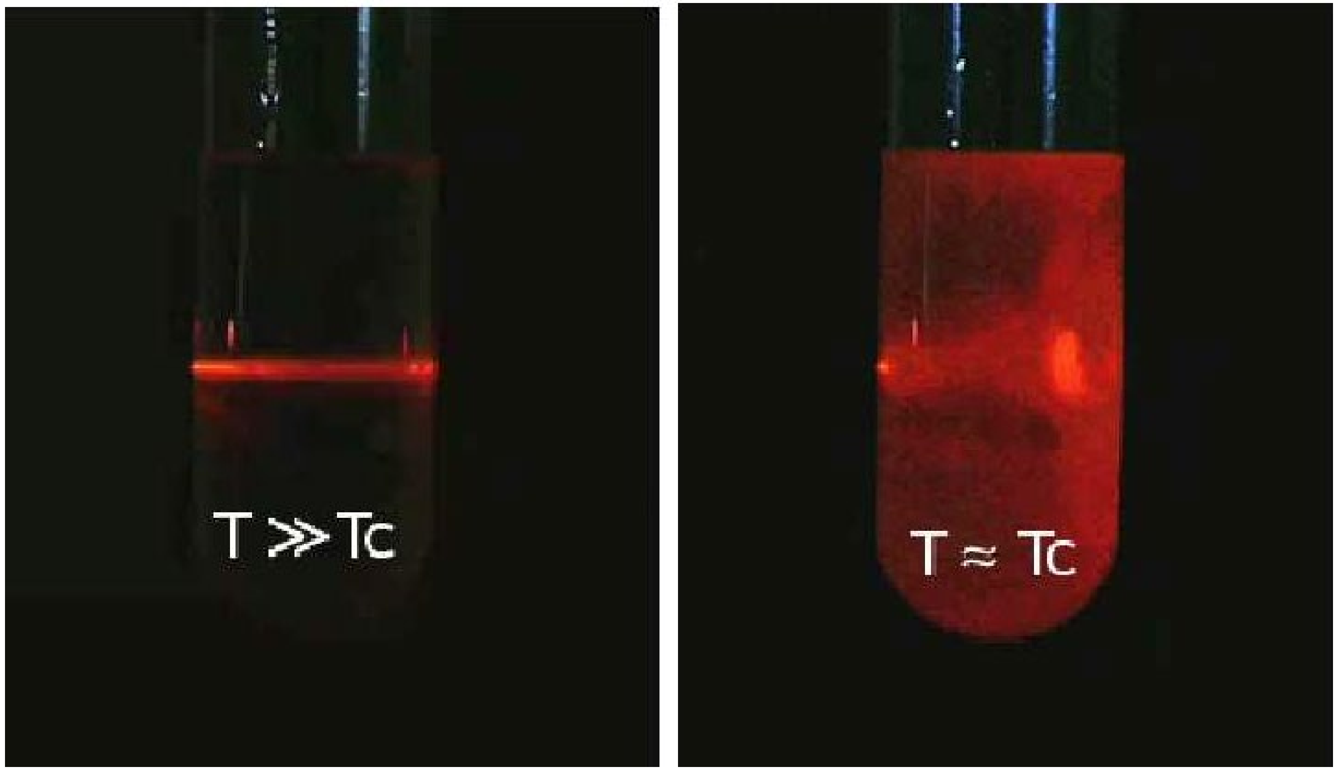
*Is there a QCD phase transition and **critical point**? Where?*

Tackle these questions with heavy-ion collisions

Critical point and fluctuations

Density fluctuations at macroscopic length scales

Critical opalescence



Unfortunately, we cannot do this in heavy-ion collisions

Event-by-event fluctuations and statistical mechanics

Consider a fluctuating number N

Cumulants: $G_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$

variance $\kappa_2 = \langle(\Delta N)^2\rangle = \sigma^2$  width

skewness $\kappa_3 = \langle(\Delta N)^3\rangle$  asymmetry

kurtosis $\kappa_4 = \langle(\Delta N)^4\rangle - 3\langle(\Delta N^2)\rangle^2$  peak shape

Experiment:

$$P(N) \sim \frac{N_{events}(N)}{N_{events}^{total}}$$

Statistical mechanics:

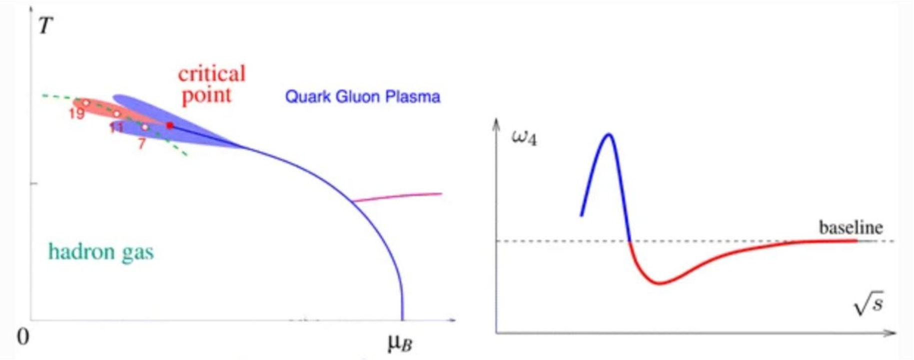
Grand partition function $$ln\, Z^{\mathrm{gce}}(T, V, \mu) = ln \left[\sum_N e^{\mu N} Z^{\mathrm{ce}}(T, V, N)\right],$$

$$\boxed{\kappa_n \propto \frac{\partial^n (\ln\, Z^{\mathrm{gce}})}{\partial(\mu_N)^n}}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

Applications

- **(QCD) critical point** – large critical fluctuations of baryon (proton) number



M. Stephanov, Phys. Rev. Lett. (2011)

Correlation length $\xi \to \infty$ diverges at the critical point

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

Looking for non-monotonic dependence of κ_4 vs $\sqrt{s_{NN}}$

- Test of (lattice) QCD at $\mu_B \approx 0$

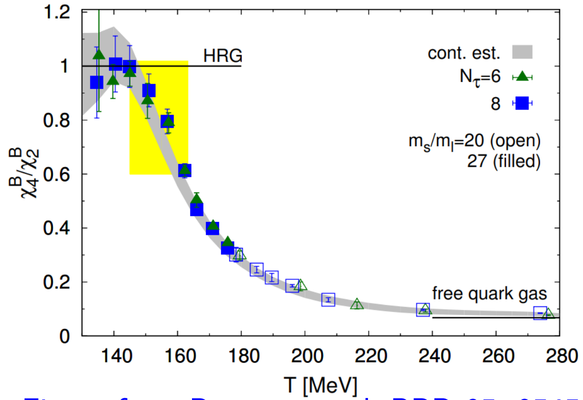
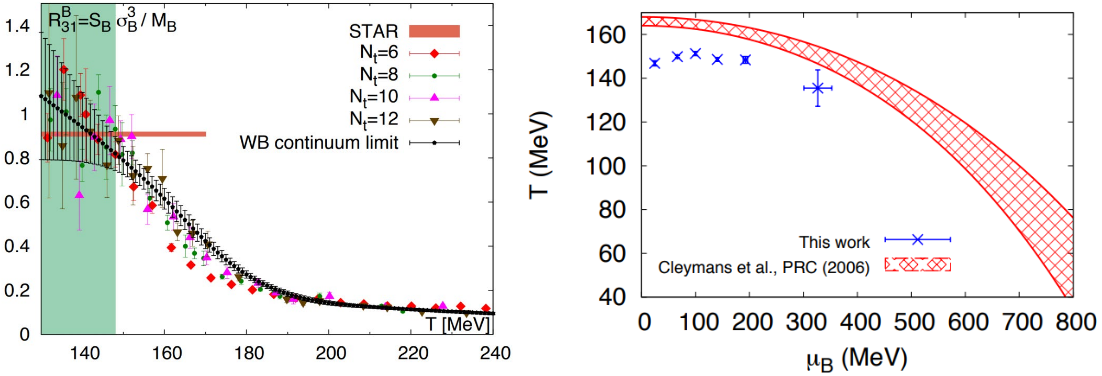


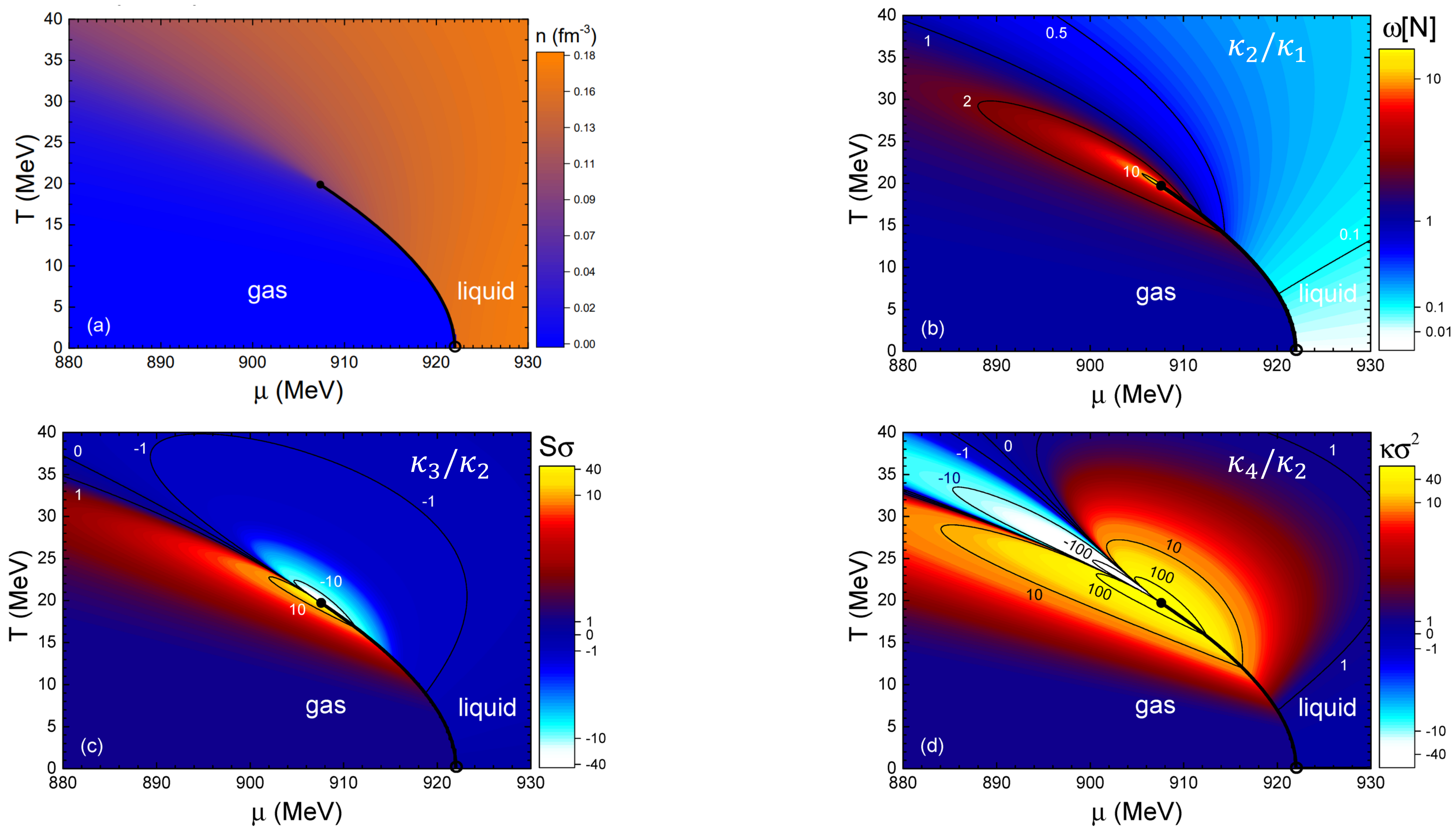
Figure from Bazavov et al. PRD 95, 054504 (2017)
Probed by LHC and top RHIC

- Freeze-out from fluctuations



Borsanyi et al. PRL 113, 052301 (2014); Bazavov et al. PRL 109, 192302 (2012)

Example: Liquid-gas transition with van der Waals equation

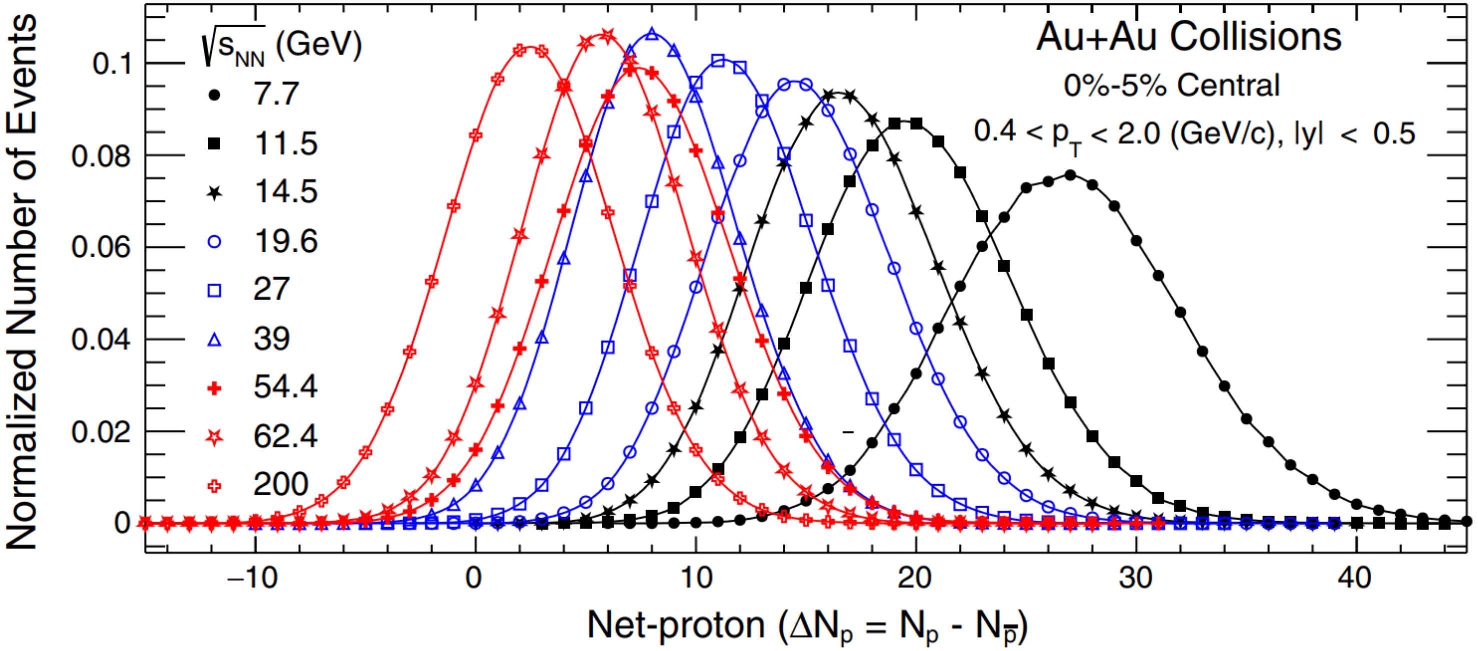


VV, Anchishkin, Gorenstein, Poberezhnyuk, Phys. Rev. C 92, 054901 (2015)

Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons $\quad P(\Delta N_p) \sim \frac{N_{\text{events}}(\Delta N_p)}{N_{events}^{total}}$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)

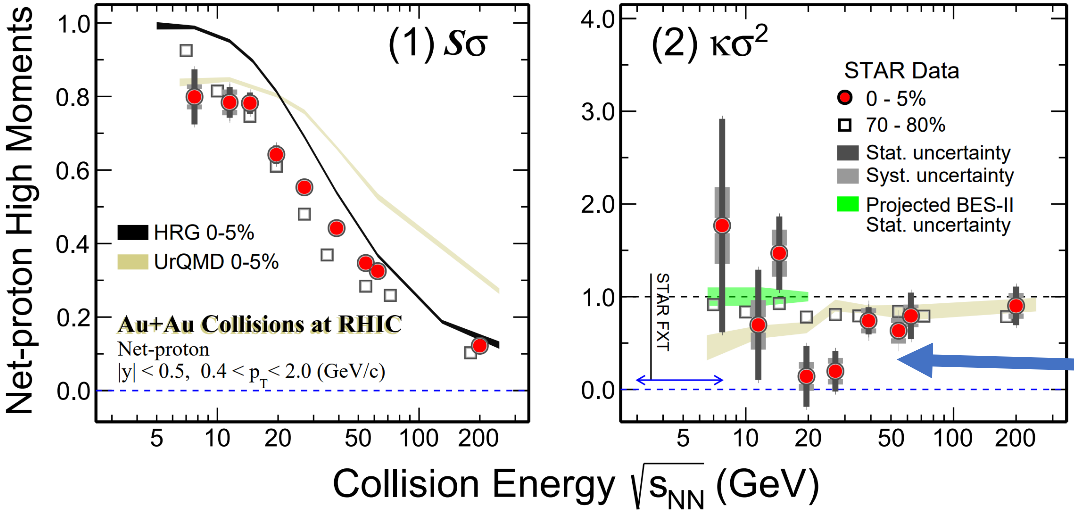


Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

$$\frac{\kappa_2}{\langle N \rangle}, \qquad \frac{\kappa_3}{\kappa_2}, \qquad \frac{\kappa_4}{\kappa_2}$$

Experimental measurements

Beam energy scan in search for the critical point (STAR Coll.)



Phys. Rev. Lett. 126, 092301 (2021)

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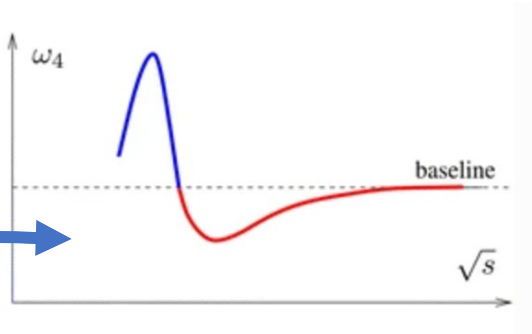


M. Stephanov, Phys. Rev. Lett. (2011)

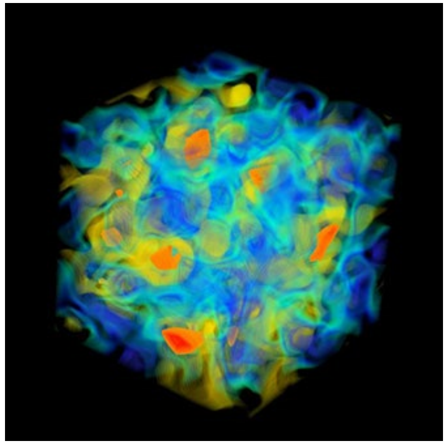
Reduced errors (better statistics) to come soon from beam energy scan II program

Can we learn more from the more accurate data available for κ_2 and κ_3?

Other measurements: LHC-ALICE, GSI-HADES & CERN-NA61/SHINE Collaborations

Theory vs experiment: Challenges for fluctuations

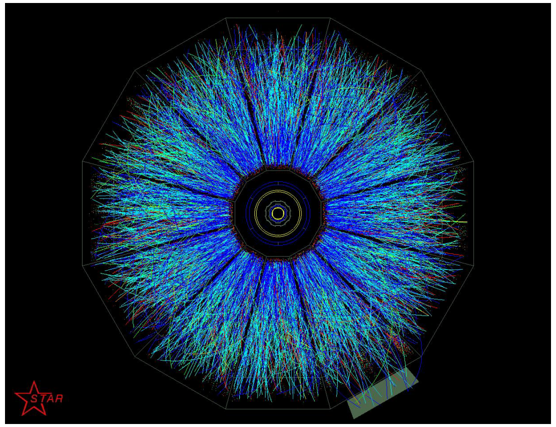
Theory



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment

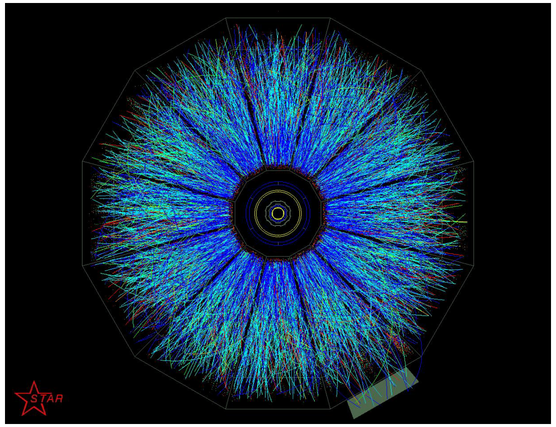


STAR event display

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

When are the measured fluctuations grand-canonical?

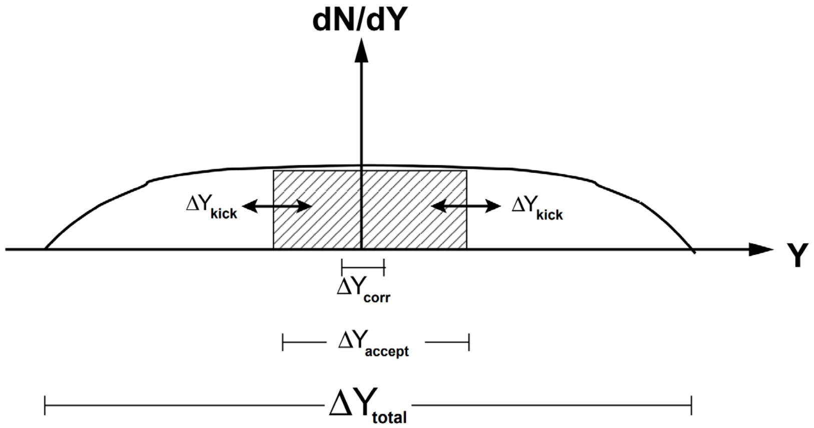
- Consider event-by-event fluctuations of particle number in acceptance ΔY_{accept} around midrapidity
- Scales
  - ΔY_{accept} – acceptance
  - ΔY_{total} – full space
  - ΔY_{corr} – rapidity correlation length (thermal smearing)
  - ΔY_{kick} – diffusion in the hadronic phase



STAR event display

- **GCE applies if $\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}, \Delta Y_{corr}$**

- In practice $\Delta Y_{total} \gg \Delta Y_{accept}$ and $\Delta Y_{accept} \gg \Delta Y_{corr}$ are not simultaneously satisfied
  - Corrections from baryon conservation are large [Bzdak et al., PRC '13]
  - $\Delta Y_{corr} \sim 1 \sim \Delta Y_{accept}$ [Ling, Stephanov, PRC '16]



V. Koch, arXiv:0810.2520

Need dynamical description

Dynamical approaches to the QCD critical point search

1. Dynamical model calculations of critical fluctuations
   - Fluctuating hydrodynamics
   - Equation of state with tunable critical point [P. Parotto et al, Phys. Rev. C 101, 034901 (2020)]
   - Predict CP signatures dependent on its location

   Under development within the Beam Energy Scan Theory (BEST) Collaboration

   BEST COLLABORATION [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]

2. **Deviations from precision calculations of the non-critical baseline**
   - Include essential non-critical contributions to (net-)proton number cumulants
   - Exact baryon conservation + hadronic interactions (hard core repulsion)
   - Based on realistic hydrodynamic simulations tuned to bulk data

   [VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

Excluded volume effect

Incorporate repulsive baryon (nucleon) hard core via excluded volume

VV, M.I. Gorenstein, H. Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

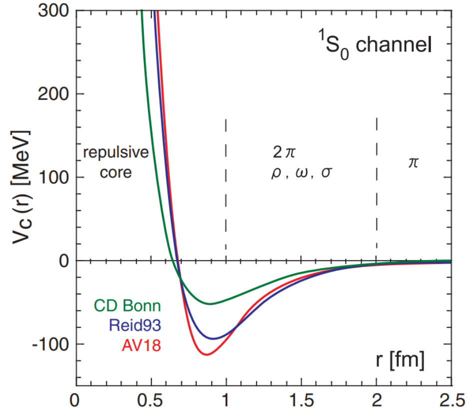
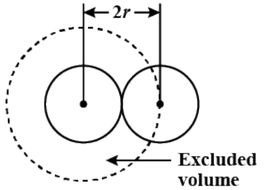


Figure from Ishii et al., PRL '07

Amounts to a van der Waals correction for baryons in the HRG model

$$V \to V - bN$$



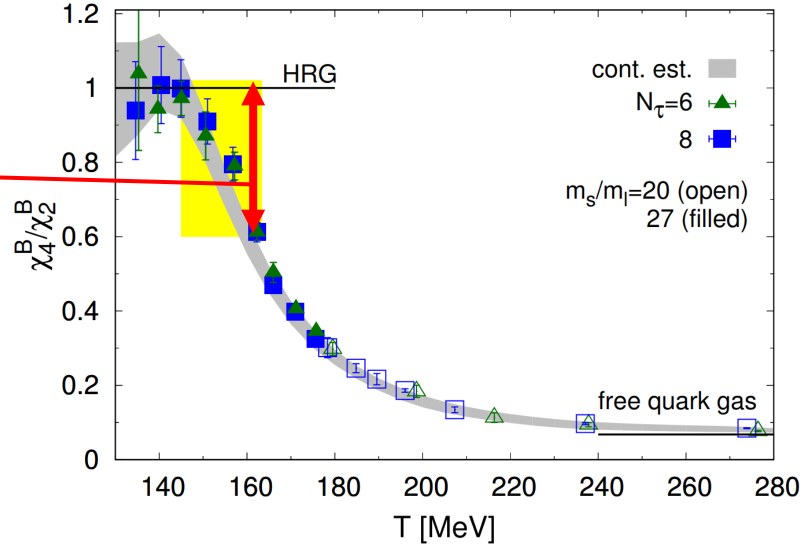
$$p_{B(\bar{B})}^{\rm ev} = p_{B(\bar{B})}^{\rm id}\, e^{-b p_{B(\bar{B})}^{\rm ev}/T}$$

- Net baryon kurtosis suppressed as in lattice QCD

$$\frac{\chi_4^B}{\chi_2^B} \simeq 1 - 12b\phi_B(T) + O(b^2)$$

- Reproduces virial coefficients of baryon interaction from lattice QCD

Excluded volume from lattice QCD: $\boxed{\text{b} \approx 1 \text{ fm}^3}$



VV, A. Pasztor, S. Katz, Z. Fodor, H. Stoecker, Phys. Lett. B 755, 71 (2017)

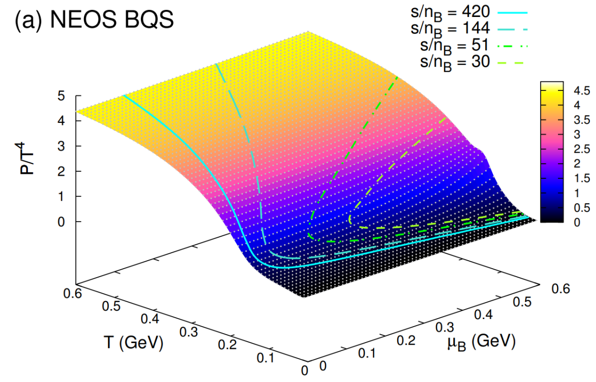
RHIC-BES: Hydrodynamic description in non-critical scenario

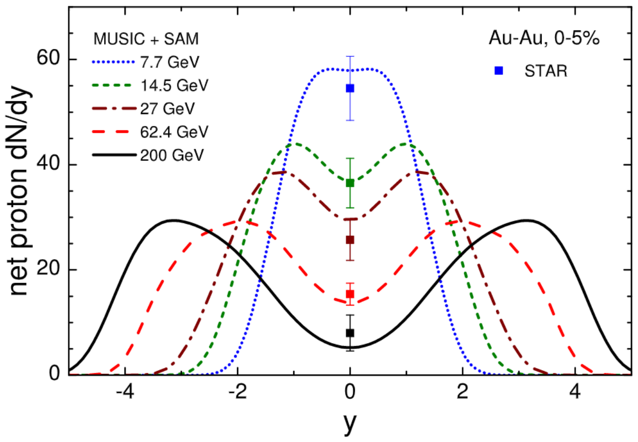
VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- Collision geometry based 3D initial state
  - Constrained to net proton distributions [Shen, Alzhrani, Phys. Rev. C '20]
- Viscous hydrodynamics evolution – MUSIC-3.0
  - Energy-momentum and baryon number conservation
  - Crossover equation of state based on lattice QCD
    [Monnai, Schenke, Shen, Phys. Rev. C '19; also Noronha-Hostler, Parotto, Ratti, Stafford, Phys. Rev. C '19]
- Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3

$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x)\, p^\mu\, \frac{d_j\, \lambda_j^{\rm ev}(x)}{(2\pi)^3}\, \exp\left[\frac{\mu_j(x) - u^\mu(x) p_\mu}{T(x)}\right].$$

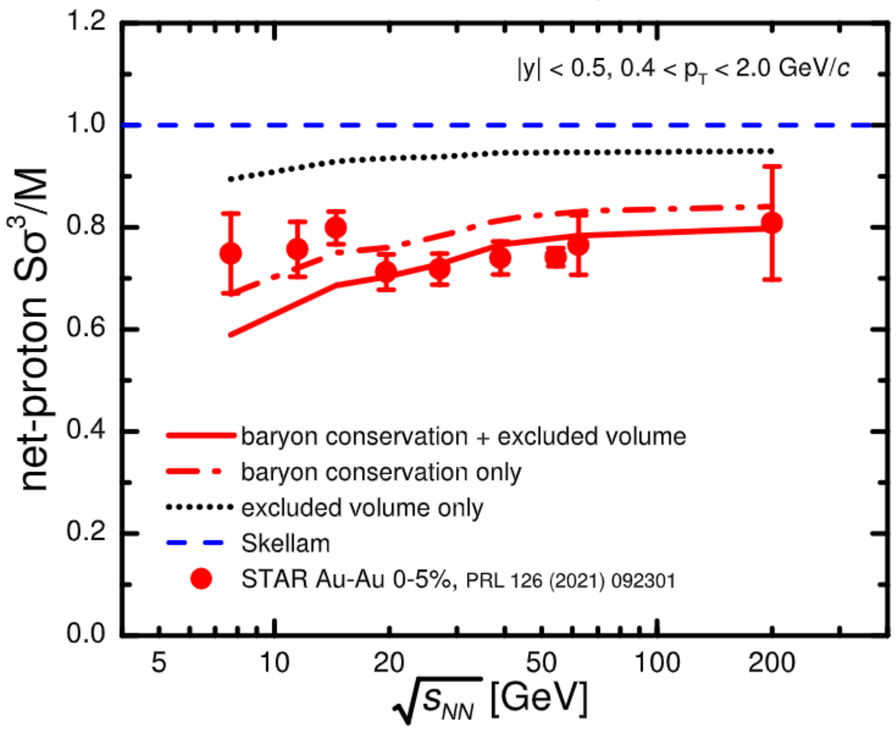
- Particlization respects QCD-based baryon number distribution
  - Incorporated via baryon excluded volume b = 1 fm^3
    [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
- Incorporates exact global baryon conservation via a novel method
  [VV, Phys. Rev. C 105, 014903 (2022)]



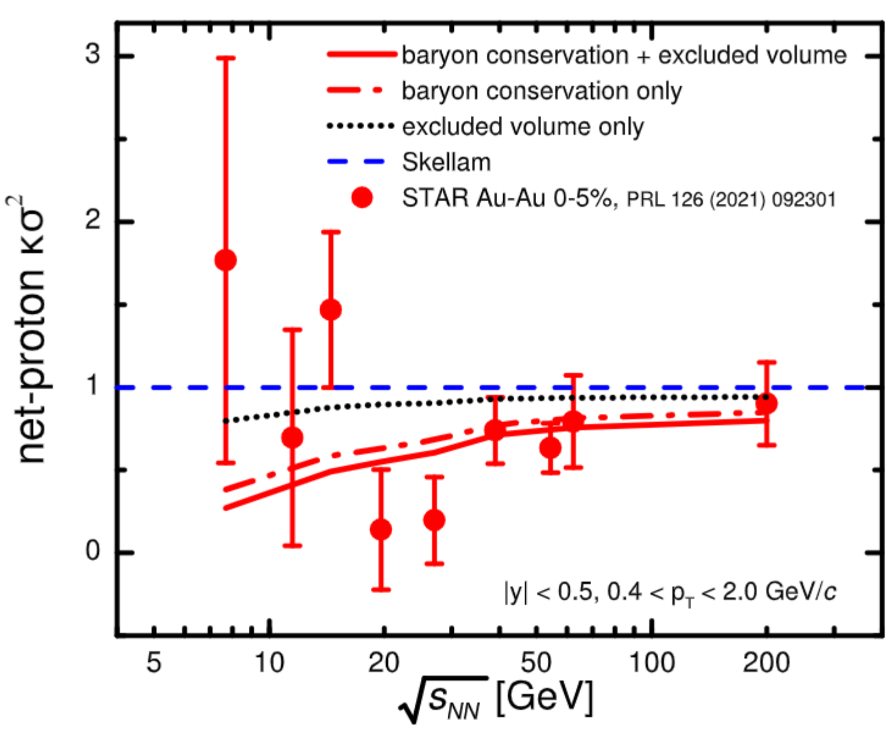


Net proton cumulant ratios

κ_3/κ_1

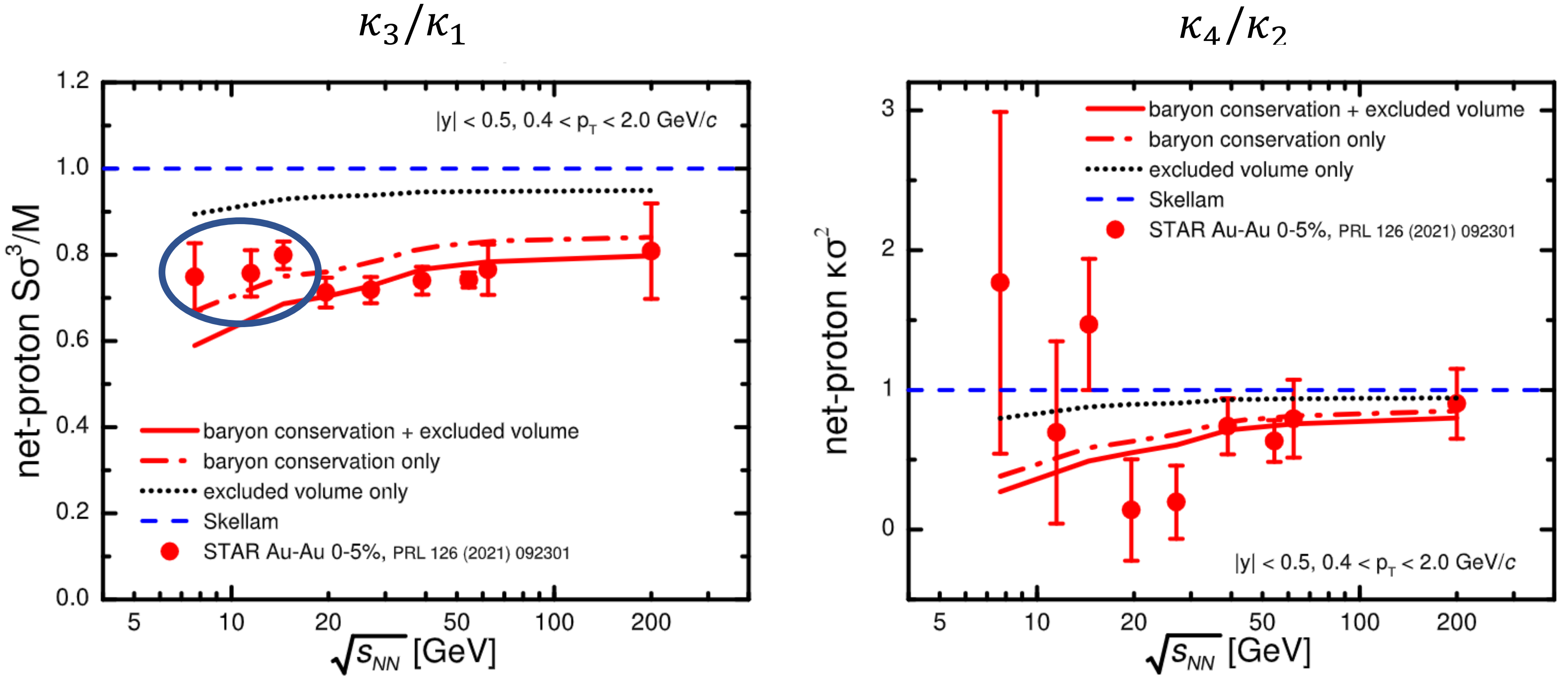


κ_4/κ_2



- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (baryon conservation and repulsion)
- Effect from baryon conservation is larger than from repulsion
- Excess of skewness in data at $\sqrt{s_{NN}} < 20$ GeV – *hint of attractive interactions?* ***Critical point?***

Net proton cumulant ratios



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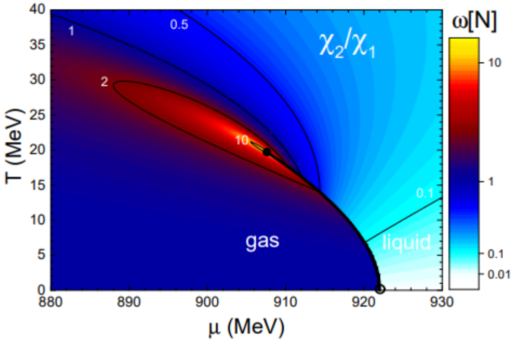
Correlation Functions

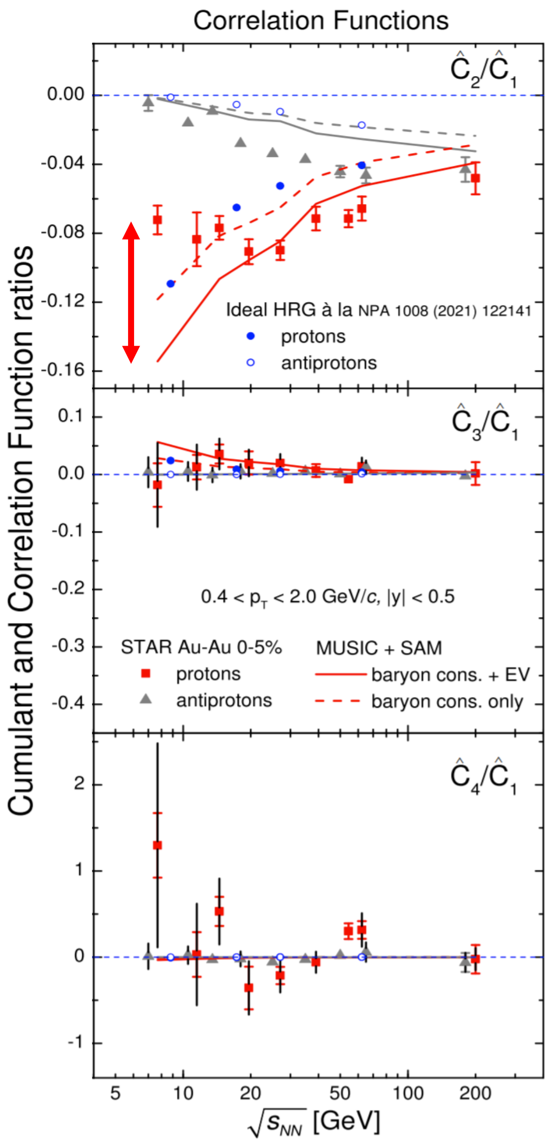
- Analyze genuine multi-particle correlations via **factorial cumulants $\hat{C}_n$** [Bzdak, Koch, Strodthoff, Phys. Rev. C '17]

$$\hat{C}_1 = \kappa_1, \qquad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$
$$\hat{C}_2 = -\kappa_1 + \kappa_2, \quad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$

- Three- and four-particle correlations are small without a CP
  - Multi-particle correlations expected near the critical point [Ling, Stephanov, PRC '15]

- Signals from the data at $\sqrt{s_{NN}} \leq 20$ GeV
  - Excess of two-proton correlations
  - Possibility of significant 4-proton correlations
  - ***Critical point?***





Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV

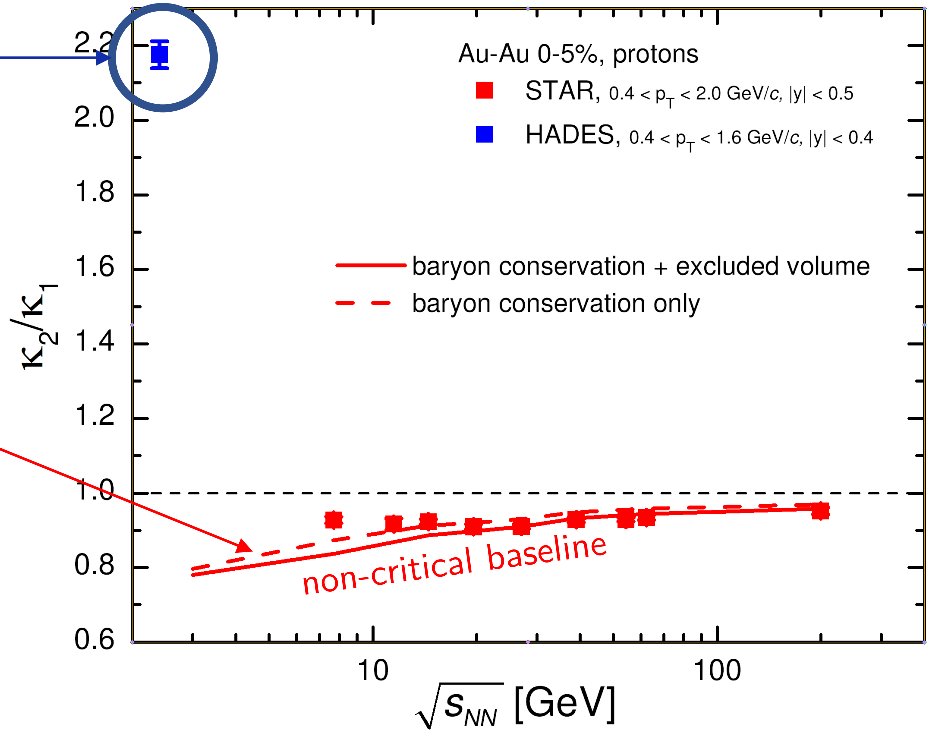
- Intriguing hint from HADES @ $\sqrt{s_{NN}} = 2.4$ GeV: huge proton variance!

[HADES Collaboration, Phys. Rev. C 102, 024914 (2020)]

- No change of trend in the non-critical baseline

- Fill the gap with ongoing/future data from STAR fixed target, future experiments like CBM-FAIR

[STAR Collaboration, arXiv:2112.00240]



Naïvely, could indicate QCD critical point near HADES freeze-out at $T \approx 70$ MeV, $\mu_B \approx 875$ MeV

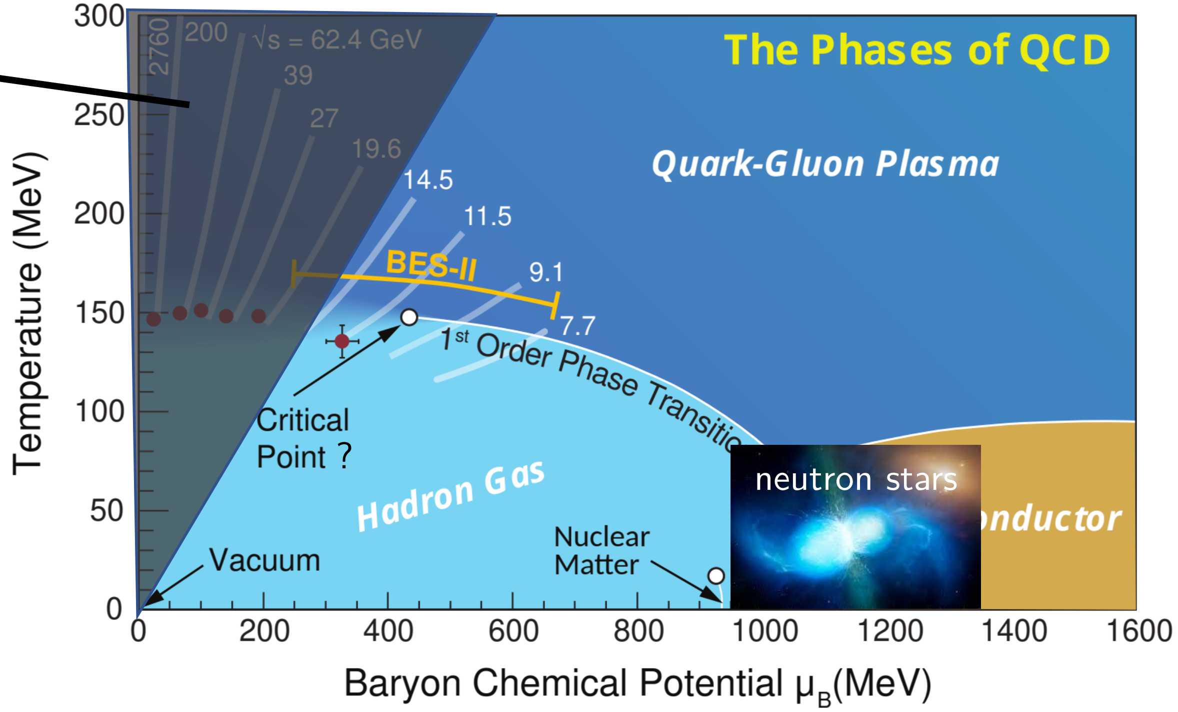
Some effective QCD approaches do predict the critical point close to that region, e.g. holography

[Critelli, Noronha, Noronha-Hostler, Portillo, Ratti, Rougemont, Phys. Rev. D 96, 096026 (2017)]

[Grefa et al., Phys. Rev. D 104, 034002 (2021)]

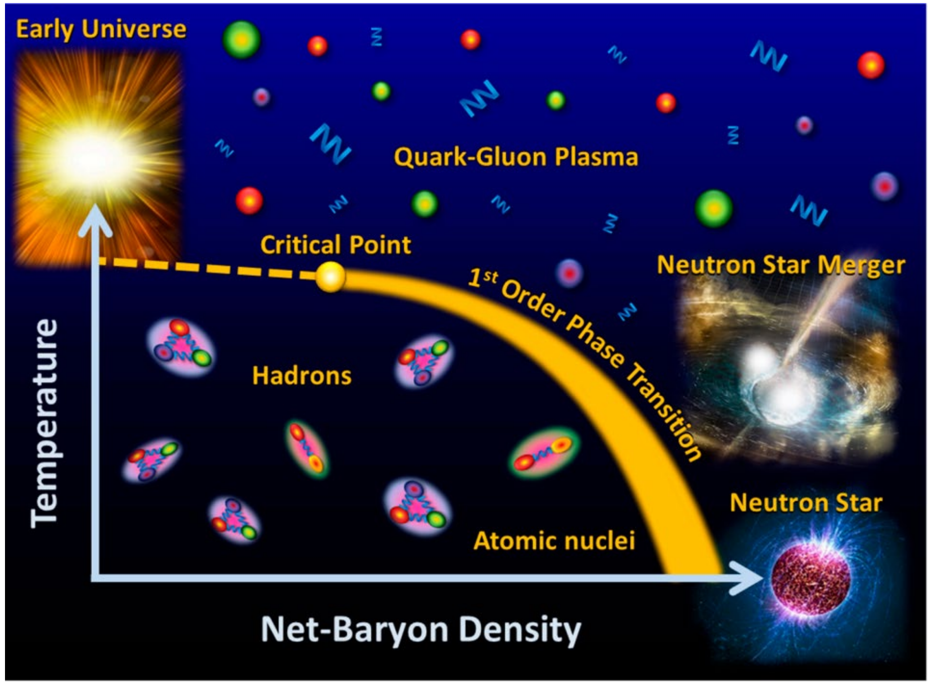
QCD phase structure: What we learned so far

Critical point disfavored

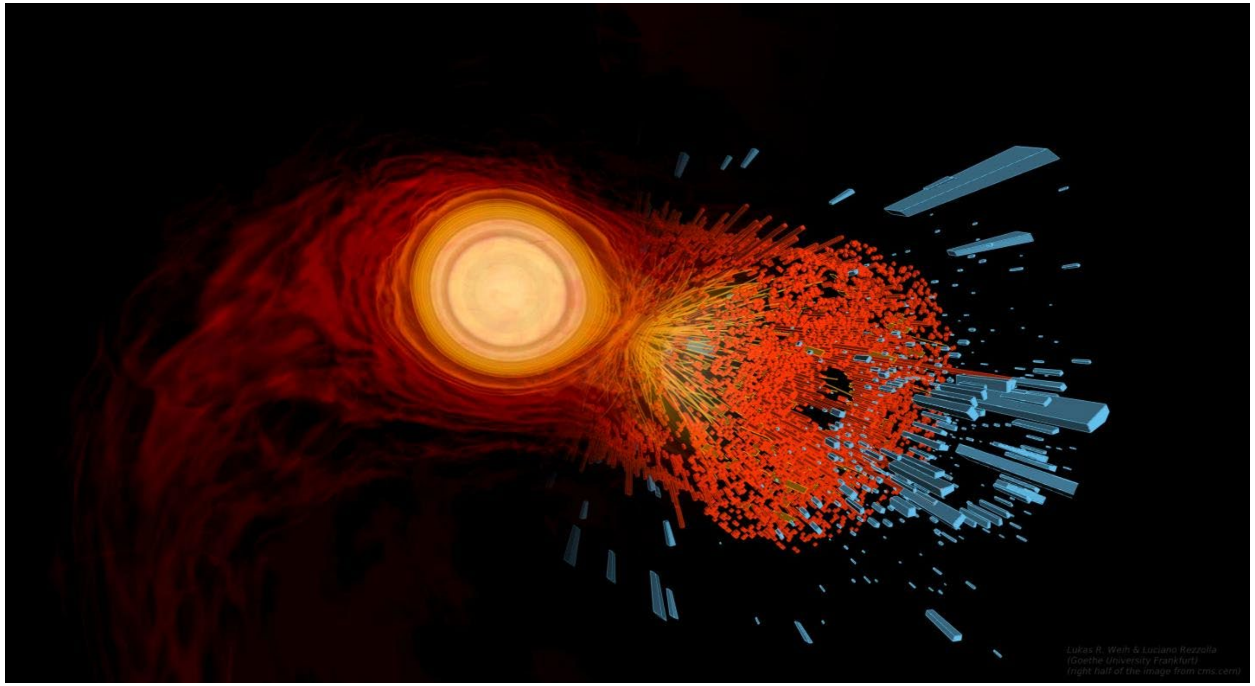


- Data at high energies ($\sqrt{s_{NN}} \geq 20$ GeV) consistent with "non-critical" physics
- Disfavors critical point at $\mu_B/T<$2-3, consistent with what we know from lattice QCD
- Interesting physics at high densities probed by future experiments, neutron stars & their mergers

Outlook: Equation of state for heavy ions and neutron stars



P. Senger (GSI)



L. Weih, L. Rezzolla (Frankfurt)

Summary

- Strongly interacting QCD matter under extreme conditions
  - Undergoes a transition to quark-gluon plasma at trillion kelvin degrees
  - Behaves like a fluid described by hydrodynamics
  - Phase structure at finite baryon density still largely unknown, no phase transition at high energies

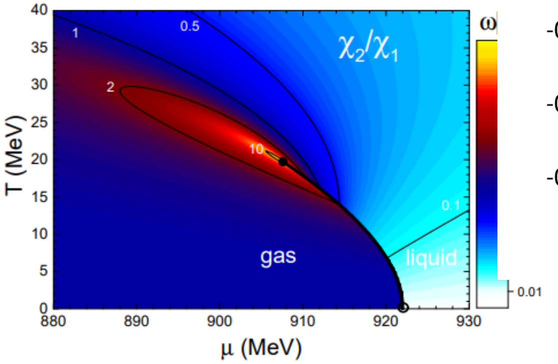
- Fluctuations are a powerful tool to explore the QCD phase diagram
  - Heavy-ion data are described quantitatively at $\sqrt{s_{NN}} \geq 20$ GeV $(\mu_B/T<3)$ without critical point
  - Possible critical point signals at $\sqrt{s_{NN}} < 14.5$ GeV
  - More evidence at lower energies to come from future experiments and connections to neutron star phenomena

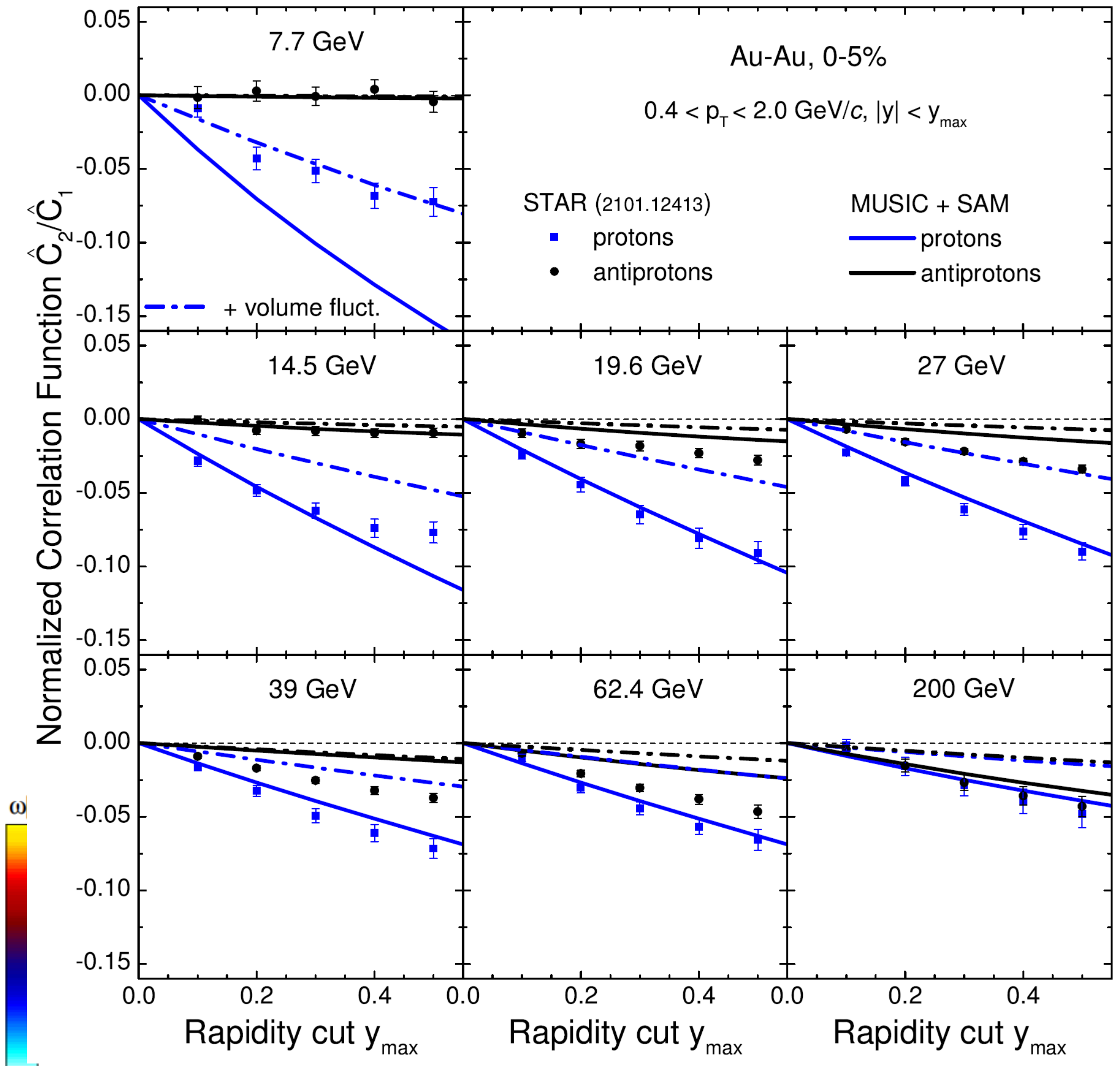
Thanks for your attention!

Backup slides

Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?
- Volume fluctuations? [Skokov, Friman, Redlich, PRC '13]
  - $C_2/C_1 \mathrel{+}= C_1 * v_2$
  - Can improve low energies but spoil high energies?
- Exact electric charge conservation?
  - Worsens the agreement at $\sqrt{s_{NN}} \leq 14.5$, higher energies virtually unaffected
- **Attractive interactions?**
  - Could work if baryon repulsion turns into attraction in the high-μ_B regime
  - **Critical point?**

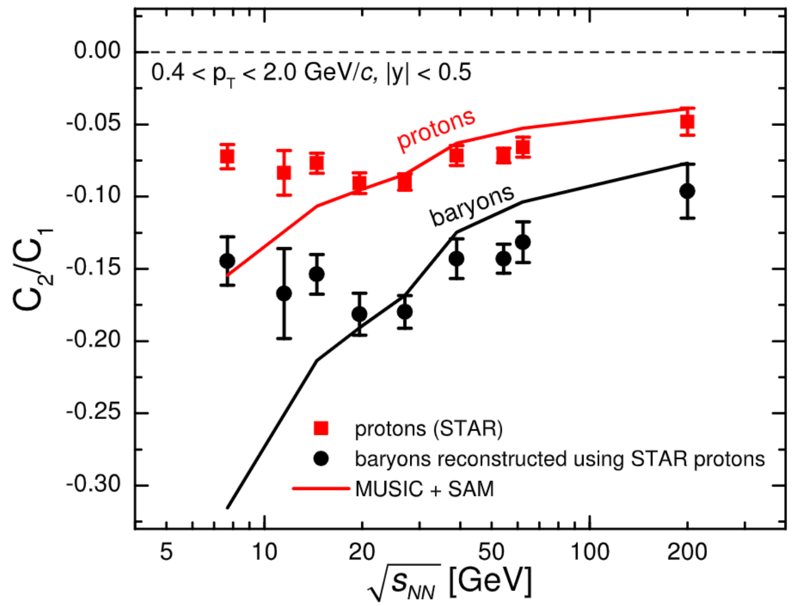




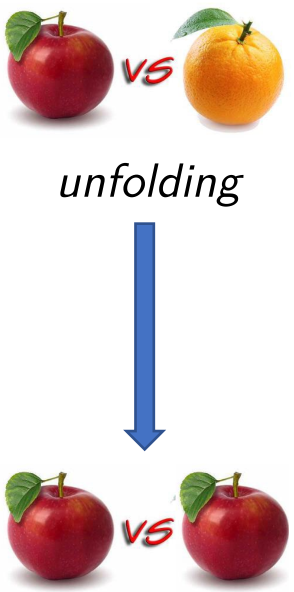
Outlook: baryon cumulants from protons

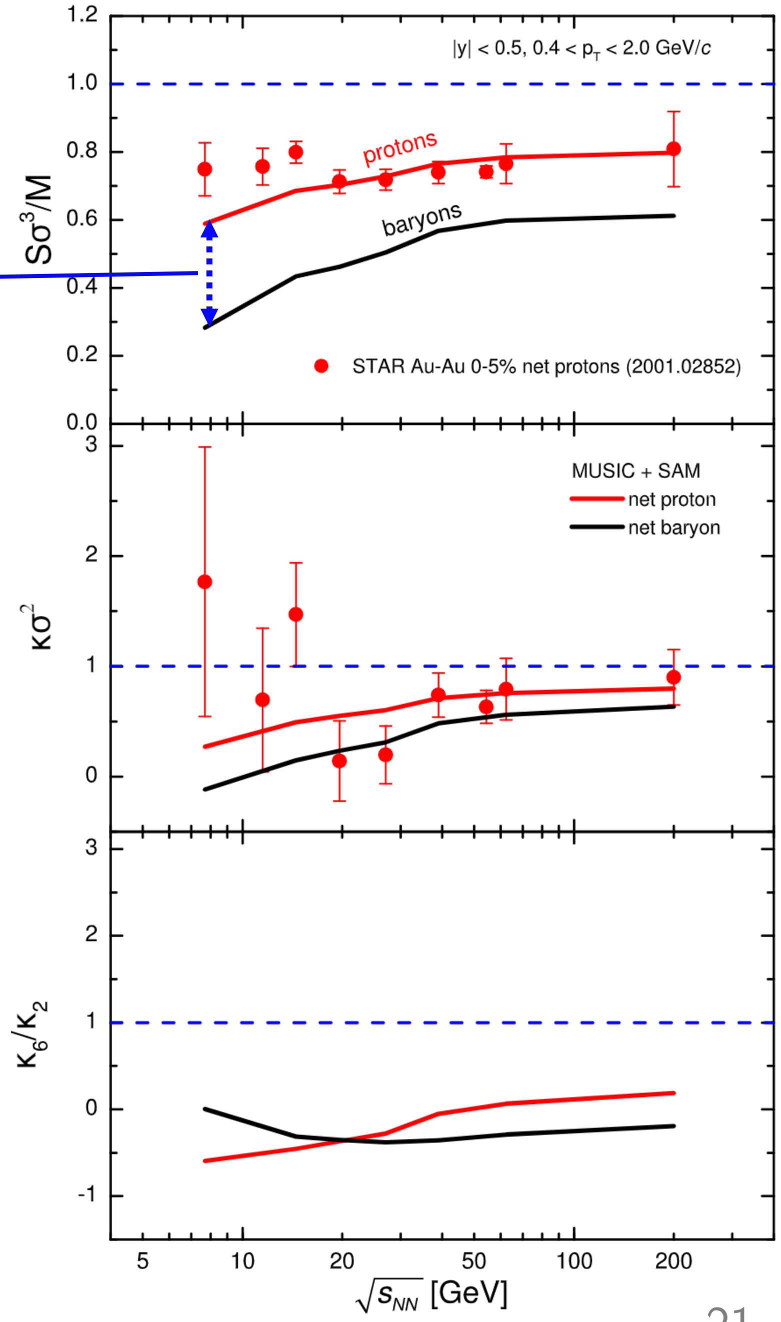
- **net baryon $\neq$ net proton**

- Baryon cumulants can be reconstructed from proton cumulants via binomial (un)folding based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]
  - Amounts to an additional "efficiency correction" and requires the use of joint factorial moments, only experiment can do it model-independently



$$\frac{\hat{C}_2^B}{\hat{C}_1^B} \approx 2\,\frac{\hat{C}_2^p}{\hat{C}_1^p}$$

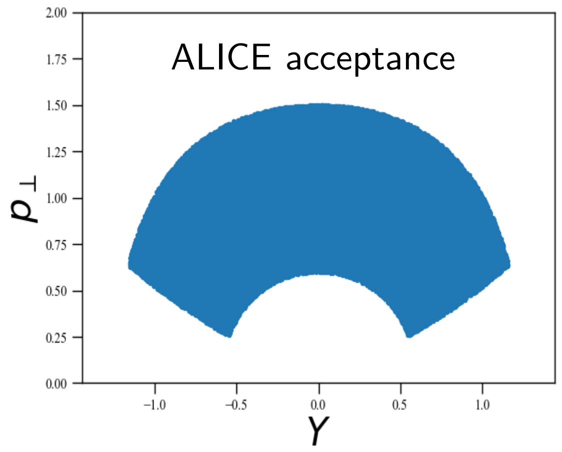




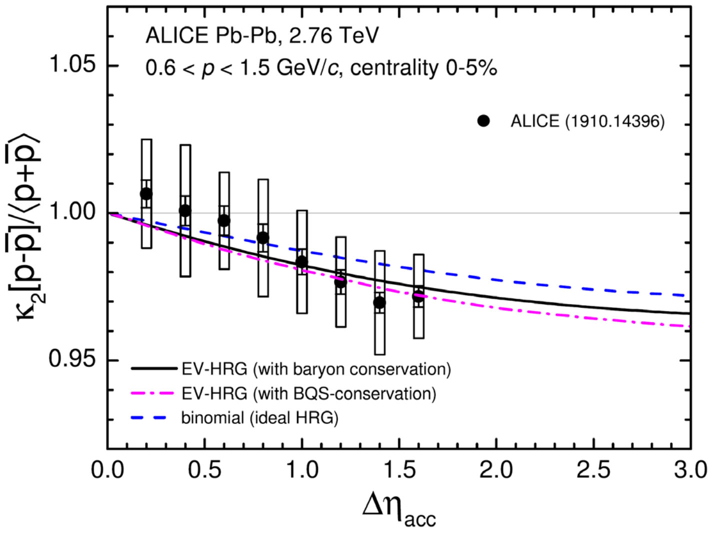
Net-particle fluctuations at the LHC

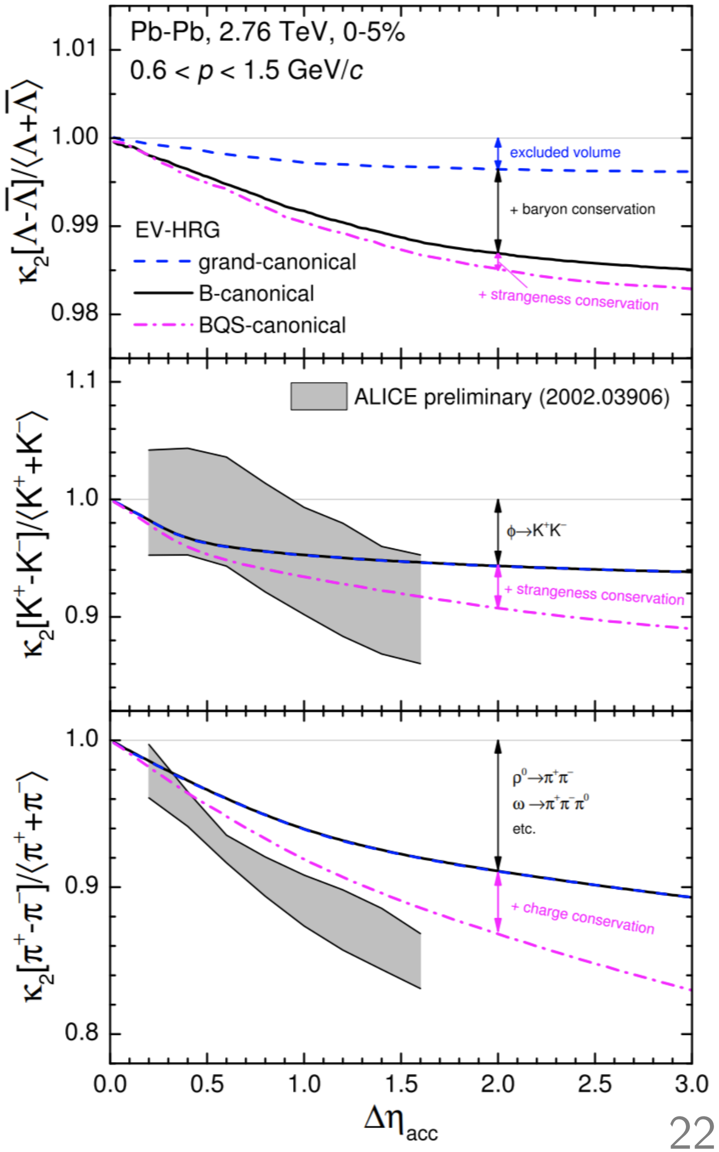
VV, Koch, Phys. Rev. C 103, 044903 (2021)

- Net protons described within errors but not sensitive to the equation of state for the present experimental acceptance
- Large effect from resonance decays for lighter particles + conservation of electric charge/strangeness
- Future measurements will require larger acceptance



$0.6 < p < 1.5$ GeV$/c$, $\Delta\eta_{acc} = 1.6$



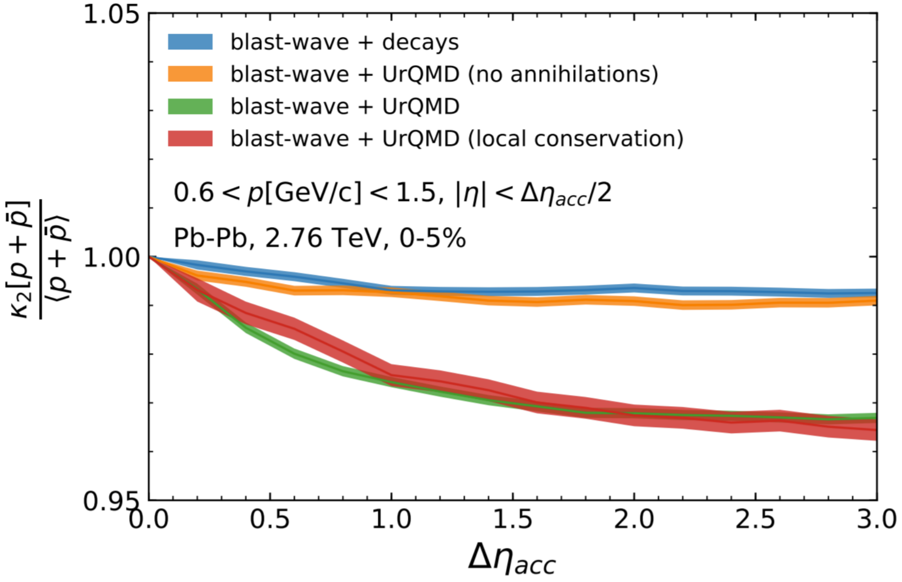


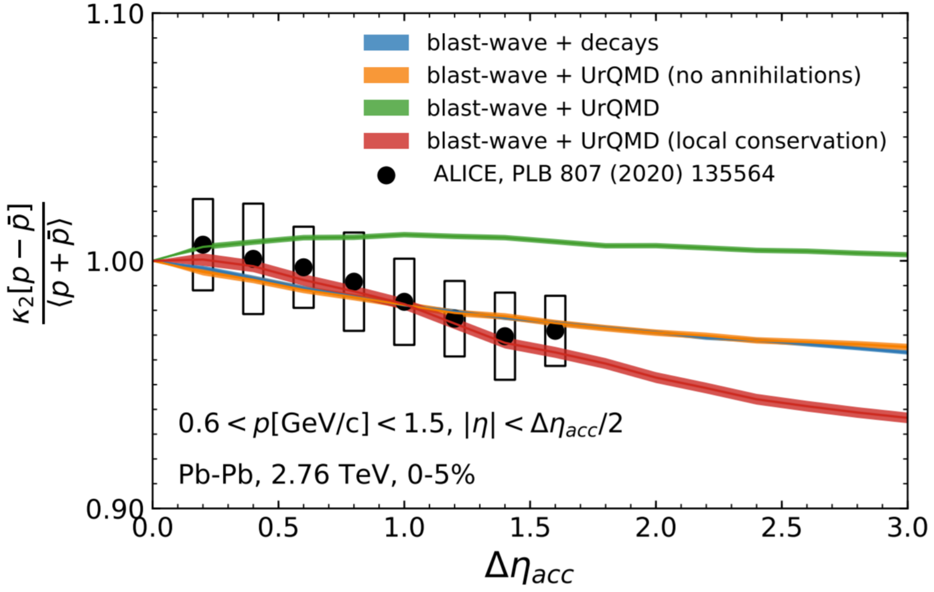
Effects of baryon annihilation and local conservation

O. Savchuk, V.V., V. Koch, J. Steinheimer, H. Stoecker, arXiv:2106.08239

Baryon annihilation $B\bar{B} \to n\pi$ in afterburners (UrQMD, SMASH) suppresses baryon yields

$$\langle p + \bar{p} \rangle \searrow \quad \Longrightarrow \quad \frac{\kappa_2[p-\bar{p}]}{\langle p+\bar{p}\rangle} \nearrow$$





- ALICE data requires local baryon conservation across $\Delta y \sim \pm 1.5$ with UrQMD annihilations (no regenerations) or global conservation ($\Delta y \sim \Delta y_{tot}$) without annihilations
- Local conservation and $B\bar{B}$ annihilation can be constrained from data through the combined analysis of $\kappa_2[p-\bar{p}]$ and $\kappa_2[p+\bar{p}]$

Thermodynamic analysis of HADES data

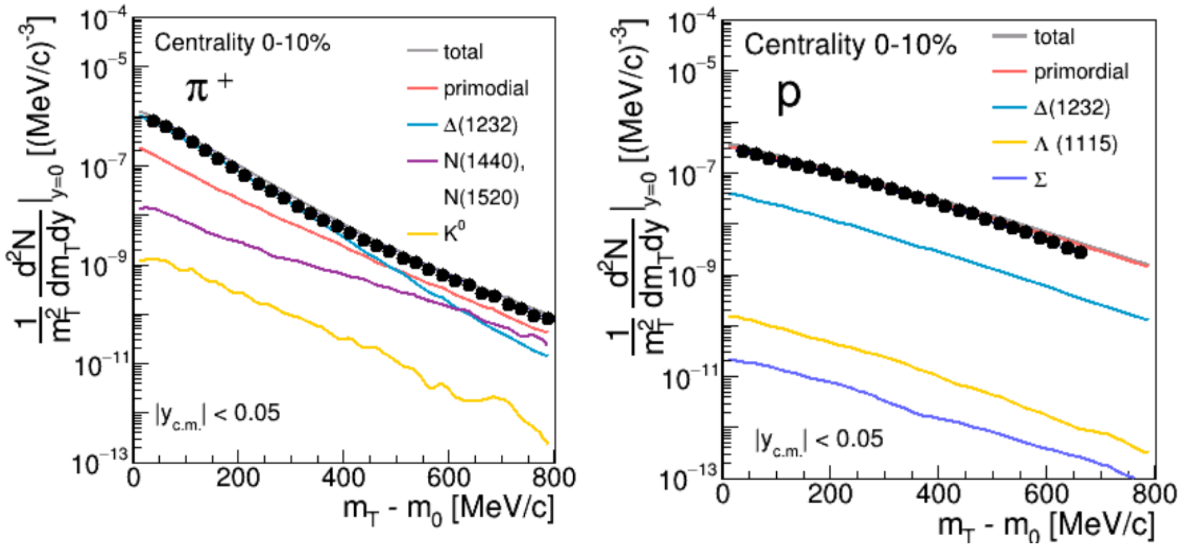
VV, Koch, in preparation

- Single freeze-out scenario: Emission from Siemens-Rasmussen hypersurface with Hubble-like flow

  $\rightarrow$ Pion and proton spectra o.k.

  [S. Harabasz et al., PRC 102, 054903 (2020)]

- Uniform $T \approx 70$ MeV, $\mu_B \approx 875$ MeV across the fireball [A. Motornenko et al., PLB 822, 136703 (2021)]



- Fluctuations:
  - Same as before but incorporate additional binomial filtering to account for protons bound in light nuclei
  - Uniform fireball $\rightarrow$ Final proton cumulants are linear combinations of baryon susceptibilities χ_n^B

Extract χ_n^B directly from experimental data

Thermodynamic analysis of HADES data

VV, Koch, in preparation

- In the grand-canonical limit (no baryon conservation) the data are described well with

$$\frac{\chi_2^B}{\chi_1^B} = 9.35 \pm 0.40, \qquad \frac{\chi_3^B}{\chi_2^B} = -39.6 \pm 7.2, \qquad \frac{\chi_4^B}{\chi_2^B} = 1130 \pm 488$$

- Could be indicative of a critical point near the HADES freeze-out at $T \approx 70$ MeV, $\mu_B \approx 875$ MeV

- However, the results are challenging to describe with baryon conservation included